Cluster Effective Field Theory calculation of electromagnetic breakup reactions with Lorentz Integral Transform method

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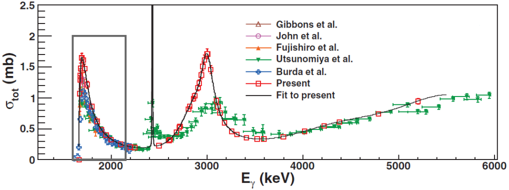
TNPI2023 - XIX Conference on Theoretical Nuclear Physics in Italy
Cortona, 11 October 2023

$$\gamma + {}^9\text{Be} \to \alpha + \alpha + n$$

Study of the reaction of *astrophysical interest* in the **low-energy regime**:

- ^{9}Be 3-body ($\alpha\alpha n$) binding energy
- Cross section



[Arnold *et al.* (2012)]

Outline

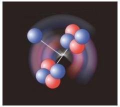
1. Model
   - Potentials from Effective Field Theory (EFT)

2. Method
   - Bound-state problem: variational and Non-Symmetrized Hyperspherical Harmonics (NSHH) method
   - Continuum problem: Lorentz Integral Transform (LIT) method

3. Application
   - $\boldsymbol{J}_{\text{1-body}}$-calculation
   - Siegert-calculation
   - Results

Model

- Effective particles: **nucleons** and α-**particles**
- Interaction: potential models from **Effective Field Theory** (EFT) [Hammer *et al.* (2017)]



P. Mueller/Argonne National Lab

Cluster-EFT approach: why?

^{9}Be binding $B_3 \approx 1.573$ MeV $<<$ α binding (≈ 20 MeV)

↓ shallow binding

$\Rightarrow$ ^{9}Be is a 3-body *effective* clustering system in the low energy regime

$\Rightarrow$ Separation of scales $\rightarrow$ EFT approach

momentum scales: M_{low}, M_{high}

$\downarrow$

EFT expansion in $\left(\frac{M_{low}}{M_{high}}\right)^\nu$

$\downarrow$

error estimate

2-Body Effective Potentials

Effective *non-local* potential in momentum space and in the partial wave ℓ

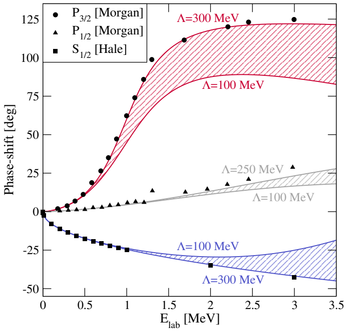
$$\mathcal{V}_\ell(p,p') = \Big[\lambda_0 + \lambda_1 (p^2 + p'^2) \Big] p^\ell p'^\ell g(p) g(p')\,, \qquad g(p) = e^{-(\frac{p}{\Lambda})^{2m}} \qquad (m = 1, 2)$$

- We calculate the on-shell $\mathcal{T}$-matrix solving the Lippmann-Schwinger equation (on-shell: $\boldsymbol{p} = \boldsymbol{p}' = \boldsymbol{k}$)
- We compare term by term the calculated $\mathcal{T}$-matrix with its Effective Range Expansion up to terms $\mathcal{O}(k^2)$ $\Rightarrow \lambda_i = \lambda_i(a_\ell, r_\ell, \Lambda)$
- For every fixed value of the cut-off Λ, we determine the LECs using the experimental values a_ℓ^{exp} and r_ℓ^{exp}

$$\lambda_i = \lambda_i(a_\ell^{\text{exp}}, r_\ell^{\text{exp}}, \Lambda)$$

The effective potentials $\mathcal{V}_\ell^{\alpha n}(p,p')$ and $\mathcal{V}_\ell^{\alpha\alpha}(p,p')$
reproduce the correct low-energy phase-shifts

αn

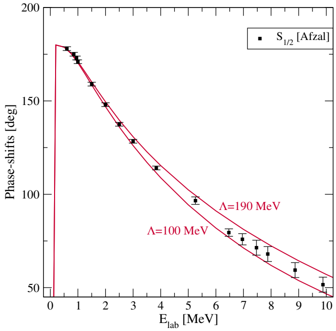


$P_{3/2}$
Resonance

$P_{1/2}$

$S_{1/2}$

$\alpha\alpha$



$S_{1/2}$
Resonance

LO : $\mathcal{V}^{\alpha n}_{P_{3/2}} + \mathcal{V}^{\alpha\alpha}_{S_{1/2}} + \mathcal{V}_3$

↑
3-body Hypercentral Potential
with a gaussian regulator (Λ_3)

Calculations: **LO** , **LO** $+ \mathcal{V}^{\alpha n}_{S_{1/2}}$

Method

Bound-state problem:

the variational method with a **Non-Symmetrized Hyperspherical Harmonics** (NSHH) basis

[Gattobigio *et al.* (2011), Deflorian *et al.* (2013)]

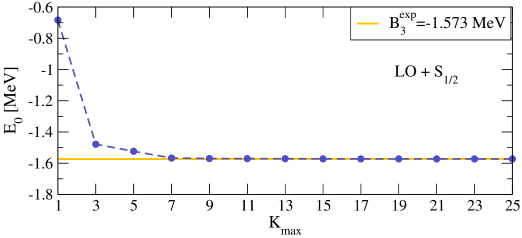
- $\hat{\mathcal{H}}$ is represented on a suitable basis in momentum space

$$\Psi = \sum_\nu c_\nu \Psi_\nu \equiv \sum_{m\{K\}} c_{m\{K\}} \underbrace{f_m(Q)}_{\text{Laguerre polynomials}} \underbrace{\mathcal{Y}_{\{K\}}(\Omega_Q)}_{\text{HH functions}}$$

- $\hat{\mathcal{H}}$ is diagonalized

$$\sum_{\nu'} \langle \Psi_\nu | \hat{\mathcal{H}} | \Psi_{\nu'} \rangle \, c_{\nu'} = E \, c_\nu \qquad E_0, \{ c_\nu^0 \} \Rightarrow \Psi_0$$

- Convergence is reached enlarging the dimension of the basis $(K_{\max}, N_{\text{lag}})$



Method

Continuum problem:

the **Lorentz Integral Transform** (LIT) method [Efros *et al.* (2007)]

Electromagnetic inclusive reactions

Cross section

$$\sigma_{\mathrm{EM}} \propto \mathcal{R}(\omega)$$

Response function

$$\mathcal{R}(\omega) = \int df \, |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2 \, \delta(E_f - E_0 - \omega)$$

$\mathcal{R}(\omega)$: states in the continuum spectrum are involved ($\hat{\mathcal{H}} |\Psi_f\rangle = E_f |\Psi_f\rangle$)

$\Rightarrow$ direct calculation is DIFFICULT

To overcome this problem we use an integral transform approach

Method

Continuum problem:

the **Lorentz Integral Transform** (LIT) method [Efros *et al.* (2007)]

- We define an Integral Transform $\mathcal{L}(\sigma)$ of the response function $\mathcal{R}(\omega)$, with a Lorentzian kernel $\mathcal{K}(\sigma, \omega)$

$$\mathcal{L}(\sigma) = \int d\omega\, \mathcal{K}(\sigma, \omega)\, \mathcal{R}(\omega)\,, \qquad \mathcal{L}(\sigma) \xrightarrow{\text{INVERSION}} \mathcal{R}(\omega)$$

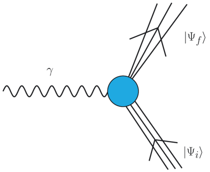
- It can be demonstrated that $\mathcal{L}(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle$, where the LIT states $|\tilde{\Psi}\rangle$ can be calculated using bound-state methods

Scattering problem "continuum problem"		Bound-state-like problem
$\mathcal{R}(\omega)$	$\xrightarrow{\text{REFORMULATION}}$	$\mathcal{L}(\sigma) = \langle \tilde{\Psi} \| \tilde{\Psi} \rangle$

Application: $\gamma + {}^{9}\text{Be} \to \alpha + \alpha + n$

Photon γ:

$\boldsymbol{A}(\boldsymbol{x})$, $\hat{\epsilon}_{\boldsymbol{q},\lambda}$



Nucleus:

$\boldsymbol{J}(\boldsymbol{x})$ [FT: $\boldsymbol{J}(\boldsymbol{q})$]

Nuclear Current operator

$\omega = |\boldsymbol{q}|$

$|\Psi_0\rangle \to |\Psi_f\rangle$ transition

$$\sigma_\gamma \propto \mathcal{R}_\gamma(\omega) \sim \langle \Psi_f | \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) | \Psi_0 \rangle$$

$\uparrow$

Nuclear Current matrix element

[Bacca and Pastore (2014)]

$$\mathcal{R}_\gamma(\omega) \sim \langle \Psi_f | \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) | \Psi_0 \rangle$$

Multipole decomposition: $\hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) \equiv J_\lambda(q) = -\sum_J \sqrt{2\pi(2J+1)} \left[\mathcal{T}^{\mathrm{E}}_{J\lambda}(q) + \lambda \mathcal{T}^{\mathrm{M}}_{J\lambda}(q) \right]$

$$\mathcal{T}^{\mathrm{E}}_{J\lambda}(q) \propto \int d\hat{\boldsymbol{q}}' \left(\hat{\boldsymbol{q}}' \times \boldsymbol{Y}^{\lambda}_{JJ1}(\hat{\boldsymbol{q}}') \right) \cdot \boldsymbol{J}(\boldsymbol{q}')$$

Dominant: $EJ = E1$

1. "$J_{\text{1-body}}$"-calculation

- The Nuclear Current operator is a sum of terms

$$\boldsymbol{J} = \boldsymbol{J}_{\text{1-body}} + \boldsymbol{J}_{\text{2-body}} + \dots$$

  We use only the one-body term $\boldsymbol{J}_{\text{1-body}}$ i.e. the Nuclear Convection current [Filandri (2022)].

- Specifically with our EFT, the continuity equation is not fully satisfied.

$$\mathcal{R}_\gamma(\omega) \sim \langle \Psi_f | \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) | \Psi_0 \rangle$$

Multipole decomposition: $\hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) \equiv J_\lambda(q) = -\sum_J \sqrt{2\pi(2J+1)} \left[\mathcal{T}^{\mathrm{E}}_{J\lambda}(q) + \lambda \mathcal{T}^{\mathrm{M}}_{J\lambda}(q) \right]$

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Dominant: $EJ = E1$

2. "Siegert"-calculation

- Continuity equation: $\omega \rho(\boldsymbol{q}) - \boldsymbol{q}\boldsymbol{J}(\boldsymbol{q}) = 0$

$$\mathcal{T}^{\mathrm{E}}_{J\lambda}(q) \propto \int d\hat{\boldsymbol{q}}' \; Y^{J}_{\lambda}(\hat{\boldsymbol{q}}') \, \rho(\boldsymbol{q}') + \text{correction} \;\equiv\; \text{"Siegert operator"} + \text{correction}$$

- In the cluster framework: Charge op. $\rho(\boldsymbol{x}) = \sum_i 2e\, \delta^3(\boldsymbol{x} - \boldsymbol{r}_{\alpha_i})$, Dipole op. $\boldsymbol{D} \equiv 2e \sum_i \boldsymbol{r}_{\alpha_i} \quad (i = 1, \ldots, N_\alpha)$

$$\mathcal{R}_\gamma(\omega) \sim \langle \Psi_f | \boldsymbol{D} | \Psi_0 \rangle$$

$$\mathcal{R}_\gamma(\omega) \sim \langle \Psi_f | \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) | \Psi_0 \rangle$$

$$\boldsymbol{J} = \boldsymbol{J}_{\text{1-body}} + \boldsymbol{J}_{\text{2-body}} + \dots$$

1. "$J_{\text{1-body}}$"-calculation $\boldsymbol{J} = \boldsymbol{J}_{\text{1-body}}$

(*p-space calculation*)

2. "Siegert"-calculation $\langle \Psi_f | \boldsymbol{D} | \Psi_0 \rangle$

(*r-space calculation*)

Which is the main difference?

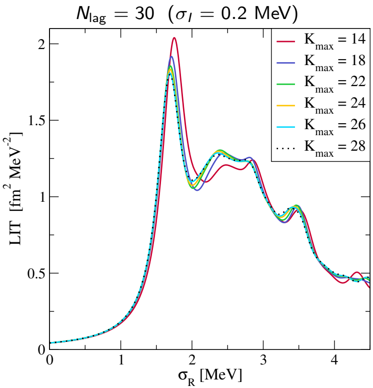
Having used the continuity equation explicitly, "Siegert"-calculation ensures that, at low energy, the **matrix element of the Dipole operator** contains the contribution also of the currents beyond $\boldsymbol{J}_{\text{1-body}}$ ($\boldsymbol{J}_{\text{2-body}}$ and $\boldsymbol{J}_{\text{3-body}}$).

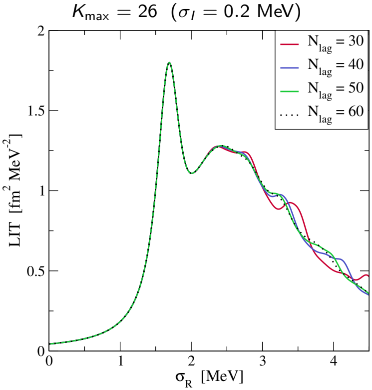
The "Siegert"-calculation in connection with the "$J_{\text{1-body}}$"-calculation provides a way to study the contributions to the cross section due to the currents beyond $\boldsymbol{J}_{\text{1-body}}$.

$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \rightarrow |f\rangle : J^{\pi} = 1/2^{+}$$

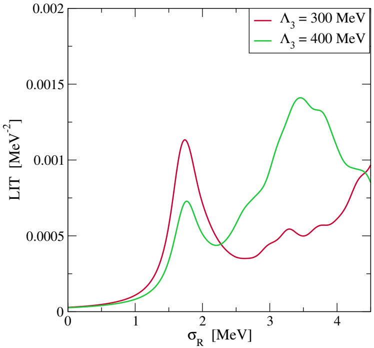
LIT: convergence studies

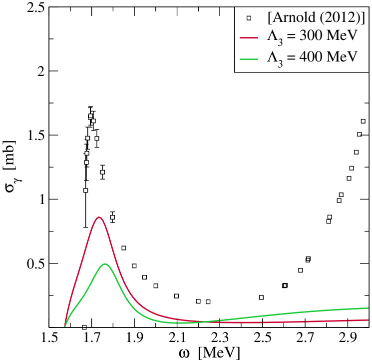
Basis $\sim f_m(Q)\, \mathcal{Y}_{\{K\}}(\Omega_Q) \quad (m = 1, \ldots, N_{\text{lag}}\,,\; K = 1, \ldots, K_{\max})$



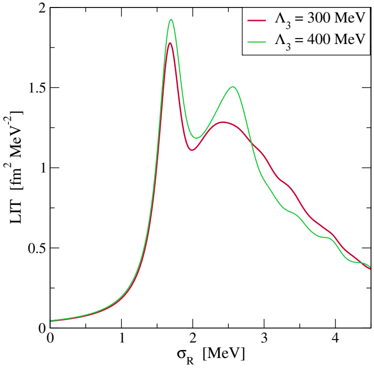


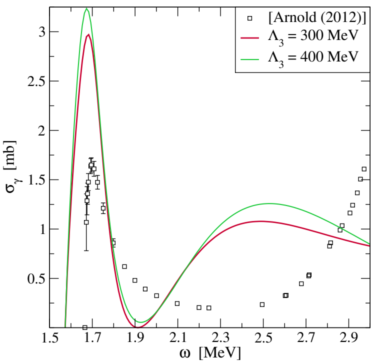
LO: $J_{\text{1-body}}$-calculation



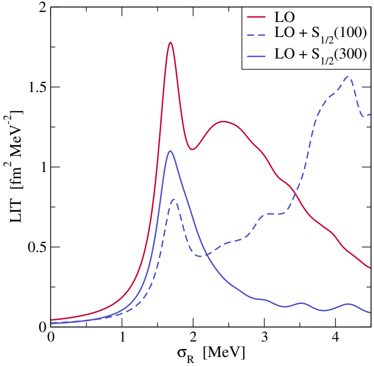


LO: Siegert-calculation





LO + $S_{1/2}$: Siegert-calculation



$\mathcal{V}^{\alpha n}_{S_{1/2}}$: we use cut-offs $\Lambda > 100$ MeV

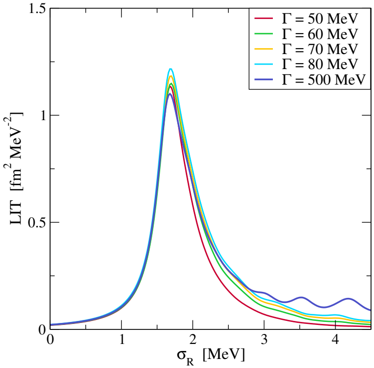
We add a **projection potential** in order to "project out" the αn deep bound state

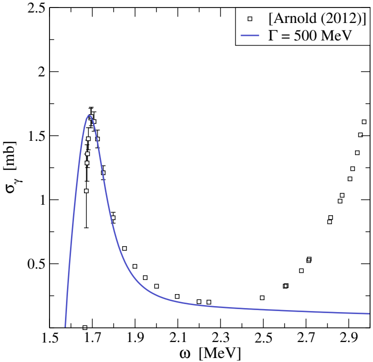
$$\mathcal{V}_\Gamma(p, p') = \psi_{S_{1/2}}(p)\, \frac{\Gamma}{2(4\pi)}\, \psi_{S_{1/2}}(p')$$

Theoretically: $\Gamma \to \infty$

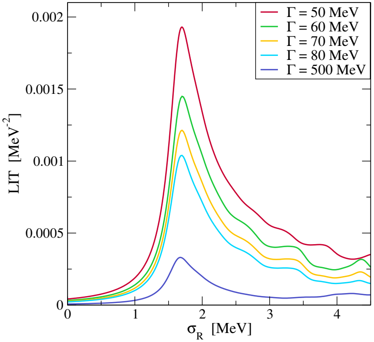
In practice: Γ-independence of the results

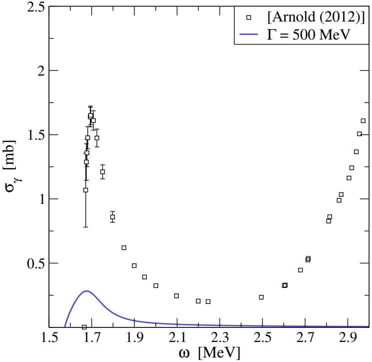
LO + $S_{1/2}$: Siegert-calculation (preliminary)





LO + $S_{1/2}$: $J_{\text{1-body}}$-calculation (preliminary)



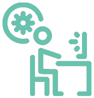


Summary

- Potentials from Effective Field Theory
- Bound states calculations
- Lorentz Integral Transform calculations

What's next?

- $\gamma + {}^{9}\text{Be} \to \alpha + \alpha + n$ $\quad$ $J^{\pi} = 5/2^{+}\,, 3/2^{+}$ contributions to σ_{γ}
- $\gamma + {}^{12}\text{C} \to \alpha + \alpha + \alpha$ $\quad$ (E2 transition: 2^{+} bound $\to$ 0^{+} resonant-state)



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Thank You!