

# The EMC effect of light nuclei within the Light-Front Hamiltonian dynamics

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INFN section of Perugia.

in collaboration with

Emanuele Pace ("Tor Vergata", Rome Univ.)

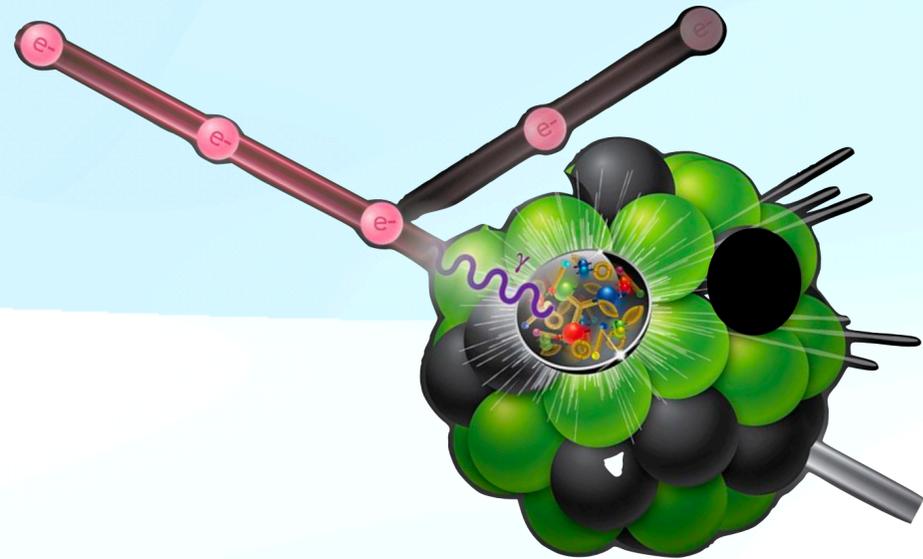
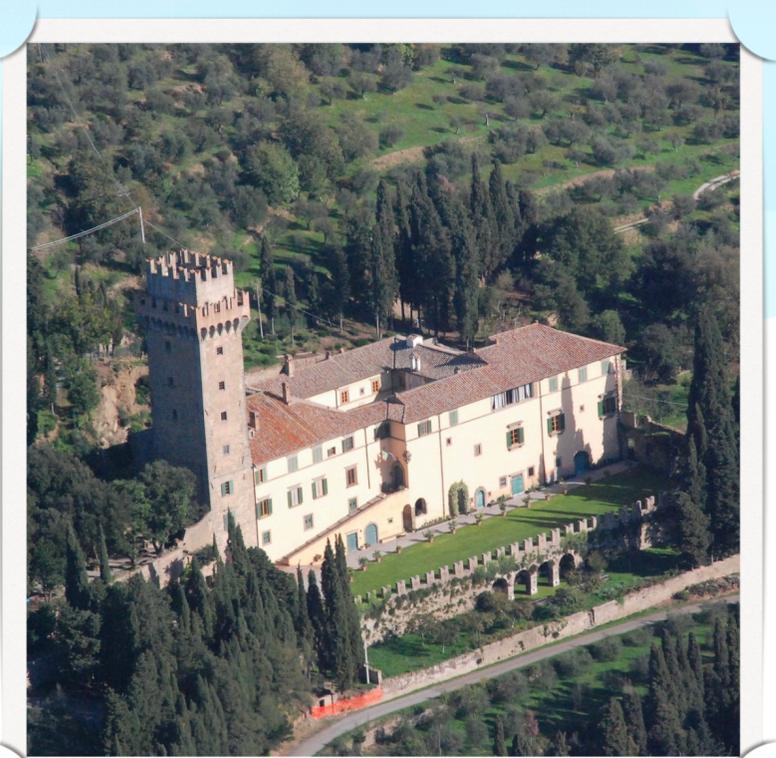
Sergio Scopetta (Perugia Univ. & INFN)

Giovanni Salmè (INFN - Rome)

Filippo Fornetti (Perugia Univ.)

Michele Viviani (INFN - Pisa)

Eleonora Proietti (Perugia Univ.; ph.d in Pisa)



# Based on

R. Alessandro, A. Del Dotto, E. Pace, G. Perna, G. Salmè and S. Scopetta,  
*Light-Front Transverse Momentum Distributions for  $\mathcal{J} = 1/2$  Hadronic Systems in Valence Approximation*

**Phys.Rev.C 104 (2021) 6, 065204**

A. Del Dotto, E. Pace, G. Salmè, and S. Scopetta,  
*Light-Front spin-dependent Spectral Function and Nucleon Momentum Distributions for a Three-Body System*, **Phys. Rev. C 95, 014001 (2017)**

E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,  
*EMC effect, few-nucleon systems and Poincaré covariance*,  
**Phys. Scr. 95, 064008 (2020)**

MARATHON Coll.

*Measurement of the Nucleon  $F_{n2}/F_{p2}$  Structure Function Ratio by the Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment*,

Phys. Rev. Lett 128 (2022) 13, 132003

E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,  
*The European Muon Collaboration effect in Light-Front Hamiltonian Dynamics*

Phys. Lett. B 839 (2023) 137810

# Based on

**F. Fornetti**, E. Pace, M. R., G. Salmè, S. Scopetta and M. Viviani,  
*The EMC for few-nucleon bound systems in Light-Front Hamiltonian dynamics*  
arXiv:2308.15925

**E. Proietti, F. Fornetti**, E. Pace, M. R., G. Salmè and S. Scopetta,  
 *$^3\text{He}$  spin dependent structure functions within the relativistic Light-Front Hamiltonian dynamics*  
in prep.  see E. Proietti's talk

# Outline

- Motivations
- The EMC effect
- The Light-Front Poincaré covariant approach
- The EMC effect within the Light-Front approach: the  $^3\text{He}$  and  $^4\text{He}$  cases
- Conclusions and Perspectives

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# Motivations: why the $^3\text{He}$

- 🌀 **Phenomenological:** a reliable flavor decomposition needs the neutron parton structure (PDFs, GPDs TMDs....)



Accurate and long-lasting experimental efforts in developing effective neutron targets to carefully investigate its electromagnetic responses.  $^3\text{H} \vec{e}$  is SPECIAL

The polarized  $^3\text{He}$  target, 90% neutron target (e.g. H. Gao et al, PR12-09-014, Chen et al, PR12-11-007, @JLab12)

- 🌀 Due to the experimental energies, the accurate theoretical description (of a polarized  $^3\text{He}$ ) has to be relativistic

- 🌀 **Theoretical:** a LF description of three body interacting systems! Bonus:

Transverse-Momentum Distributions (TMDs) for addressing in a novel way the nuclear dynamics

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# Motivations: why the ${}^4\text{He}$

- Goal: extend the approach applied for  ${}^3\text{He}$  to **any nuclei  $A$**  and calculate the **EMC ratio** for  ${}^4\text{He}$  and  ${}^3\text{H}$  [E. Pace, M. Rinaldi, G. Salmè, S. Scopetta, *Phys. Lett. B* 839 (2023) 137810]
- Since  ${}^4\text{He}$  is a **strongly bound system** this could provide a challenging test to our approach
- Compare EMC effect for  ${}^3\text{He}$ ,  ${}^4\text{He}$  and  ${}^3\text{H}$  obtained by different **modern NN and NNN interactions** (Argonne V18+UIX, NVIa+3N, NVIb+3N)
- Compare the EMC effect for  ${}^4\text{He}$  obtained by different choice of  $F_2^p$  and  $F_2^n$  **parametrization**

[R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, *Phys. Rev. C* 51 (1995) 38–51]

[R. B. Wiringa et al., *Phys. Rev. Lett.* 74 (1995) 4396–4399]

[M. Viviani et al., *Phys. Rev. C* 107 (1) (2023) 014314]

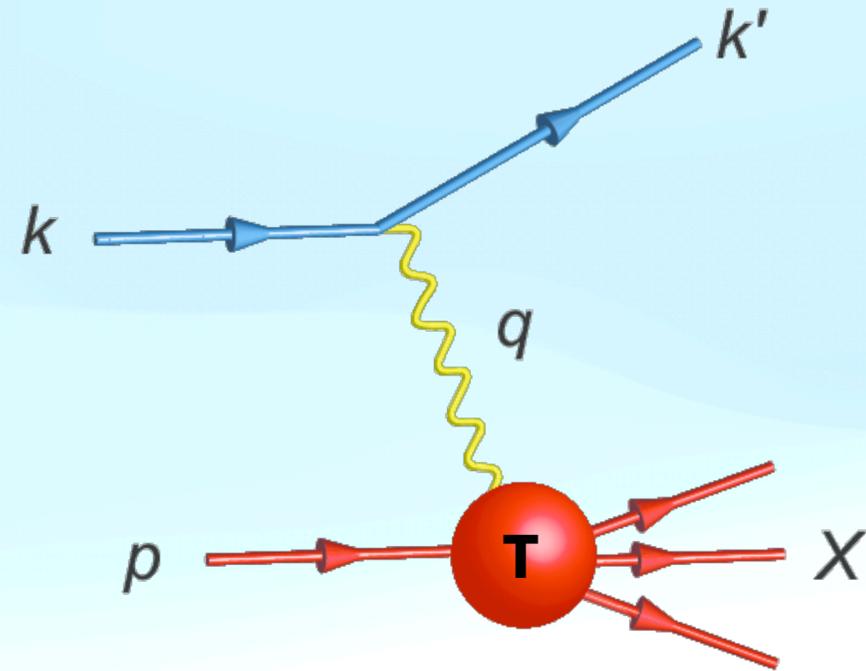
[M. Piarulli et al., *Phys. Rev. Lett.* 120 (5) (2018) 052503]

[M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim, *Phys. Rev. C* 107 (1) (2023) 014314]

\*F. Fornetti's slide

# The EMC effect

Almost 40 years ago, the European Muon Collaboration (**EMC**) measured in deep inelastic scattering (DIS) processes:

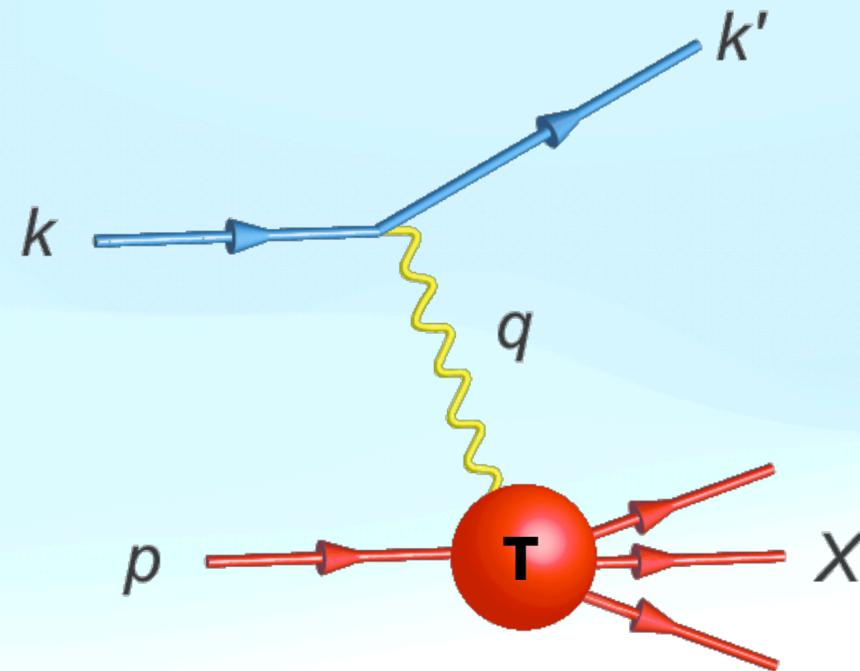


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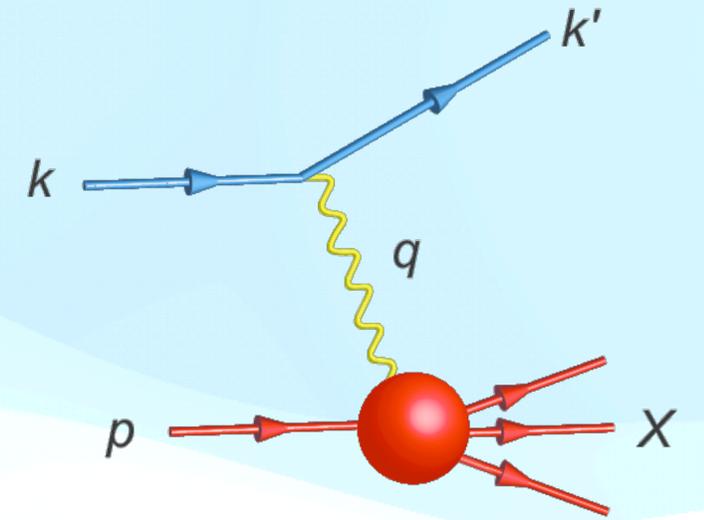
they extracted the *Structure Function* of the target

$$F_2^A(x)$$

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$$R(x) = F_2^{56\text{Fe}}(x) / F_2^{2\text{H}}(x)$$

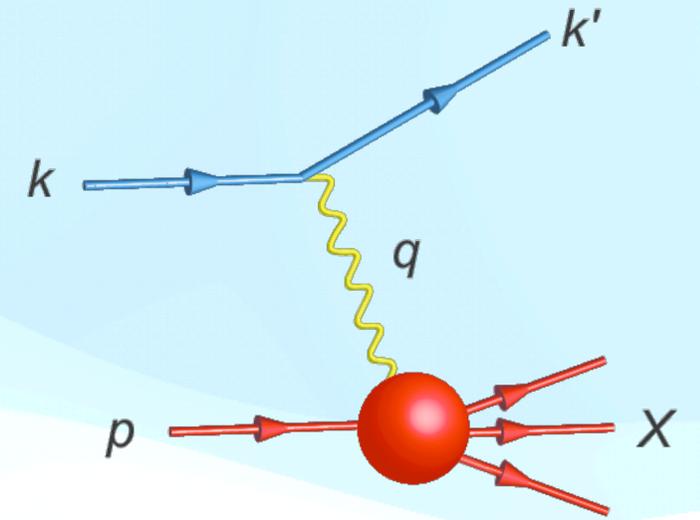


At high energies, the expected result is  $R(x) = 1$  up to corrections of the Fermi motion. **BUT**

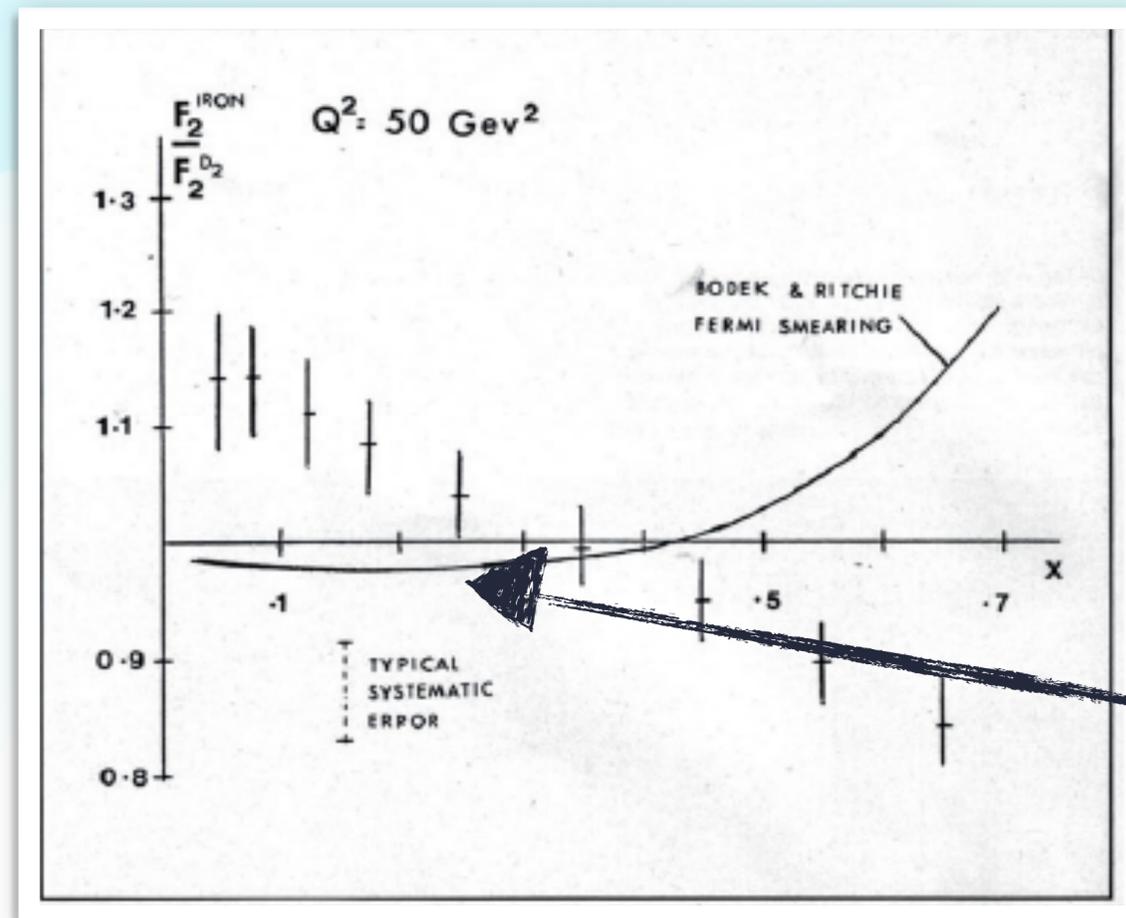
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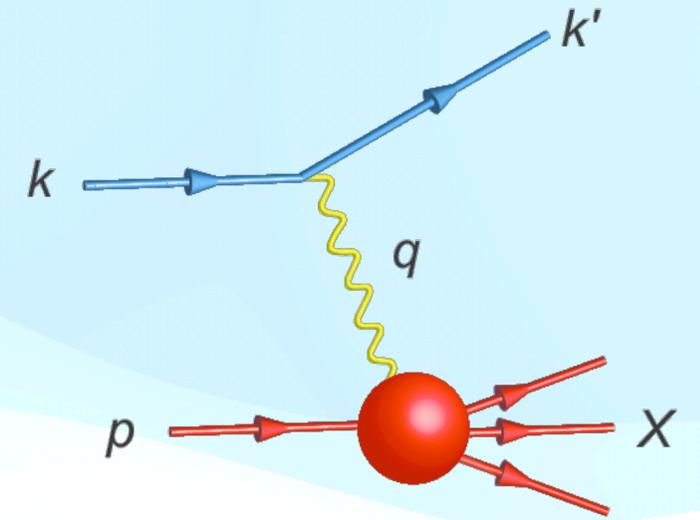
Aubert et al. Phys.Lett. B123 (1983) 275  
1488 citations (inSPIRE)

Few % deviation from what expected.  
WHY?

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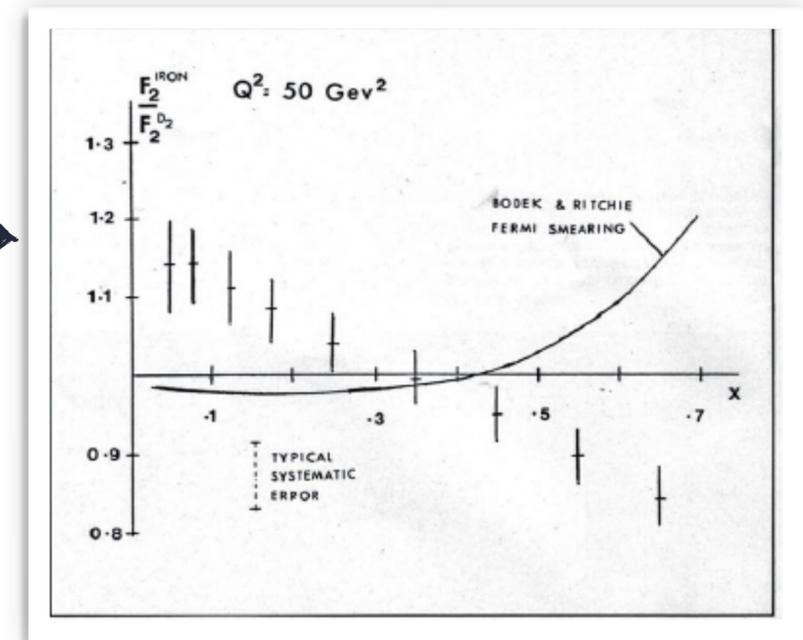
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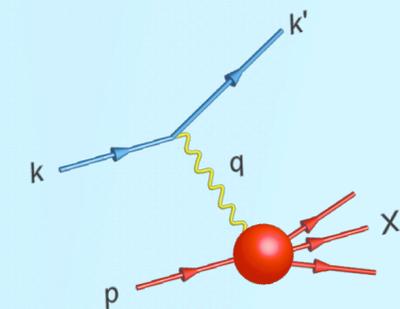
Naive parton model interpretation:

“Valence quarks, in the bound nucleon, are in average slower than in the free nucleon”

Is the bound proton bigger than the free one??

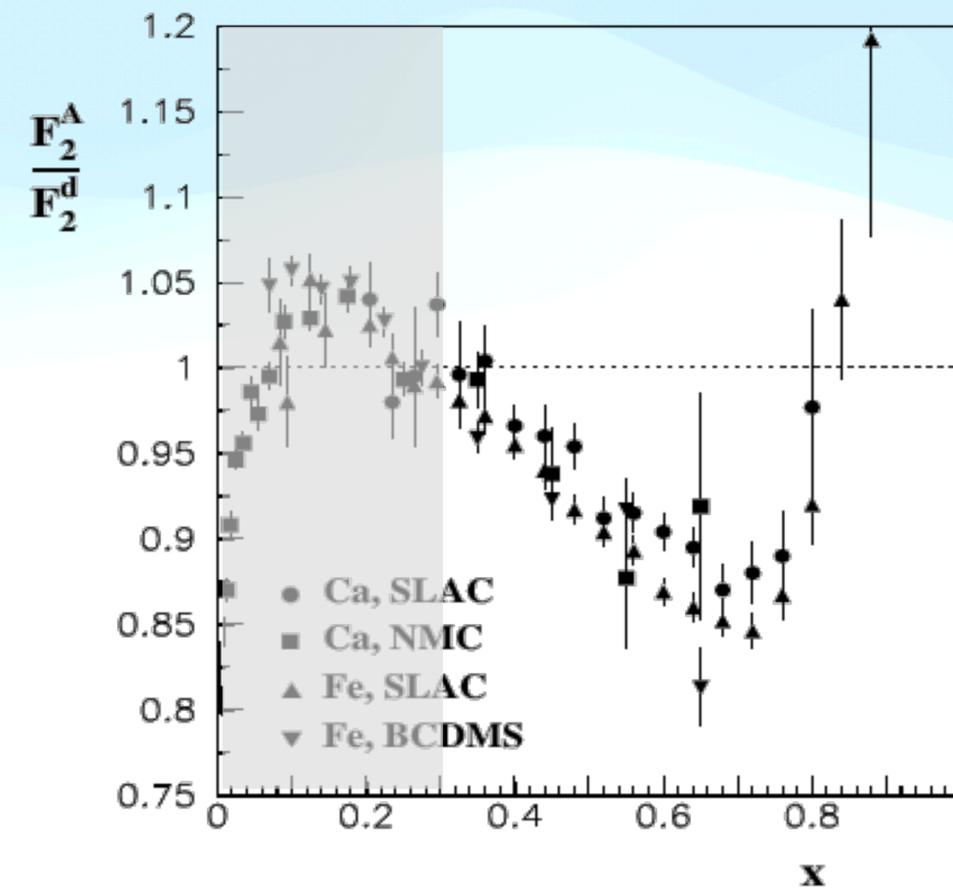


# The EMC effect in details

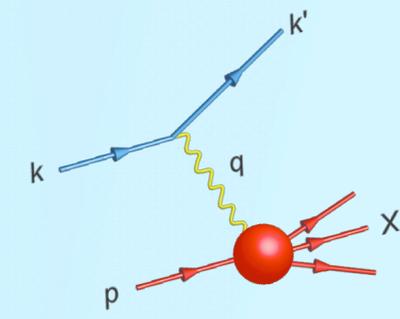


We remind that for DIS off nuclei:  $0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$

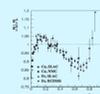
  $x \leq 0.3$  "Shadowing region": coherence effects, the photon interacts with partons belonging to different nucleons



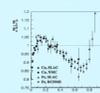
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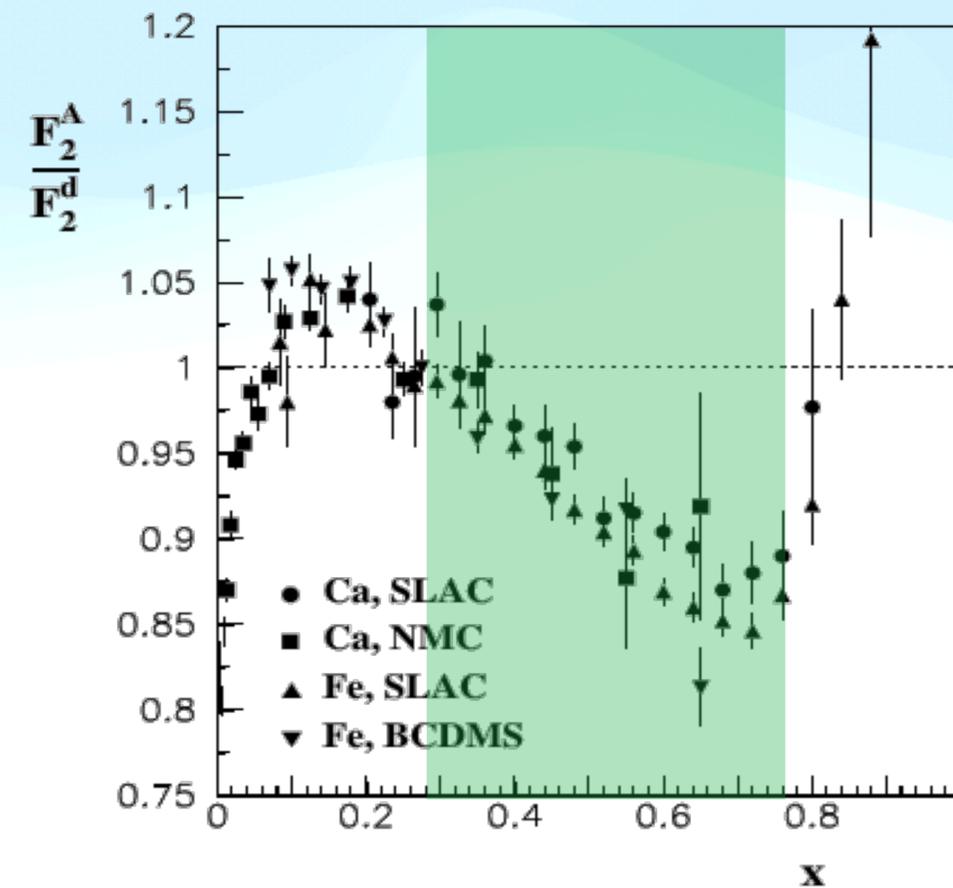
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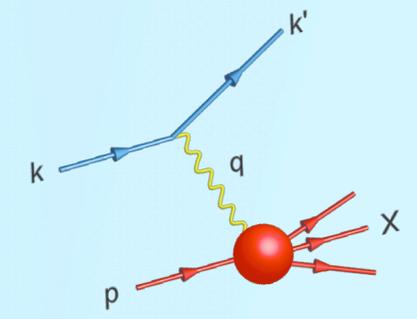
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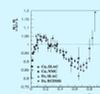
$0.2 \leq x \leq 0.8$  "EMC (binding) region": mainly valence quarks involved

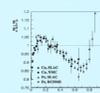


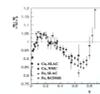
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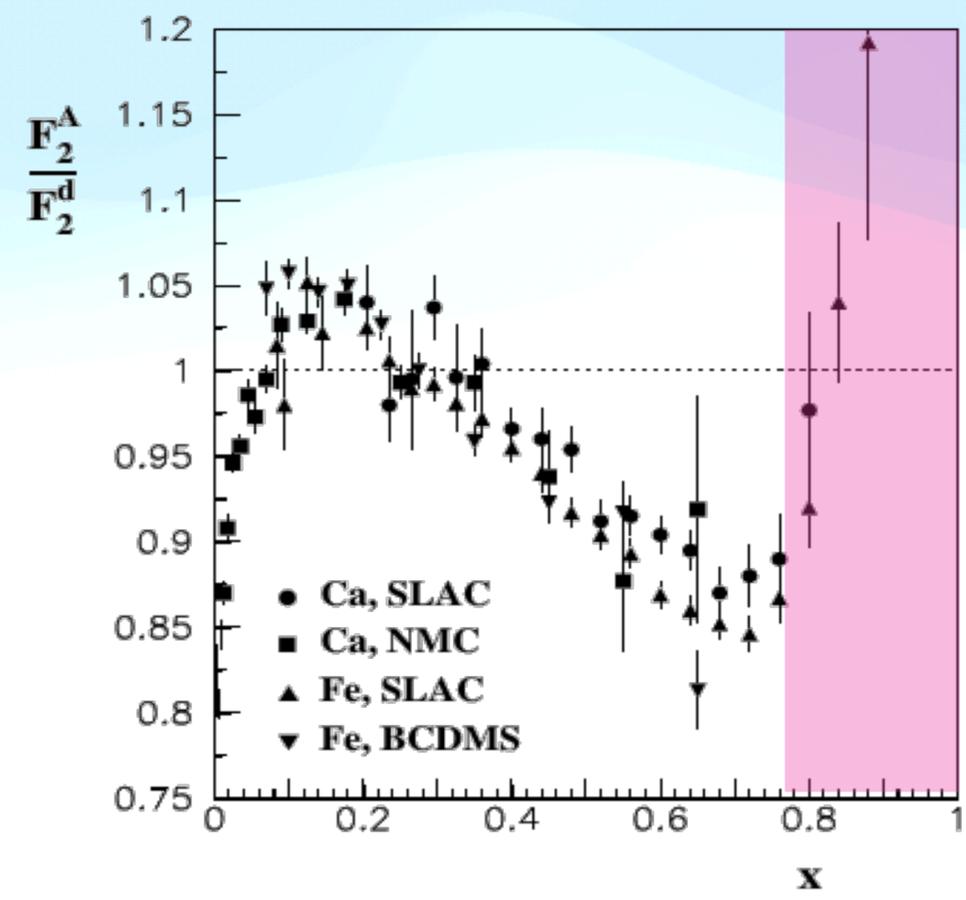


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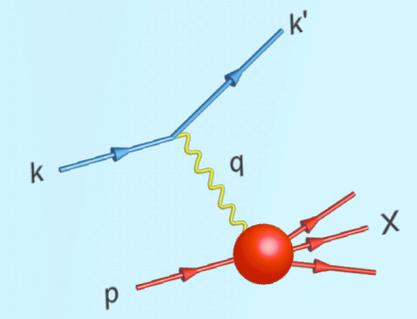
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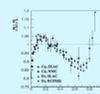
  $0.8 \leq x \leq 1$  "Fermi motion region"

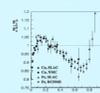


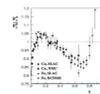
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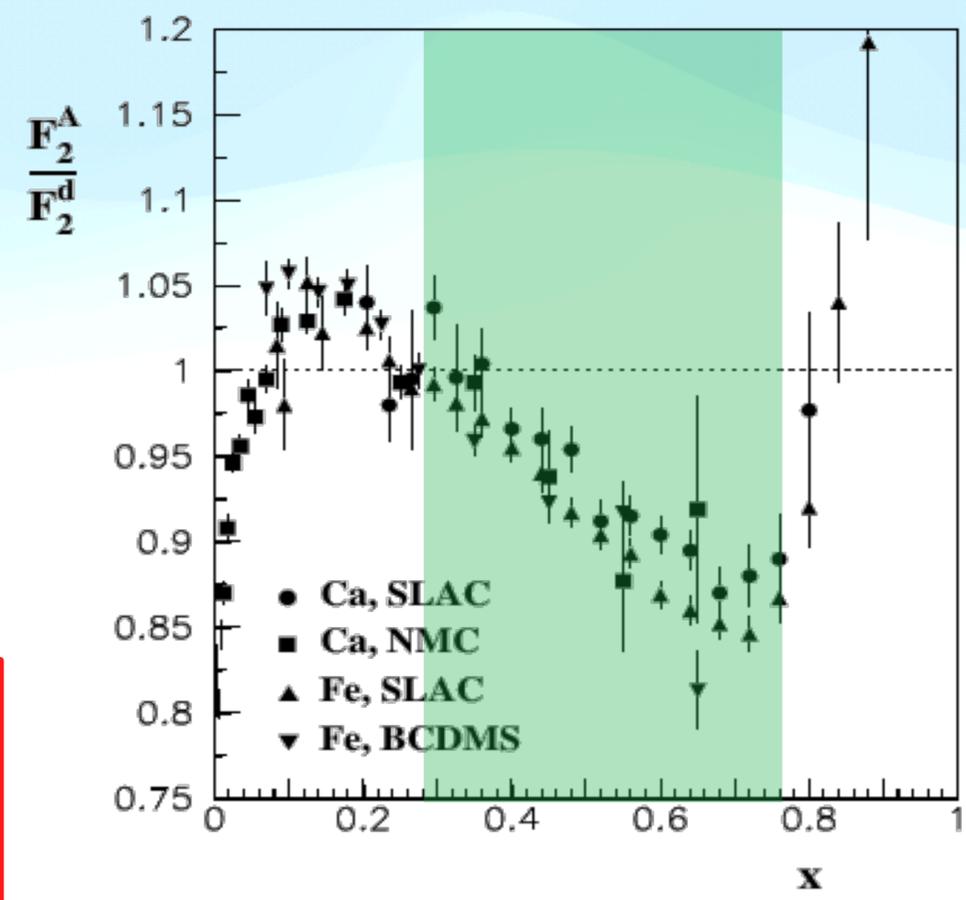
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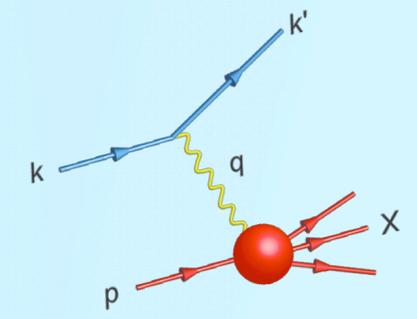
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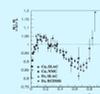
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**Or due to correlations...Local...**

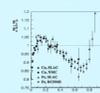


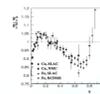
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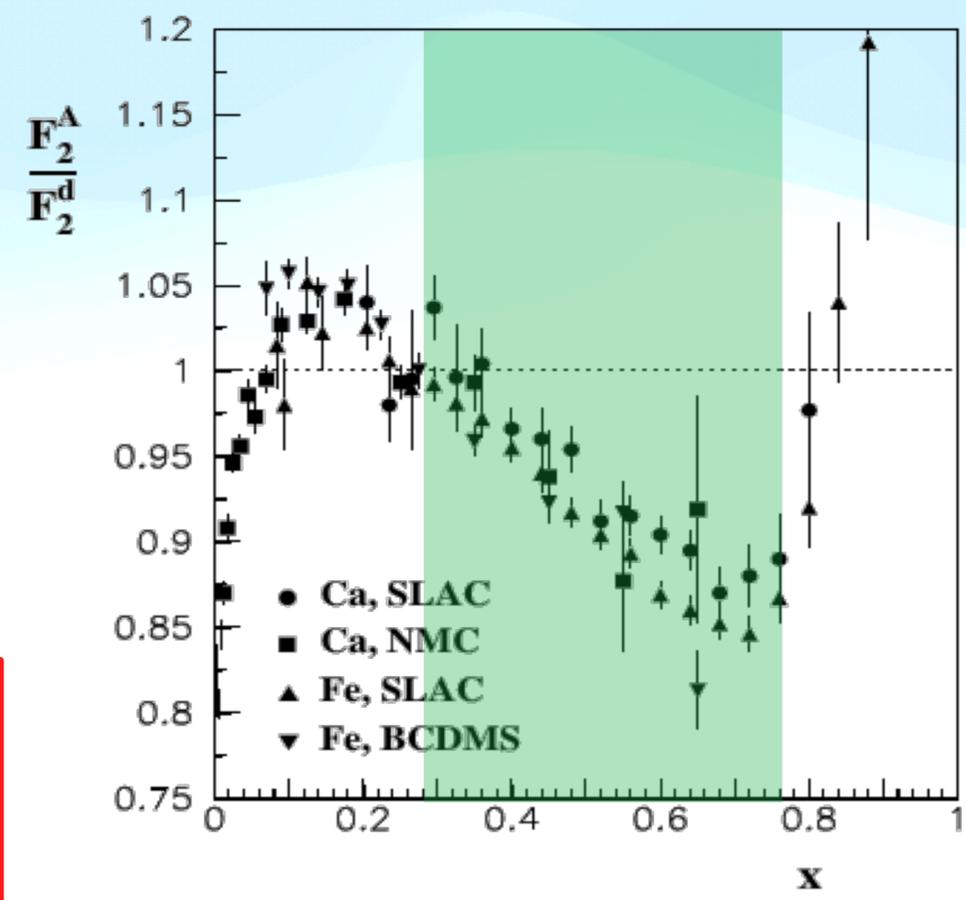
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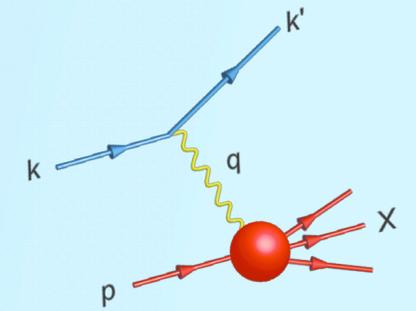
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**Explanation (exotic) advocated:** confinement radius bigger for bound nucleons, quarks in bags with 6, 9, ..., 3A quark, pion cloud effects... Alone or mixed with conventional ones...



# The EMC effect explanations and perspective?



Situation: basically not understood. Very unsatisfactory. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:



the knowledge of nuclear PDFs is crucial for the analysis of heavy ions collisions;



neutron parton structure measured with nuclear targets; several QCD sum rules involve the neutron information (Bjorken SR, for example): importance of Nuclear Physics for QCD

Inclusive measurements cannot distinguish between models

One has probably to go beyond (not treated here...) (R. Dupré and S.Scopetta, EPJA 52 (2016) 159)

- Hard Exclusive Processes (GPDs)
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Status of "Conventional" calculations for light nuclei:

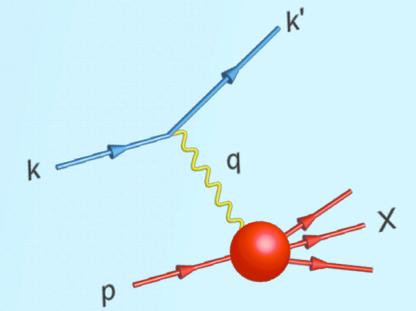


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A fully Poincarè covariant approach to perform the calculation is essential to embed relativistic effects and fulfill sum rules!

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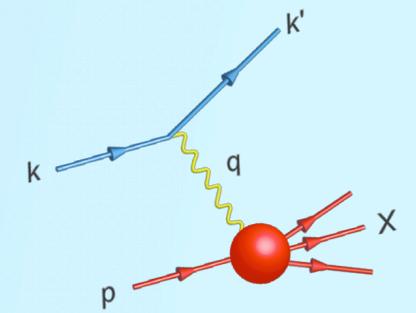


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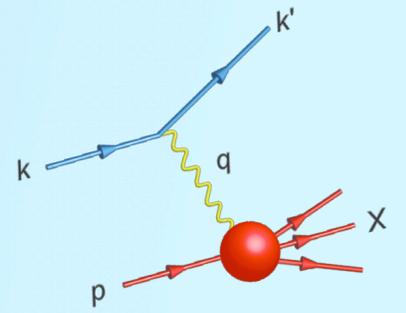


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# The relativistic Hamiltonian dynamics framework

## Why do we need a relativistic treatment ?

General answer: to develop an advanced scheme, appropriate for the kinematics of JLAB12 and of EIC

- The **Standard Model of Few-Nucleon Systems**, with nucleon and meson degrees of freedom within a non relativistic (NR) framework, has achieved **high sophistication** [e.g. the NR  $^3\text{He}$  and  $^3\text{H}$  Spectral Functions in Kievsky, Pace, Salmè, Viviani PRC 56, 64 (1997)].
- Covariance wrt the Poincaré Group,  $G_P$ , needed for nucleons at large 4-momenta and pointing to high precision measurements. Necessary if one studies, e.g., i) nucleon structure functions; ii) nucleon GPDs and TMDs, iii) signatures of short-range correlations; iv) exotics (e.g. 6-bag quarks in  $^2\text{H}$ ), etc
- At least, one should carefully treat the **boosts** of the nuclear states,  $|\Psi_i\rangle$  and  $|\Psi_f\rangle$ !

Our definitely preferred **framework for embedding** the successful NR phenomenology:

Light-front Relativistic Hamiltonian Dynamics (RHD, fixed dof) + Bakamjian-Thomas (BT) construction of the Poincaré generators for an interacting theory.

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- Covariance wrt the Poincaré Group,  $G_P$ , needed for nucleons at large 4-momenta and pointing to high precision measurements. Necessary if one studies, e.g., i) nucleon structure functions; ii) nucleon GPDs and TMDs, iii) signatures of short-range correlations; iv) exotics (e.g. 6-bag quarks in  $^2\text{H}$ ), etc
- At least, one should carefully treat the boosts of the nuclear states,  $|\Psi_i\rangle$  and  $|\Psi_f\rangle$ !

Our definitely preferred framework for embedding the successful NR phenomenology:

Light-front Relativistic Hamiltonian Dynamics (RHD, fixed dof) + Bakamjian-Thomas (BT) construction of the Poincaré generators for an interacting theory.

# The relativistic Hamiltonian dynamics framework

In RHD+BT, one can address both **Poincaré covariance** and **locality**, general principles to be implemented **in presence of interaction**:

- **Poincaré covariance** → The 10 generators,  $P^\mu \rightarrow$  4D displacements and  $M^{\nu\mu} \rightarrow$  Lorentz transformations, have to fulfill

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = -i(g^{\mu\rho} P^\nu - g^{\nu\rho} P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho} M^{\nu\sigma} + g^{\nu\sigma} M^{\mu\rho} - g^{\mu\sigma} M^{\nu\rho} - g^{\nu\rho} M^{\mu\sigma})$$

Also  $\mathcal{P}$  and  $\mathcal{T}$  have to be taken into account !

- **Macroscopic locality** ( $\equiv$  cluster separability (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of **large** spacelike separation (i.e. causally disconnected), rather than for arbitrary (microscopic-locality) spacelike separations. In this way, **when a system is separated into disjoint subsystems by a sufficiently large spacelike separation, then the subsystems behave as independent systems.**

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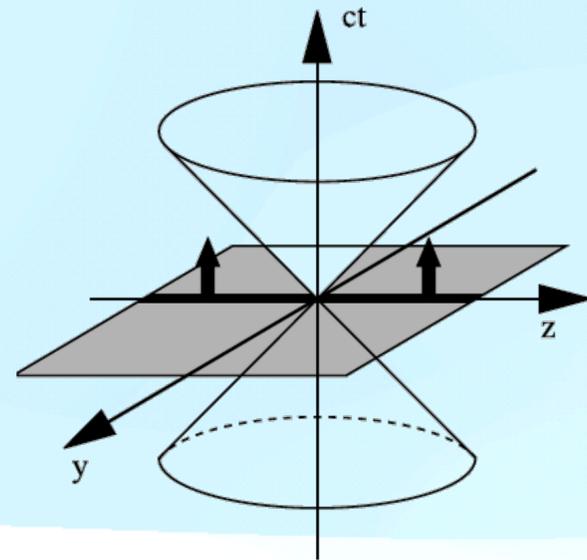
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# Forms of relativistic Dynamics

P.A.M. Dirac, 1949

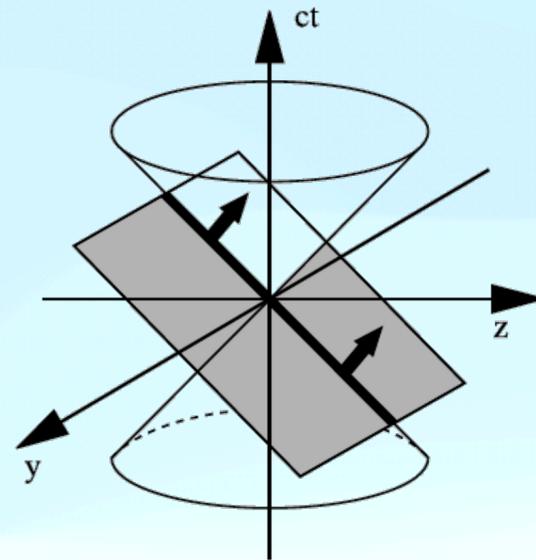


The instant form

$$\begin{aligned}\tilde{x}^0 &= ct \\ \tilde{x}^1 &= x \\ \tilde{x}^2 &= y \\ \tilde{x}^3 &= z\end{aligned}$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\Sigma : t = 0$$

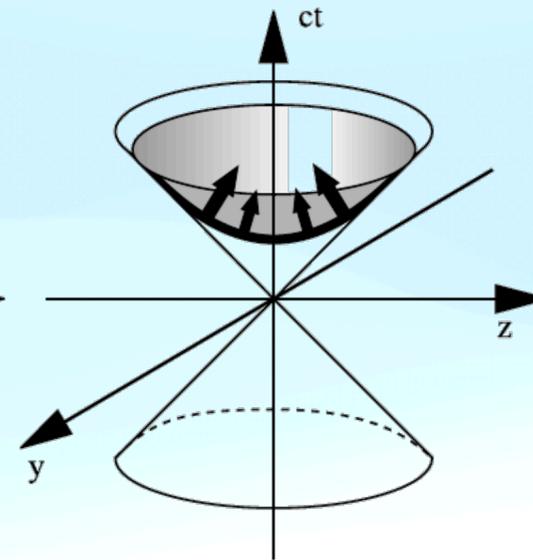


The front form

$$\begin{aligned}\tilde{x}^0 &= ct+z \\ \tilde{x}^1 &= x \\ \tilde{x}^2 &= y \\ \tilde{x}^3 &= ct-z\end{aligned}$$

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$$\Sigma : x^+ = x^0 + x^3 = 0$$

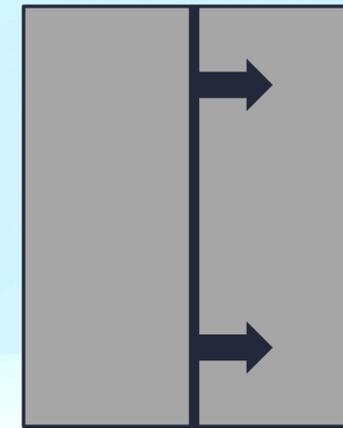


The point form

$$\begin{aligned}\tilde{x}^0 &= \tau, & ct &= \tau \cosh \omega \\ \tilde{x}^1 &= \omega, & x &= \tau \sinh \omega \sin \theta \cos \phi \\ \tilde{x}^2 &= \theta, & y &= \tau \sinh \omega \sin \theta \sin \phi \\ \tilde{x}^3 &= \phi, & z &= \tau \sinh \omega \cos \theta\end{aligned}$$

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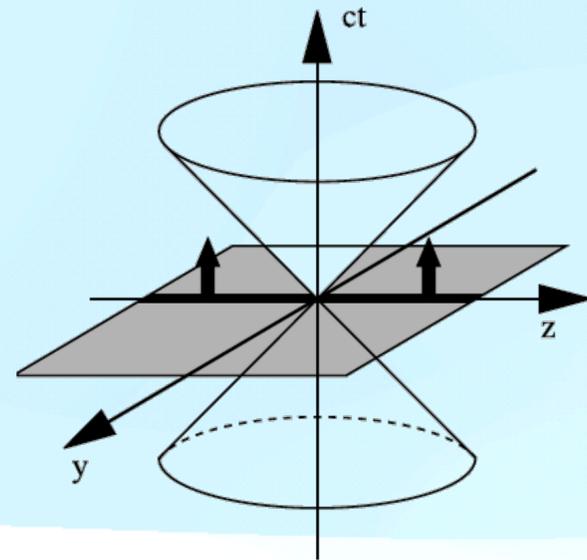
$$\Sigma : (x^0)^2 - x_i x^i = k^2$$



$\Sigma =$  hyperplane of the initial conditions

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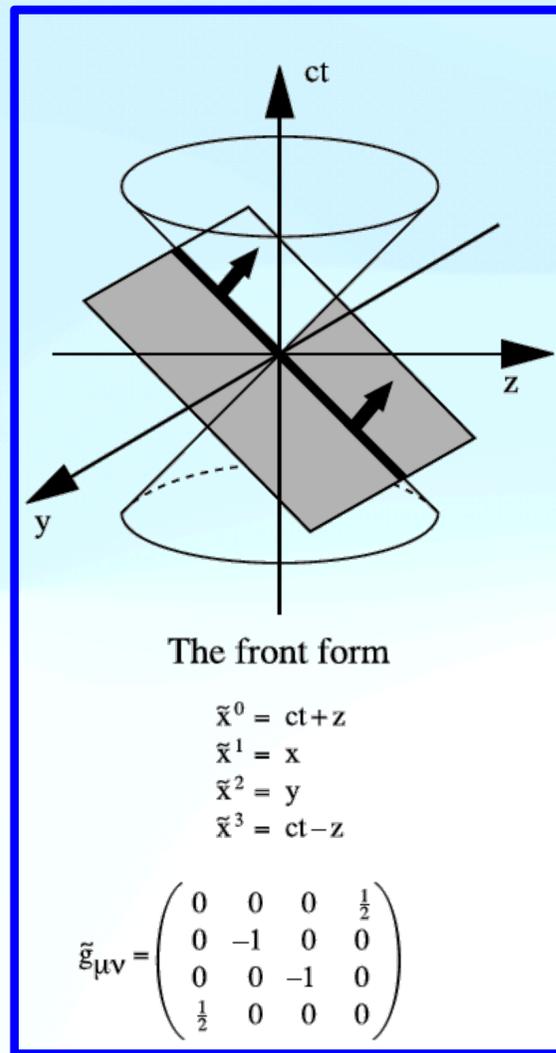


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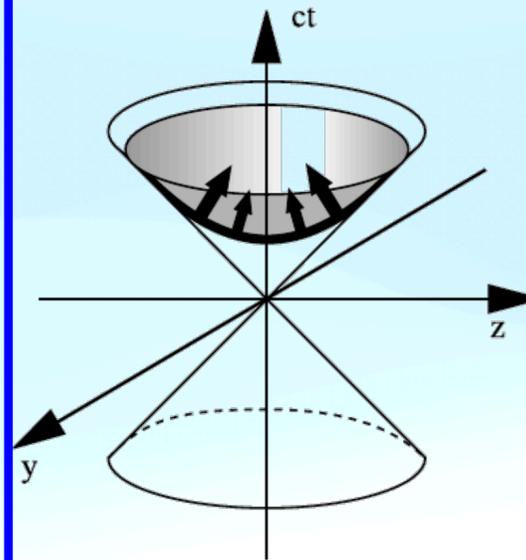


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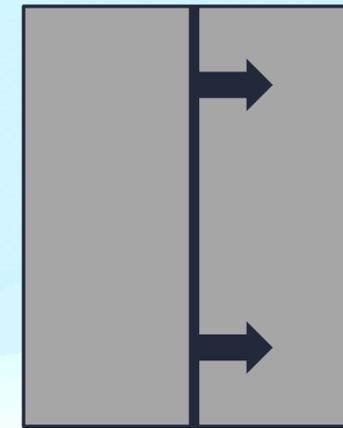


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The [Light-Front framework](#) has several advantages:

- 7 Kinematical generators: i) [three LF boosts](#) ( In instant form they are dynamical!), ii)  $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$ , iii) [Rotation](#) around the [z-axis](#).
- The LF boosts have a subgroup structure : trivial Separation of [intrinsic and global](#) motion, as in the NR case. [important to correctly treat the boost between initial and final states !](#)
- $P^+ \geq 0 \rightarrow$  meaningful Fock expansion, once massless constituents are absent
- No square root in the dynamical operator  $P^-$ , propagating the state in the LF-time.
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**Drawback: the transverse LF-rotations are dynamical**

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# Bakamjian-Thomas construction and LFHD

**Bakamjian and Thomas** (PR 92 (1953) 1300) proposed an explicit construction of 10 Poincaré generators in presence of interactions.

• The key ingredient is the **mass operator**:

i) only the **mass operator**  $M$  contains the interaction

ii) it generates the dependence of the 3 dynamical generators upon the interaction

• The **mass operator** is given by the sum of  $M_0$  with an interaction  $V$ , or  $M_0 + U$ . The interaction,  $U$  or  $V$ , must commute with all the kinematical generators and with the non-interacting angular momentum, as in the **NR** case.

In the **Few-body case**, one can easily embed the **NR phenomenology**:

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# The BT Mass operator for an A=3 system

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$$M_0(123) = \sum_{i=1}^3 \sqrt{m^2 + k_i^2}$$

free mass operator

momenta in the intrinsic reference frame  $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$

● The commutation rules impose to  $V^{BT}$  invariance for translations and rotations as well as independence on the total momentum, as it occurs for  $V^{NR}$ .

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The formalism has been extended also for  ${}^4\text{He}$

# Reference frames

R. Alessandro, A. del Dotto, E. Pace, G. Perna, S. Scopetta and G. Salmè, PRC 104 (2021) 6, 065204

E. Pace, M. Rinaldi, S. Scopetta and G. Salmè, Phys. Scr. 95, 064008 (2020)

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- The intrinsic LF frame of the cluster, (1; 23), where  $\tilde{P} = (\mathcal{M}_0(1, 23), \vec{0}_\perp)$ , with  $\kappa^+(1; 23) = \xi \mathcal{M}_0(1, 23)$  and  $\mathcal{M}_0(1, 23) = \sqrt{m^2 + |\kappa|^2} + \sqrt{M_S^2 + |\kappa|^2}$   
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while  $\mathbf{p}_\perp(lab) = \mathbf{k}_\perp(123) = \boldsymbol{\kappa}_\perp(1, 23)$

The formalism has been extended also for A-nucleus

# The spin-dependent LF spectral function

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

The **Spectral Function**: probability distribution to find inside a bound system a particle with a given  $\tilde{\kappa}$  when the rest of the system has energy  $\epsilon$ , with a **polarization vector**  $S$ :

$$\mathcal{P}_{\sigma'\sigma}^T(\tilde{\kappa}, \epsilon, S) = \rho(\epsilon) \sum_{JJ_z \alpha} \sum_{Tt} {}_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} | \Psi_{\mathcal{M}}; ST_z \rangle \langle ST_z; \Psi_{\mathcal{M}} | \tilde{\kappa}, \sigma\tau; JJ_z; \epsilon, \alpha; Tt \rangle_{LF}$$

$|\Psi_{\mathcal{M}}; ST_z\rangle = \sum_m |\Psi_m; S_z T_z\rangle D_{m,\mathcal{M}}^{\mathcal{J}}(\alpha, \beta, \gamma)$ 
→ Euler angles of rotation from the z-axis to the **polarization vector**  $S$

three-body bound eigenstate of  $M_{BT}(123) \sim M^{NR}$

$|\tilde{\kappa}, \sigma\tau; JJ_z; \epsilon, \alpha; T\tau\rangle_{LF}$  tensor product of a plane wave for particle 1 with LF momentum  $\tilde{\kappa}$  in the intrinsic reference frame of the [1 + (23)] cluster times the fully interacting state of the (23) pair of energy eigenvalue  $\epsilon$ . It has eigenvalue:

$$\mathcal{M}_0(1, 23) = \sqrt{m^2 + |\kappa|^2} + E_S \quad E_S = \sqrt{M_S^2 + |\kappa|^2} \quad M_S = 2\sqrt{m^2 + m\epsilon}$$

and fulfills the macroscopic locality (Keister, Polyzou, Adv. N. P. 20, 225 (1991)).

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The spectral function is written in terms of the overlap  ${}_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} \Psi_{\mathcal{M}}; ST_z \rangle_{LF}$

The tensor product of the plane wave of the interacting particle and the state of the spectator system

In the intrinsic reference frame of the cluster  $[1; 2, 3, \dots, A - 1]$

The wave function of the nucleus A (i.e. the eigenstate of  $M[1, 2, \dots, A] \sim M^{NR}$ )

In the intrinsic frame of the system  $[1, 2, \dots, A]$

We can express the **LF overlap** in terms of the **IF overlap** using **Melosh rotations** and then we can approximate the IF overlap into a NR overlap thanks to the **BT construction**:

$$\{\alpha\}; \phi \rangle_{LF} \rightarrow \{\alpha\}; \phi \rangle_{IF} \sim \{\alpha\}; \phi \rangle_{NR}$$

The **LF spectral function** contains the determinant of the Jacobian of the transformation between the intrinsic frames  $[1; 2, 3, \dots, A - 1]$  and  $[1, 2, \dots, A]$ , connected each other by a **LF boost**

\*F. Fornetti's slide

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The wave function of the nucleus A (i.e. the eigenstate of  $M[1, 2, \dots, A] \sim M^{NR}$ )

In the intrinsic frame of the system  $[1, 2, \dots, A]$

We can express the **LF overlap** in terms of the **IF overlap** using **Melosh rotations** and then we can approximate the IF overlap into a NR overlap thanks to the **BT construction**:

$$\{\alpha\}; \phi \rangle_{LF} \rightarrow \{\alpha\}; \phi \rangle_{IF} \sim \{\alpha\}; \phi \rangle_{NR}$$

The **LF spectral function** contains the determinant of the Jacobian of the transformation between the intrinsic frames  $[1; 2, 3, \dots, A - 1]$  and  $[1, 2, \dots, A]$ , connected each other by a **LF boost**

\*F. Fornetti's slide

# The spin-dependent LF spectral function

The spectral function is written in terms of the overlap  ${}_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} \Psi_{\mathcal{M}}; ST_z \rangle_{LF}$

The tensor product of the plane wave of the interacting particle and the state of the spectator system

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\*F. Fornetti's slide

# The nuclear structure function $F_2$

The hadronic tensor, in **Impulse approximation** is found to be (Pace, M.R., Salmè and S. Scopetta, Phys. Scri. 2020)

$$W_A^{\mu\nu}(P_A, T_{Az}) = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{(2\pi)^3 2\kappa^+} \frac{1}{\xi} \mathcal{P}^N(\tilde{\mathbf{k}}, \epsilon) w_{N,\sigma}^{\mu\nu}(p, q)$$

hadronic tensor of the bound nucleon

In the Bjorken limit the nuclear structure function can be obtained from the hadronic tensor:

$$F_2^A(x) = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{(2\pi)^3 2\kappa^+} \frac{1}{\xi} \mathcal{P}^N(\tilde{\mathbf{k}}, \epsilon) (-x) g_{\mu\nu} w_{N,\sigma}^{\mu\nu}(p, q) = \sum_N \int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{(2\pi)^3 2\kappa^+} \mathcal{P}^N(\tilde{\mathbf{k}}, \epsilon) \frac{P_A^+}{p^+} \frac{x}{z} F_2^N(z)$$

$$x = \frac{Q^2}{2P_A \cdot q} \quad \text{Bjorken variable} \quad \xi = \frac{\kappa^+}{M_0(1,23)} \neq x \quad z = \frac{Q^2}{2p \cdot q}$$

nucleon structure function  
 $F_2^N(z) = -z g_{\mu\nu} \sum_\sigma w_{N,\sigma}^{\mu\nu}(p, q)$

One should notice that:  $\int d\epsilon \int d\kappa^+ \neq \int d\kappa^+ \int d\epsilon$  but in the BJ limit  $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$

therefore,  $F_2$  and the EMC effect can be evaluated the LC momentum distribution directly!

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therefore,  $F_2$  and the EMC effect can be evaluated the LC momentum distribution directly!

$$F_2^A(x) = \sum_N \int_{\xi_{min}}^1 d\xi F_2^N\left(\frac{m x}{\xi M_A}\right) f_1^N(\xi)$$

Light-cone momentum distribution

Free nucleon structure function

Determinant of the Jacobian matrix. LF boost: effect of a Poincaré covariance approach

Squared nuclear wave function. Thanks to the BT construction, one is allowed to use the NR one

$$\text{With: } f_1^N(\xi) = \int d\mathbf{k}_\perp n^N(\xi, \mathbf{k}_\perp)$$

LF momentum distribution:

$$n^N(\xi, \mathbf{k}_\perp) = \frac{1}{2\pi} \int \prod_{i=2}^{A-1} [d\mathbf{k}_i]$$

$$\frac{\partial k_z}{\partial \xi}$$

$$\mathcal{N}^N(\mathbf{k}, \mathbf{k}_2, \dots, \mathbf{k}_{A-1})$$

\*F. Fornetti's slide

# LC momentum distributions

From the normalization of the Spectral Function one has

$$f_{\tau}^A(\xi) = \int dk_{\perp} n^{\tau}(\xi, \mathbf{k}_{\perp}) \longrightarrow \int_0^1 d\xi f_{\tau}^A(\xi) = 1$$

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We can define the essential sum rules that must be satisfied:

$$N_A = \int d\xi \left[ Z f_p^A(\xi) + (A - Z) f_n^A(\xi) \right] = 1$$

Baryon number sum rule

$$MSR = \int d\xi \xi \left[ Z f_p^A(\xi) + (A - Z) f_n^A(\xi) \right] = 1$$

Momentum sum rule

Within the LFHD we are able to fulfill **both sum rules at the same time!**

**E. Pace, M.R., G. Salmè and S. Scopetta, Phys. Lett. B (2023) 137810**

**A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)**

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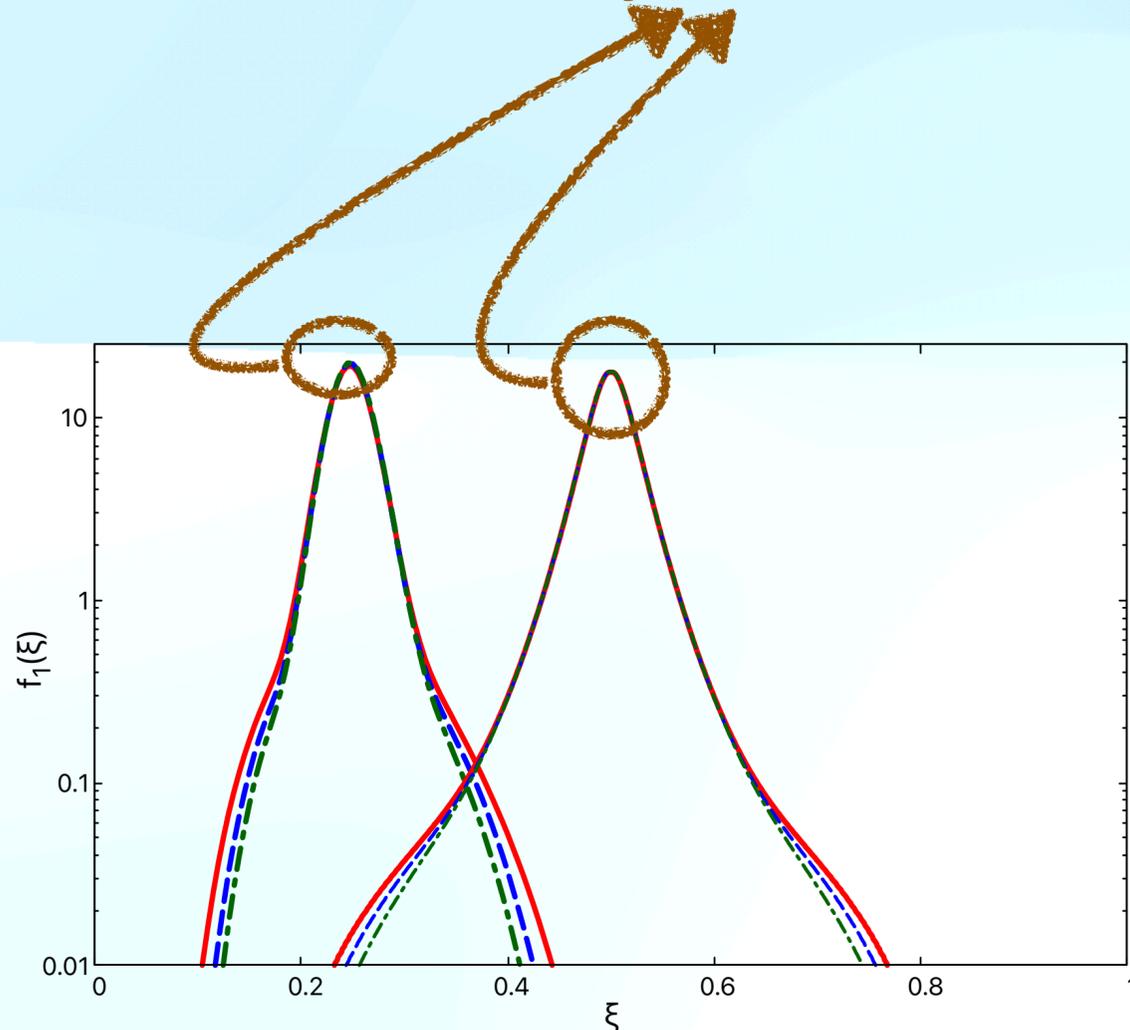
Not possible within the IF! (Frankfurt & Strikman; Miller;....80's)

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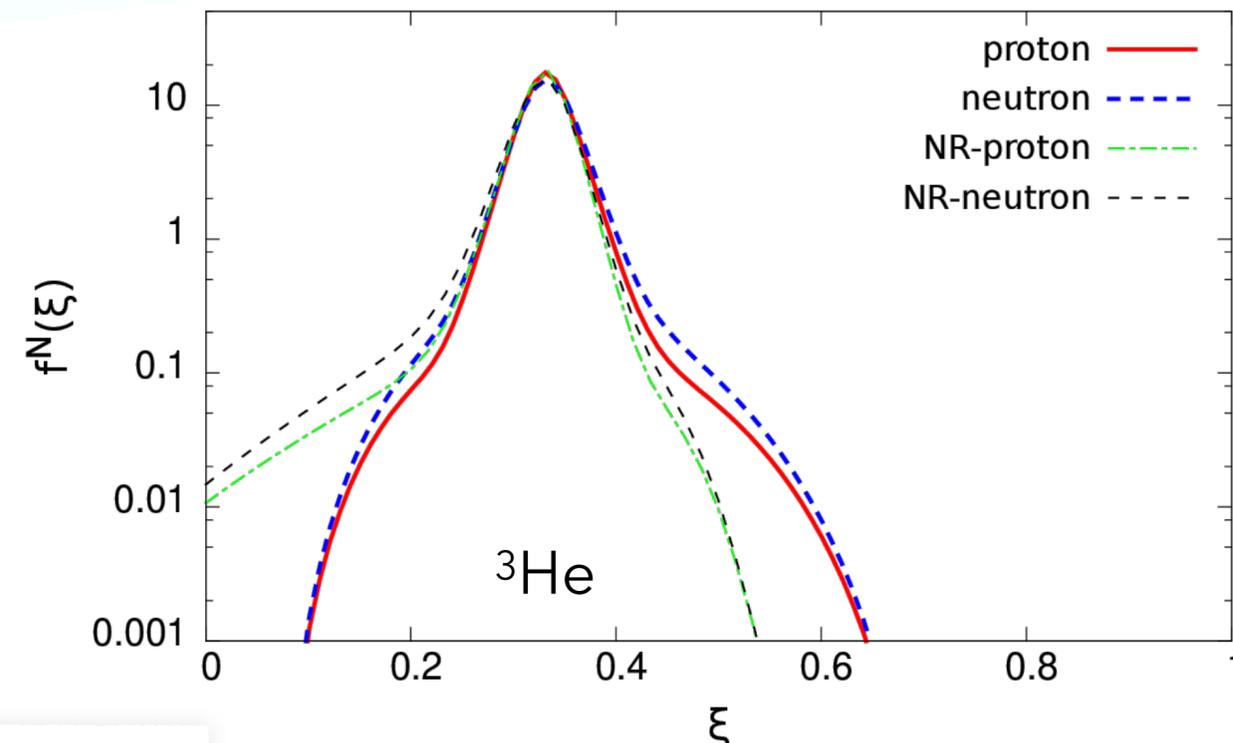
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The distributions are peaked in  $1/A$  with an accuracy of  $1/1000$ :  
**MSR and Number of baryon sum rules are numerically satisfied**



Matteo Rinaldi

- The tails of the distributions are generated by the **short range correlations (SRC)** induced by the potentials (i.e the **high-momentum content** of the 1-body momentum distribution)
- The tails of the LC momentum distribution calculated by the **Av18/UIX** potential is larger than the ones obtained by the  $\chi$ EFT potentials for both  ${}^4\text{He}$  and deuteron
- This difference will partially cancel out on the EMC ratio



TNPI2023

# LC momentum distributions

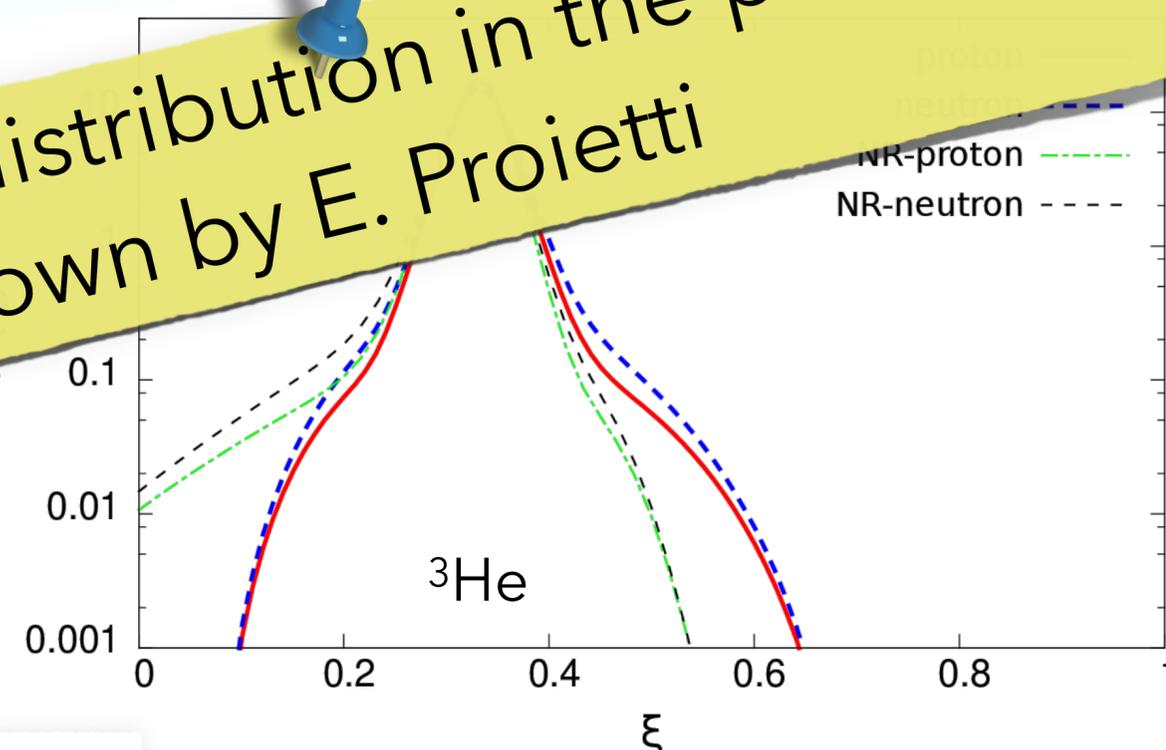
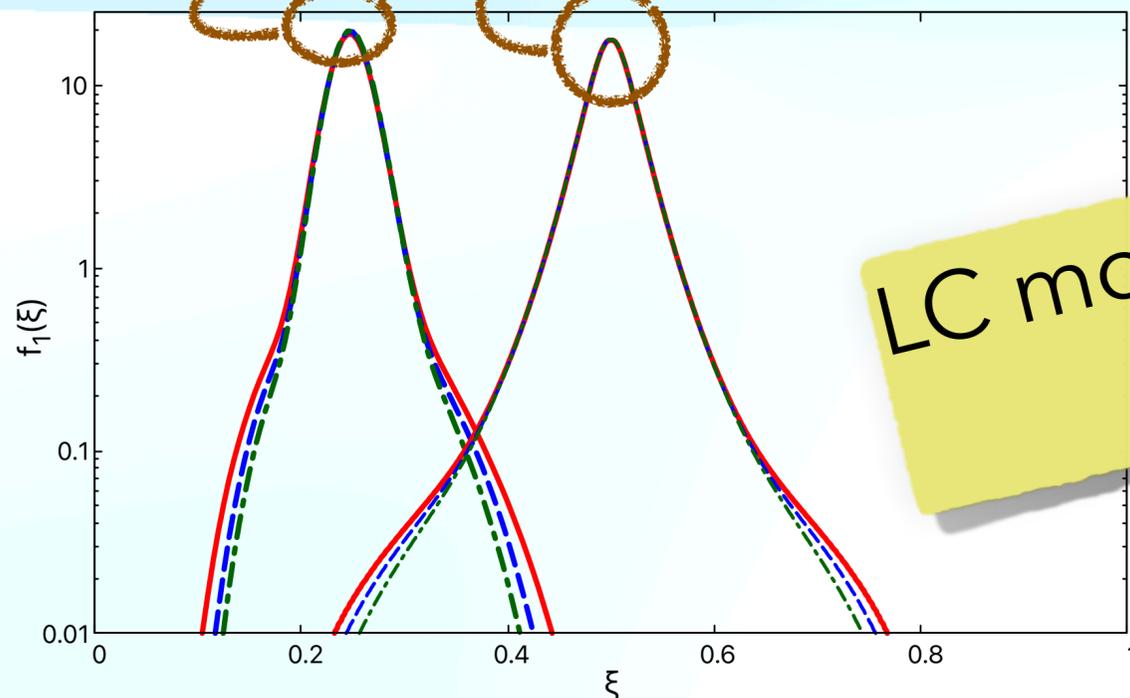
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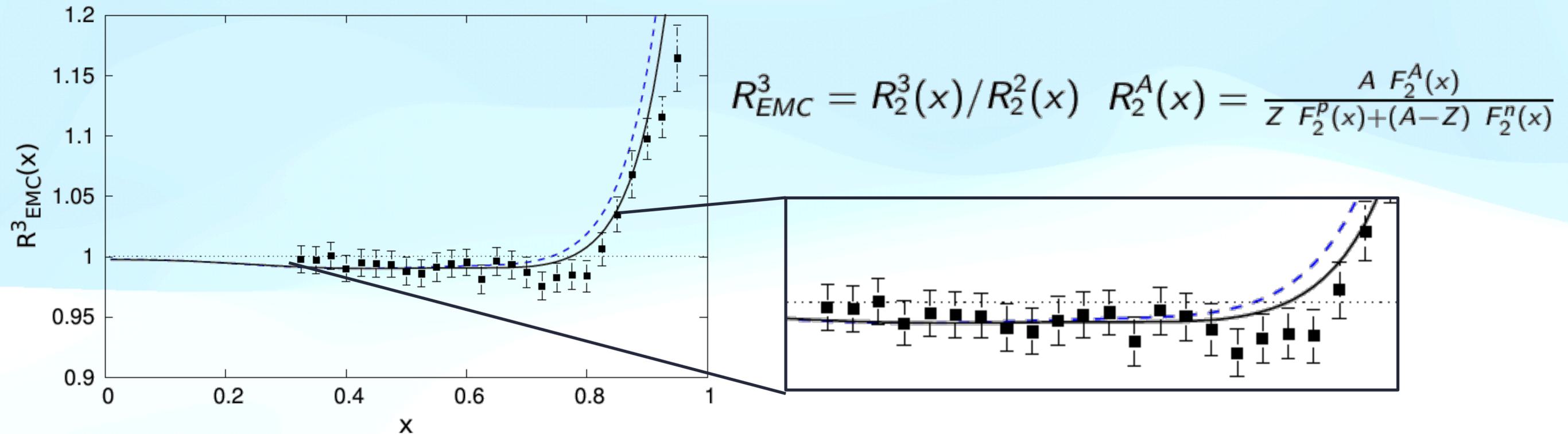
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- The tails of the LC momentum distribution calculated by the **Av18/UIX** potential is larger than the ones obtained by the  $\chi$ EFT potentials for both  ${}^4\text{He}$  and deuteron
- This difference will not be visible in the unpolarized case

LC momentum distribution in the polarized case shown by E. Proietti



# The $^3\text{He}$ EMC effect within the LFHD

E. Pace, M.R., G. Salmè and S. Scopetta, Phys. Lett. B (2023) 137810



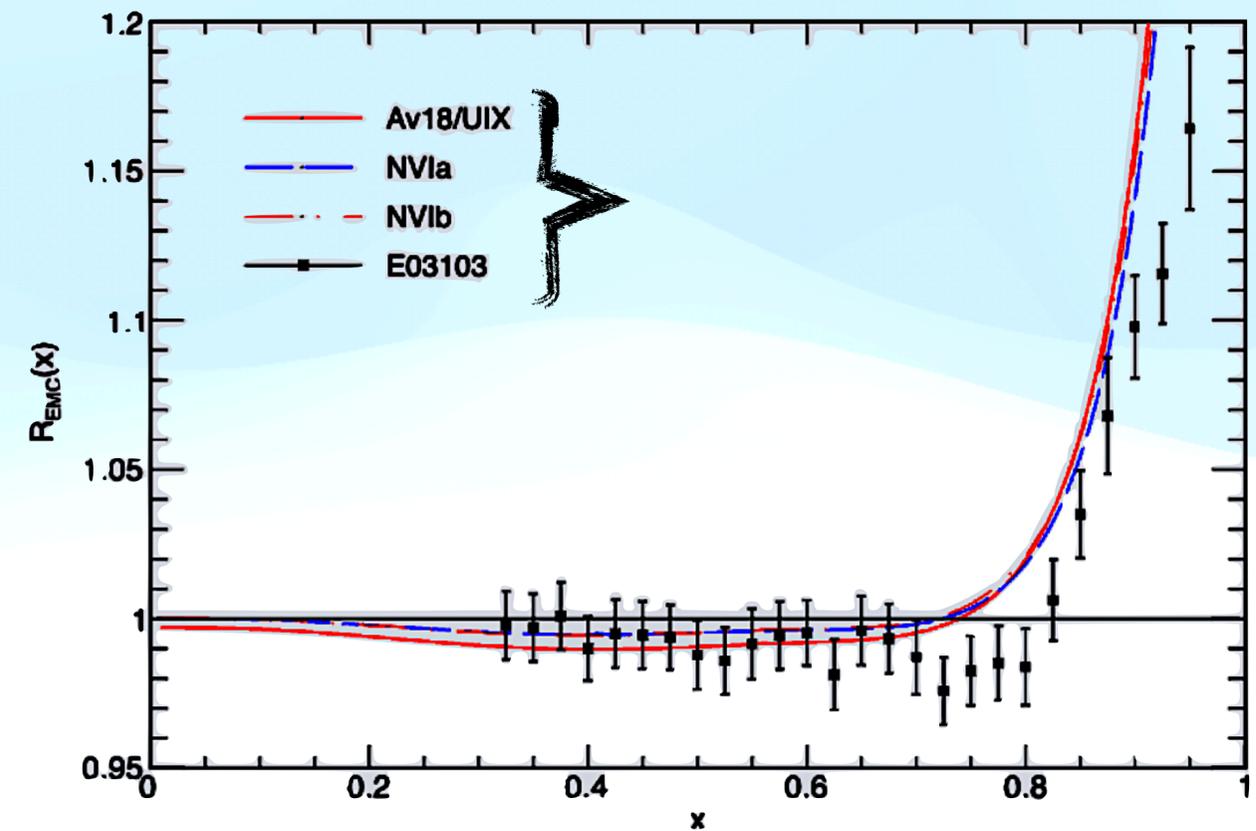
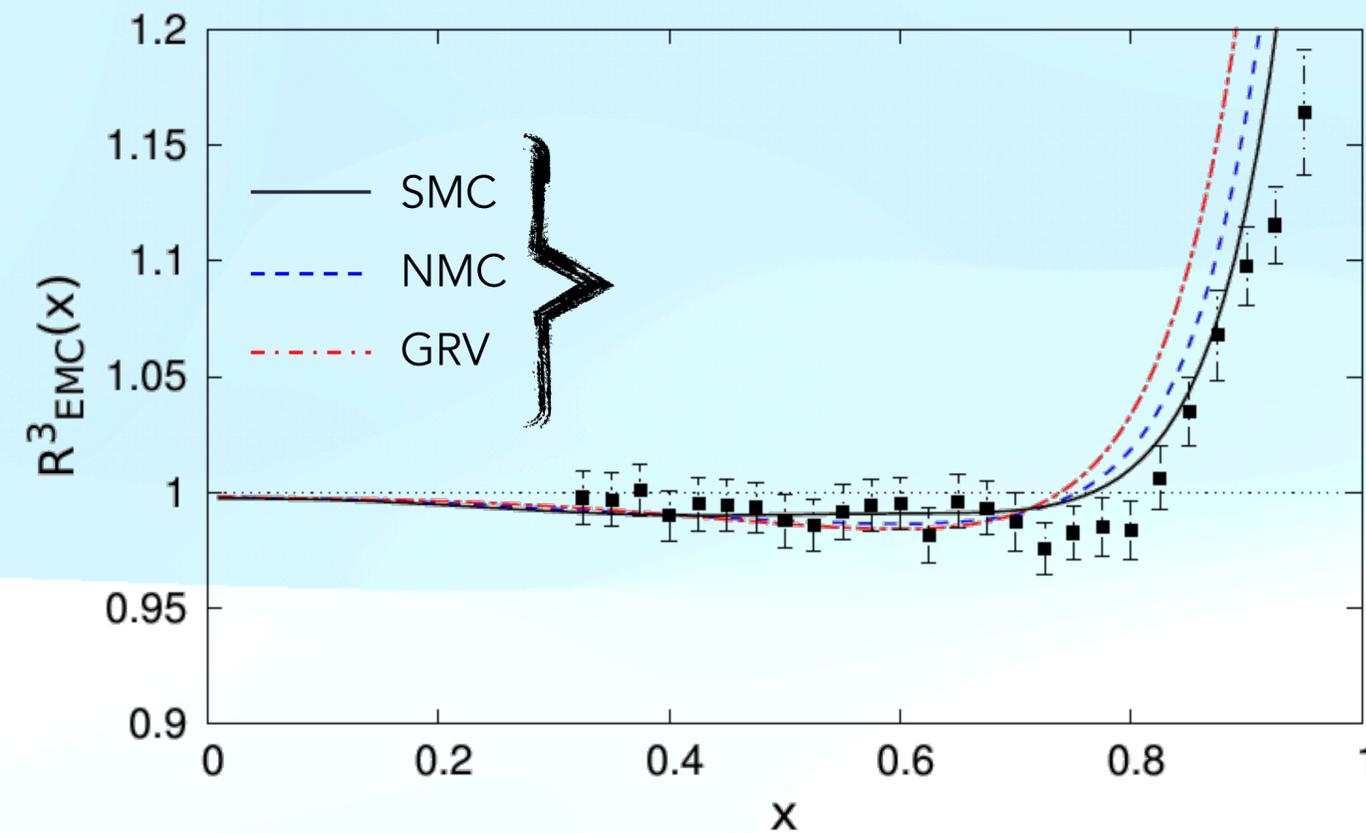
 Solid line: with Av18 description of  $^3\text{He}$ , Dashed line: including three-body forces (U-IX) with "SMC" nucleon structure functions (Adeva et al PLB 412, 414 (1997)).

 Full squares: data from J. Seely et al., PRL. 103, 202301 (2009) reanalyzed by S. A. Kulagin and R. Petti, PRC 82, 054614 (2010)

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F. Fornetti, E. Pace, M. Rinaldi, S. Scopetta, G. Salmè and M. Viviani, arXiv:2308.15925



$F_2^n(x)$  extracted from the MARATHON data  
[MARATHON, PRL 128,132003 (2022)]

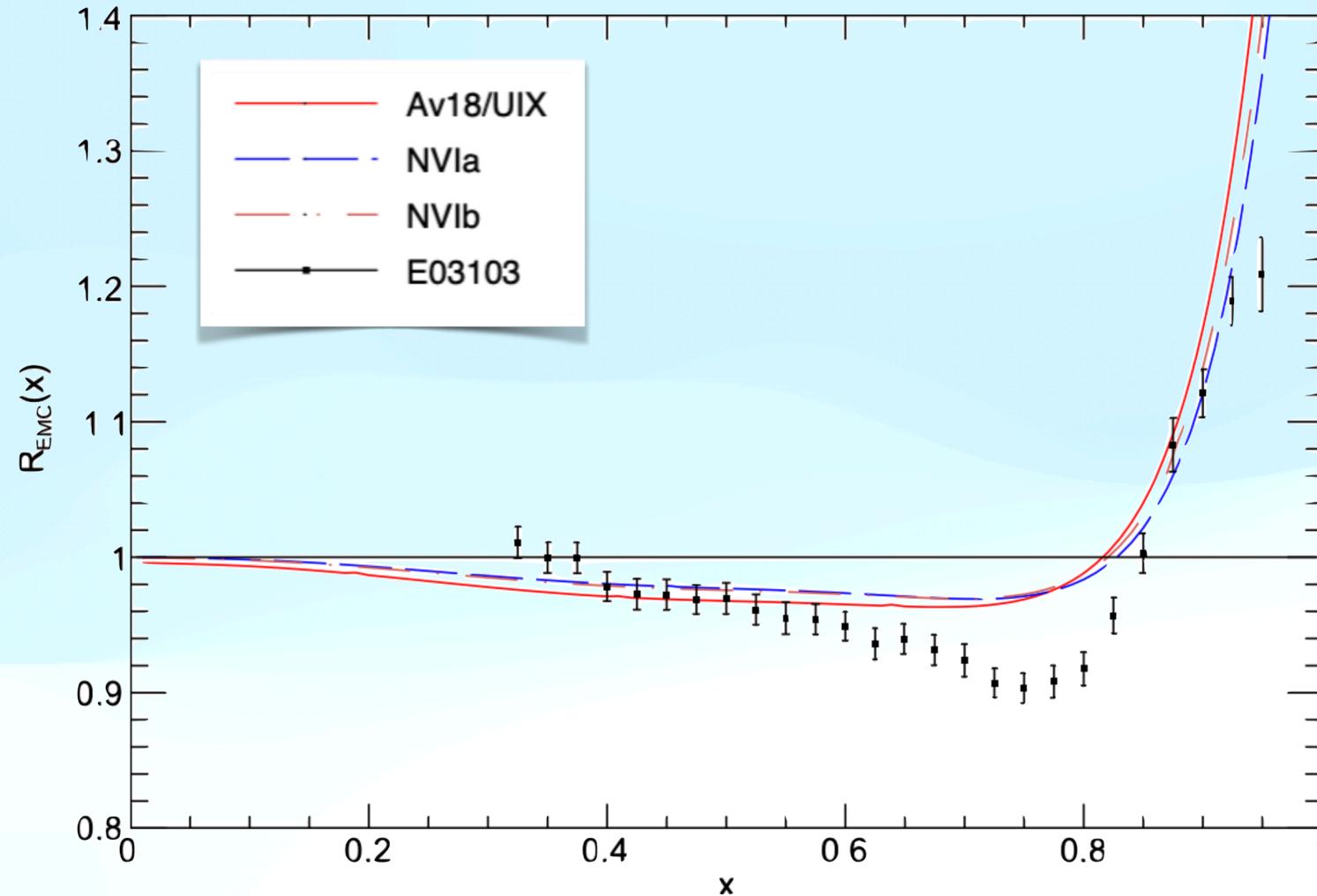
$F_2^p(x)$  SMC

# The $^4\text{He}$ EMC effect within the LFHD

arXiv:2308.15925 [nucl-th]

Full squares: JLab data from  
experiment E03103

[J. Arrington, et al, Phys. Rev. C 104 (6)  
(2021) 065203]



Analogous results obtained  
also for  $^3\text{He}$  and  $^4\text{He}$

\*F. Fornetti's slide

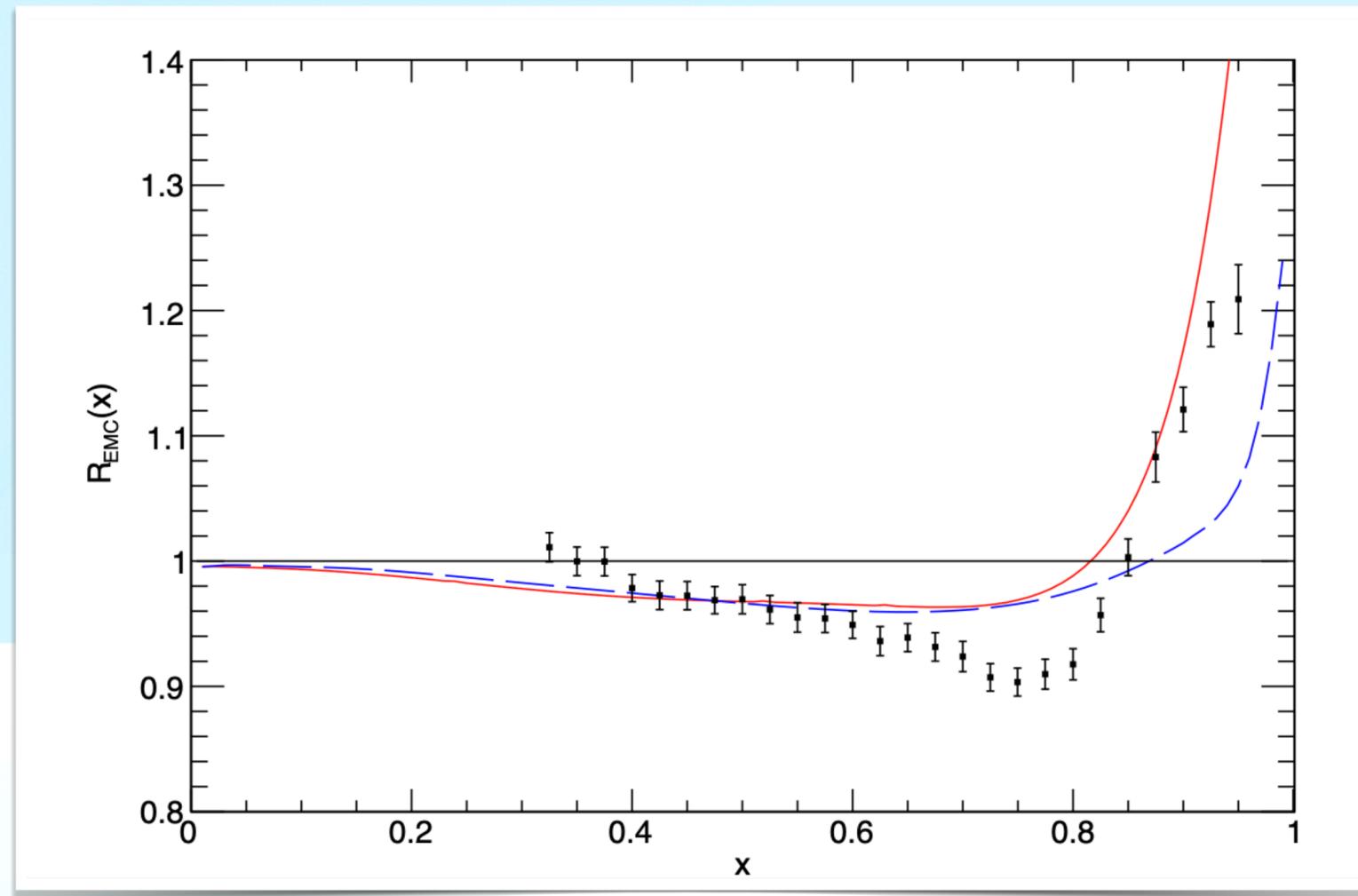
- The differences between the calculations from different potentials are of the **same order for both nuclei**
- They are definitely **smaller than the difference between data and theoretical prediction**

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arXiv:2308.15925 [nucl-th]

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[J. Arrington, et al, Phys. Rev. C 104 (6) (2021) 065203]



Both lines calculated with Av18/UIX  
Solid line: SMC parametrization of  $F_2^p$

Dashed line: NV1b+3N: CJ15 + TMC  
Parametrization of  $F_2^p$

$F_2^n$  extracted from MARATHON data

[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]

[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, Phys. Rev. D 93 (11) (2016) 114017]

[MARATHON, PRL 128,132003 (2022) ]

[E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810]

The dependance on the ratio  $F_2^n/F_2^p$  is **largely under control** as well the dependance on the parametrization of  $F_2^p$  in the properly EMC region

\*F. Fornetti's slide

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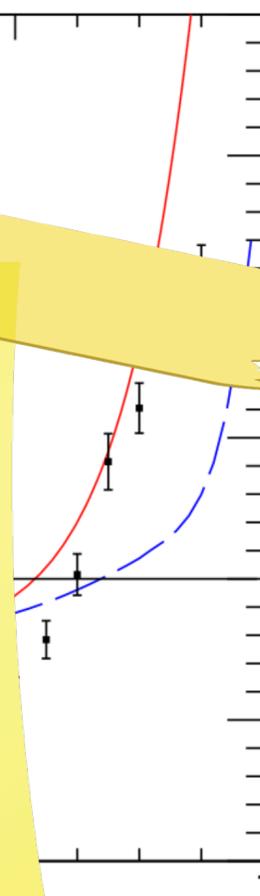
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We calculated the valence contribution to the EMC effect within an approach:

- i) able to include relativistic effects
- ii) fulfill number and momentum sum rules at the same time!
- iii) including conventional nuclear effects



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\*F. Fornetti's slide

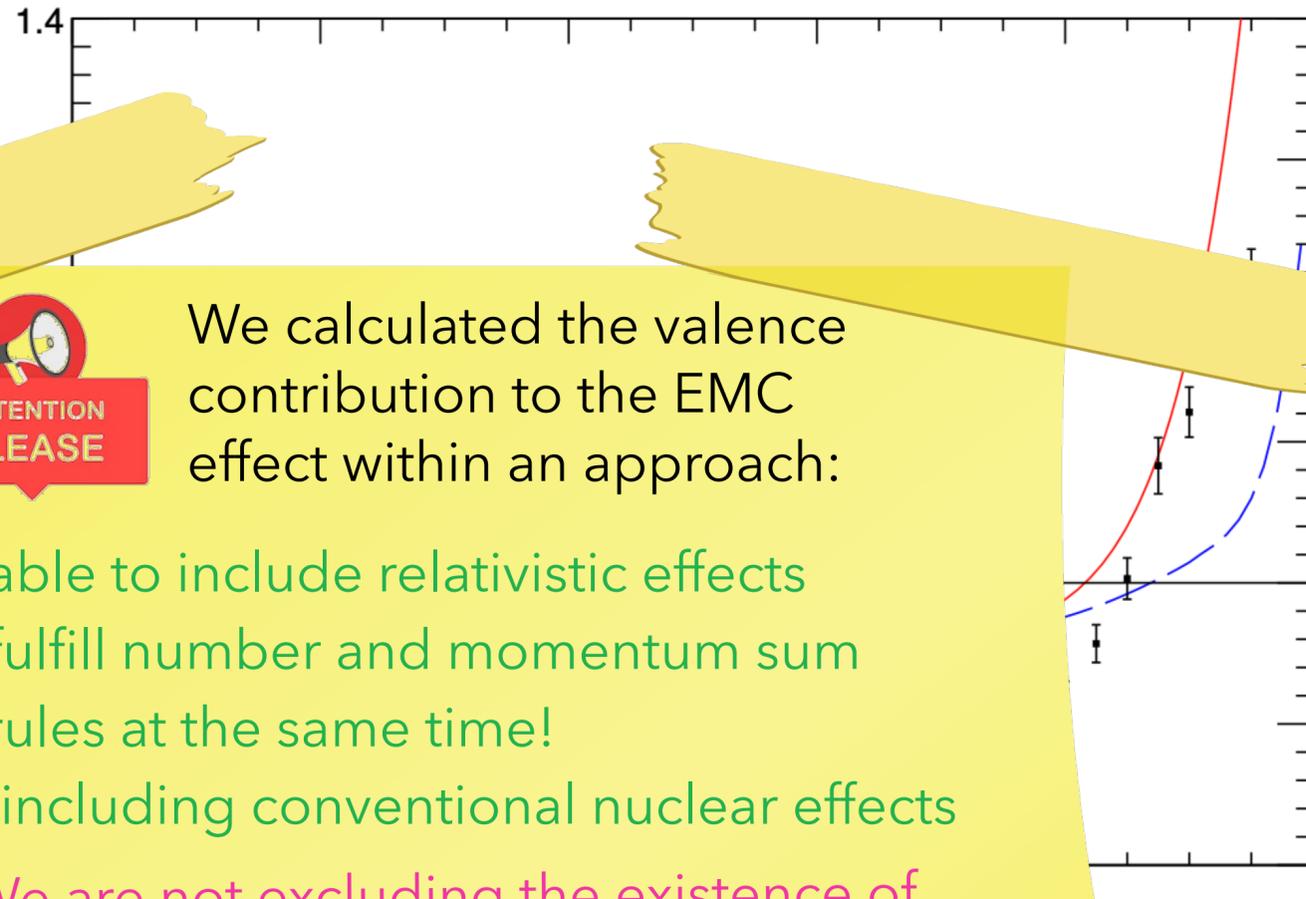
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 $F_2^n$  extracted from MARATHON data



ATTENTION PLEASE

We calculated the valence contribution to the EMC effect within an approach:

- i) able to include relativistic effects
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- iii) including conventional nuclear effects

We are not excluding the existence of effects beyond the conventional ones!



We need to test the approach with heavier nuclei

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parametr

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\*F. Fornetti's slide

# CONCLUSIONS

- ✓ A Poincaré covariant description of nuclei, based on the light-front Hamiltonian dynamics, has been proposed. The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework. N.B. Normalization and momentum sum rule are both automatically fulfilled.
- ✓ LC spin-dependent momentum distributions are available, for both longitudinal and transverse polarizations of the nucleon. (see E. Proietti's talk)
- ✓ Encouraging calculation of  $^3\text{He}$  and  $^4\text{He}$  EMC, shedding light on the role of a reliable description of the nucleus.



Analyses of  $A(e,e',p)X$  reactions, with polarized initial and final states, for accessing nuclear TMD's in  $^3\text{He}$  are in progress

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# Rescaling

## Structure Functions for Light Nuclei

S. A. Kulagin<sup>1,\*</sup> and R. Petti<sup>2,†</sup>

<sup>1</sup>*Institute for Nuclear Research of the Russian Academy of Sciences, 117312 Moscow, Russia*

<sup>2</sup>*Department of Physics and Astronomy,  
University of South Carolina, Columbia SC 29208, USA*

### Abstract

We discuss the nuclear EMC effect with particular emphasis on recent data for light nuclei including  $^2\text{H}$ ,  $^3\text{He}$ ,  $^4\text{He}$ ,  $^9\text{Be}$ ,  $^{12}\text{C}$  and  $^{14}\text{N}$ . In order to verify the consistency of available data, we calculate the  $\chi^2$  deviation between different data sets. We find a good agreement between the results from the NMC, SLAC E139, and HERMES experiments. However, our analysis indicates an overall **normalization offset** of about 2% in the data from the recent JLab E03-103 experiment with respect to previous data for nuclei heavier than  $^3\text{He}$ . **We also discuss the extraction of the neutron/proton structure function ratio  $F_2^n/F_2^p$  from the nuclear ratios  $^3\text{He}/^2\text{H}$  and  $^2\text{H}/^1\text{H}$ .** Our analysis shows that the E03-103 data on  $^3\text{He}/^2\text{H}$  require a renormalization of about 3% **in order to be consistent with the  $F_2^n/F_2^p$  ratio obtained from the NMC experiment.** After such a renormalization, the  $^3\text{He}$  data from the E03-103 data and HERMES experiments are in a good agreement. Finally, we present a detailed comparison between data and model calculations, which include a description of the nuclear binding, Fermi motion and off-shell corrections to the structure functions of bound proton and neutron, as well as the nuclear pion and shadowing corrections. Overall, a good agreement with the available data for all nuclei is obtained.

# Backup Slides: effective polarizations

## Effective polarizations

Key role in the extraction of **neutron polarized structure functions** and **neutron Collins and Sivers single spin asymmetries**, from the corresponding quantities measured for  $^3\text{He}$

Effective longitudinal polarization (axial charge for the nucleon)

$$p_{||}^{\tau} = \int_0^1 dx \int d\mathbf{p}_{\perp} \Delta f^{\tau}(x, |\mathbf{p}_{\perp}|^2)$$

Effective transverse polarization (tensor charge for the nucleon)

$$p_{\perp}^{\tau} = \int_0^1 dx \int d\mathbf{p}_{\perp} \Delta'_{\tau} f^{\tau}(x, |\mathbf{p}_{\perp}|^2)$$

Effective polarizations	proton	neutron
LF longitudinal polarization	-0.02299	0.87261
LF transverse polarization	-0.02446	0.87314
non relativistic polarization	-0.02118	0.89337

- The difference between the LF polarizations and the non relativistic results are **up to 2% in the neutron case** (larger for the proton ones, but it has an overall small contribution), and should be **ascribed to the intrinsic coordinates**, implementing the **Macro-locality**, and not to the Melosh rotations involving the spins.
- N.B. Within a NR framework:  $p_{||}^{\tau}(NR) = p_{\perp}^{\tau}(NR)$



# Backup Slides: effective polarizations

The BT Mass operator for A=3 nuclei - II

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

and must obey to the commutation rules proper of the Galilean group, leading to translational invariance and independence of total 3-momentum.

Those properties are analogous to the ones in the BT construction. This allows us to consider the standard non-relativistic mass operator as a sensible BT mass operator, and embed it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

The 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger equation, like the Schrödinger one, has a suitable structure for the BT construction.

Therefore **what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.**

The eigenfunctions of  $M^{NR}$  do not fulfill the cluster separability, but we take care of Macro-locality in the spectral function.



# Diagrams and infographics

