MAP 22: a new global (SIDIS-DY) fit of unpolarised TMDs at $N^3LL^{(-)}$

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(MAP collaboration)

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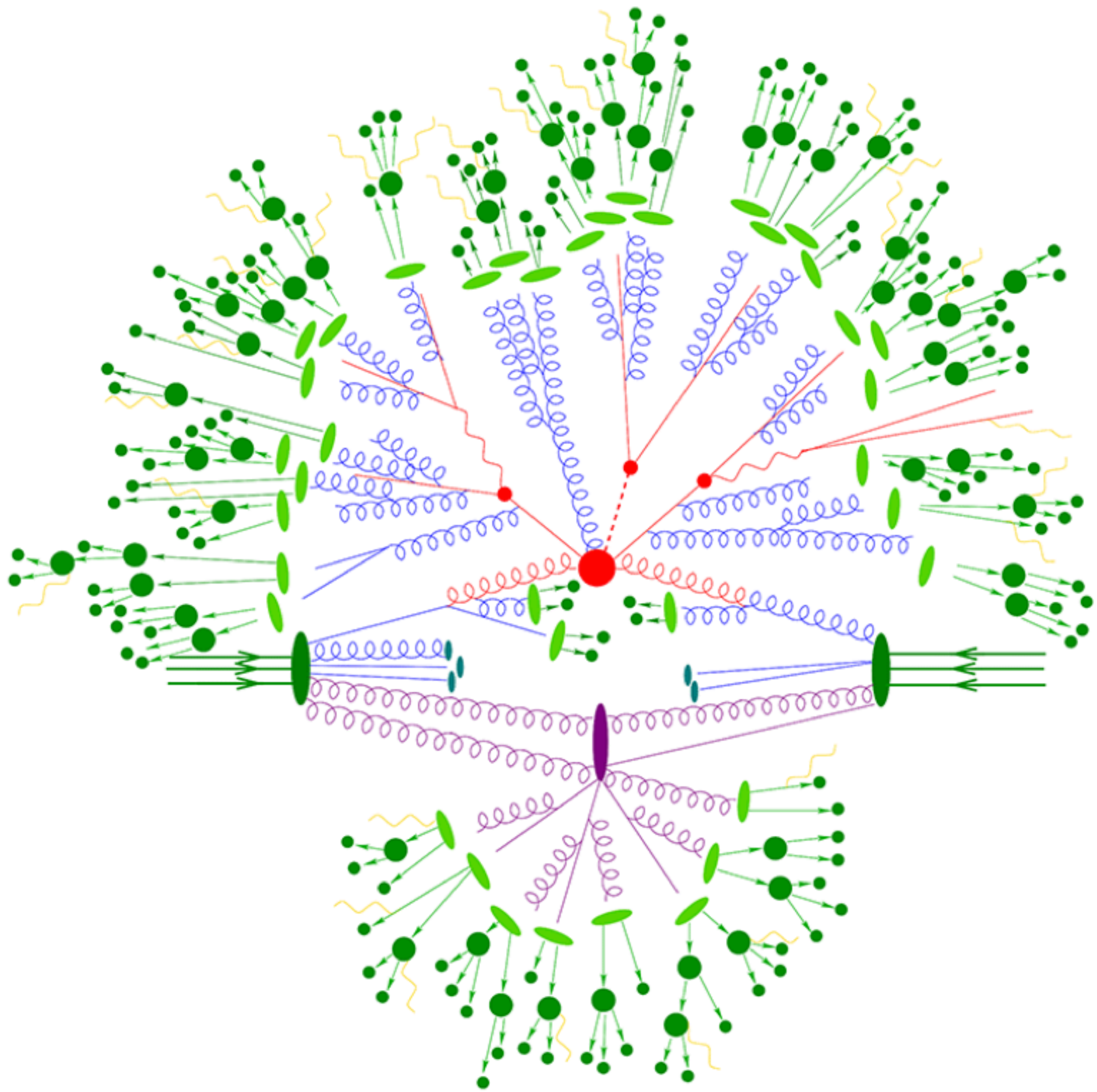
Istituto Nazionale di Fisica Nucleare

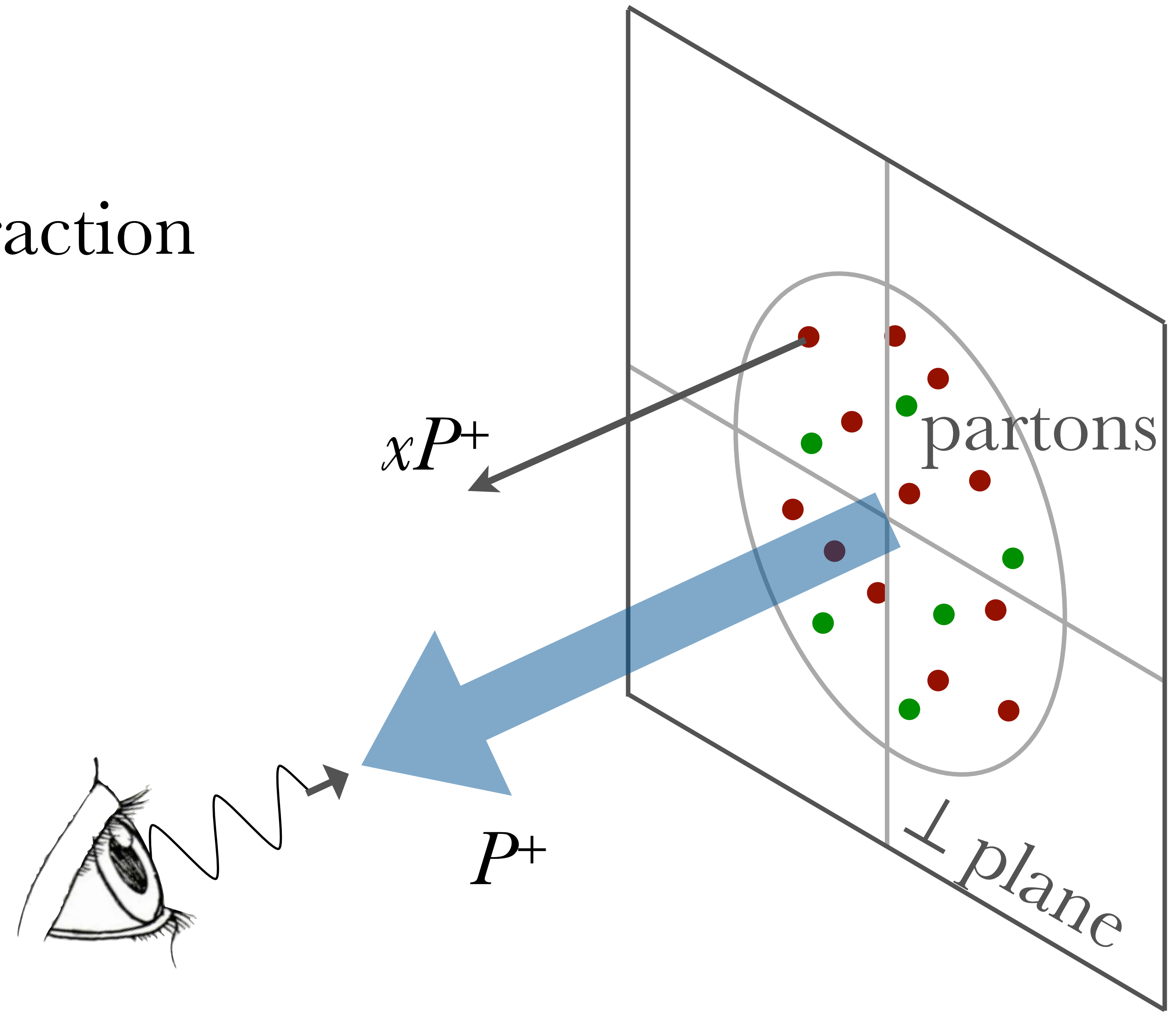
Sezione di Cagliari

Typical event at hadron collider



Collinear PDF (FF)

Collinear PDF $f(x)$
depend on:
x = longitudinal-momentum fraction



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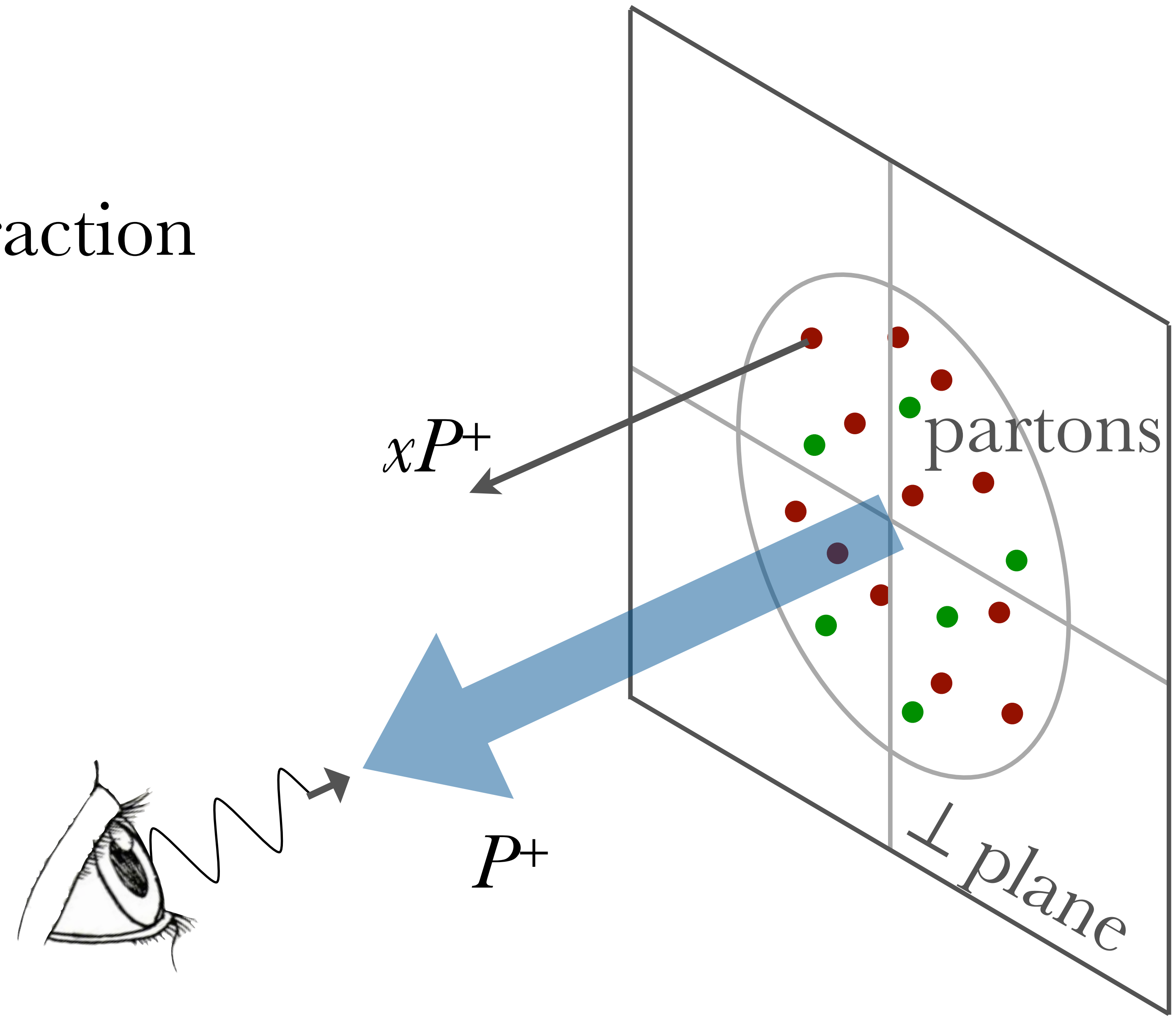

$$\sigma \sim f_a \otimes f_b \otimes \sigma_{ab}$$

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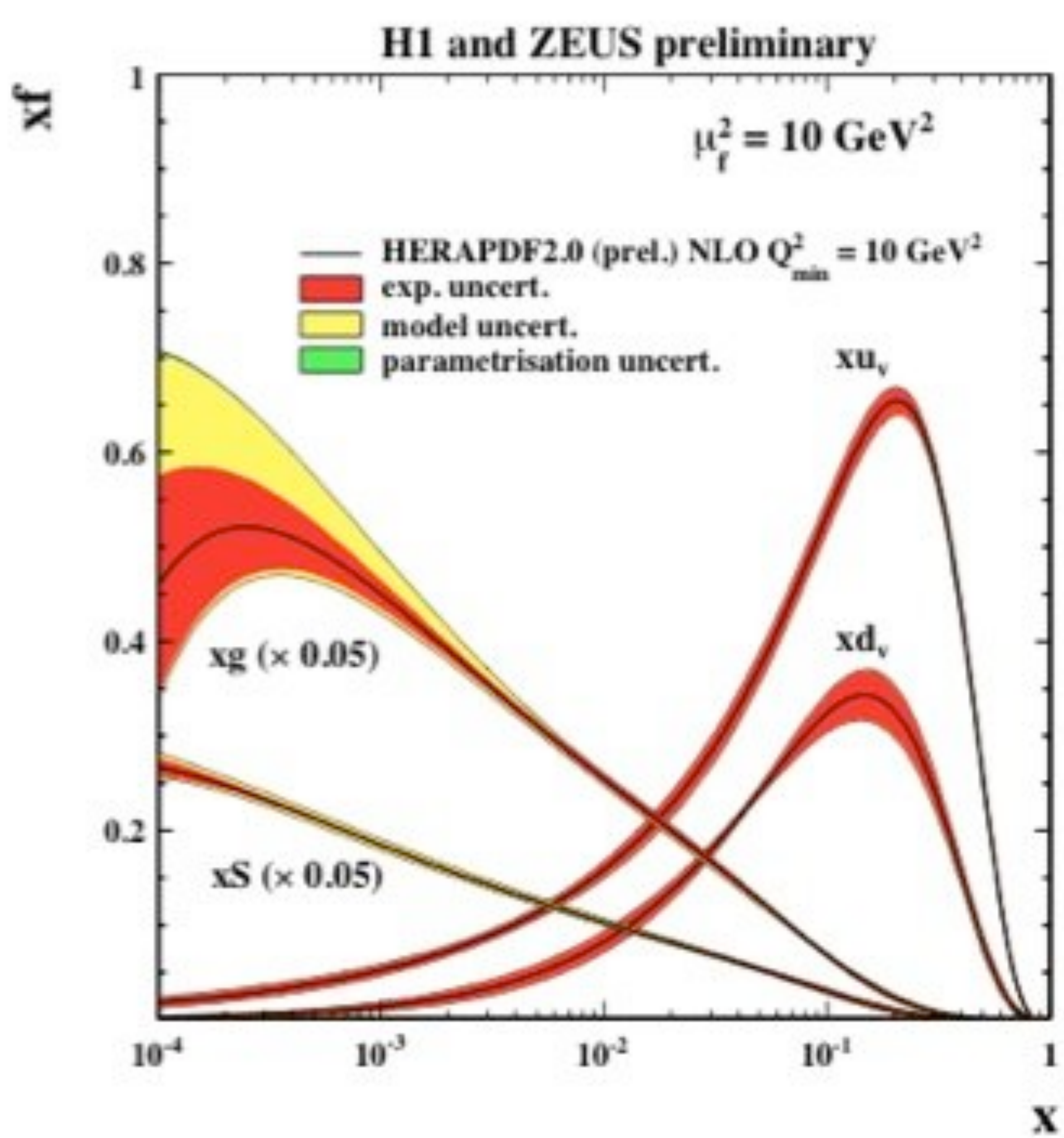
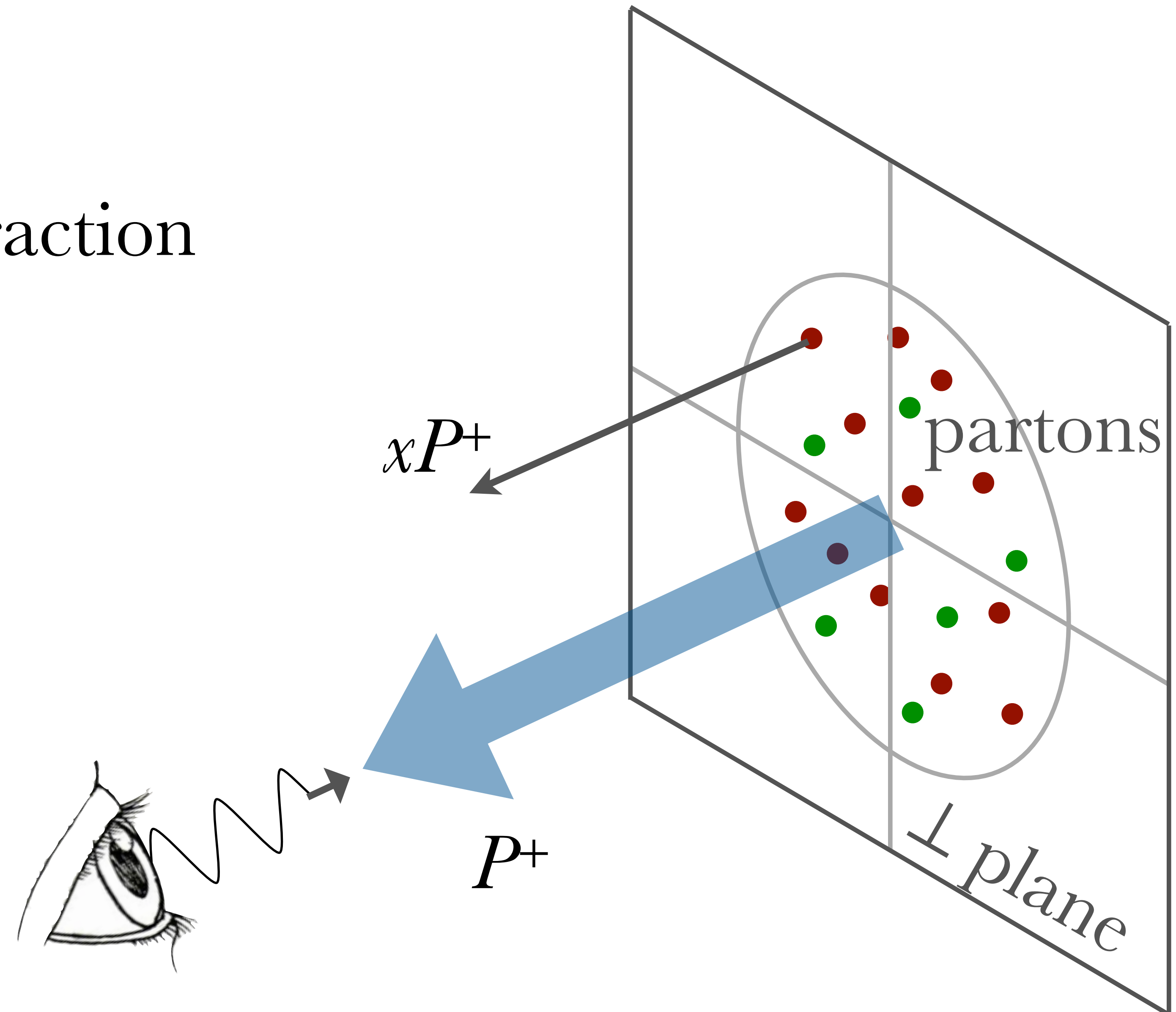
1-dim imaging





$$\sigma \sim f_a \otimes f_b \otimes \sigma_{ab}$$

The need for resummation

$$\sigma^{f.o.}(Q, Q_i) \sim \alpha_S^k \left[\sigma_{LO}(Q, Q_i) + \alpha_S \sigma_{NLO}(Q, Q_i) + \alpha_S^2 \sigma_{NNLO}(Q, Q_i) + \ldots\right]$$

The need for resummation

fixed order reliable only when $Q \sim Q_i \;\; \forall i$
otherwise, $\alpha_s \log(Q_i/Q)$ terms **spoil** perturbative results

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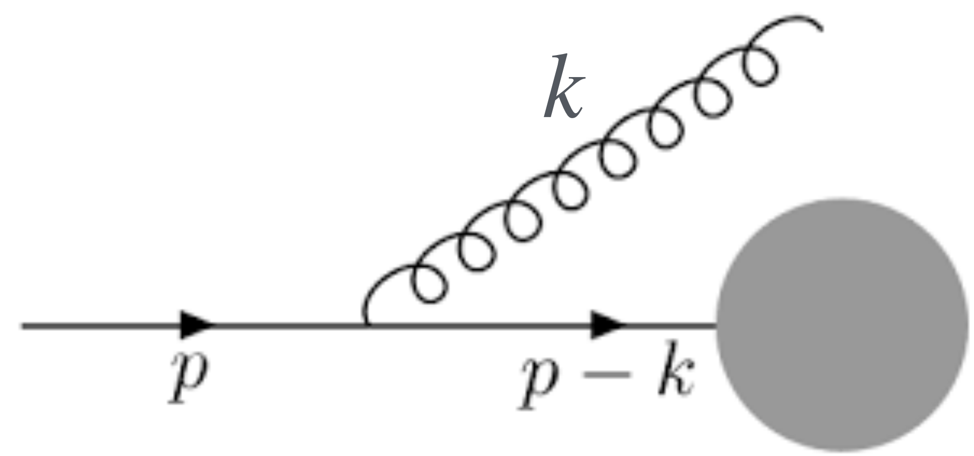
The origin of logs:



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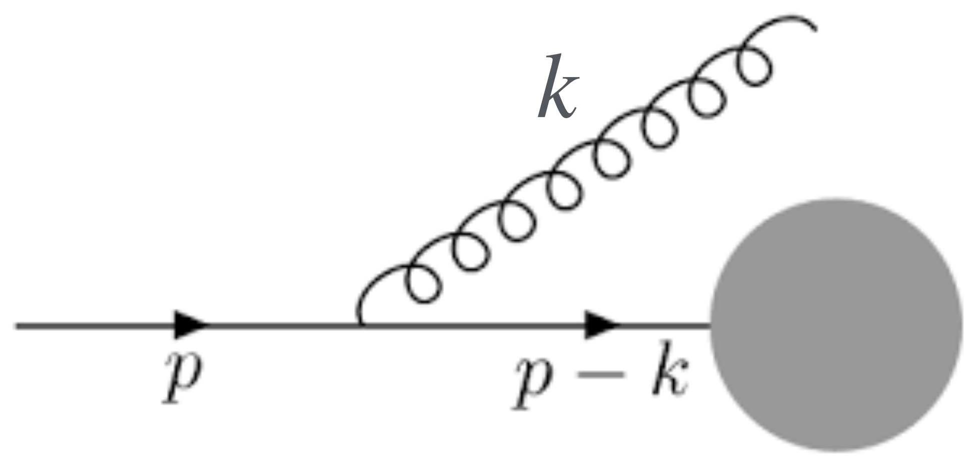
$$\frac{1}{(p-k)^2} = \frac{1}{-2p \cdot k} = \frac{1}{-2E_q E_g (1 - \cos\theta_{gq})}$$



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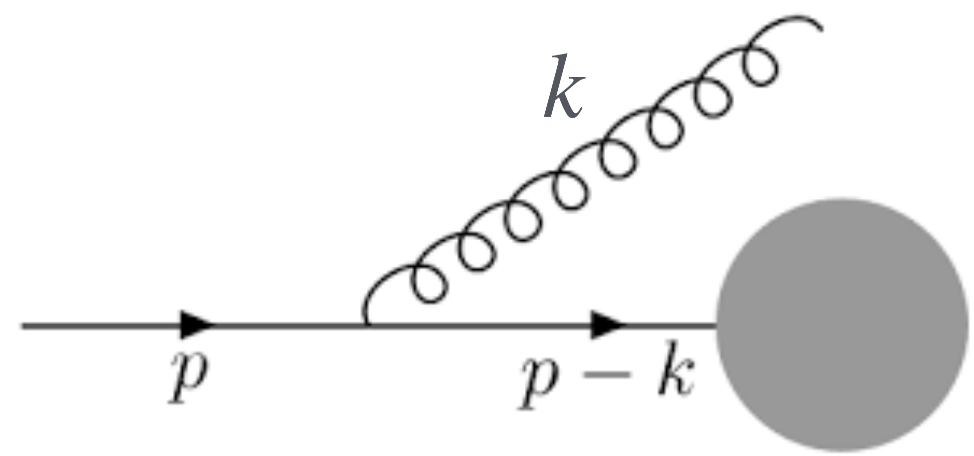
$E_g \to 0$ (soft) $\theta_{gq} \to 0$ (collinear)



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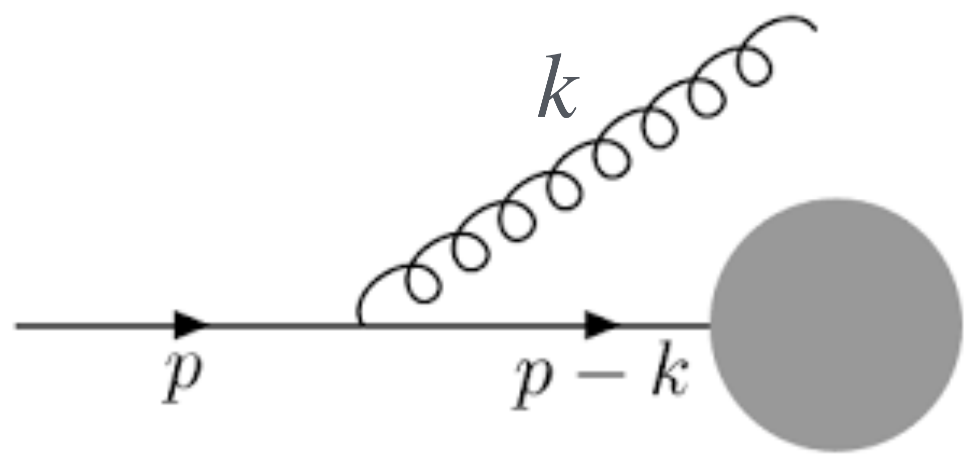

$\alpha_s L^2$	$\alpha_s L$	$\ldots$	$\ldots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\ldots$	$\mathcal{O}(\alpha_s^n)$	(N^nLO)
LL	NLL	NNLL	$\ldots$	$\ldots$	

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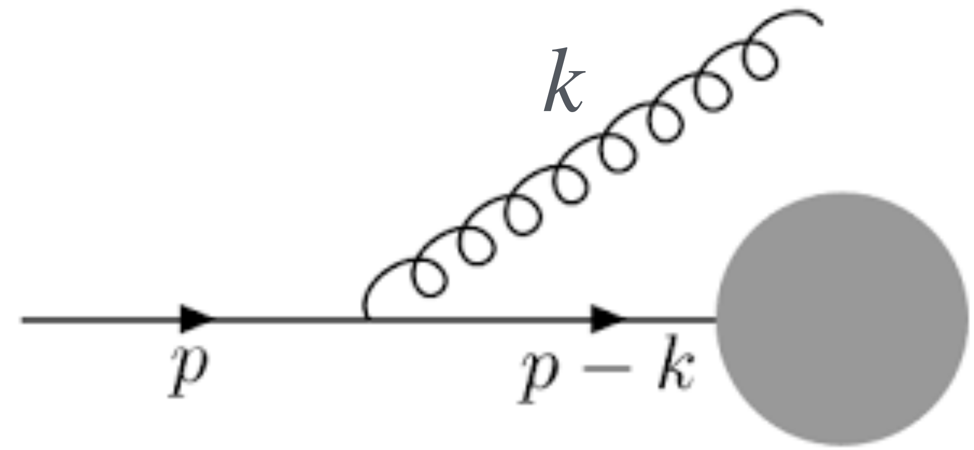

Resummation

$\alpha_s L^2$	$\alpha_s L$	$\dots$	$\dots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\dots$	$\mathcal{O}(\alpha_s^n)$	$(N^n LO)$
LL	NLL	NNLL	$\dots$	$\dots$	

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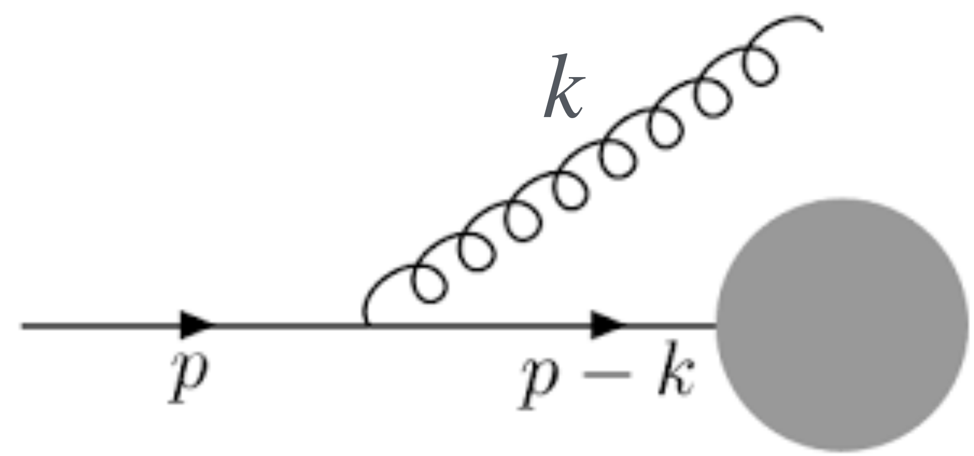

Resummation

- reorganisation into towers of logs

$\alpha_s L^2$	$\alpha_s L$	$\ldots$	$\ldots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\ldots$	$\mathcal{O}(\alpha_s^n)$	$(N^n LO)$
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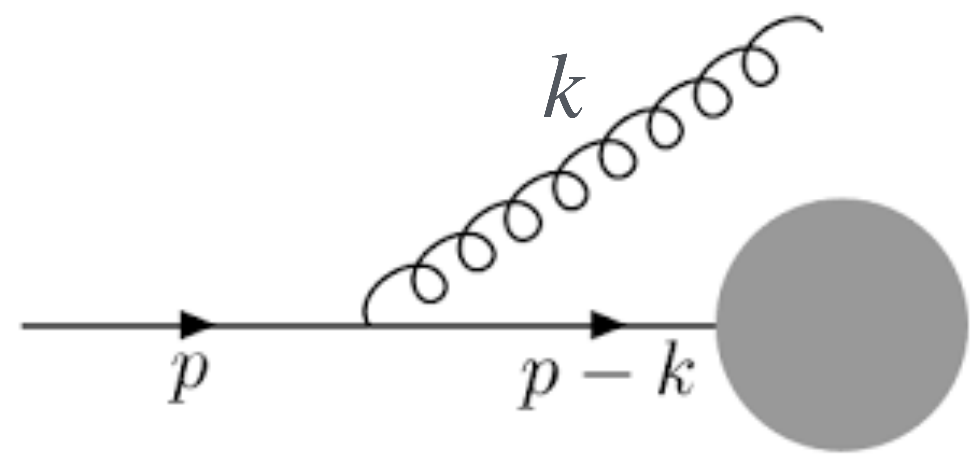

Resummation

- reorganisation into towers of logs
- all-order summation of each tower

$\alpha_s L^2$	$\alpha_s L$	$\ldots$	$\ldots$	$\mathcal{O}(\alpha_s)$	(LO)
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$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\ldots$	$\mathcal{O}(\alpha_s^n)$	$(N^n LO)$
LL	NLL	NNLL	$\ldots$	$\ldots$	

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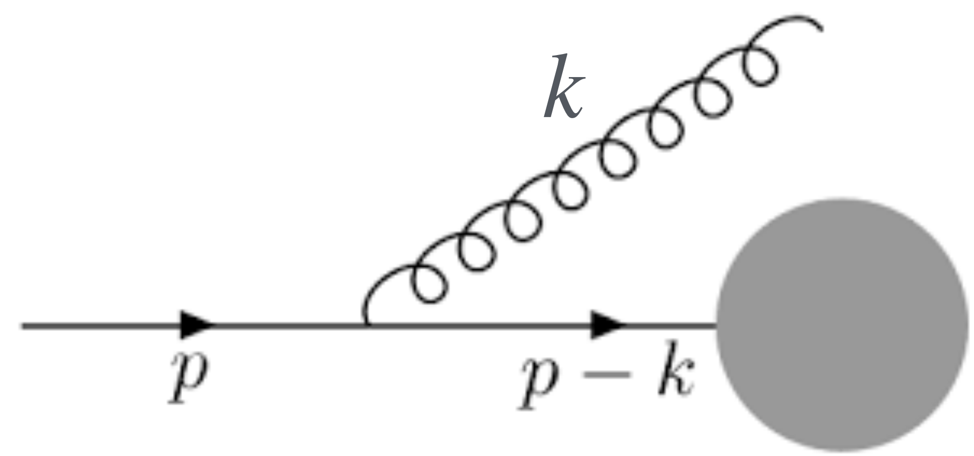

Resummation

- reorganisation into towers of logs
- all-order summation of each tower
- key-point: exponentiation!

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LL	NLL	NNLL	$\ldots$	$\ldots$	

$$\sigma^{res} \sim \alpha_S^k \sigma_{LO} \exp\left[L g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) + \ldots\right]$$

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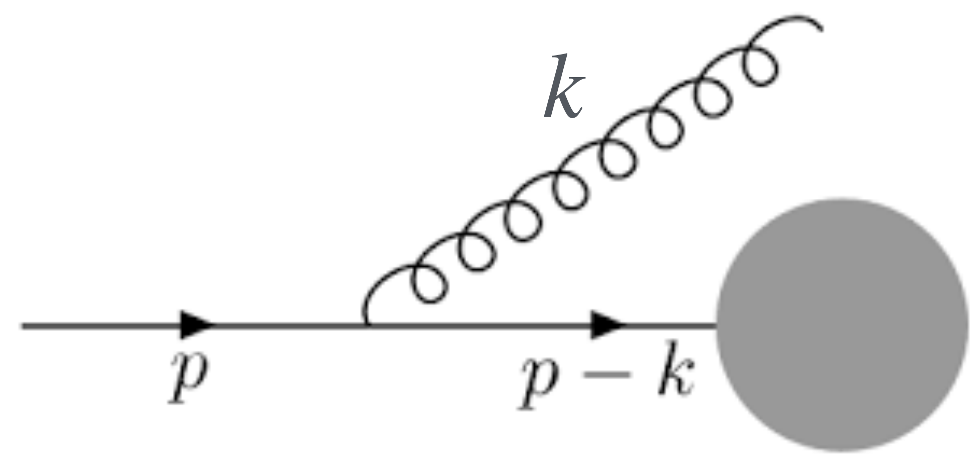

Resummation

- reorganisation into towers of logs
- all-order summation of each tower
- key-point: exponentiation!
- improved expansion

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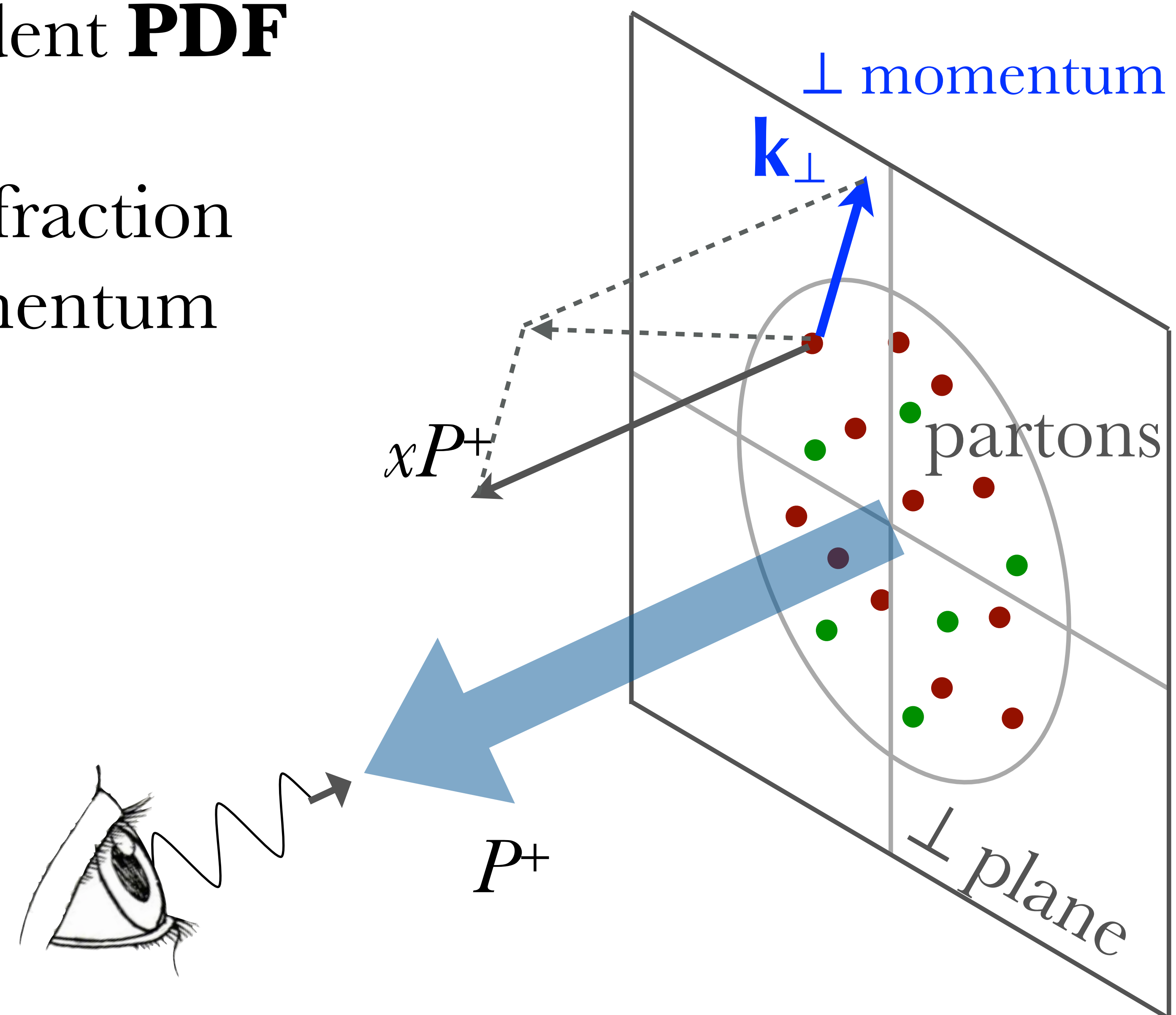
TMD PDF (FF)

Transverse **M**omentum **D**ependent **PDF**

$F(x, k_\perp)$ depend on:

x = longitudinal-momentum fraction

$k_\perp$ = *(intrinsic)* transverse-momentum



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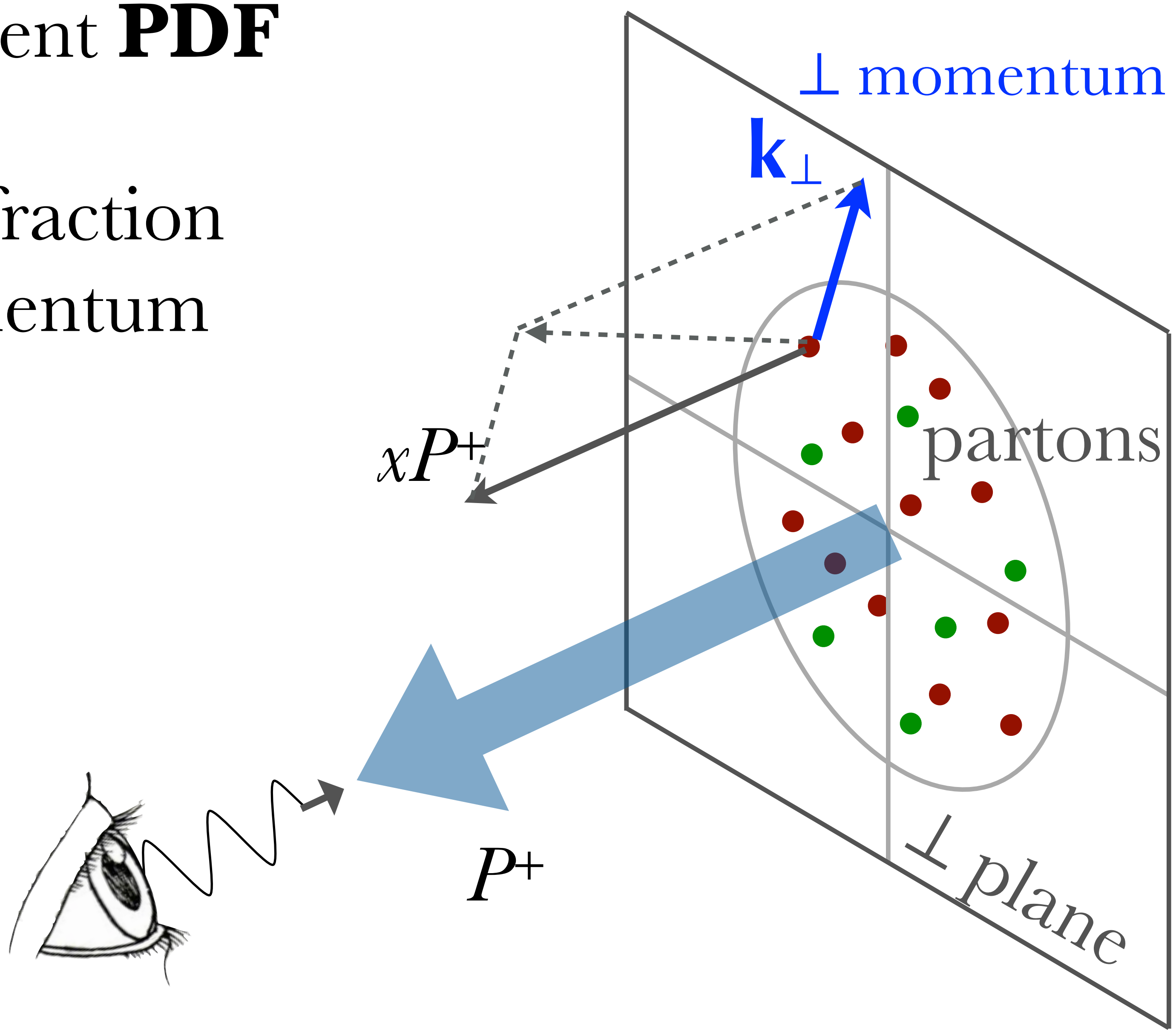

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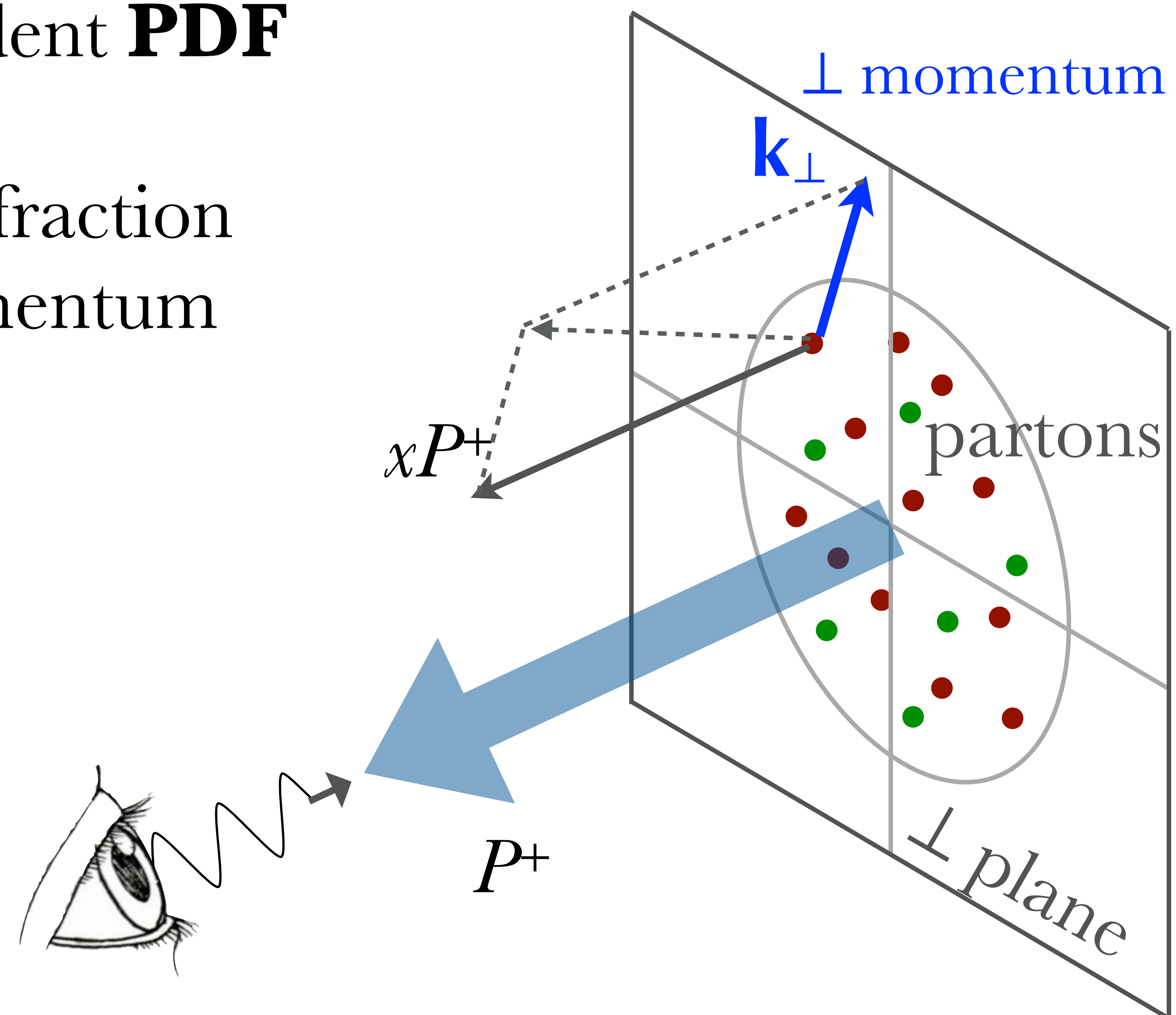

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I will focus on

<u>unpolarised</u> <u>quark</u> TMDs

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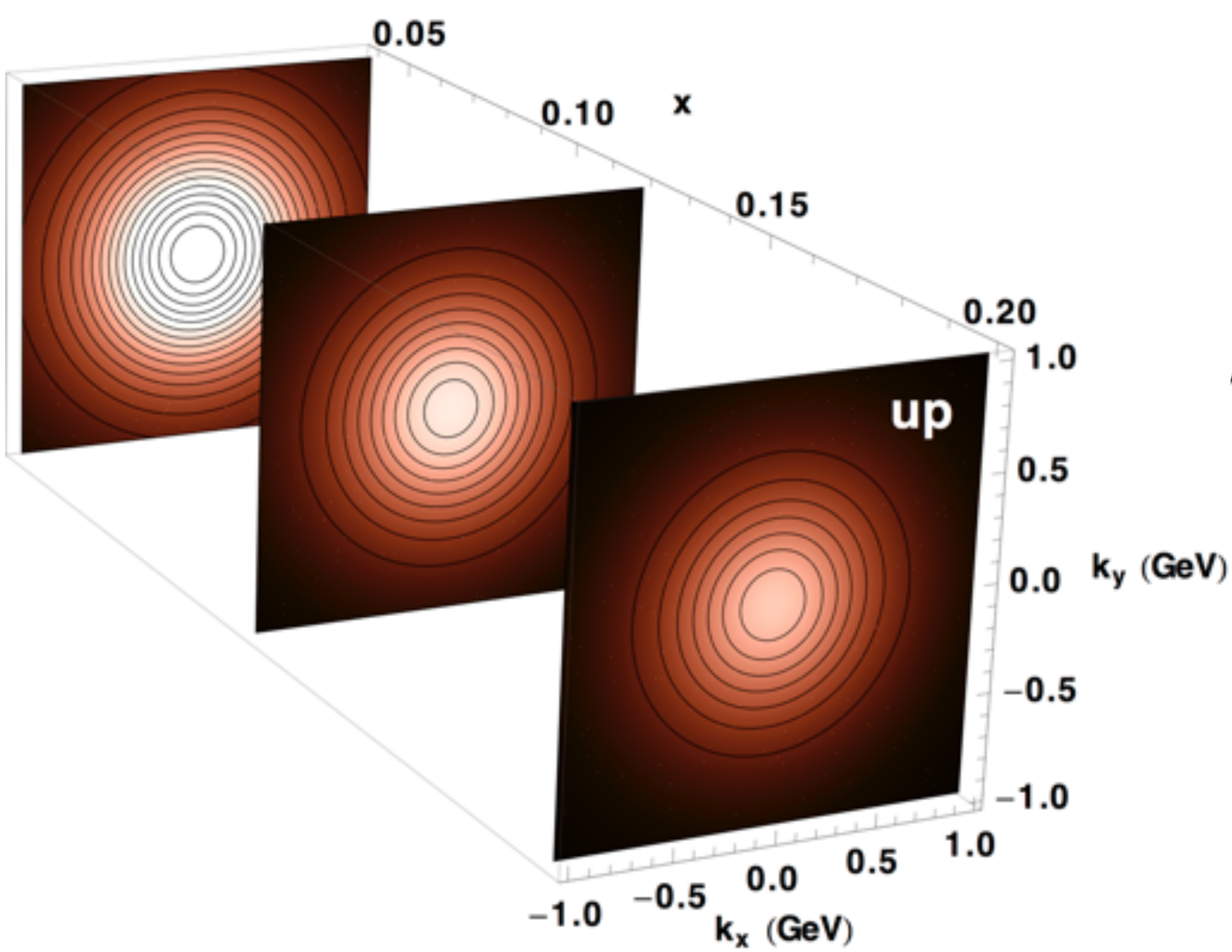

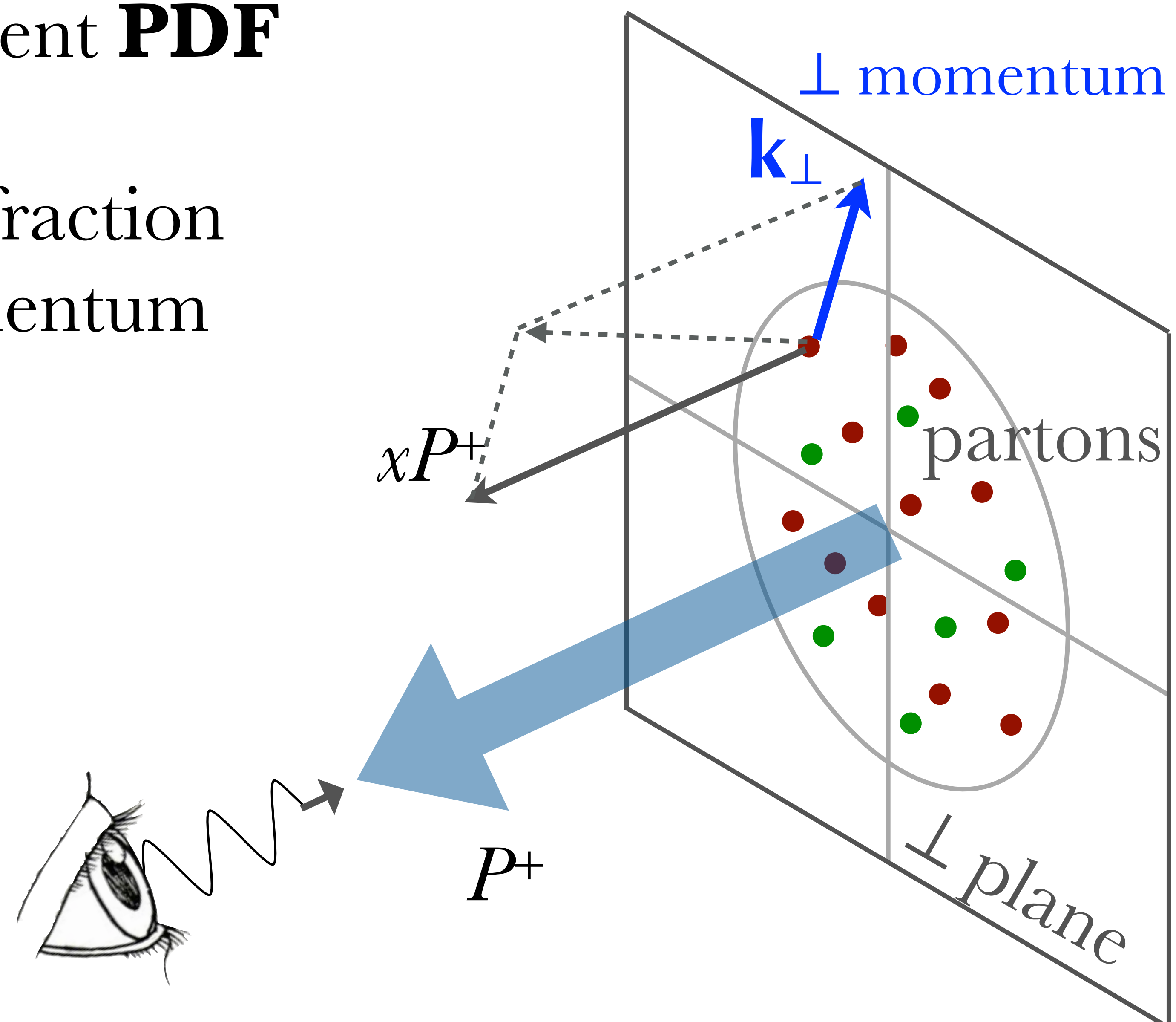

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Factorising processes

Processes for which TMD factorisation has been **proven**:

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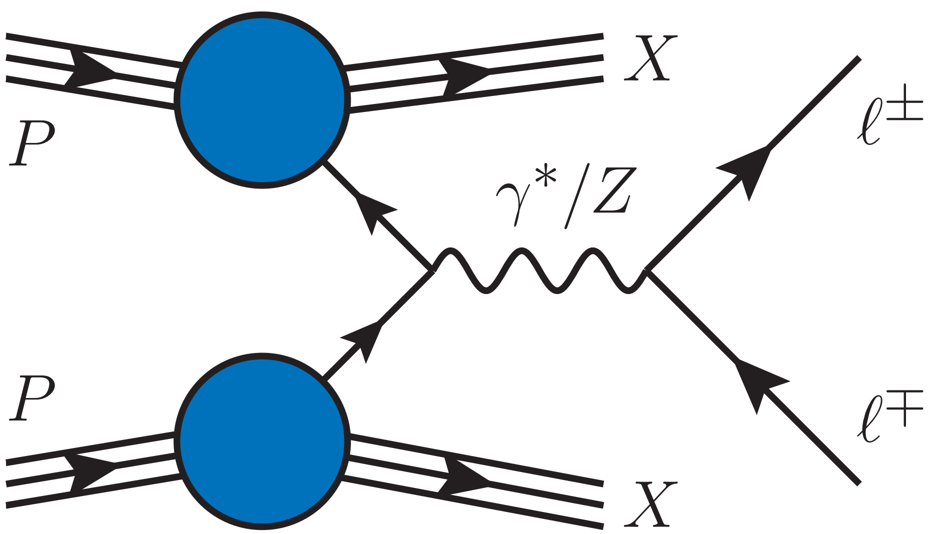
Drell-Yan



$PP \longrightarrow \ell^{\pm}\ell^{\mp}\, X$

- **Two** TMD **PDFs**
- Lots of data:
  - low-energy: FNAL
  - mid-energy: RHIC
  - high-energy: Tevatron, LHC

Factorising processes

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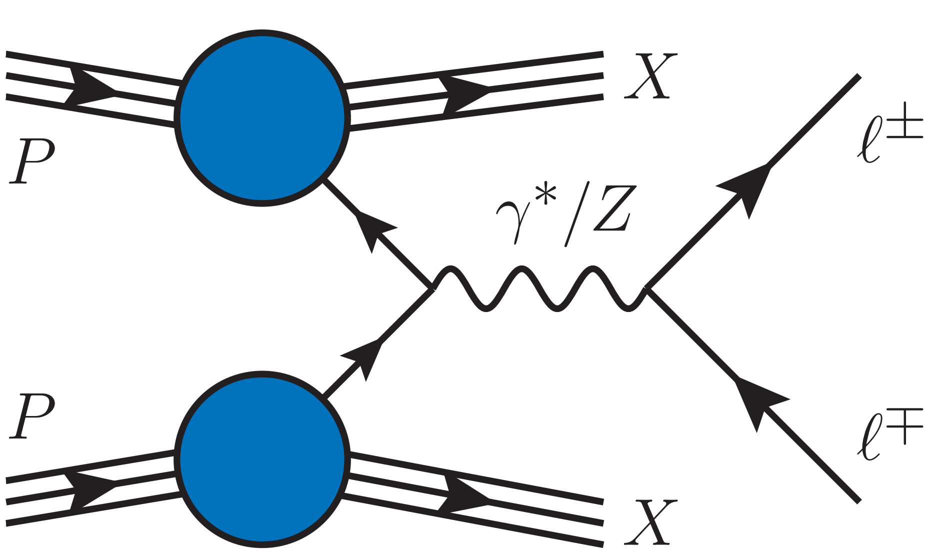
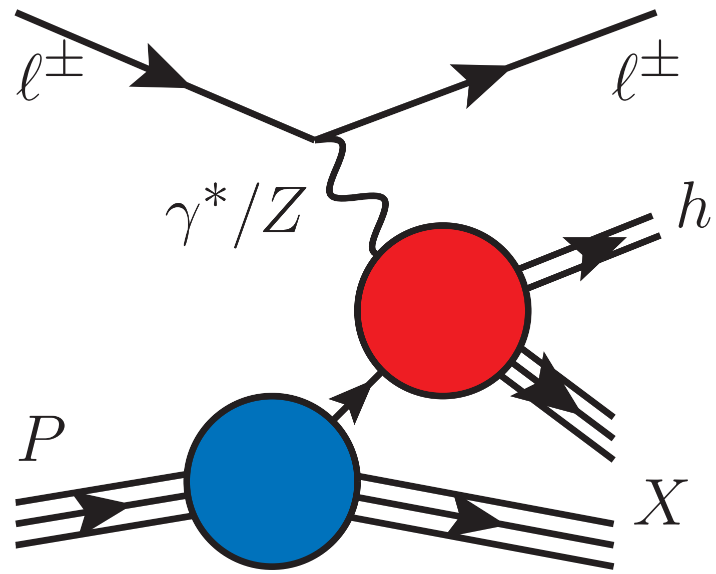


$PP \longrightarrow \ell^\pm \ell^\mp \; X$

Semi-inclusive DIS



$P\ell^\pm \longrightarrow \ell^\pm h \; X$

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  - mid-energy: RHIC
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- many precise data points:
  - HERMES at DESY
  - COMPASS at CERN

Factorising processes

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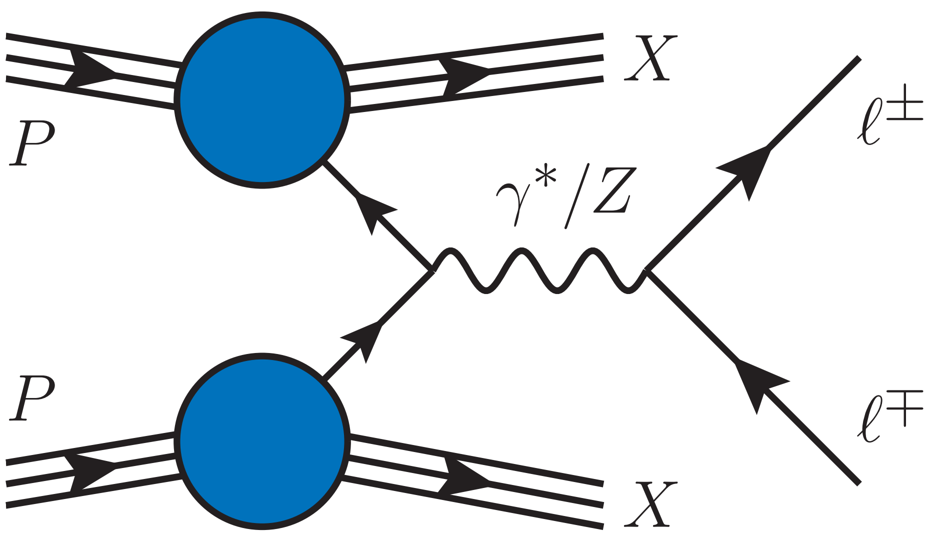

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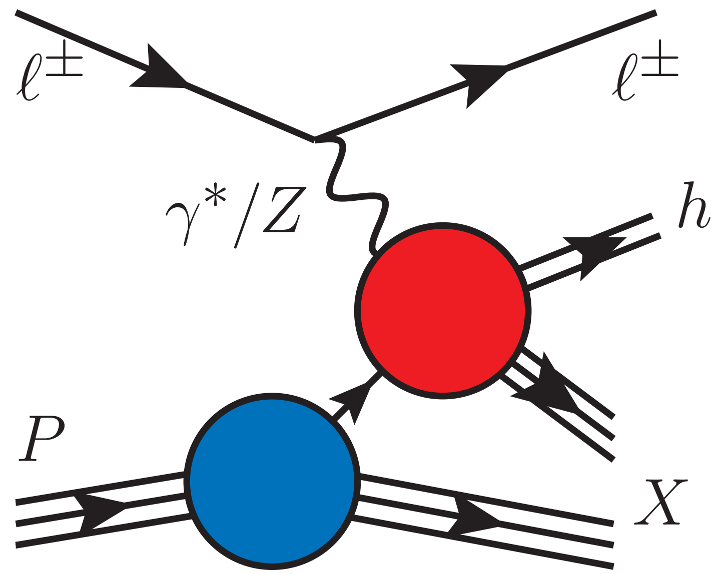

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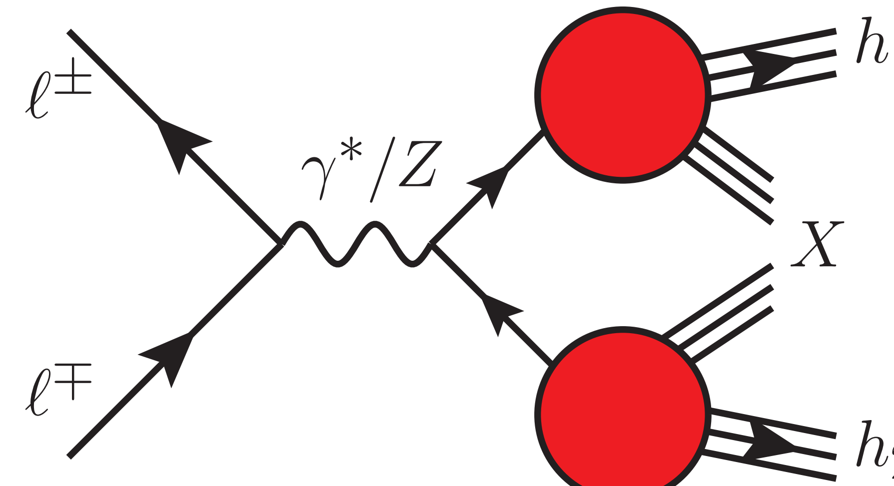
e^+e^- annihilation

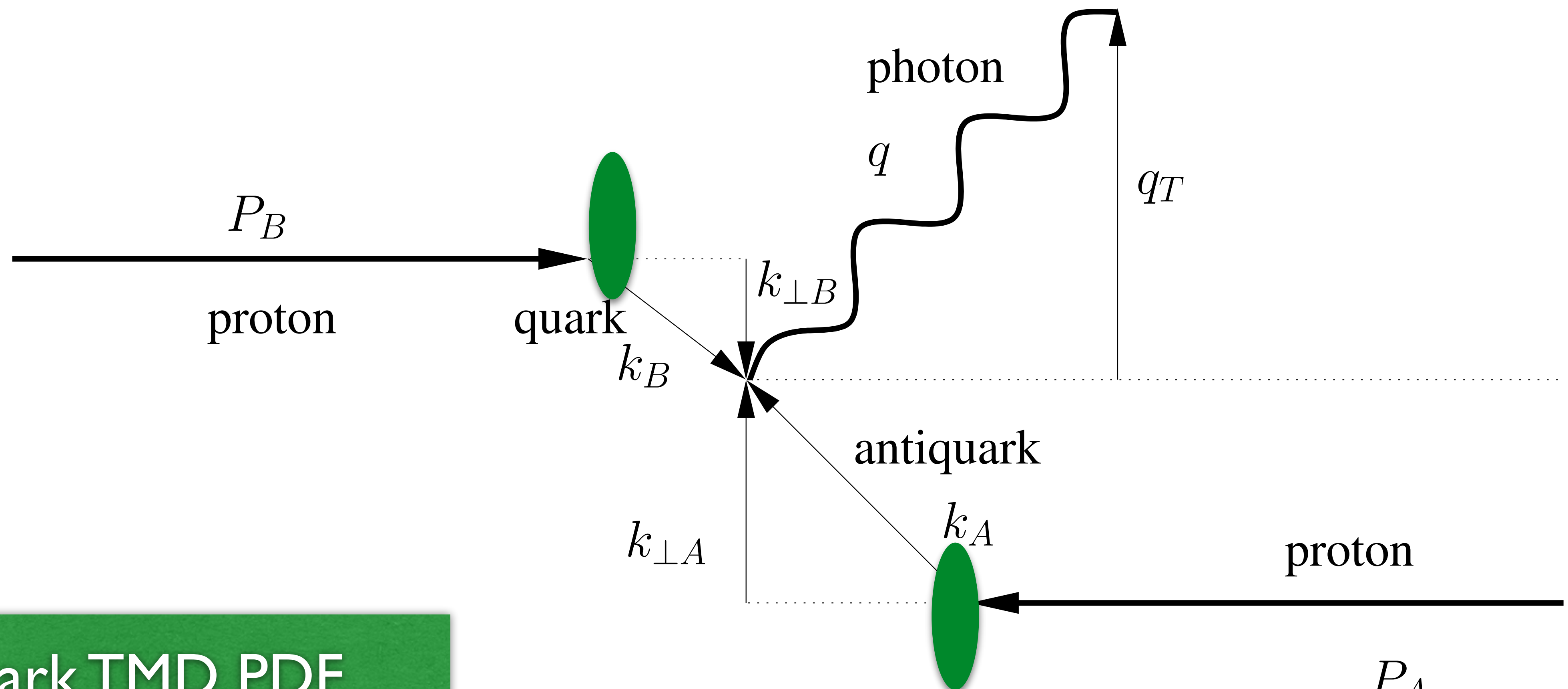


$$\ell^\pm \ell^\mp \rightarrow h_1 h_2 X$$

- **Two** TMD **FFs**
- di-hadron prod. from:
  - BELLE at KEK
  - BABAR at SLAC

missing!

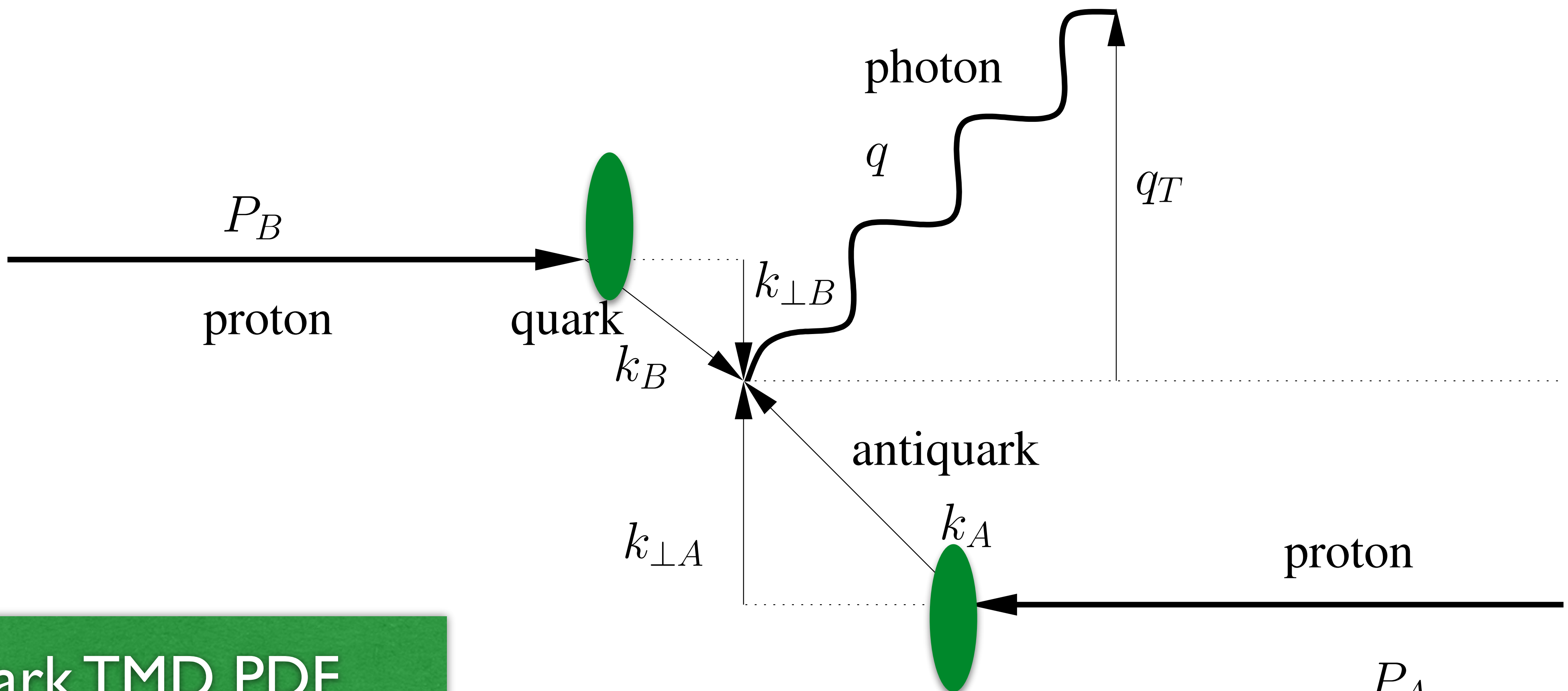
TMD factorisation for DY



quark TMD PDF

$$\frac{d\sigma}{dq_T\,dy\,dQ} \propto x_A x_B H^{DY}(Q,\mu) \sum_q c_q(Q^2) \int d^2\mathbf{k}_{\perp A}\, d^2\mathbf{k}_{\perp B}\, F^{\bar{q}}(x_A, \mathbf{k}_{\perp A}^2; \mu, \zeta_A)\, F^{q}(x_B, \mathbf{k}_{\perp B}^2; \mu, \zeta_B)\, \delta^{(2)}(\mathbf{k}_{\perp A} + \mathbf{k}_{\perp B} - \mathbf{q}_T)$$

TMD factorisation for DY



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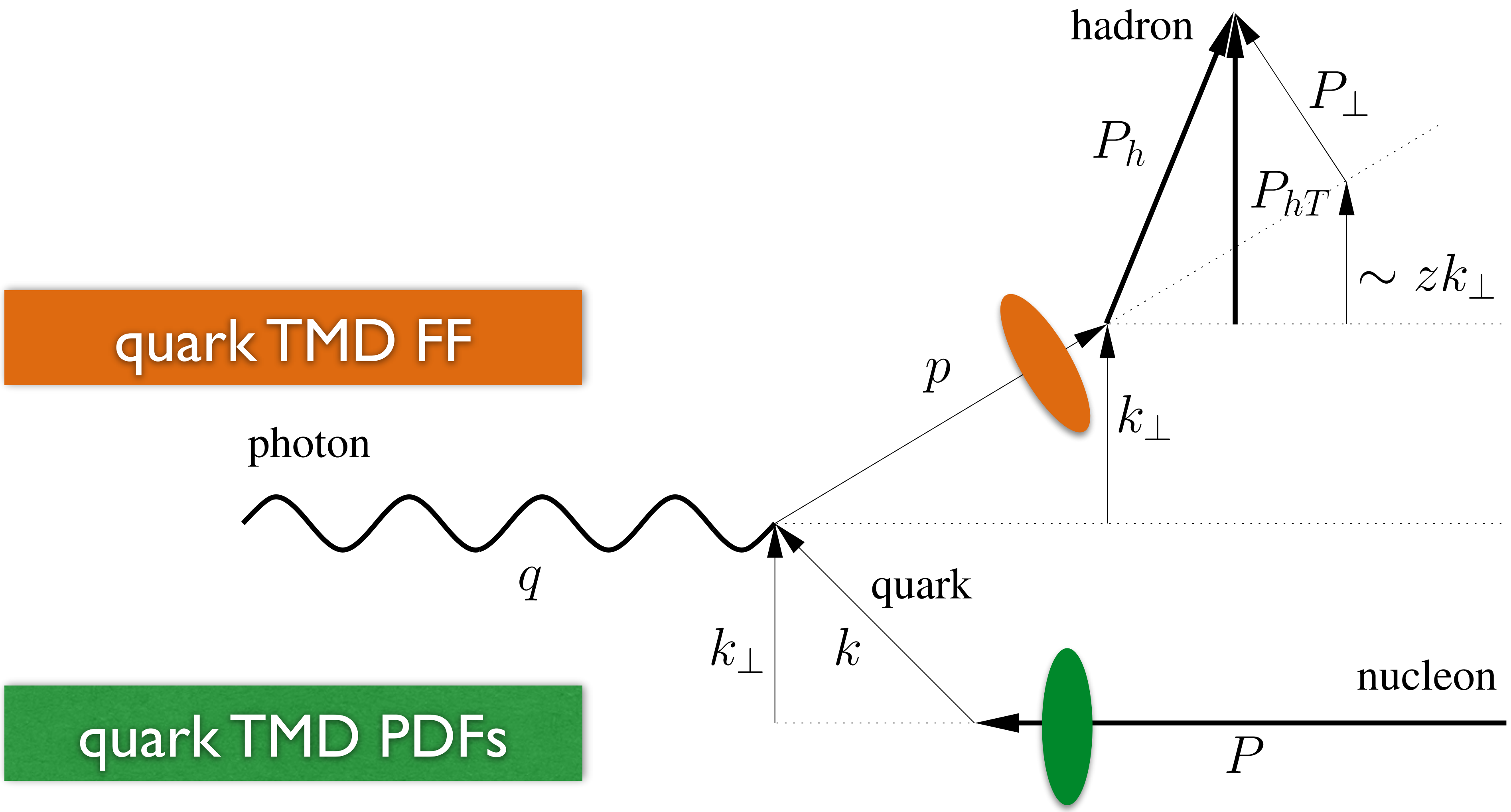
$$= x_A x_B H^{DY}(Q,\mu) \sum_q c_q(Q^2) \int \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \hat{F}^{\bar{q}}(x_A, b_T^2; \mu, \zeta_A)\, \hat{F}^q(x_B, b_T^2; \mu, \zeta_B)$$

TMD factorisation for SIDIS

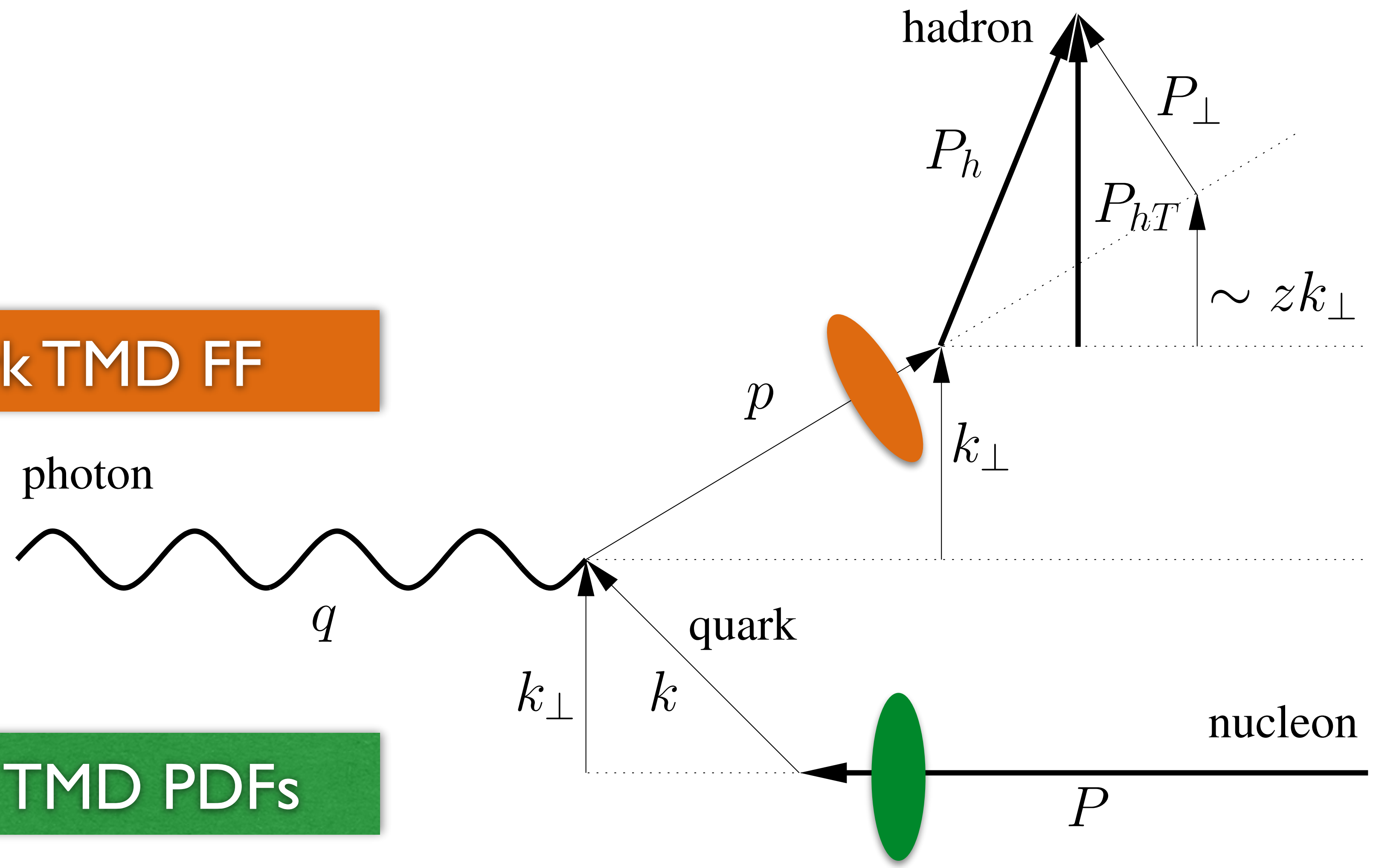


$$\frac{d\sigma}{dx\,dz\,dq_T\,dQ} \propto xH^{SIDIS}(Q,\mu)\sum_q e_q(Q^2)\int d^2\mathbf{k}_\perp \int \frac{d^2\mathbf{P}_\perp}{z^2}\, F^q(x,\mathbf{k}_\perp^2;\mu,\zeta_A)\, D^{q\to h}(z,\mathbf{P}_\perp^2;\mu,\zeta_B)\;\delta^{(2)}(\mathbf{k}_\perp + \mathbf{P}_\perp/z + \mathbf{q}_T)$$

TMD factorisation for SIDIS

quark TMD FF

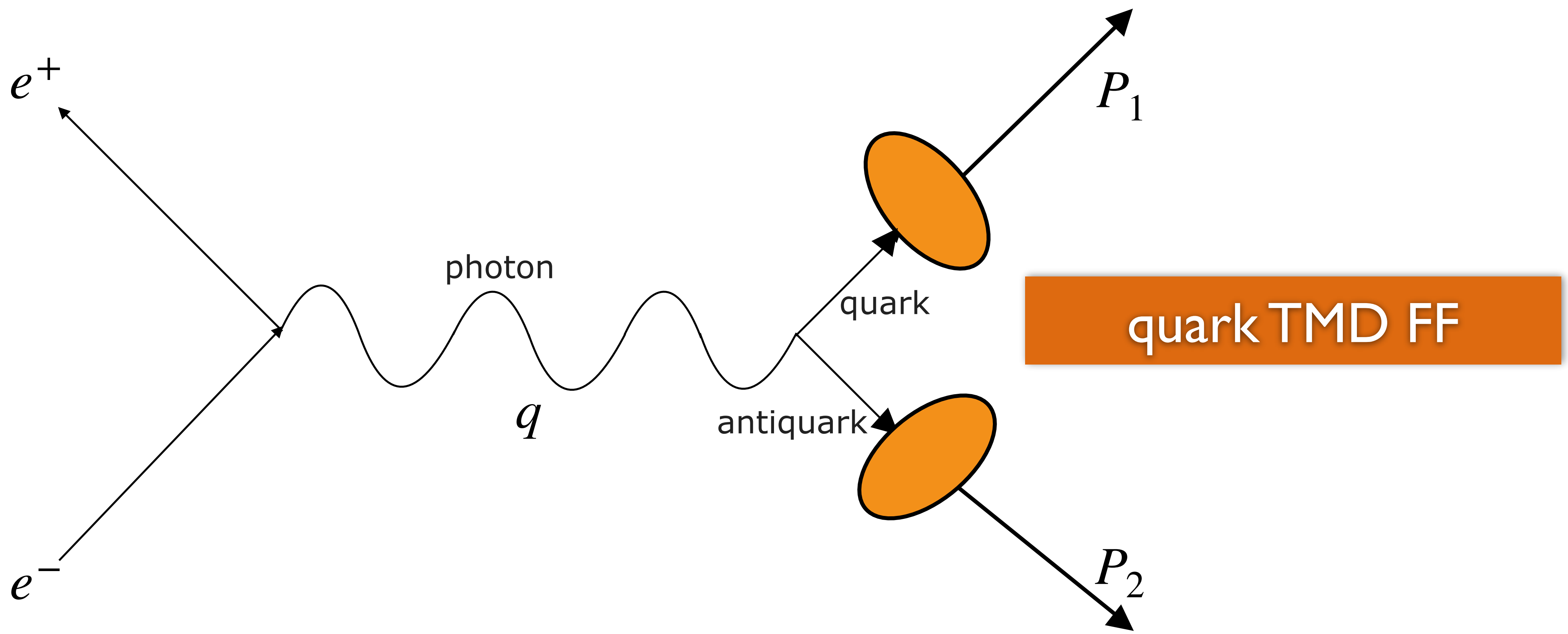
quark TMD PDFs



$$\frac{d\sigma}{dx\,dz\,dq_T\,dQ} \propto xH^{SIDIS}(Q,\mu)\sum_q e_q(Q^2)\int d^2\mathbf{k}_\perp \int \frac{d^2\mathbf{P}_\perp}{z^2}\, \boxed{F^q(x,\mathbf{k}_\perp^2;\mu,\zeta_A)}\, \boxed{D^{q\to h}(z,\mathbf{P}_\perp^2;\mu,\zeta_B)}\;\delta^{(2)}(\mathbf{k}_\perp + \mathbf{P}_\perp/z + \mathbf{q}_T)$$

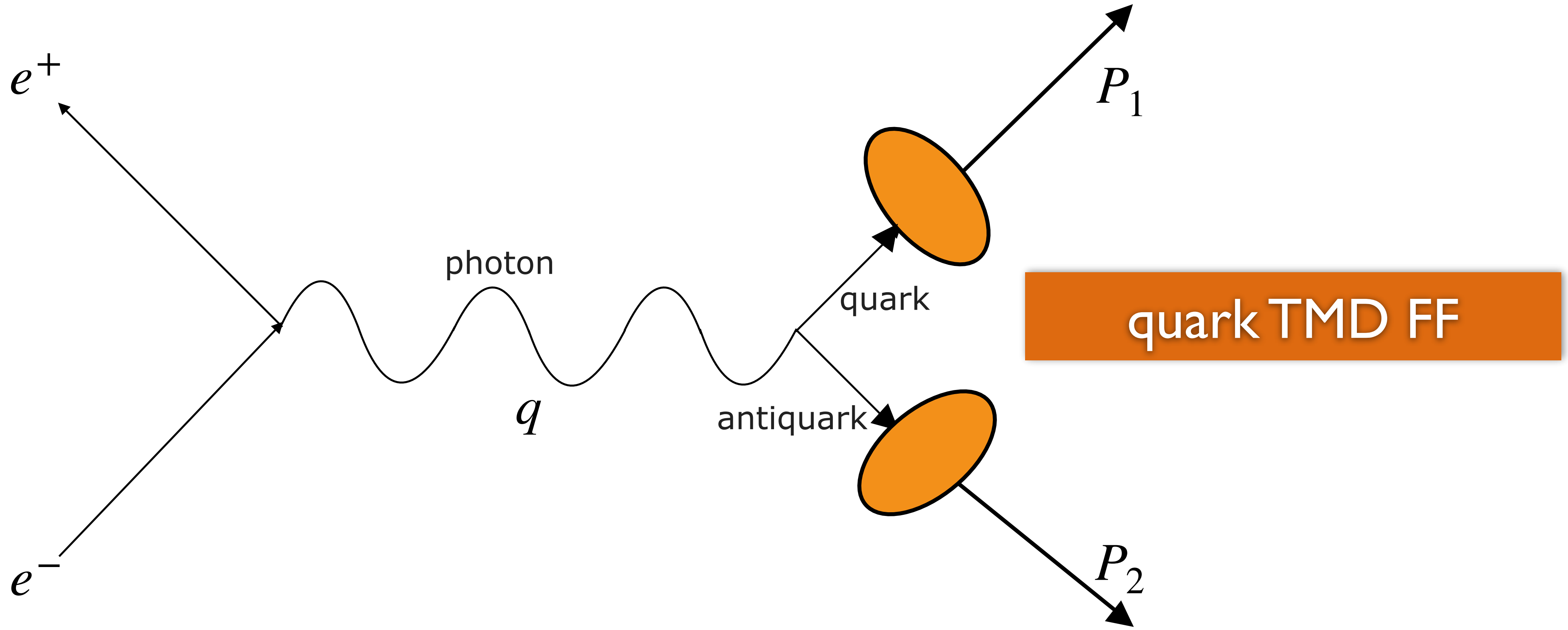
$$= xH^{SIDIS}(Q,\mu)\sum_q e_q(Q^2)\int \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \boxed{\hat{F}^{\bar{q}}(x,b_T^2;\mu,\zeta_A)}\, \boxed{\hat{D}^q(z,b_T^2;\mu,\zeta_B)}$$

TMD factorisation for DIA



$$\frac{d\sigma}{dq_T\, dy\, dQ} \propto A(y)\, H^{DY}(Q,\mu) \sum_q c_q(Q^2) \int \frac{d^2\mathbf{P}_{\perp 1}}{z_1^2} \frac{d^2\mathbf{P}_{\perp 2}}{z_2^2}\, D^{\bar{q}}(z_A, \mathbf{P}^2_{\perp 1}; \mu, \zeta_1)\, D^{q}(z_B, \mathbf{P}^2_{\perp 2}; \mu, \zeta_2)\, \delta^{(2)}(\mathbf{P}_{\perp 1}/z_1 + \mathbf{P}_{\perp 2}/z_2 + \mathbf{q}_T)$$

TMD factorisation for DIA



$$\frac{d\sigma}{dq_T\,dy\,dQ} \propto A(y)\,H^{DY}(Q,\mu)\sum_q c_q(Q^2)\int \frac{d^2\mathbf{P}_{\perp 1}}{z_1^2}\,\frac{d^2\mathbf{P}_{\perp 2}}{z_2^2}\,\boxed{D^{\bar{q}}(z_A,\mathbf{P}^2_{\perp 1};\mu,\zeta_1)}\,\boxed{D^{q}(z_B,\mathbf{P}^2_{\perp 2};\mu,\zeta_2)}\;\delta^{(2)}(\mathbf{P}_{\perp 1}/z_1+\mathbf{P}_{\perp 2}/z_2+\mathbf{q}_T)$$

$$= A(y)\,H^{DY}(Q,\mu)\sum_q c_q(Q^2)\int \frac{db_T}{2\pi}\,b_T\,J_0(b_Tq_T)\,\boxed{\hat{D}^{\bar{q}}(z_1,b_T^2;\mu,\zeta_1)}\,\boxed{\hat{D}^{q}(z_2,b_T^2;\mu,\zeta_2)}$$

TMD structure

$$
\begin{aligned}
F_{f/P}(x,\mathbf{b}_T;\mu,\zeta) &= \sum_j C_{f/j}(x,b_*;\mu_b,\zeta_F)\otimes f_{j/P}(x,\mu_b) && :A \\
&\times \exp\left\{K(b_*;\mu_b)\ln\frac{\sqrt{\zeta_F}}{\mu_b}+\int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F-\gamma_K\ln\frac{\sqrt{\zeta_F}}{\mu'}\right]\right\} && :B \\
&\times \exp\left\{g_{j/P}(x,b_T)+g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\} && :C
\end{aligned}
$$

TMD structure

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

$$\times \quad \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \qquad : B$$

$$\times \quad \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \qquad : C$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

TMD structure

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \quad : A$$

$$\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \quad : B$$

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- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
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- CS and RGE evolution to large b_T
- **perturbative**

TMD structure

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \quad : A$$

$$\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \quad : B$$

$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \quad : C$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL	2	3	4	NNLO

- CS and RGE evolution to large b_T
- **perturbative**

TMD structure

$$F_{f/P}(x,\mathbf{b}_T;\mu,\zeta) = \sum_j C_{f/j}(x,b_*;\mu_b,\zeta_F) \otimes f_{j/P}(x,\mu_b) \qquad : A$$

$$\times \exp\left\{K(b_*;\mu_b)\ln\frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta_F}}{\mu'}\right]\right\} \qquad : B$$

$$\times \exp\left\{g_{j/P}(x,b_T) + g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\} \qquad : C$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
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Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
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N^3LL	2	3	4	NNLO

- CS and RGE evolution to large b_T
- **perturbative**

$N^3LL^- = N^3LL$ with NLO FF

TMD structure

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

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$$\times \exp\left\{g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\} \qquad : C$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

$$(\mu_b = 2e^{-\gamma_E}/b_*)$$

- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole

$N^3LL^- = N^3LL$ with NLO FF

TMD structure

$$F_{f/P}(x,\mathbf{b}_T;\mu,\zeta) = \sum_j C_{f/j}(x,b_*;\mu_b,\zeta_F) \otimes f_{j/P}(x,\mu_b) \quad : A$$

$$\times \exp\left\{K(b_*;\mu_b)\ln\frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta_F}}{\mu'}\right]\right\} \quad : B$$

$$\times \exp\left\{g_{j/P}(x,b_T) + g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\} \quad : C$$

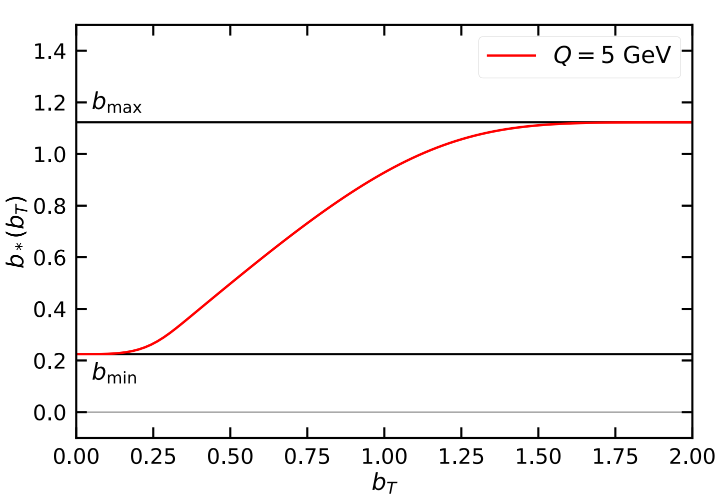
- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

$$\left(\mu_b = 2e^{-\gamma_E}/b_*\right)$$

$$b_*(b) = b_{\max}\left(\frac{1-\exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$



- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole

$N^3LL^- = N^3LL$ with NLO FF

TMD structure

$$F_{f/P}(x,\mathbf{b}_T;\mu,\zeta) = \sum_j C_{f/j}(x,b_*;\mu_b,\zeta_F) \otimes f_{j/P}(x,\mu_b) \qquad : A$$

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$$\times \exp\left\{g_{j/P}(x,b_T) + g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\} \qquad f_{NP} \qquad : C$$

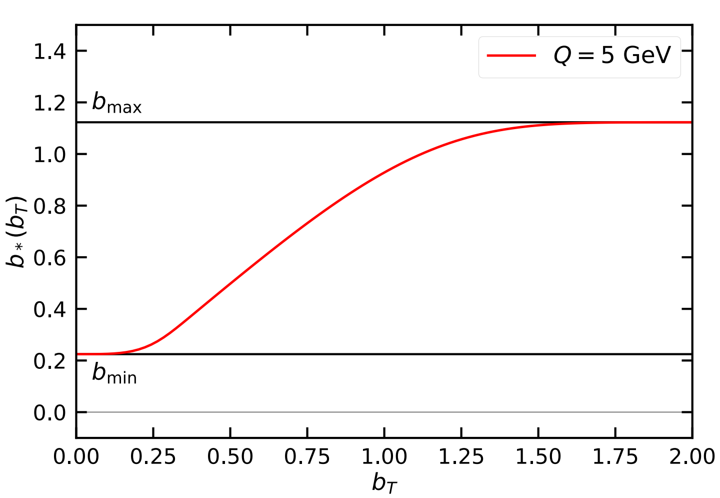
- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

$$\left(\mu_b = 2e^{-\gamma_E}/b_*\right)$$

$$b_*(b) = b_{\max}\left(\frac{1-\exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$



- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole
- f_{NP} "parametrises" the **non-perturbative** transverse modes
- **fit** f_{NP} to data

$N^3LL^- = N^3LL$ with NLO FF

TMD global fits

	Accuracy	HERMES	COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✔	✔	✔	✔	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✔	✔	✔	✔	2031	1.06

Non-perturbative: f_{NP}

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- depends on choice of b* and collinear PDFs
- requires definition of a functional form

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- depends on choice of b* and collinear PDFs
- requires definition of a functional form

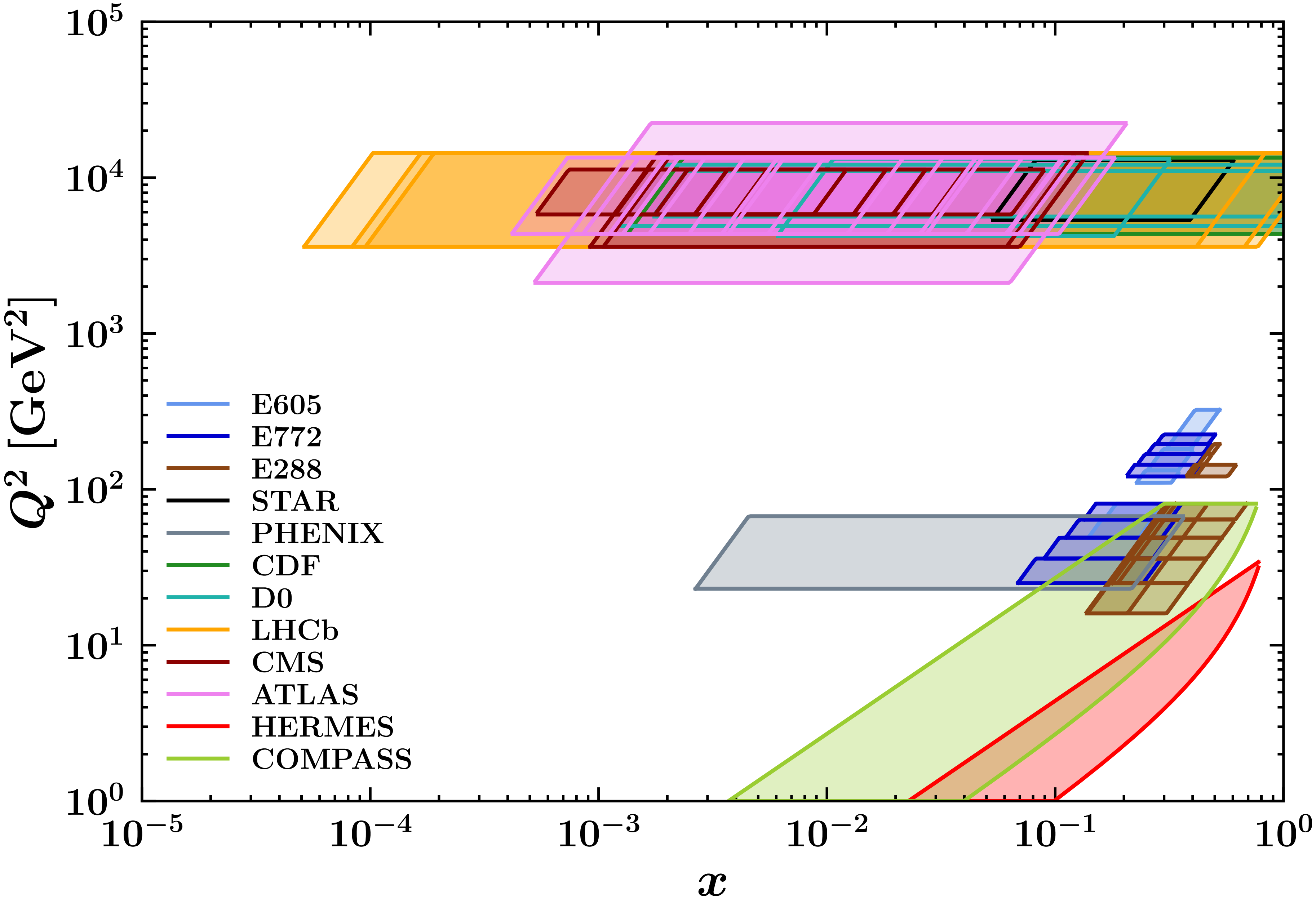
Our functional form

$$f_{1NP}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_T^2}{g1}} + \lambda^2 k_T^2 e^{-\frac{k_T^2}{g1B}} + \lambda_2^2 e^{-\frac{k_T^2}{g1C}} \right)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha^2_{\{1,2,3\}}}}$$

- similar form for TMD FF
- 21 free parameters

Datasets

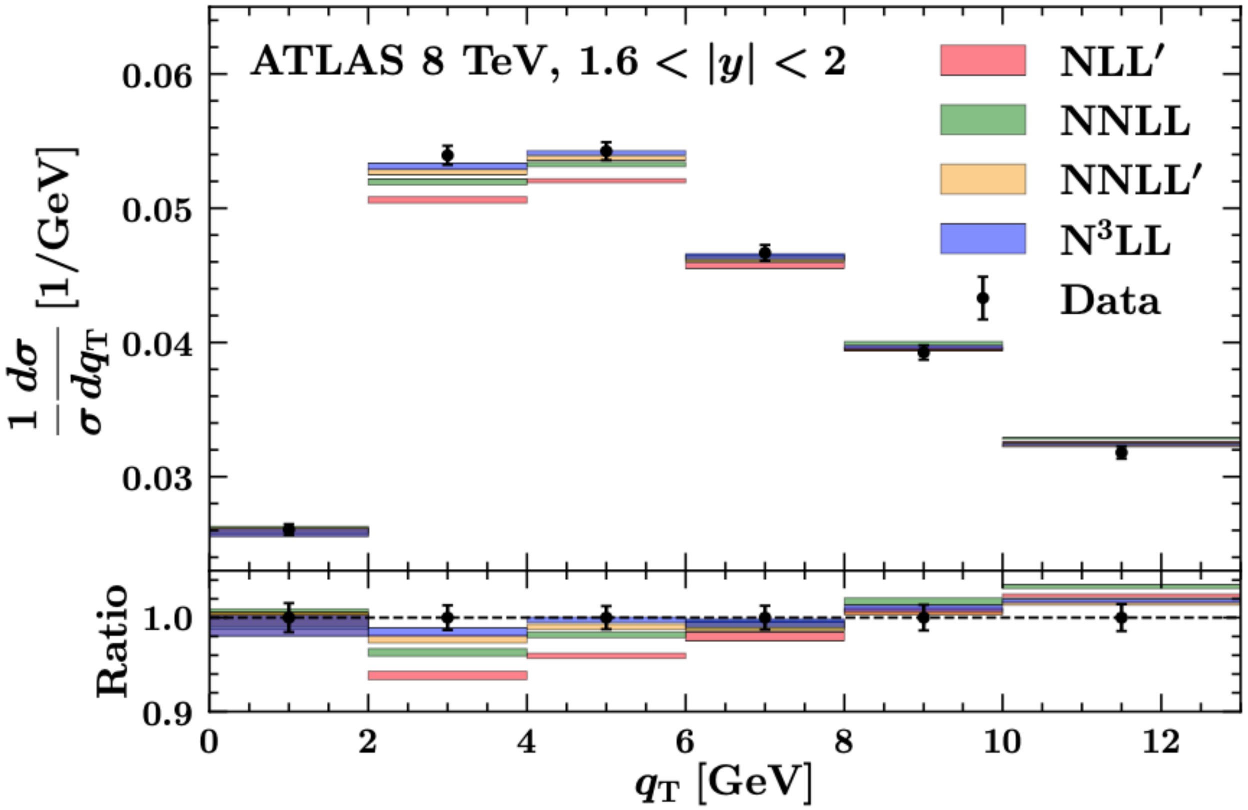


Cuts on kinematics

- $\langle Q \rangle > 1.3$ GeV
- $0.2 < \langle z \rangle < 0.7$
- $q_T/Q \leq 0.2$ (Drell-Yan)
- $P_{hT}|_{max} = min[min[\,0.2Q,\, 0.5zQ\,] + 0.3\text{ GeV},\, zQ\,]$ (SIDIS)

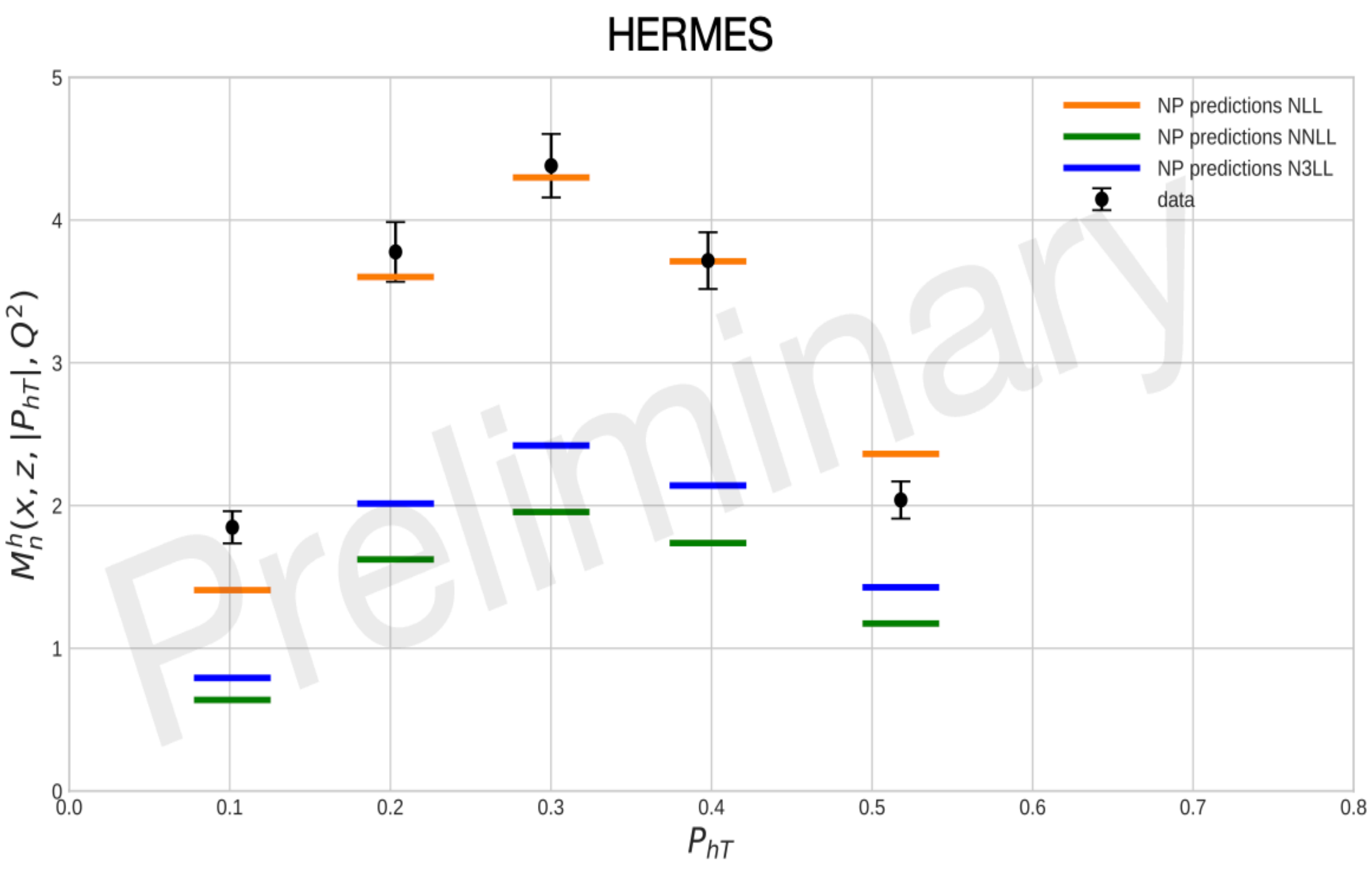
SIDIS normalisation

DY beyond NLL

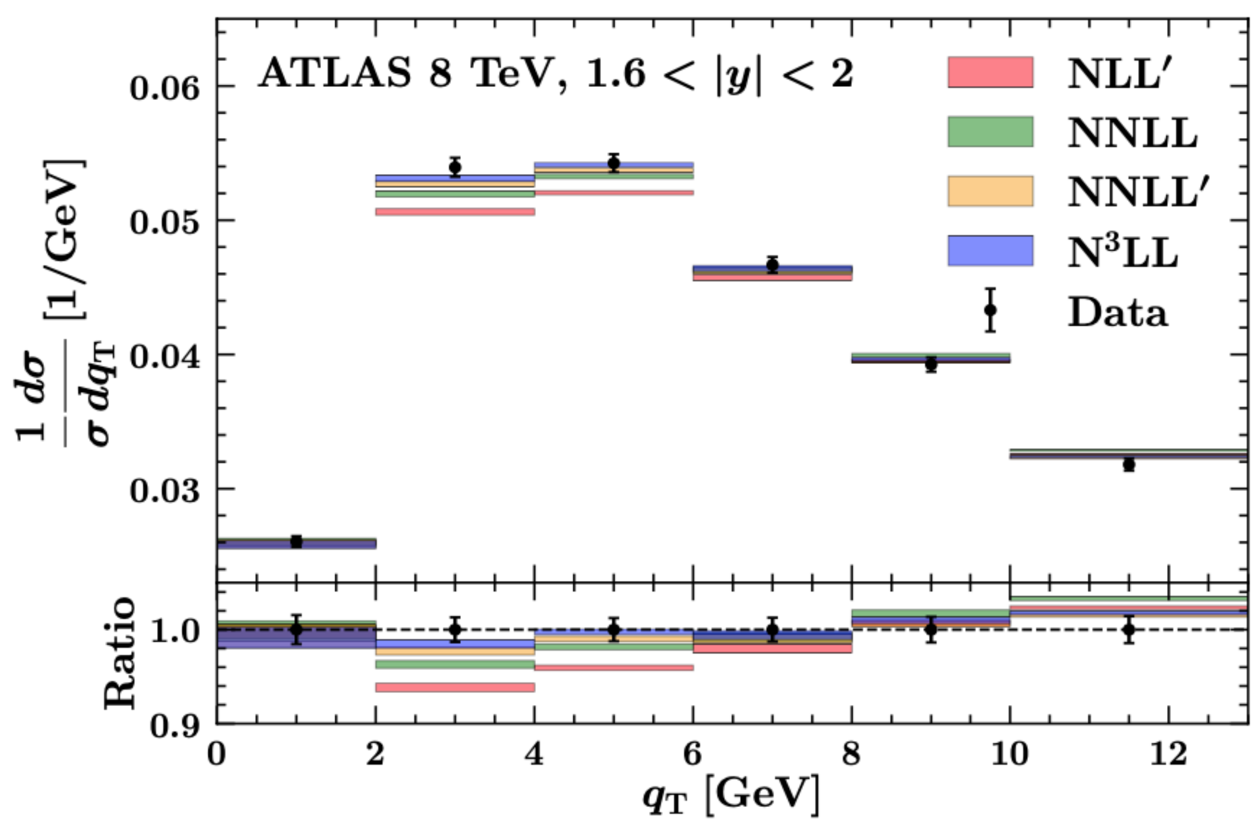


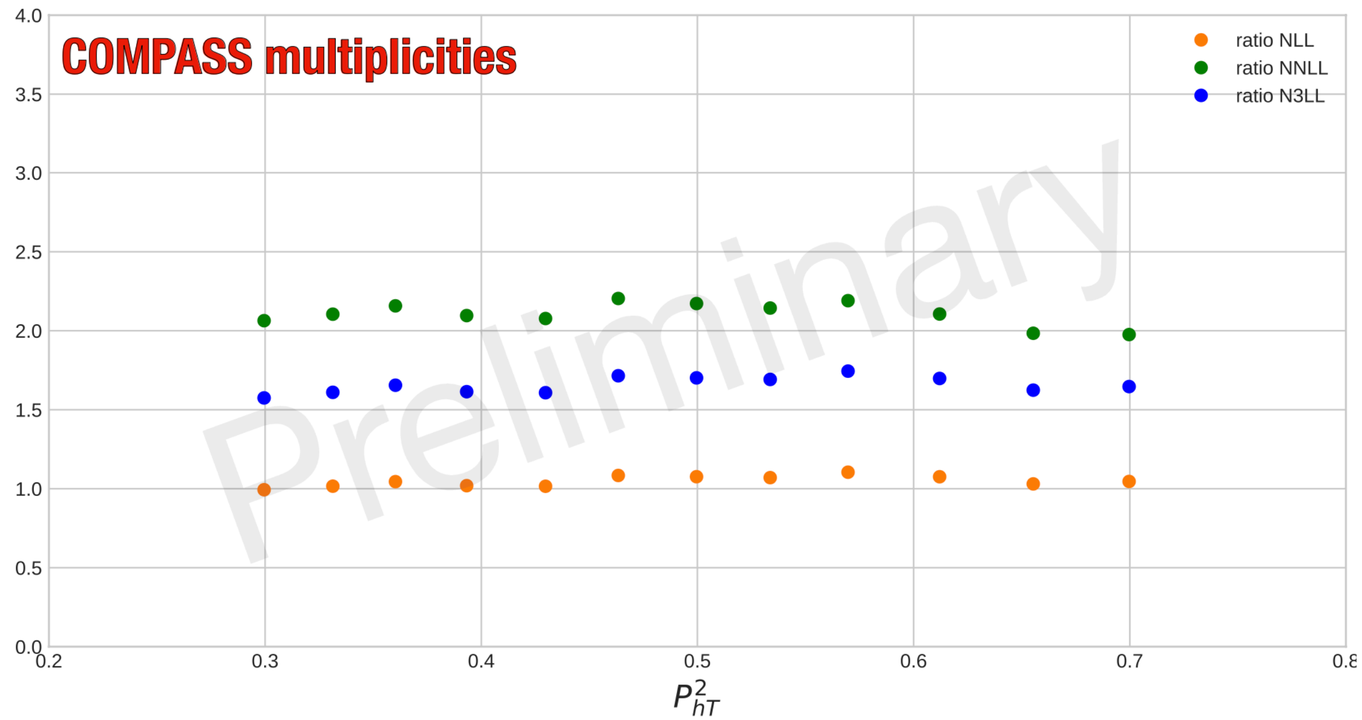
SIDIS normalisation

SIDIS beyond NLL



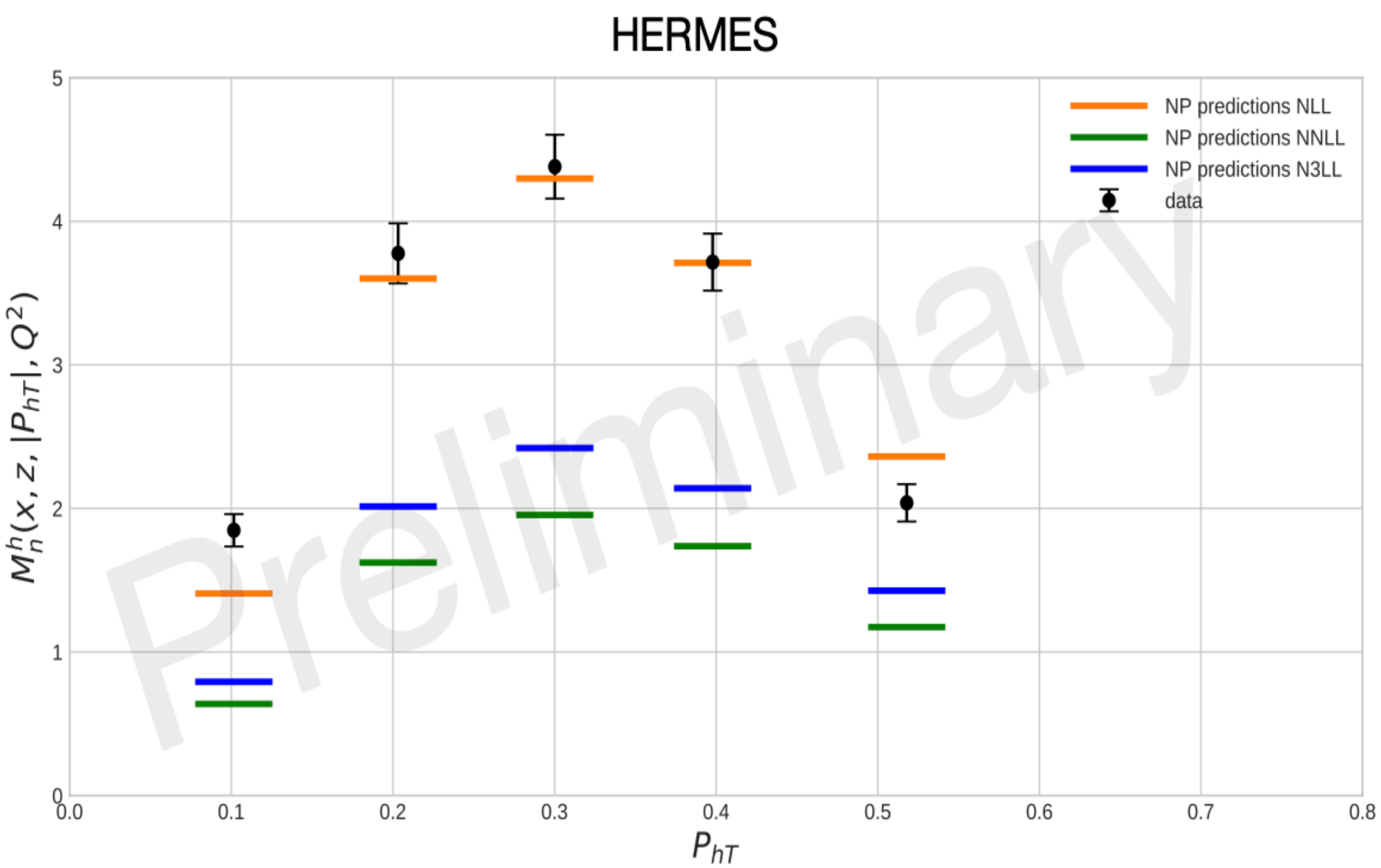
DY beyond NLL



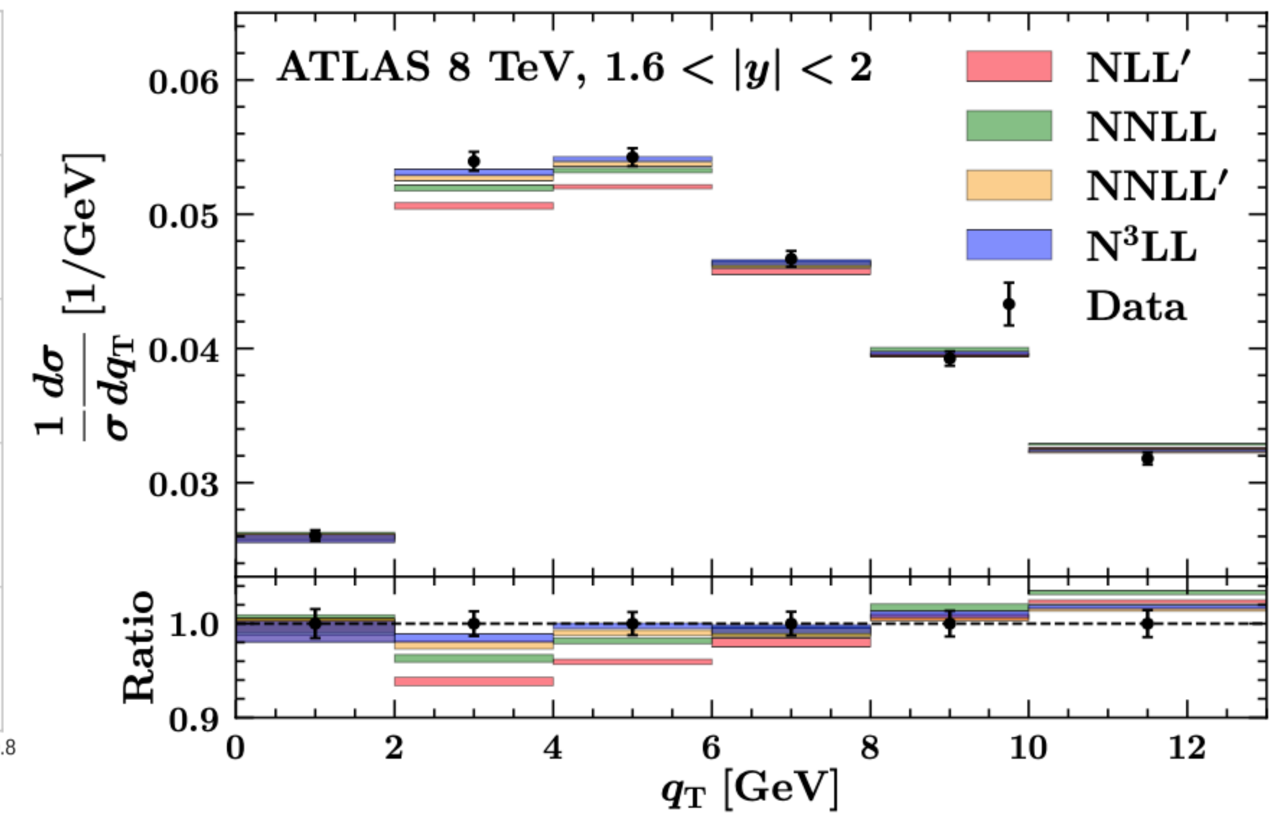


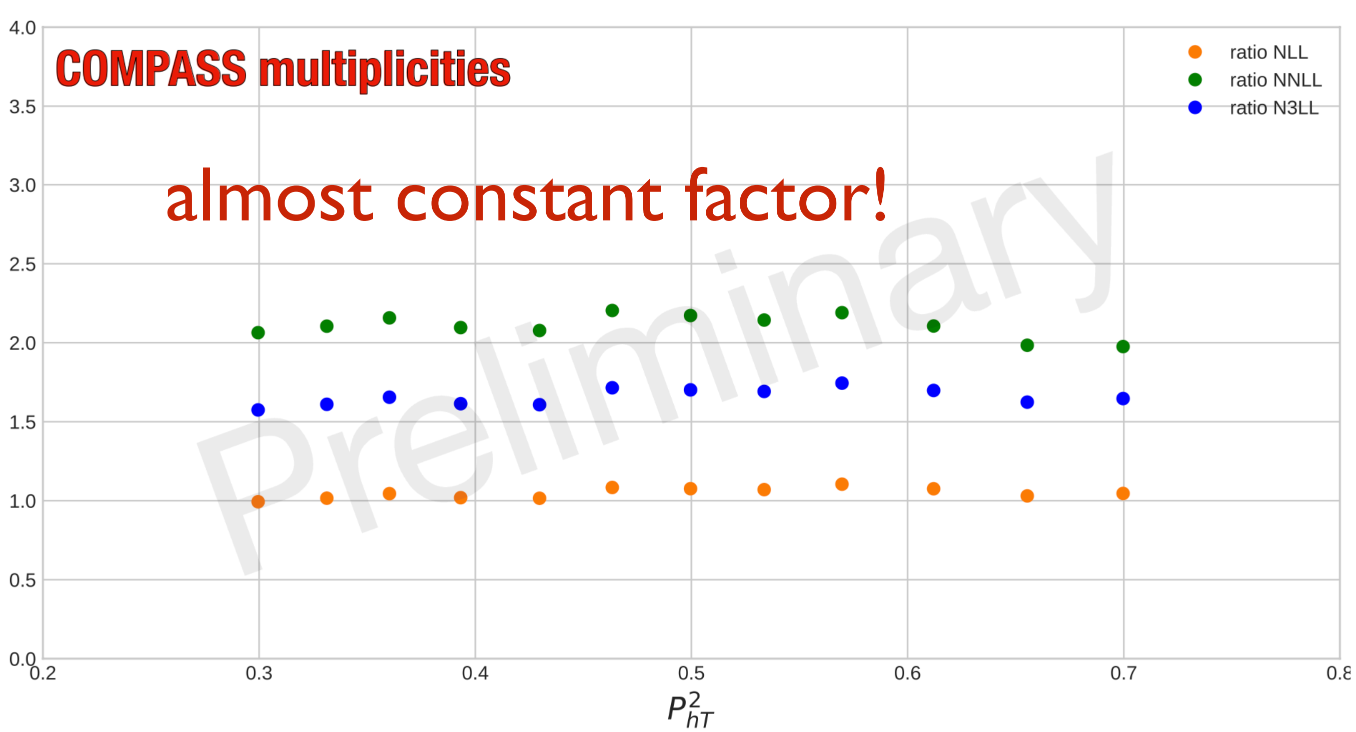
SIDIS normalisation

SIDIS beyond NLL



DY beyond NLL



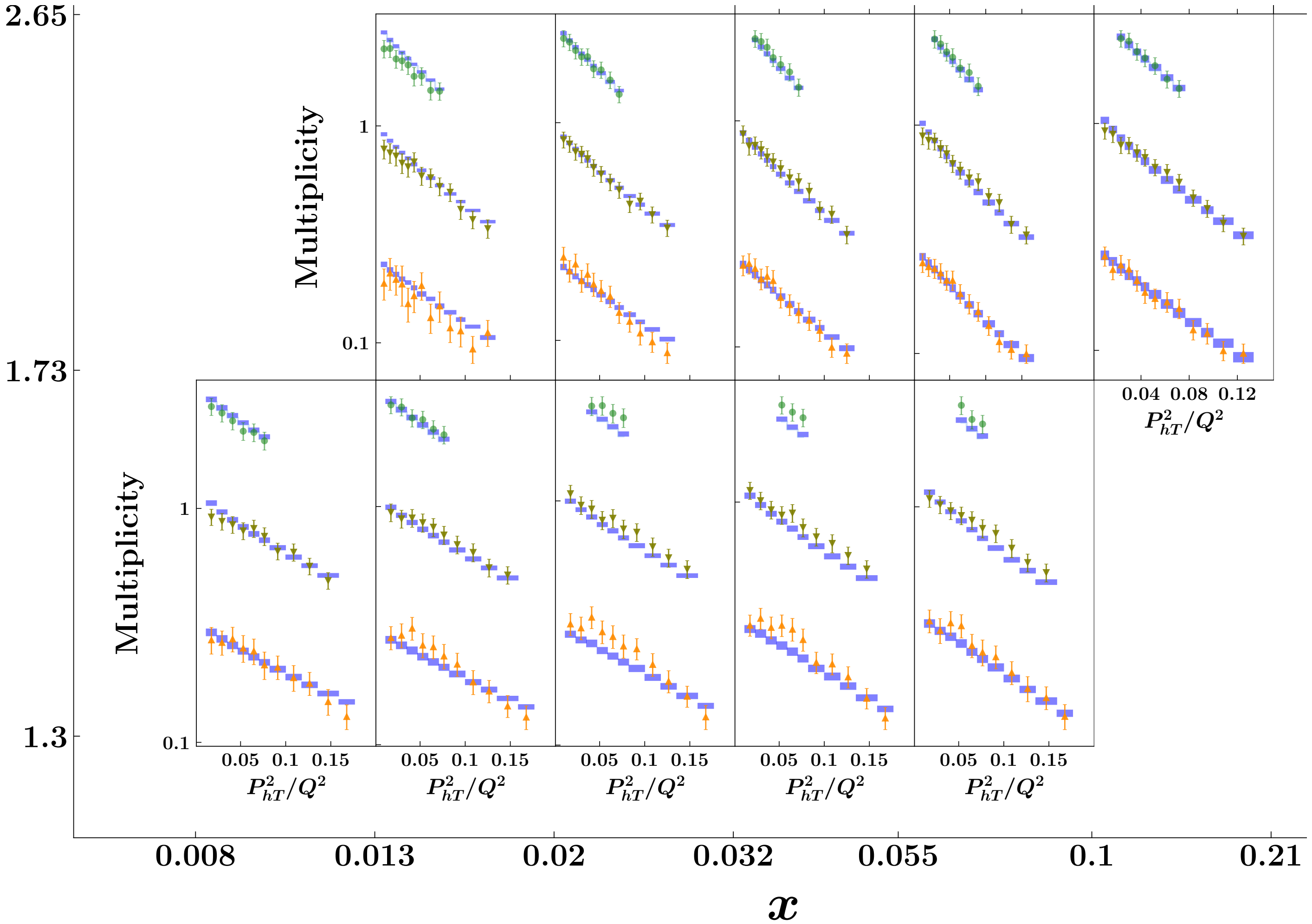


<u>normalisation factor for SIDIS</u>

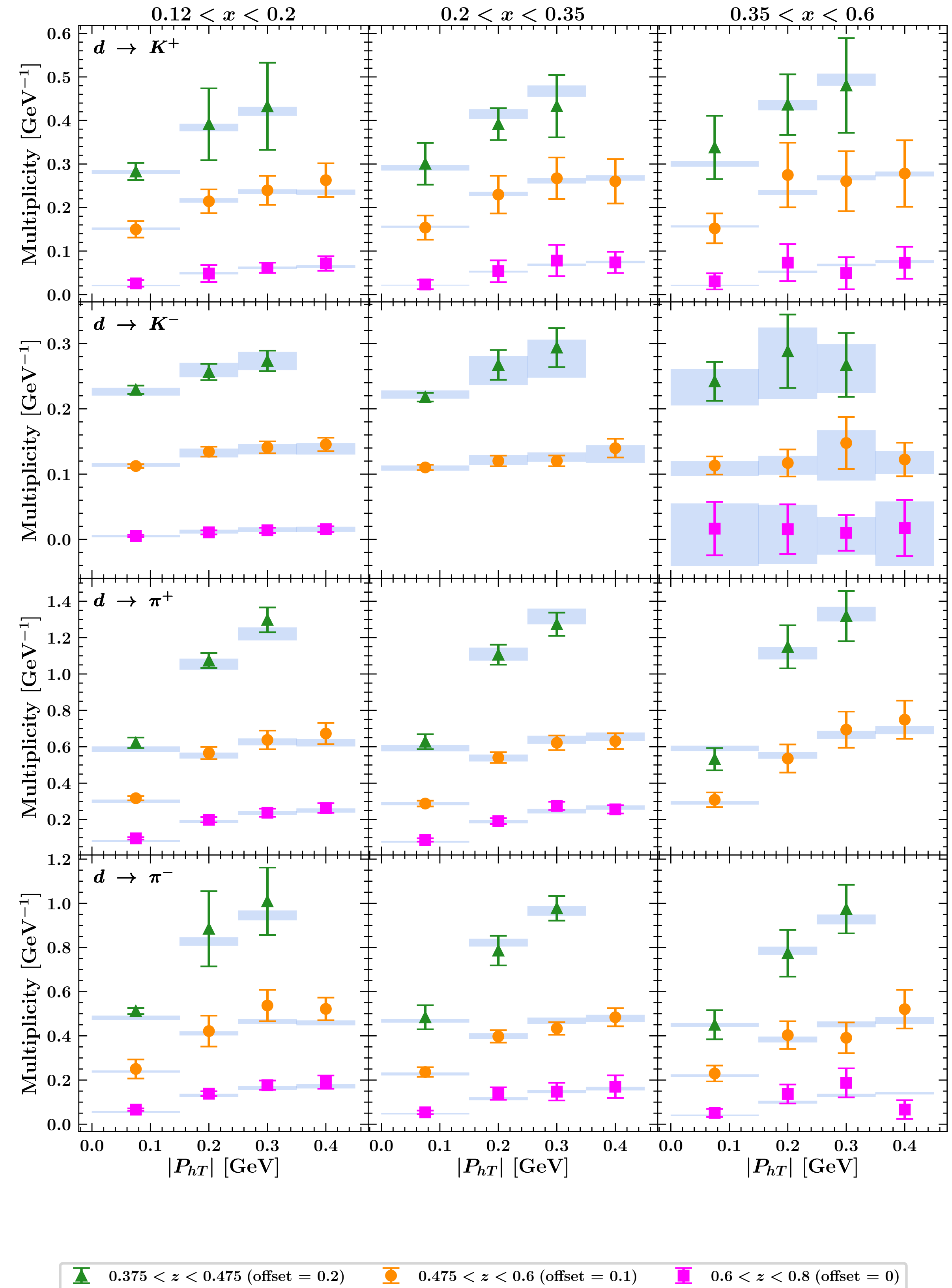
- computed a priori, before the fit
- independent on the fitting parameters
- dependent on collinear PDFs

Fit quality: SIDIS

COMPASS



HERMES

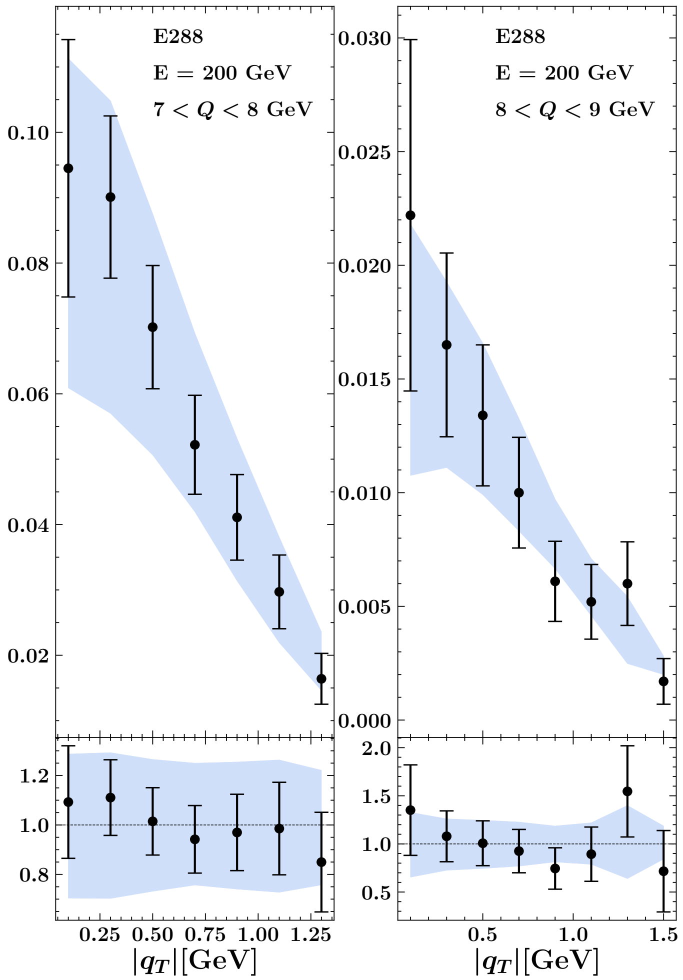


Good agreement for almost all bins

$\chi^2/N_{data} = 0.87$ (SIDIS total)

Fit quality: Drell-Yan

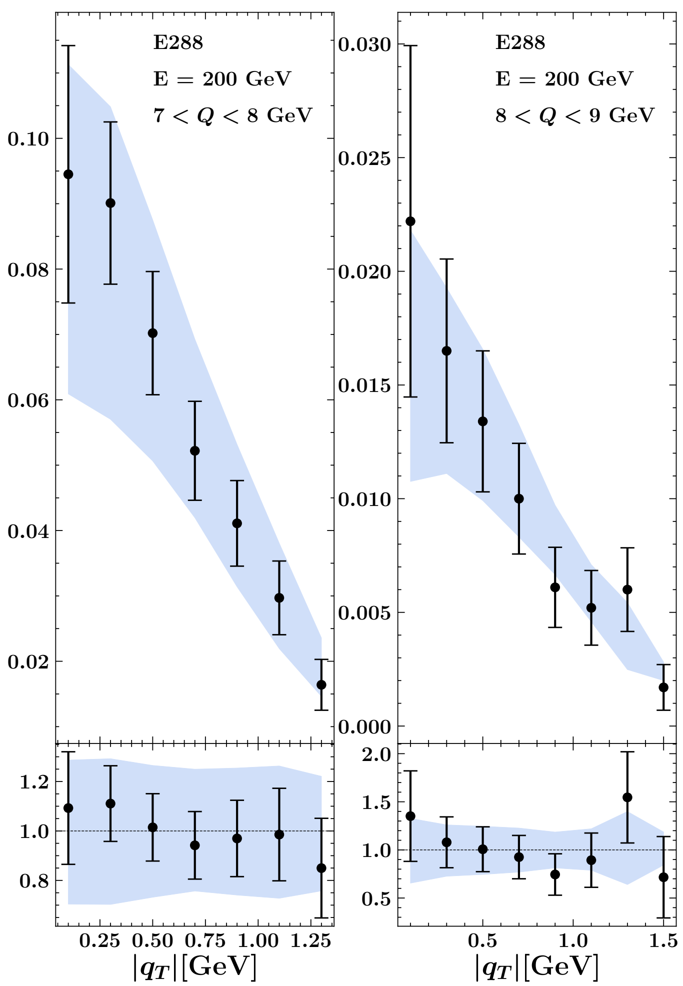
E288



$\chi^2/N_{data} = 1.24$
(DY fixed-target)

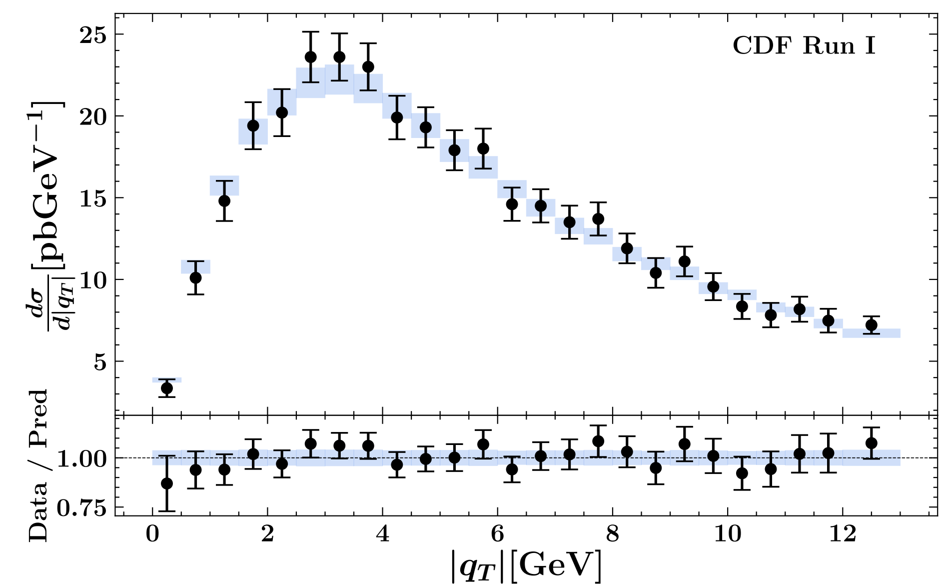
Fit quality: Drell-Yan

E288



$\chi^2/N_{data} = 1.24$
(DY fixed-target)

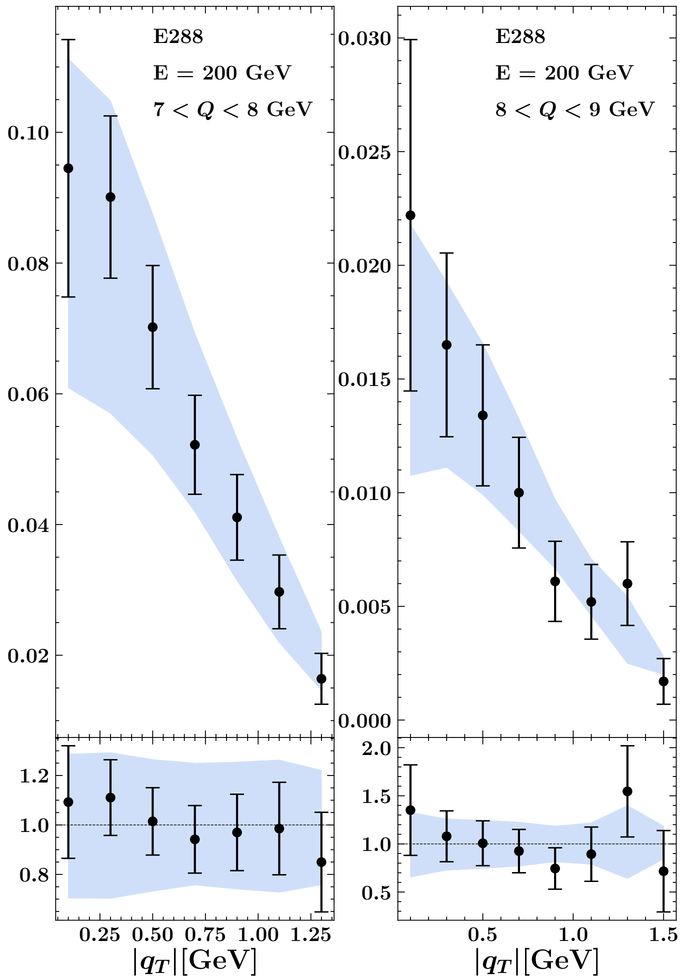
CDF



$\chi^2/N_{data} = 0.93$
(DY Tevatron)

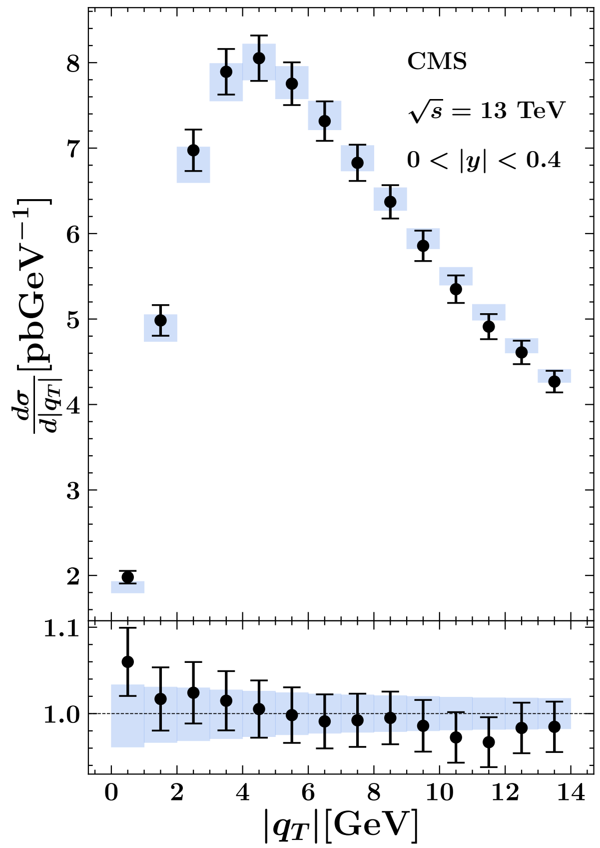
Fit quality: Drell-Yan

E288



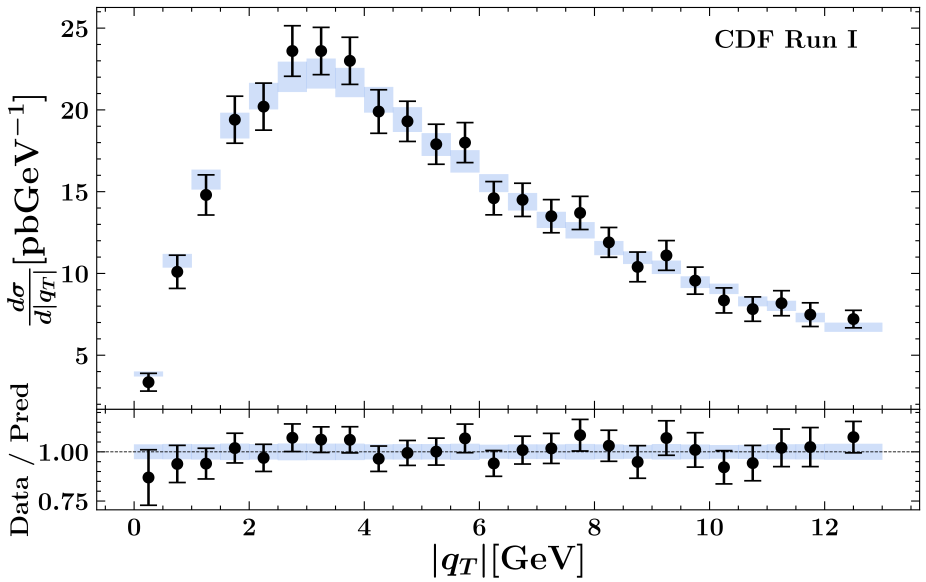
$\chi^2/N_{data} = 1.24$
(DY fixed-target)

CMS



$\chi^2/N_{data} = 0.55$
(DY CMS)

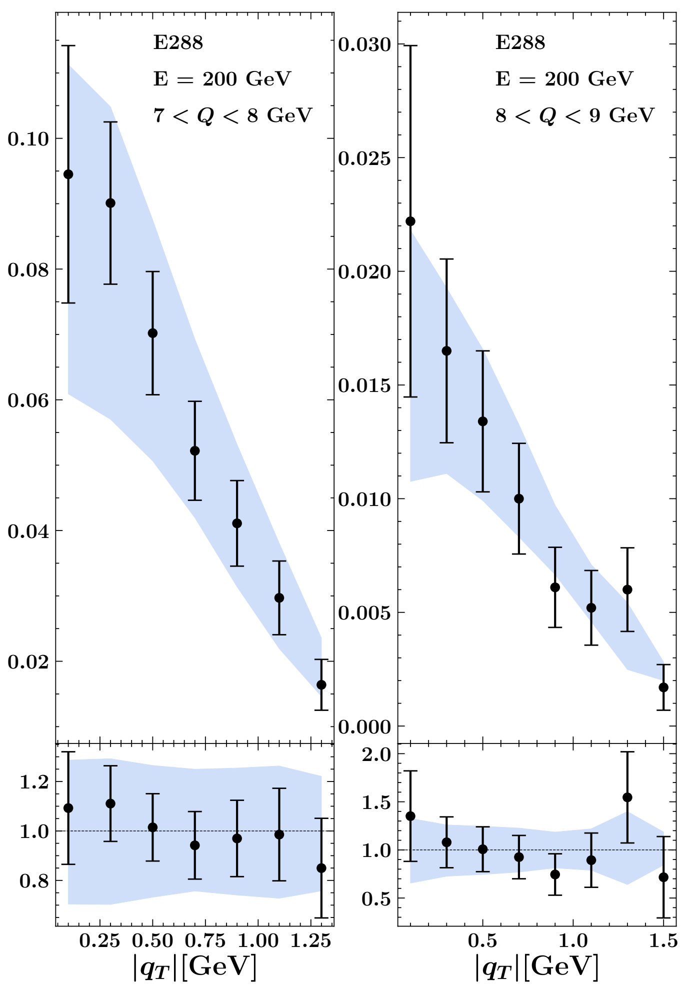
CDF



$\chi^2/N_{data} = 0.93$
(DY Tevatron)

Fit quality: Drell-Yan

E288



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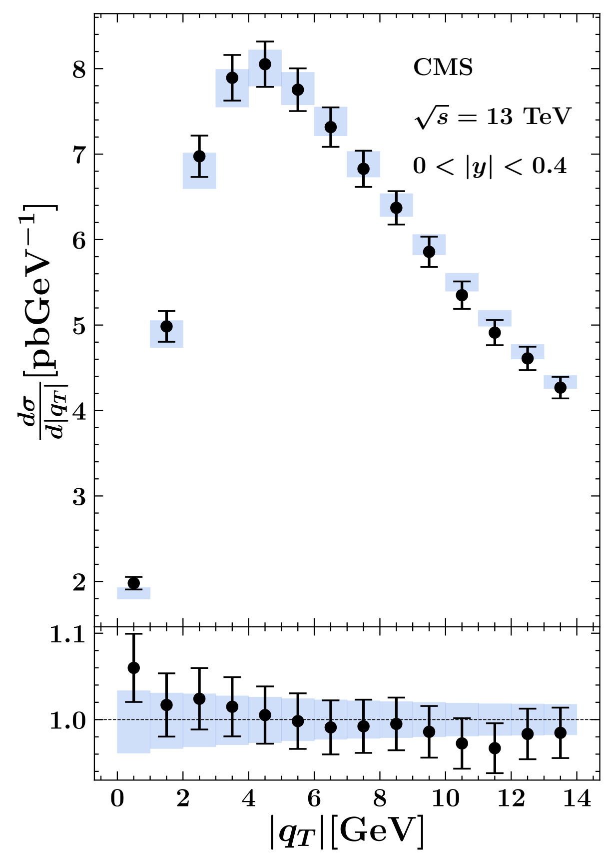
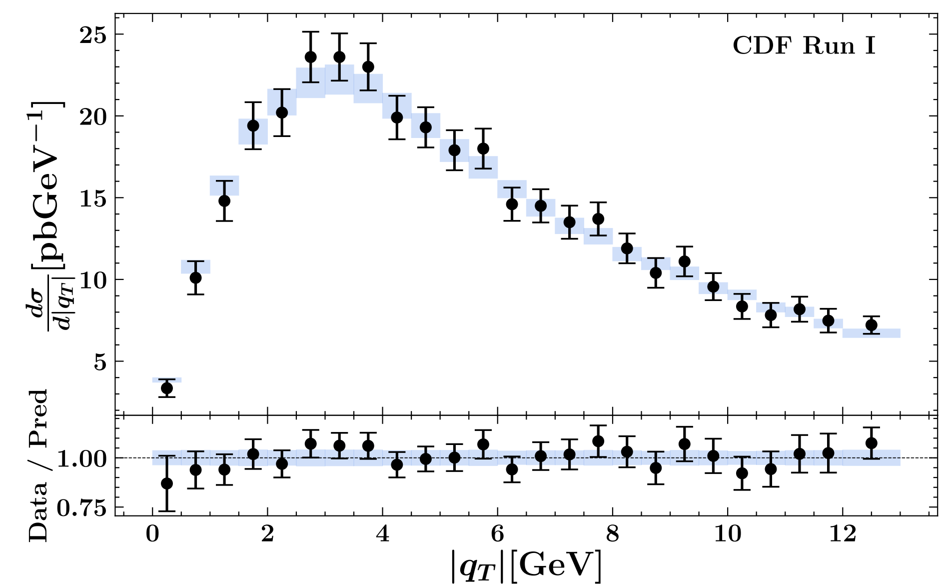


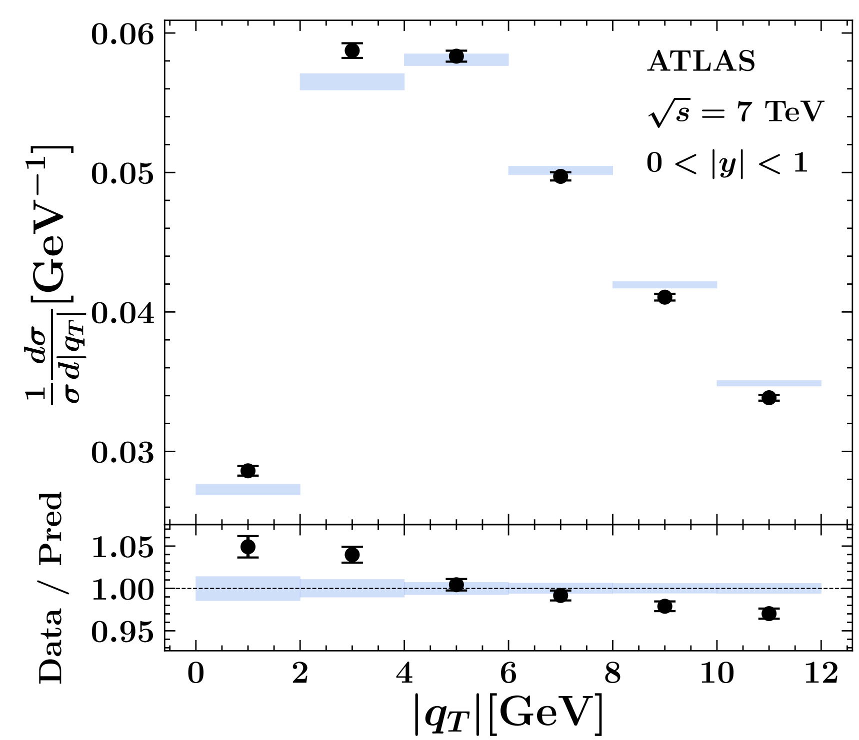
$\chi^2/N_{data} = 0.55$
(DY CMS)

CDF



$\chi^2/N_{data} = 0.93$
(DY Tevatron)

ATLAS



$\chi^2/N_{data} = 5.05$
(DY ATLAS)

Fitting parameters

Parameter	Average over replicas
g_2 [GeV]	0.248 ± 0.008
N_1 [GeV^2]	0.316 ± 0.025
α_1	1.29 ± 0.19
σ_1	0.68 ± 0.13
λ [GeV^{-1}]	1.82 ± 0.29
N_3 [GeV^2]	0.0055 ± 0.0006
β_1	10.23 ± 0.29
δ_1	0.0094 ± 0.0012
γ_1	1.406 ± 0.084
λ_F [GeV^{-2}]	0.078 ± 0.011
N_{3B} [GeV^2]	0.2167 ± 0.0055
N_{1B} [GeV^2]	0.134 ± 0.017
N_{1C} [GeV^2]	0.0130 ± 0.0069
λ_2 [GeV^{-1}]	0.0215 ± 0.0058
α_2	4.27 ± 0.31
α_3	4.27 ± 0.13
σ_2	0.455 ± 0.050
σ_3	12.71 ± 0.21
β_2	4.17 ± 0.13
δ_2	0.167 ± 0.006
γ_2	0.0007 ± 0.0110

Fitting parameters

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σ_3	12.71 ± 0.21
β_2	4.17 ± 0.13
δ_2	0.167 ± 0.006
γ_2	0.0007 ± 0.0110

- $\lambda \sim 2$: weighted Gaussian important

Fitting parameters

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β_1	10.23 ± 0.29
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N_{1C} [GeV^2]	0.0130 ± 0.0069
λ_2 [GeV^{-1}]	0.0215 ± 0.0058
α_2	4.27 ± 0.31
α_3	4.27 ± 0.13
σ_2	0.455 ± 0.050
σ_3	12.71 ± 0.21
β_2	4.17 ± 0.13
δ_2	0.167 ± 0.006
γ_2	0.0007 ± 0.0110

- $\lambda \sim 2$: weighted Gaussian important
- $\lambda_2 \neq 0$: third Gaussian non-negligible

Fitting parameters

Parameter	Average over replicas
g_2 [GeV]	0.248 ± 0.008
N_1 [GeV^2]	0.316 ± 0.025
α_1	1.29 ± 0.19
σ_1	0.68 ± 0.13
λ [GeV^{-1}]	1.82 ± 0.29
N_3 [GeV^2]	0.0055 ± 0.0006
β_1	10.23 ± 0.29
δ_1	0.0094 ± 0.0012
γ_1	1.406 ± 0.084
λ_F [GeV^{-2}]	0.078 ± 0.011
N_{3B} [GeV^2]	0.2167 ± 0.0055
N_{1B} [GeV^2]	0.134 ± 0.017
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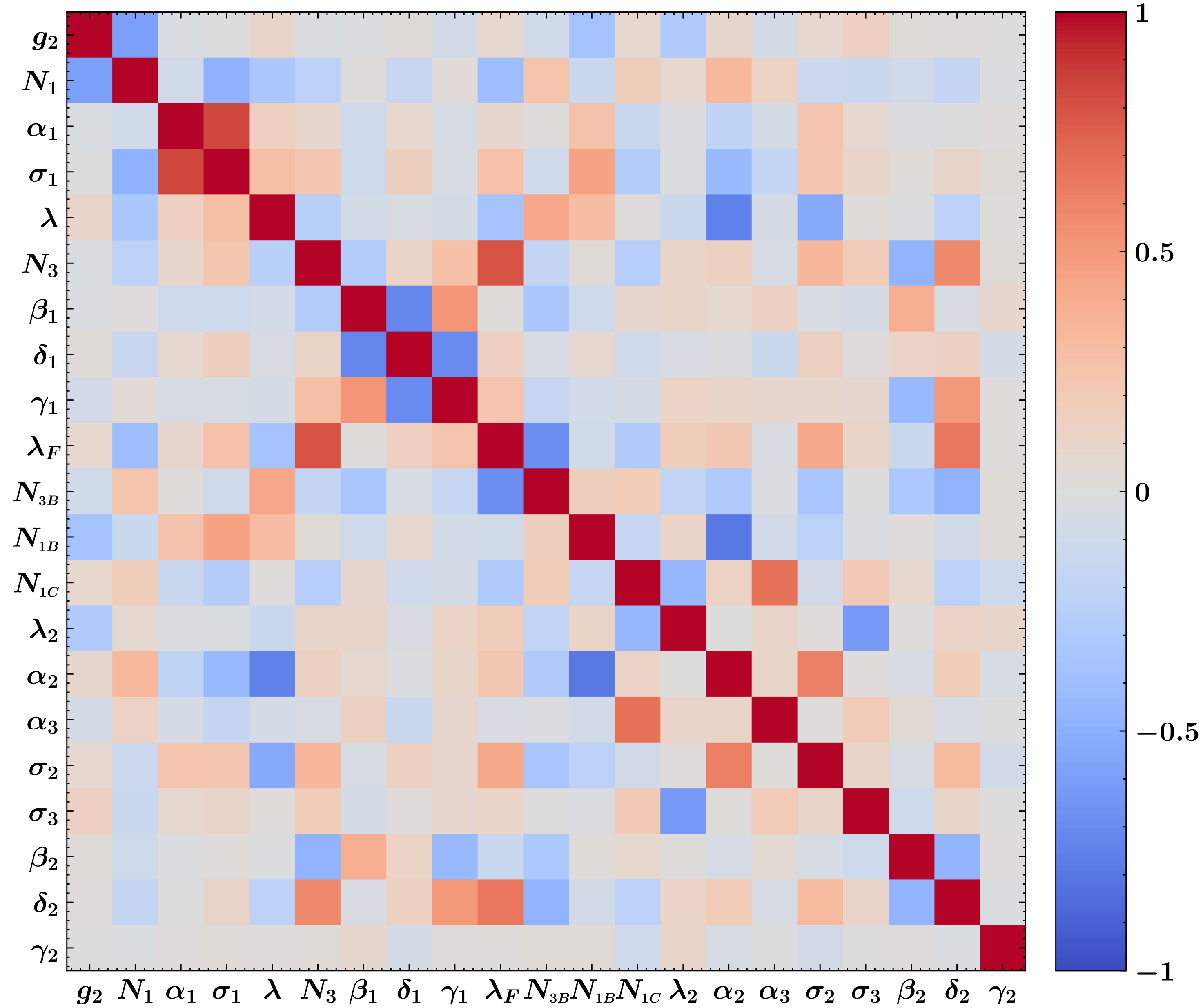
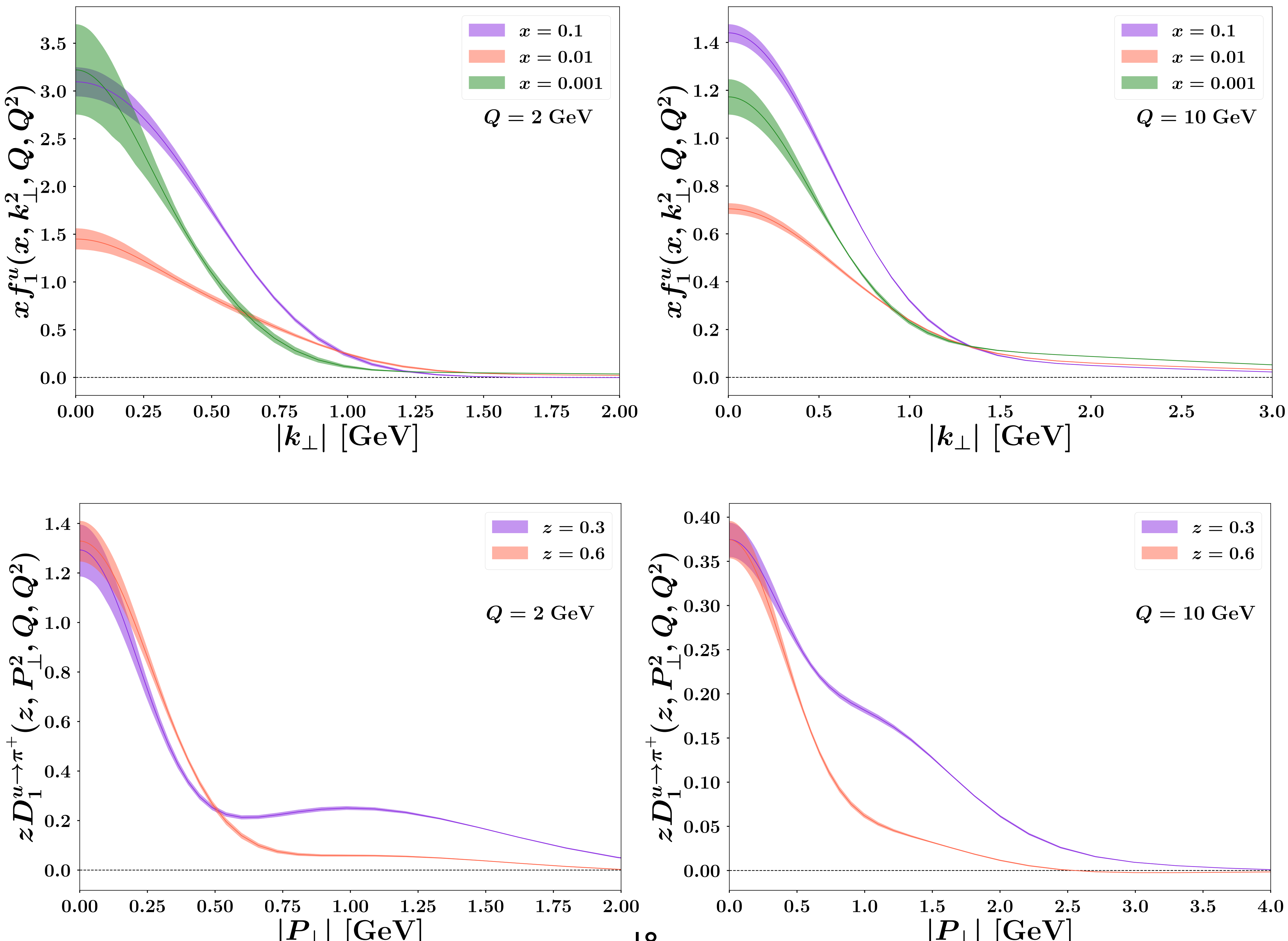
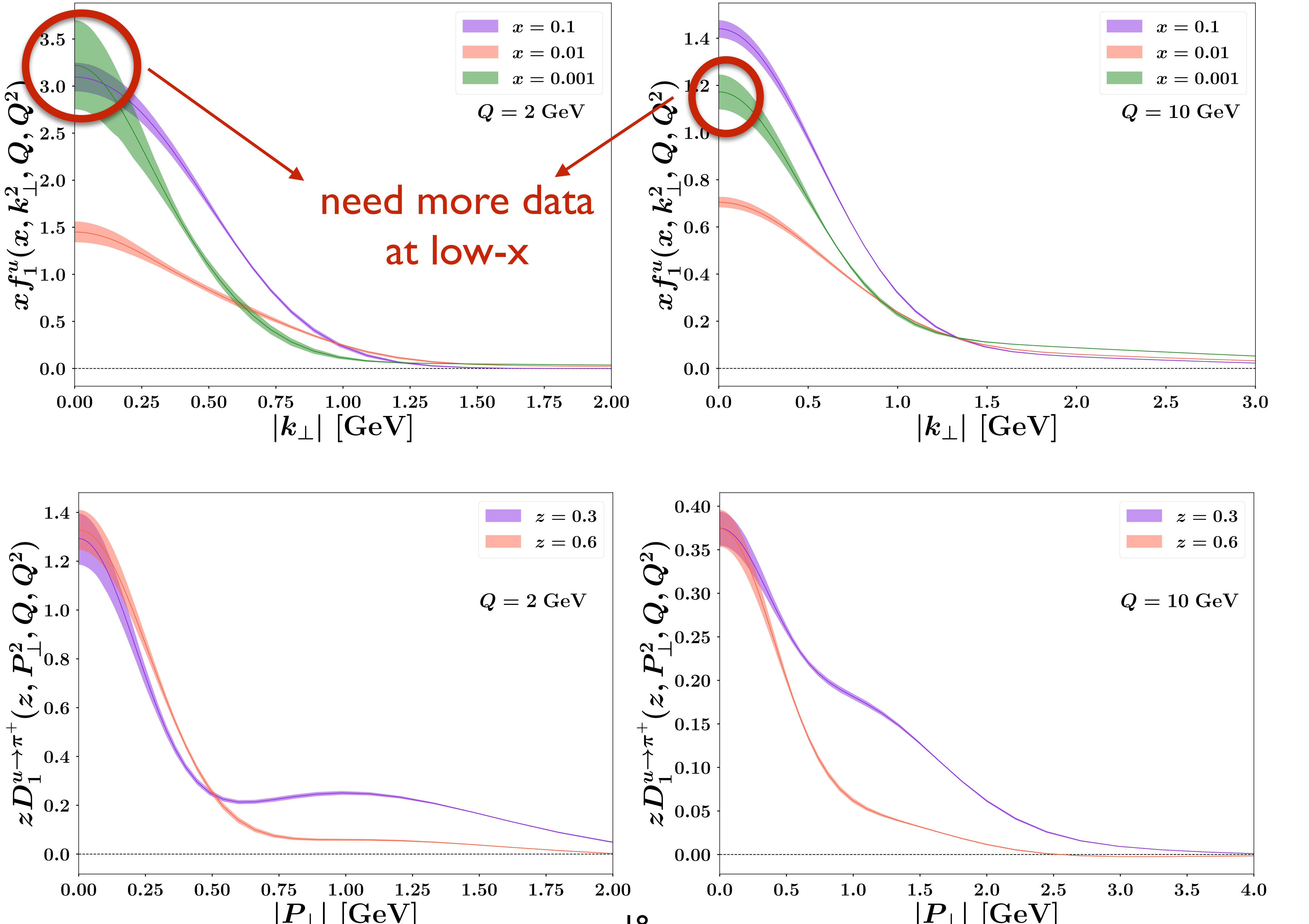


- $\lambda \sim 2$: weighted Gaussian important
- $\lambda_2 \neq 0$: third Gaussian non-negligible
- g_2 very small standard deviation
- correlation matrix nearly diagonal

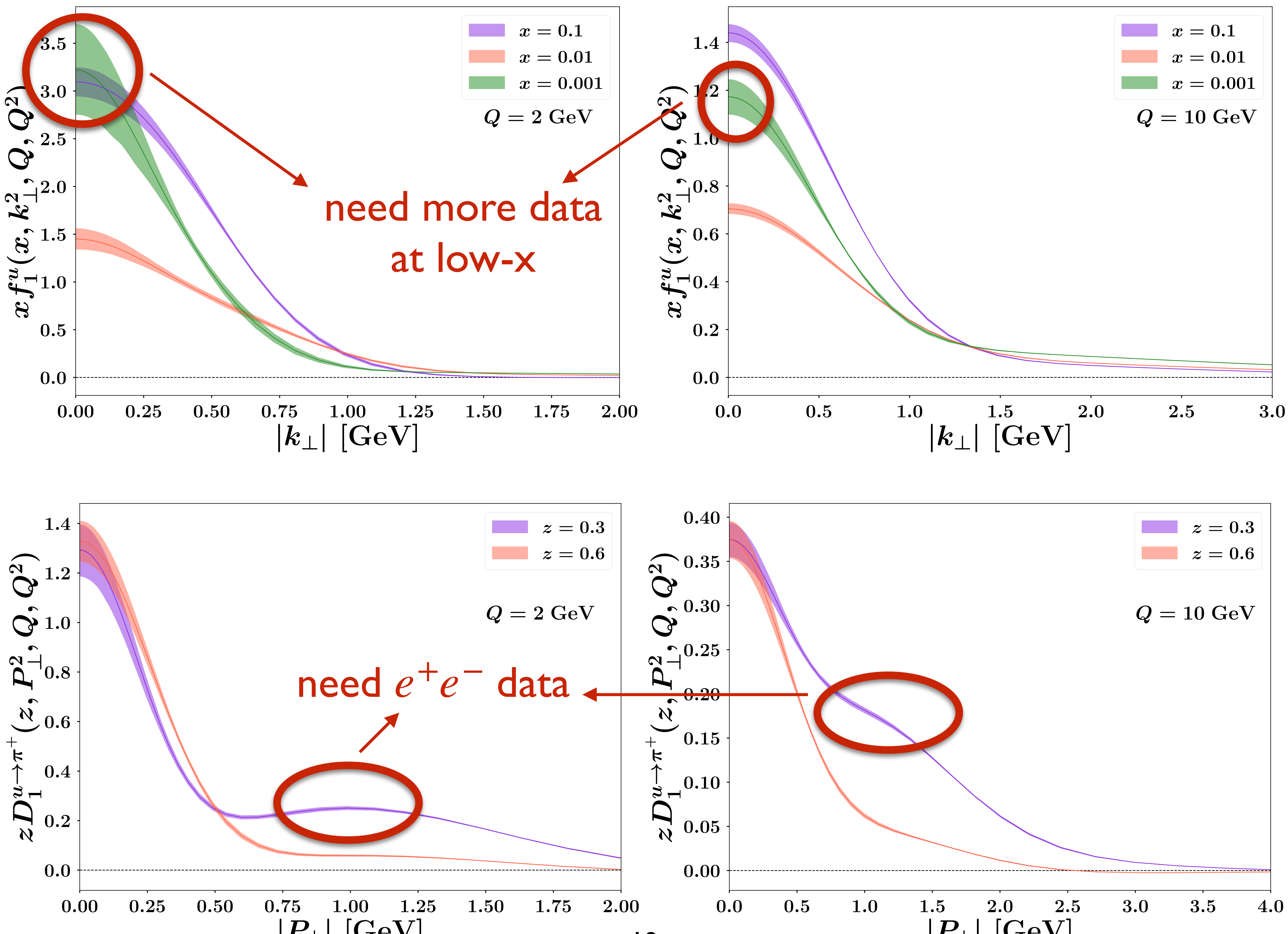
TMD PDFs and FFs



TMD PDFs and FFs



TMD PDFs and FFs



The Nanga Parbat framework

Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

Conclusions

- Extraction of **TMD PDFs and FFs** from DY and SIDIS data at **N³LL⁽⁻⁾**
- 484 DY (Fermilab, LHC, RHIC) + 1547 SIDIS (COMPASS, Hermes): **2031 data points**
- **Normalisation factors** used for SIDIS data
- Very good description of entire dataset ($\chi^2 = 1.06$) **except for ATLAS data**
- **Code and TMD grids** available at the NangaParbat website
- **Plans for the future**:
  - improve perturbative accuracy
  - matching with fixed order
  - include theoretical uncertainties
  - flavour dependence
- **We need e^+e^- data to:**
  - verify TMD factorisation for $e^+e^- \to 2h$
  - perform a "true" global fit
  - de-correlate TMD FF and TMD PDF
  - asymptotically understand QCD!

Backup

Normalisation of SIDIS multiplicities

Introduction of a normalisation prefactor

$$\text{PREFACTOR}(x, z, Q) = \frac{\left.\frac{d\sigma^h}{dx dQ^2 dz}\right|_{\text{nonmix.}}}{\int W \, d^2 q_T}$$

$$\left.\frac{d\sigma^h}{dx dQ^2 dz}\right|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} \left.\left\{ \left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N} \right](x, z, Q) \right\}\right|_{\text{nonmix.}}$$

$$\int W \bigg|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} \left[D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N} \right](x, z, Q)$$

Independent of the fitting parameters!!

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Slide by M. Cerutti

Non-mixed terms in collinear SIDIS cross section

$$\frac{d\sigma^h}{dx\,dQ^2\,dz}\bigg|_{O(\alpha_s^1)} = \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2}\left(\delta_{f'f}+\delta_{f'g}\right)\frac{\alpha_s}{\pi}\Bigg\{\left[D_1^{h/f'}\otimes C_1^{f'f}\otimes f_1^{f/N}\right](x,z,Q)$$

$$+\frac{1-y}{1+(1-y)^2}\left[D_1^{h/f'}\otimes C_L^{f'f}\otimes f_1^{f/N}\right](x,z,Q)\Bigg\},$$

$$C_1^{qq} = \frac{C_F}{2}\Bigg\{-8\delta(1-x)\delta(1-z)$$

$$+\delta(1-x)\left[P_{qq}(z)\ln\frac{Q^2}{\mu_F^2}+L_1(z)+L_2(z)+(1-z)\right]$$

$$+\delta(1-z)\left[P_{qq}(x)\ln\frac{Q^2}{\mu^2}+L_1(x)-L_2(x)+(1-x)\right]$$

$$+2\frac{1}{(1-x)_+}\frac{1}{(1-z)_+}-\frac{1+z}{(1-x)_+}-\frac{1+x}{(1-z)_+}+2(1+xz)\Bigg\},$$

Slide by M. Cerutti

Experimental uncertainties

$$m_i \pm \sigma_{i,\text{stat}} \pm \sigma_{i,\text{unc}} \pm \sigma^{(1)}_{i,\text{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\text{corr}}$$

uncorrelated **correlated**

additive **multiplicative**

$$\chi^2 = \sum_{i,j=1}^{n} (m_i - t_i) V^{-1}_{ij} (m_j - t_j)$$

$$\sigma^{(l)}_{i,\text{corr}} \equiv \delta^{(l)}_{i,\text{corr}} m_i$$

covariance matrix

$$V_{ij} = s_i^2 \delta_{ij} + \left(\sum_{l=1}^{k_a} \delta^{(l)}_{i,\text{add}} \delta^{(l)}_{j,\text{add}} + \sum_{l=1}^{k_m} \delta^{(l)}_{i,\text{mult}} \delta^{(l)}_{j,\text{mult}} \right) m_i m_j$$

Slide by C. Bissolotti

χ^2 chisquare

systematic shift

$$d_i = \sum_{\alpha=1}^{k} \lambda_\alpha \sigma_{i,\mathrm{corr}}^{(\alpha)}$$

shift

$$\frac{\partial \chi^2}{\partial \lambda_\alpha} = 0$$

nuisance parameters

$$\bar{t}_i = t_i + d_i$$

shifted prediction

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^{k} \lambda_\alpha^2$$

recover the form of the uncorrelated definition

penalty term

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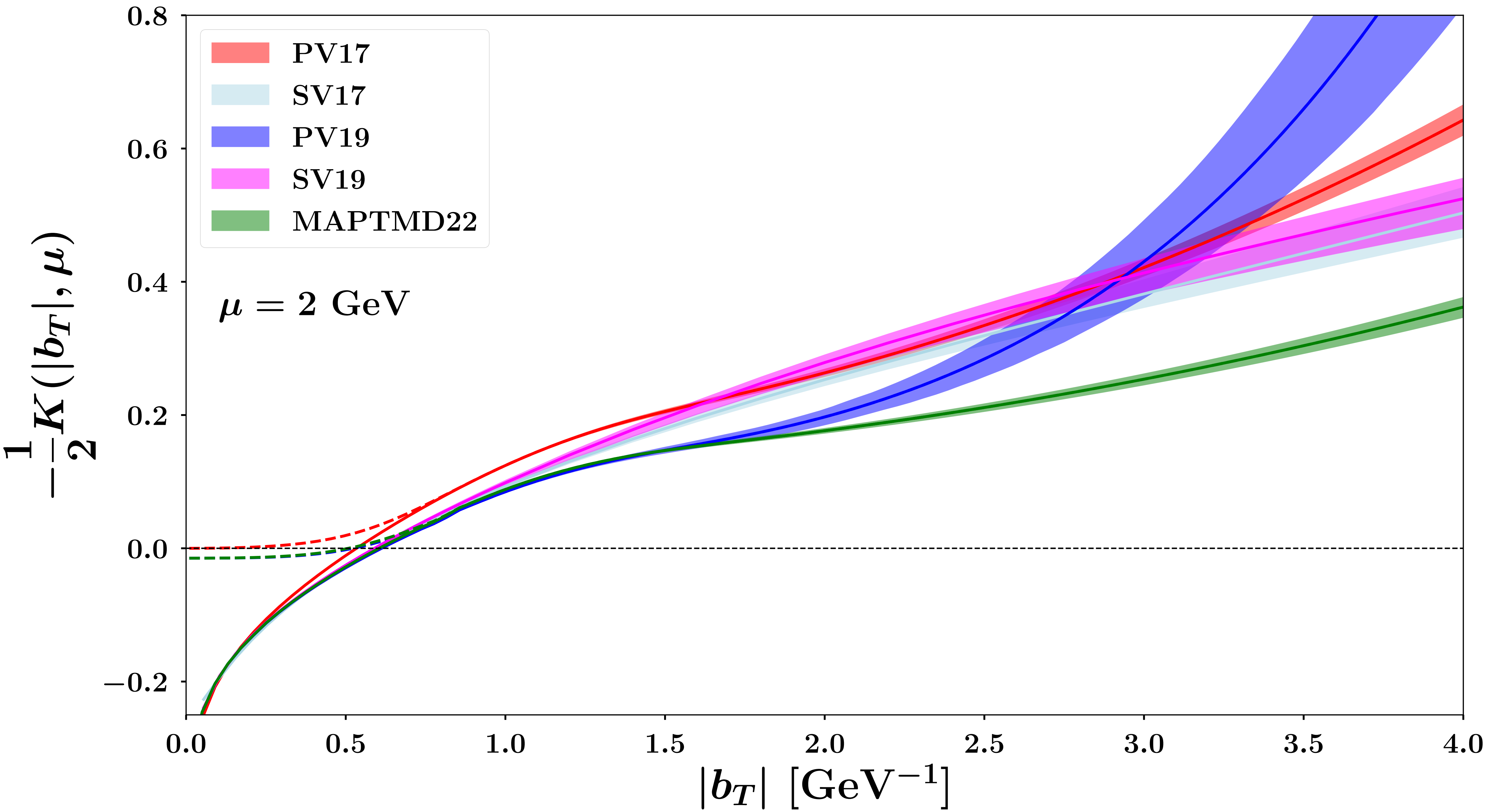
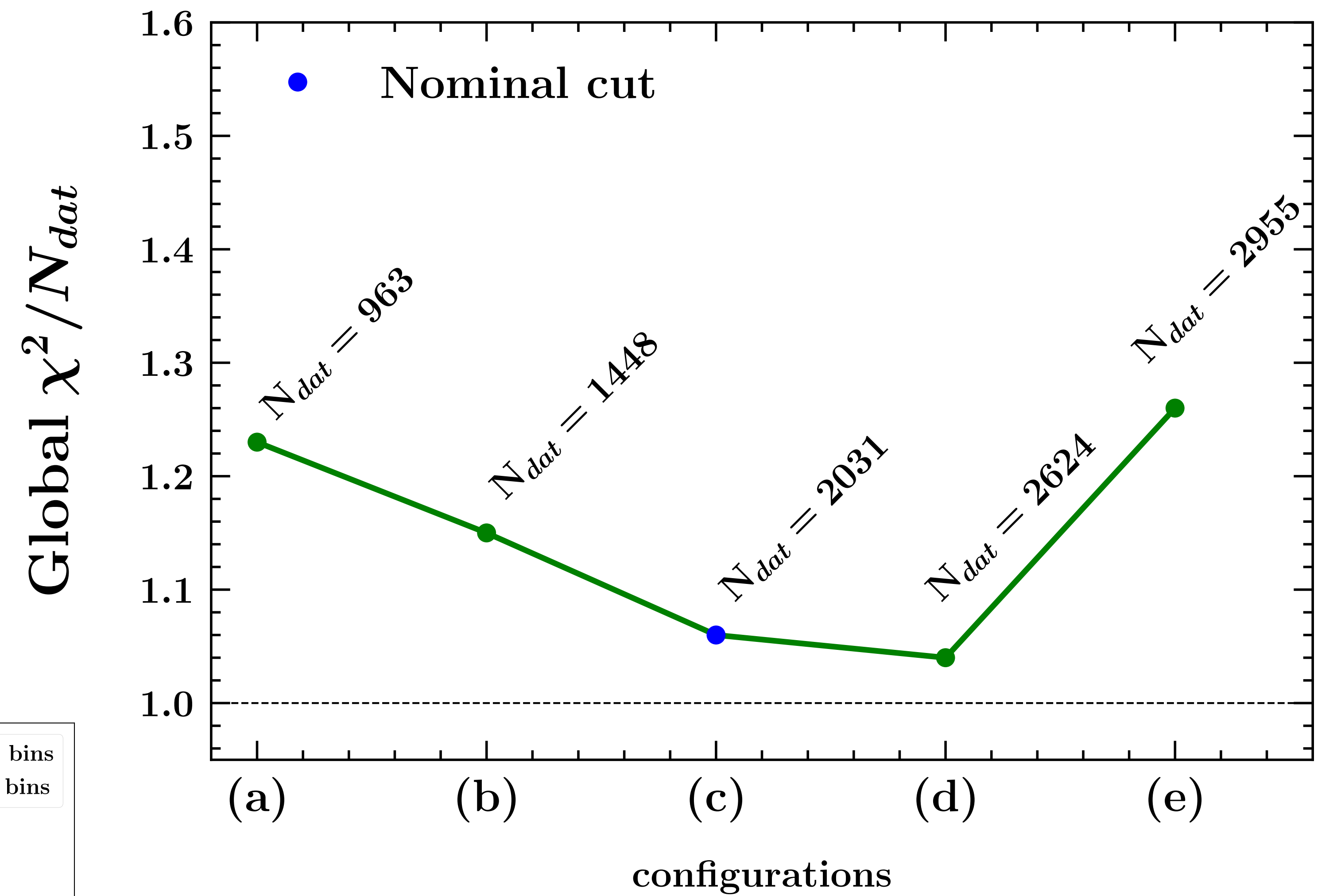
Collins-Soper kernel

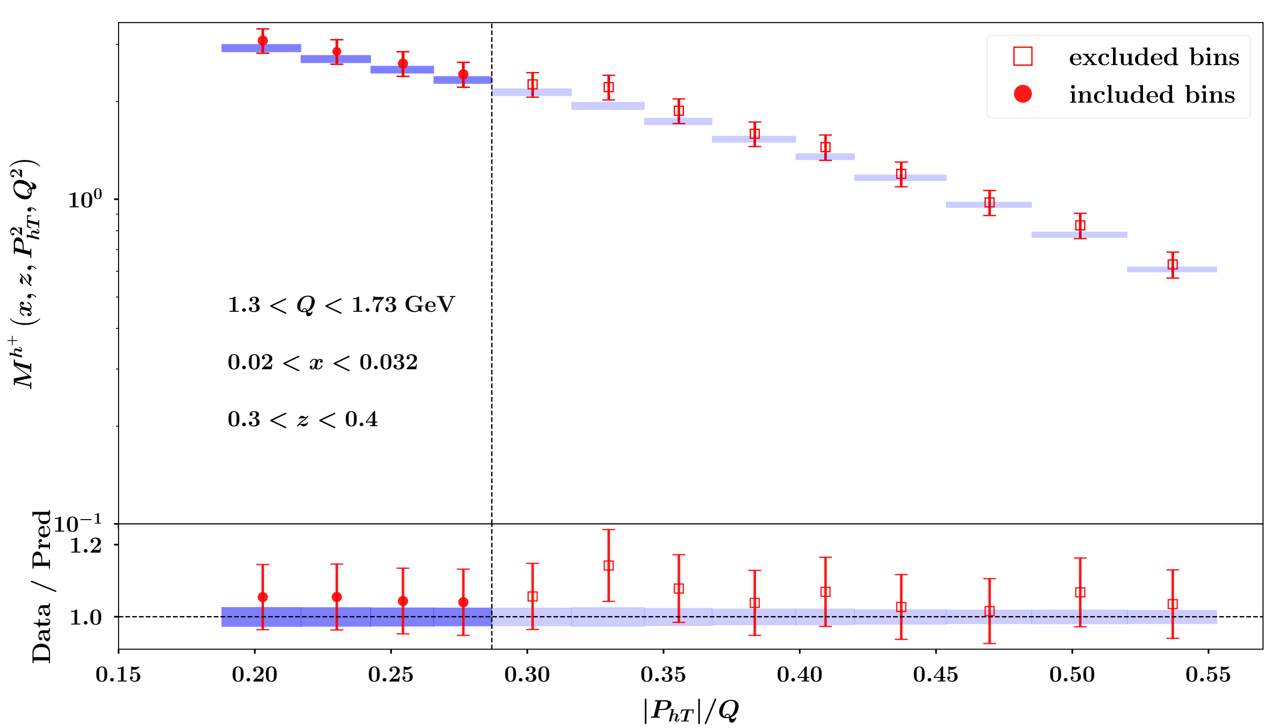


Different SIDIS cuts

$$P_{hT}|_{max} = min[min[\,0.2Q,\, 0.5zQ\,] + 0.3 \text{ GeV},\, zQ\,]$$

- (a) $c_1 = 0.4, c_2 = 0.4, c_3 = 0$
- (b) $c_1 = 0.15, c_2 = 0.4, c_3 = 0.2$
- (c) $c_1 = 0.2, c_2 = 0.5, c_3 = 0.3$ (baseline)
- (d) $c_1 = 0.2, c_2 = 0.6, c_3 = 0.4$
- (e) $c_1 = 0.2, c_2 = 0.7, c_3 = 0.5$





Different logarithmic orders

	N^3LL^-		NNLL		NLL	
Data set	N_{dat}	$\langle\chi^2\rangle \pm \delta\langle\chi^2\rangle$	N_{dat}	$\langle\chi^2\rangle \pm \delta\langle\chi^2\rangle$	N_{dat}	$\langle\chi^2\rangle \pm \delta\langle\chi^2\rangle$
ATLAS	72	5.01 ± 0.26	/	/	/	/
PHENIX 200	2	3.26 ± 0.31	2	0.81 ± 0.11	/	/
STAR 510	7	1.16 ± 0.04	7	0.99 ± 0.03	/	/
Other sets	170	0.83 ± 0.01	170	2.37 ± 0.11	/	/
DY collider	251	2.06 ± 0.07	179	2.3 ± 0.1	/	/
E772	53	2.48 ± 0.12	53	2.05 ± 0.22	/	/
Other sets	180	0.87 ± 0.04	180	0.71 ± 0.04	180	0.81 ± 0.04
DY fixed-target	233	1.24 ± 0.04	233	1.01 ± 0.05	180	0.81 ± 0.04
HERMES	344	0.71 ± 0.04	344	1.1 ± 0.06	344	0.51 ± 0.02
COMPASS	1203	0.95 ± 0.02	1203	0.6 ± 0.06	1203	0.41 ± 0.01
SIDIS	1547	0.89 ± 0.02	1547	0.71 ± 0.05	1547	0.43 ± 0.01
Total	2031	1.08 ± 0.01	1959	0.89 ± 0.01	1727	0.47 ± 0.01