

Neutron-star astrophysics with GWs: neutron-star mountains, r-modes and post-merger signals

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Outline

- ★ Astrophysical motivation
- ★ Methods and challenges
- ★ Current results and outlook

Continuous GW sources vs other types of sources



(Hokusai "The Great Wave off Kanagawa")

One-time cataclysmic events, e.g. last moments of binary systems of

- ★ black holes (GW150914 etc.) and neutron stars (GW170817),



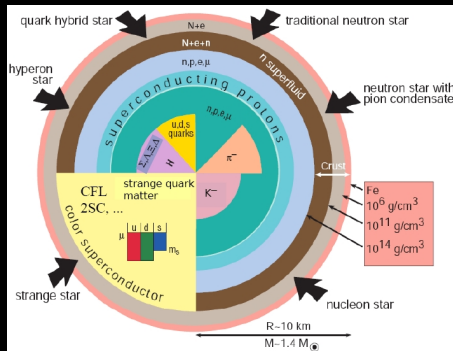
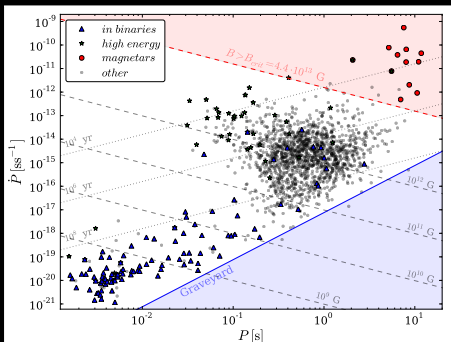
(Shoson "Cranes landing")

Persistent phenomena, e.g.

- ★ rotating asymmetric neutron stars ("*gravitational pulsars*"),
- ★ low-mass binary systems,
- ★ boson clouds.

Neutron stars = very dense, magnetized stars

The **most relativistic material objects** in the Universe: compactness $M/R \simeq 0.5$, observed in all EM spectrum as pulsars, magnetars, in supernovæ remnants, in accreting systems, in double neutron star binaries...



About 2500 NS observed to date, $\sim 10^8 - 10^9$ in the Galaxy.

Continuous GWs from spinning neutron stars

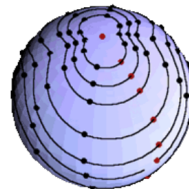
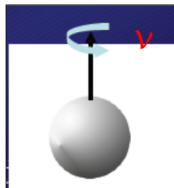
Characteristics:

- ★ Long-lived: $T > T_{obs}$,
- ★ Nearly periodic: $f_{GW} \propto f_{rot}$

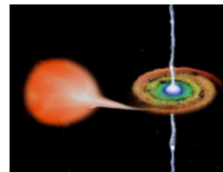
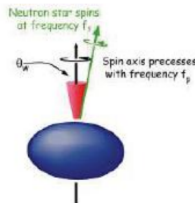
Mechanisms that can create time-varying quadrupole moment:

- ★ "Mountains" (elastic and/or magnetic stresses, $f_{GW} = 2f_{rot}$),
- ★ Oscillations (r-modes, $f_{GW} = 4/3f_{rot}$),
- ★ Free precession ($f_{GW} \propto f_{rot} + f_{prec}$)
- ★ Accretion (drives deformations from r-modes, thermal gradients, magnetic fields, $f_{GW} \simeq f_{rot}$)

(see PASA 32, 34 2015; Universe 5(11), 217 2019)



Courtesy: B. J. Owen

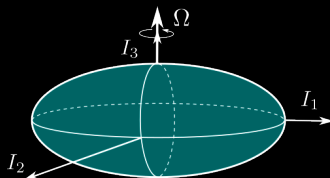


Courtesy: McGill U.

GW amplitude and the spindown limit

$$\text{GW strain } h_0 = \frac{4\pi^2 G}{c^4} \frac{I_3 \epsilon^2 f_{\text{GW}}^2}{d}$$

with the distance d and the deformation $\epsilon = (I_1 - I_2)/I_3$. Depending on the dense matter model, $\epsilon_{\text{max}} = 10^{-3} - 10^{-6}$.



Rotational energy loss: $\dot{E}_{\text{rot}} \propto f \dot{f}$

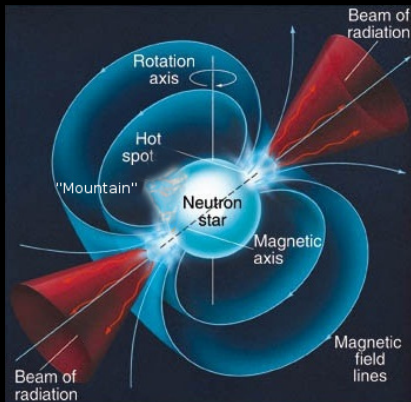
Energy emitted in GWs: $\dot{E}_{\text{GW}} \propto f^6 I_3^2 \epsilon^2$

Spindown upper limit: observed spindown fully due to GWs, $\dot{E}_{\text{rot}} = \dot{E}_{\text{GW}}$:

Assuming the knowledge of I_3 and d

$$\rightarrow \text{upper limit } h_0^{\text{sd}} = \frac{1}{d} \sqrt{\frac{5G}{2c^3} \frac{I_1}{f}} I_3$$

$$\epsilon_{\text{sd}} = 0.237 I_{38}^{-1} \left[\frac{h_{\text{sd}}}{10^{-24}} \right] \left[\frac{\text{Hz}}{f_{\text{rot}}} \right]^2 \left[\frac{d}{1 \text{ kpc}} \right]$$



”Collider” vs ”table top” experiments

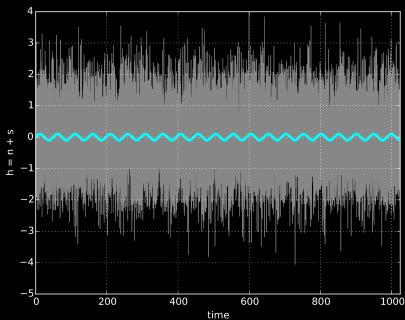
- ★ Many potential sources, but the GW ‘engine’ is not guaranteed
 - ★ ”opposite problem” to transient compact binary coalescences (BH events).
- ★ GW amplitude is small: $\lesssim 10^{-25}$ vs 10^{-21} (GW140915)
- ★ Discovery of a persistent source will be the capstone of GW astronomy:
 - ★ repeatable studies,
 - ★ access to ‘cold’ dense-matter equation of state of NSs, but also
 - ★ testing GR (polarizations etc.),
 - ★ searches for dark matter & exotic particles,
 - ★ detectors’ calibration, ”distance ladder”/cosmography.

Example: a monochromatic signal



In this case a Fourier transform is sufficient to detect the signal (simplest **matched filter method**):

$$F = \left| \int_0^{T_0} x(t) \exp(-i\omega t) dt \right|^2$$



T_0 - time series duration, S_0 - spectral density of the data.

$$\text{Signal-to-noise } SNR = h_0 \sqrt{\frac{T_0}{S_0}}$$

Example: the \mathcal{F} -statistic

\mathcal{F} -statistic estimates how well the amplitude and phase modulated model matches the data $x(t)$

$$\mathcal{F} = \frac{2}{S_0 T_0} \left(\frac{|F_a|^2}{\langle a^2 \rangle} + \frac{|F_b|^2}{\langle b^2 \rangle} \right)$$

where S_0 is the spectral density, T_0 is the observation time, and

$$F_a = \int_0^{T_0} x(t) a(t) \exp(-i\phi(t)) dt, \quad F_b = \dots$$

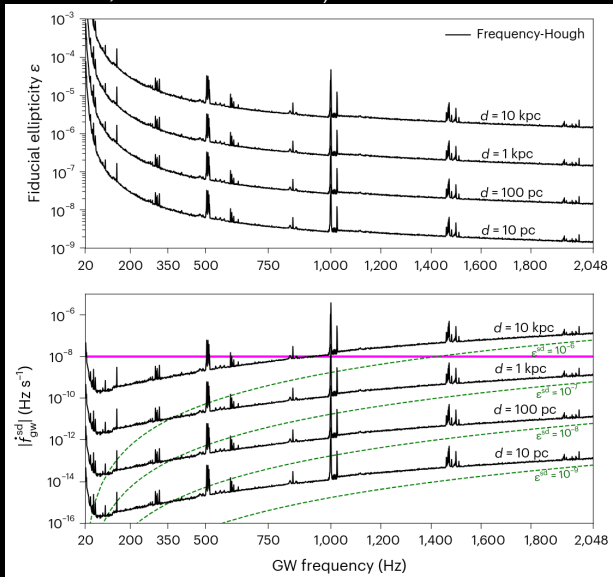
$a(t)$, $b(t)$ - amplitude modulation functions that depend on the sources' sky position (α, δ) ,

$\phi(t)$ - phase modulation function that depends on $(f, \dot{f}, \alpha, \delta)$

(PRD 58, 063001, 1998)

All-sky CW search in LIGO O3a and O3 data

(PRD **103**, 064017 2021, arXiv:2201.00697)



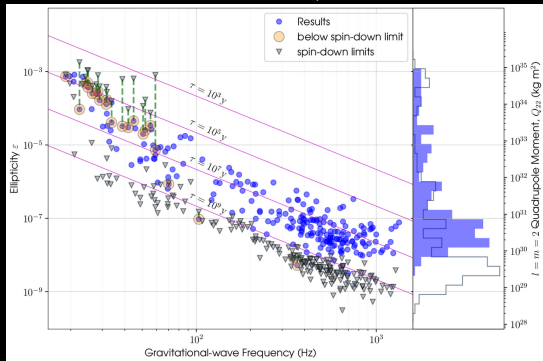
O3 search for GWs from known pulsars (ApJ 935, 1 2022)

236 known pulsars analyzed at $l = m = 2$ and $l = 2, m = 1$ mode.

★ 23 targets surpass the spin-down limit.

Highlights:

- ★ Crab: less than $0.009\% \dot{E}_{rot}$ in GW,
- ★ Vela: less than $0.052\% \dot{E}_{rot}$.

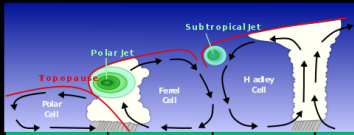


★ Two millisecond pulsars below the spin-down limit, including J0711-6830 with $f_{rot} \simeq 182$ Hz \rightarrow ellipticity $\epsilon < 5.26 \times 10^{-9}$,

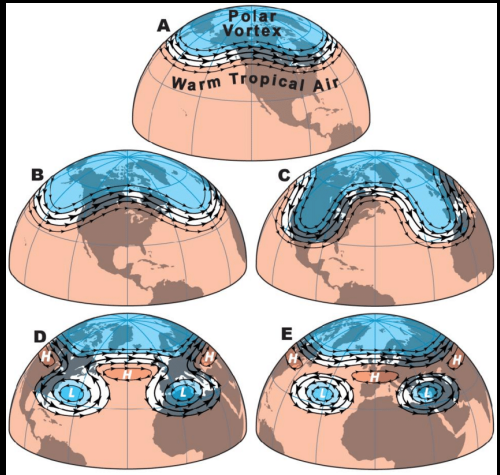
★ Limits for dipole GW emission in the Brans-Dicke theory.

Rossby waves in planetary atmospheres

- ★ Carl-Gustaf Rossby (1898 - 1957),
- ★ A type of inertial planetary wave, driven by Coriolis force (→ present in rotating systems),
- ★ On Earth, associated with high-altitude winds (→ jet stream),



(Wikipedia)



(S. Harris, AIMS Environmental Science 6(1):14-40 2019)

Rossby waves on neutron stars: r-modes

- ★ R-modes belong to a subset of inertial modes supported by rotation (Coriolis force as a restoring mechanism),
- ★ **Retrograde** in frame co-rotating with the star, **prograde** in inertial frame → unstable to Chandrasekhar-Friedman-Schutz (CFS) instability,

- ★ r-mode frequency in the rotating frame:

$$\omega_r = \kappa \Omega, \text{ with } \kappa = \frac{2m}{l(l+1)}.$$

→ for $l = m = 2$, $\omega_r = 2/3\Omega$,

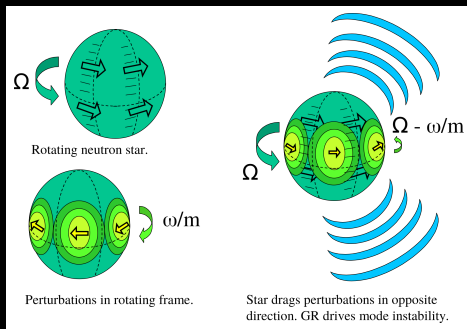
- ★ In the inertial frame:

$$\omega_i = \omega_r - m\Omega = -4/3\Omega.$$

$l = m = 2$ mode frequency in Newtonian approximation is $\omega_i = 4/3\Omega$, in GR corrections related to NS mass and radius (→ EoS):

$$\omega_i = \frac{4}{3}\Omega \left(1 + C_1 \frac{GM}{Rc^2} - C_2 \left(\frac{GM}{Rc^2} \right)^2 \right),$$

where the $(1 + \dots)$ corrections are due to GR, rapid rotation, NS crust, matter stratification, magnetic fields. . . (Idrisy et al. 2015, Phys. Rev. D 91, 024001)



Relation between the GW and spin frequencies

The r-mode GW frequency f depends on the NS spin frequency ν and NS structure (e.g., [Idrisy et al. 2015](#)), which can be expressed as ([Caride et al. 2019, Phys. Rev. D, 100, 064013](#)):

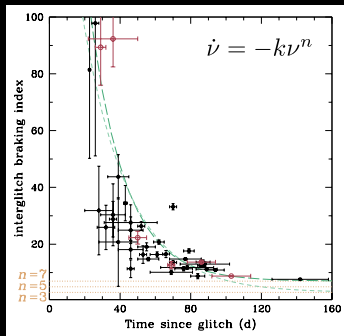
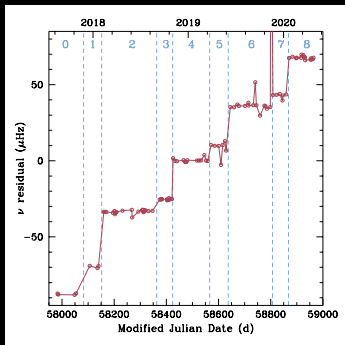
$$\begin{aligned}f &= A\nu - B \left(\frac{\nu^2}{\nu_K^2} \right) \nu, \\ \dot{f} &= A\dot{\nu} - 3B \left(\frac{\nu^2}{\nu_K^2} \right) \dot{\nu}, \\ \ddot{f} &= A\ddot{\nu} - \left(3 + \frac{6}{n} \right) B \frac{\nu^2}{\nu_K^2} \ddot{\nu}.\end{aligned}$$

with constants A and B dependent on the EoS (assumption: they don't change between the glitches). A describes the effects of GR, B the effects of rotation; ν_K denotes the Keplerian (mass-shedding) frequency.

$$\begin{aligned}1.39 < A < 1.57, \\ 0 < B < 0.195.\end{aligned}$$

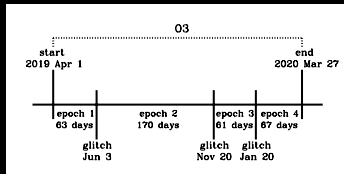
PSR J0537–6910 a.k.a. the Big Glitcher

Young (1-5 kyrs) energetic X-ray pulsar in the LMC, rotating at $\nu = 62$ Hz:



★ Timing (ν , $\dot{\nu}$, $\ddot{\nu}$) solution provided by the NICER instrument, covering the O3 LIGO-Virgo observing run (Ho et al., 2020, MNRAS 498, 4605-4614).

★ Pulsar is associated with a SN remnant N157B \rightarrow orientation: polarization $\psi = 2.2864$ and inclination angles $\iota = 1.522$.

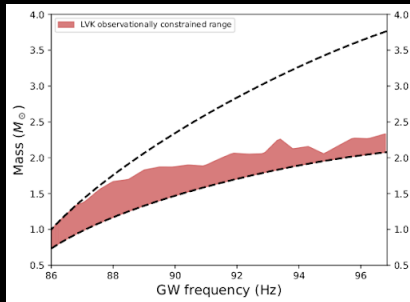
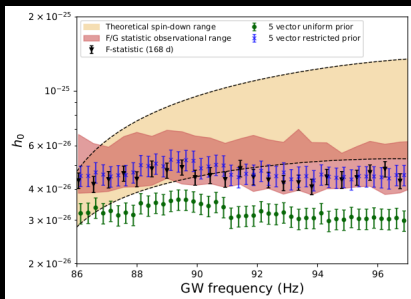


(Figs. 1 and 6 of Ho et al., 2020; Fig. 1, arXiv:2104.14417)

Frequently glitching X-ray pulsar J0537-6910

J0537 is an energetic X-ray pulsar rotating at $f = 62$ Hz & rapidly spinning-down. Is some of the spin-down due to GW emission?

Search for r-modes, $f_{GW} \approx 4/3f \in 86 - 97$ Hz GW emission (ApJ **922**, 71 2021):



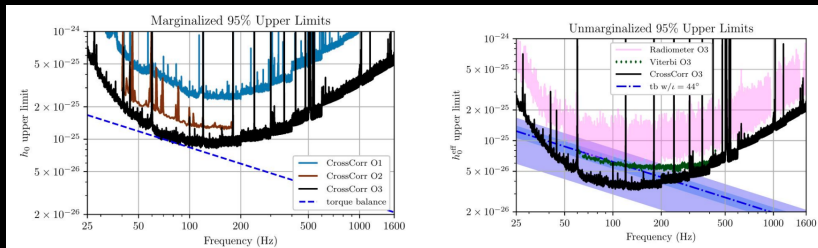
GW frequency of r-mode is a function of mass and radius of the NS \rightarrow equation of state.

Sco X-1

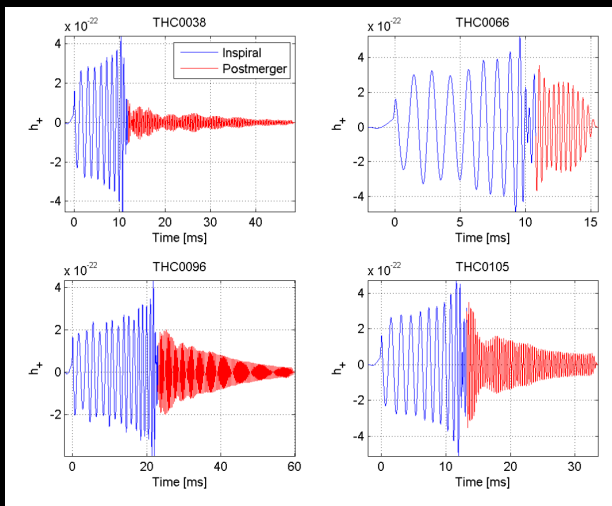
Two searches for GWs from the low-mass X-ray binary Scorpius X-1 (NS in a binary system, with unknown orientation and frequency parameters):

- ★ [PRD 106, 062002 \(2022\)](#) - Hidden-Markov model search
- ★ [arXiv:2209.02863](#) - semi-coherent CrossCorrelation search with stronger astrophysical model assumptions

Polarization-averaged upper limit reaches conservative torque balance prediction. (“accretion from binary companion is completely balanced by GWs”)

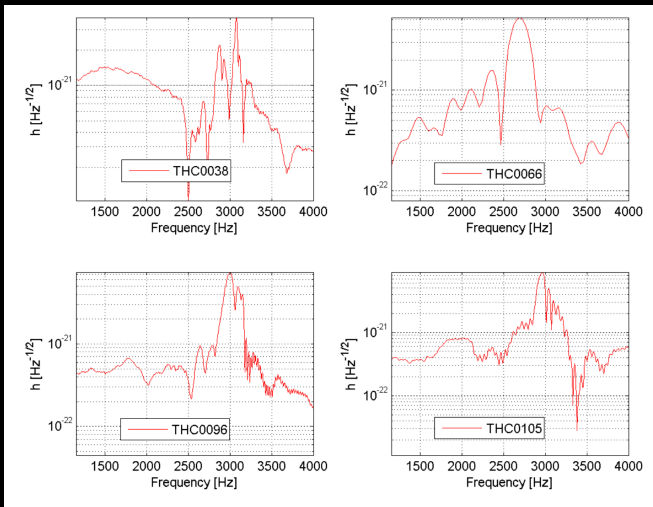


Search for post-merger signals (CQG 40 215008 2023)



Waveforms consistent with GW170817 event, available in the 2nd release of CoRe database (CQG 40 085011 2023)

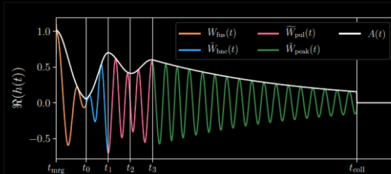
Post-merger signals



Postmerger spectra dominated by one frequency, f_{max} , related to the rotation of the bar-deformed high mass neutron star.

Post-merger signal: an approximation

Signal can be approximated by a sum of exponentially damped sinusoids



Morphology of NRPMw model
(from Fig. 2 of Breschi et al.).

We approximate the postmerger signal by **one** damped sinusoid with 3 intrinsic parameters: **frequency** (f), **frequency drift** (γ), and **damping time** (τ) (we display here waveform of single + or \times polarization of the PM signal),

$$h(t) = h_o e^{-t/\tau} \cos(2\pi f t + 2\pi \gamma t^2 + \phi_o).$$

Post-merger signals

A universal relation between frequency f_{max} and an average compactness \mathcal{C} of the binary neutron star:

$$f_{max} \simeq b_0 + b_1\mathcal{C} + b_2\mathcal{C}^2 \text{ [kHz]},$$

where $b_0 = -3.12$, $b_1 = 51.90$ and $b_2 = -89.07$ and

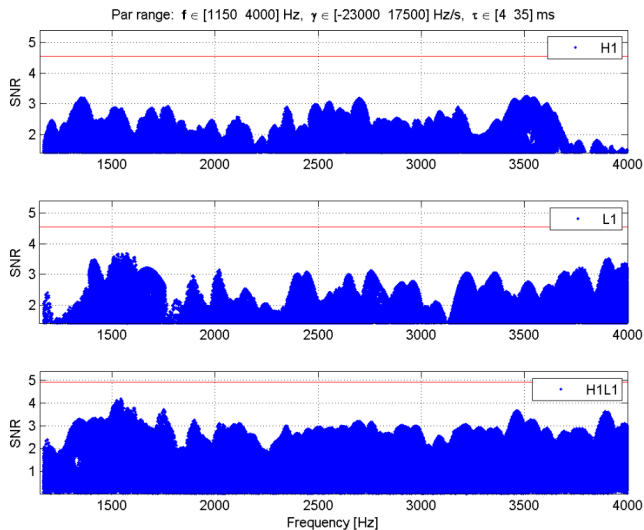
$$\mathcal{C} = \frac{\bar{M}}{\bar{R}},$$

where \bar{M} and \bar{R} are average mass and radius of the binary respectively.

\bar{M} can be accurately determined from the inspiral phase and thus \bar{R} can be determined from f_{max} estimate in the postmerger phase¹.

¹Bose et al., Phys. Rev. Lett., 120:031102 (2018)

Triggers of the GW170817 postmerger search



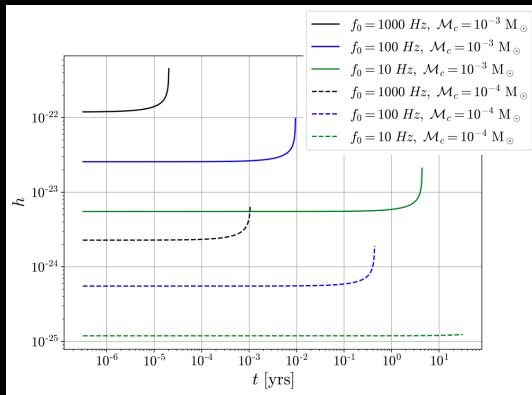
Horizontal red line — 1% false alarm probability threshold.

Other CW sources: low-mass primordial BHs

Inspiral of low-mass binary, with $\mathcal{M}_c = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5} \approx 10^{-3} - 10^{-5} M_\odot$ results in slow evolution of chirp frequency \approx CW signal

Evolution of GW frequency:

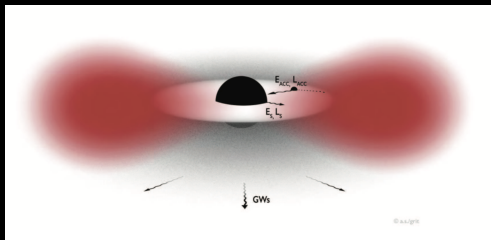
$$\frac{df}{dt} = \frac{96}{5} \pi^{8/3} \left(\frac{G \mathcal{M}_c}{c^3} \right)^{5/3} f^{11/3} \times (1 + \text{PN corrections})$$



(Credit: Marc Andrés-Carcasona & Ornella Piccinni)

Other CW sources: boson clouds near BHs

- ★ CW emission from (hypothetical) light boson particles forming clouds around spinning BHs,
- ★ bosons will collide with each other over time, resulting in GW emission at almost constant frequency.



(Credit: Richard Brito, Vitor Cardoso, Paolo Pani)

(LVK analyses with O3 data: "Search for continuous gravitational wave emission from the Milky Way center in O3 LIGO-Virgo data", [PRD 106, 042003 2022](#), "All-sky search for gravitational wave emission from scalar boson clouds around spinning black holes in LIGO O3 data", [PRD 105, 102001 2022](#))

Summary and O4 outlook

- ★ Searches for signals with **more complicated morphology, transient aspects, loosely-coherent approach**:
 - ★ accounting for NSs glitches (sudden changes in spin frequency),
 - ★ hierarchical follow-up of transient CW-like candidates,
 - ★ using machine-learning algorithms.
 - ★ NS spin frequency wandering,
 - ★ mismatch between the GW frequency (and spindown) and the parameters inferred from EM observations
 - ★ Focus on interesting targets like Sco X-1, supernovæ remnants (CasA, Vela Jr, G347, Crab)
 - ★ GWs from r-modes and at multiple frequencies at once,
 - ★ post-merger emission,
 - ★ Dark matter constituents (also dark photon dark matter interacting directly with the detectors (as in PRD **105**, 063030 2022)
 - ★ non-tensorial GWs.

(Astro)physics questions

- ★ Magneto-elastic 'mountains': elastic properties of the crust, braking strain,
 - ★ Thermally induced quadrupole: accretion processes, heating reactions in the crust,
 - ★ Instabilities (*r*-modes): heating & cooling, rotational evolution,
 - ★ Superfluidity.
-
- ★ Conditions at birth: SN ↔ NS deformation connection,
 - ★ Long-term evolution of NS asymmetry,
-
- ★ Dark matter: PBHs, boson clouds, dark photons. . .

See Haskell & Bejger, Nature Astronomy (2023):
<https://rdcu.be/dmIeq>

Extra slides

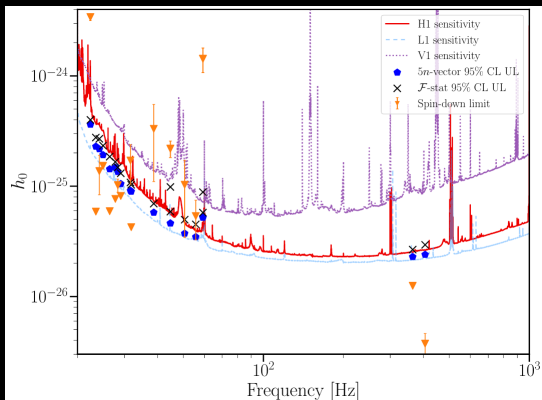
O3 narrowband known pulsars search (ApJ 932, 133 2022)

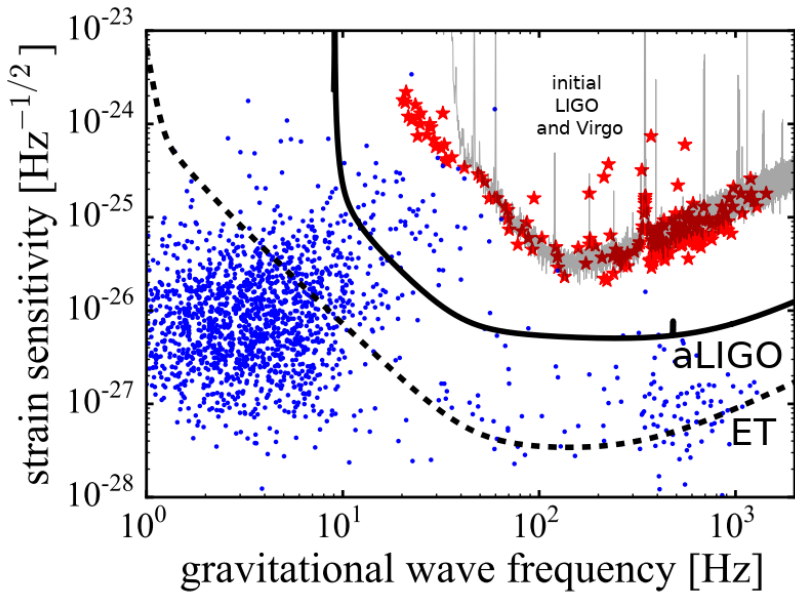
Usually assumed that GW signal is phase-locked with the pulsar EM emission.

Here, a small possible mismatch between the assumed and true signal phase evolution:

$$\Delta f_{gw} = f_{gw} (1 + \delta),$$
$$\delta \sim 10^{-3} - 10^{-4}$$

- ★ 18 pulsars, 7 surpassing the spin-down limit,
- ★ 6 glitching pulsars targeted for long-duration (hours-months) transient GWs





Example: computational cost for an all-sky search

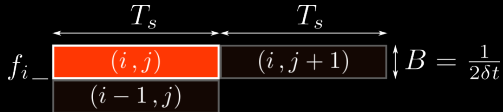
In order to optimally cover a range of $(f, \dot{f}, \alpha, \delta)$ parameters,

$$\text{computing power} \propto \underbrace{T_0^2}_f \times \underbrace{T_0^{[0-3]}}_{\alpha, \delta} \times \underbrace{T_0 \log(T_0)}_{f \text{ by FFT}} = T_0^{[3-6]} \log(T_0).$$

(see PRD **90**, 122010, 2014). Coherent search of $T_0 \simeq 1$ yr of data would require **zettaFLOPS** (10^{21} FLOPS) scale computers \rightarrow **currently impossible**

☹

Solution: divide data into shorter length time frames ($T_s \simeq$ days)



- ★ Perform a search in narrow frequency bands: sampling time $\delta t = 1/2B$, number of data points $N_p = T_s/\delta t = 2T_sB$

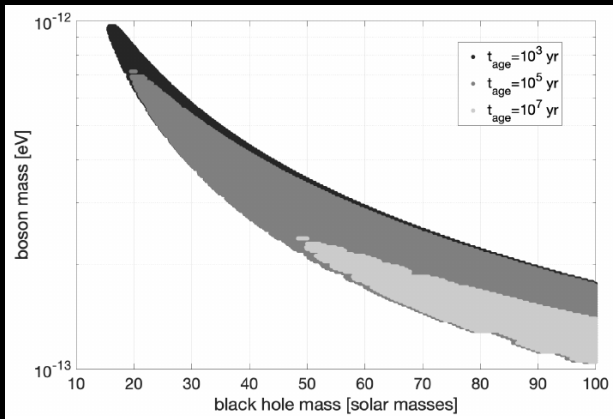
\rightarrow feasible on a petaFLOP computer.

Second stage: look for coincidences between different T_s segments.

Third stage: Analyze interesting outliers ("targeted search").

O3 constraints on boson clouds

"Search for continuous gravitational wave emission from the Milky Way center in O3 LIGO-Virgo data", [PRD 106, 042003 2022](#)



Constraints in the BH mass–boson mass plane for sources at the Galactic center. Clouds with different ages are considered (younger clouds emit stronger GWs). Shaded areas are excluded by observations.