Simulation of Transfer Function
and
Improved Analytical Understanding of Filter

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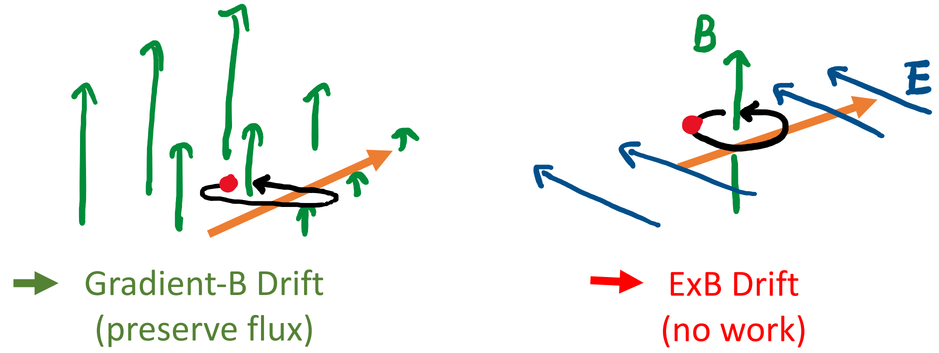
Transverse Drift Filter - Principles of Operation

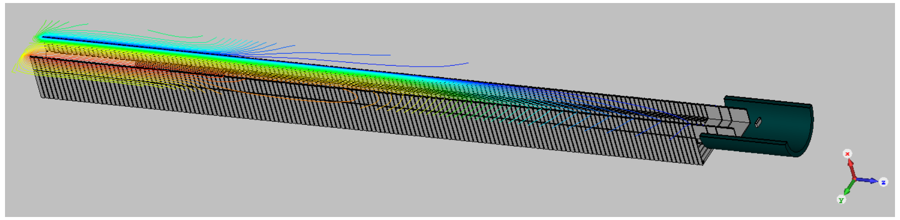
$$\rho_c \ll \left|\frac{B}{\nabla B}\right|, \left|\frac{E}{\nabla E}\right| \text{ ; and}$$

$$\tau_c \ll \left|\frac{B}{dB/dt}\right|, \left|\frac{E}{dE/dt}\right| \text{ ;}$$

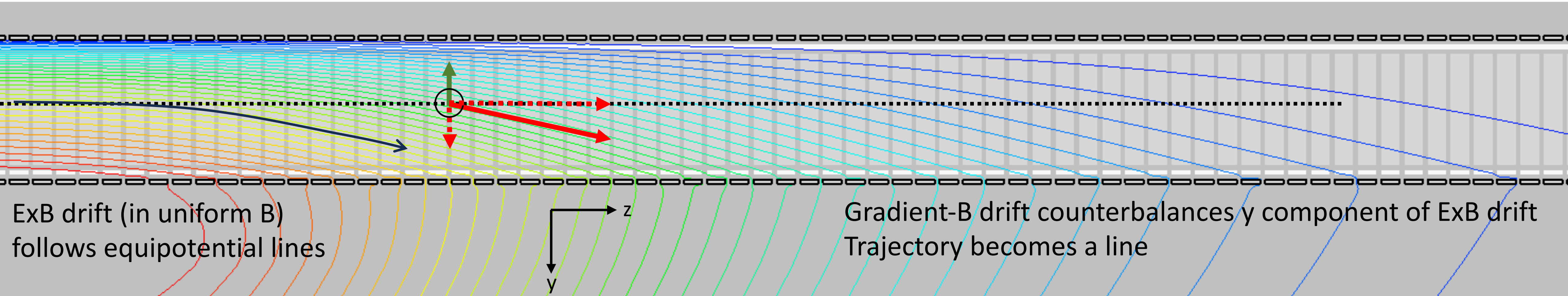
Adiabatic Field Conditions

- Cyclotron motion of charged particles in a magnetic field
  - Work in "Guiding Center System" (GCS) of cyclotron motion
- GCS drift terms in <u>adiabatic</u> field conditions
  - 0^{th} order ExB drift
  - 1^{st} order Gradient-B and Curvature drifts
- Balance drifts to produce a linear trajectory transverse to B
  - Potential increases along trajectory, draining transverse KE
  - Drifts are not balanced for particles with incorrect transverse KE
    - ➢ Gradient-B drift dependent on transverse momentum
- Everything is Lorentz force law





Equipotential lines in yz plane of filter



Analytical Filter Fields and Drift Balancing

- Given the following field configuration:

$$V(x,y,z) = T'_\perp \sin\left(\frac{y}{\lambda}\right) e^{-z/\lambda}$$

$$E_x = 0$$

$$E_y = \frac{T'_\perp}{\lambda} \cos\left(\frac{y}{\lambda}\right) e^{-z/\lambda}$$

$$E_z = -\frac{T'_\perp}{\lambda} \sin\left(\frac{y}{\lambda}\right) e^{-z/\lambda}$$

$$B_x = B_0 \cos\left(\frac{x}{\lambda}\right) e^{-z/\lambda}$$

$$B_y = 0$$

$$B_z = -B_0 \sin\left(\frac{x}{\lambda}\right) e^{-z/\lambda}$$

- The drift balancing condition along z is:

$$\boldsymbol{V}^y_{E\times B}(z) = \boldsymbol{V}_{\nabla B}(z)$$

$$\frac{\boldsymbol{E}\times\boldsymbol{B}}{B_x^2} = -\frac{\boldsymbol{\mu}\times\boldsymbol{\nabla}_\perp \boldsymbol{B(z)}}{qB(z)}$$

- With ***x*** « ***λ***,

$$\frac{E_z}{B_x}\hat{\boldsymbol{y}} = -\frac{\mu}{qB_x}\frac{dB_x}{dz}\hat{\boldsymbol{y}}$$

$$\approx \frac{E_z}{B} = -\frac{\mu}{qB}\frac{\partial B_x}{\partial z}$$

$$-\frac{T'_\perp}{B_0\lambda}\sin\left(\frac{y_0}{\lambda}\right) = \frac{\mu}{q\lambda}$$

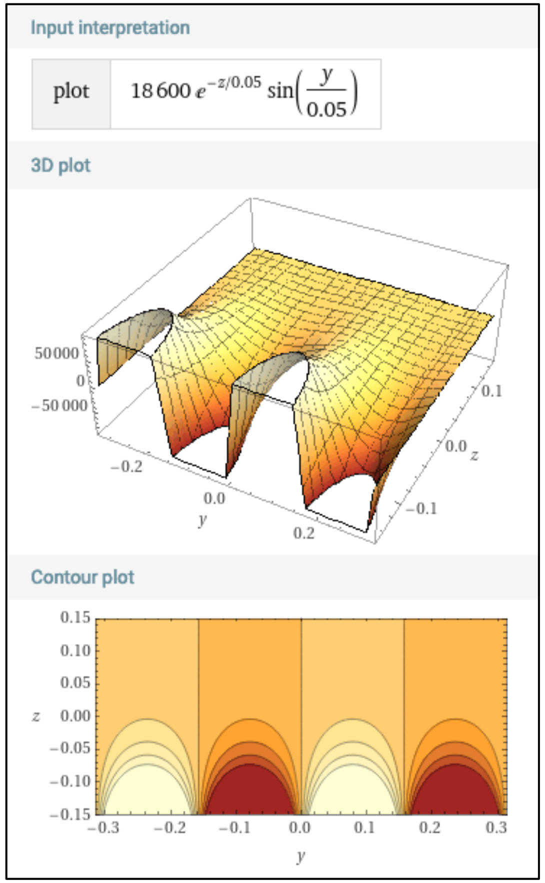
- Solving for $T'_\perp$ and rewriting $\mu = \dfrac{T_{\perp 0}}{B_0 e^{-z_0/\lambda}}$,

$$T'_\perp = -\frac{T_{\perp 0} e^{z_0/\lambda}}{q\sin\left(\frac{y_0}{\lambda}\right)} = -\frac{T_{\perp 0} e^{z_0/\lambda}}{\sin\left(\frac{y_0}{\lambda}\right)}\ [\mathrm{eV}]$$

- Rearranging,

$$\boxed{T_{\perp 0} = -T'_\perp \sin\left(\frac{y_0}{\lambda}\right) e^{-z_0/\lambda}}$$

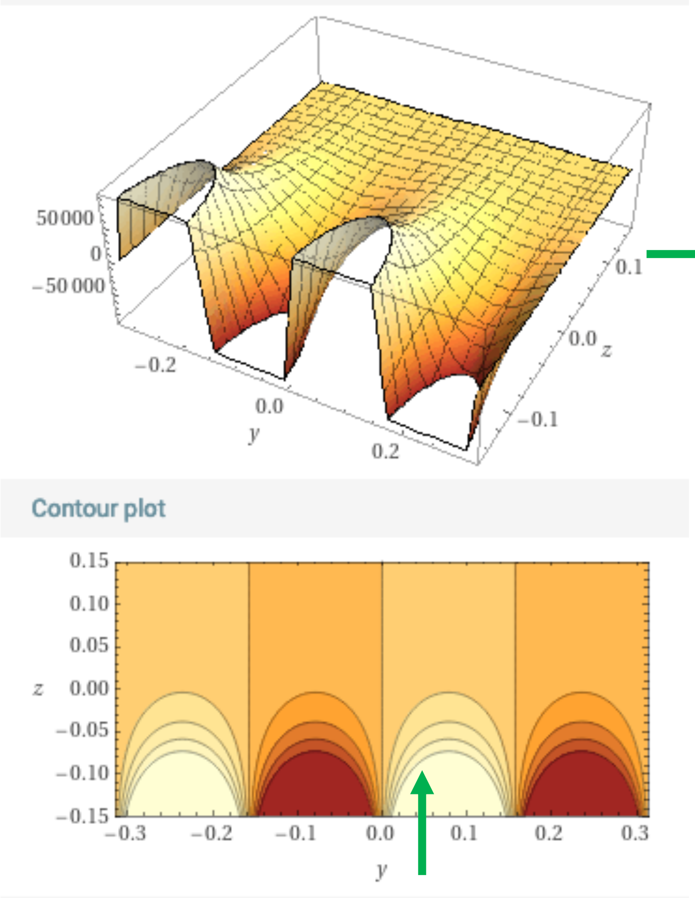
- Which makes clear that the initial $T_{\perp 0}$ needed for a straight trajectory is a function of $T'_\perp$ of the field definition and the initial position (y_0, z_0).



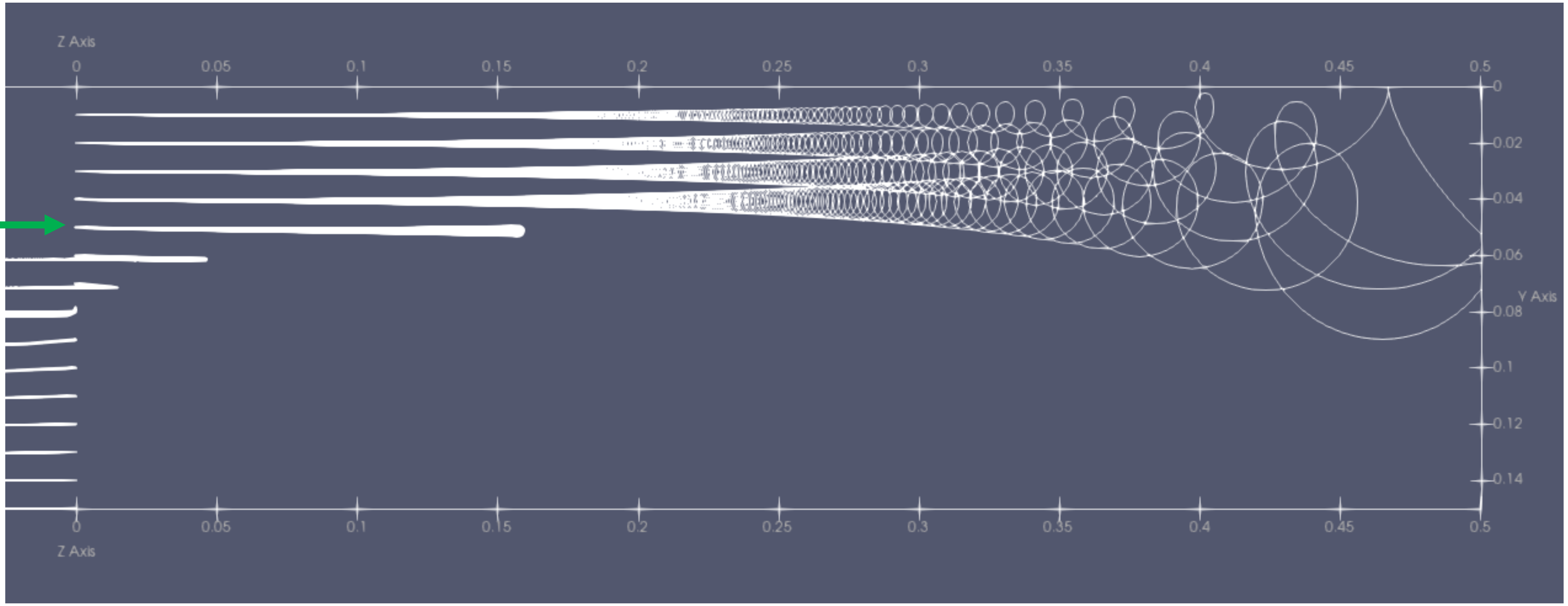
3D/Contour plots of potential

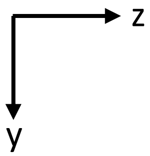
Analytical Forms (cont.)

- Vary y_0 depending on $T_{\perp 0}$ to find straight trajectory
- Only works from $0 < \frac{y_0}{\lambda} < \frac{\pi}{4}$ $(E_y/E_z > 1)$ to direction of z-component of ExB
  - Analogy: Airplane stalling



Kassiopeia Simulation

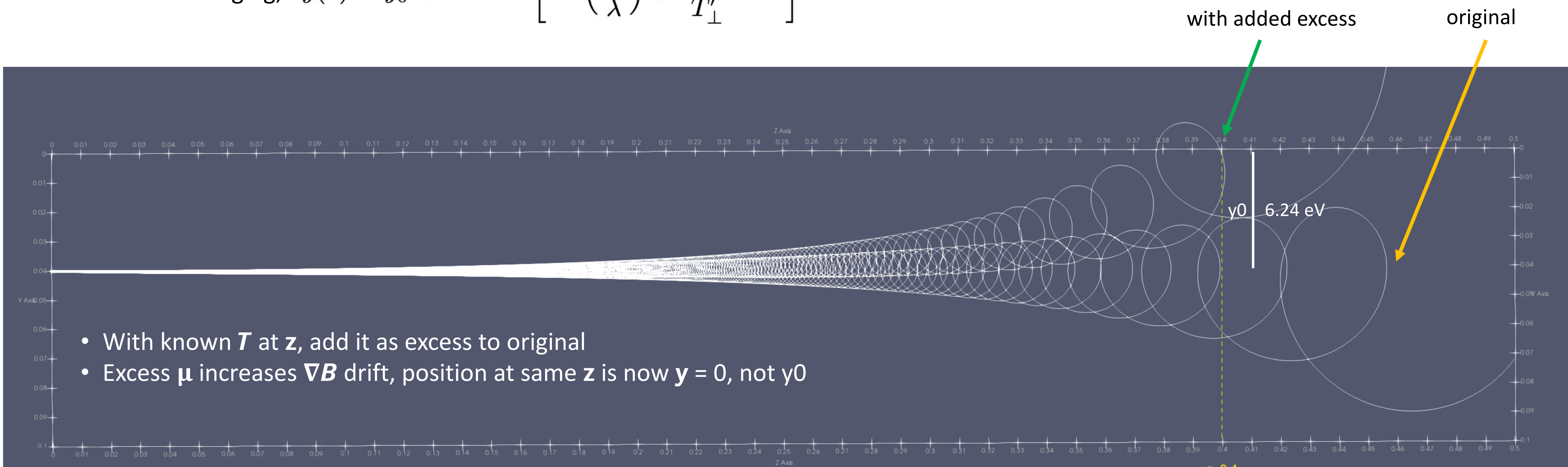




$$T_{\perp 0} = -T'_{\perp} \sin\left(\frac{y_0}{\lambda}\right) e^{-z_0/\lambda}$$

Analytical Trajectory

- For an excess $\Delta T_{\perp} = T_{\perp 0} - T'_{\perp} \sin\left(\frac{y_0}{\lambda}\right) e^{-z_0/\lambda}$
- Express $\Delta T_{\perp}$ along trajectory as $\Delta T_{\perp} = T'_{\perp} e^{-z/\lambda} \left[\sin\left(\frac{y}{\lambda}\right) - \sin\left(\frac{y_0}{\lambda}\right)\right]$
- Rearranging, $y(z) = y_0 + \lambda \arcsin\left[\sin\left(\frac{y_0}{\lambda}\right) + \frac{\Delta T_{\perp}}{T'_{\perp}} e^{z/\lambda}\right]$



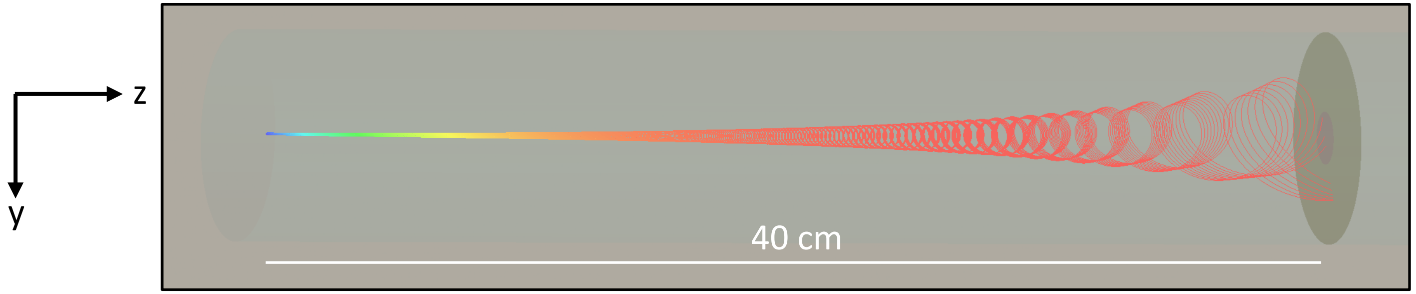
- With known ***T*** at **z**, add it as excess to original
- Excess **μ** increases **∇*B*** drift, position at same **z** is now **y** = 0, not y0

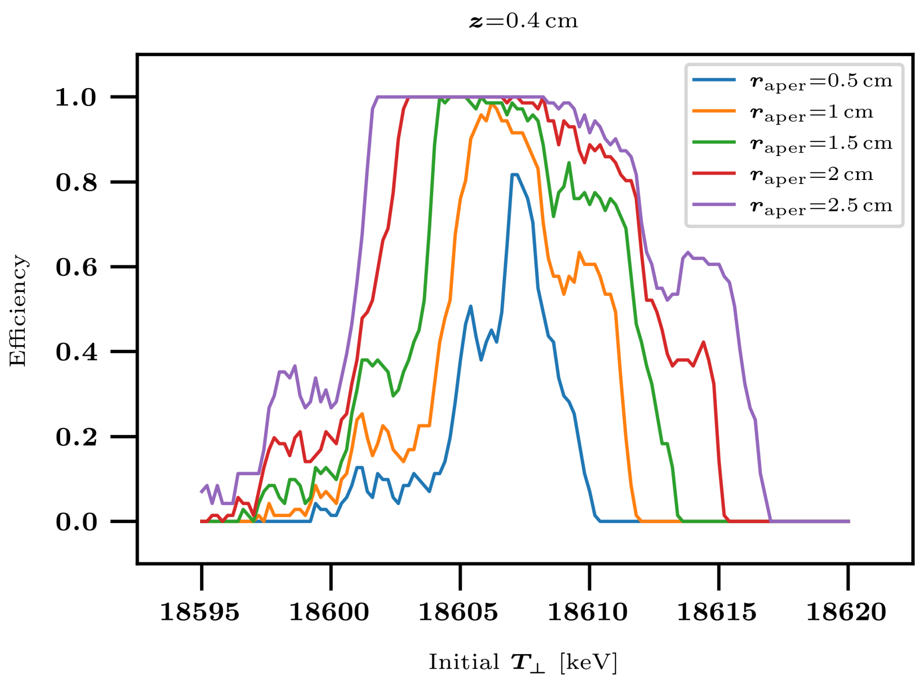
$$37200\ e^{-0.4/0.05} \sin\left(\frac{2.618\ \text{cm}}{0.05\ \text{m}}\right) = 6.24\ \text{eV in } 2.618\ \text{cm at z=0.4 m}$$

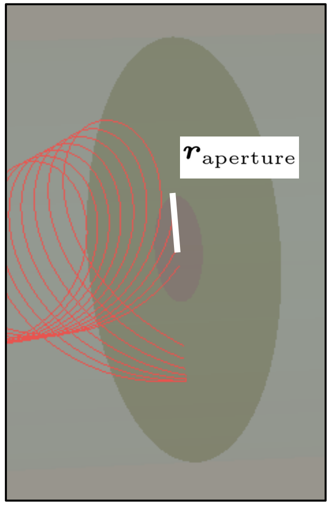
z=0.4

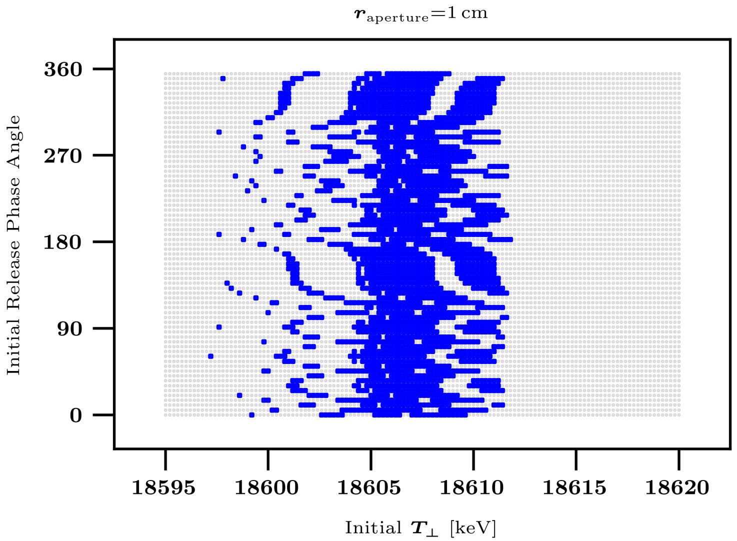
Transfer Function Simulation

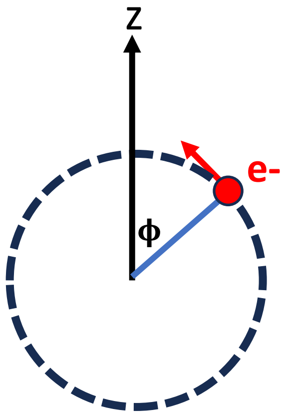
- Release at y_0 at initial phase angle 0-360° in 5° steps (pitch 90°)
- 18595-18620 eV in 0.2 eV steps, N = 8946
- Aperture of radius r_{aperture} centered on y_0 at $z{=}0.4$ m
- Count electrons making it into aperture (no constraints on entry angle)



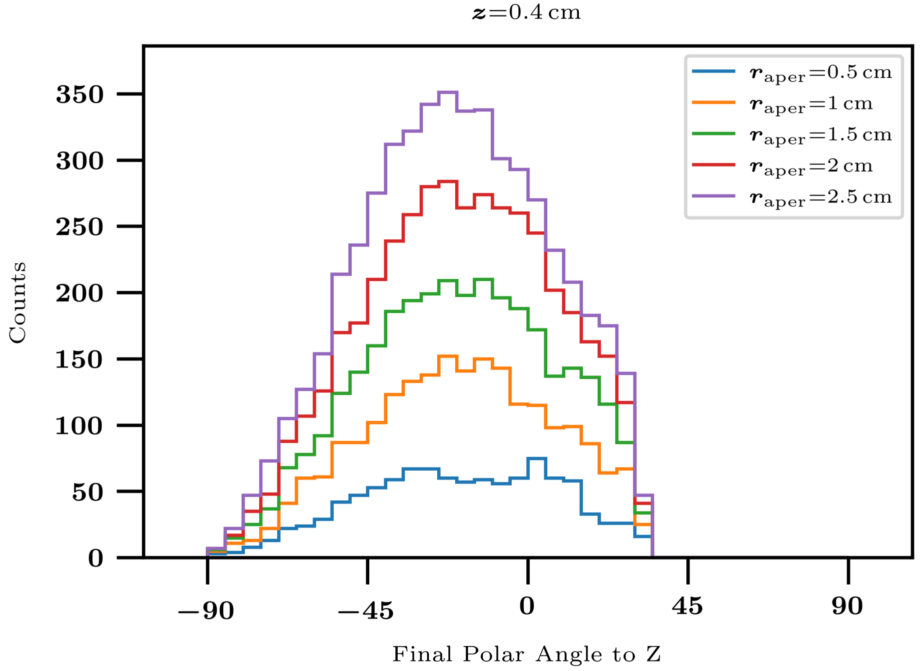








Phase Angle



Summary

- Better understanding of analytical filter leading to simulated transfer function
- Study of analytical forms for parallel field in progress
- Chance to expand PTOLEMY toolkit (Lorentz4) for fast solving of E fields with relaxation methods for numerical studies of parallel momentum acceptance