Axion dark matter:
A perspective for the post-inflationary scenario

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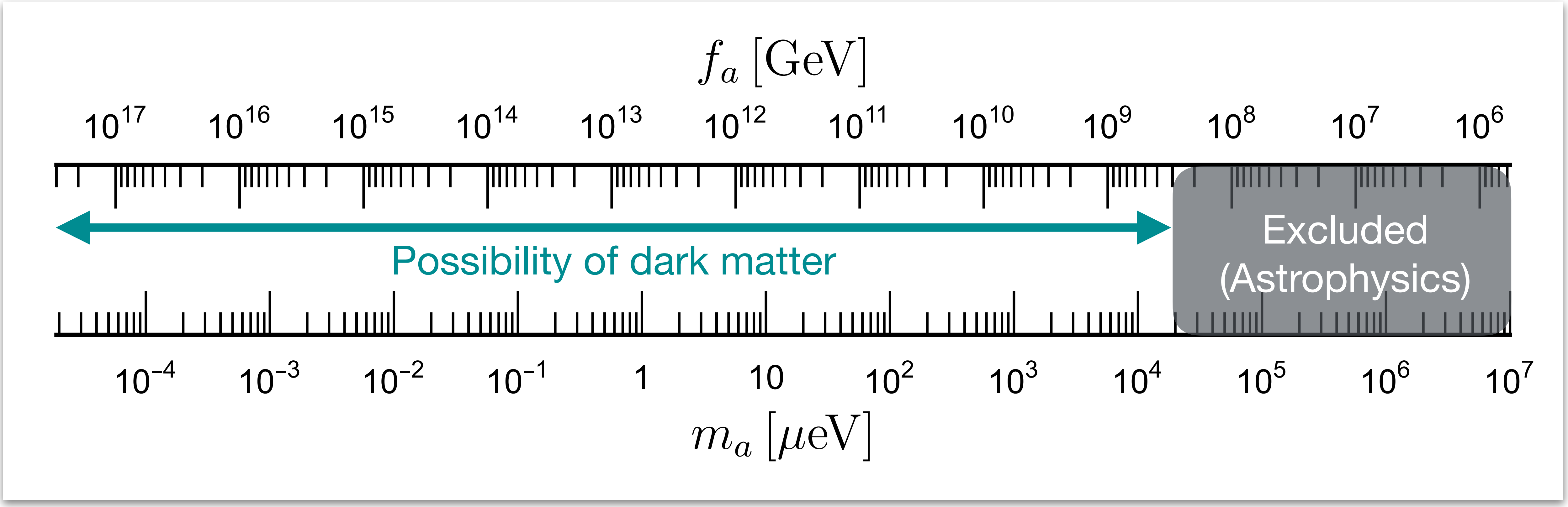
31 May 2023, Colloquium for COST Action "*COSMIC WISPERS*" (zoom)

Axion dark matter mass?

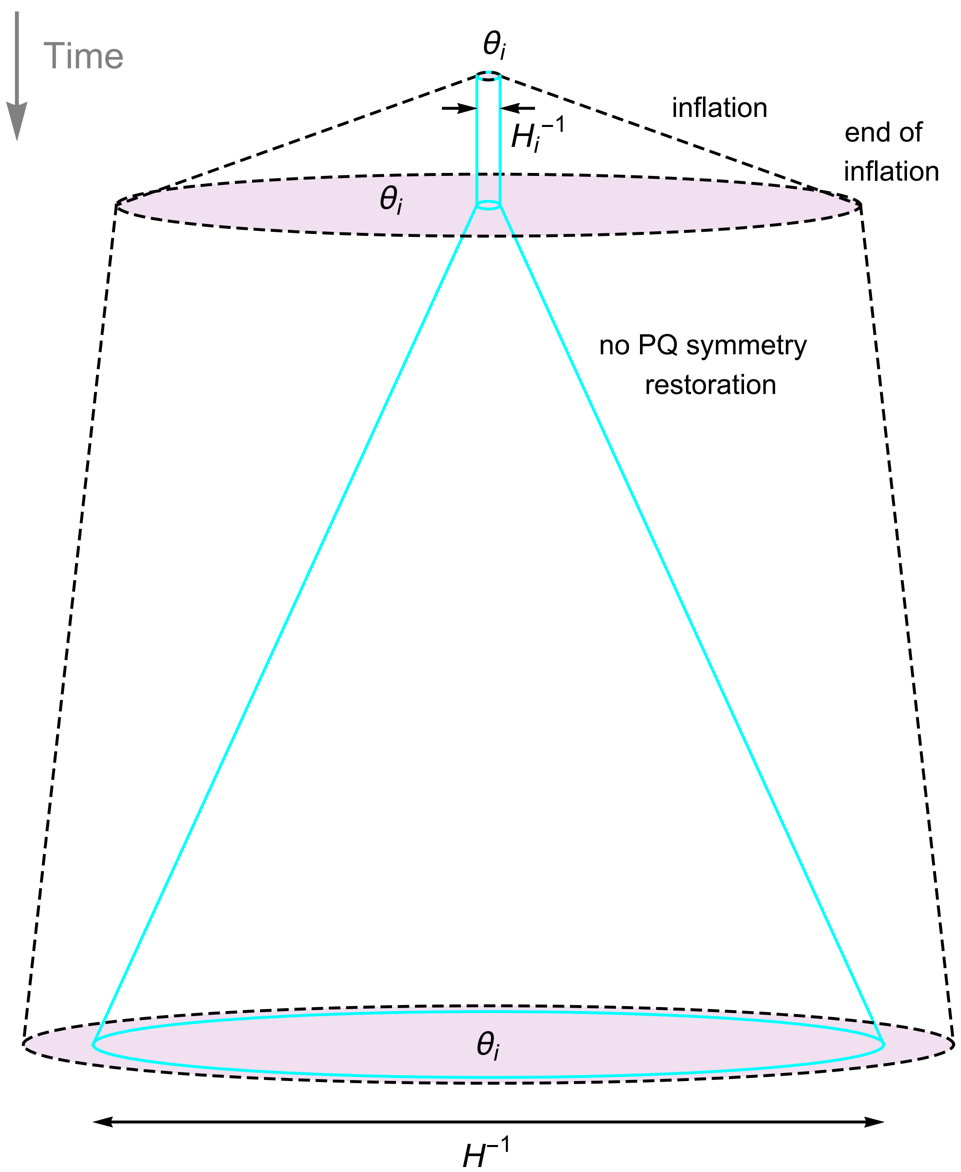


- What is the "typical mass" of the axion dark matter?
- Use cosmology to answer this question...

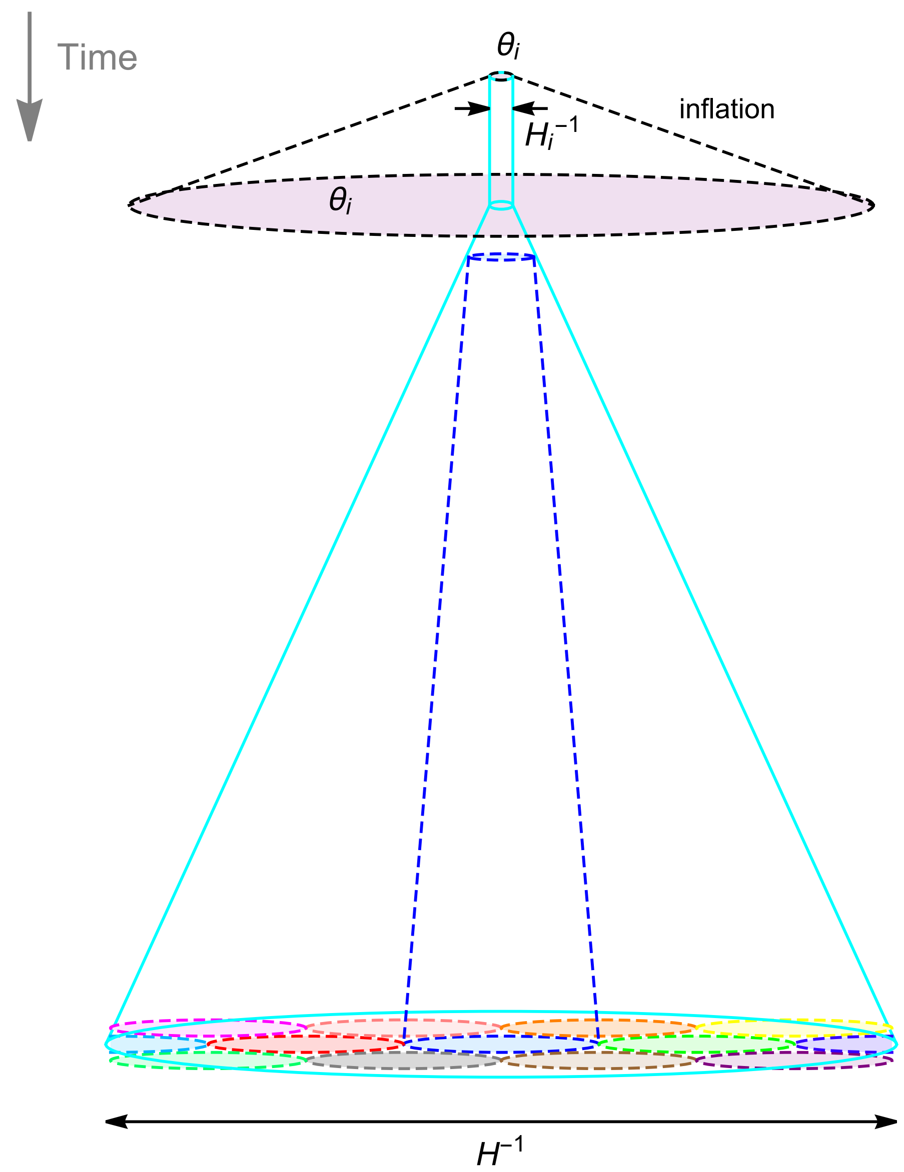
$$\Omega_a(m_a) = \Omega_{\mathrm{CDM}} \quad \Rightarrow \quad m_a = ???\,\mu\mathrm{eV}$$

Initial conditions

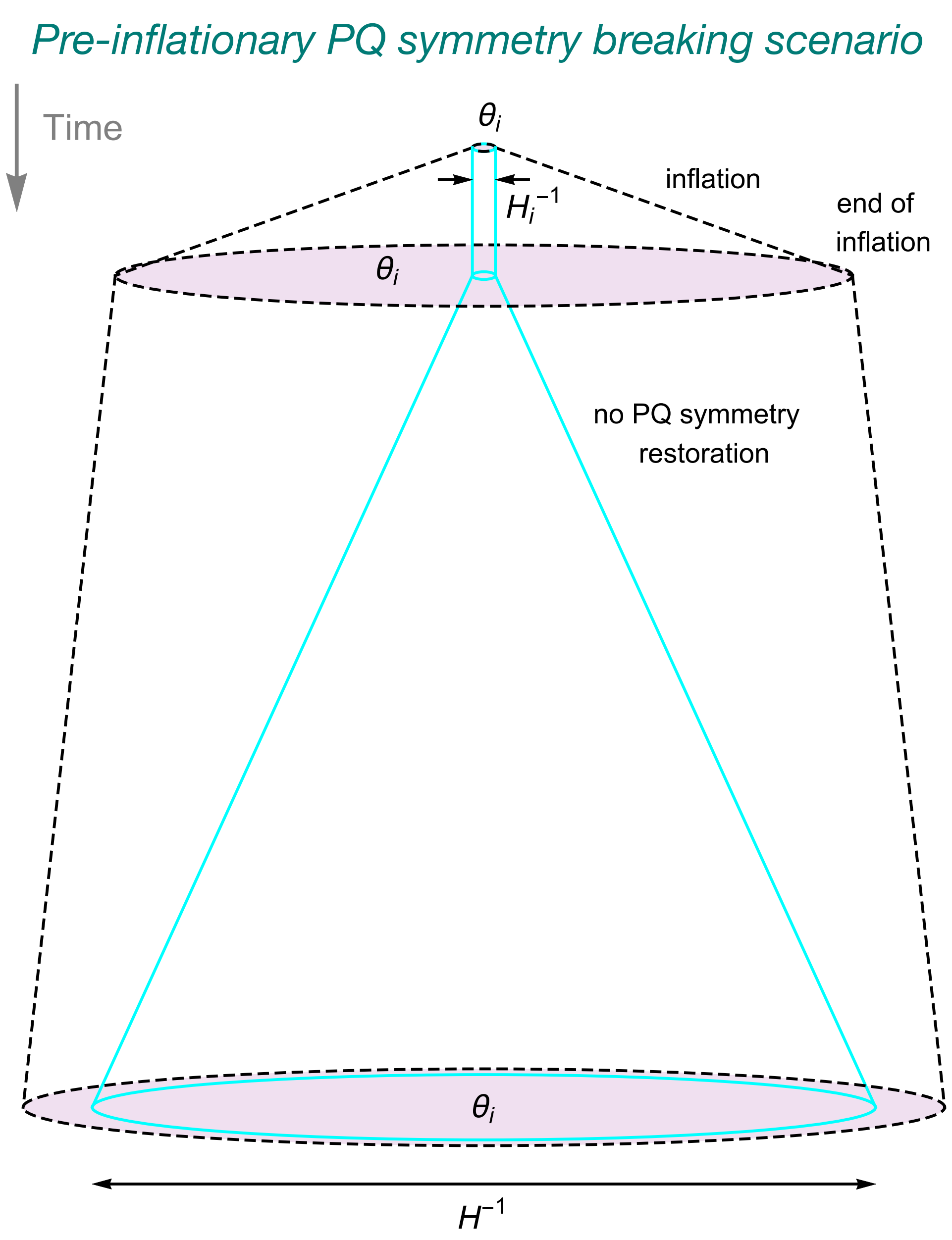
Pre-inflationary PQ symmetry breaking scenario

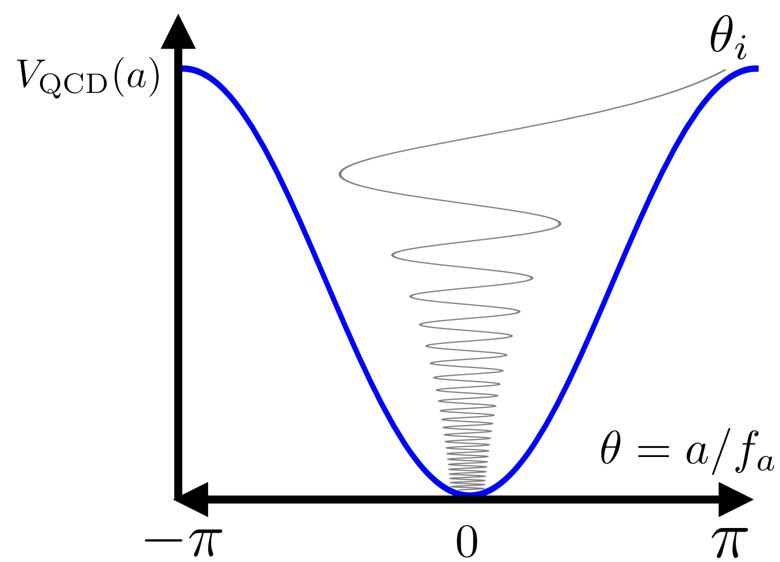


Post-inflationary PQ symmetry breaking scenario



Initial conditions





$$\Omega_a h^2 \simeq 0.14\theta_i^2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{1.17}$$

- Pre-inflationary scenario:
  - Well understood production mechanism (vacuum realignment).
  - Uncertainty due to the unknown initial angle θ_i .

Post-inflationary scenario

- Topological defects
  - Strings

    formed at the PQ phase transition
    ($T \lesssim f_a$)

  - Domain walls

    formed at around the QCD phase transition
    ($T_{\rm QCD} \sim 1\,{\rm GeV}$)

- Inhomogeneity at

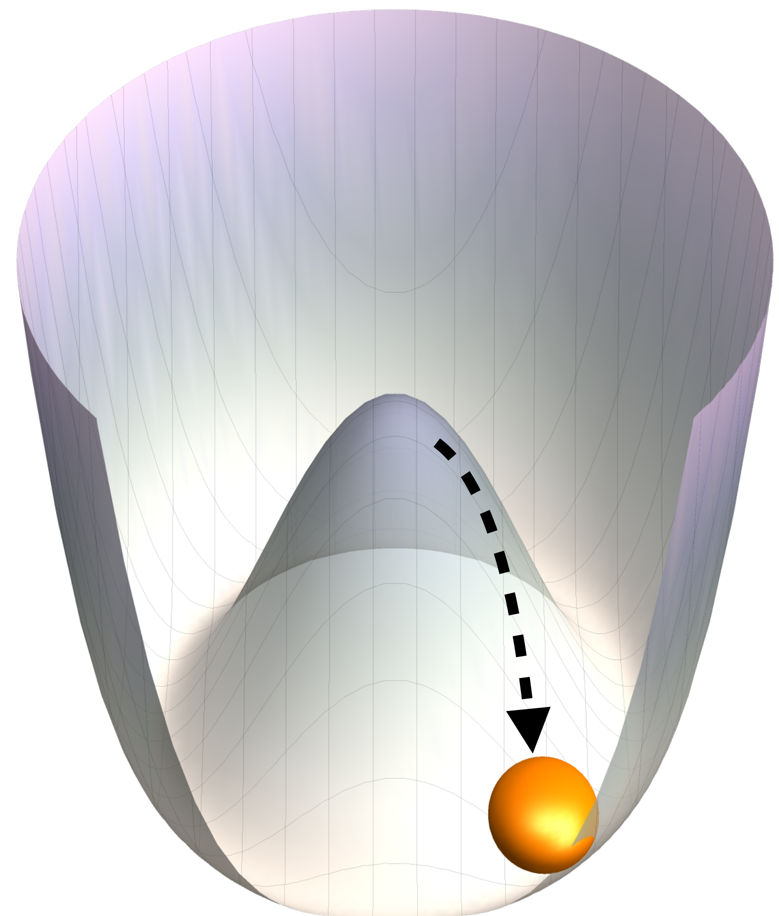
$$L = \frac{R_0}{R_{\rm QCD} H_{\rm QCD}} = 0.036\,{\rm pc}\left(\frac{50\,\mu{\rm eV}}{m_a}\right)^{0.167}$$

➡ Seeds of miniclusters

[Kolb, Tkachev, astro-ph/9311037]
[Vaquero, Redondo, Stadler, 1809.09241]

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi)$$

$$V(\phi) = \lambda\left(|\phi|^2 - \frac{v_{\rm PQ}^2}{2}\right)^2$$



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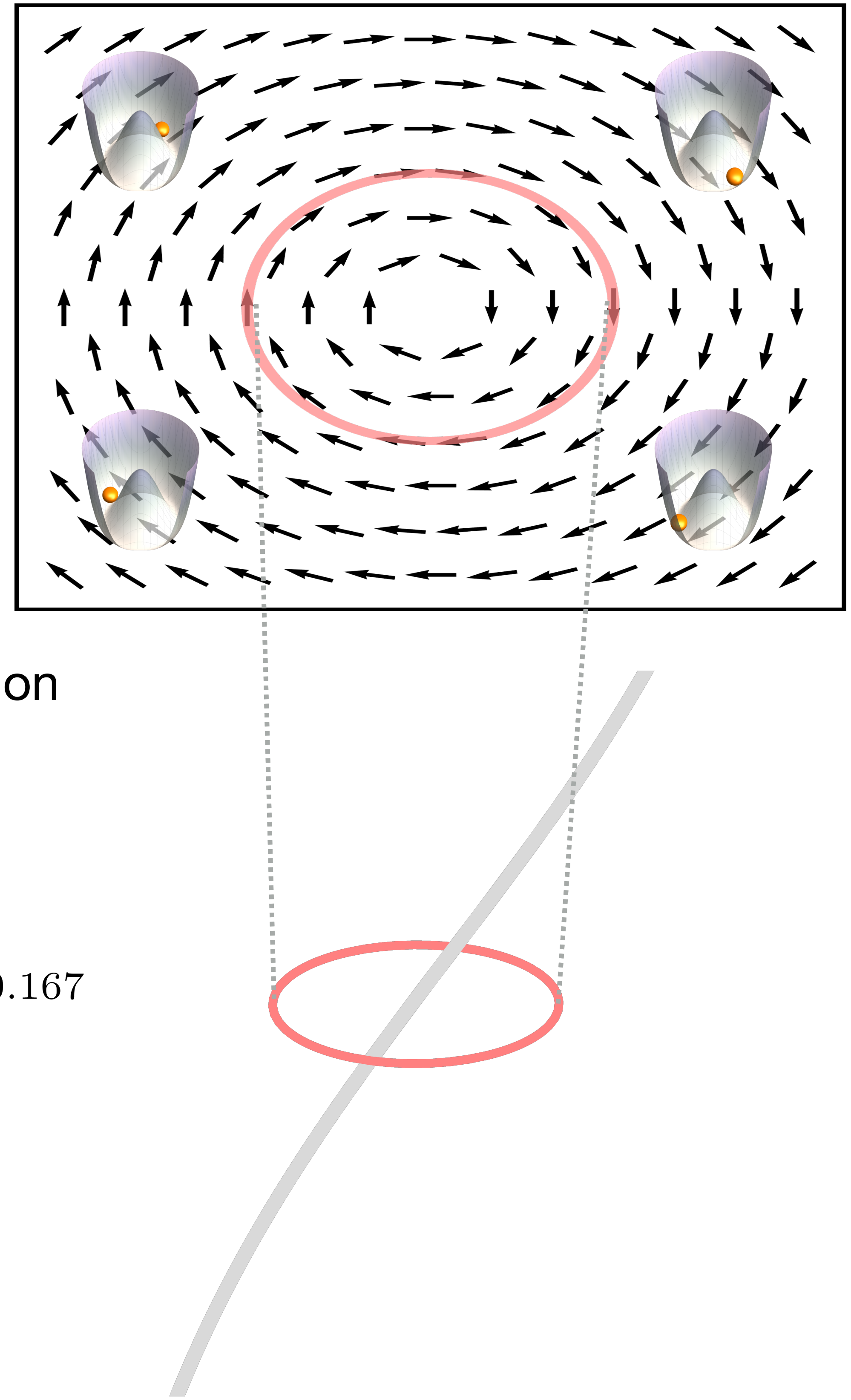

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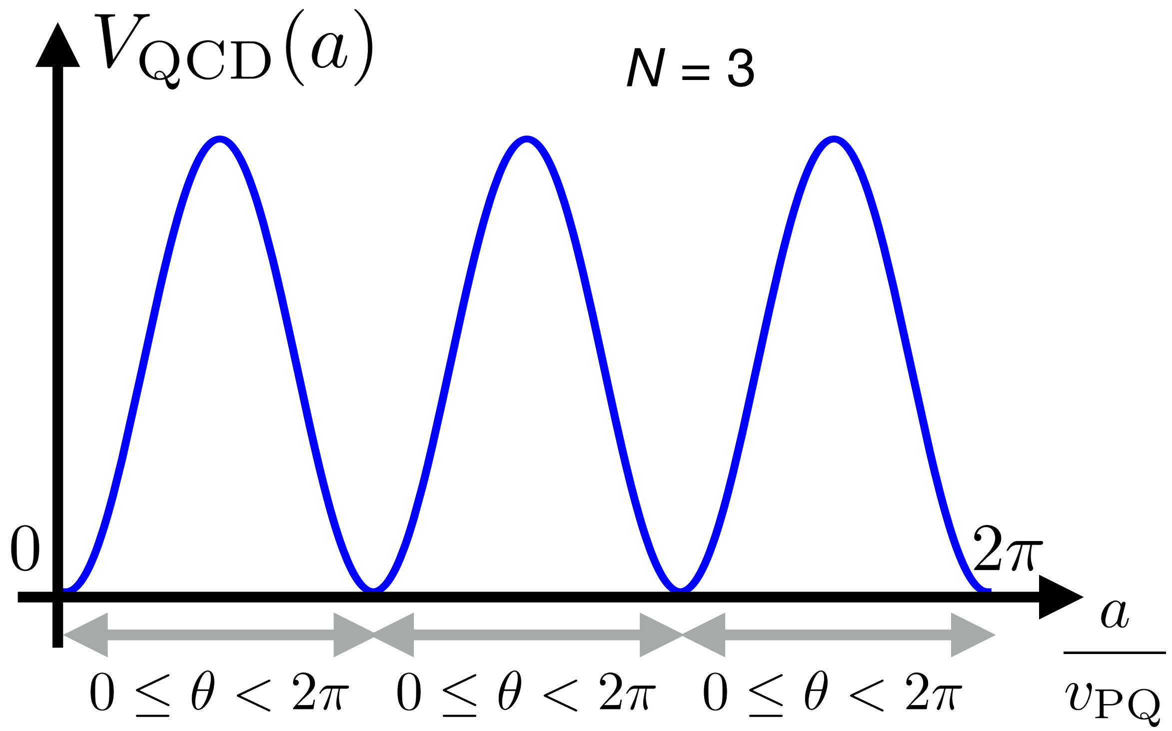
Domain wall formation after the QCD phase transition

- Axion is a periodic angular field defined in the range

$$a/v_{\rm PQ} \in (-\pi, \pi).$$

$v_{\rm PQ}$: symmetry breaking scale

- There can be a difference in periodicity between $a/v_{\rm PQ}$ and θ, creating *N* degenerate minima at low energies.



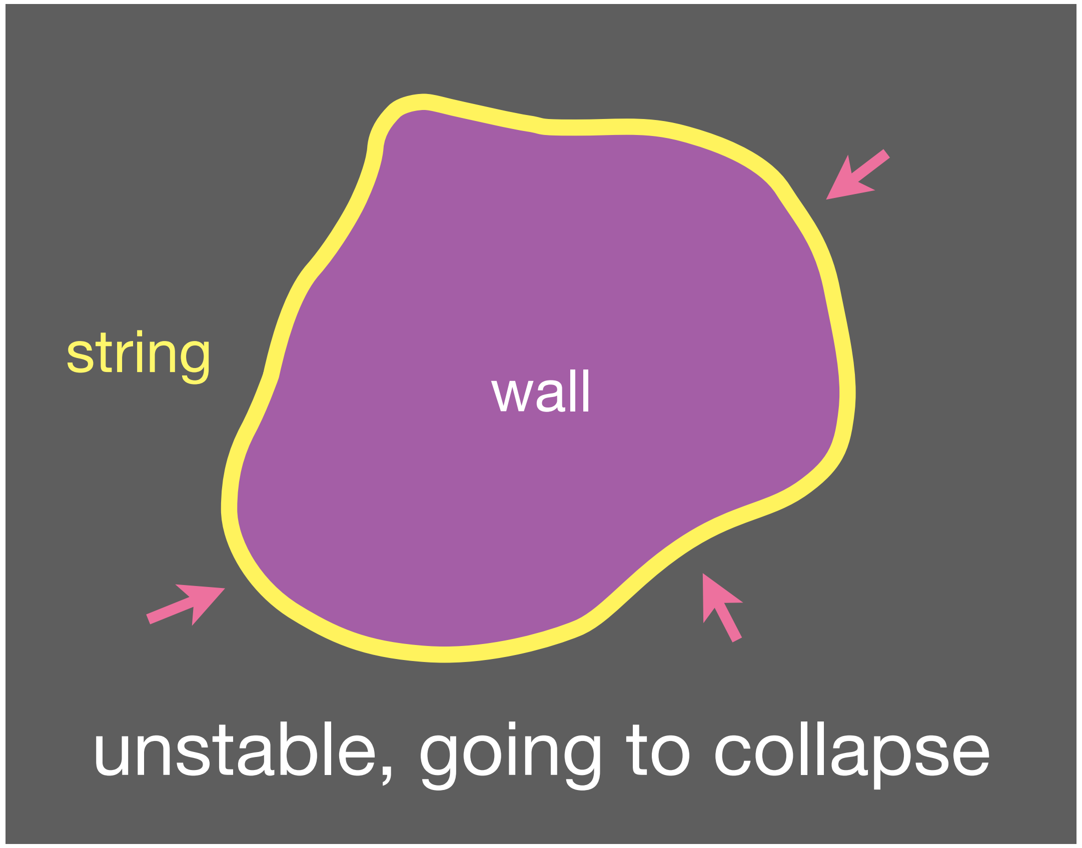
- The "domain wall number" *N* depends on models.

e.g. *N* = 1 for (the simplest version of) KSVZ models
N = 6 for DFSZ models

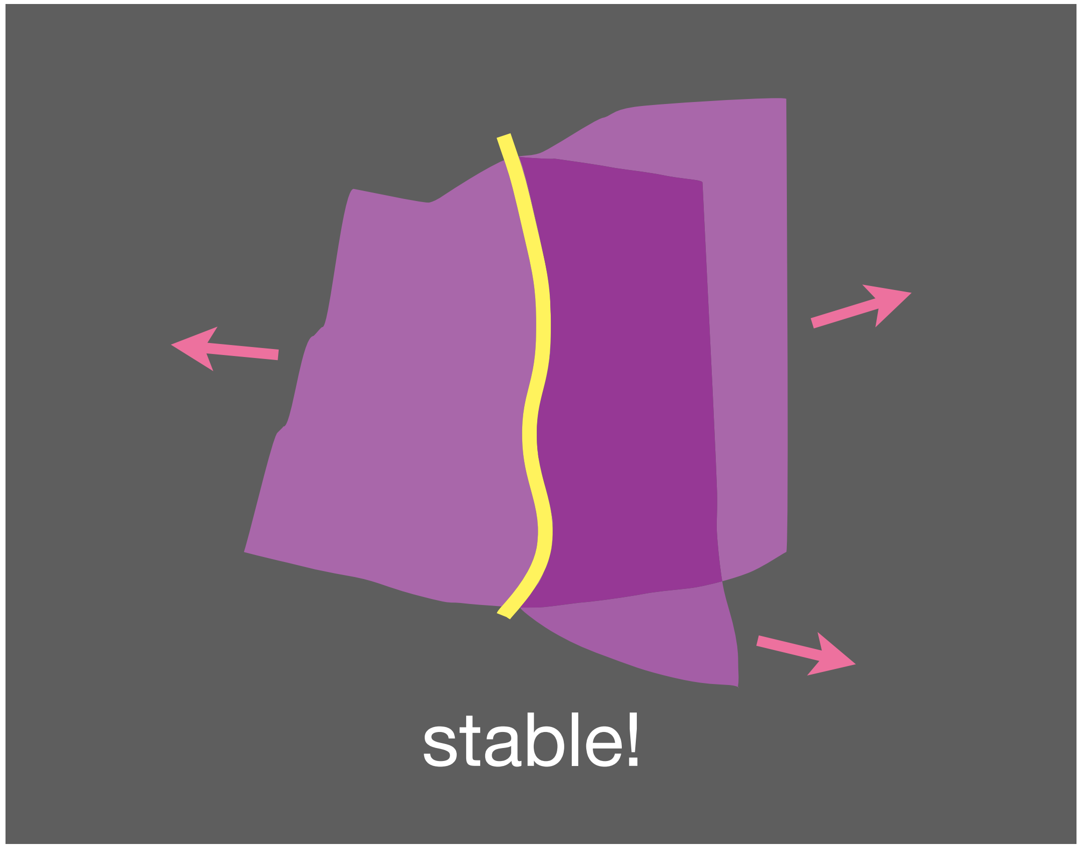
Domain wall problem

[Sikivie (1982)]

$N = 1$



$N = 3$

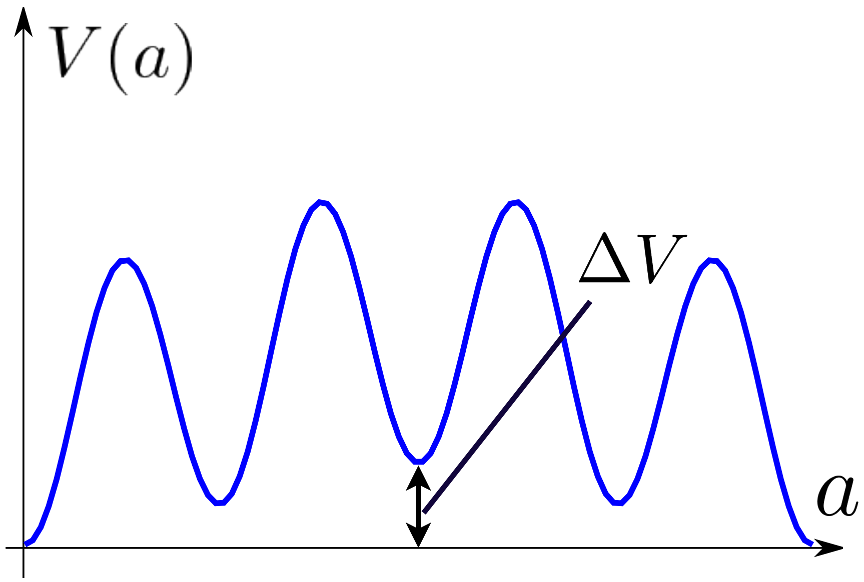


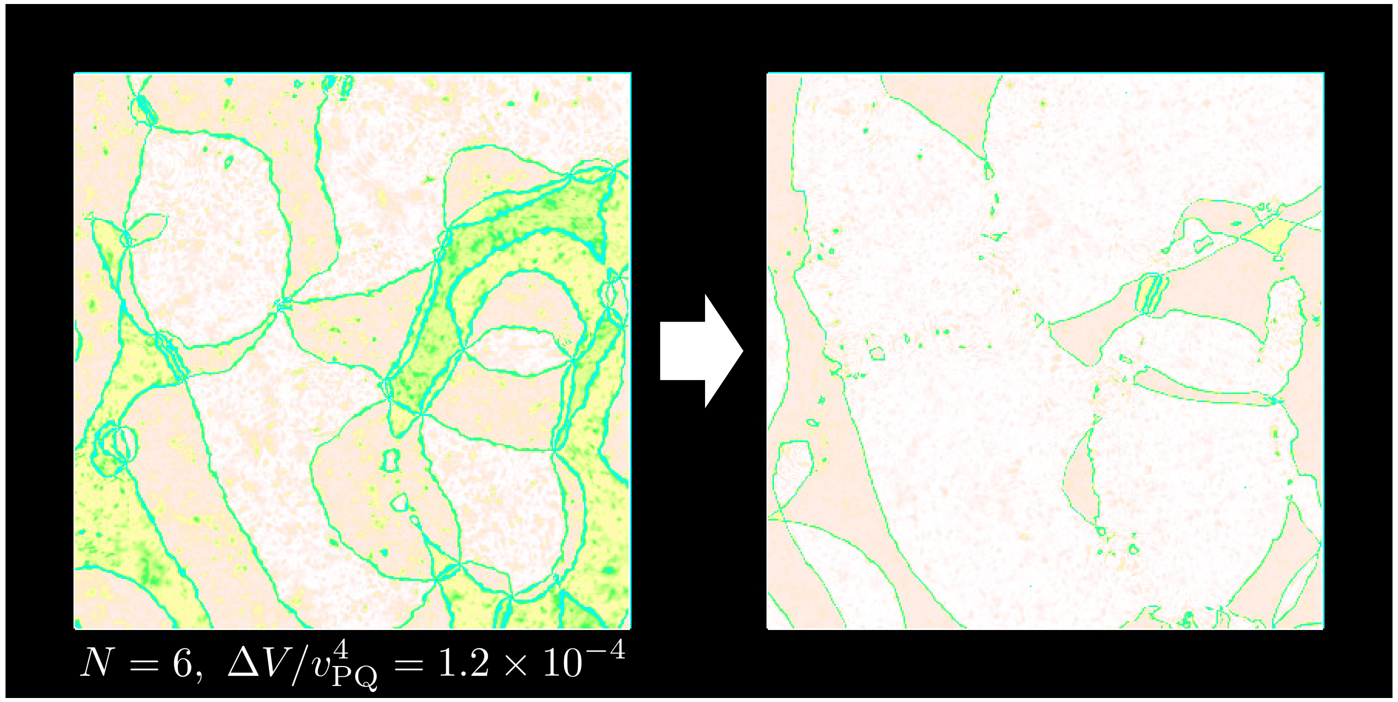
- Models with $N > 1$ are basically ruled out since domain walls are stable and come to overclose the universe.

- But the problem can be avoided if there exists an additional term which explicitly breaks the PQ symmetry and lifts degenerate vacua, making walls unstable.

Domain wall problem

[Sikivie (1982)]





[Kawasaki, KS, Sekiguchi, 1412.0789]

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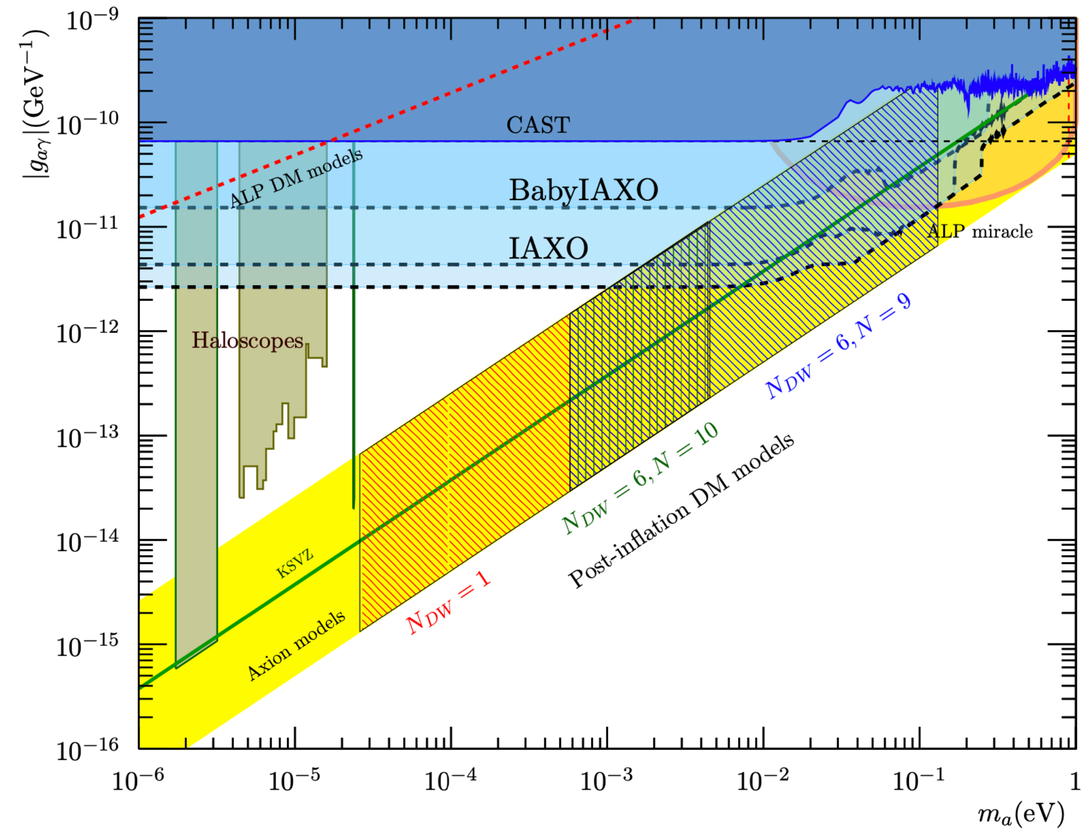
$N > 1$: Signatures from long-lived domain walls?

- Higher DM mass can be predicted, up to the lifetime of domain walls. Potential target of searches at higher mass ranges.
  [Kawasaki, KS, Sekiguchi, 1412.0789; Ringwald, KS, 1512.06436]

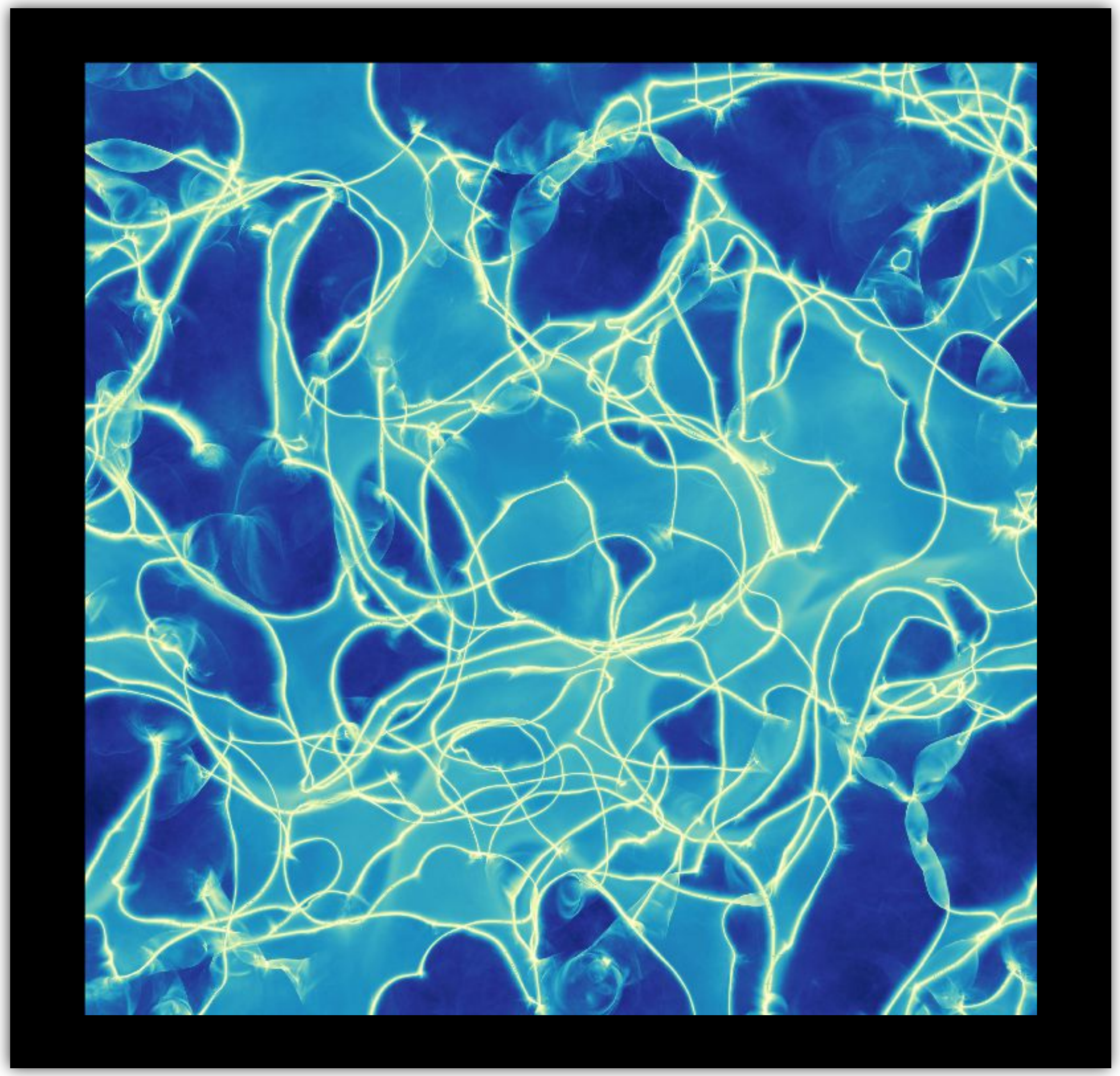


[Armengaud et al., 1904.09155]

- Observational signatures
  - Gravitational waves
    - Too small for QCD axion models. [Hiramatsu, Kawasaki, KS, Sekiguchi, 1207.3166]
    - Observable signatures predicted in ALP models or non-standard QCD axion models. [Gelmini, Simpson, Vitagliano, 2103.07625; Ferreira, Notari, Pujolàs, Rompineve, 2107.07542]
    - But incompatible with isocurvature constraints? [Gorghetto, Hardy, 2212.13263]
  - Possibility of primordial black hole formation

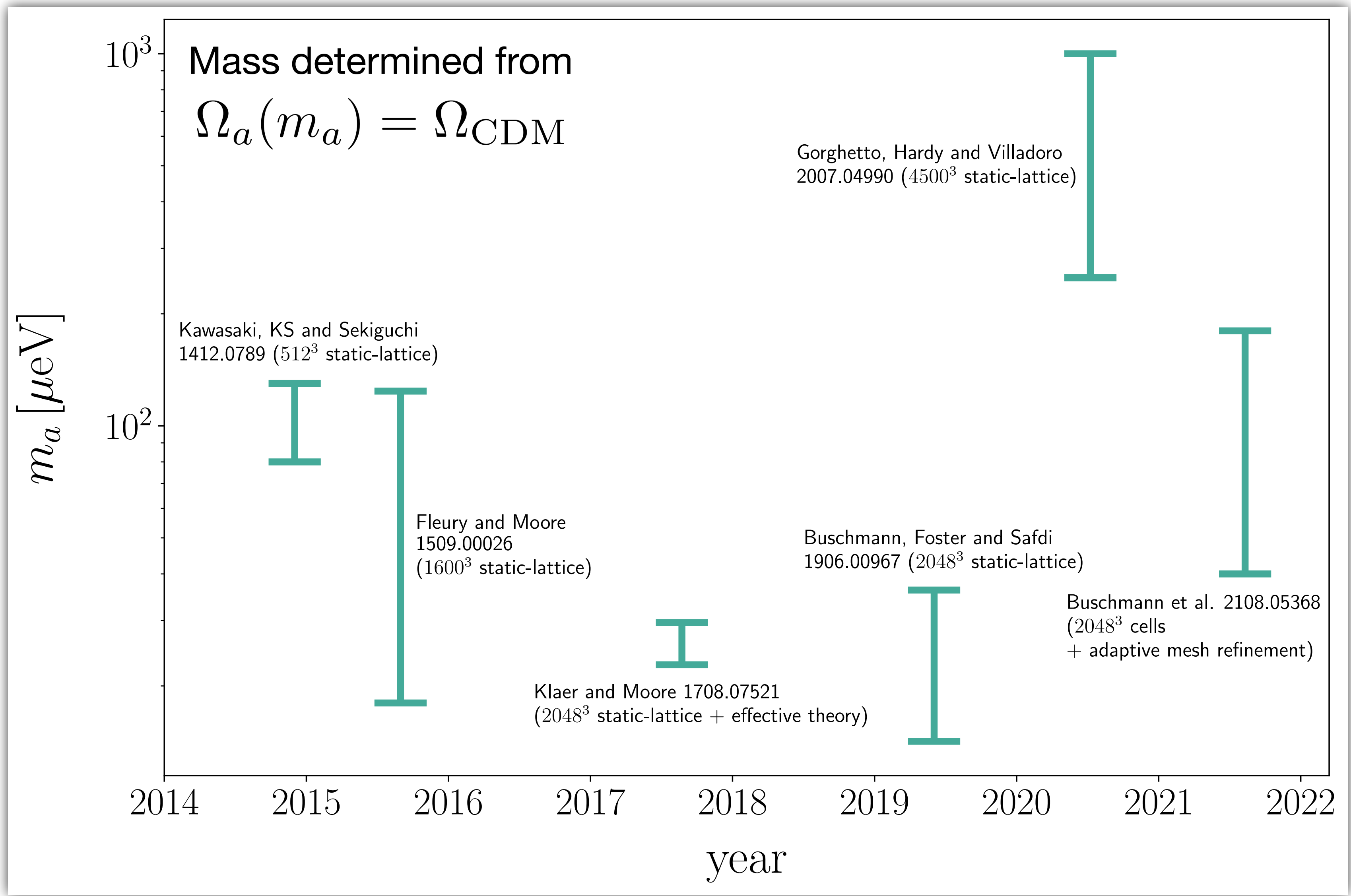
[Ferrer, Masso, Panico, Pujolas, Rompineve, 1807.01707; Gelmini, Simpson, Vitagliano, 2207.07126]

$N = 1$: The simplest scenario?



- No domain wall problem.
- One should be able to predict the dark matter mass uniquely (only one free parameter, f_a).
- Controversy on the interpretation of results from axion string simulations.

$N = 1$: The simplest scenario?



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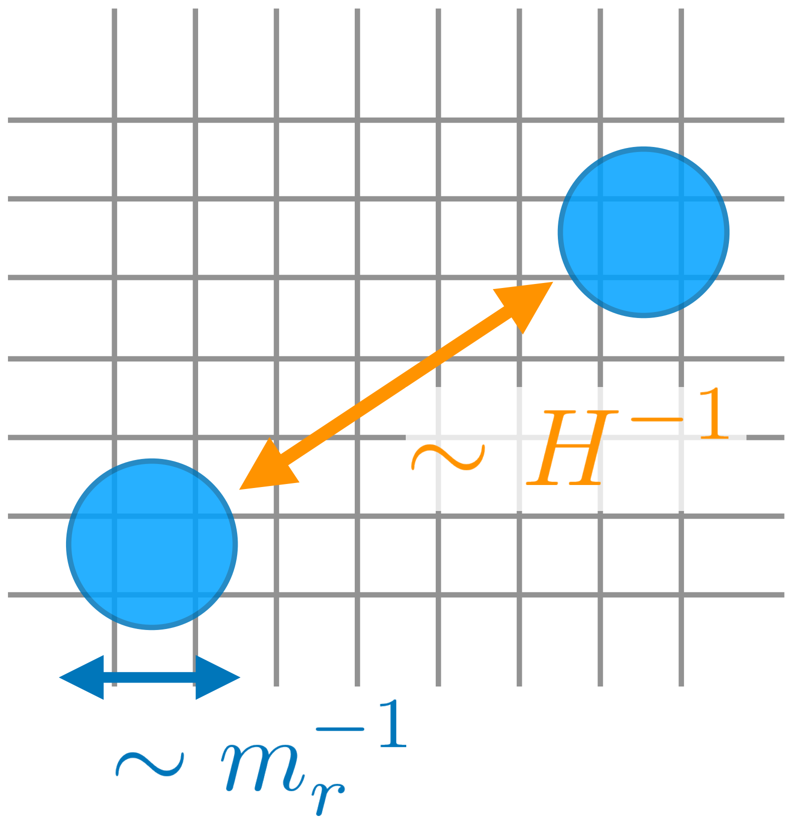
How to simulate axion strings

- Solve the classical EOM for a complex scalar field (PQ field) in comoving coordinates, discretized as static lattice.

$$\partial_\tau^2 \phi - \nabla^2 \phi + \lambda \phi (|\phi|^2 - \tau^2) = 0$$



- Simulation requires a proper resolution of two different length scales.
  - String core radius

$$\sim m_r^{-1} \sim f_a^{-1}$$

m_r : mass of the radial direction

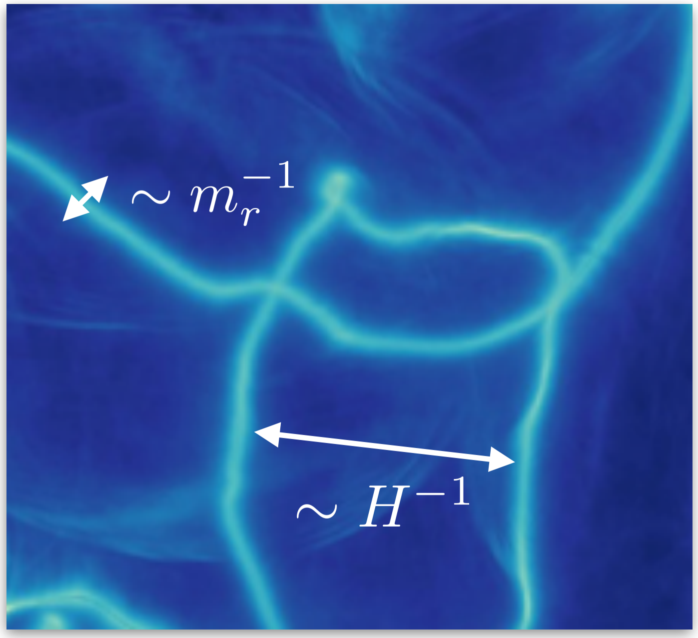
  - Hubble radius

$$\sim H^{-1}$$

Difficulty in string dynamics

- String tension acquires a logarithmic correction.

$$\mu = \frac{\text{energy}}{\text{length}} \simeq \pi f_a^2 \log\left(\frac{m_r}{H}\right)$$



- Realistic value

$$f_a/H_{\rm QCD} \sim 10^{30} \quad \Rightarrow \quad \log(m_r/H) \sim 70$$

- Difficult to reach it in simulations with limited dynamical ranges.

- Actual strings may be "heavier" than what we observe in simulations. (affects dynamics?)

Attractor/scaling solution

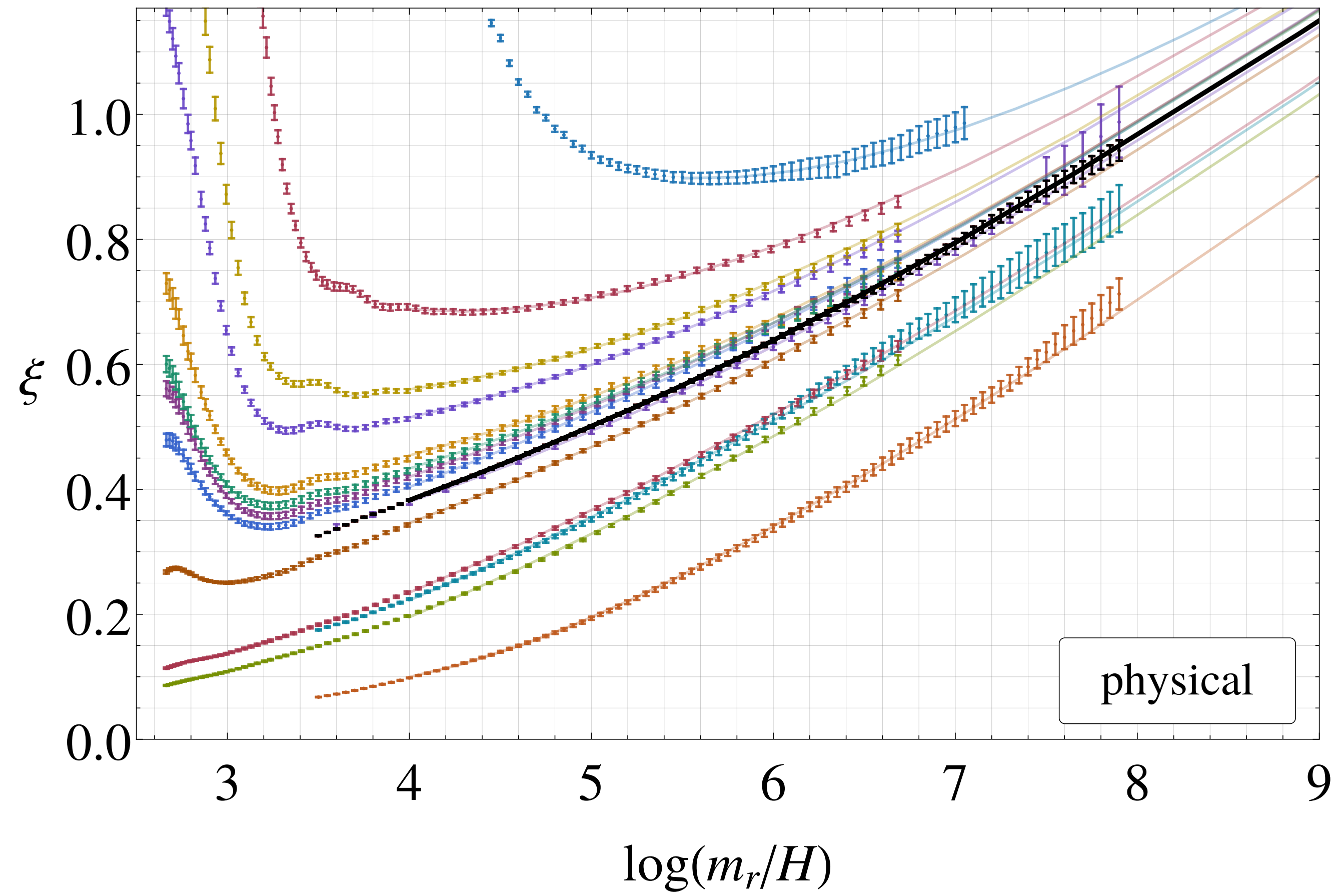
String density parameter

$$\xi = \frac{\ell_s}{\mathcal{V}} t^2$$

ℓ_s : string length

$\mathcal{V}$: spatial volume

[Gorghetto, Hardy, Villadoro, 2007.04990]



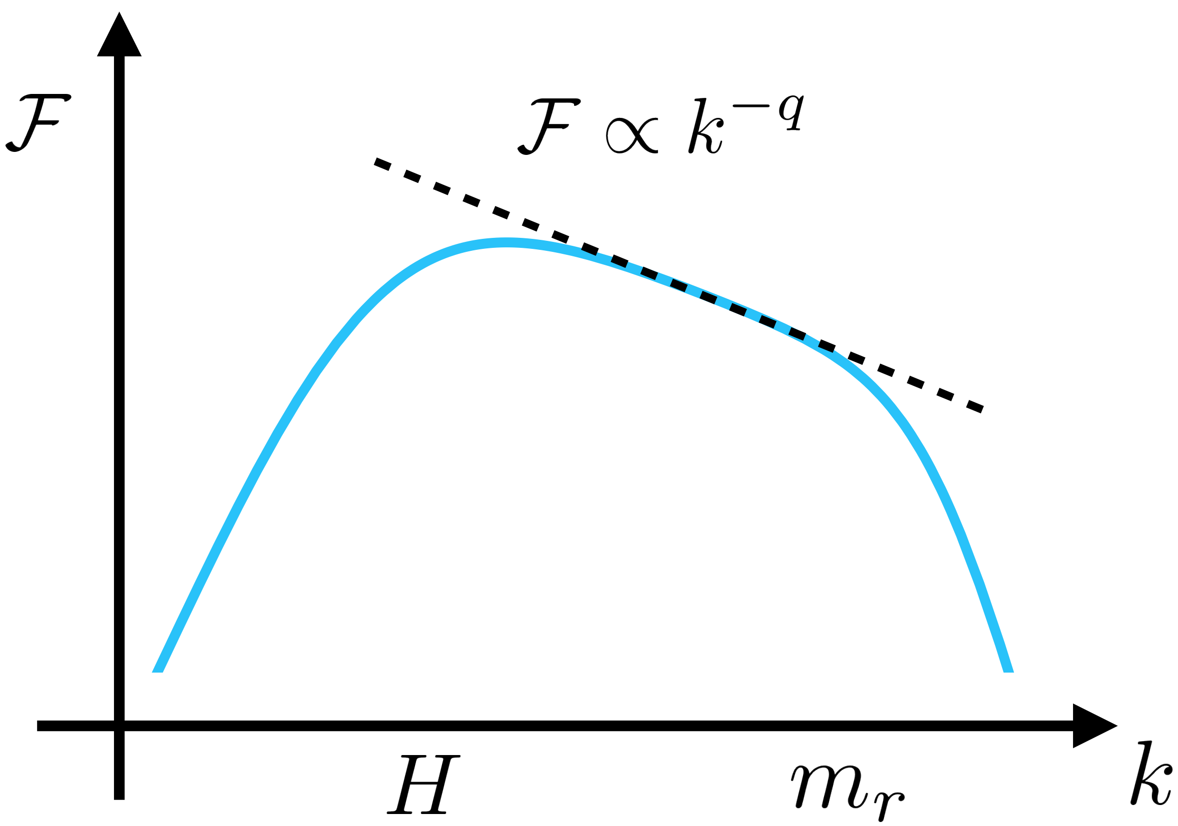
- Different initial conditions appear to converge.
- Characterize the attractor, and use it for extrapolation.
- Most of recent simulations observe a logarithmic growth in the string density. [Fleury, Moore, 1509.00026; Gorghetto et al., 1806.04677; 2007.04990; Kawasaki et al., 1806.05566] (but full consensus has not been reached, cf. [Hindmarsh et al., 1908.03522; 2102.07723])

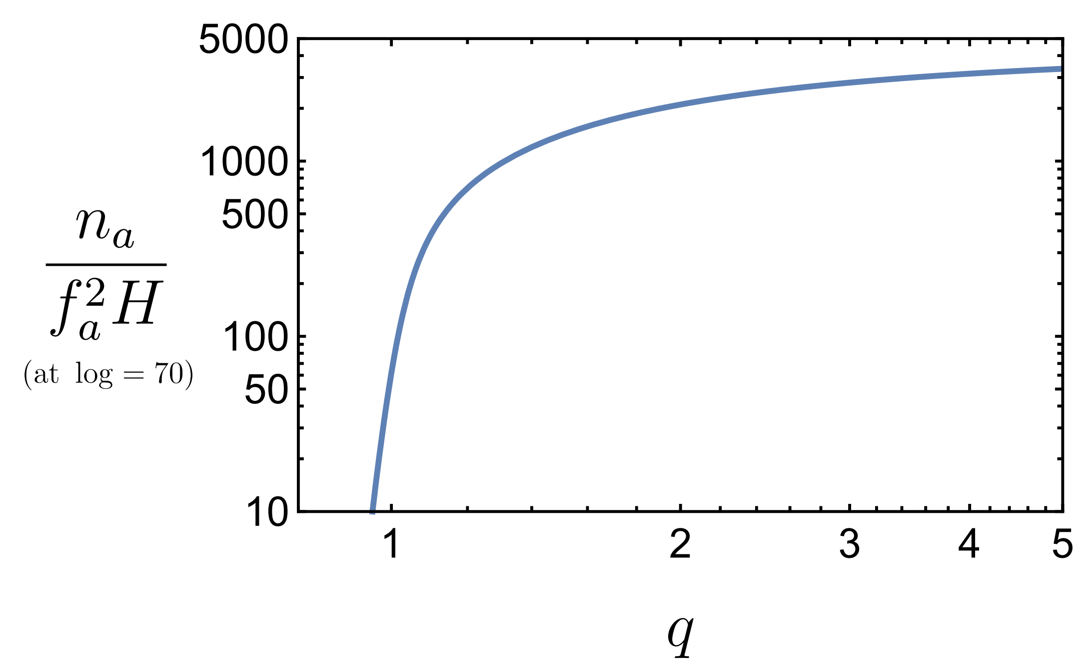
Spectrum of radiated axions

- Differential energy transfer rate

$$\mathcal{F}\left(\frac{k}{RH},\frac{m_r}{H}\right) \equiv \frac{1}{(f_a H)^2}\frac{1}{R^3}\frac{\partial}{\partial t}\left(R^4 \frac{\partial \rho_a}{\partial k}\right)$$

- $\mathcal{F}$ seems to be well approximated by a simple power law, whose exponent is important. [Gorghetto, Hardy, Villadoro, 1806.04677]



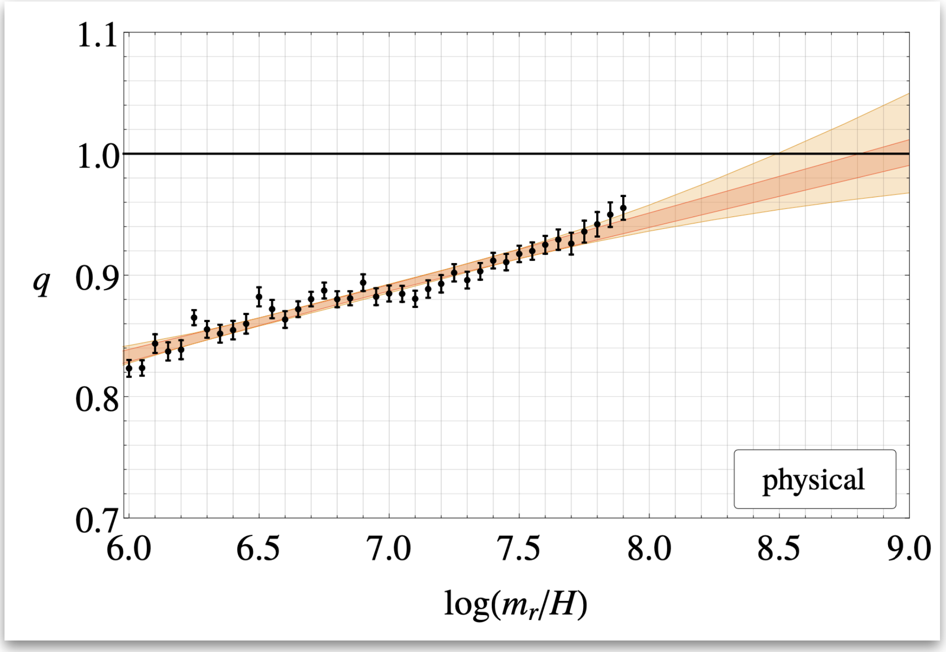


Discrepancy in the spectrum

- Simulations by [Gorghetto et al., 2007.04990]

$$q \gg 1 \ (\text{for} \ \log \gg 1)$$

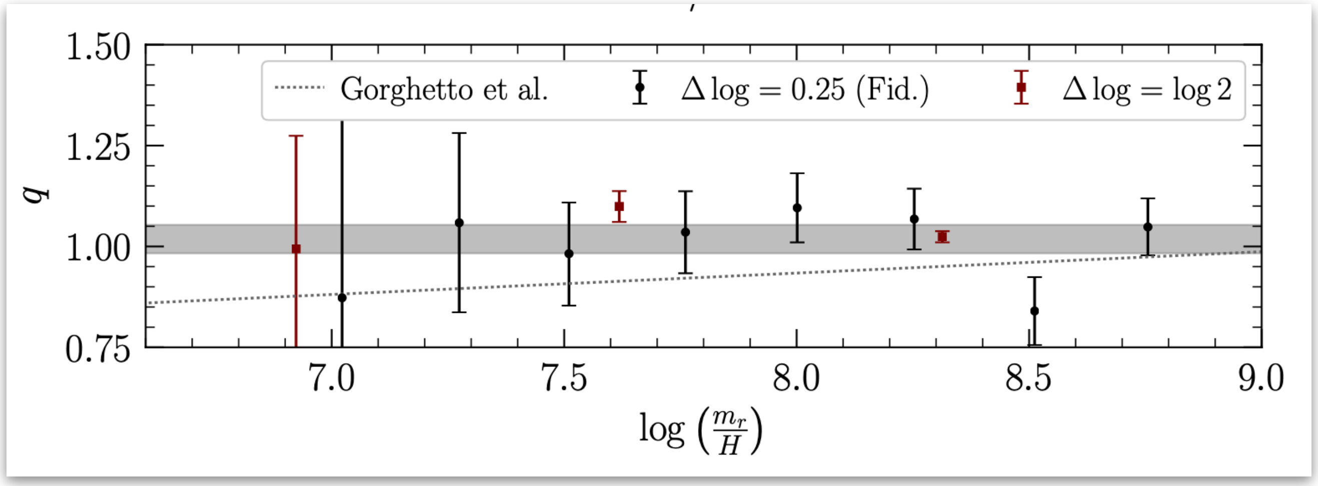
$$\frac{n_a}{n_{\text{misalign}}} \approx 13\text{–}66$$



- Simulations by [Buschmann et al., 2108.05368]
(using Adaptive Mesh Refinement (AMR))

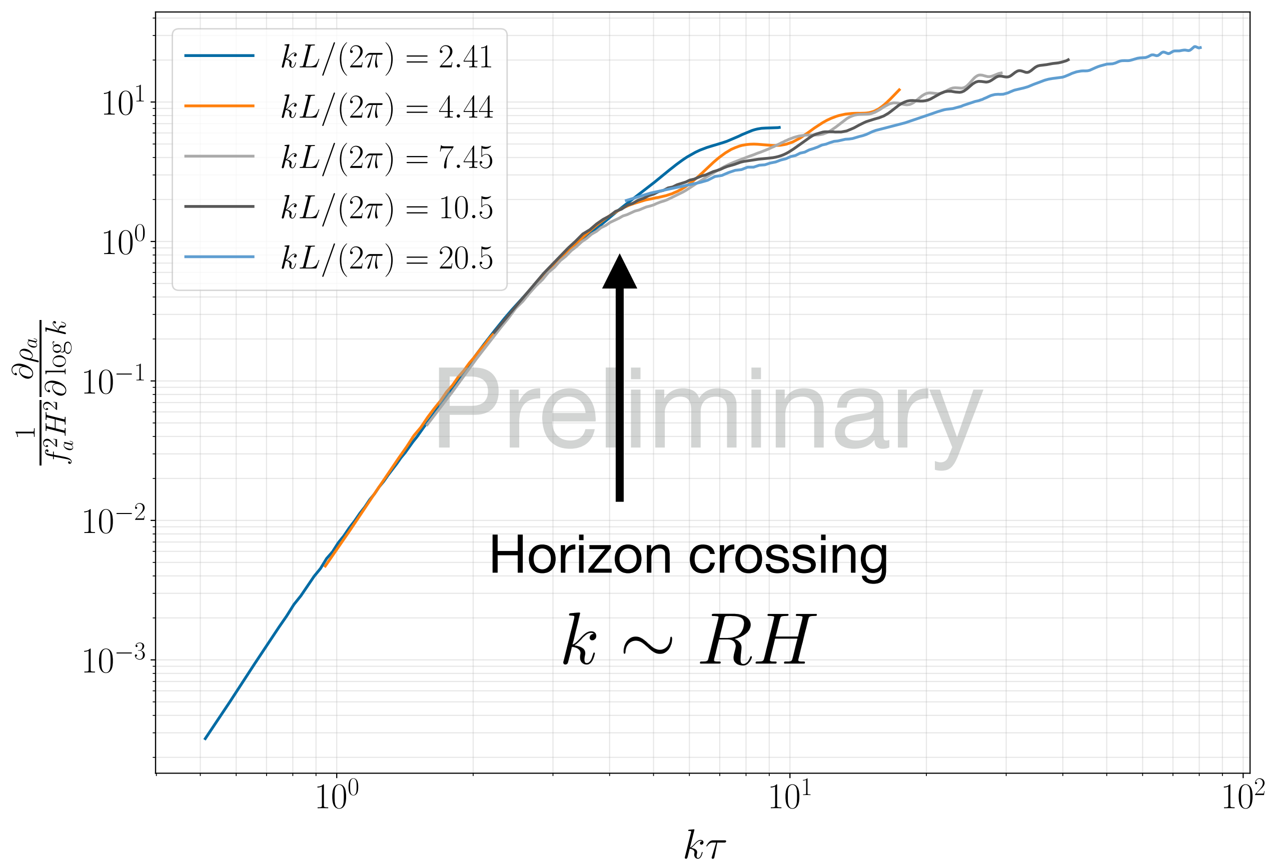
$$q \approx 1$$

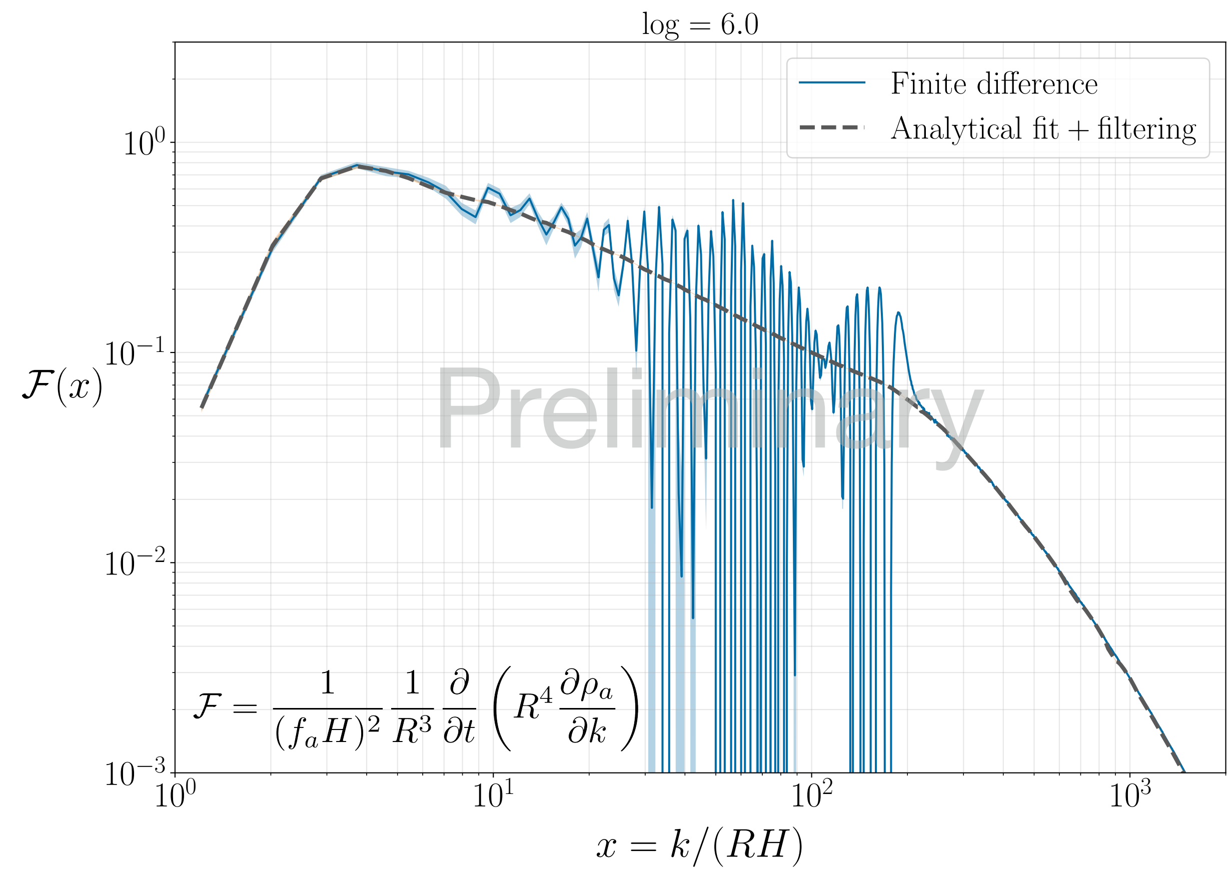
$$\frac{n_a}{n_{\text{misalign}}} \approx 1.5\text{–}8.9$$



Discrepancy in the index of the axion radiation spectrum...

Oscillation in the axion spectrum





[Redondo, KS, Vaquero, appearing soon]

- Coherent oscillation with frequency $\sim 2k$, interpreted as axion field oscillations after the horizon entry.
- Simple finite difference leads to a lot of contaminations from axion field oscillations.
- Reducing them by applying a filter to remove high frequency components in the mode evolution data.

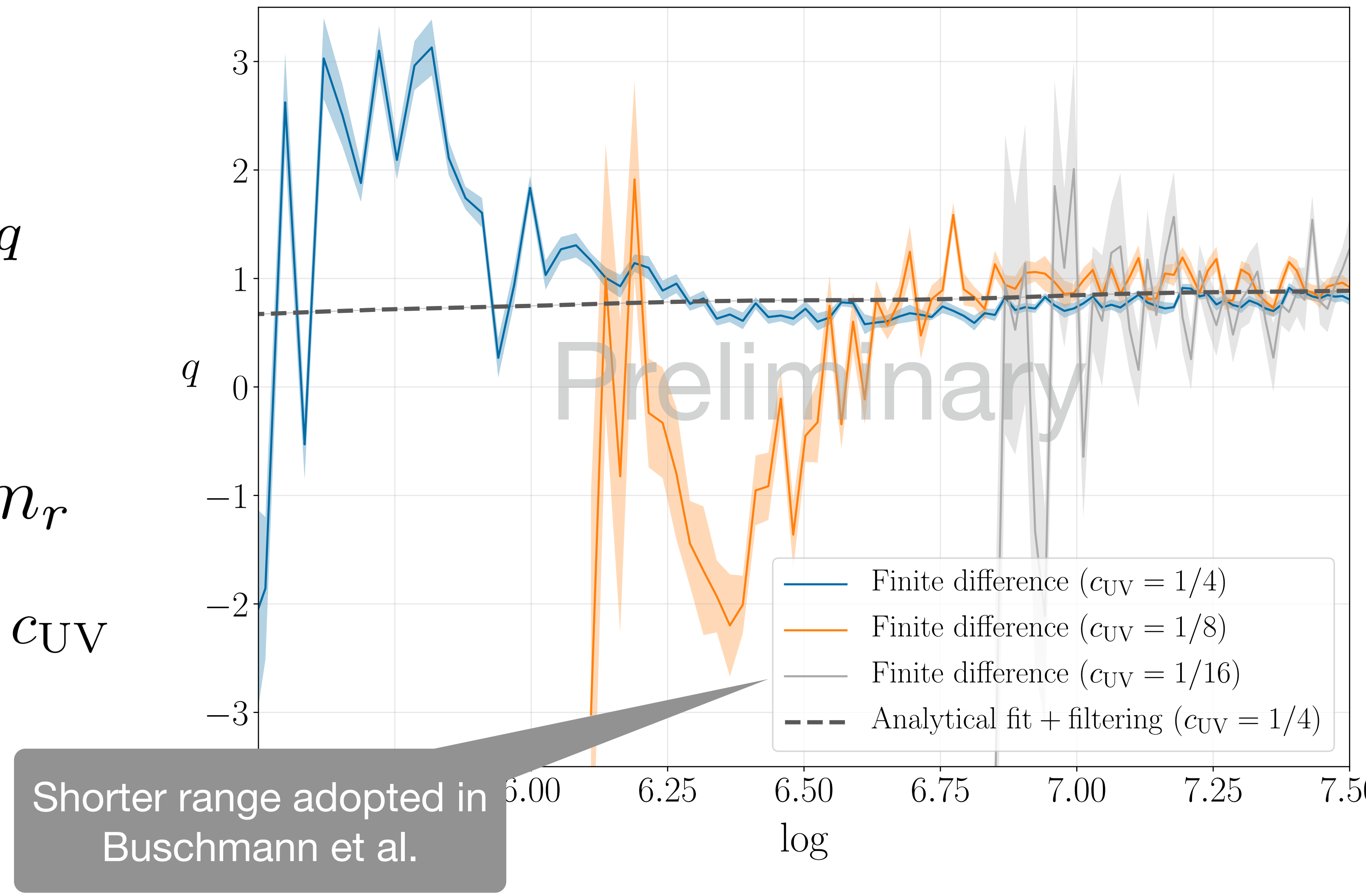
Oscillation in the axion spectrum

Fit a power law $\mathcal{F} \propto k^{-q}$

to the data in a range

$$c_{\rm IR} H < k/R < c_{\rm UV} m_r$$

with some coefficients $c_{\rm IR}, c_{\rm UV}$

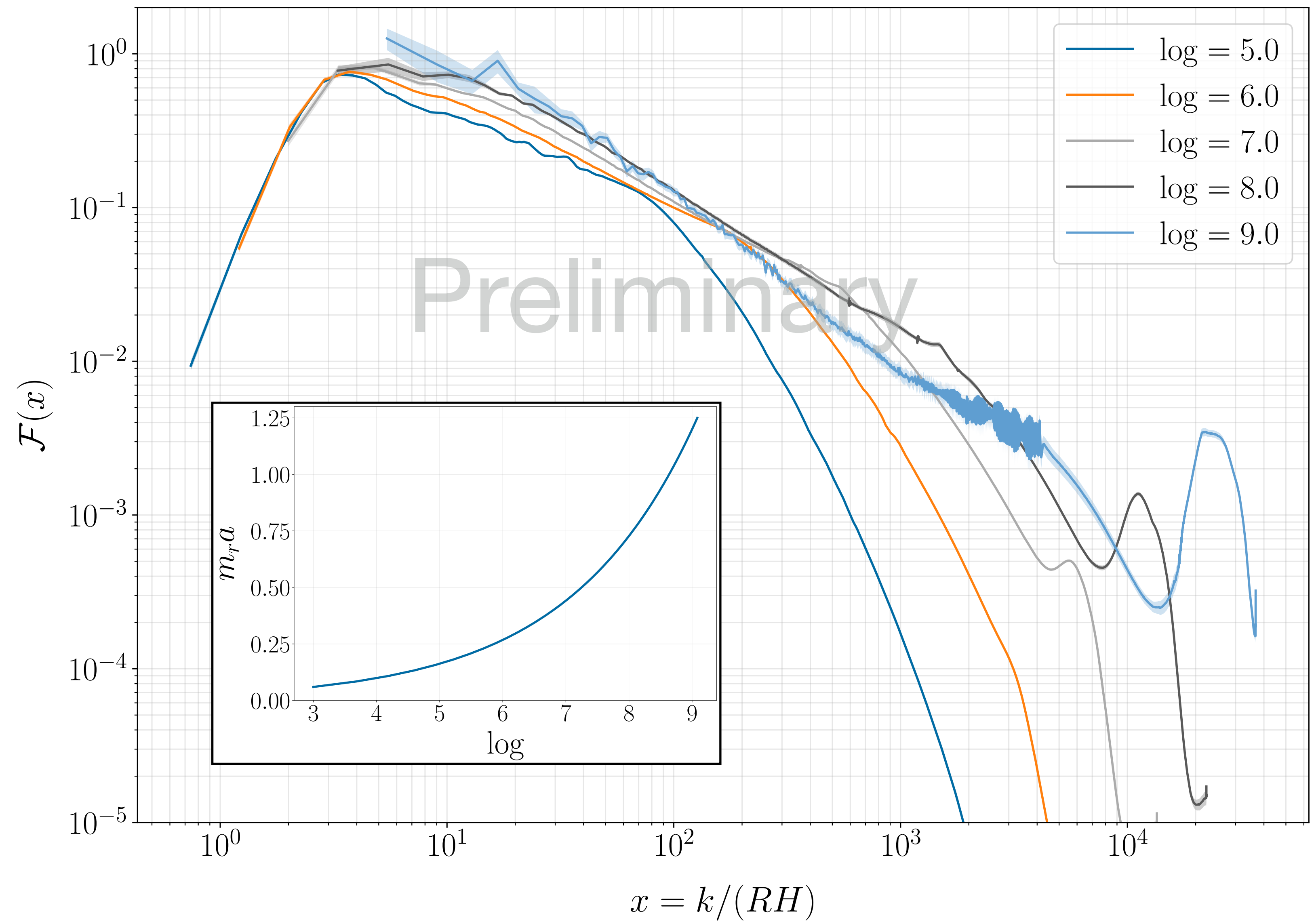


[Redondo, KS, Vaquero, appearing soon]

- Bias due to the oscillations in the IR modes gives rise to larger fluctuations in q.
- Possible source of discrepancy in the literature?

Discretization effects

[Redondo, KS, Vaquero, appearing soon]



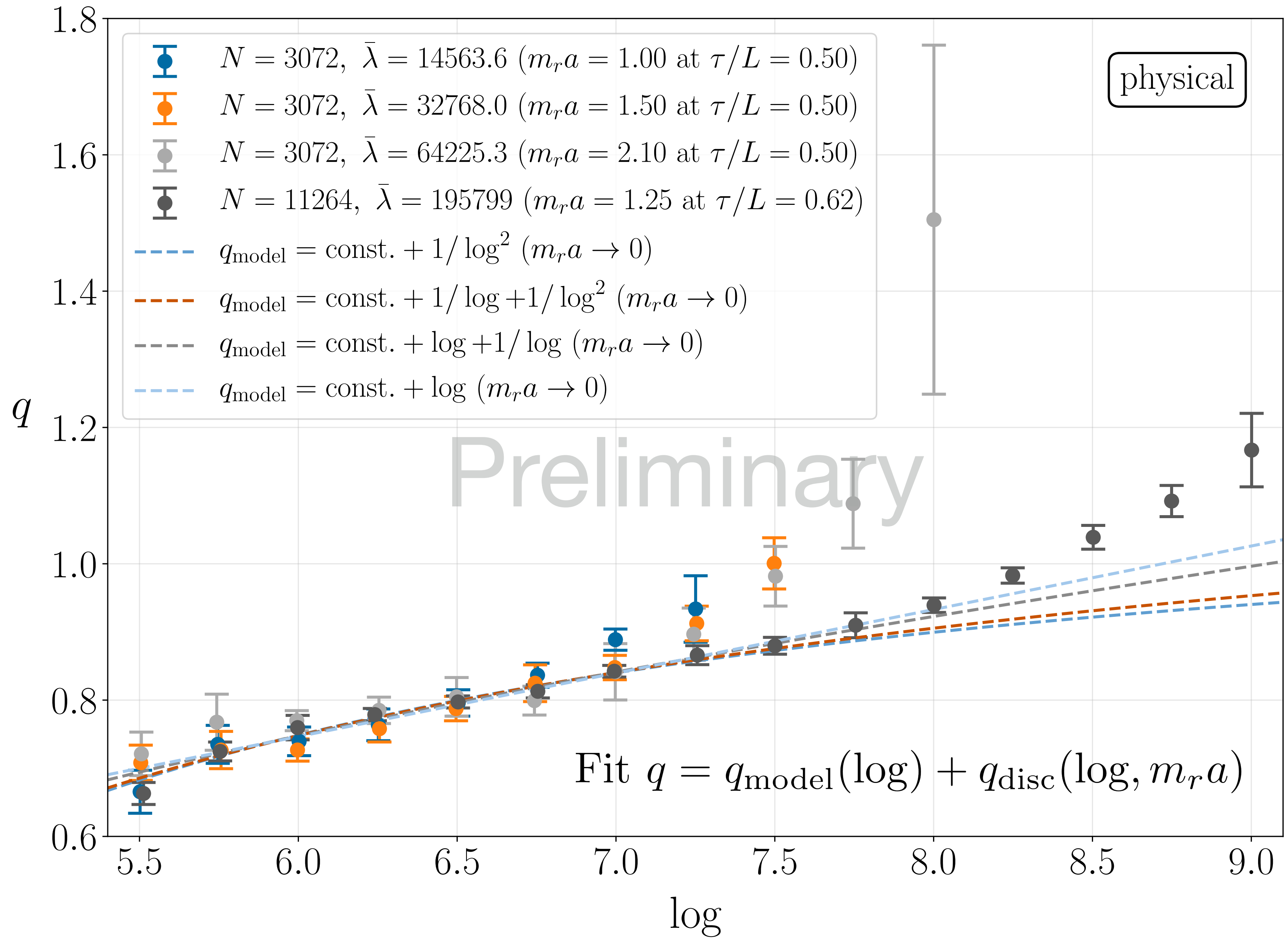
The largest simulation with 11264^3 static lattice performed at RAVEN (MPCDF, Garching) and SQUID (Osaka)

- Effect of the resolution of the string core parameterized by $m_r a$.

$a = L/N$: lattice spacing

- The effect appears to blow up faster at larger logs, leading to a significant distortion of the spectrum.

Discretization effects



[Redondo, KS, Vaquero, appearing soon]

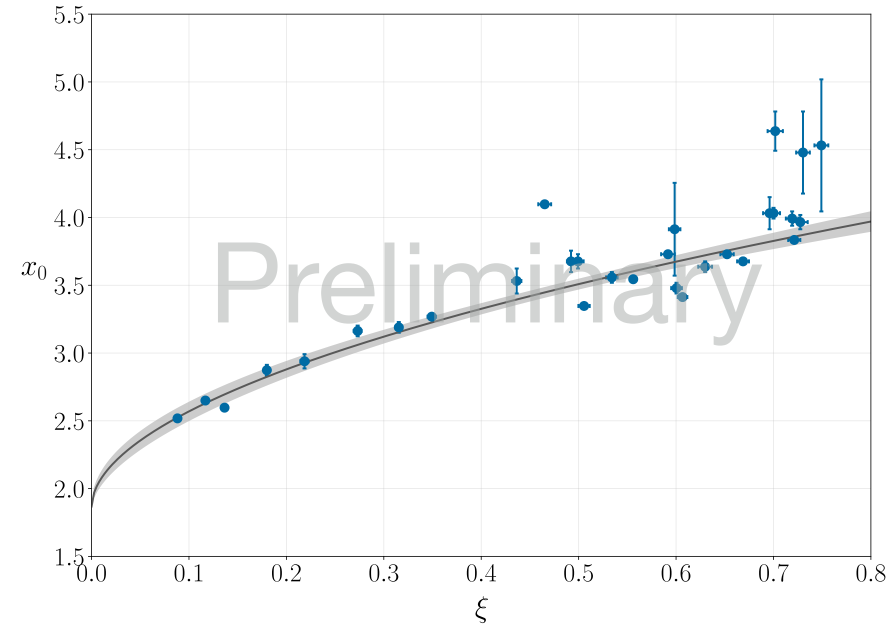
- Large $m_r a$ biases q towards larger values.
- Model the part that is independent of $m_r a$, and use it for extrapolation.

Extrapolation to large log

- Evolution of the IR peak
  $x_0 \equiv k_{\rm peak}/(RH)$ is observed,
  which is relevant if $q \sim 1$.

$$\frac{n_a}{f_a^2 H} \sim \frac{8\pi\xi}{x_0}$$

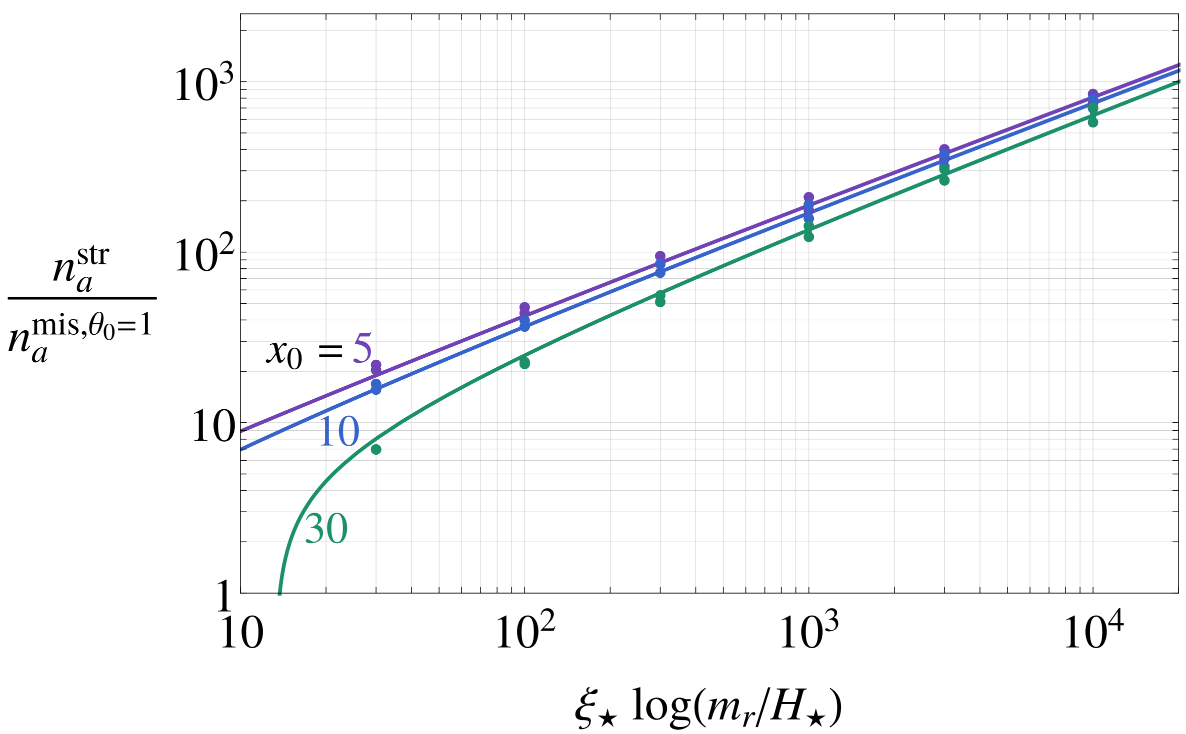
$$x_0 = 1.86 + (2.36 \pm 0.11)\xi^{0.53\pm0.06}$$



[Redondo, KS, Vaquero, appearing soon]

- If $q \gg 1$, the non-linear effect around the QCD phase transition is relevant, which alleviates the log-enhancement of the axion number.

$$\frac{n_a^{\rm string}}{n_a^{\rm misalign}} \propto (\xi \log)^{1/2}$$

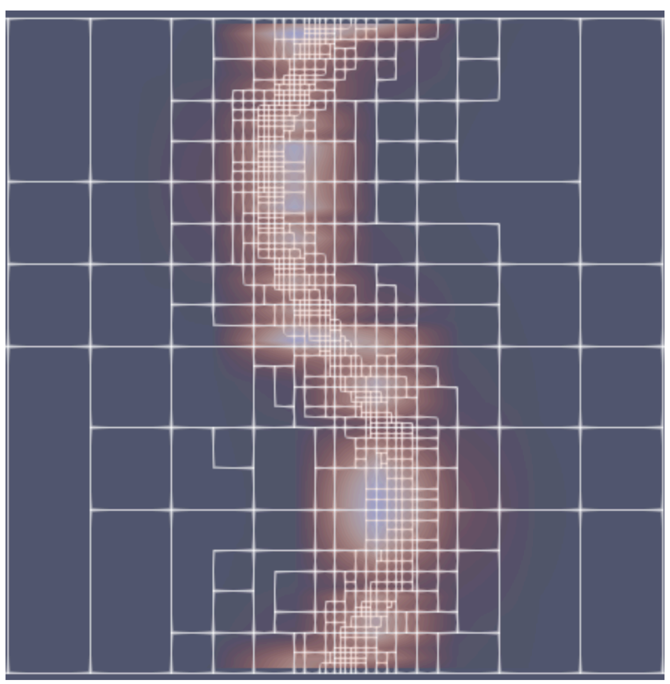


[Gorghetto, Hardy, Villadoro, 2007.04990]

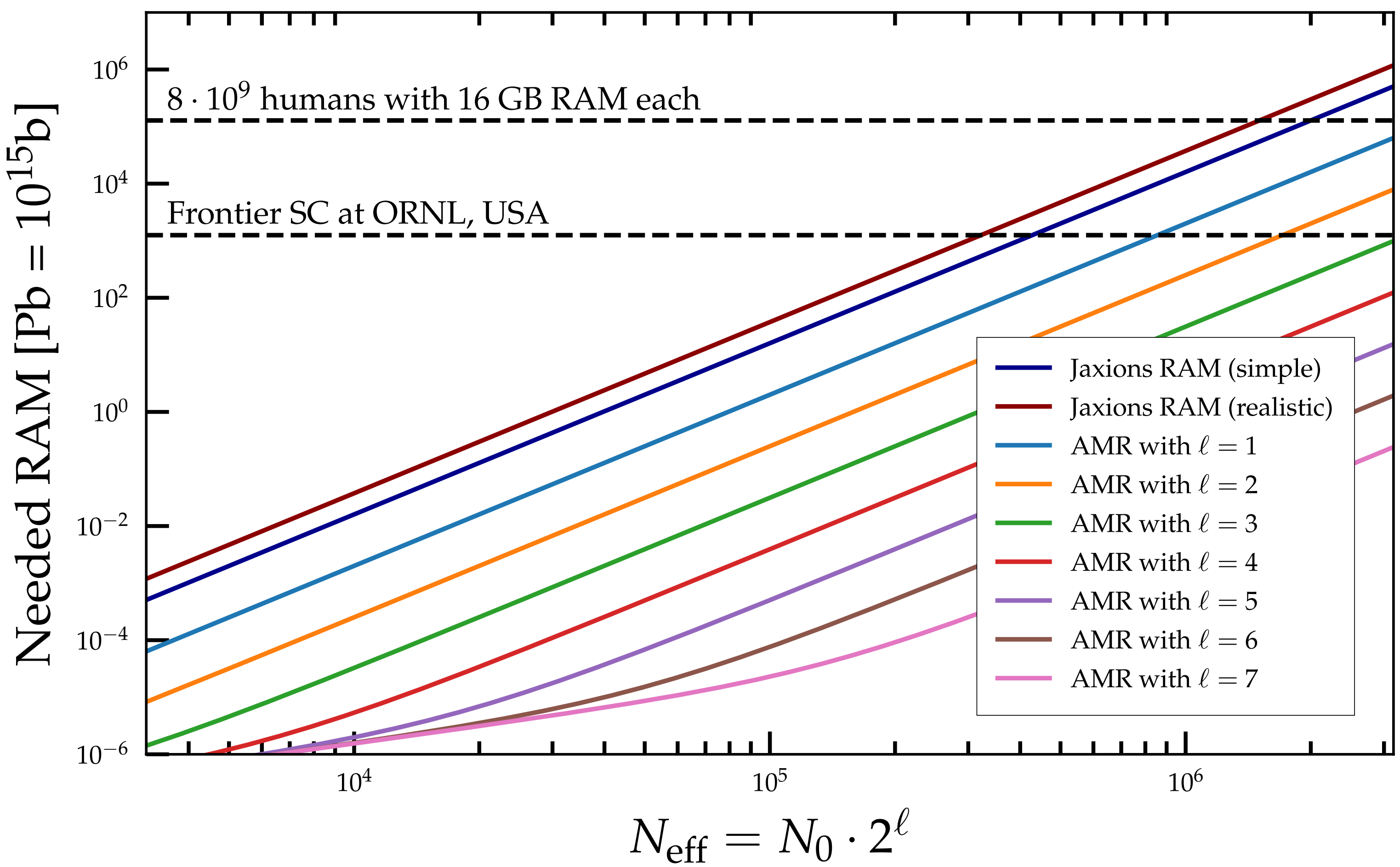
Future: Simulations with Adaptive Mesh Refinement

- Instead of using a uniform grid, selectively increase the resolution in the region of interest (string core).

- Potential to substantially increase the dynamical range by using computational resources more efficiently.



[Drew, Shellard, 1910.01718]



[Kaltschmidt, Redondo]

Figure by M. Kaltschmidt, presented at the workshop "DM beyond the weak scale", Liverpool (2023)

Alternative approach: Direct simulations with effective theory

[Klaer, Moore, 1707.05566; 1708.07521]

- Clever method to effectively increase the string tension by considering a theory with 1 vector field + 2 complex scalars.

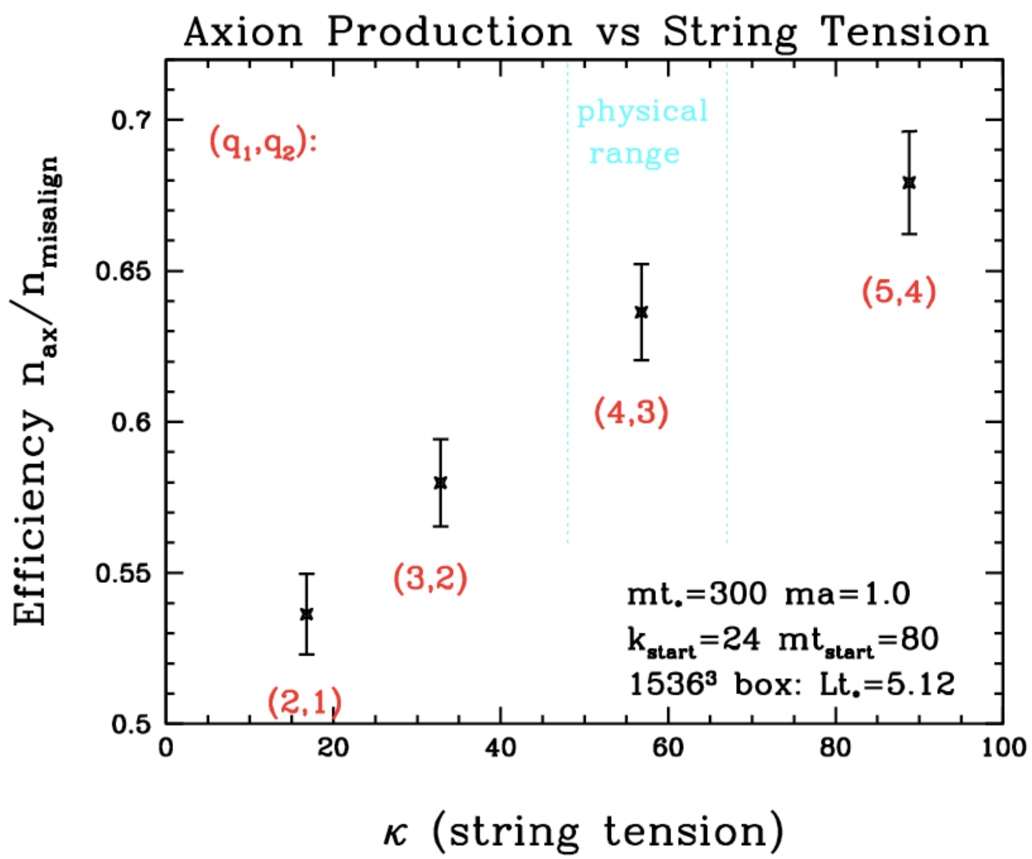
$$\mu \simeq 2\pi v^2 \quad \text{with} \quad f_a = \frac{v}{\sqrt{q_1^2 + q_2^2}}$$

$$\kappa \equiv \frac{\mu}{\pi f_a^2} \simeq 2(q_1^2 + q_2^2) \gg 1$$

$$\begin{aligned} -\mathcal{L} = & \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + |(\partial_\mu - i q_1 e A_\mu)\phi_1|^2 \\ & + |(\partial_\mu - i q_2 e A_\mu)\phi_2|^2 \\ & + \lambda \left[\left(|\phi_1|^2 - \frac{v^2}{2} \right)^2 + \left(|\phi_2|^2 - \frac{v^2}{2} \right)^2 \right] \end{aligned}$$

- Axion production turned out to be less efficient than the misalignment estimate, predicting smaller DM mass.

- Is the axion spectrum consistent with the results from indirect simulations at smaller logs?



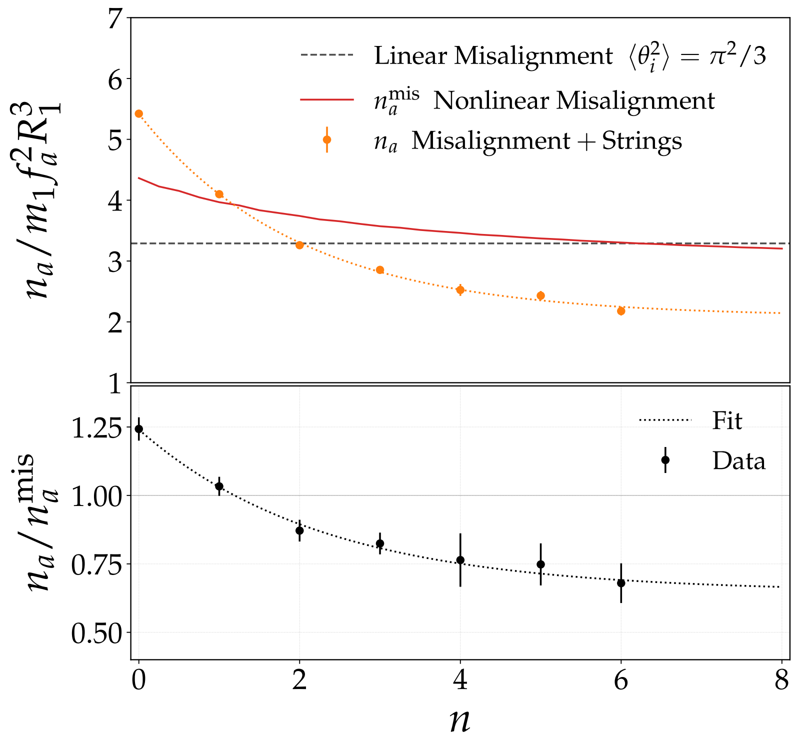
Simulations for other scenarios

- Generic ALP mass

[Chaumet, Moore, 2108.06203]
[O'Hare, Pierobon, Redondo, Wong, 2112.05117]

$$m_a^2(T) \propto T^{-n}$$

n = 0 may lead to a larger abundance.



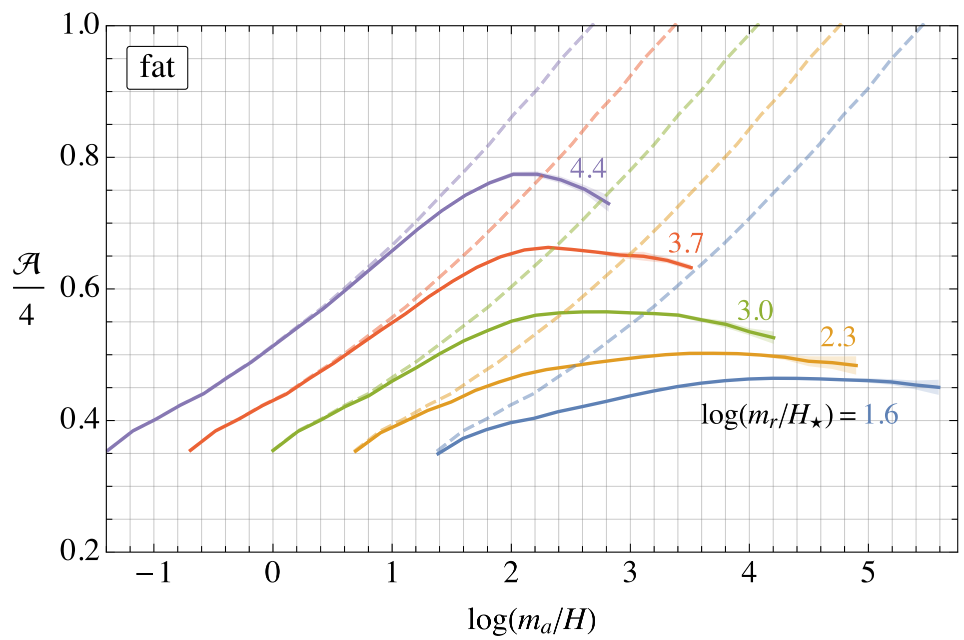
- $N > 1$

More challenging because of
3 different scales.

$$H \ll m_a \ll m_r$$

Recent simulations show a logarithmic increase in the area of domain walls.

[Gorghetto, Hardy, 2212.13263]



Conclusions

- New generation of numerical simulations allow us to have a better understanding of the nature of axion dark matter in the post-inflationary scenario.

- Main problem is dynamical range.

  - Measurement of the axion spectrum is still uncertain due to severe discretization effects.

  - There are several rooms for improvement in simulation or analysis methods, which are expected to provide a shaper prediction for the axion dark matter mass.