Dynamical nucleon-nucleon clusterization
and in-medium cross section by means of Bayesian Inference

Featured in Physics

Examination of cluster production in excited light systems at Fermi energies from new experimental data and comparison with transport model calculations

C. Frosin *et al.* (INDRA-FAZIA Collaboration)
Phys. Rev. C **107**, 044614 – Published 28 April 2023

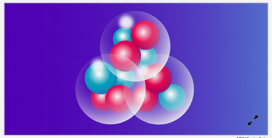
Physics See synopsis: Characterizing Clusters in Nuclear Collisions

SYNOPSIS

Characterizing Clusters in Nuclear Collisions

April 28, 2023 • *Physics* 16, s50

High-energy collisions result in the formation of clusters of neutrons and protons inside atomic nuclei and in the emission of energetic ions.



APS/Carin Cain

A. Camaiani
and thanks to
S. Piantelli,
G. Casini
G. Poggi

On 2019 (?) we proposed, following the work of Tian et al on clusterization during HIC in AMD,

Cluster formation and decay in very excited light systems beyond Fermi energies

INDRA-FAZIA setup GANIL

Spokesperson(s): A. Camaiani and D. Gruyer
for the INDRA-FAZIA collaboration

12C+12C @ 95 MeV/u with INDRA+FAZIA

- Detection of events complete in charge (Zdet = Zsys)
- Detailed balance of the reaction channels
- Detailed kinematics of the reaction channels
- Particle correlations to back trace parent E*

PHYSICAL REVIEW C **95**, 044613 (2017)

Nuclear stopping and light charged particle emission in $^{12}C + ^{12}C$ at 95 MeV/nucleon

G. Tian (田国玉),[1,2] R. Wada,[1,3] Z. Chen (陈志强),[1,*] R. Han (韩瑞),[1] W. Lin (林炜平),[1] X. Liu (刘星泉),[1] P. Ren (任培培),[1,2] F. Shi (石福栋),[1] F. Luo (罗飞),[1] Q. Sun (孙琪),[1,2] L. Song (宋林),[1,2] and G. Q. Xiao (肖国青)[1]

PHYSICAL REVIEW C **97**, 034610 (2018)

Cluster correlation and fragment emission in $^{12}C + ^{12}C$ at 95 MeV/nucleon

G. Tian,[1,2] Z. Chen,[1,2,*] R. Han,[1] F. Shi,[1] F. Luo,[1] Q. Sun,[1,2] L. Song,[1,2] X. Zhang,[1,2] G. Q. Xiao,[1] R. Wada,[3,†] and A. Ono[4]

PHYSICAL REVIEW C **102**, 064617 (2020)

Effects of cluster correlations on fragment emission in $^{12}C + ^{12}C$ at 50 MeV/nucleon

R. Han,[1] Z. Chen,[1,2,*] R. Wada,[3,†] A. Ono,[4] G. Tian,[1] F. Shi,[1] X. Zhang,[1,2] B. Liu,[1,2] and H. Sun[1,2]

PHYSICAL REVIEW C **107**, 044602 (2023)

Reaction dynamics and in-medium nucleon-nucleon cross section with $^{12}C + ^{1}H$ at 95 MeV/nucleon

G. Tian,[1] Z. Chen,[1,2,*] R. Wada,[3,†] X. Liu,[4] W. Lin,[4] M. Huang,[5] H. Zheng,[6] Q. Hu,[1,2] R. Han,[1,2] F. Shi,[1] X. Zhang,[1,2] B. Liu,[1,2] and H. Sun[1,2]

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INDRA-FAZIA setup. While there is no doubt that the experiment can run successfully and lead to publishable results, the proposal was unclear on what precisely will be the crucial advance in understanding any of the specific aspects related to the cluster formation in nuclei. It was also not clear how the observations would be linked to the fundamental interactions between the constituent particles. At this stage, the comparison of experiment and theory appears rather simplified with no discussion on the physics that can drive the clustering. Among the discussed topics, the possibility to study the decay of excited clusters with complete kinematical reconstruction seemed to the PAC the most innovative and promising aspect and could be of interest in nuclear astrophysics. However, because of the lack of clear physical

The physics behind clusterization in HIC

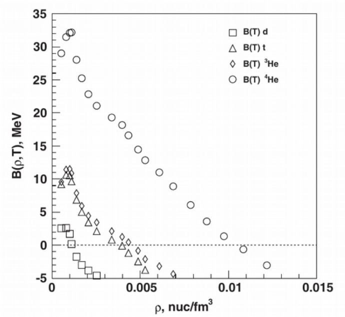
APPROVED

- E818: clusterization of warm dilute nuclear matter

$V_f \sim \exp(B)$

- Mott effect: in-medium modifications of the cluster binding energies

K. Hagel et al, PRL 108, 062702 (2012)



EXTENDING OUR KNOWLEDGE OF WARM DENSE NUCLEAR MATTER IN THE LOW DENSITY REGION.

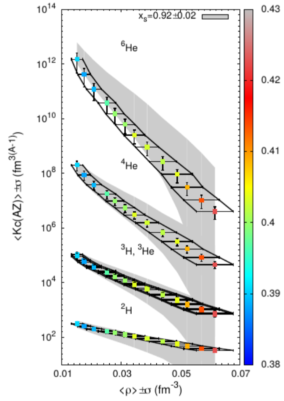


Figure 2. ^{124}Xe+^{112}Sn 32 A MeV system (INDRA data). The chemical equilibrium constants as a function of the density. The grey bands are the chemical equilibrium constants from a calculation where we consider homogeneous matter with five light clusters, calculated at the average value of (T, ρ, Y_p), and considering different cluster-meson scalar coupling constants $g_{s_i} = x_{s_i} A_i g_s$ with $x_{s_i} = 0.92 \pm 0.02$. The color code represents the global proton fraction.

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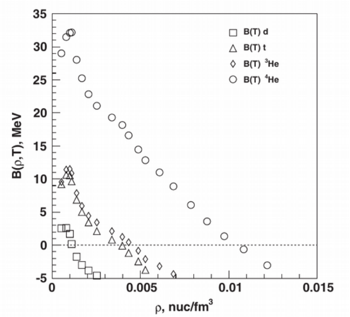

- Mott effect as responsible for α-particle enhancement in HIC arXiv:2305.02988

Modified pBUU to take into account Mott effect: Au+Au 0.12 – 1 A GEV with FOPI

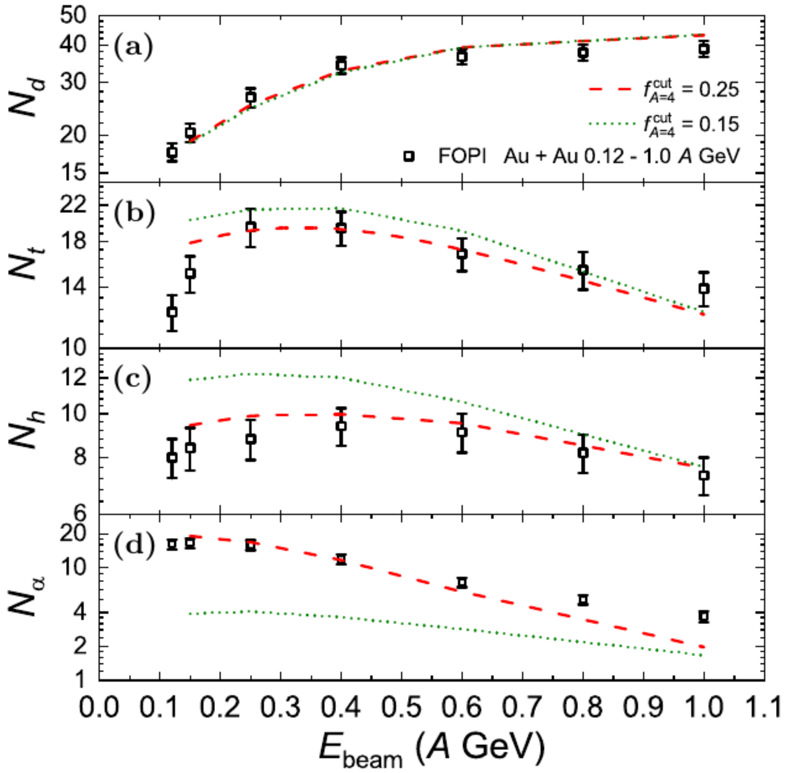

FIG. 4. Incident-energy dependence of light-nuclei yields from the kinetic approach with $f^{cut}_{A=2} = 0.11$, $f^{cut}_{A=3} = 0.16$ and $f^{cut}_{A=4} = 0.25$. The results for a smaller $f^{cut}_{A=4} = 0.15$ are also included for comparison. The experimental data are from the FOPI Collaboration [16].

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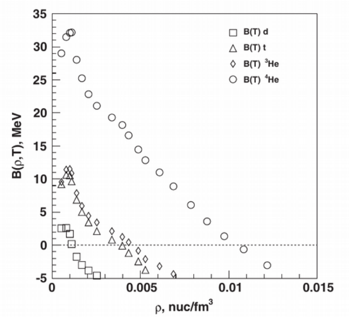

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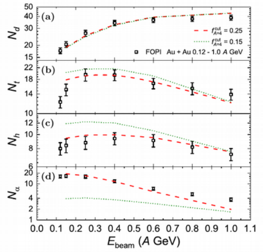

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- Cluster rate production A. Ono, Prog. Part. Nucl. Physics 105 (2019) 139–179

$$\frac{\partial f_x}{\partial t} + \frac{\partial H_x}{\partial \boldsymbol{p}} \cdot \frac{\partial f_x}{\partial \boldsymbol{r}} - \frac{\partial H_x}{\partial \boldsymbol{r}} \cdot \frac{\partial f_x}{\partial \boldsymbol{p}} = I_x^{\mathrm{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$v_i d\sigma(C_1, C_2, p_{\mathrm{rel}}, \Omega) = \frac{2\pi}{\hbar} P_1(C_1, p_{\mathrm{rel}}, \Omega) P_2(C_2, p_{\mathrm{rel}}, \Omega) |\mathcal{M}|^2 \delta\big(E_f(C_1, C_2, p_{\mathrm{rel}}, \Omega) - E_i\big) \frac{p_{\mathrm{rel}}^2 dp_{\mathrm{rel}} d\Omega}{(2\pi\hbar)^3}$$

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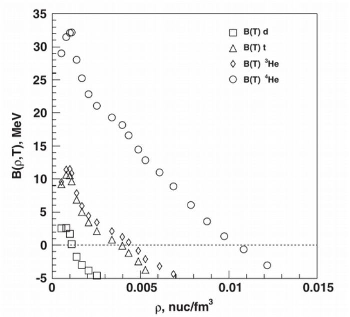

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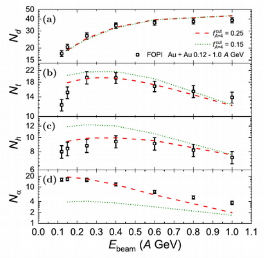

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$$N_1 + N_2 + B_1 + B_2 \rightarrow C_1 + C_2$$

$$\sim f(\sigma^*_{NN})$$

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$$\frac{\partial f_x}{\partial t} + \frac{\partial H_x}{\partial \boldsymbol{p}} \cdot \frac{\partial f_x}{\partial \boldsymbol{r}} - \frac{\partial H_x}{\partial \boldsymbol{r}} \cdot \frac{\partial f_x}{\partial \boldsymbol{p}} = I_x^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

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The in-medium NN cross section: σ^*_{NN}

$\sigma^*_{NN} = \sigma^{free}_{NN} \times f_{Pauli} \times \eta$ M. Henri et al PRC 101, 064622 (2020)

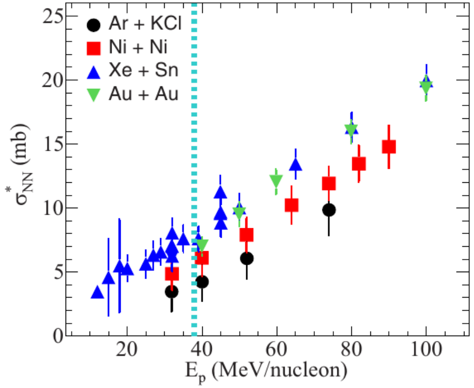
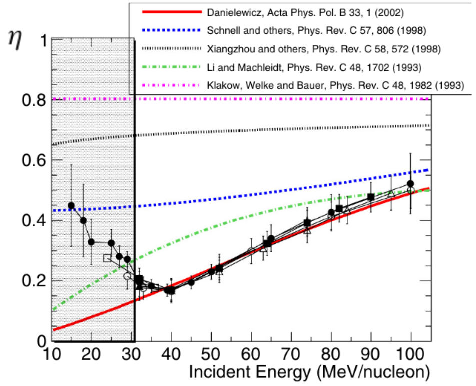


FIG. 2. Evolution of the in-medium nucleon-nucleon cross section σ^*_{NN} as a function of the incident projectile energy E_P. The vertical dotted line corresponds to the Fermi energy.

O. Lopez. PRC 90, 064602 (2014)



- Several parametrizations are available in the literature
- σ^*_{NN} scales with the mass systems ($\sim A^{1/3}_{tot}$), linked to the number of NN

DDS Coupland PRC 84, 054603 (2011)

$$\sigma = \sigma_0 \tanh(\sigma_{free}/\sigma_0),$$
$$\sigma_0 = y\rho^{-2/3}, \quad y = 0.85$$

A. Schnell, et al. Phys. Rev. C 57, 806 (1998).

$$\sigma = \sigma_{free} \exp\left(-0.6 \frac{\rho}{\rho_0} \frac{1}{1 + (T_{c.m.}/150 \text{ MeV})^2} \right)$$

Li and Macheidt PRC 1993, 1994

$$\sigma_{np}(E_{lab}, \rho) = [31.5 + 0.092\,\text{abs}(20.2 - E_{lab}^{0.53})^{2.9}] \times \frac{1.0 + 0.0034 E_{lab}^{1.51}\rho^2}{1.0 + 21.55\rho^{1.34}},$$

$$\sigma_{pp}(E_{lab}, \rho) = [23.5 + 0.0256(18.2 - E_{lab}^{0.5})^{4.0}] \times \frac{1.0 + 0.1667 E_{lab}^{1.05}\rho^3}{1.0 + 9.704\rho^{1.2}},$$

Q. Li et al, PLB 828 (2022) 137019

$$\sigma_{el}^{in\text{-}med} = \mathcal{F}(\rho, p) \cdot \sigma_{el}^{free}$$

with

$$\mathcal{F}(\rho, p) = \begin{cases} f_0, & p_{NN} > 1 \text{ GeV}/c, \\ \frac{\lambda + (1-\lambda)e^{-\frac{\rho}{\zeta\rho_0}} - f_0}{1 + (p_{NN}/p_0)^\kappa} + f_0, & p_{NN} \le 1 \text{ GeV}/c. \end{cases}$$

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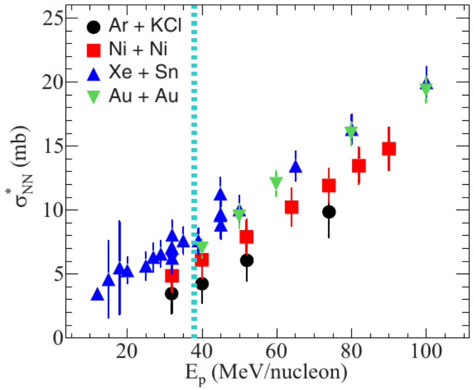

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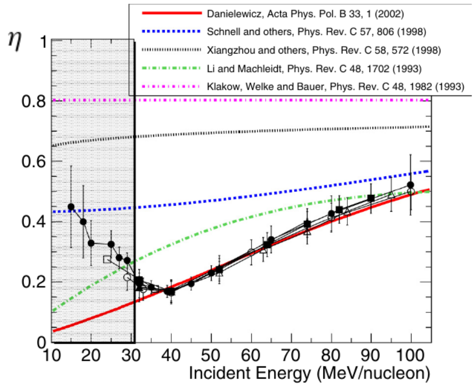

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A. Ono, Prog. Part. Nucl. Physics 105 (2019)

- Cluster production
- Isospin observables (n/p, R, ...)

P. Li et al, PRC C 97, 044620 (2018)
P. Li et al, PLB 828 (2022)
Liu et al, PRL 86 (2001)

- Nuclear stopping
- Collective Flow

Baran et al, Nuclear Physics A 730 (2004)

- Target-projectile mixing
- Neck formation

R. Wang et al, PLB 807 (2020)

- GDR strength

- Astro (?) - NS, to explore

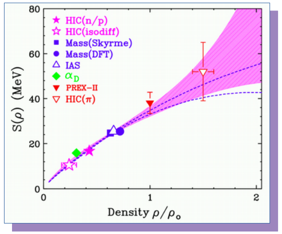
The golden question: why?

- Clustering in HIC is not well known,
  it is not easy to include it in transport model,
  we need a new data with top apparatus

  ⟶ This is not enough for ~~D. Lacroix~~ the PAC

- Eos at sub-saturation densities?

  ⟶ See the S. Huth Nature,
  or latest Lynch PLB



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  ⟶ B.-A. Li, Phys. Rev. Lett. 88 (19) (2002)
  N. Ikeno Phys. Rev. C 93 (4) (2016) 044612

A. Ono, Prog. Part. Nucl. Physics 105 (2019)

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- **Isospin observables (n/p, R, ...)**

P. Li et al, PRC C 97, 044620 (2018)
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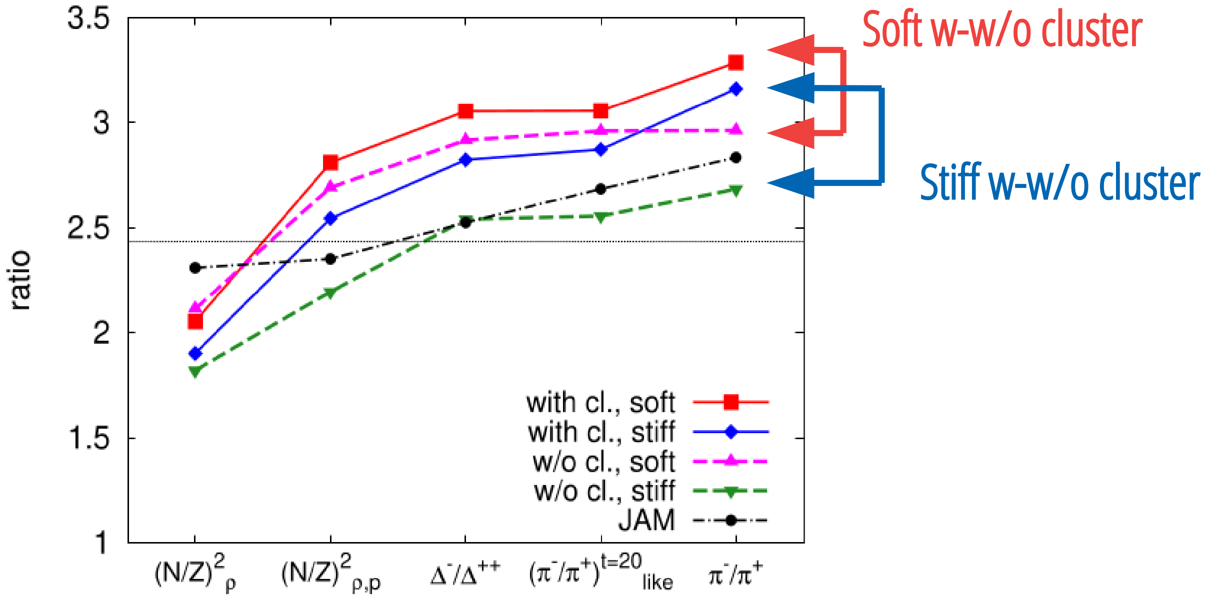

Fig. 27. Relation of various isospin ratios in $^{132}Sn + ^{124}Sn$ central collisions at 300 MeV/nucleon calculated with AMD+JAM approach, from the $(N/Z)^2$ ratio in the high-density region to the final π^-/π^+ yield ratio. The meaning of line styles is the same as in Fig. 26.

- Eos at supra-saturation densities?

→ B.-A. Li, Phys. Rev. Lett. 88 (19) (2002)
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- Astro (?) - NS, to explore

The proposed approach: Bayesian inference via transport model

32S+24Mg @ 25 & 50 MeV/u

"Light systems like 32S + 24Mg should be fine, even though it is not really an infinite nuclear matter" (cit. A. Ono)

- Comprehensive analysis of the σ^*_{NN} related observables to fully constraint it in **inclusive way**
- **Exclusive** detection of events complete in charge (HIPSE on going) to obtained a detailed balance of channels and their characterization (aiming also at E*)

A. Ono, Prog. Part. Nucl. Physics 105 (2019)

- **Cluster production**
- **Isospin observables (n/p, R, ...)**

P. Li et al, PRC C 97, 044620 (2018)
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R. Wang et al, PLB 807 (2020)

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- Astro (?) - NS, to explore

The proposed approach: Bayesian inference via transport model

32S+24Mg @ 25 & 50 MeV/u - INCLUSIVE

OBSERVABLES

Clusters production
Isospin observables (t/3He, R, ...)
Stopping
Ternary vs Binary events
....

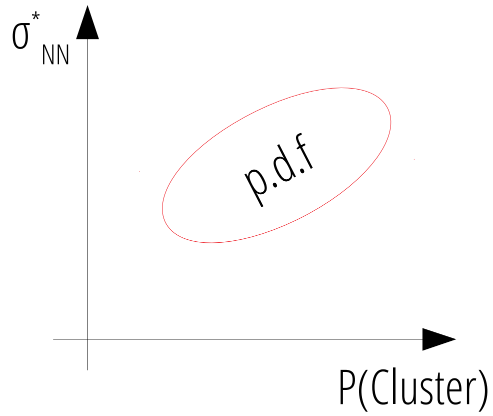
LIKELYHOOD

POSTERIOR

The Model

A grid exploring the full parameter space

- Sigma receipt and/or parameter
- Clusterization Probability



From inclusive posterior on σ_{NN}, P

PRIOR

The proposed approach: Bayesian inference via transport model

32S+24Mg @ 25 & 50 MeV/u - EXCLUSIVE

OBSERVABLES

Channel detailed balance
Channel detailed kinematics
...

LIKELYHOOD

POSTERIOR

The Model

A grid exploring the full parameter space

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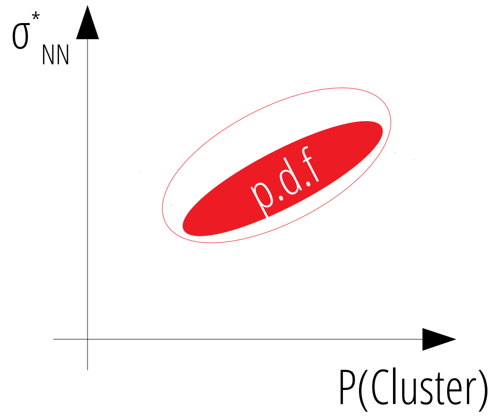
- Clusterization Probability



From inclusive posterior on σ_{NN}, P

PRIOR

And now?

1. Take a decision: is it worth spending effort on it?

And now?

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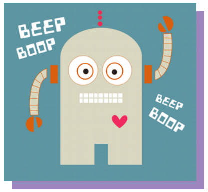
NO →

And now?

YES

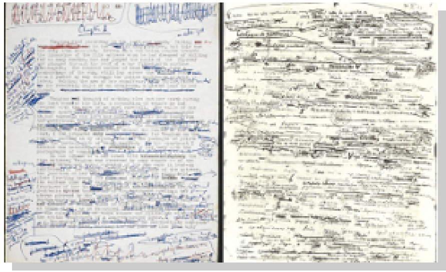
1. Take a decision: is it worth spending effort on it?

2. **HIPSE simulation**: filtering via KalivedaSim, efficiency estimation for (almost)-complete events



3. **MC simulation**: run the AMD grid (?)

4. **Prepare a draft**



5. Study

6. Update the draft

7. Study...

8. Update the draft ....

Proc. 13th Int. Conf. on Nucleus-Nucleus Collisions
JPS Conf. Proc. **32**, 010076 (2020)
https://doi.org/10.7566/JPSCP.32.010076

Impacts of Cluster Correlations on Heavy-Ion Collision Dynamics

Akira Ono

Department of Physics, Tohoku University, Sendai 980-8578, Japan

Unlike widely adopted choices, we assume that it depends on a kind of phase-space density $\rho' = (\rho_1'^{(\text{ini})}\rho_2'^{(\text{ini})}\rho_1'^{(\text{fin})}\rho_2'^{(\text{fin})})^{1/4}$, where

$$\rho_i'^{(\text{ini/fin})} = \left(\frac{2\nu}{\pi}\right)^{3/2} \sum_{k(\neq i)} \theta(p_{\text{cut}} > |\boldsymbol{P}_i^{(\text{ini/fin})} - \boldsymbol{P}_k|) e^{-2\nu(\boldsymbol{R}_i - \boldsymbol{R}_k)^2}, \qquad (4)$$

with a momentum cut at $p_{\text{cut}} = (375\ \text{MeV}/c)\ e^{-\epsilon/(225\ \text{MeV})}$, are introduced for the initial and final momenta, $\boldsymbol{P}_i^{(\text{ini})}$ and $\boldsymbol{P}_i^{(\text{fin})}$, of the scattered two nucleons $i = 1$ and 2, with a weak dependence on the collision energy ϵ in the two-nucleon center-of-mass system. If the clusters are not considered, we observe very weak stopping at 50 MeV/nucleon as shown by the orange filled square, as is often seen in usual AMD calculations without extensions.

Stopping may be reduced with further reduction of σ_{NN}. However, we here explore another possibility to suppress clusters in medium by forming clusters only in low phase-space density region. With the condition $\rho' < 0.125\ \text{fm}^{-3}$ for cluster formation, stopping is reduced as shown by the red filled circles in Fig. 1. With the same choice of these in-medium effects, the degree of stopping and the fragment yields at 250–270 MeV/nucleon are also described well, as shown in Figs. 2 and 3.