In ricordo di Enrico

Le masse dei quark sul reticolo

Vittorio Lubicz

Dipartimento di Fisica, Sapienza Università di Roma

23 maggio 2023

ELSEVIER

Nuclear Physics B 431 (1994) 667–685

NUCLEAR
PHYSICS B

Quark masses from lattice QCD at the next-to-leading order

C.R. Allton[a], M. Ciuchini[a,b], M. Crisafulli[a], E. Franco[a], V. Lubicz[a],
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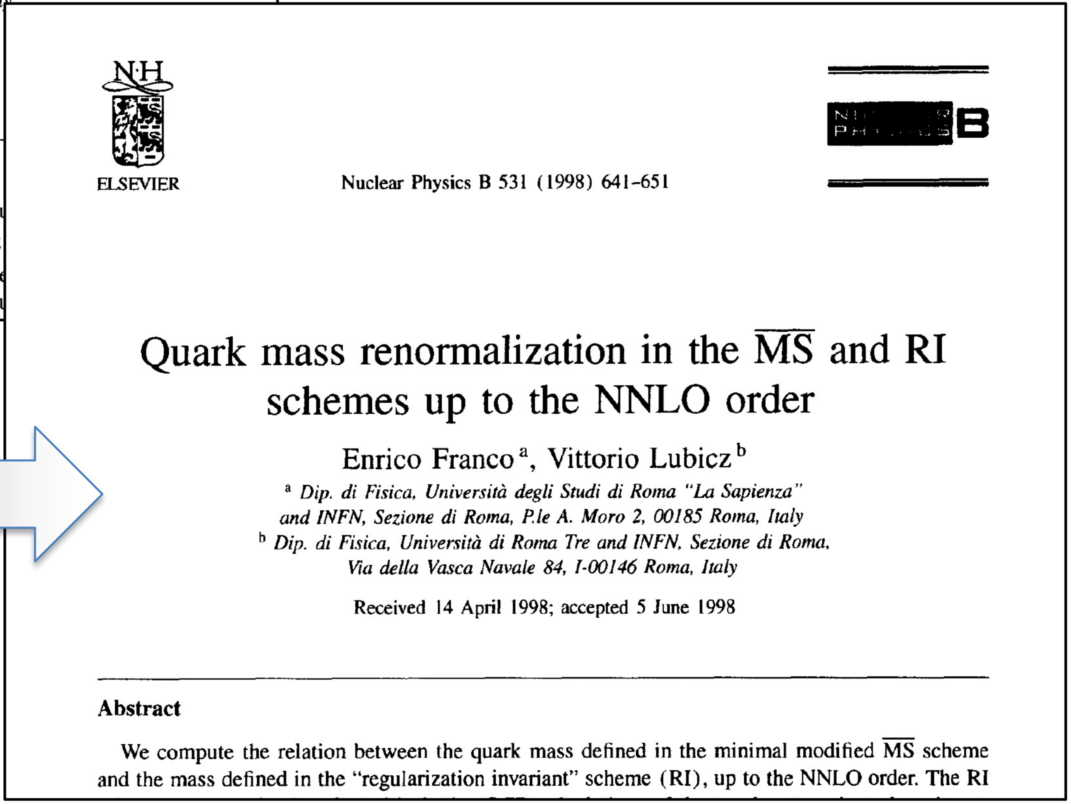
Received 15 June 1994; accepted 21 September 1994

Abstract

Using the results of several quenched lattice simulations, we predict the valu
charm quark masses in the continuum at the next-to-leading order, $m_s^{\overline{MS}}(\mu = 2$
and $m_{ch}^{\overline{MS}}(\mu = 2\,\mathrm{GeV}) = (1.48 \pm 0.28)\,\mathrm{GeV}$. The errors quoted above hav
taking into account the original statistical error of the lattice results and the u

Il mio primo lavoro
con Enrico
(1994)

Il mio lavoro
solo con Enrico
(1998)



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Nuclear Physics B 531 (1998) 641–651

B

Quark mass renormalization in the $\overline{\mathrm{MS}}$ and RI schemes up to the NNLO order

Enrico Franco[a], Vittorio Lubicz[b]

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Abstract

We compute the relation between the quark mass defined in the minimal modified $\overline{\mathrm{MS}}$ scheme and the mass defined in the "regularization invariant" scheme (RI), up to the NNLO order. The RI

Una collaborazione trentennale, molto bella e da cui ho imparato molto

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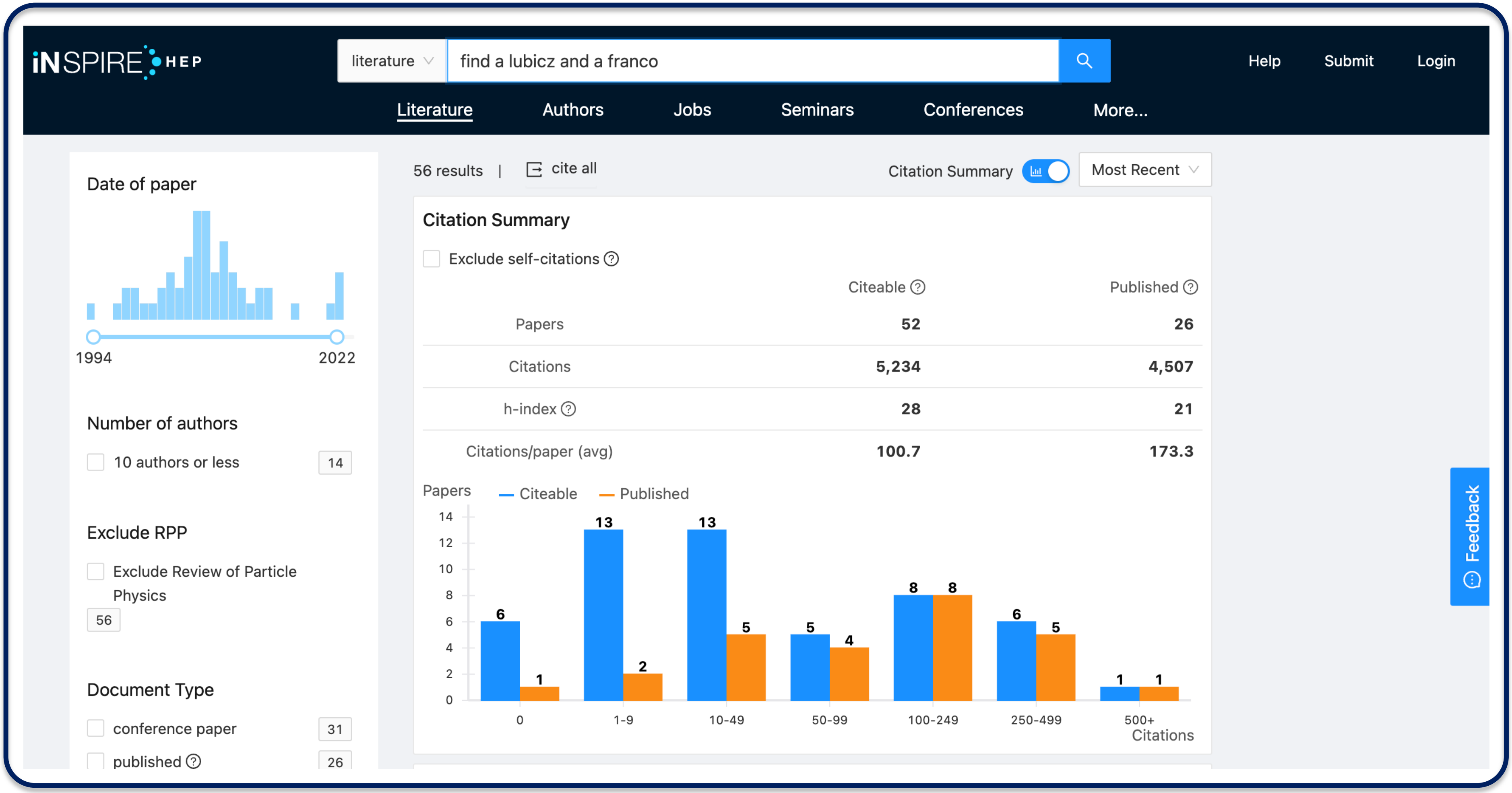

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Papers	**52**	**26**
Citations	**5,234**	**4,507**
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Citations/paper (avg)	**100.7**	**173.3**

Lattice determination of quark masses

- Quark masses cannot be directly measured in the experiments, because quarks are confined inside hadrons

- Being free parameters of the Standard Model, quark masses cannot be determined by theoretical considerations only

→ Quark masses can be determined by combining together a theoretical and an experimental input

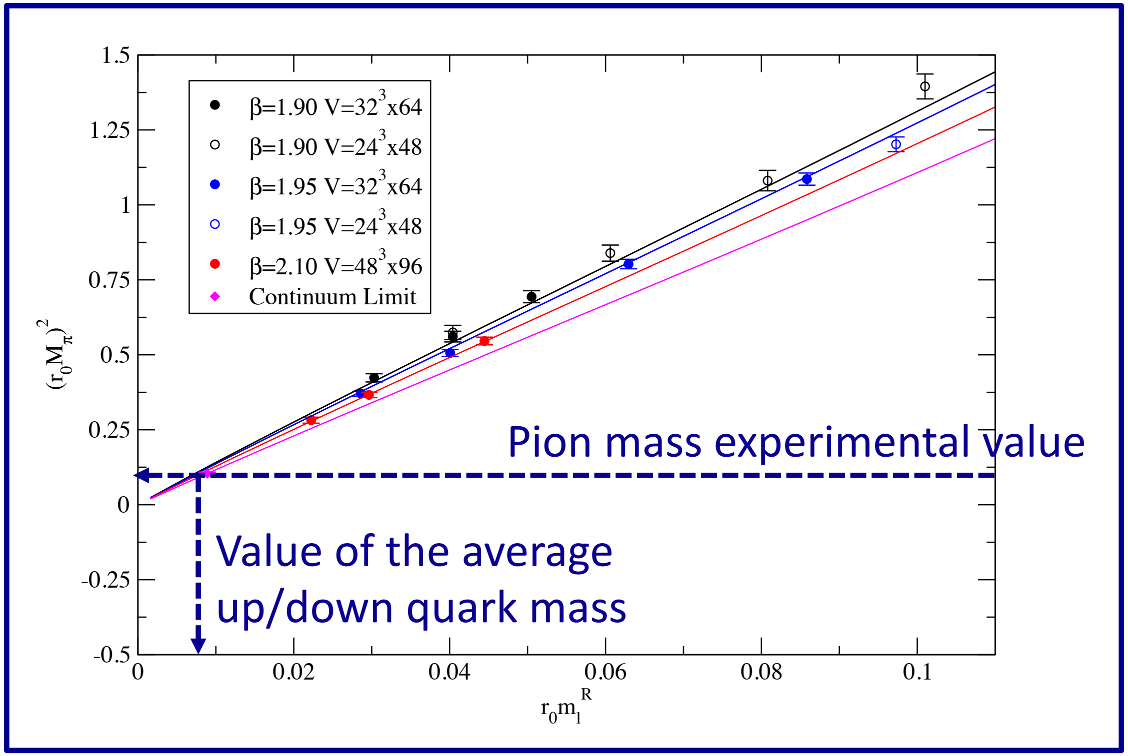
LATTICE
QCD

$$M_H(\Lambda_{QCD}, m_q)^{LAT} = M_H^{EXP}$$

(H = PGB)

Lattice determination of quark masses

Pion mass squared



Pion mass experimental value

Value of the average up/down quark mass

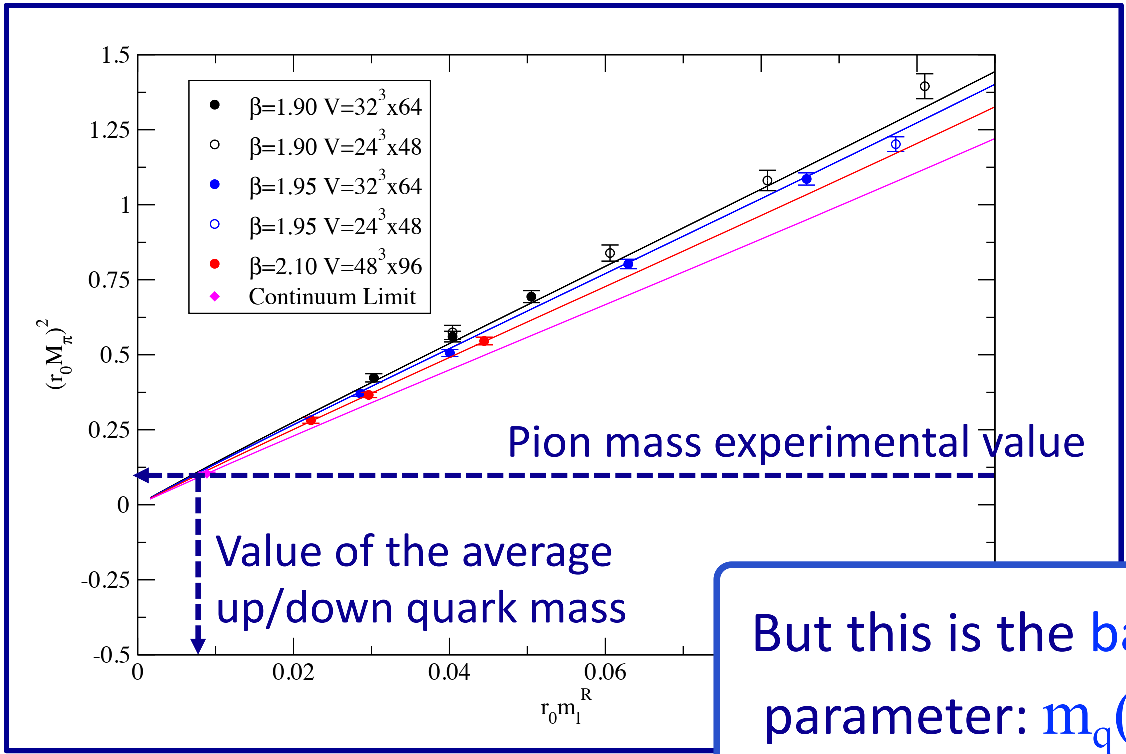
Average up/down quark mass

LATTICE QCD

$$M_H(\Lambda_{QCD}, m_q)^{LAT} = M_H^{EXP}$$

Lattice determination of quark masses

Pion mass squared



Pion mass experimental value

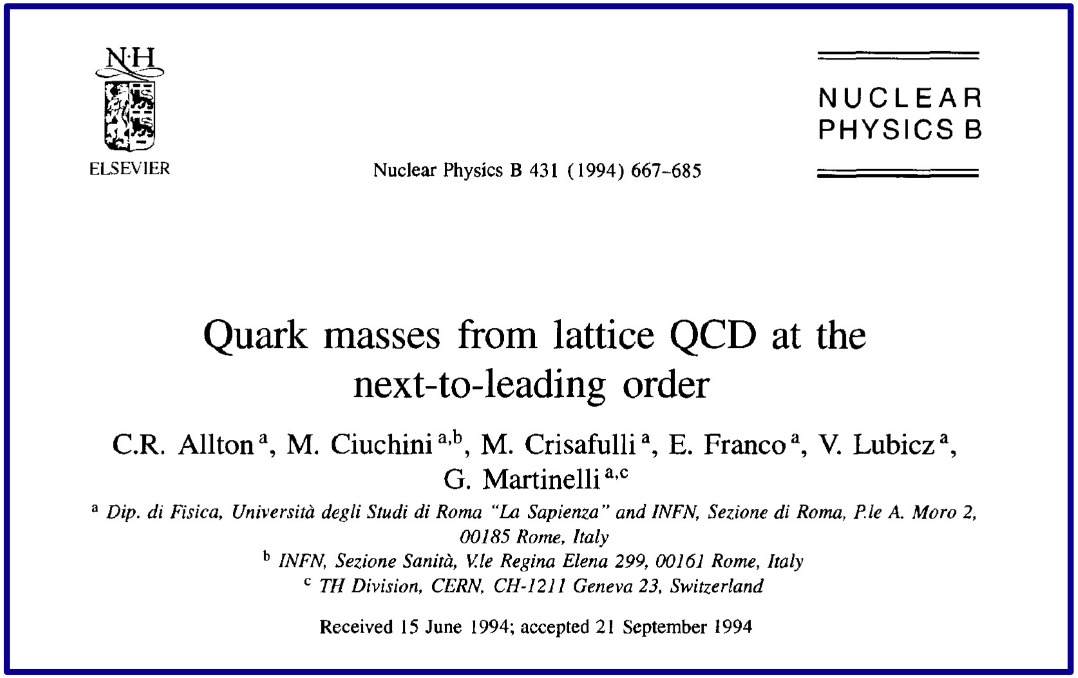
Value of the average
up/down quark mass

Average up/down quark mass

But this is the bare parameter: $m_q(a)$

LATTICE
QCD

$$M_H(\Lambda_{QCD}, m_q)^{LAT} = M_H^{EXP}$$



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NUCLEAR PHYSICS B

Quark masses from lattice QCD at the next-to-leading order

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Il mio primo lavoro con Enrico (1994)

Prima determinazione sul reticolo delle masse dei quark al NLO in QCD

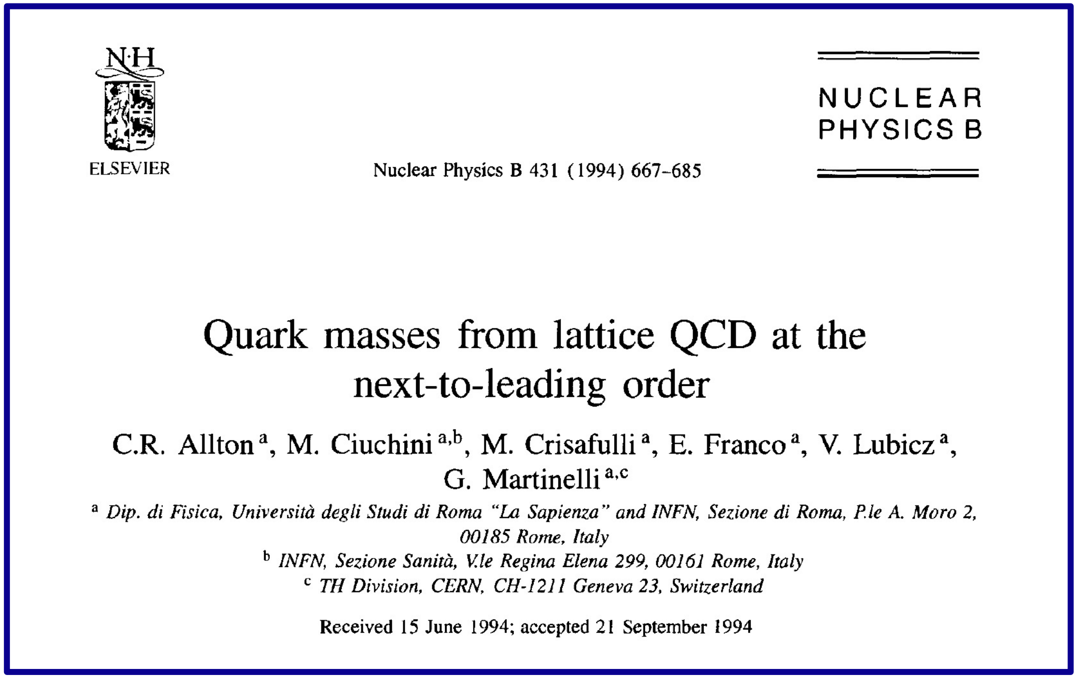
$$m^{\overline{\rm MS}}(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\pi/a)}\right)^{\gamma^{(0)}/2\beta_0}\left[1 + \frac{\alpha_s(\mu) - \alpha_s(\pi/a)}{4\pi}\left(\frac{\gamma^{(1)}}{2\beta_0} - \frac{\gamma^{(0)}\beta_1}{2\beta_0^2}\right) + \frac{\alpha_s(\pi/a)}{4\pi}K_m\right] m(a)$$

Massa del quark nello schema $\overline{\text{MS}}$ alla scala μ

La costante K_m
è determinata dal confronto
del propagatore bare
sul reticolo e $\overline{\text{MS}}$ nel continuo

(Gonzalez Arroyo, Martinelli, Yndurain, 1982)

Massa del quark bare sul reticolo



N·H
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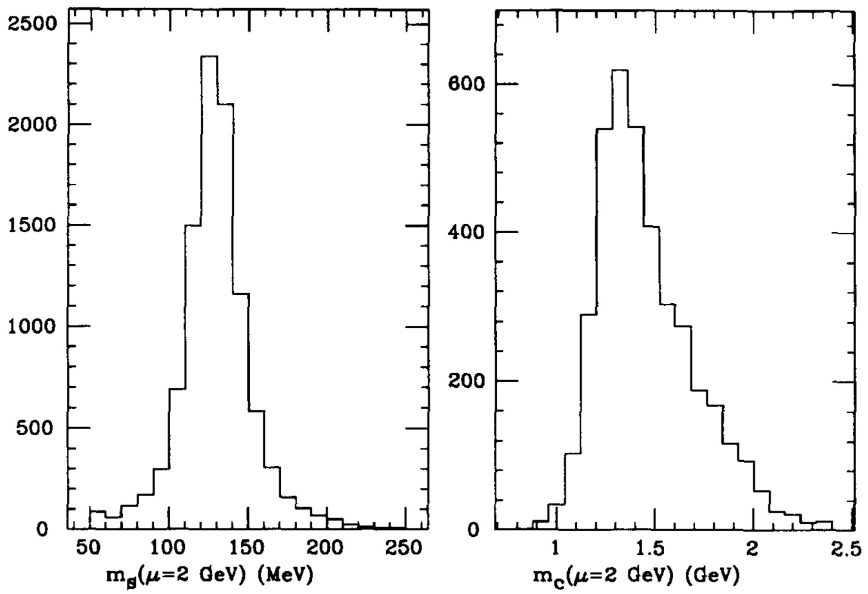
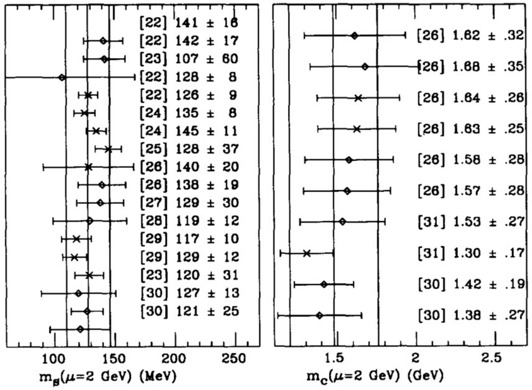


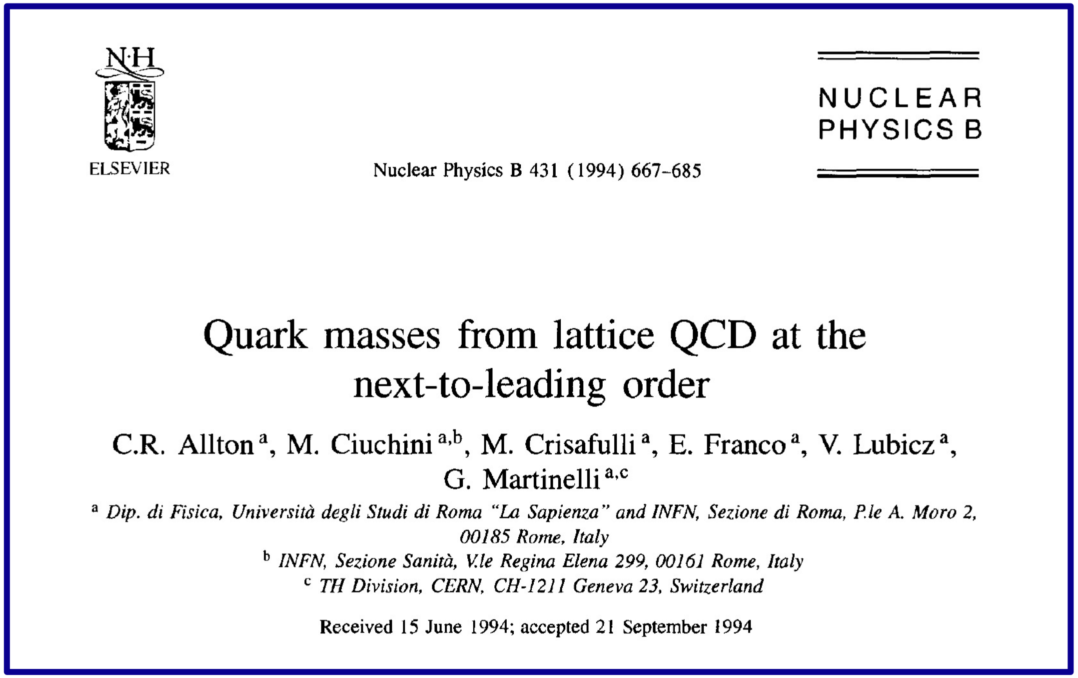


15-20%

$$m_s^{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV}) = (128 \pm 18)\ \mathrm{MeV}\,,$$

$$m_{\mathrm{ch}}^{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV}) = (1.48 \pm 0.28)\ \mathrm{GeV}.$$

Il mio primo lavoro con Enrico (1994)



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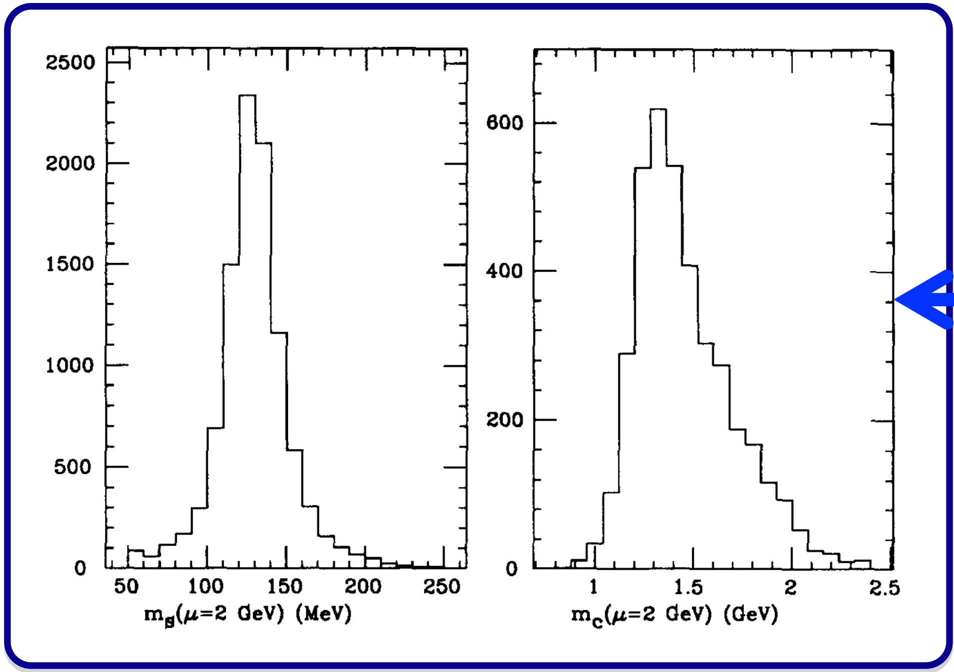
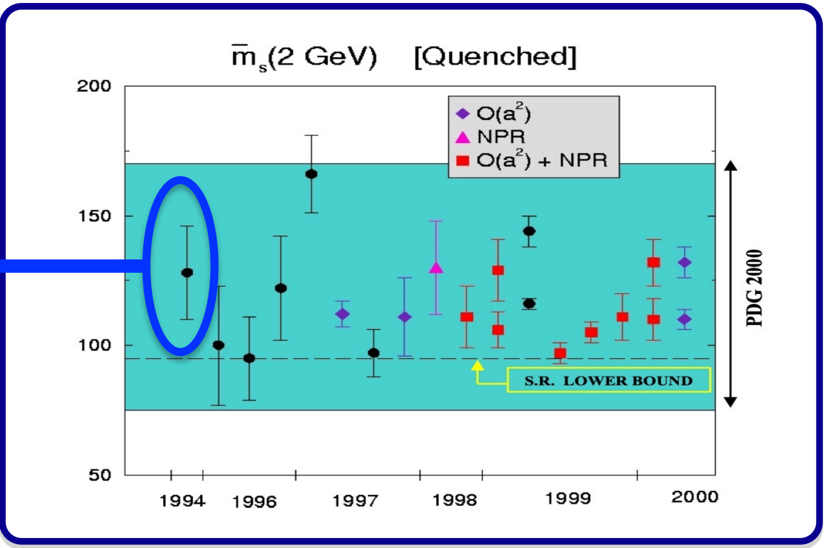
Dopo questo lavoro,
la determinazione delle masse dei quark sul reticolo
è diventata un intenso campo di attività





VL, plenary talk at Lattice 2000, Bangalore

Non-perturbative renormalization

- The convergence rate of the asymptotic series of lattice perturbation theory is often unsatisfactory

$$m^{\overline{\mathrm{MS}}}(\mu)=\left(\frac{\alpha_s(\mu)}{\alpha_s(\pi/a)}\right)^{\gamma^{(0)}/2\beta_0}\left[1+\frac{\alpha_s(\mu)-\alpha_s(\pi/a)}{4\pi}\left(\frac{\gamma^{(1)}}{2\beta_0}-\frac{\gamma^{(0)}\beta_1}{2\beta_0^2}\right)+\frac{\alpha_s(\pi/a)}{4\pi}K_m\right]m(a)$$

Wilson action

K_m = 8.11

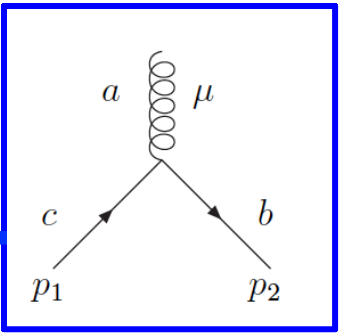
Wilson-clover

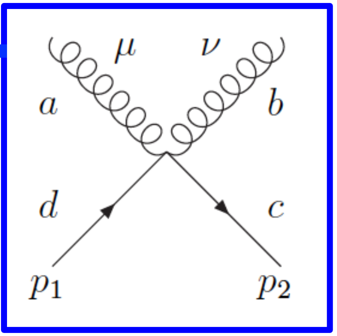
K_m = 16.4

- Moreover, lattice perturbation theory is notoriously difficult, and most of results are known only at the 1-loop level

$$(V_1^a)^{bc}_\mu(p_1,p_2)=-g_0(T^a)^{bc}\left(\mathrm{i}\gamma_\mu\cos\frac{a(p_1+p_2)_\mu}{2}+r\sin\frac{a(p_1+p_2)_\mu}{2}\right)$$

$$(V_2^{ab})^{cd}_{\mu_1\mu_2}(p_1,p_2)=-\frac{1}{2}ag_0^2\,\delta_{\mu_1\mu_2}\left(\frac{1}{N_c}\delta^{ab}+d^{abe}T^e\right)^{cd}\,.$$
$$\cdot\left(-\mathrm{i}\gamma_\mu\sin\frac{a(p_1+p_2)_\mu}{2}+r\cos\frac{a(p_1+p_2)_\mu}{2}\right)$$

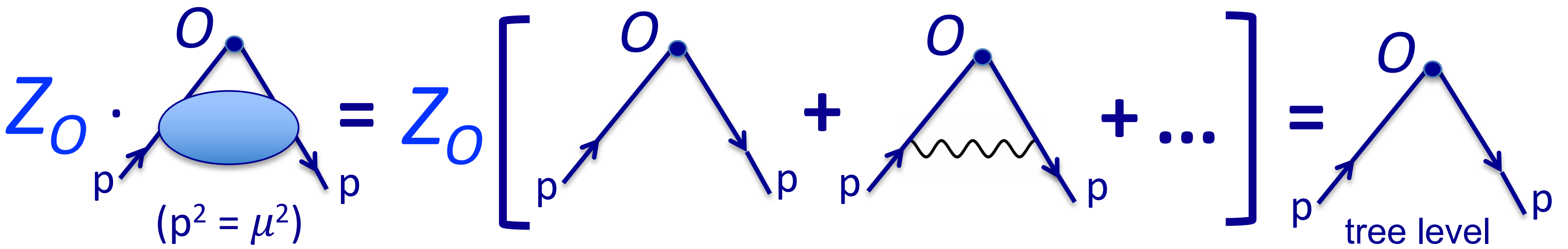


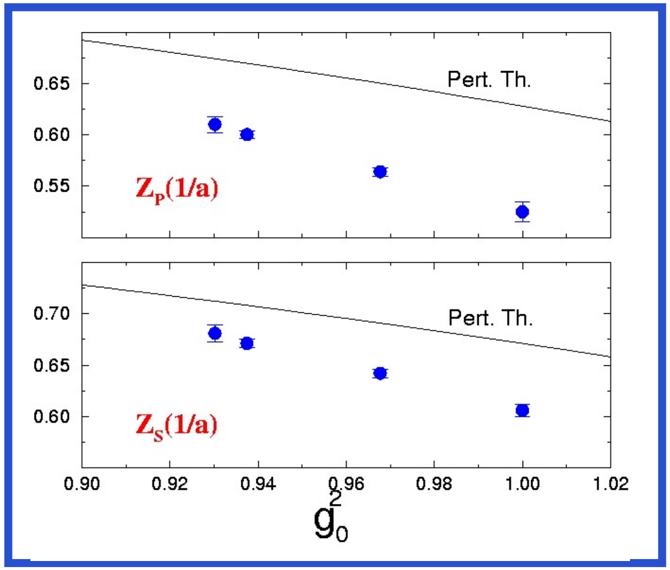


Non-perturbative renormalization

The RI-MOM scheme

G. Martinelli, C. Pittori, C.T. Sachrajda,
M. Testa, A. Vladikas, 1995





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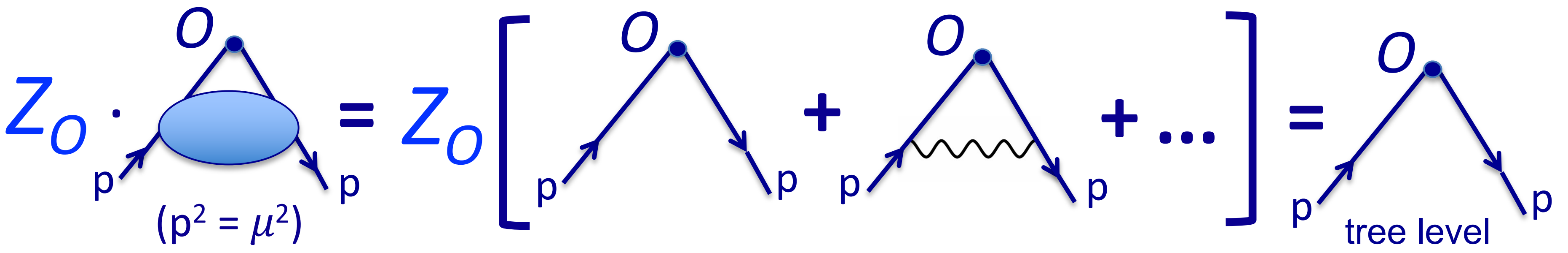


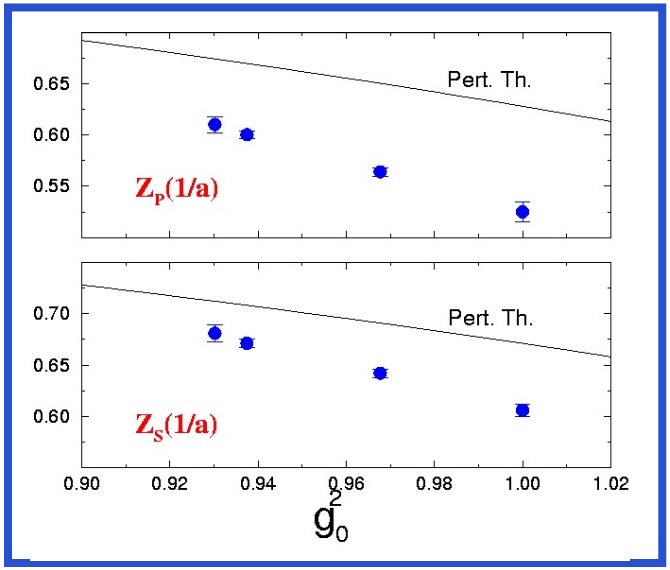
- Nel lavoro con Enrico del 1994 abbiamo anche fornito la relazione tra la massa del quark nello schema $\overline{\text{MS}}$ e la massa nello schema RI-MOM

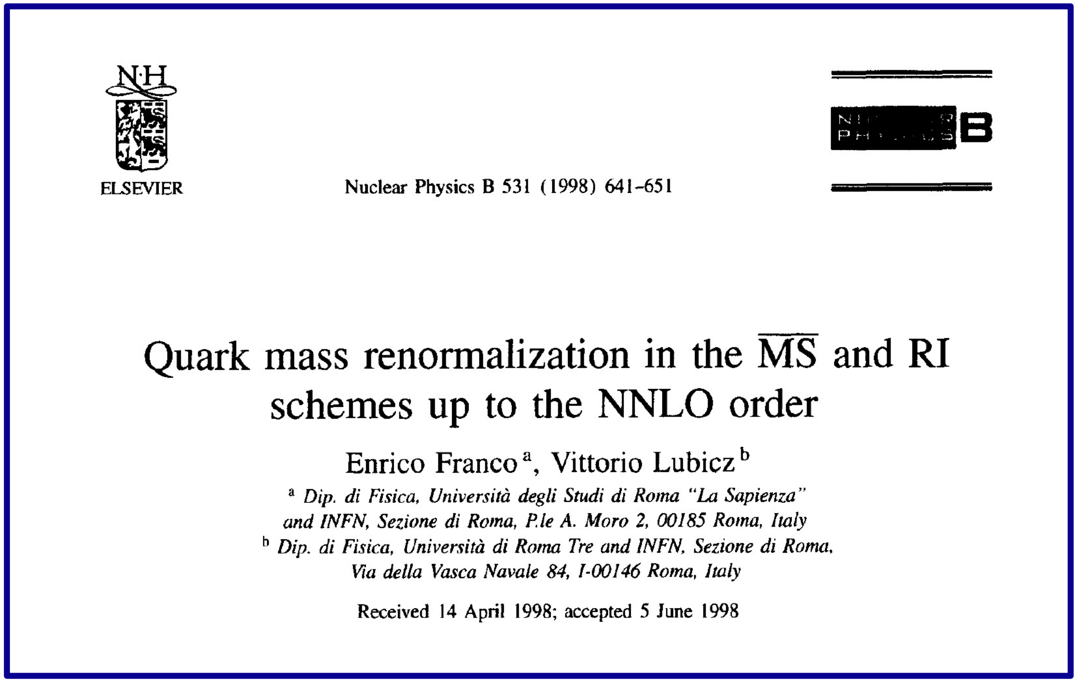
$$m^{\text{fey,lan}}(\mu) = \left[1 - \frac{\alpha_s(\mu)}{4\pi} C_m^{\text{fey,lan}}\right] m^{\overline{\text{MS}}}(\mu)$$

Massa nello schema RI-MOM

$$C_m^{\text{lan}} = -\frac{N^2-1}{2N} 4$$

Massa nello schema $\overline{\text{MS}}$





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Il mio lavoro solo con Enrico (1998)

Relazione tra le masse nello schema MS e RI-MOM al NNLO in QCD:

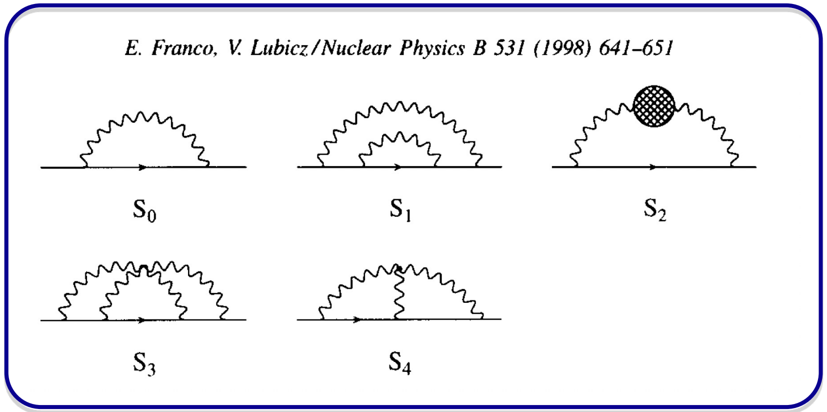
$$R_m(\mu) = m^{\overline{MS}}(\mu) \,/\, m^{RI}(\mu)$$

$$R_m^{\mathrm{LAN}}(\mu) = 1 - \frac{16}{3}\frac{\alpha_s(\mu)}{(4\pi)} - \left(\frac{1990}{9} - \frac{152}{3}\zeta_3 - \frac{89}{9}n_f\right)\frac{\alpha_s^2(\mu)}{(4\pi)^2}$$

C_m^{lan}C

Questo lavoro

NLO = -13 % , NNLO = -7 % , μ = 2 GeV



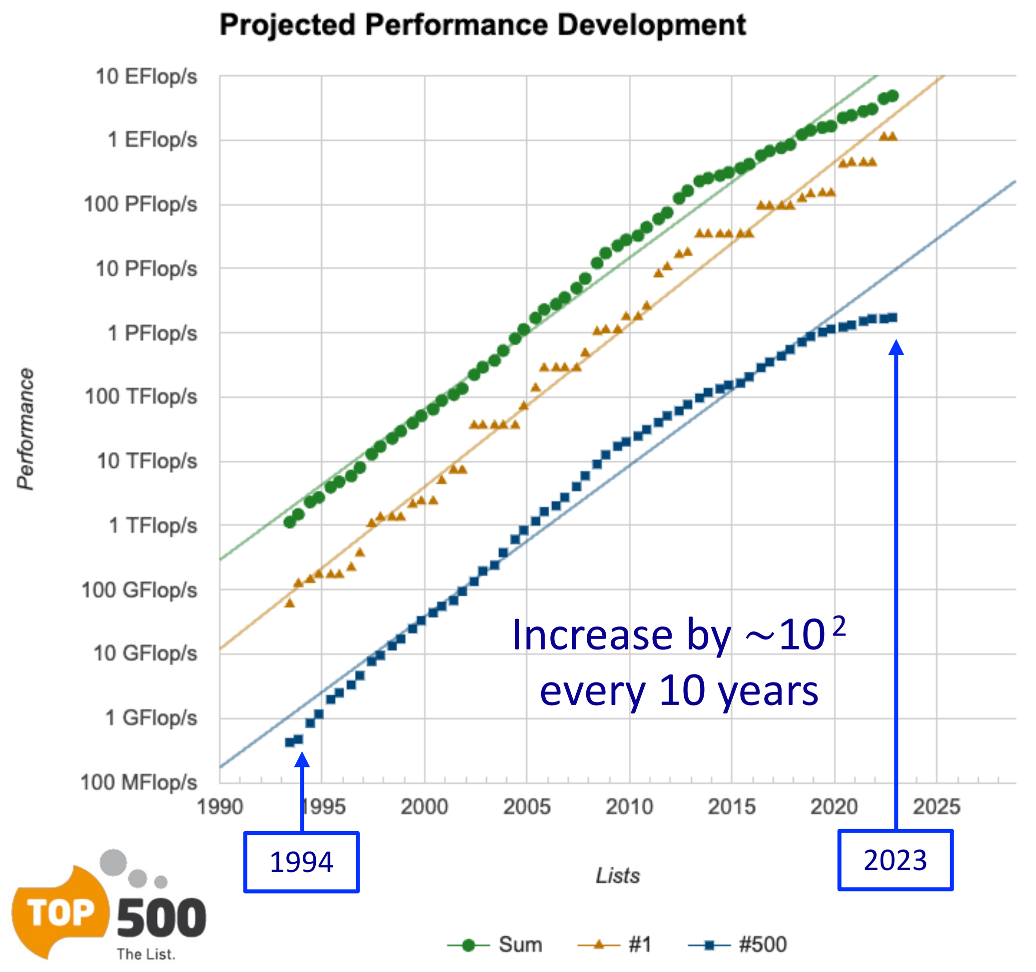
E. Franco, V. Lubicz/Nuclear Physics B 531 (1998) 641–651

Il fattore di conversione è stato poi calcolato:

- 3 loop: Chetyrkin. Rétey, 2000
- 4 loop: Gracey, 2023

QUARK MASSES IN THE PRECISION ERA OF LQCD

The increasing of computational power



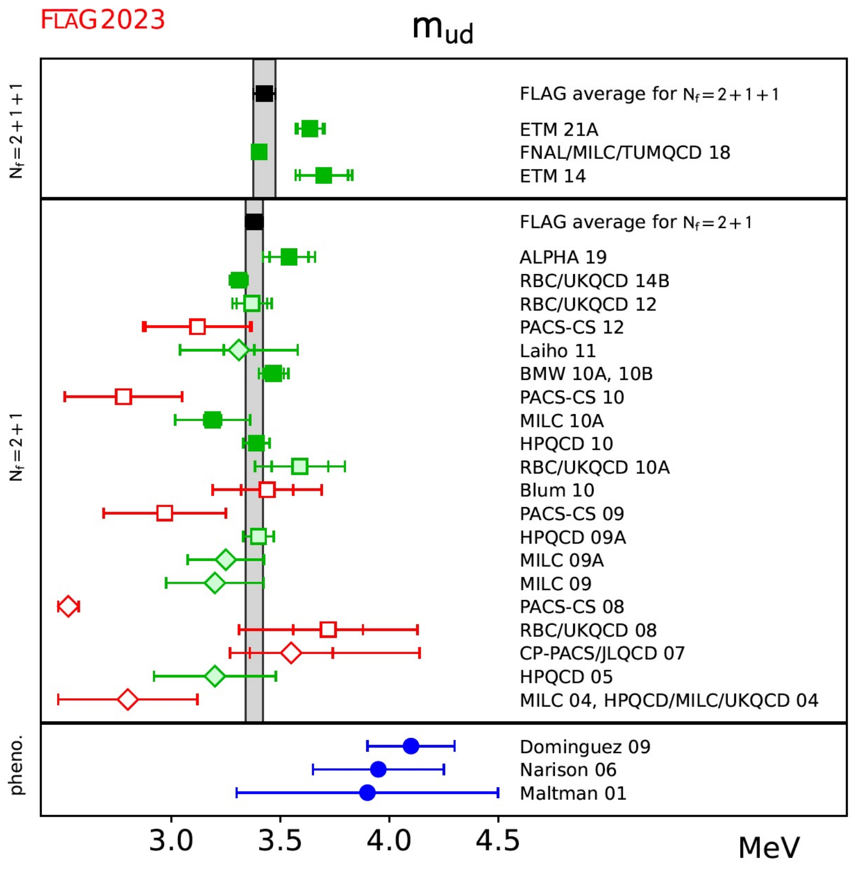
Increase by $\sim 10^2$ every 10 years

1994

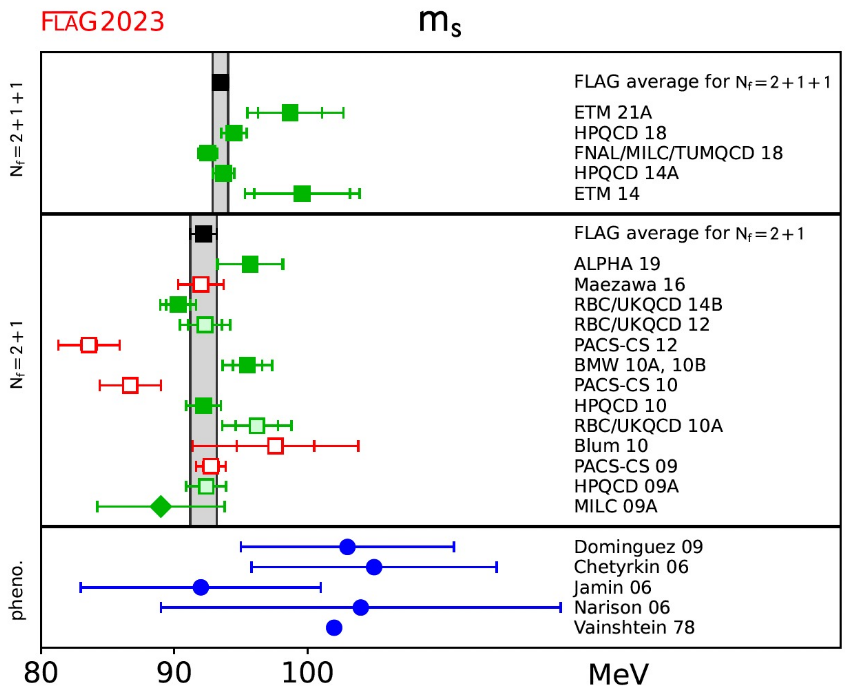
2023

Rank	Site
1	DOE/SC/Oak Ridge National Laboratory United States Frontier Rpeak = 1,686 TFlop/s
2	RIKEN Center for Computational Science Japan Fugaku Rpeak = 537 TFlop/s
3	EuroHPC/CSC Finland LUMI Rpeak = 429 TFlop/s
4	EuroHPC/CINECA Italy Leonardo Rpeak = 256 TFlop/s

QUARK MASSES IN THE PRECISION ERA OF LQCD



The accuracy is at the 1% level



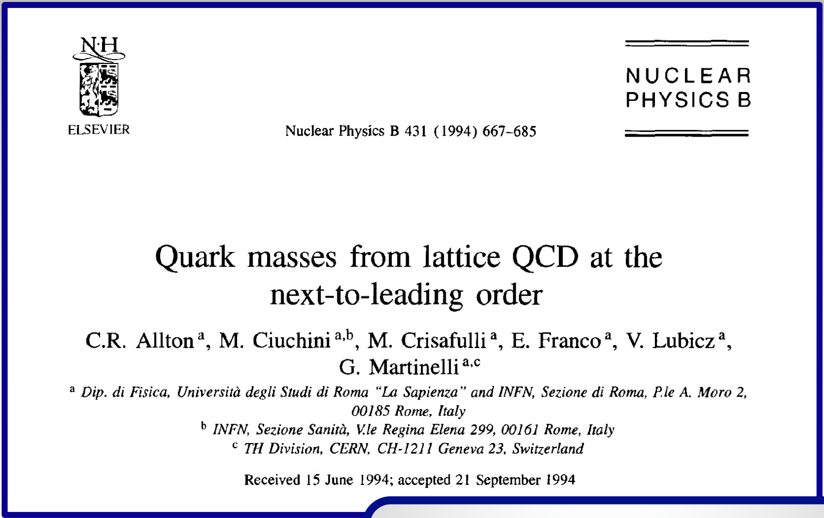
$\overline{m}_{ud} = 3.427(51)$ MeV (Nf=2+1+1)

$\overline{m}_{ud} = 3.381(40)$ MeV (Nf=2+1)

$\overline{m}_{s} = 93.46(58)$ MeV (Nf=2+1+1)

$\overline{m}_{s} = 92.2(1.0)$ MeV (Nf=2+1)

QUARK MASSES IN THE PRECISION ERA OF LQCD



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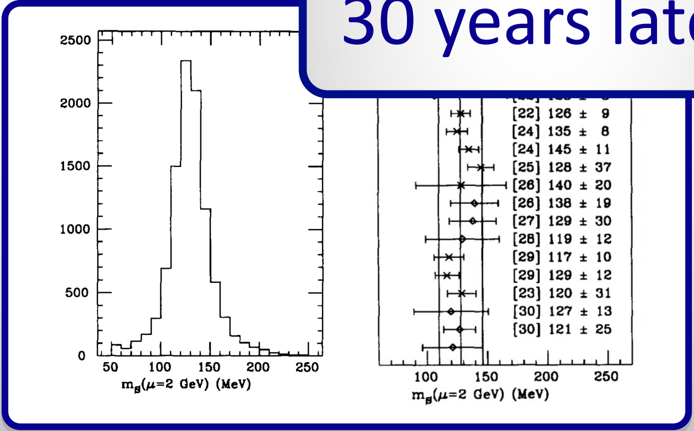
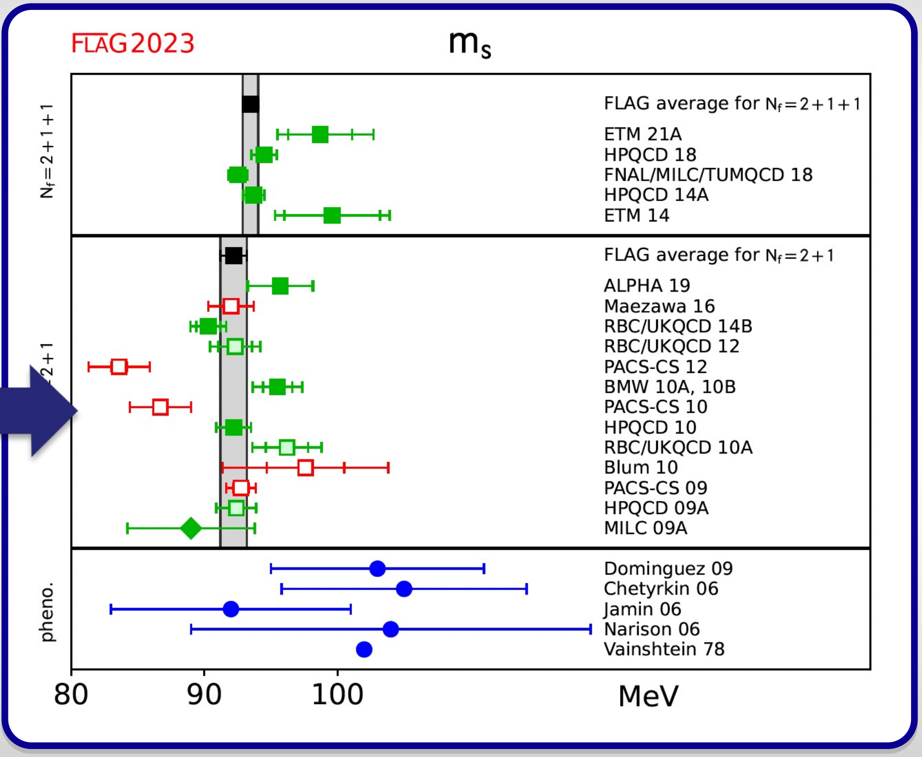
30 years later



$$m_s^{\overline{MS}}(\mu = 2\ \text{GeV}) = (128 \pm 18)\ \text{MeV}$$

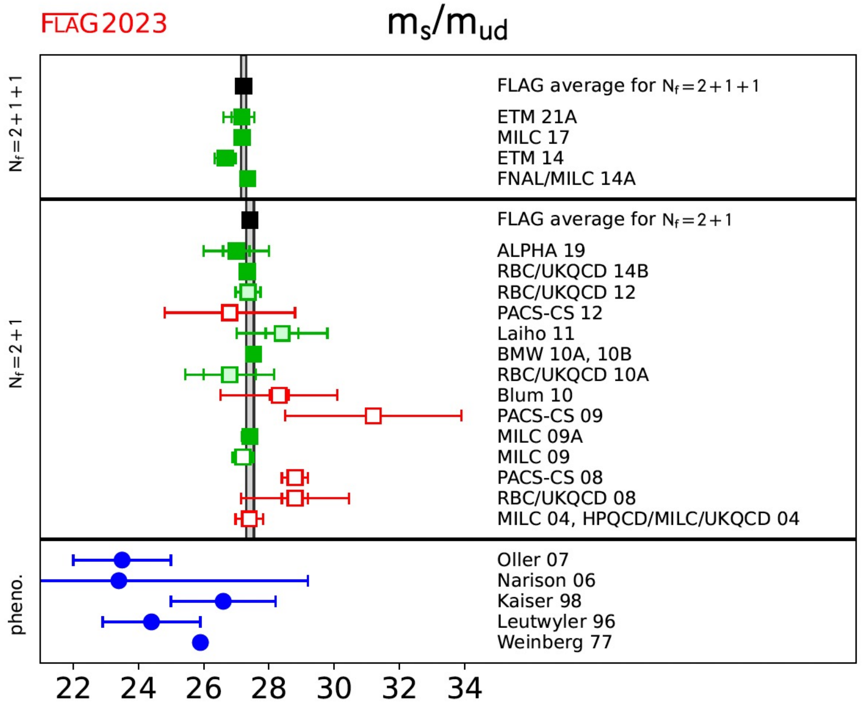
The accuracy is at the 1% level



$\overline{m}_s$ = 93.46(58) MeV (Nf=2+1+1)

$\overline{m}_s$ = 92.2(1.0) MeV (Nf=2+1)

QUARK MASSES IN THE PRECISION ERA OF LQCD



In the ratio of quark masses, several sources of uncertainties (renormalization, lattice scale, ...), either cancel out or are largely reduced

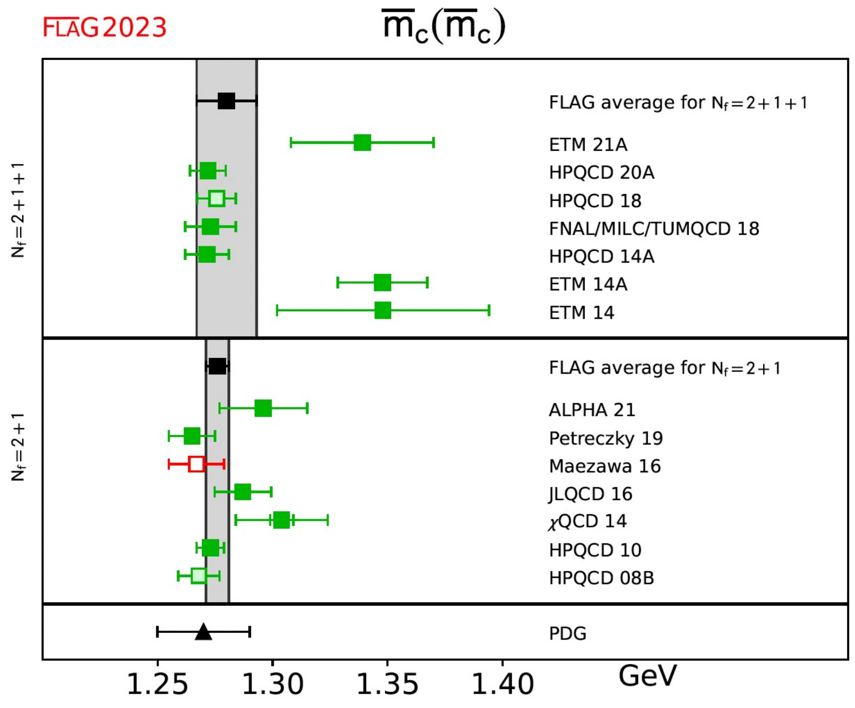
$\overline{m}_s / \overline{m}_{ud} = 27.227(81)$ (Nf=2+1+1)

$\overline{m}_s / \overline{m}_{ud} = 27.42(12)$ (Nf=2+1)

The accuracy is at the 3-4 permille level

QUARK MASSES IN THE PRECISION ERA OF LQCD

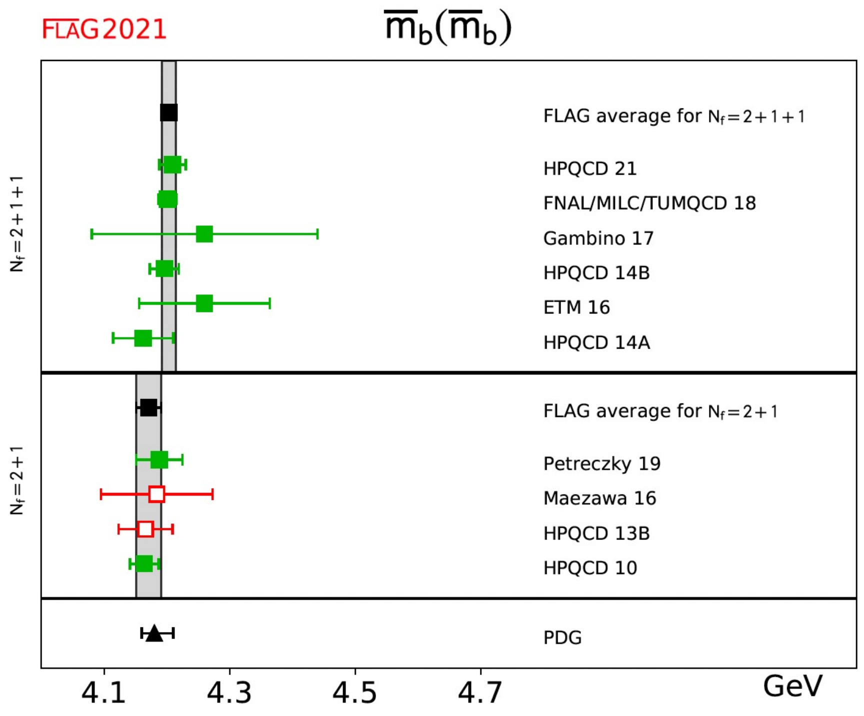
The charm quark mass



$\overline{m}_c = 1.280(13)$ GeV (Nf=2+1+1)

$\overline{m}_c = 1.276(5)$ GeV (Nf=2+1)

The bottom quark mass

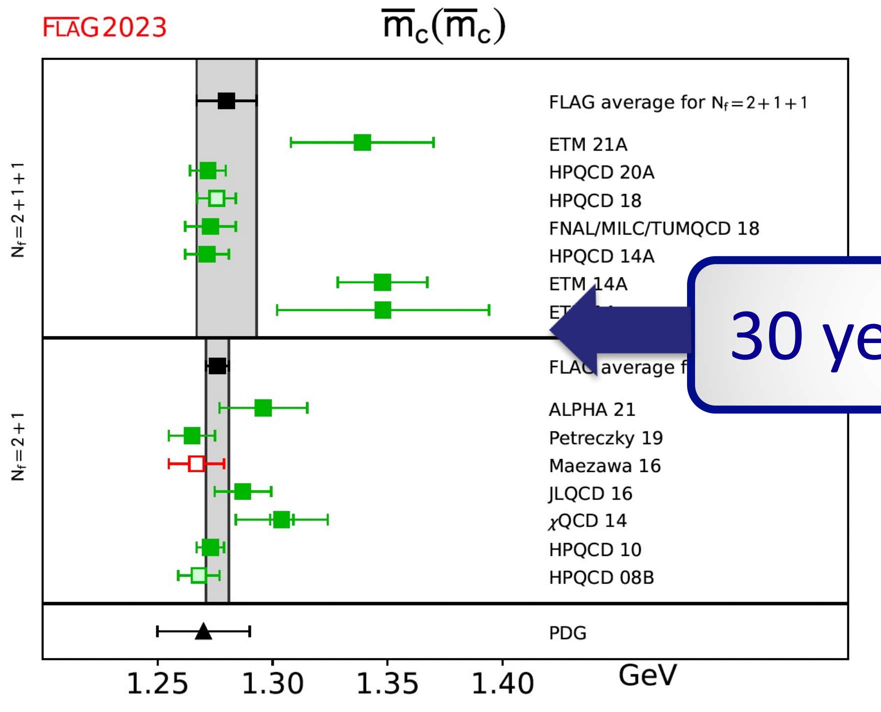


$\overline{m}_b = 4.203(11)$ MeV (Nf=2+1+1)

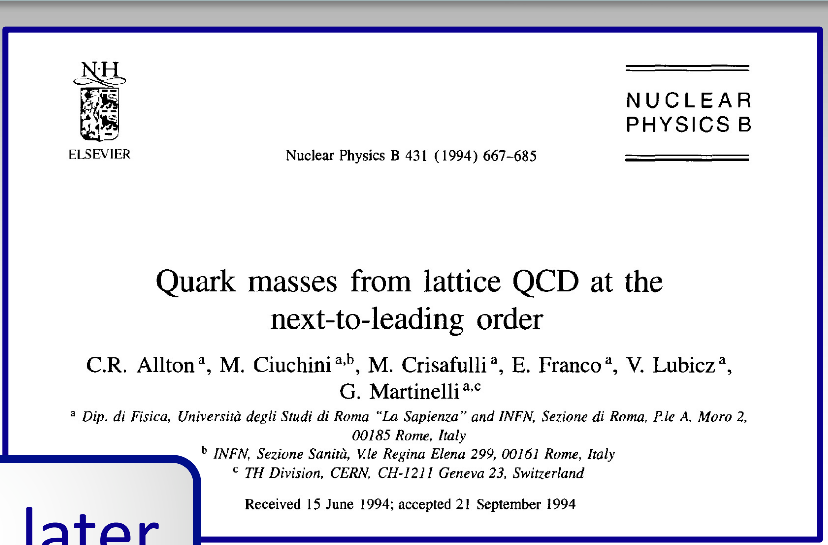
$\overline{m}_b = 4.171(20)$ GeV (Nf=2+1)

QUARK MASSES IN THE PRECISION ERA OF LQCD

The charm quark mass



30 years later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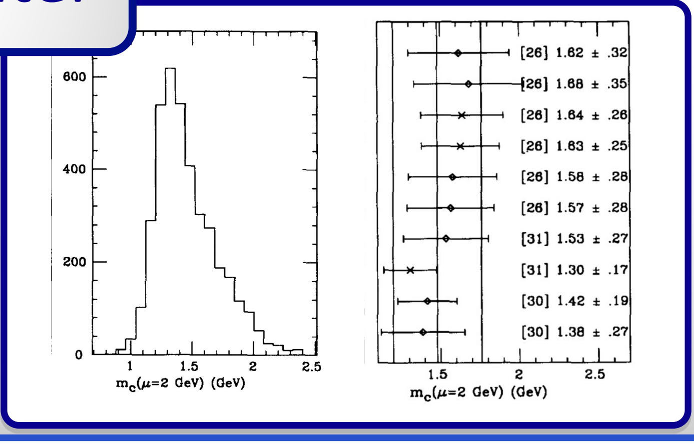


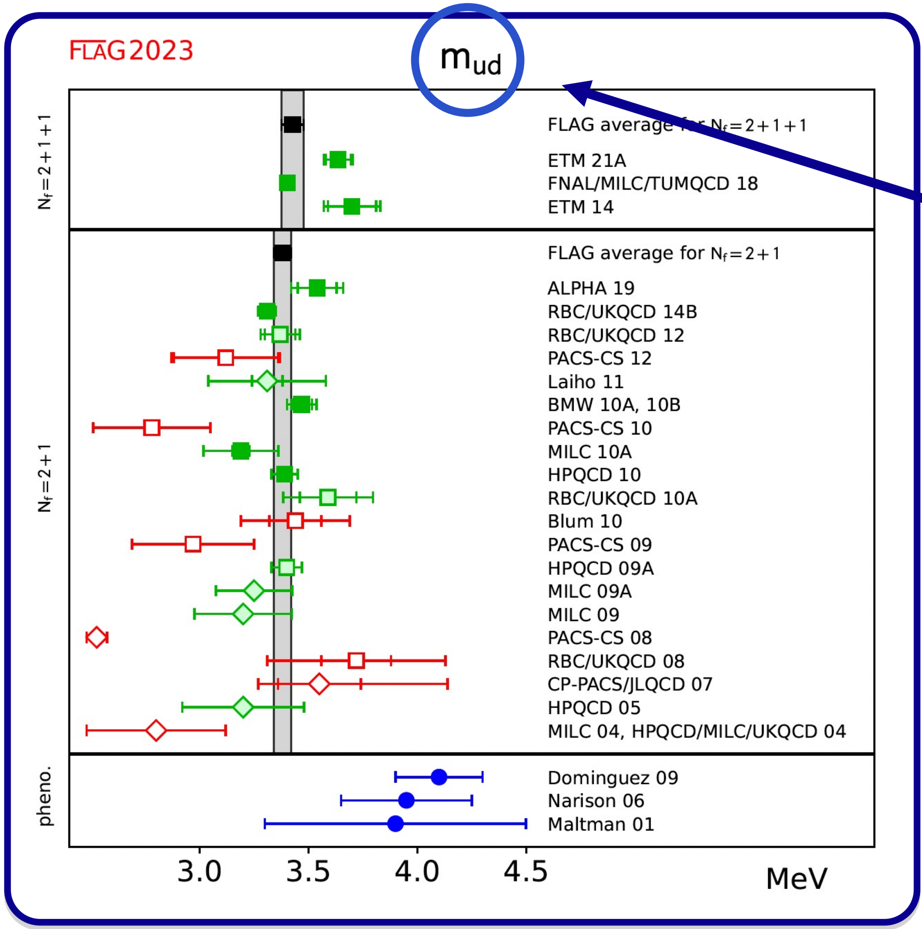
$\overline{m}_c = 1.280(13)$ GeV (Nf=2+1+1)

$\overline{m}_c = 1.276(5)$ GeV (Nf=2+1)

$$m_{ch}^{\overline{MS}}(\mu = 2\ \mathrm{GeV}) = (1.48 \pm 0.28)\ \mathrm{GeV}$$

$$\overline{m}_c(\overline{m}_c) = (1.59 \pm 0.30)\ \mathrm{GeV}$$

ISOSPIN BREAKING EFFECTS and mu/md



m_{ud} is the average up-down quark mass

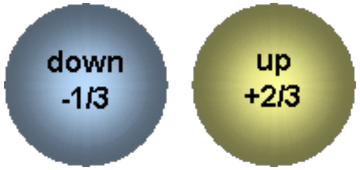
In order to evaluate m_u and m_d separately, isospin breaking effects must be introduced in lattice calculations

$\overline{m}_{ud} = 3.427(51)$ MeV (Nf=2+1+1)

$\overline{m}_{ud} = 3.381(40)$ MeV (Nf=2+1)

ISOSPIN BREAKING EFFECTS

Isospin symmetry is an almost exact property of the strong interactions



Isospin breaking effects are induced by:

$m_u \neq m_d$: $O[(m_d-m_u)/\Lambda_{QCD}] \approx 1/100$ "Strong"

$Q_u \neq Q_d$: $O(\alpha_{em}) \approx 1/100$ "Electromagnetic"

Since electromagnetic interactions renormalize quark masses the two corrections are intrinsically related

Though small, IB effects play often a very important role

ISOSPIN BREAKING EFFECTS

- The knowledge of mu and md (besides m_{ud}) is important for our understanding of flavor physics at the fundamental level

mu ≃ 2.5 MeV md ≃ 5 MeV

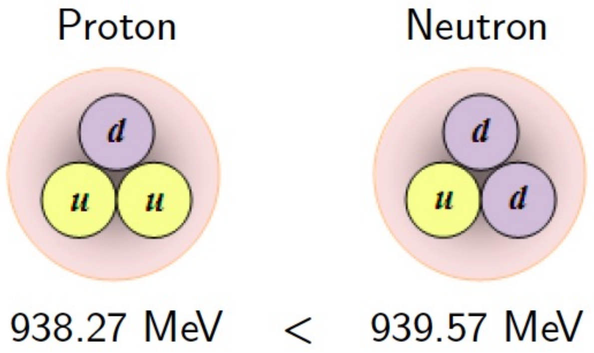
mc ≃ 1.2 GeV ms ≃ 100 MeV

mt ≃ 175 GeV mb ≃ 4.3 GeV

A remarkable relation:

$$\left(\frac{m_d}{m_s}\right)^{1/2} \simeq \left(\frac{m_u}{m_c}\right)^{1/4} \simeq V_{us} \simeq 0.22$$

- The actual values of the mass difference md-mu and quark charges Qd, Qu implies Mn > Mp and guarantees the stability of matter



M(n) – M(p) = 1.3 MeV = 0.14%

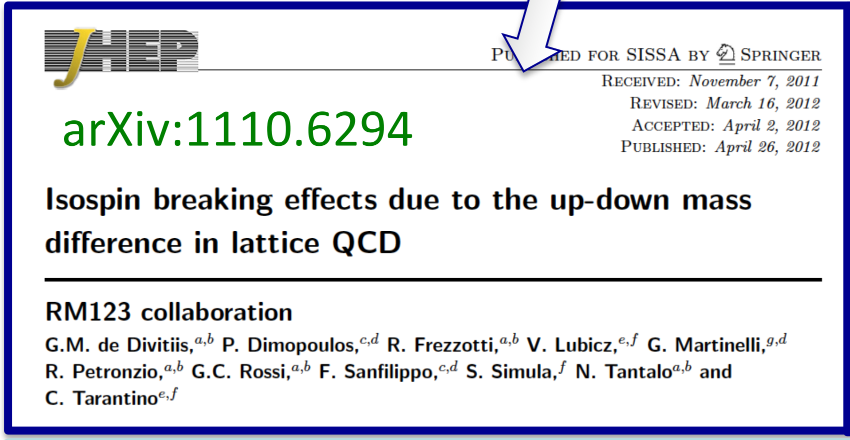
ISOSPIN BREAKING EFFECTS

A strategy for Lattice QCD :

The isospin breaking part of the Lagrangian

is treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}



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Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

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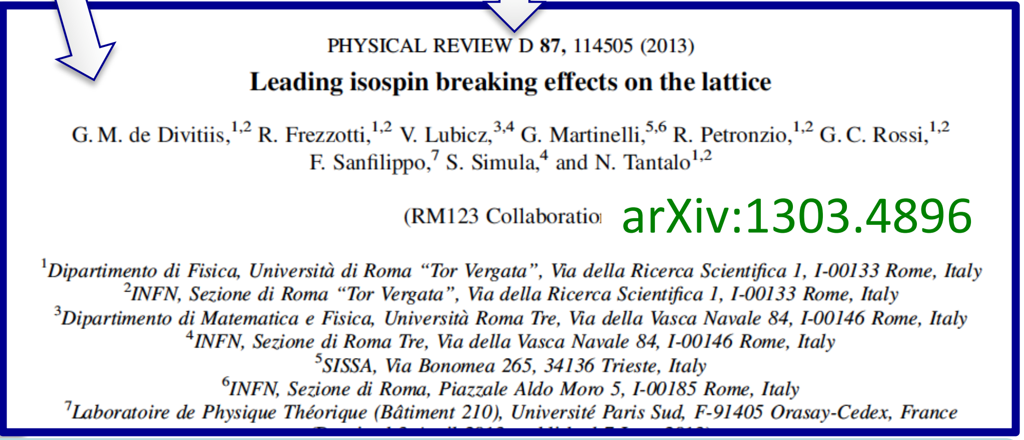

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Leading isospin breaking effects on the lattice

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arXiv:1303.4896

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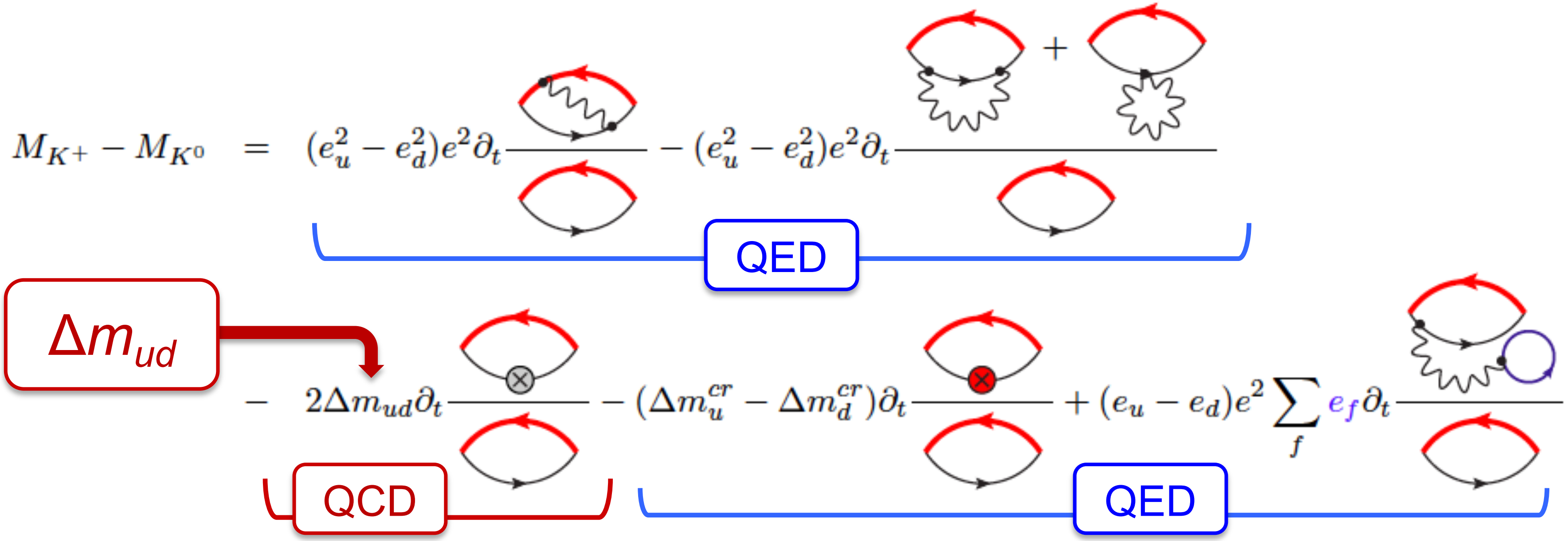
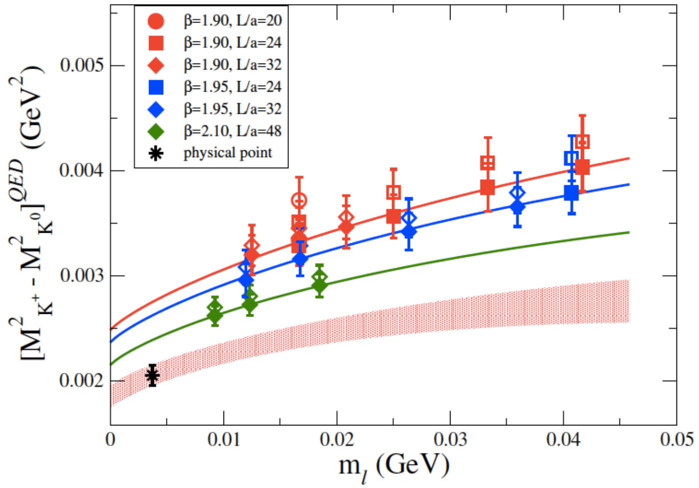
RM123 Collaboration

mu-md from the K^+ and K^0 masses





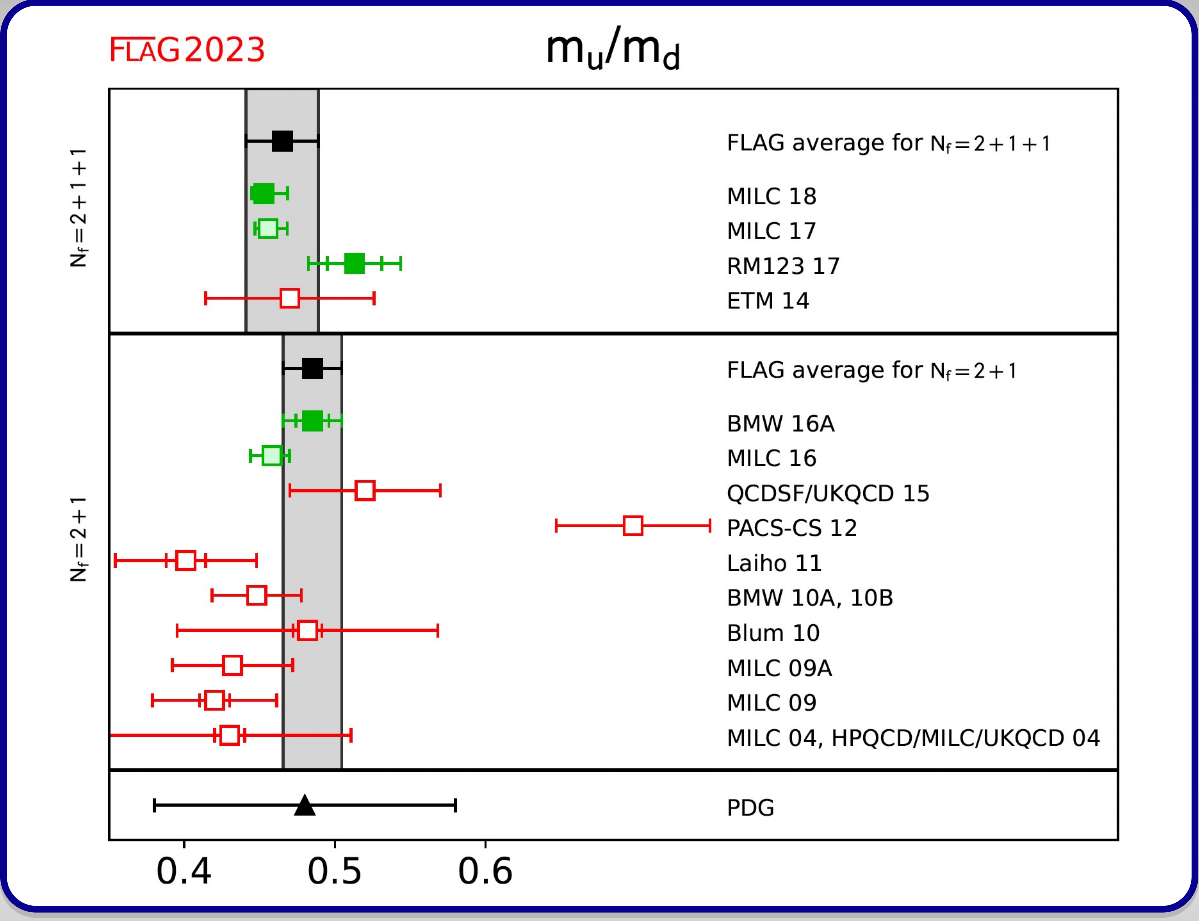
RM123 Collaboration, arXiv:1704.06561:

$$\left[M_{K^+} - M_{K^0}\right]^{QED} = 2.07(15)\ \text{MeV}$$

and from the experimental value

$$\left[M_{K^+} - M_{K^0}\right]^{QCD} = -6.00(15)\ \text{MeV}$$

ISOSPIN BREAKING EFFECTS and mu/md



$\overline{m}_u$ = 2.14(8) MeV $\overline{m}_d$ = 4.70(5) MeV $\overline{m}_u/\overline{m}_d$ = 0.465(24) (Nf=2+1+1)

$\overline{m}_u$ = 2.27(9) MeV $\overline{m}_d$ = 4.67(9) MeV $\overline{m}_u/\overline{m}_d$ = 0.485(19) (Nf=2+1)

QUARK MASSES FROM LATTICE QCD

$\overline{m}_u = 2.14(8)$ MeV

$\overline{m}_d = 4.70(5)$ MeV

$\overline{m}_{ud} = 3.427(51)$ MeV

$\overline{m}_u / \overline{m}_d = 0.465(24)$

$\overline{m}_s = 93.46(58)$ MeV

$\overline{m}_s / \overline{m}_{ud} = 27.227(81)$

$\overline{m}_c = 1.280(13)$ GeV

$\overline{m}_b = 4.203(11)$ MeV

E come ho cercato di raccontare, anche a questi risultati Enrico ha dato un importante contributo

Grazie Enrico

per la bellissima collaborazione di tutti questi anni

Slides di riserva

The (mu-md) expansion

- Identify the isospin breaking term in the QCD action

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) - \frac{1}{2}(m_d - m_u)(\bar{u}u - \bar{d}d) \right] =$$

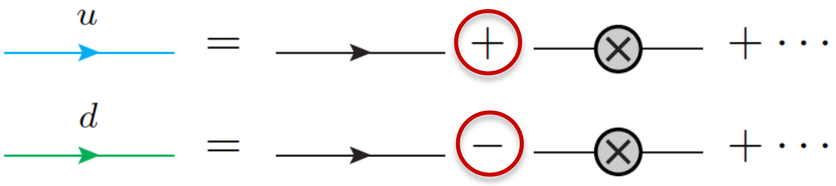
$$= \sum_x \left[m_{ud}(\bar{u}u + \bar{d}d) - \Delta m(\bar{u}u - \bar{d}d) \right] = S_0 - \Delta m \hat{S} \quad \longleftarrow$$

- Expand the functional integral in powers of Δm

Advantage: factorized out

$$\langle O \rangle = \frac{\int D\phi\, O\, e^{-S_0 + \Delta m\, \hat{S}}}{\int D\phi\, e^{-S_0 + \Delta m\, \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi\, O\, e^{-S_0}\left(1 + \Delta m\, \hat{S}\right)}{\int D\phi\, e^{-S_0}\left(1 + \Delta m\, \hat{S}\right)} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O\, \hat{S} \rangle_0}{1 + \Delta m \cancel{\langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O\, \hat{S} \rangle_0$$

- At leading order in Δm the corrections only appear in the valence quark propagators:



(disconnected contractions of ūu and d̄d vanish due to isospin symmetry)