Time-dependent and eikonal approximations to analyse breakup of halo nuclei

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Breakup reaction

Breakup used to study exotic nuclear structures e.g. halo nuclei: halo dissociates from core by interaction with target.

Coulomb breakup used to infer radiative-capture rates of astrophysical interest \([^7\text{Be}(p,\gamma)^8\text{B}, ^{14}\text{C}(n,\gamma)^{15}\text{C}, \ldots]\)

Outline

- Description of breakup models: CDCC, time-dependent (TD), eikonal (DEA)
- Comparison.
  - When and why chose a particular model?
Framework

**Projectile** \( (P) \) modelled as a two-body system: core \( (c) \)+loosely bound **neutron** \( (n) \) described by

\[
H_0 = T_r + V_{cn}(r)
\]

\( V_{cn} \) adjusted to reproduce bound state \( \Phi_0 \) and resonances

Target \( T \) seen as structureless particle

\( P-T \) interaction simulated by optical potentials

⇒ breakup reduces to **three-body** scattering problem:

\[
\left[ T_R + H_0 + V_{cT} + V_{nT} \right] \Psi(R, r) = E_T \Psi(R, r)
\]

with initial condition \( \Psi(r, R) \xrightarrow{Z \to -\infty} e^{ikZ} + \ldots \Phi_0(r) \)
Solve the three-body scattering problem:

\[
[T_R + H_0 + V_{cT} + V_{nT}] \Psi(r, R) = E_T \Psi(r, R)
\]

by expanding \( \Psi \) on eigenstates of \( H_0 \):

\[\Psi(r, R) = \sum_i \chi_i(R) \Phi_i(r) \quad \text{with} \quad H_0 \Phi_i = \epsilon_i \Phi_i\]

Leads to set of coupled-channel equations (hence CC)

\[
[T_R + \epsilon_i + V_{ii}] \chi_i + \sum_{j \neq i} V_{ij} \chi_j = E_T \chi_i,
\]

with \( V_{ij} = \langle \Phi_i | V_{cT} + V_{nT} | \Phi_j \rangle \)

The continuum has to be discretised (hence CD)

[Tostevin, Nunes, Thompson, PRC 63, 024617 (2001)]

**Fully quantal approximation**

No approx. on \( P-T \) motion, no restriction on energy

But expensive computationally (at high energies)
**Time-dependent model**

$P-T$ motion described by classical trajectory $R(t)$

[Esbensen, Bertsch and Bertulani, NPA 581, 107 (1995)]


$P$ structure described quantum-mechanically by $H_0$

Time-dependent potentials simulate $P-T$ interaction

Leads to the resolution of time-dependent Schrödinger equation (TD)

$$i\hbar \frac{\partial}{\partial t} \Psi(r, b, t) = [H_0 + V_{cT}(t) + V_{nT}(t)]\Psi(r, b, t)$$

Solved for each $b$ with initial condition $\Psi \rightarrow \Phi_0$

Many programs have been written to solve TD

Lacks quantum interferences between trajectories
Dynamical Eikonal Approximation

Three-body scattering problem:

\[ [T_R + H_0 + V_{cT} + V_{nT}] \Psi(\mathbf{r}, \mathbf{R}) = E_T \Psi(\mathbf{r}, \mathbf{R}) \]

with condition \( \Psi \xrightarrow{Z \to -\infty} e^{iKZ} \Phi_0 \)

Eikonal approximation: factorise \( \Psi = e^{iKZ} \hat{\Psi} \)

\( T_R \Psi = e^{iKZ} [T_R + vP_Z + \frac{\mu PT}{2} v^2] \hat{\Psi} \)

Neglecting \( T_R \) vs \( P_Z \) and using \( E_T = \frac{1}{2} \mu PT v^2 + \epsilon_0 \)

\[ i\hbar v \frac{\partial}{\partial Z} \hat{\Psi}(\mathbf{r}, \mathbf{b}, Z) = [H_0 - \epsilon_0 + V_{cT} + V_{nT}] \hat{\Psi}(\mathbf{r}, \mathbf{b}, Z) \]

solved for each \( \mathbf{b} \) with condition \( \hat{\Psi} \xrightarrow{Z \to -\infty} \Phi_0(\mathbf{r}) \)

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

Same equation as TD with straight line trajectories
Comparison of CDCC, TD, and DEA

[PC, Esbensen, and Nunes, accepted in PRC]

d$\sigma_{bu}/dE$

d$\sigma_{bu}/d\Omega$

All models agree
Data: [Nakamura et al.
PRC 79, 035805 (2009)]

DEA agrees with CDCC
TD reproduces trend
but lacks oscillations
$^{15}\text{C} + \text{Pb} \ @ \ 20\text{AMeV}$

$d\sigma_{\text{bu}}/dE$

$d\sigma_{\text{bu}}/d\Omega$

**TD** $\equiv$ **CDCC**

**DEA** too high

**TD** gives trend of **CDCC**

(lacks oscillations)

**DEA peaks too early**

**DEA$\neq$CDCC** due to Coulomb deflection

(TD straight lines)
## Comparison

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<th>TD</th>
<th>DEA</th>
<th>Eikonal</th>
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### Other reactions

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Conclusion and outlook

Good understanding of reaction process
Next step: improving projectile description
CDCC is by no means the only one reaction model
TD and DEA (and eikonal) are reliable (within their range of validity) AND are faster
⇒ use them to improve description of the projectile:
- core excitation
- two-neutron haloes (cf. E. C. Pinilla Beltran’s poster)
- microscopic description

Range of validity can be extended
- describe stripping with CDCC (cf. K. Minomo’s talk)
- low-$E$ Coulomb breakup within DEA
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