Gauge Theories

and

Non-Commutative Geometry

A Review

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June 21, 2023

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Historical Introduction

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- This solution is partial : It applies only to the static approximation and only to the $1/r$ potential.
- Not surprisingly, it was Heisenberg who, in 1930
  (Heisenberg $\rightarrow$ Peierls $\rightarrow$ Pauli $\rightarrow$ Oppenheimer)
  suggested non-commutativity in x-space as a "solution" to all short distance singularities.

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- In fact, as history evolved, Pauli was probably right. The motivation based on short distance singularities did not prove fruitful for elementary particle physics. With the development of the renormalisation program the problem of ultraviolet divergences took a completely different turn. It is not finiteness but rather absence of sensitivity to unknown physics at very short distances that turned out to be the important criterion.

Historical Introduction

Motivation II. : External fluxes.

- Landau (1930) Electron in an external magnetic field

$[v_x, v_y] = \mathrm{i}(e\hbar/m^2 c)B \qquad x_c = \frac{cp_y}{eB} + x \qquad y_c = -\frac{cp_x}{eB}$

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- Since the presence of non-vanishing magnetic-type external fields is a common feature in many modern supergravity and string models, the study of field theories formulated on spaces with non-commutative geometry has become quite fashionable.

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- The construction of gauge theories using the techniques of non-commutative geometry.
- Gauge theories and quantum gravity

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- Define a * product
  $f * g = \mathrm{e}^{\frac{\mathrm{i}}{2}\frac{\partial}{x_\mu}\theta_{\mu\nu}\frac{\partial}{y_\nu}} f(x)g(y)|_{x=y}$

All computations can be viewed as expansions in θ
expansions in the external field

More efficient ways?

Quantum field theory in a space with non-commutative geometry?

Large N field theories

- $\phi^i(x)\ i = 1, ..., N\ ;\ N \to \infty$

  $\phi^i(x) \to \phi(\sigma, x)\ 0 \leq \sigma \leq 2\pi$

  $\sum_{i=1}^{\infty} \phi^i(x)\phi^i(x) \to \int_0^{2\pi} d\sigma(\phi(\sigma, x))^2$

  but

  $\phi^4 \to (\int)^2$

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- For a Yang-Mills theory, the resulting expression is local

Gauge theories on surfaces

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- A simple algebraic result: J. Hoppe

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  At large N :

  The $SU(N)$ algebra $\rightarrow$ The algebra of the area preserving diffeomorphisms of a closed surface.

- The structure constants of $[SDiff(S^2)]$ are the limits for large N of those of $SU(N)$.

Alternatively: For the sphere

E. G. Floratos, J.I. and G. Tiktopoulos

$$x_1 = \cos\phi \ \sin\theta, \qquad x_2 = \sin\phi \ \sin\theta, \qquad x_3 = \cos\theta$$

$$Y_{l,m}(\theta, \phi) = \sum_{\substack{i_k=1,2,3 \\ k=1,\ldots,l}} \alpha^{(m)}_{i_1\ldots i_l} \ x_{i_1}\ldots x_{i_l}$$

where $\alpha^{(m)}_{i_1\ldots i_l}$ is a symmetric and traceless tensor.
For fixed l there are $2l+1$ linearly independent tensors $\alpha^{(m)}_{i_1\ldots i_l}$, $m = -l, ..., l$.

Choose, inside $SU(N)$, an $SU(2)$ subgroup.

$[S_i, S_j] = \mathrm{i}\epsilon_{ijk} S_k$

A basis for $SU(N)$:

$$S^{(N)}_{l,m} = \sum_{\substack{i_k=1,2,3\\ k=1,\ldots,l}} \alpha^{(m)}_{i_1\ldots i_l}\ S_{i_1}\ldots S_{i_l}$$

$$[S^{(N)}_{l,m},\ S^{(N)}_{l',m'}] = \mathrm{i} f^{(N)l'',m''}_{l,m;\ l',m'}\ S^{(N)}_{l'',m''}$$

The three $SU(2)$ generators S_i, rescaled by a factor proportional to $1/N$, will have well-defined limits as N goes to infinity.

$$S_i \to T_i = \frac{2}{N} S_i$$
$$[T_i, T_j] = \frac{2\mathrm{i}}{N} \epsilon_{ijk} T_k$$
$$T^2 = T_1^2 + T_2^2 + T_3^2 = 1 - \frac{1}{N^2}$$

In other words: under the norm $\|x\|^2 = \mathrm{Tr} x^2$, the limits as N goes to infinity of the generators T_i are three objects x_i which commute and are constrained by

$$x_1^2 + x_2^2 + x_3^2 = 1$$

$$\frac{N}{2\mathrm{i}}\,[f,g] \to \quad \epsilon_{ijk}\; x_i\; \frac{\partial f}{\partial x_j}\frac{\partial g}{\partial x_k}$$

$$\frac{N}{2\mathrm{i}}\,[T^{(N)}_{l,m}, T^{(N)}_{l',m'}] \to \;\{Y_{l,m}, Y_{l',m'}\}$$

$$N[A_\mu, A_\nu] \to \quad \{A_\mu(x,\theta,\phi), A_\nu(x,\theta,\phi)\}$$

$\Rightarrow$ The classical d-dim. $SU(N)$ Yang-Mills theory for $N \to \infty$

$\equiv$

A classical theory on a $d+2$-dim space with the extra two dimensions forming a closed surface. The gauge invariance is mapped into area preserving diffeomorphisms of the surface.

The classical Y-M action

$$S_{YM} \sim \int \mathrm{d}^4 x \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} \quad \Rightarrow \quad \int_{S^2} \mathrm{d}\Omega \int \mathrm{d}^4 x F_{\mu\nu}(x,\theta,\phi) F^{\mu\nu}(x,\theta,\phi)$$

with

$$F_{\mu\nu}(x,\theta,\phi) = \\ \partial_\mu A_\nu(x,\theta,\phi) - \partial_\nu A_\mu(x,\theta,\phi) + \{A_\mu(x,\theta,\phi), A_\nu(x,\theta,\phi)\}$$

The quantum theory ??

Gauge theories on surfaces - Finite N

E.G. Floratos and J.I.

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$$A_\mu(x) \ = \ A_\mu^a(x) \ t_a$$

Gauge theories on surfaces - Finite N

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- Given an $SU(N)$ Yang-Mills theory in a $d-$dimensional space

$$A_\mu(x) \;=\; A_\mu^a(x)\; t_a$$

- there exists a reformulation in $d+2$ dimensions

$$A_\mu(x) \to \mathcal{A}_\mu(x, z_1, z_2) \qquad F_{\mu\nu}(x) \to \mathcal{F}_{\mu\nu}(x, z_1, z_2)$$

with $\qquad [z_1, z_2] = \frac{2\mathrm{i}}{N}$

$$[A_\mu(x), A_\nu(x)] \rightarrow \{\mathcal{A}_\mu(x, z_1, z_2), \mathcal{A}_\nu(x, z_1, z_2)\}_{Moyal}$$

$$[A_\mu(x), \Omega(x)] \rightarrow \{\mathcal{A}_\mu(x, z_1, z_2), \Omega(x, z_1, z_2)\}_{Moyal}$$

$$\int d^4x \; Tr\,(F_{\mu\nu}(x) F^{\mu\nu}(x)) \quad \rightarrow \quad \int d^4x dz_1 dz_2 \; \mathcal{F}_{\mu\nu}(x, z_1, z_2) * \mathcal{F}^{\mu\nu}(x, z_1, z_2)$$

These expressions are defined for *all* N

Not necessarily integer ???

We can parametrise the T_i's in terms of two operators, z_1 and z_2.

$$T_+ = T_1 + \mathrm{i}\, T_2 = \mathrm{e}^{\frac{\mathrm{i}z_1}{2}} \left(1 - z_2^2\right)^{\frac{1}{2}} \mathrm{e}^{\frac{\mathrm{i}z_1}{2}}$$
$$T_- = T_1 - \mathrm{i}\, T_2 = \mathrm{e}^{-\frac{\mathrm{i}z_1}{2}} \left(1 - z_2^2\right)^{\frac{1}{2}} \mathrm{e}^{-\frac{\mathrm{i}z_1}{2}}$$
$$T_3 = z_2$$

If we assume that z_1 and z_2 satisfy:

$[z_1, z_2] = \frac{2i}{N}$

The T_i's satisfy the $SU(2)$ algebra.

If we assume that the T_i's satisfy the $SU(2)$ algebra, the z_i's satisfy the Heisenberg algebra

For the torus

Choose, inside $SU(N)$, a quantum $U(1) \times U(1)$

$$g = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \omega & 0 & \dots & 0 \\ 0 & 0 & \omega^2 & \dots & 0 \\ . & . & . & . & 0 \\ . & . & . & . & 0 \\ . & . & . & . & 0 \\ 0 & 0 & 0 & \dots & \omega^{N-1} \end{pmatrix} \quad ; \quad h = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \end{pmatrix}$$

(N odd), $\omega = e^{4\pi i/N}$

$$g^N = h^N = 1 \quad ; \quad hg = \omega gh$$

For the torus

We can use the integer mod$N$ powers of these matrices to express the $SU(N)$ generators:

$$S_{m_1,m_2} = \omega^{m_1 m_2/2} g^{m_1} h^{m_2} \quad ; \quad S^{\dagger}_{m_1,m_2} = S_{-m_1,-m_2}$$

$$[S_{\mathrm{m}}, S_{\mathrm{n}}] = 2\mathrm{i}\sin\left(\frac{2\pi}{N}\mathrm{m}\times\mathrm{n}\right) S_{\mathrm{m}+\mathrm{n}}$$

$\mathrm{n} = (n_1, n_2)$ and $\mathrm{n}\times\mathrm{m} = n_1 m_2 - m_1 n_2$

$$SU(N)|_{N\to\infty} = \mathrm{SDiff}(T^2)$$

z_1 , z_2 the two angular variables:

$$h = \mathrm{e}^{\mathrm{i}z_1} \quad g = \mathrm{e}^{-2\mathrm{i}\pi z_2} \Rightarrow [z_1, z_2] = \frac{2\mathrm{i}}{N} \to hg = \omega g h$$

For the torus

The generators of the Heisenberg algebra z_1 and z_2,
as well as the group elements $h = \mathrm{e}^{\mathrm{i}z_1}$ and $g = \mathrm{e}^{-2\mathrm{i}\pi z_2}$

are infinite dimensional operators

but we can represent the $SU(N)$ algebra by the finite dimensional matrices g , h and S_{m_1,m_2}

They form a discrete subgroup of the Heisenberg group
$\Rightarrow$
quantum mechanics on a discrete phase space

We can define two new operators
$\hat{q}$ ("position" in the discrete space) and $\hat{p}$ (its FFT):
They are represented by finite matrices but, obviously, they do not satisfy the Heisenberg algebra.

The topology of the 2-dim. surface and the form of the matrices.

I only state the result:

A. Sphere

For given N, we define $N^2 - 1$ matrix spherical harmonics as polynomials in the $SU(2)$ generators T_i as follows: Let z_+ and z_- be two independent complex variables,

$$\frac{(-z_+^2 S_+ + z_-^2 S_- + 2z_+ z_- S_3)^l}{2^l \ l!} = \sqrt{\frac{4\pi}{2l+1}} \sum_{m=-l}^{l} \frac{z_+^{l+m} z_-^{l-m}}{\sqrt{(l+m)!(l-m)!}} \hat{Y}_{lm}$$

The matrices $\hat{Y}_{lm}$, appropriately rescaled, go to the spherical harmonics Y_l^m. Two results:

1) $\hat{Y}_{l,m}$ with negative m have non-vanishing elements in the lower diagonals.

2) $\hat{Y}_{l,m}$ with positive m have non-vanishing elements in the upper diagonals.

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B. Torus

The matrices which form a fuzzy torus have non-vanishing elements in the k-upper and the $N-k$-lower diagonals.

C. Higher genus surfaces

The corresponding matrices have non-vanishing elements in upper and lower diagonals with appropriate symmetry properties.

The techniques of non-com. geometry

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- Diffeomorphisms *space-time*
- Internal symmetries
- Question: Is there a space on which internal symmetry transformations act as diffeomorphisms?
- Answer: Yes, but it is a space with non-commutative geometry. A space defined by an algebra of matrix-valued functions

The techniques of non-com. geometry

Ali H. Chamseddine, Alain Connes, Viatcheslav Mukhanov et al.

The construction involves A fundamental spectral triplet :

Given a spin manifold M, the triplet consists of:

1. A Hilbert space $\mathcal{H}$
2. An algebra of functions $\mathcal{A}$ which are $C^\infty(M)$
3. The Dirac operator $\mathcal{D}$

(If we ignore gravity, $\mathcal{D}$ can be replaced by the chirality operator)

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- Decorations: Chirality, CPT
- $\Rightarrow$ Gauge theories emerge naturally with General Relativity

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- *New predictions for the Standard Model parameters?*

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- Answer: Compute the corresponding β-function.

$$16\pi^2 \beta_{g_1} = g_1^3 \frac{1}{10}$$
$$16\pi^2 \beta_{g_2} = -g_2^3 \frac{43}{6}$$
$$16\pi^2 \beta_{\lambda} = 12\lambda^2 - \frac{9}{5} g_1^2 \lambda - 9 g_2^2 \lambda + \frac{27}{100} g_1^4 + \frac{9}{10} g_1^2 g_2^2 + \frac{9}{4} g_2^4$$

$$\beta_z = \beta_{\eta_1} + \beta_{\eta_2} =$$
$$= \frac{-\lambda w}{16\pi^2 \rho z}\left[\left(\frac{27}{100}\rho^2 + \frac{9}{10}\rho + \frac{9}{4}\right) z^2 - \left(2\rho^2 + \frac{54}{5}\rho - \frac{16}{3}\right) z\right.$$
$$\left. +12(\rho+1)^2\right]$$

$$\eta_1 = \frac{g_1^2}{\lambda} \quad ; \quad \eta_2 = \frac{g_2^2}{\lambda} \quad ; \quad z = \eta_1 + \eta_2 \quad ; \quad \rho = \frac{\eta_1}{\eta_2} \quad ; \quad w = \eta_1 \eta_2$$

- β_z has no zeroes! $\Rightarrow$ The Standard Model is irreducible.

Related question: Is there a B.R.S. symmetry for the model on non-com. geometry?

The spectacular accuracy reached by experiments, as well as theoretical calculations, made particle physics a precision science

Example: $m_W = 80.385 \pm 0.015\text{GeV}$
$\Rightarrow$ "Approximate" theories are no more sufficient!

A discrepancy by a few percent implies that we do not have the right theory!

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- But, for the moment, we see no corner!

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- Whether it will turn out to be convenient for us to use, is still questionable.
- It will depend on our ability to simplify the mathematics sufficiently, or to master them deeply, in order to get new insights
- In the meantime, it is fun. to play with matrices.