

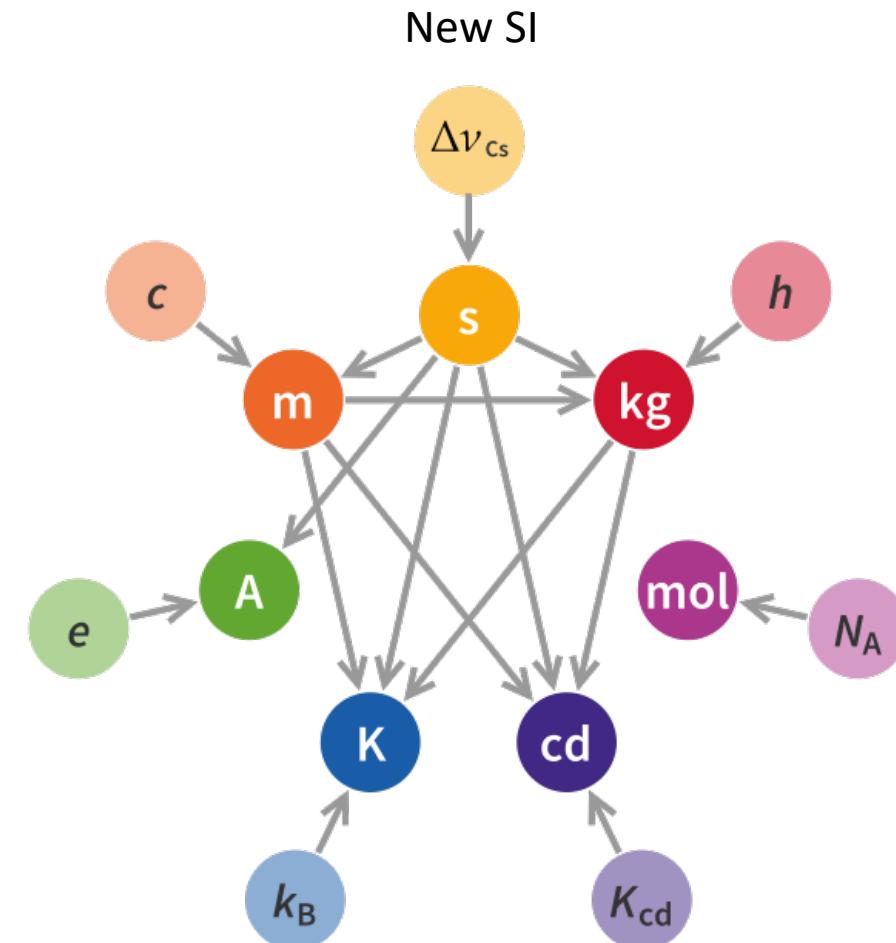
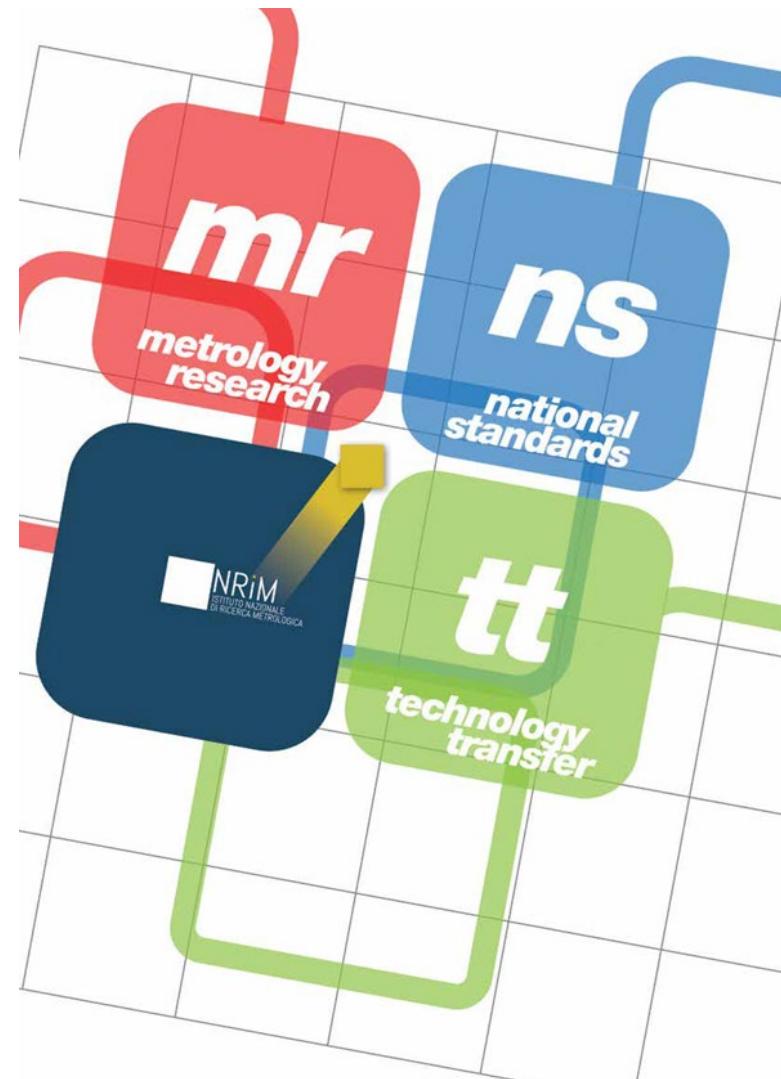
[www.inrim.it](http://www.inrim.it)

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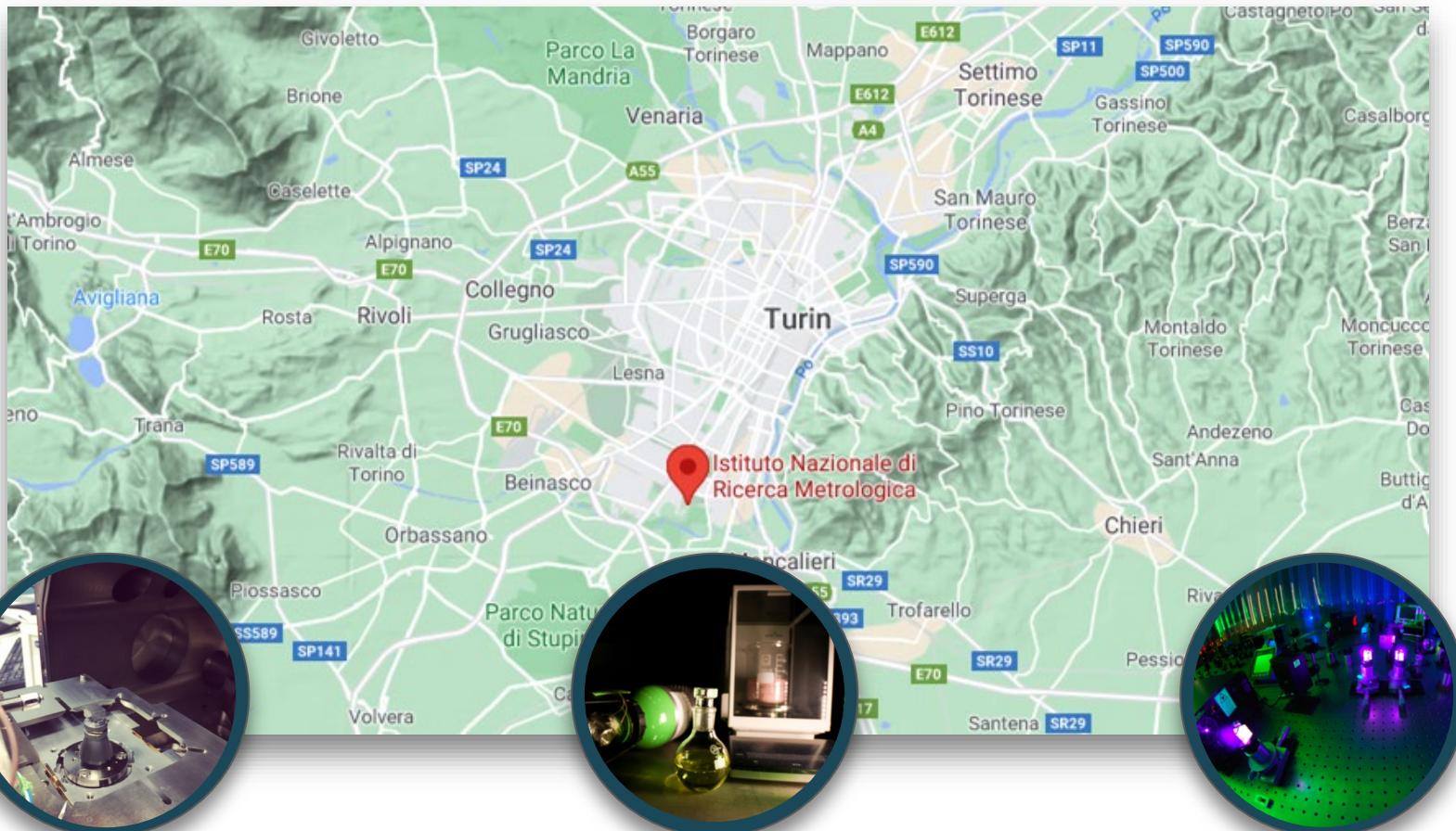
Traveling Wave Parametric Amplifiers come  
metamateriali per le tecnologie quantistiche  
Workshop INFN CSN4&5 - Dipartimento di Fisica su Tecnologie Quantistiche

*Emanuele ENRICO*

# Introduction



# Introduction

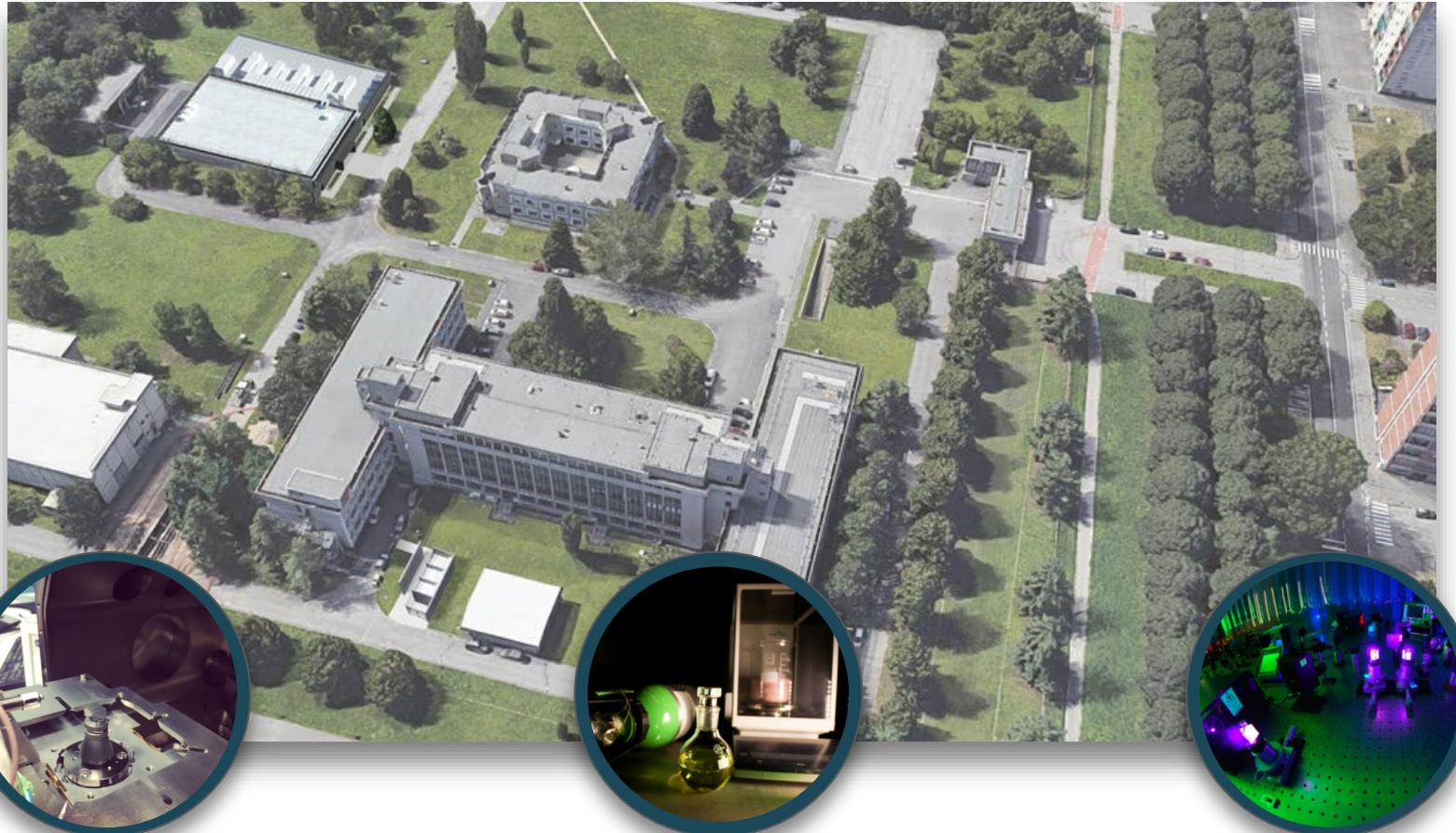


Advanced materials  
metrology and life science

Applied metrology and  
engineering

Quantum metrology and  
nano technologies

# Introduction

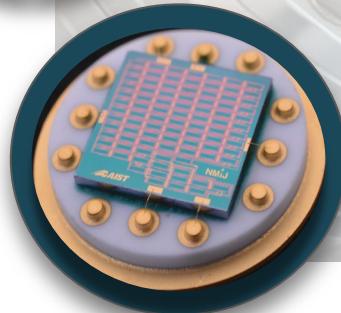


Advanced materials  
metrology and life science

Applied metrology and  
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Quantum metrology and  
nano technologies

# Quantum Electronics - NanoTech

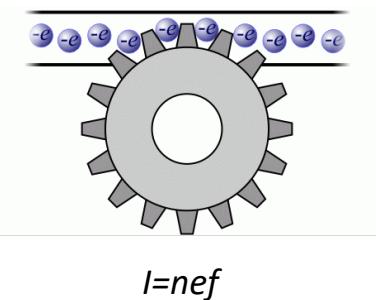


Quantum Hall Array  
Resistance Standard

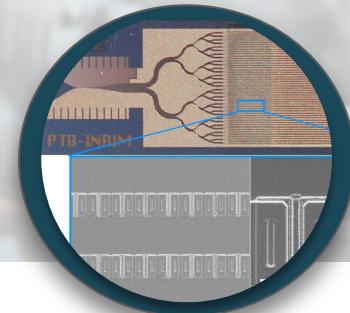
$$R_H = R_K/i$$

with  $R_K = h/e^2$

and  $i = 1, 2, 3, 4, \dots$



Single-electron transistor low  
DC current standard

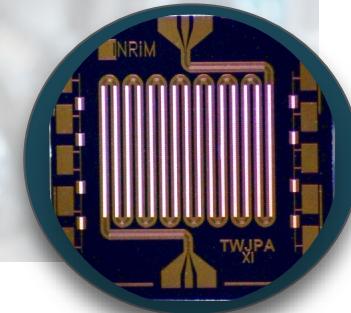


Josephson  
Voltage Standard

$$V_J = n K_J f$$

with  $K_J = h/2e$

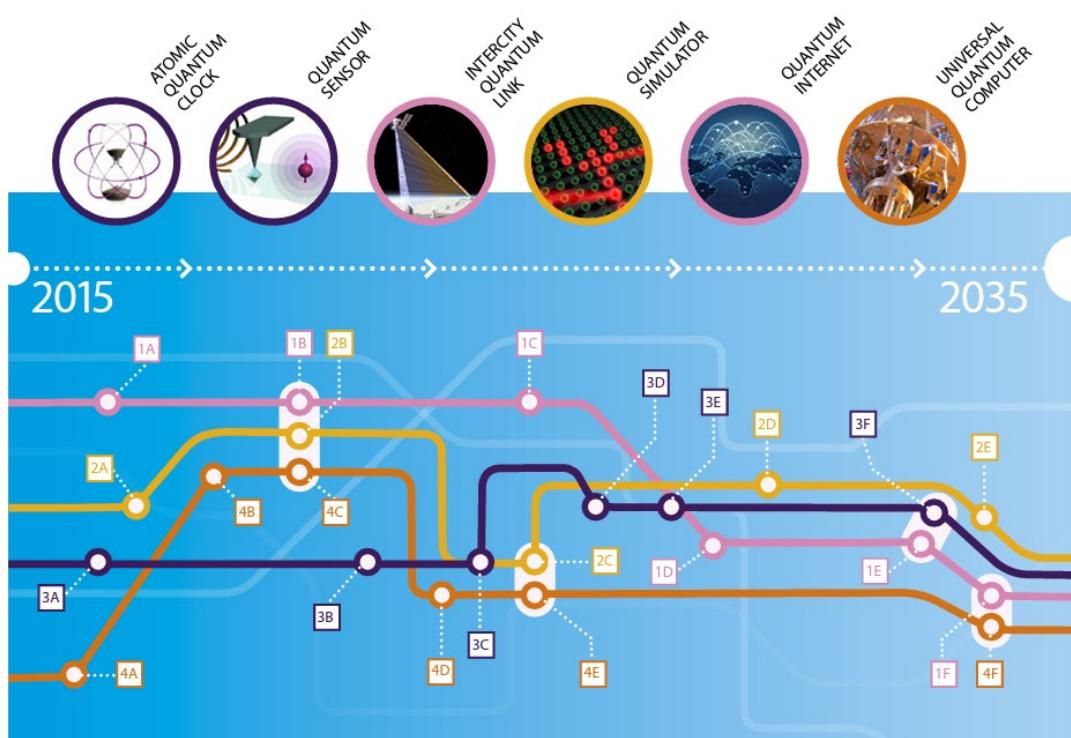
and  $k = 1, 2, 3, 4, \dots$



Microwave photonics

$\nu$	1 GHz → 20 GHz
$\lambda = c/\nu$	300 cm → 15 cm
$E = h/\nu$	4 μeV → 80 μeV
$T = E/k_B$	50 mK → 1 K

# Quantum Electronics - Applications



- 1. Communication**
- 0 – 5 years
- A Core technology of quantum repeaters
  - B Secure point-to-point quantum links
- 5 – 10 years
- C Quantum networks between distant cities
  - D Quantum credit cards
- > 10 years
- E Quantum repeaters with cryptography and eavesdropping detection
  - F Secure Europe-wide internet merging quantum and classical communication
- 2. Simulators**
- A Simulator of motion of electrons in materials
  - B New algorithms for quantum simulators and networks
- 3. Sensors**
- A Quantum sensors for niche applications (incl. gravity and magnetic sensors for health care, geosurvey and security)
  - B More precise atomic clocks for synchronisation of future smart networks, incl. energy grids
  - C Quantum sensors for larger volume applications including automotive, construction
  - D Handheld quantum navigation devices
- 4. Computers**
- A Operation of a logical qubit protected by error correction or topologically
  - B New algorithms for quantum computers
  - C Small quantum processor executing technologically relevant algorithms
  - D Solving chemistry and materials science problems with special purpose quantum computer > 100 physical qubit
  - E Gravity imaging devices based on gravity sensors
  - F Integrate quantum sensors with consumer applications including mobile devices
  - G General purpose quantum computers exceed computational power of classical computers

# Circuit QED Toolbox

- Transmission lines
- Resonators / Cavities
- Qubits
- Isolators / Circulators
- Bias-tee
- Directional couplers
- Quantum limited linear amplifiers
- Detectors (transducers or counters eg. single microwave photon detectors)
- Nonclassical radiation sources

**Superconducting metamaterials** are engineered circuits characterized by mixing processes promoting:

- Parametric amplification  
Quantum limited added-noise
- Parametric downconversion  
Nonclassical radiation

# Solid State QuBit (dispersive) Readout

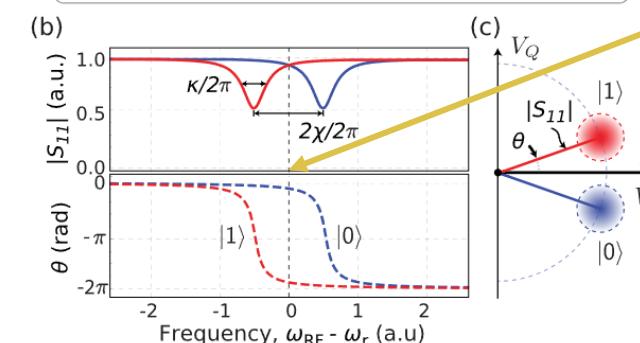
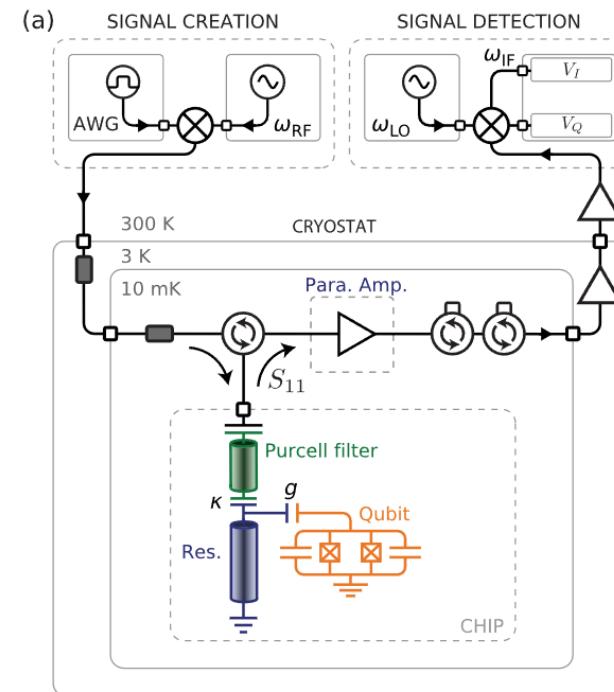
A quantum measurement can be described as an **entanglement of the qubit degree of freedom with a “pointer variable”** of a measurement probe with a quantum Hamiltonian[1] followed by classical measurement of the probe

## Example:

In circuit QED, the **qubit** (the quantum system) is entangled with an observable of a **superconducting resonator** (the probe) allowing us to gain information about the qubit state by interrogating the resonator, rather than directly interacting with the qubit.

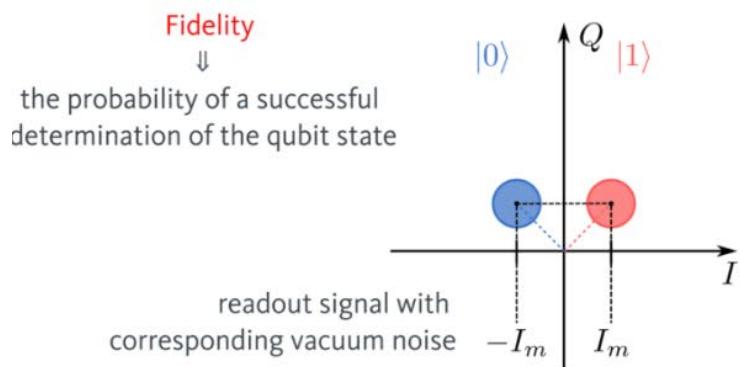
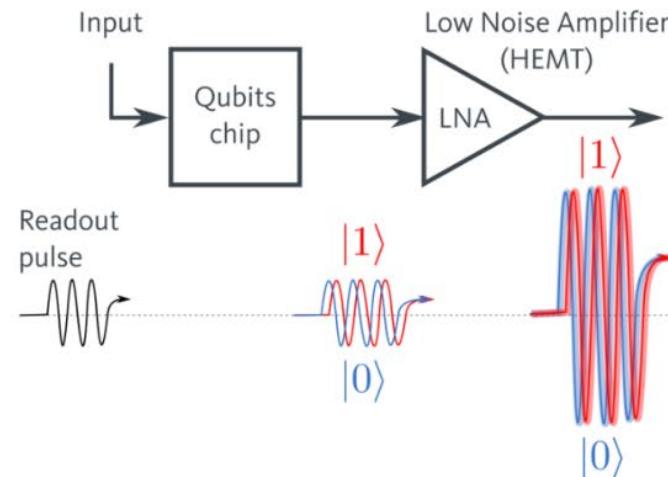
Therefore, the optimization of the readout performance is translated to **maximizing the signal-to-noise ratio** of a microwave probe tone sent to the resonator, while **minimizing the unwanted “back-action”** on the qubit.

[1] Braginsky, V. B; Ya Khalili, F. (1996). Quantum non demolition measurements: the route from toys to tools, Rev. Mod. Phys, (68) 1

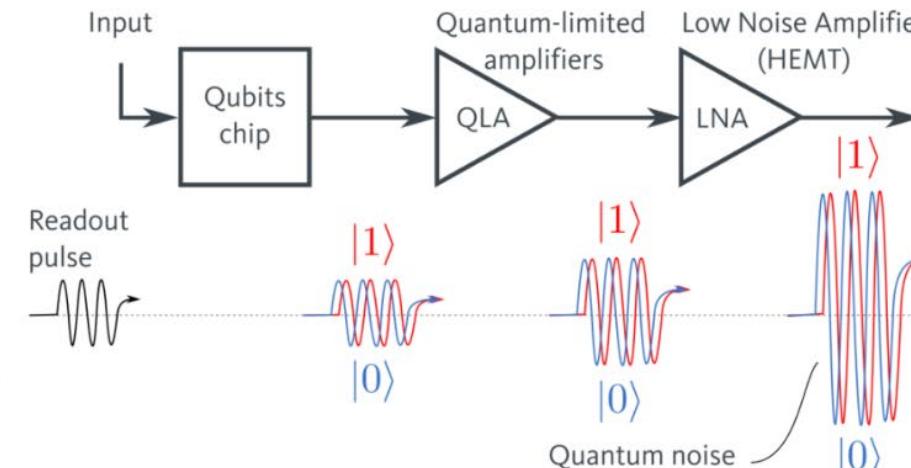


Highest state discrimination,  
in-between the two  
resonances

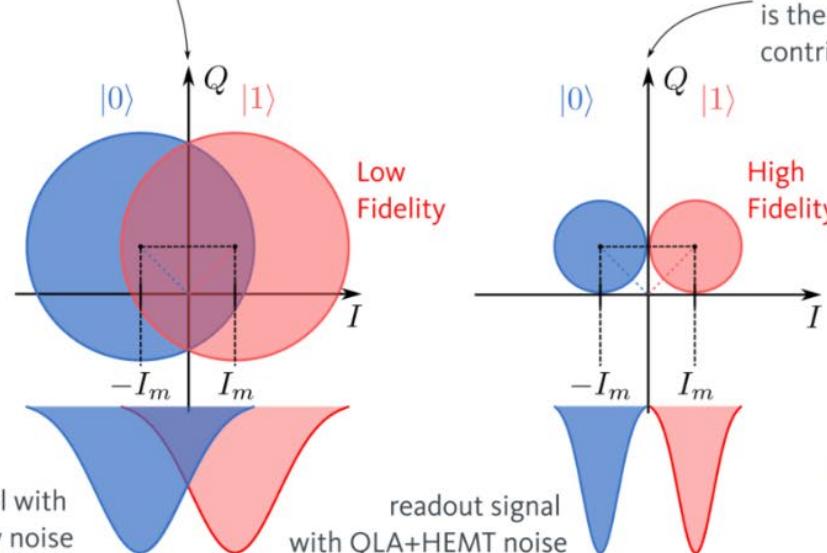
# e.g. QuBits and Amplifiers



readout signal with HEMT-amplifier-only noise



Quantum noise is the dominant contribution



more details on  
*J. Phys. Conf. Ser. 834 (2017) 012003*  
[www.ibm.com/blogs](http://www.ibm.com/blogs)  
[thequantumaviary.blogspot.com](http://thequantumaviary.blogspot.com)

# DARTWARS Project

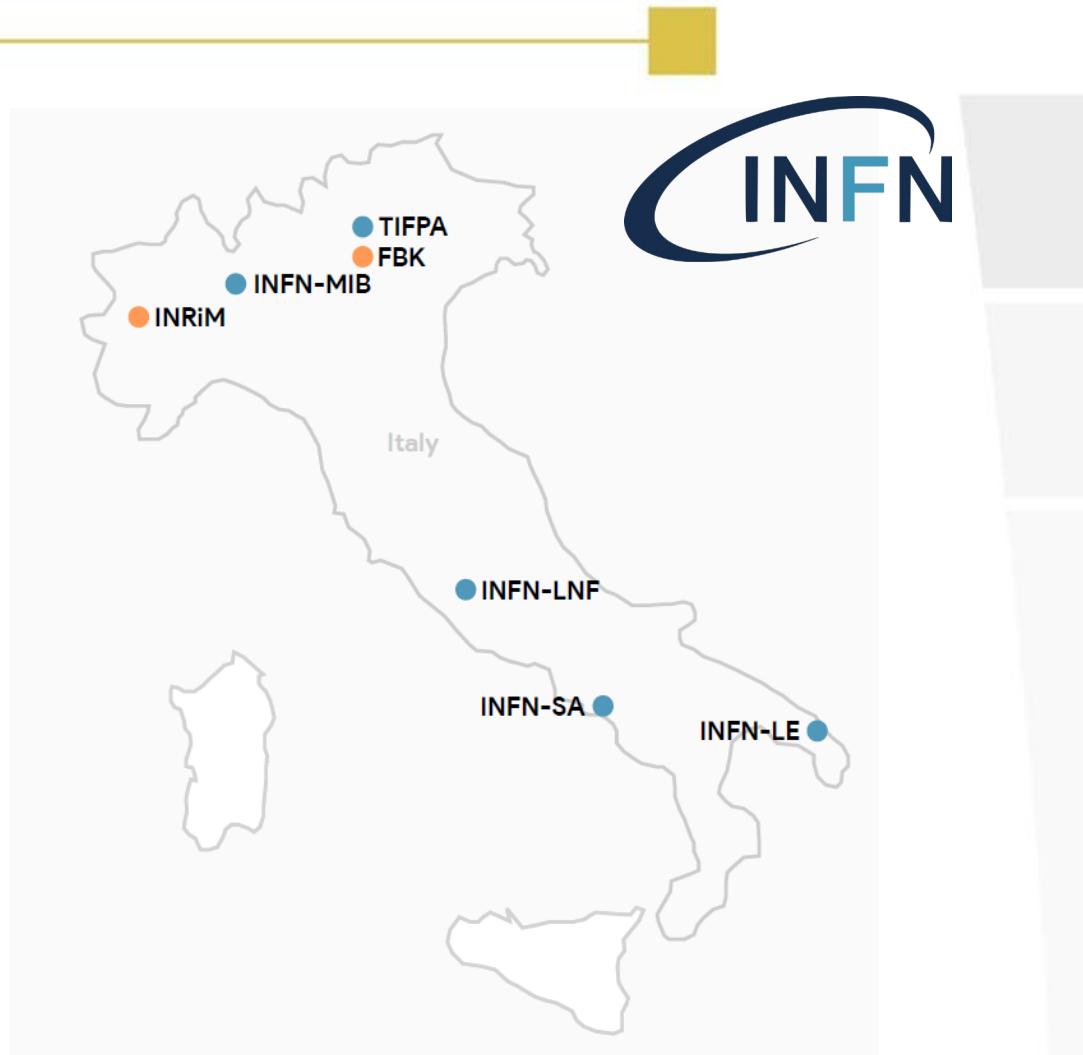
- JPA has typically been used only for single frequency measurements due to lower bandwidth and saturation power
- New approach: **microwaves travelling along a transmission line with embedded non-linear elements**  
*Phys. Rev. B 87, 144301*
- The **nonlinear reactive element** can be implemented by **Josephson Junction (JJ) or Kinetic Inductance (KI)** of superconductors. The relationship is, at the first order

$$L(I) = L_0 \left[ 1 + \left( \frac{I}{I_c} \right)^2 \right]$$

$I_c$  is the superconductor critical current for KI

$I_c$  is the junction critical current for JJ

At  $I < I_c$  junctions are dissipationless and act as a nonlinear inductor.



# PiQuET Laboratory



It-fab

The Italian Network for Micro and Nano Fabrication



UNIVERSITA  
DEGLI STUDI  
DI TORINO

# Nonlinear (meta) materials

Dipole electric momentum of a nonlinear material under electromagnetic stimulus

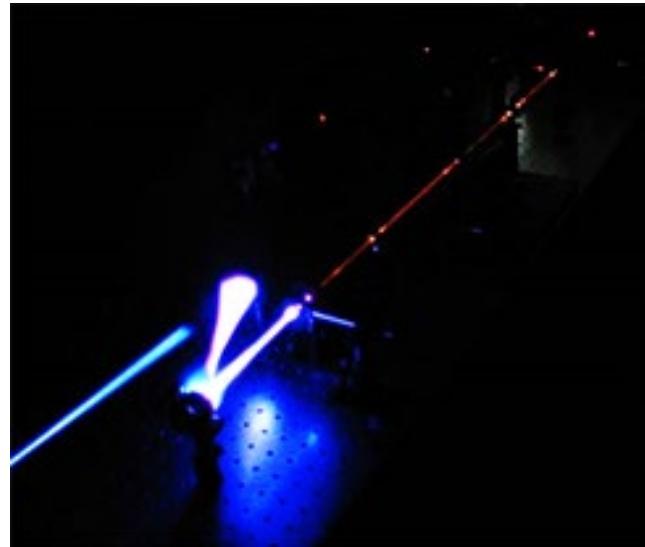
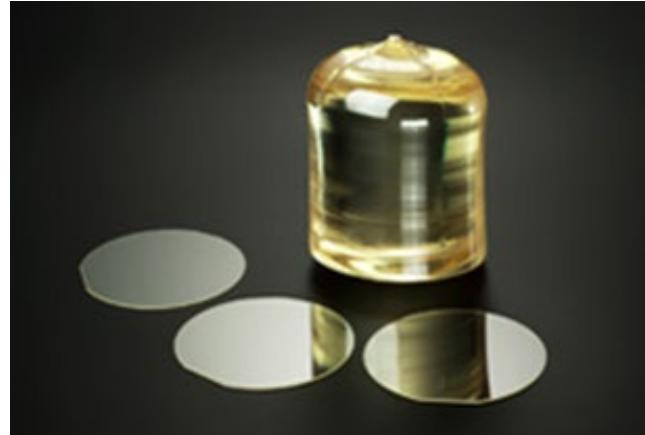
$$P(t)=\varepsilon_0 (\chi^{(1)}E(t)+\chi^{(2)}E^2(t)+\chi^{(3)}E^3(t)+\dots)$$

When a (weak) signal is pumped by a (strong) one

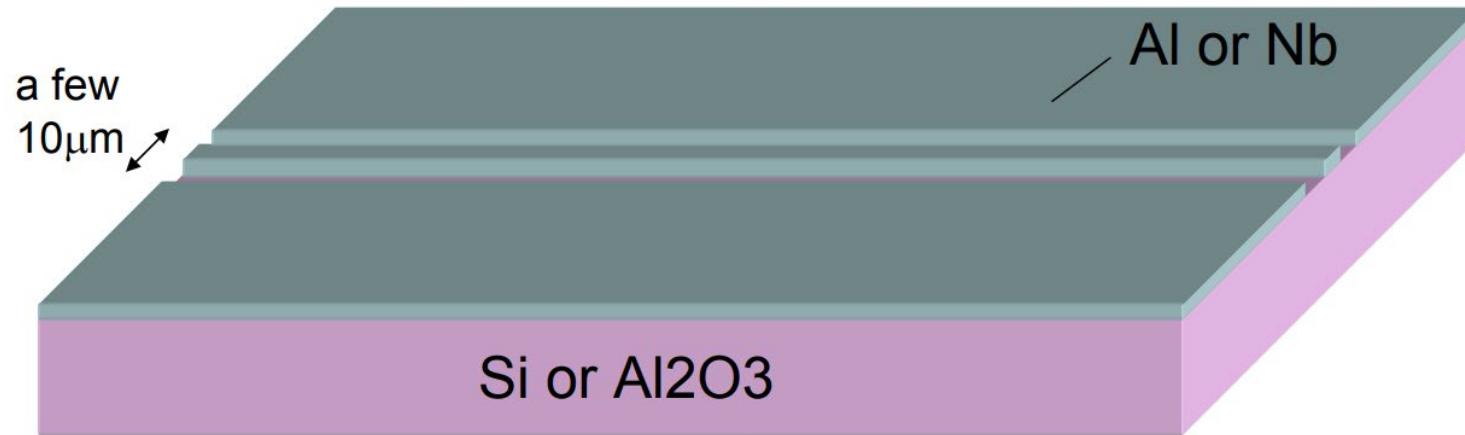
$$E(t)=E_p \cos(\omega_p t) + E_s \cos(\omega_s t)$$

It generates

- Second harmonics (SHG)
- Sum frequency
- Different frequency or Parametric Down Conversion (PDC)



# Superconducting transmission line Coplanar Waveguide



1/e propagation length  $\sim 10\text{km}$  !

Attenuation comparable to optical fibers

(1 mm dia. Copper wire : 700 m )

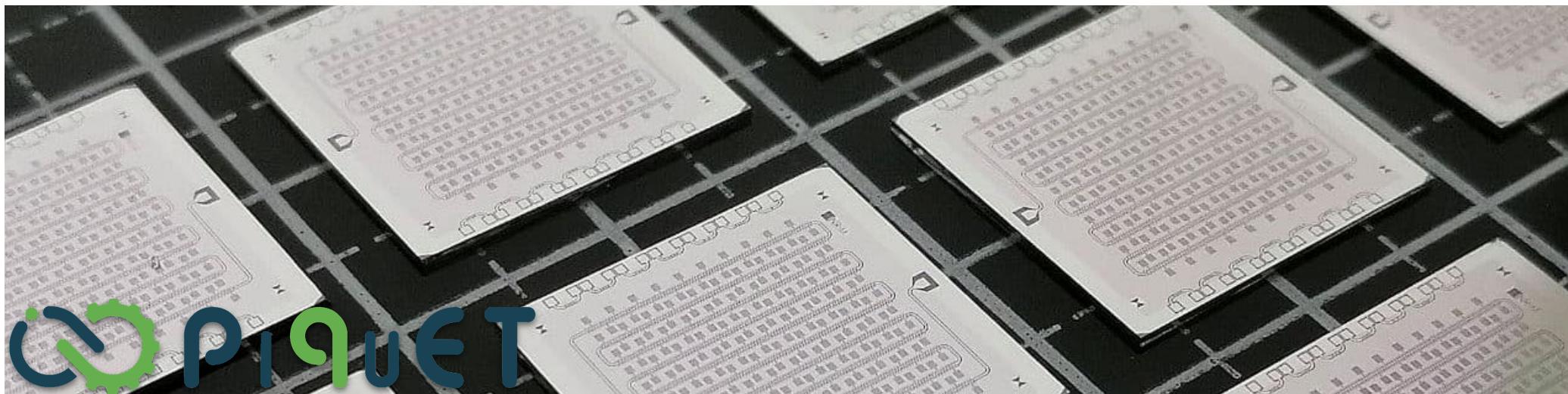
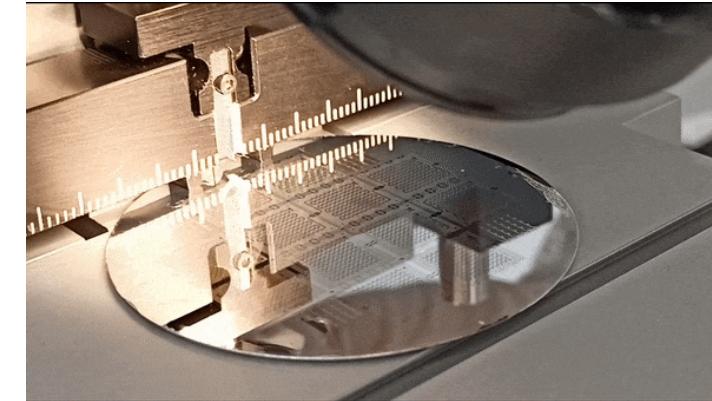
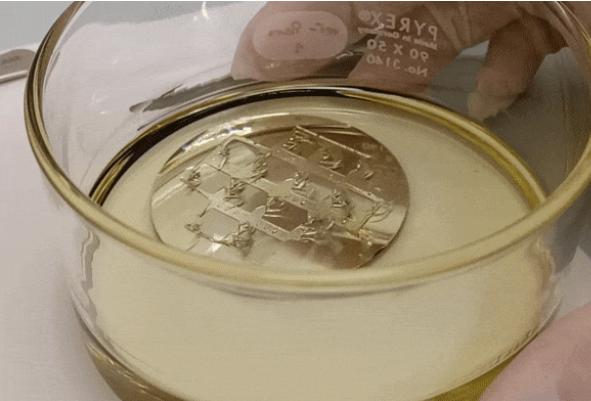
Add some spices

# 10 SPICES WITH SURPRISING HEALTH BENEFITS



Brian David Josephson

# Standard JJs fabrication processes



 **PIQUET**  
PIEMONTE QUANTUM ENABLING TECHNOLOGY

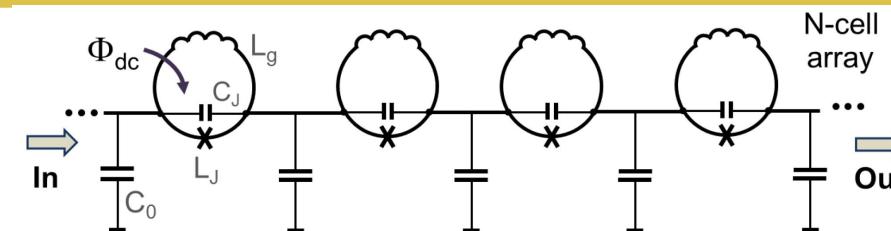
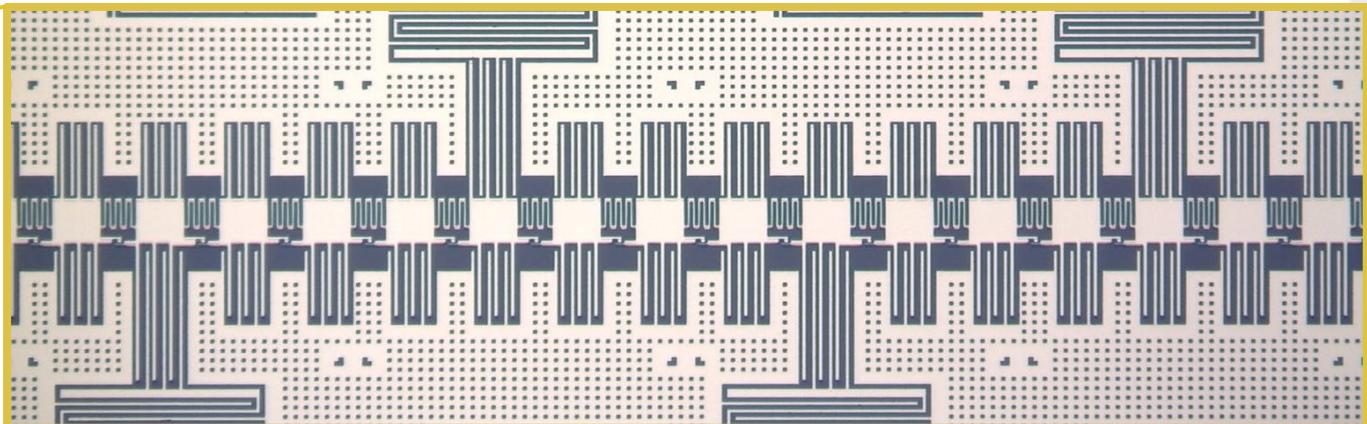
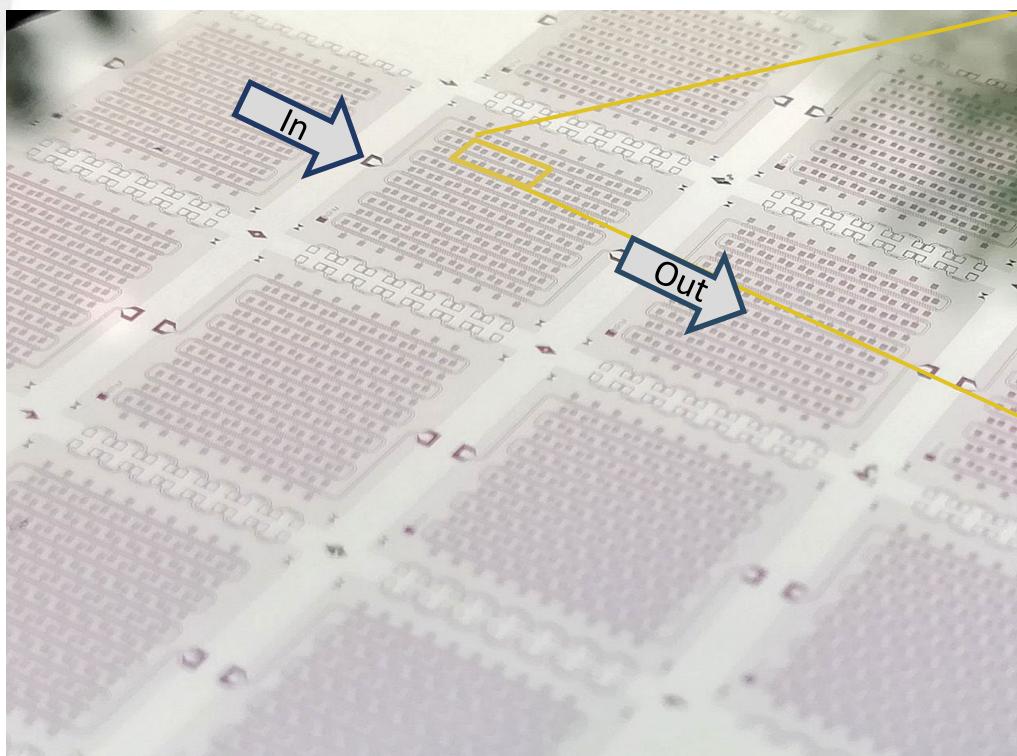
Traveling Wave Parametric Amplifiers come metamateriali per le tecnologie quantistiche  
Workshop INFN CSN4&5 - Dipartimento di Fisica su Tecnologie Quantistiche – 07 giugno 2023 - E. ENRICO

 **DART  
WARS**

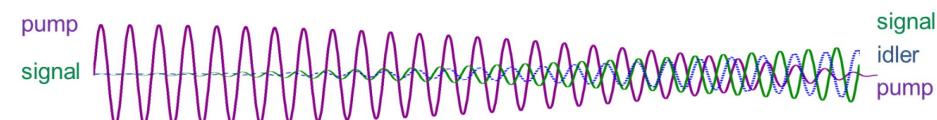
 **INRIM**  
ISTITUTO NAZIONALE  
DI RICERCA METROLOGICA

# Nonlinear (meta) materials - $\mu$ Waves

- Transmission line (eg. CPW or stripline) + **identical** meta-atom (with JJ nonlinearity)
- Effects of the interaction with the single cell are perturbative -> avoid abrupt changes that acts like point defects or scattering sites (crystal analogy)



Very small phase mismatch  $\Rightarrow$  almost exponential gain,  $\sqrt{G} \propto e^{gN}$

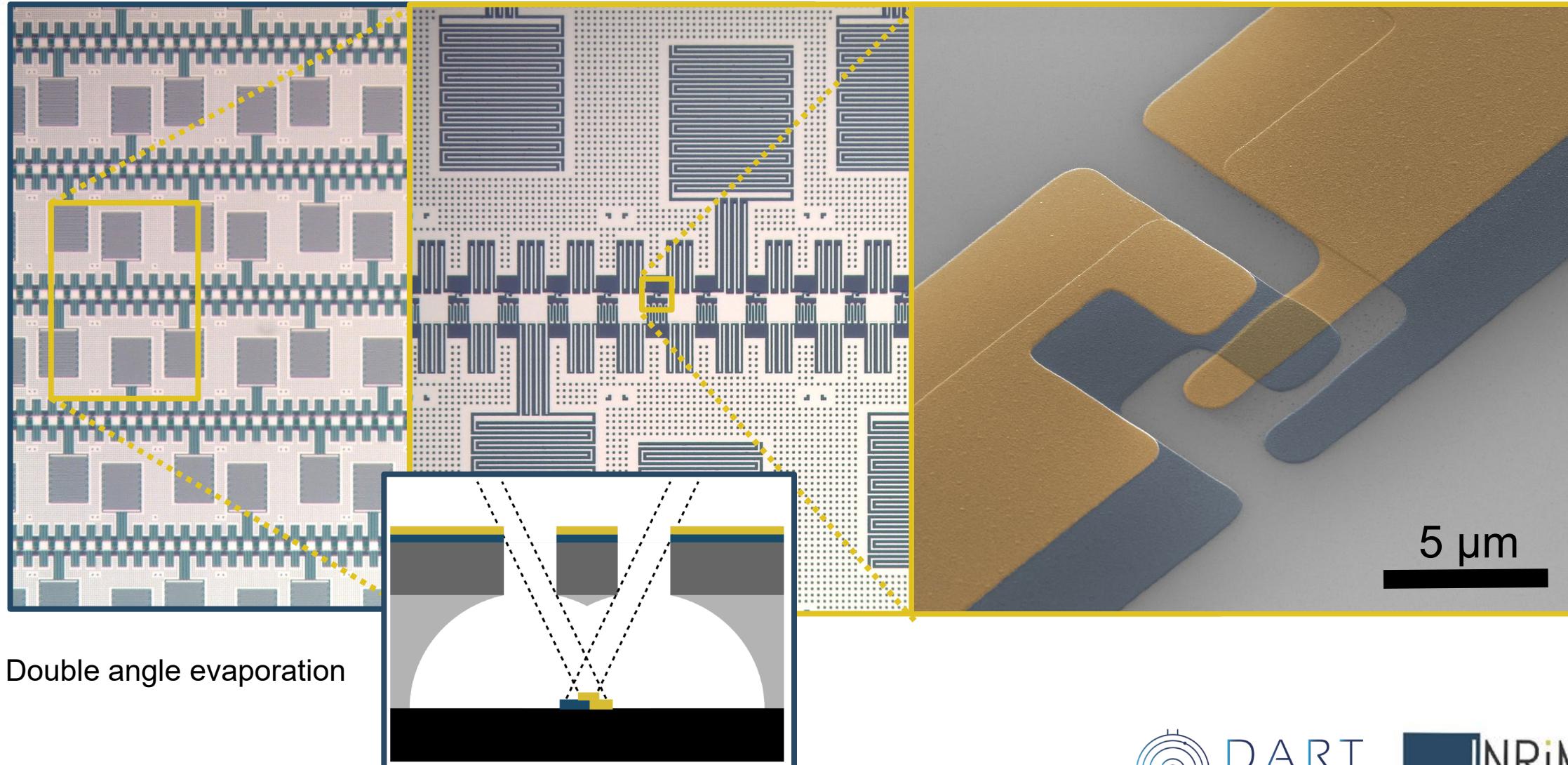


Zorin, Phys. Rev. Appl. 6, 034006 (2016)



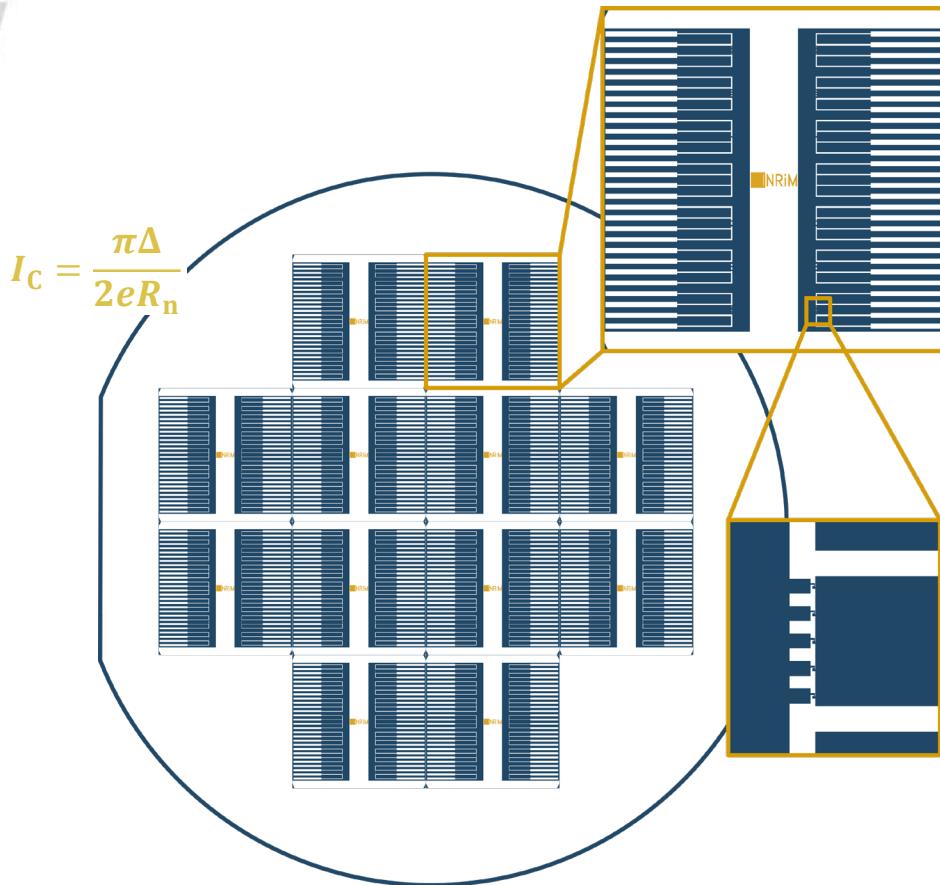
S. Pagano et al., Development of Quantum Limited Superconducting Amplifiers for Advanced Detection, IEEE Trans. Appl. Supercond, 32, 4 (2022)

# JTWPA by 2-steps optical lithography

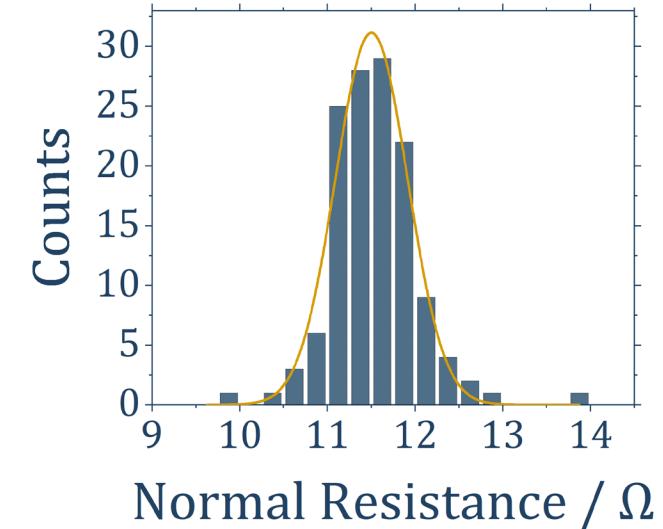
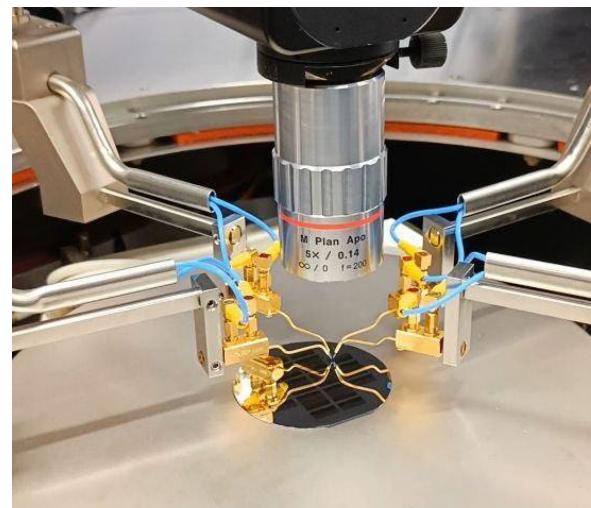


# Josephson junction testing

Test on a 2" wafer scale of JJs **normal resistance spread** by means of a 4-terminal measurement with a semi-automatic probe.



Each wafer contains 480 JJs organized in 144 series  
of different length (1-5)



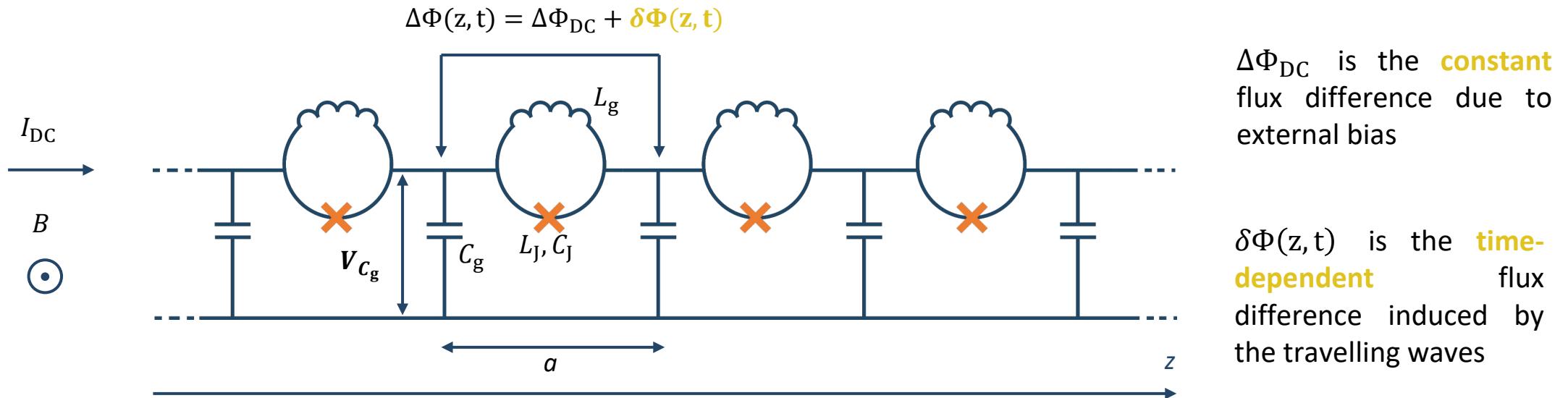
After several improvement of the fabrication protocol, we obtain a relative standard deviation

$$\sigma_{\text{rel}} \sim 3.6\%$$



# First Quantization Hamiltonian

We adapt and further extend the work presented in [1] to an **rf-SQUID based Josephson Traveling Wave Parametric Amplifier** [2]



The **first quantization Hamiltonian** can be written as the sum of the electromagnetic energy stored in each component of the transmission line

$$H = \frac{1}{a} \int_0^{aN} \left[ \frac{1}{2L_g} \Delta\Phi(z, t)^2 + \varphi_0 I_c \left( 1 - \cos \left( \frac{\Delta\Phi(z, t)}{\varphi_0} \right) \right) + \frac{C_J}{2} \left( \frac{\partial \Delta\Phi(z, t)}{\partial t} \right)^2 + \frac{C_g}{2} V_{cg}^2(z, t) \right] dz$$

[1] T. H. A. van der Reep, "Mesoscopic Hamiltonian for Josephson traveling-wave parametric amplifier", Phys. Rev. A **99**, 063838 (2019)

[2] A. B. Zorin, "Josephson Traveling-Wave Parametric Amplifier with Three-Wave Mixing", Phys. Rev. App. **6**, 034006 (2016)



# Noise Temperature

The **effective temperature  $T_{\text{eff}}(\omega)$**  of the amplifier is the temperature that a **Bose-Einstein distribution** should have to equal the output  $\omega$  mode occupancy generated by a vacuum input state [5]:

$$\frac{1}{e^{\hbar\omega/k_B T_{\text{eff}}(\omega)} - 1} = |v(\omega)|^2$$

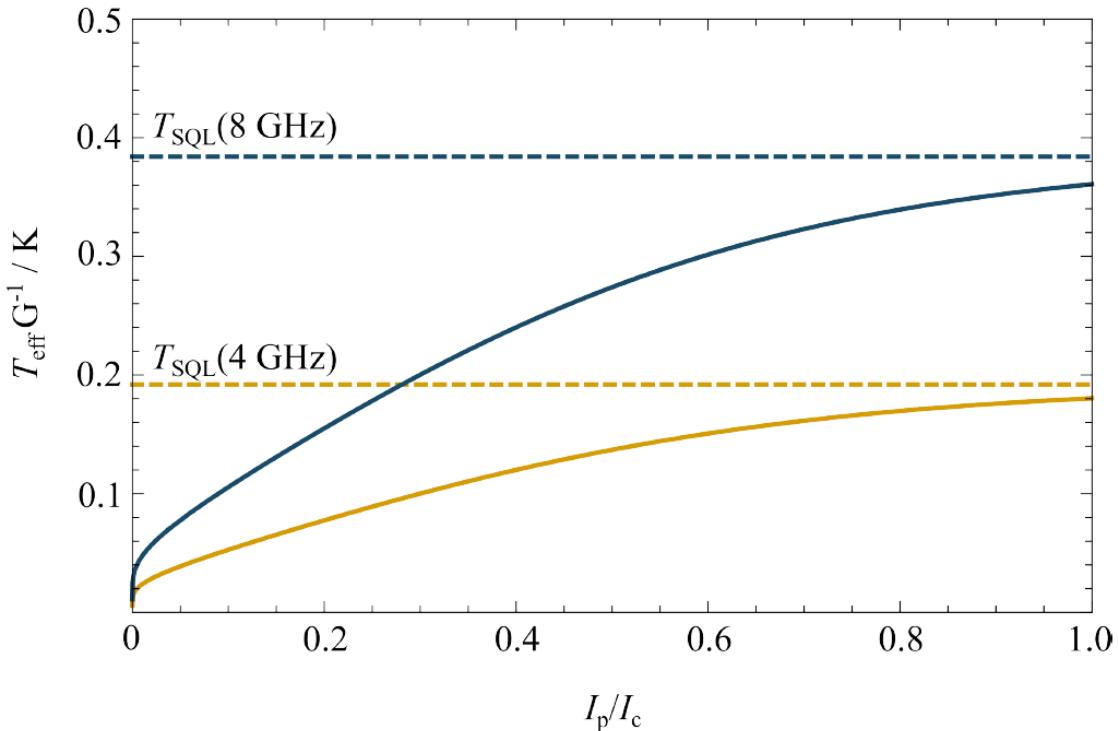
The **noise temperature  $T_n(\omega)$**  is the effective temperature normalized on the gain minus the contribution given by the fluctuation of the input vacuum state:

$$T_n(\omega) = \frac{T_{\text{eff}}(\omega)}{G(\omega)} - \frac{1}{2} \frac{\hbar\omega}{k_B}$$

For high gain  $T_n(\omega)$  approaches the **Standard Quantum Limit**:

$$T_{n,\text{SQL}} = \frac{\hbar\omega}{k_B}$$

[5] A. A. Clerk *et al.*, "Introduction to quantum noise, measurement, and amplification", *Rev. Mod. Phys.* **82**, 1155 (2010)



# For more details on the model ...

PHYSICAL REVIEW B **104**, 184517 (2021)

## Quantum model for rf-SQUID-based metamaterials enabling three-wave mixing and four-wave mixing travelling-wave parametric amplification

Angelo Greco<sup>✉</sup> and Luca Fasolo<sup>✉</sup>

INRIM, Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy  
and Department of Electronics and Telecommunications, PoliTo, Corso Castelfidardo 39, 10129 Torino, Italy

Alice Meda and Luca Callegaro<sup>✉</sup>

INRIM, Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy

Emanuele Enrico<sup>✉\*</sup>

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and INFN, Trento Institute for Fundamental Physics and Applications, I-38123 Povo, Trento, Italy

(Received 30 April 2021; revised 22 October 2021; accepted 25 October 2021; published 24 November 2021)

A quantum model for Josephson-based metamaterials working in the three-wave mixing (3WM) and four-wave mixing (4WM) regimes at the single-photon level is presented. The transmission line taken into account, namely Josephson traveling wave parametric amplifier (JTWPA), is a bipole composed of a chain of rf-SQUIDs, which can be biased by a DC current or a magnetic field to activate the 3WM or 4WM nonlinearities. The model exploits a Hamiltonian approach to analytically determine the time evolution of the system both in the Heisenberg and interaction pictures. The former returns the analytic form of the gain of the amplifier, while the latter allows recovering the probability distributions vs time of the photonic populations in the initial Fock and coherent input states. The dependence of the metamaterial's nonlinearities on the circuit parameters in a lumped model framework while evaluating the effects of the biasing on the model validity.

DOI: [10.1103/PhysRevB.104.184517](https://doi.org/10.1103/PhysRevB.104.184517)



A. Greco *et al.*, Phys. Rev. B (2021)

IEEE TRANSACTIONS ON APPLIED SUPERCONDUCTIVITY, VOL. 32, NO. 4, JUNE 2022

1700306

## Bimodal Approach for Noise Figures of Merit Evaluation in Quantum-Limited Josephson Traveling Wave Parametric Amplifiers

L. Fasolo<sup>✉</sup>, C. Barone<sup>✉</sup>, M. Borghesi, G. Carapella, A. P. Caricato, I. Carusotto, W. Chung, A. Cian<sup>✉</sup>, D. Di Gioacchino<sup>✉</sup>, E. Enrico<sup>✉</sup>, P. Falferi, M. Faverzani<sup>✉</sup>, E. Ferri, G. Filatrella, C. Gatti<sup>✉</sup>, A. Giachero, D. Giubertoni<sup>✉</sup>, A. Greco<sup>✉</sup>, Ç. Kutlu, A. Leo, C. Ligi<sup>✉</sup>, P. Livreri<sup>✉</sup>, G. Maccarrone, B. Margesin<sup>✉</sup>, G. Maruccio, A. Matlashov, C. Mauro, R. Mezzena<sup>✉</sup>, A. G. Monteduro, A. Nucciotti<sup>✉</sup>, L. Oberto<sup>✉</sup>, S. Pagano<sup>✉</sup>, V. Pierro<sup>✉</sup>, L. Piersanti<sup>✉</sup>, M. Rajteri<sup>✉</sup>, A. Rettaroli<sup>✉</sup>, S. Rizzato<sup>✉</sup>, Y. K. Semertzidis<sup>✉</sup>, S. Uchaikin, and A. Vinante<sup>✉</sup>

**Abstract**—The advent of ultra-low noise microwave amplifiers revolutionized several research fields demanding quantum-limited technologies. Exploiting a theoretical bimodal description of a linear phase-preserving amplifier, in this contribution we analyze some of the intrinsic properties of a model architecture (i.e., an rf-SQUID based Josephson Traveling Wave Parametric Amplifier) in terms of amplification and noise generation for key case study input states (Fock and coherent). Furthermore, we present an analysis of the output signals generated by the parametric amplification mechanism when thermal noise fluctuations feed the device.

**Index terms**—Josephson junctions, microwave photonics, noise figure, superconducting quantum interference devices, superconducting quantum computation, superconducting radio frequency, superconducting transistors, superconducting waveguides.

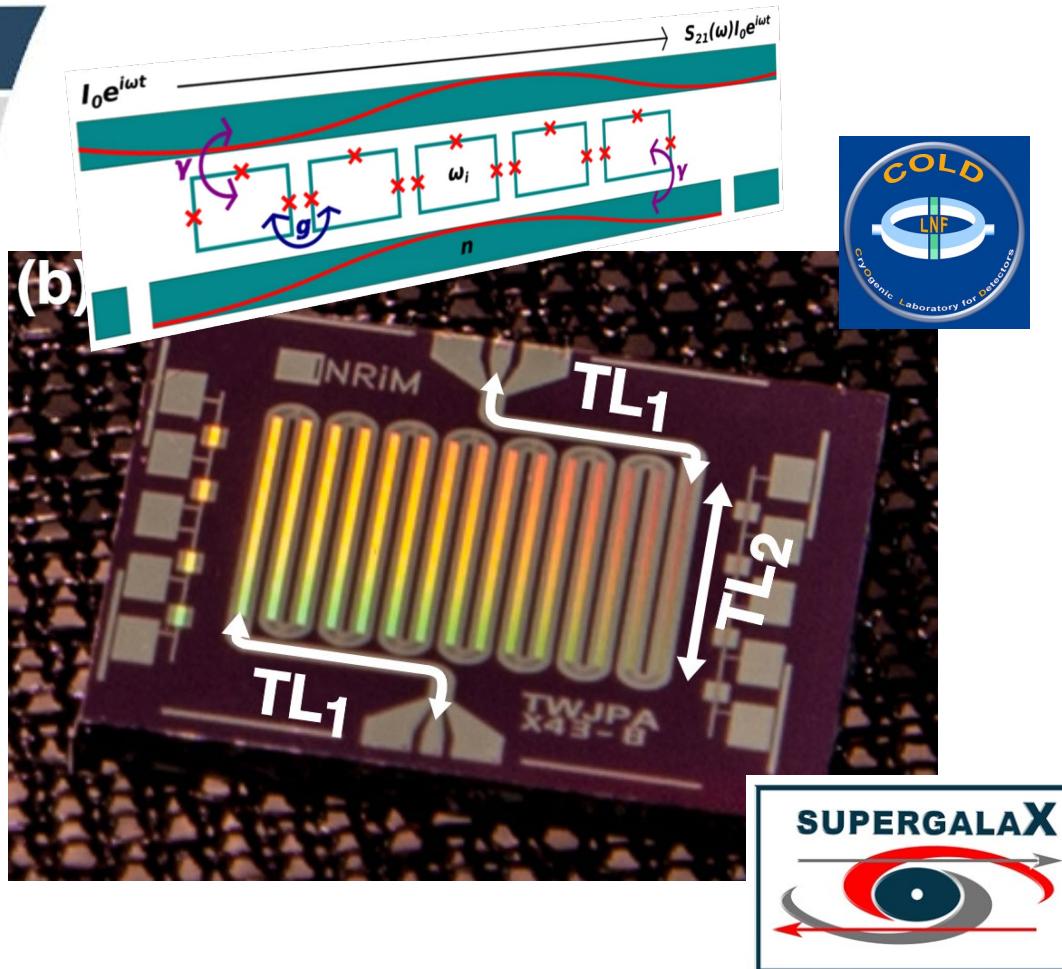


the amplifier is considered as a two-ports black-box driven at a pump frequency  $\omega_p$  that amplifies a bosonic input mode at frequency  $\omega$ . The amplification is associated with the creation of a second mode at frequency  $\omega' = \omega_p - \omega$  (the so-called idler mode of a three-wave mixing parametric amplification [15]) that is commonly considered as an internal mode of the amplifier that causes the onset of noise at the output port. Here, we extend and give a different perspective of this description considering the case in which an uncorrelated idler mode is already present at the input port (i.e., considering a bimodal input field), analyzing the effect of the interaction between these modes inside the amplifier in terms of typical noise estimators. This operative condition may arise in real measurement setups where the amplifier is exploited, for instance, for the multiplexed readout of broadband signals [16] or for the joint detection and amplification of probing signals in a microwave quantum illumination

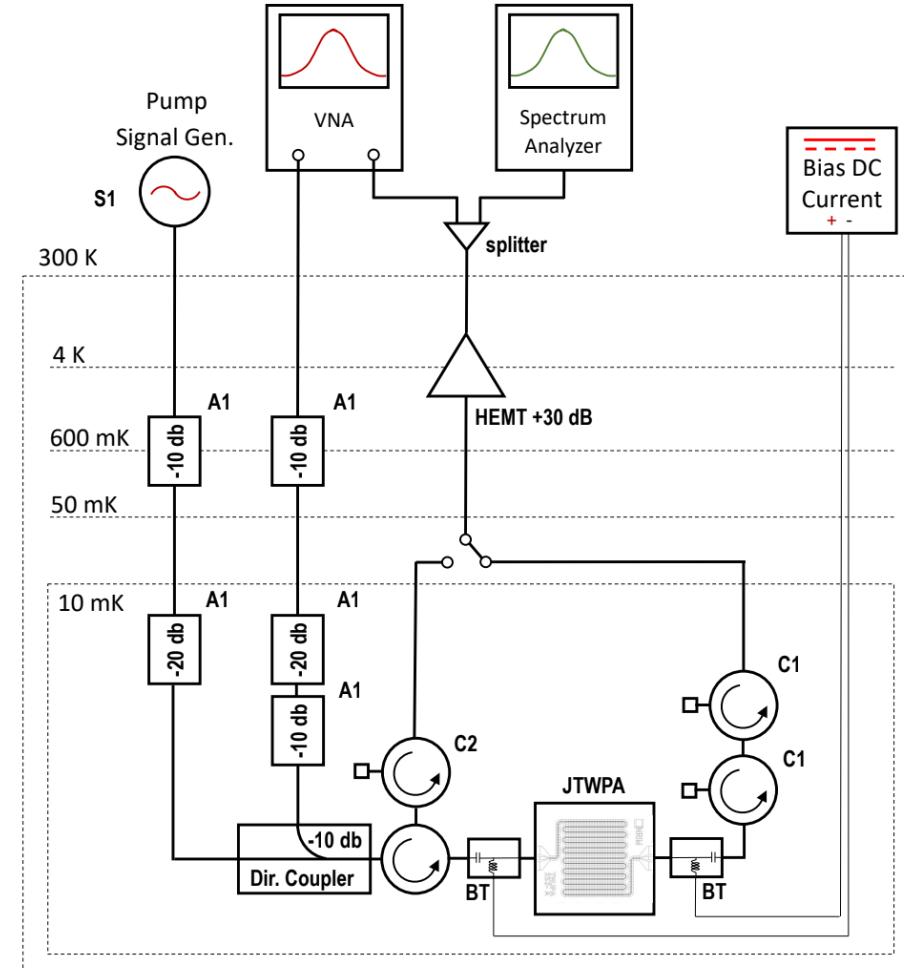
L. Fasolo *et al.*, IEEE TAS (2022)



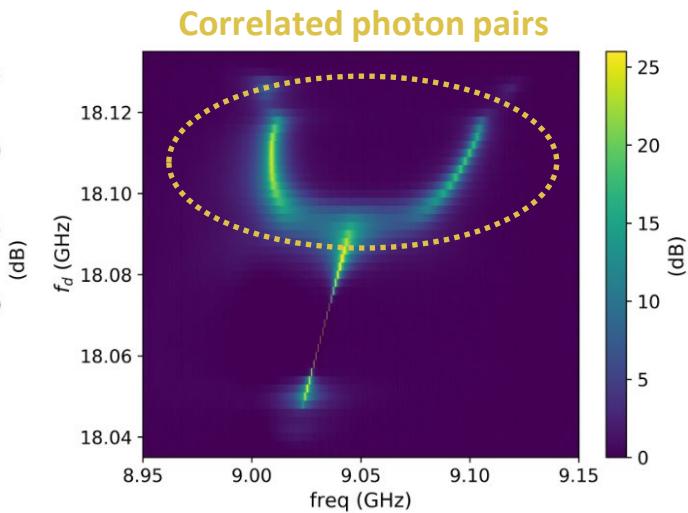
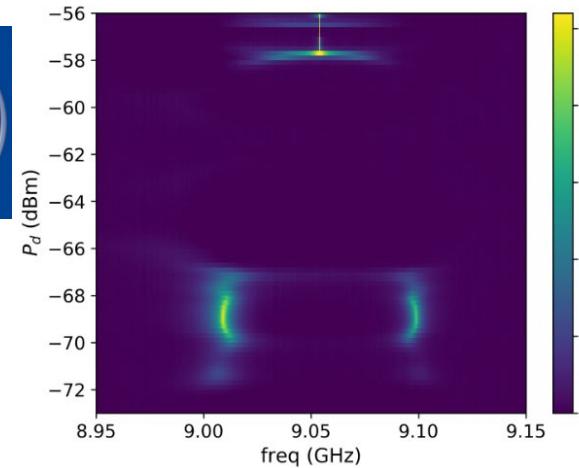
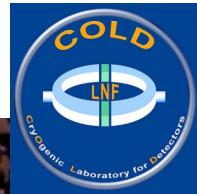
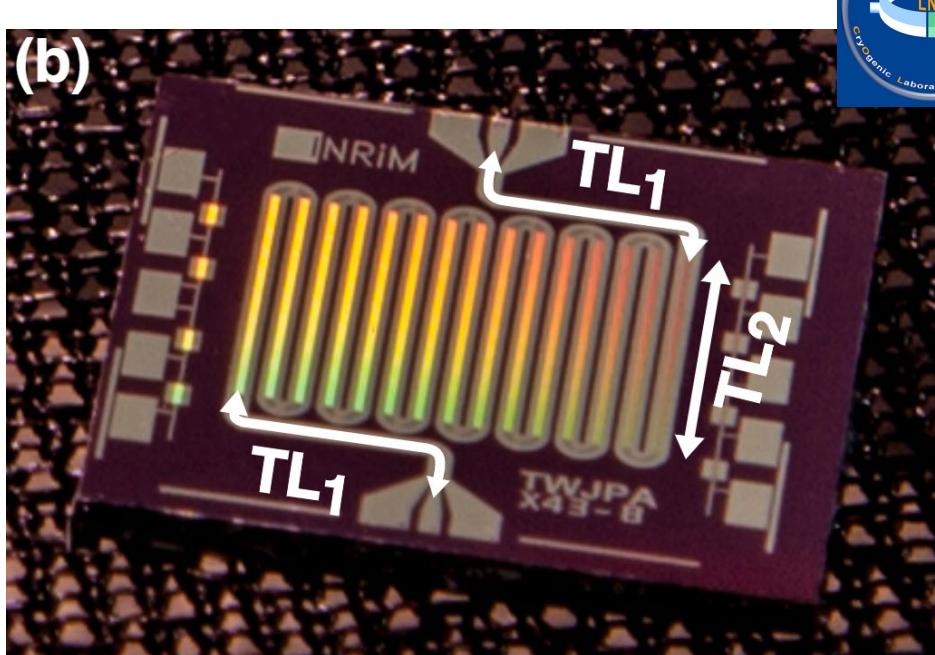
# Dynamical Casimir effect fingerprint



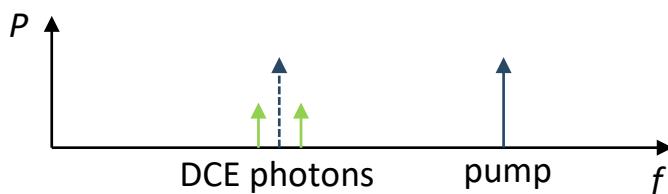
Highly sensitive detection of single microwave photons with coherent quantum network of superconducting qubits for searching galactic axions



# Dynamical Casimir effect fingerprint



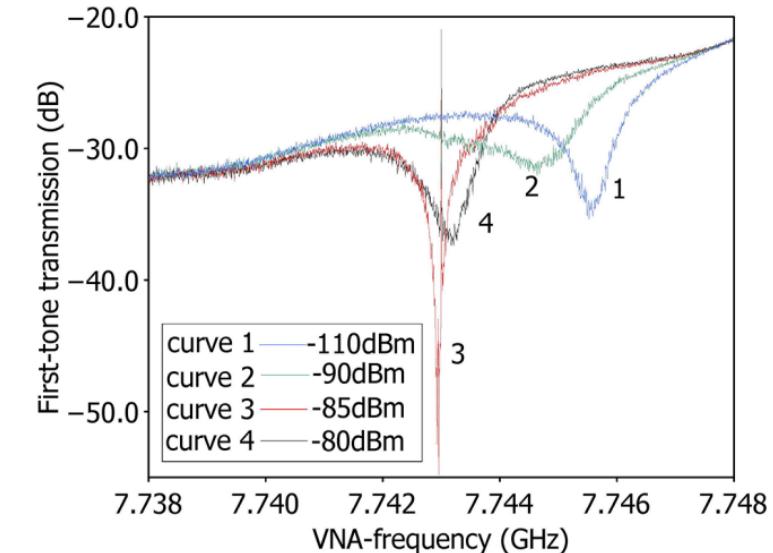
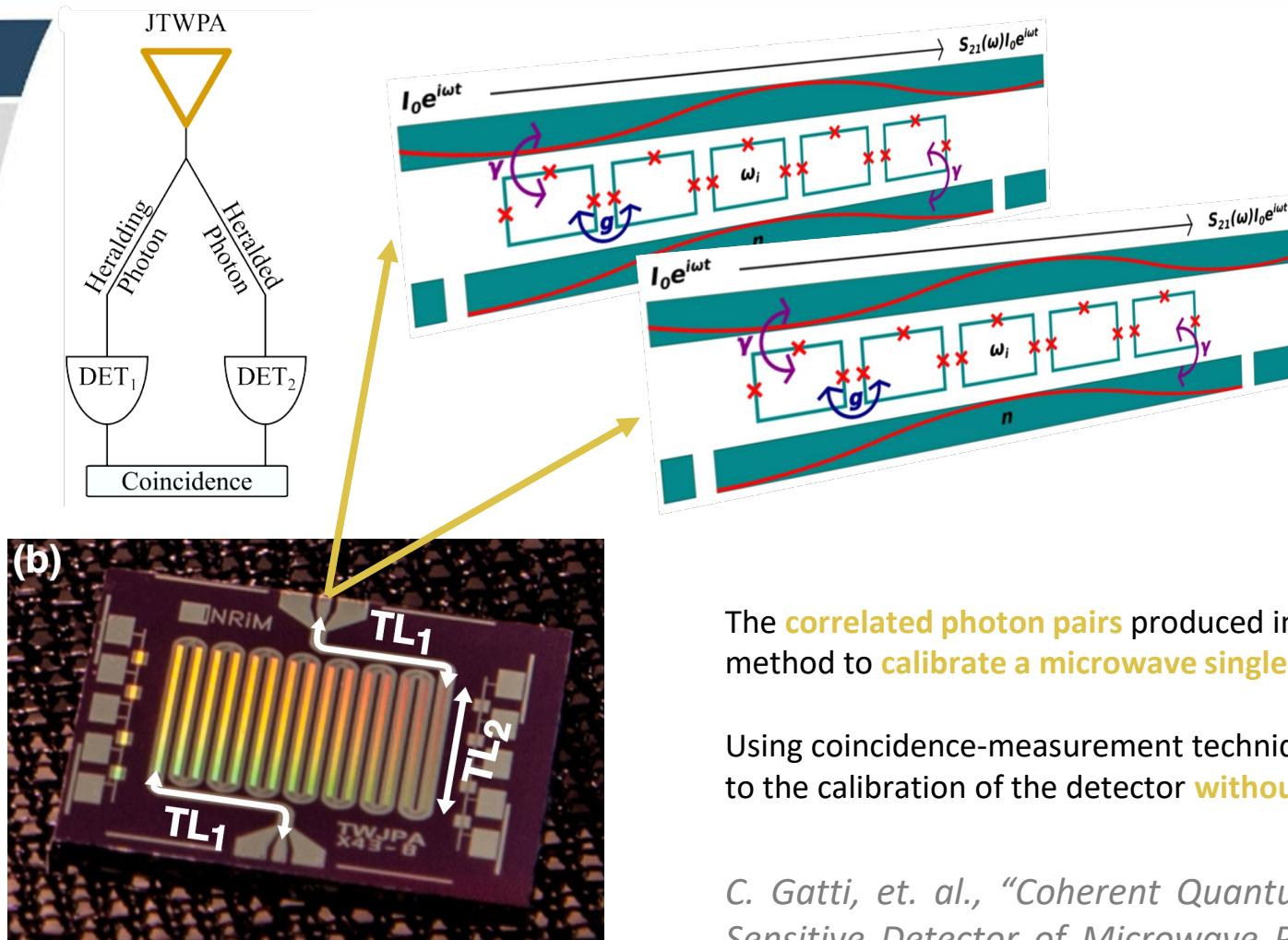
- **photon splitting**, reasonably attributed to **time-varying internal reflections driven by the pump**
- **transitions** between the linear 15 sections and the 50 Ohm CPW curves.
- series of **semi-transparent moving mirrors** that are 'seen' by the traveling tones in a metamaterial with a time-varying refractive index (impedance)



F. Chiarello, et. al., "Microwave photon emission in superconducting circuits", (in preparation)



# Heralded source of microwave photons



The **correlated photon pairs** produced in JTWPA open the way to a fundamentally absolute method to **calibrate a microwave single-photon detector**:

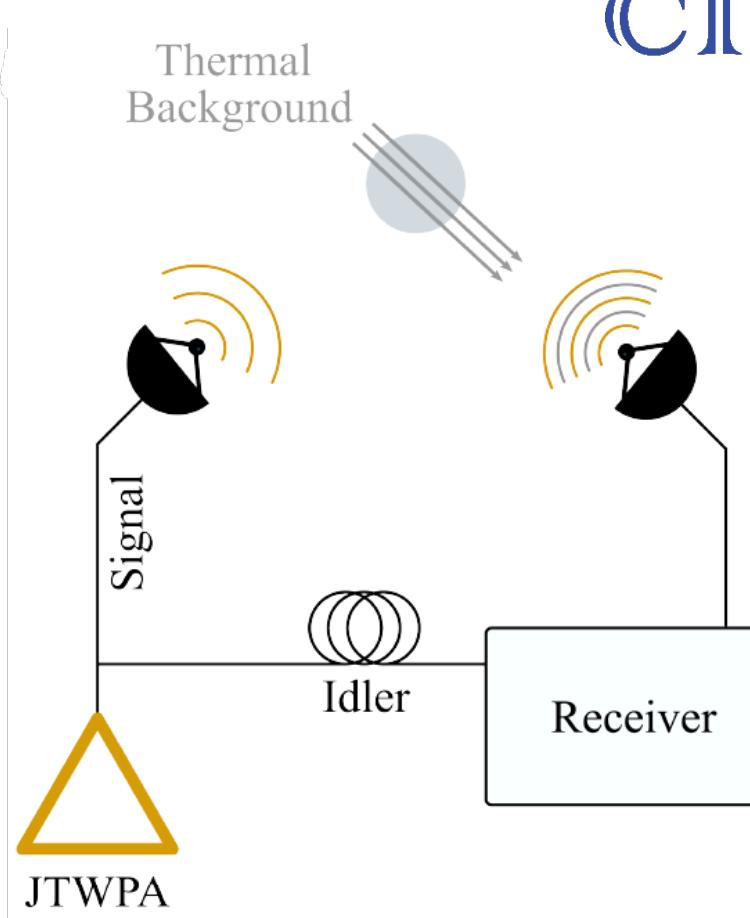
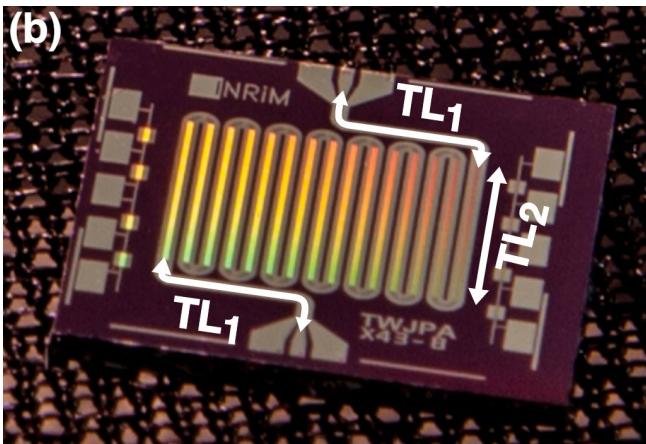
Using coincidence-measurement techniques, the **heralded single-photon sources** can be applied to the calibration of the detector **without the need for an absolute radiometric reference**.

C. Gatti, et. al., “Coherent Quantum Network of Superconducting Qubits as a Highly Sensitive Detector of Microwave Photons for Searching of Galactic Axions”, IEEE TAS (2023)

# Source of entangled microwave photons

## Quantum illumination (Quantum Radar)

L. Fasolo, A. Greco, E. Enrico, F. Illuminati, R. Lo Franco, D. Vitali, P. Livreri, "Josephson Traveling Wave Parametric Amplifiers as non-classical light source for Microwave Quantum Illumination", Measurements: Sensors, 18, 100349 (2021)



Italian national  
interuniversity consortium  
for telecommunications



UNIVERSITÀ  
DI CAMERINO



# Quantum Signals Processing Laboratory



Model: Leiden Cryogenics CF-CS110-500



Cooling Power > 500 uW @ 120 mK

2 Warm-Insertable 110 mm diameter probes

- 8x RF lines + 24 DC wirings
- 12x insulated coax cables for QHE experiments

9 T Magnet

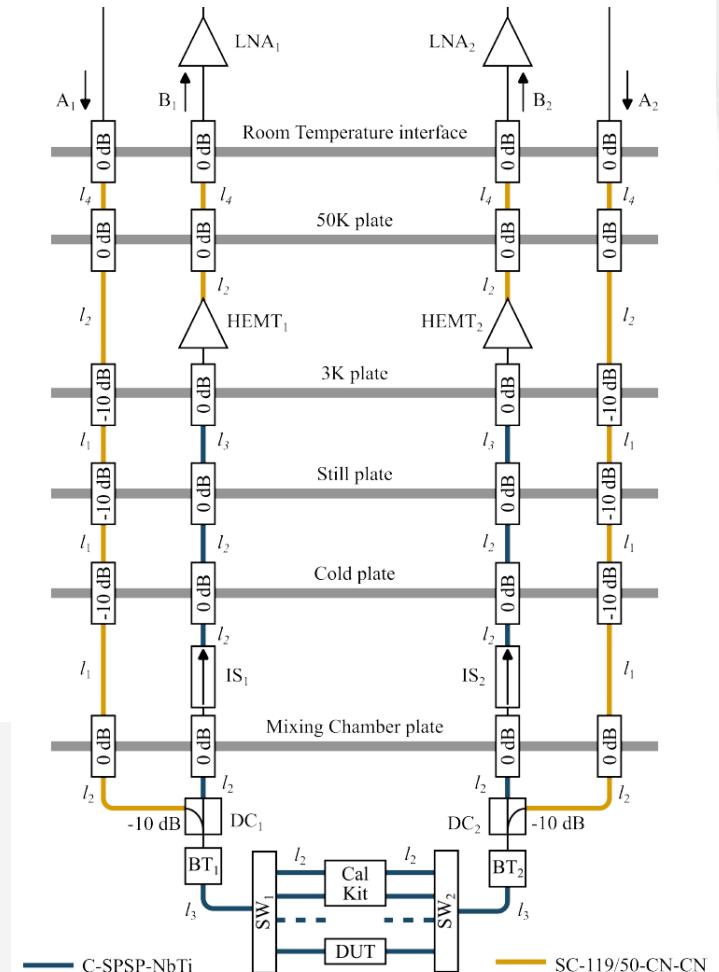
Mu-metal shield

Luca Oberto (INRiM)

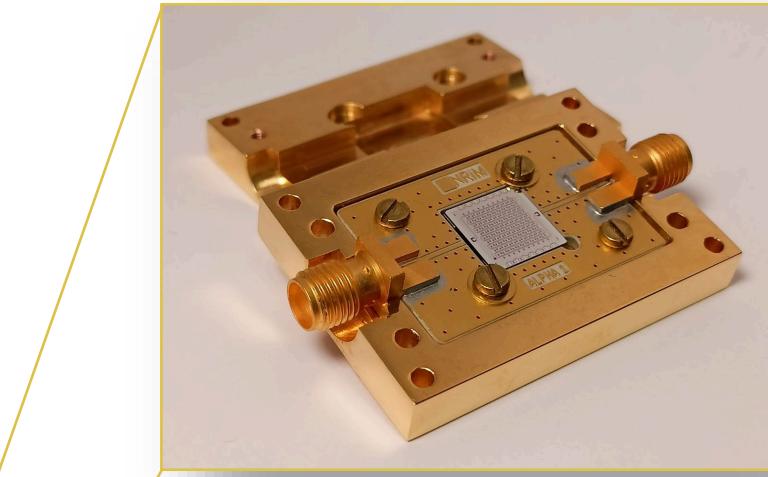
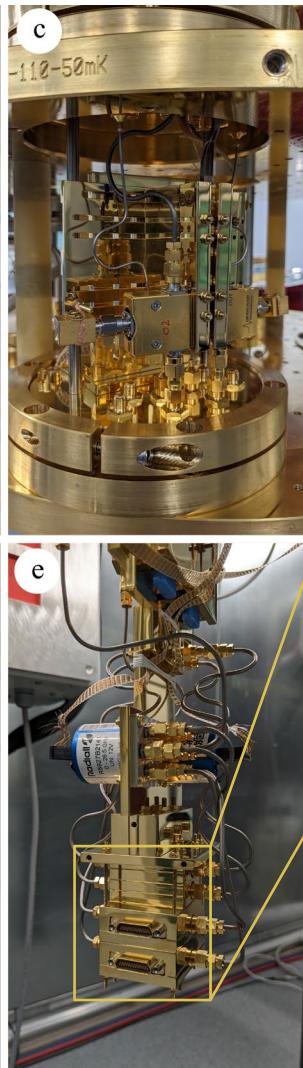
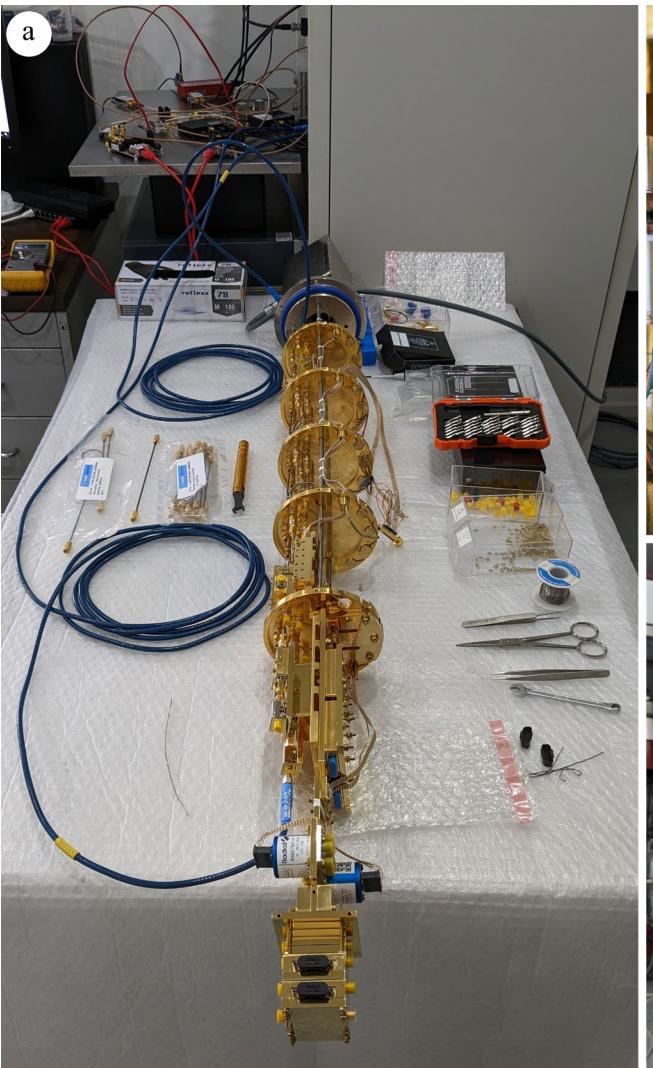


Traveling Wave Parametric Amplifiers come metamateriali per le tecnologie quantistiche

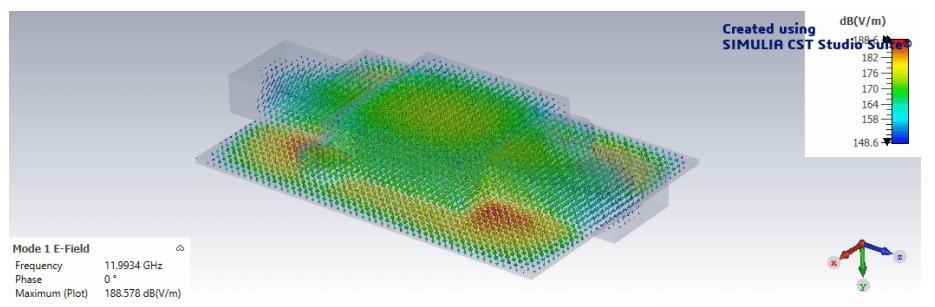
Workshop INFN CSN4&5 - Dipartimento di Fisica su Tecnologie Quantistiche – 07 giugno 2023 - E. ENRICO



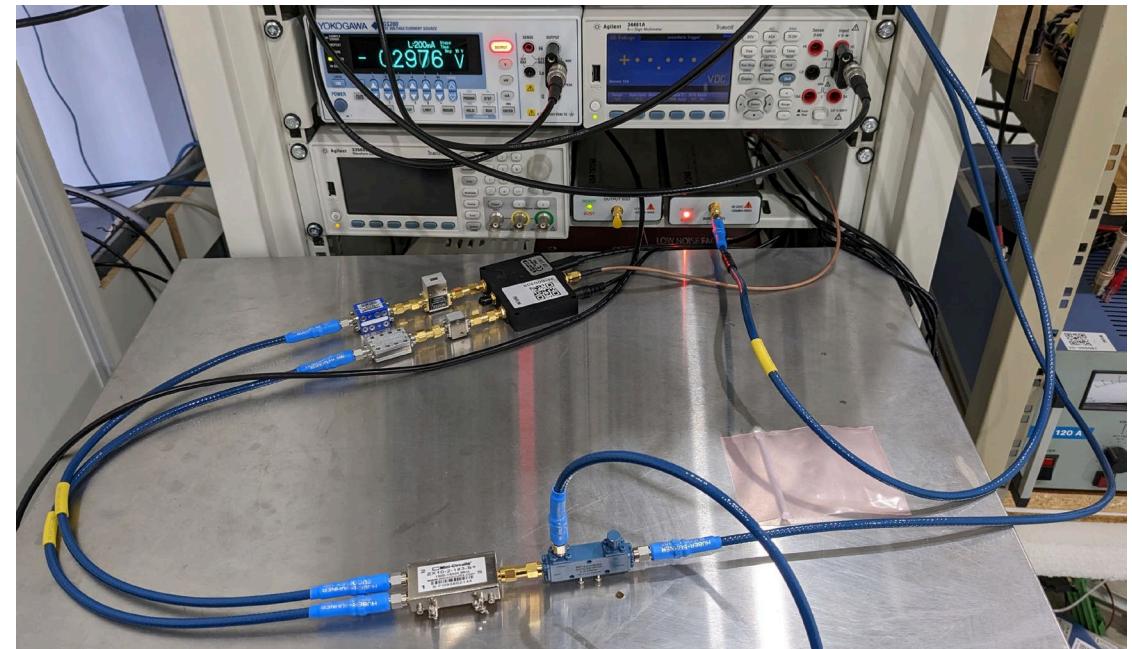
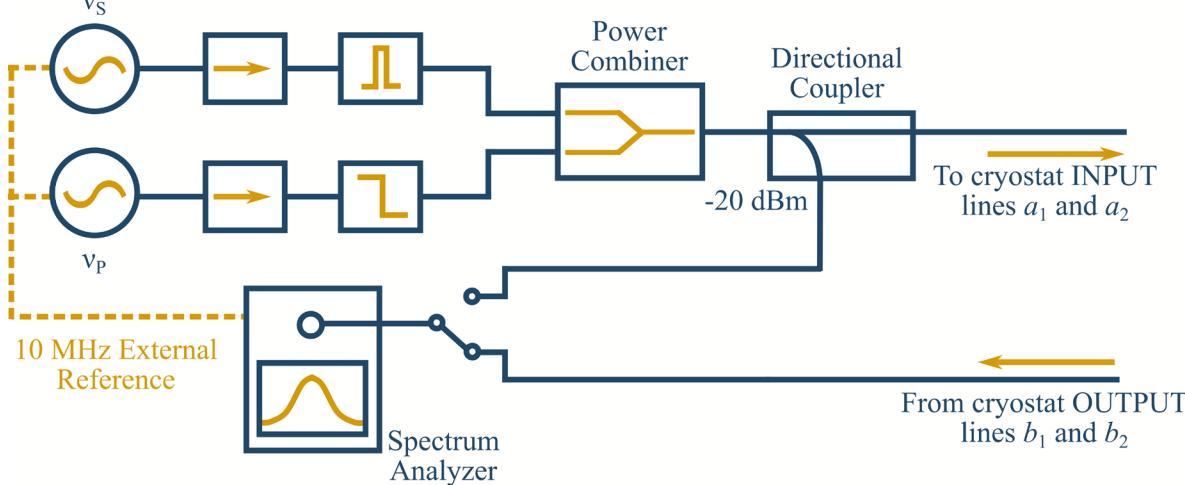
# Quantum Signals Processing Laboratory



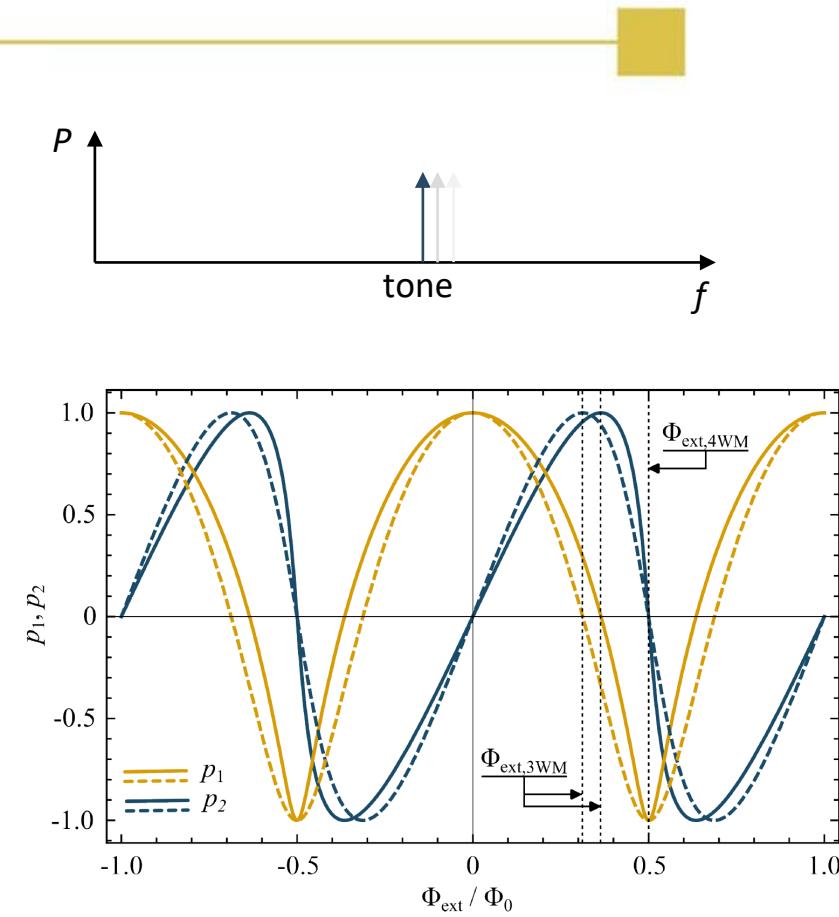
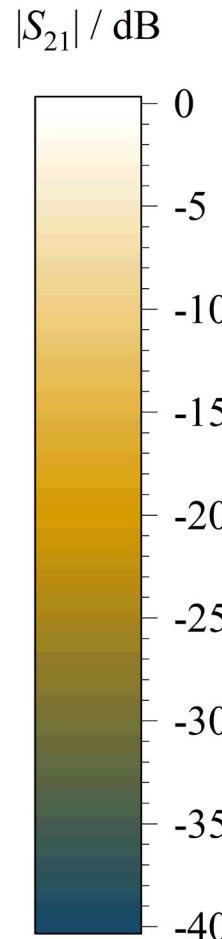
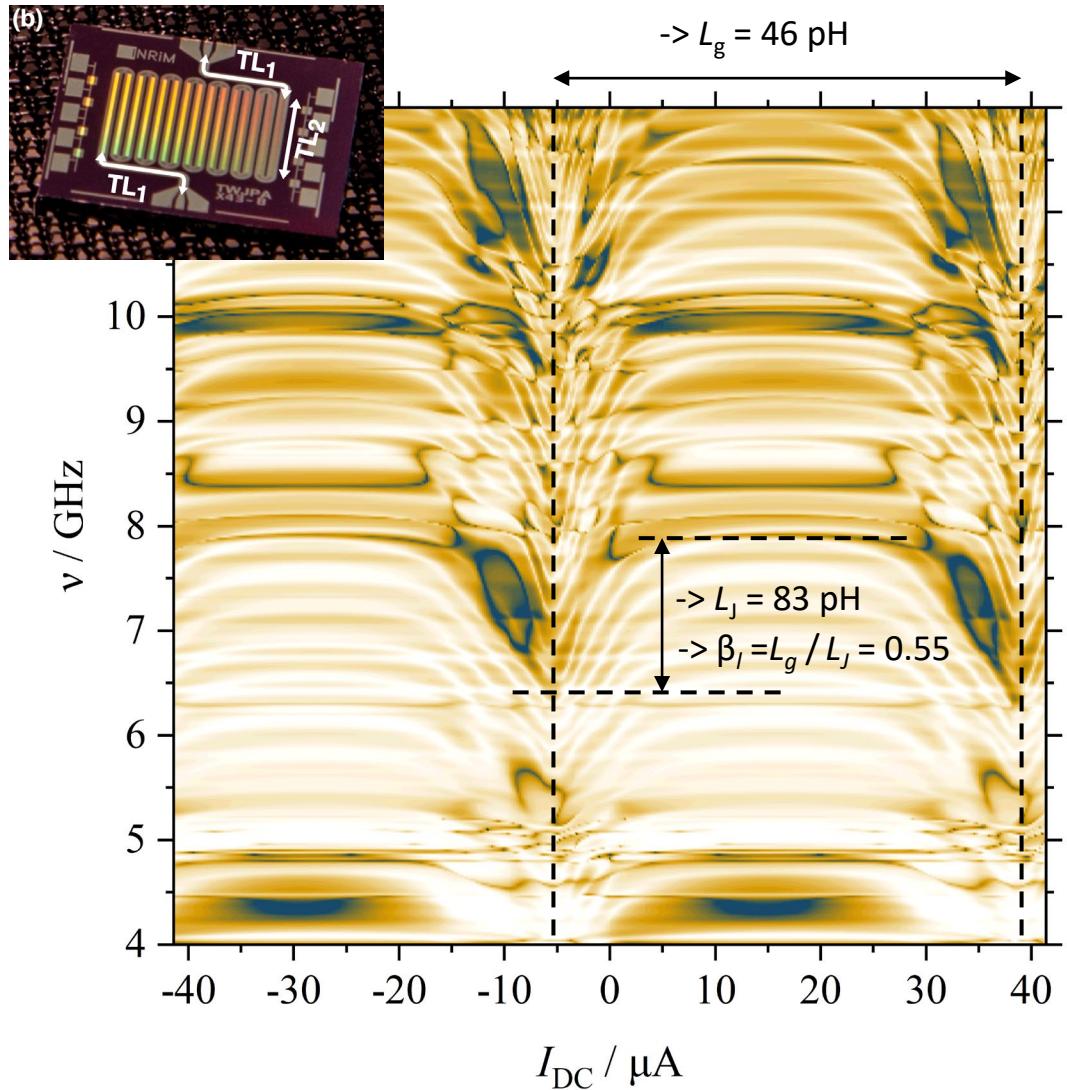
Two-port sample holder  
(lowest resonant cavity mode at 12 GHz)



# Room Temperature Setup



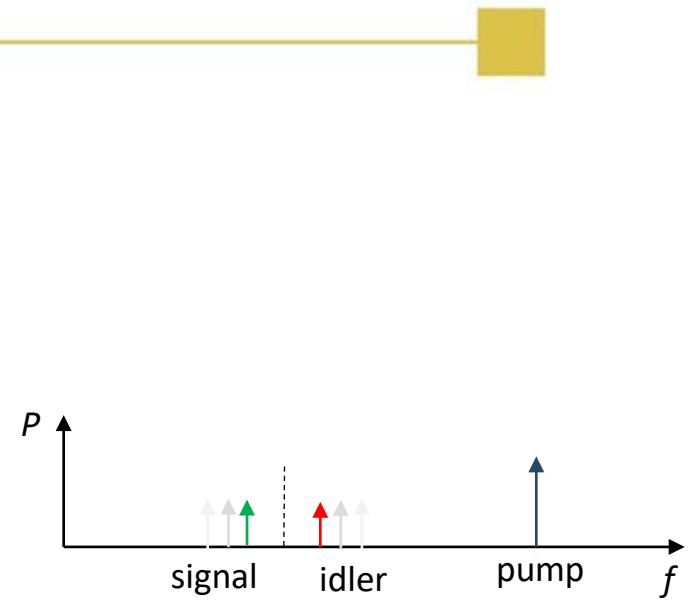
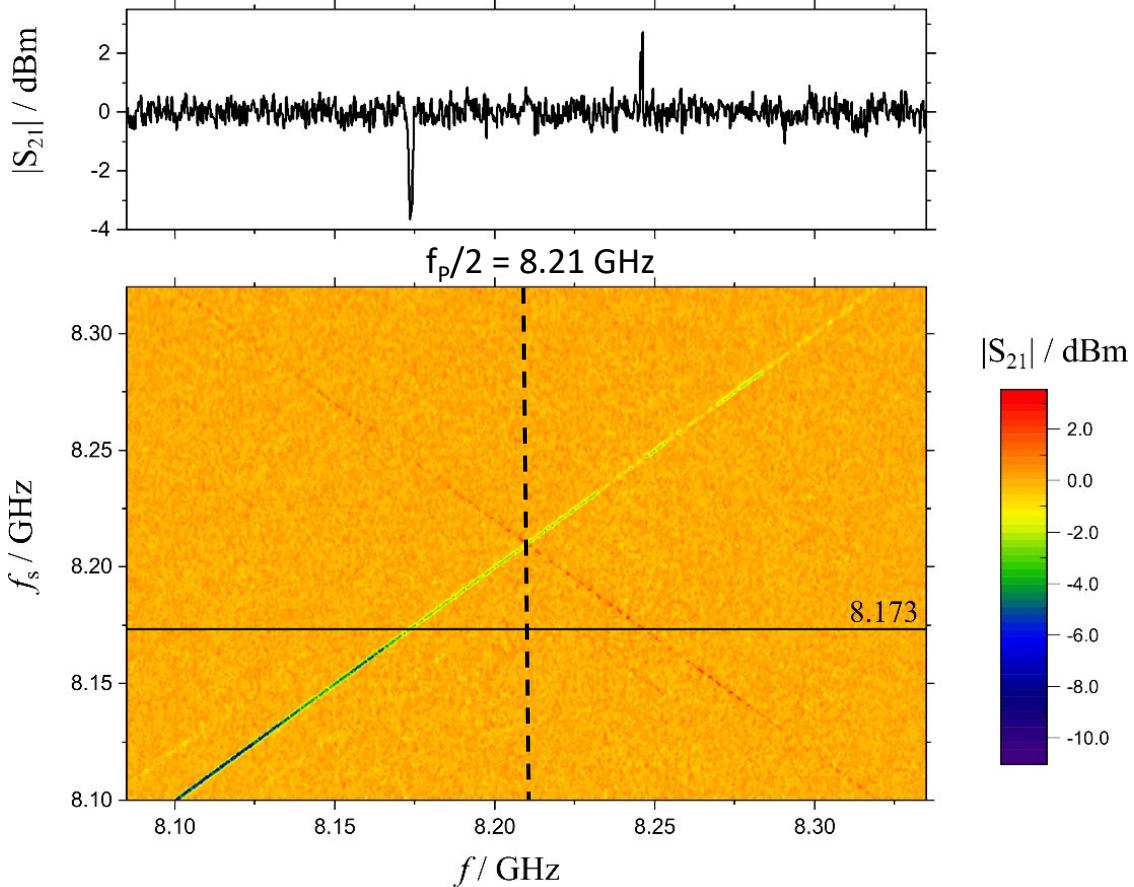
# Single tone spectroscopy



L. Fasolo, (in preparation)

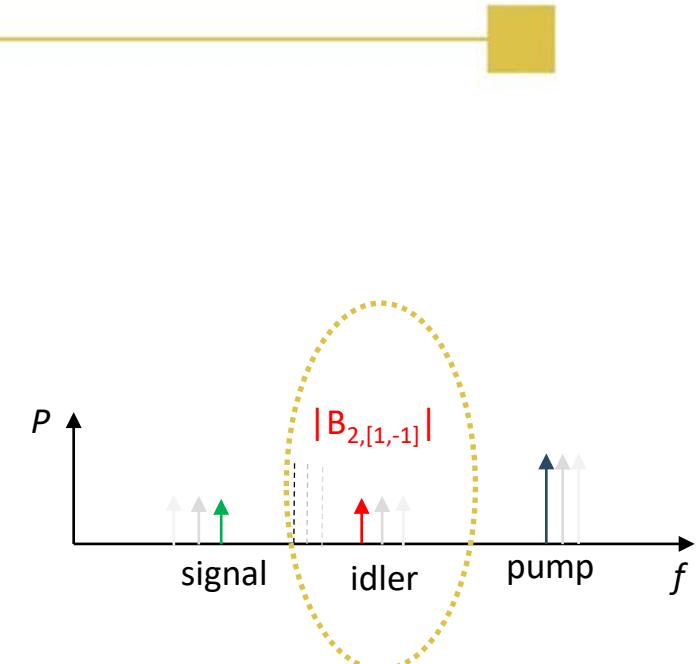
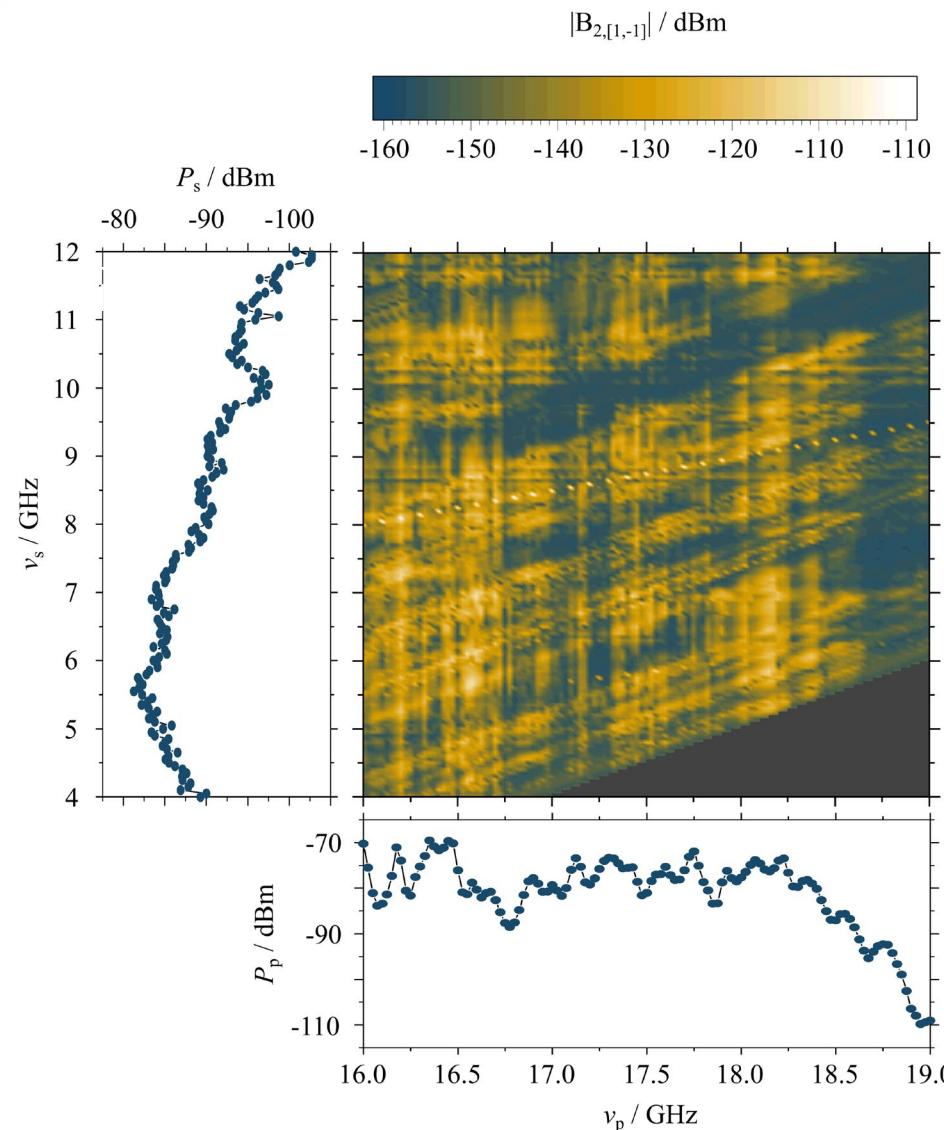
# S/I frequency correlation

fixed  $f_p = 16.42$  GHz



L. Fasolo, (in preparation)

# Double tone spectroscopy

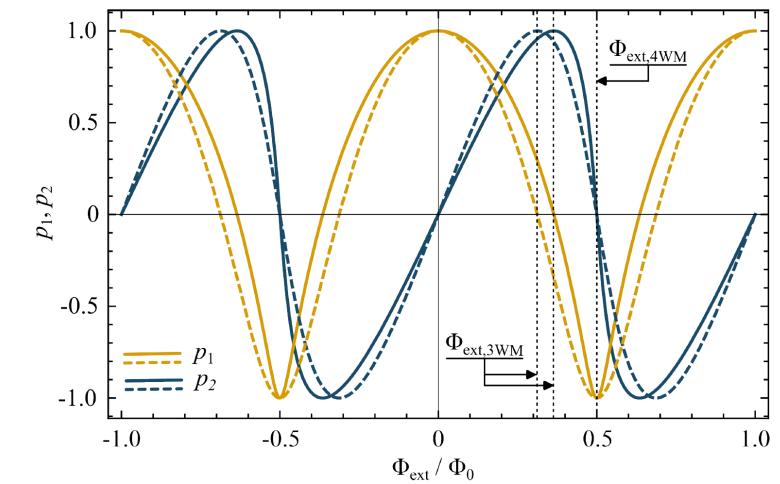
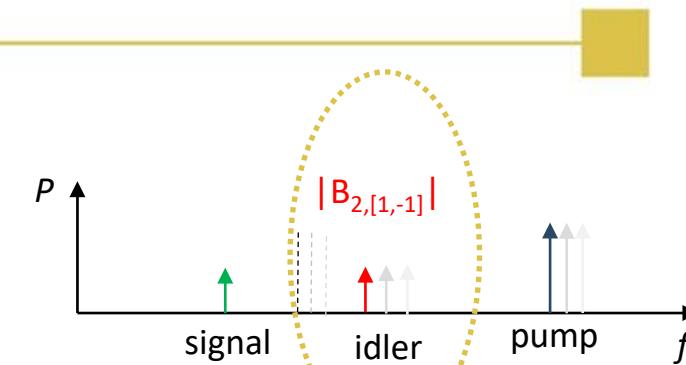
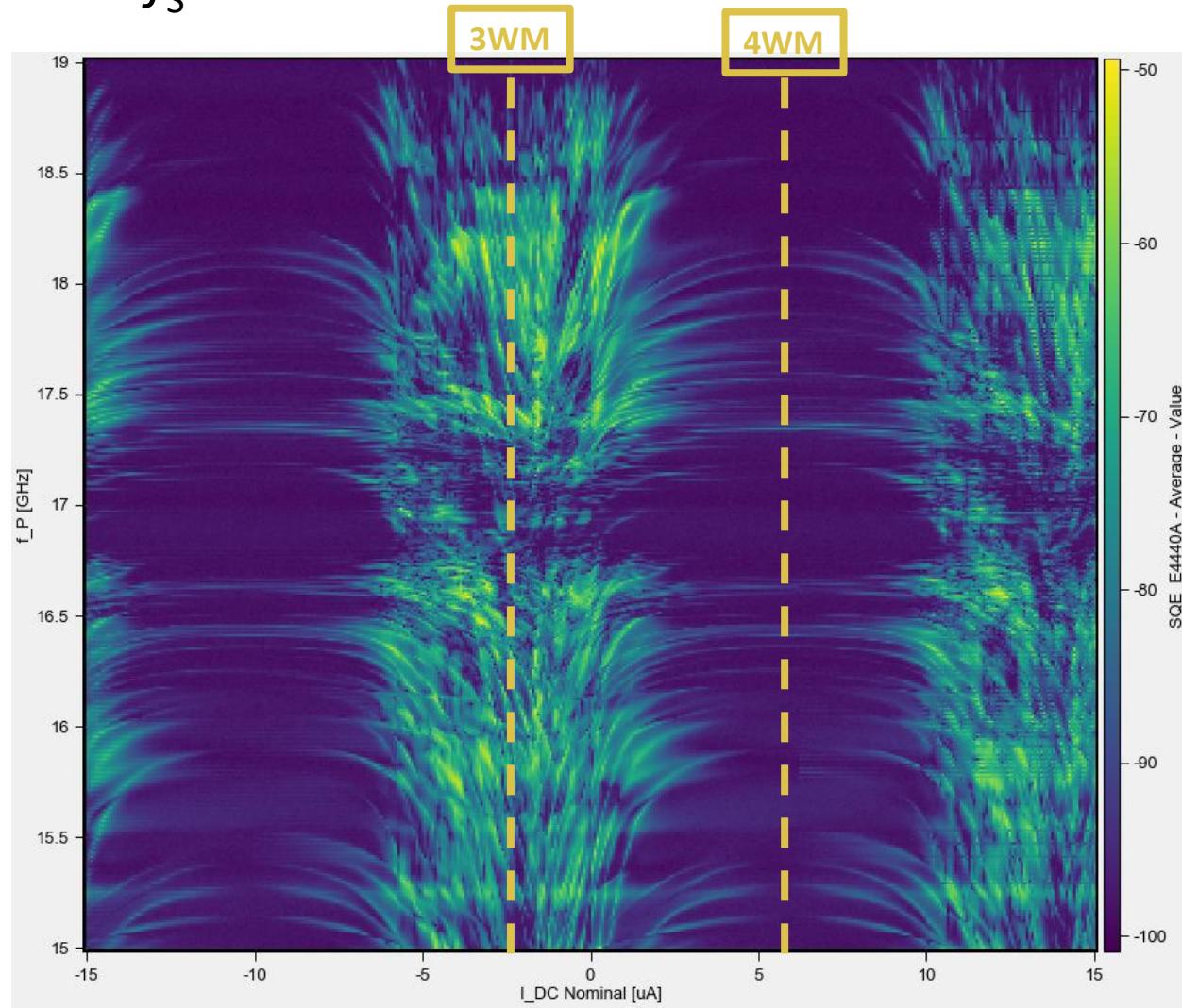


L. Fasolo, (in preparation)



# Double tone spectroscopy

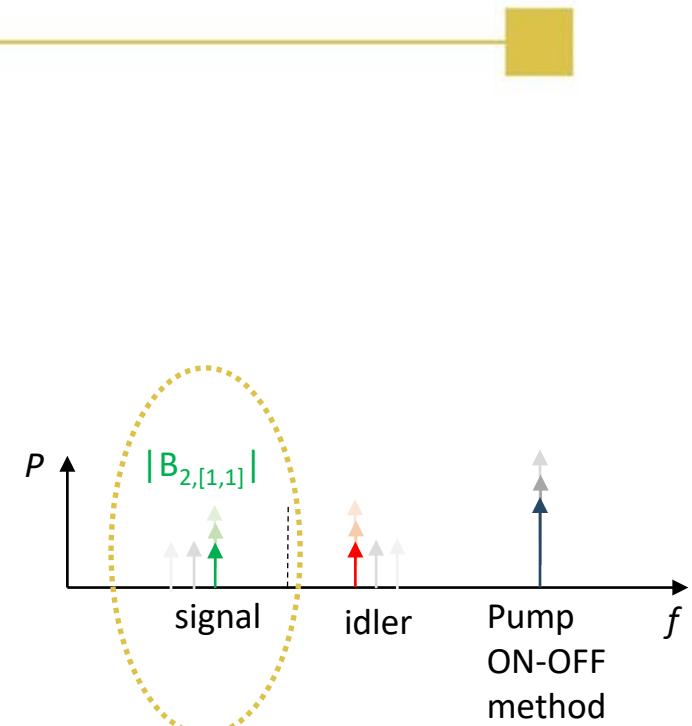
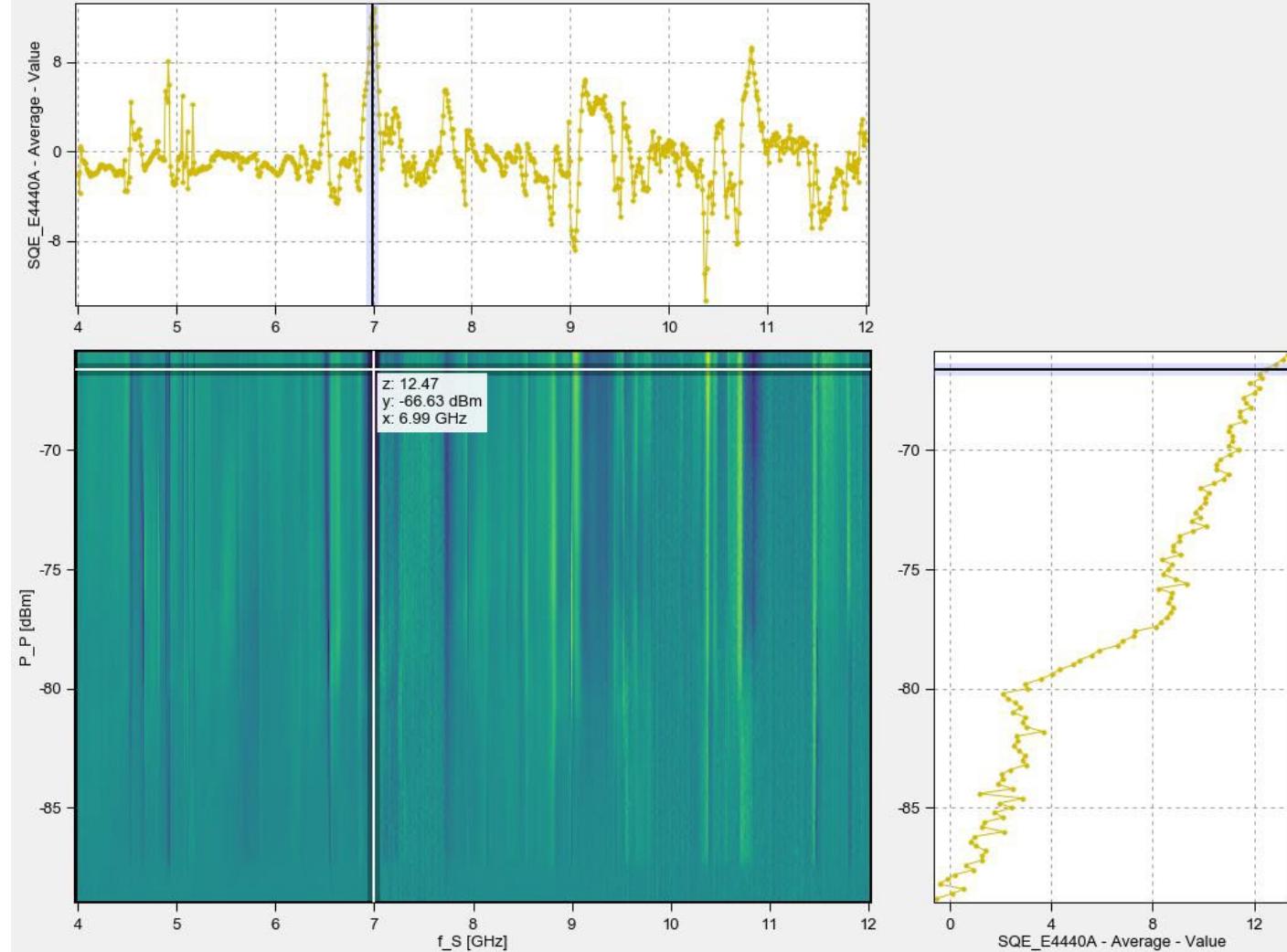
fixed  $f_s = 7 \text{ GHz}$



L. Fasolo, (in preparation)

# Signal gain vs $P_p$

fixed  $f_p = 18$  GHz



L. Fasolo, (in preparation)



Thanks for your attention!