Quantum metrology… and an application to HEP

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Quantum metrology?

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Metrology: estimation of a parameter, through measurements.

The estimation is always performed by averaging over N measurements, so that (central limit theorem), the error of the average goes as $1/\sqrt{N}$

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Quantum Metrology: estimation of a parameter with increased precision (thanks to quantum effects, e.g. entanglement)

Usually: $\sqrt{N}$ enhancement:
the error goes as $1/N$

Measurements

Want to estimate a parameter φ written onto a probe by a transformation U_φ

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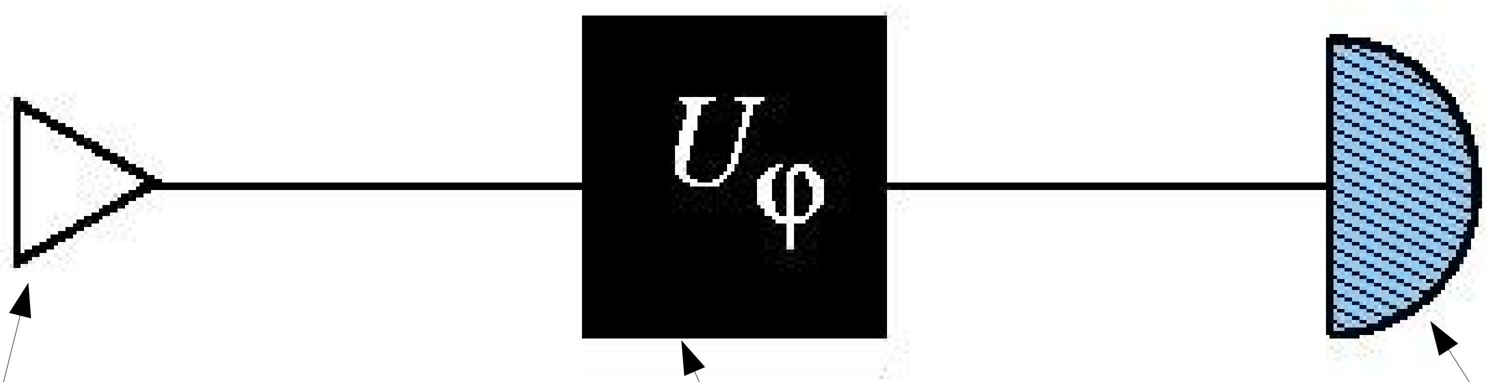
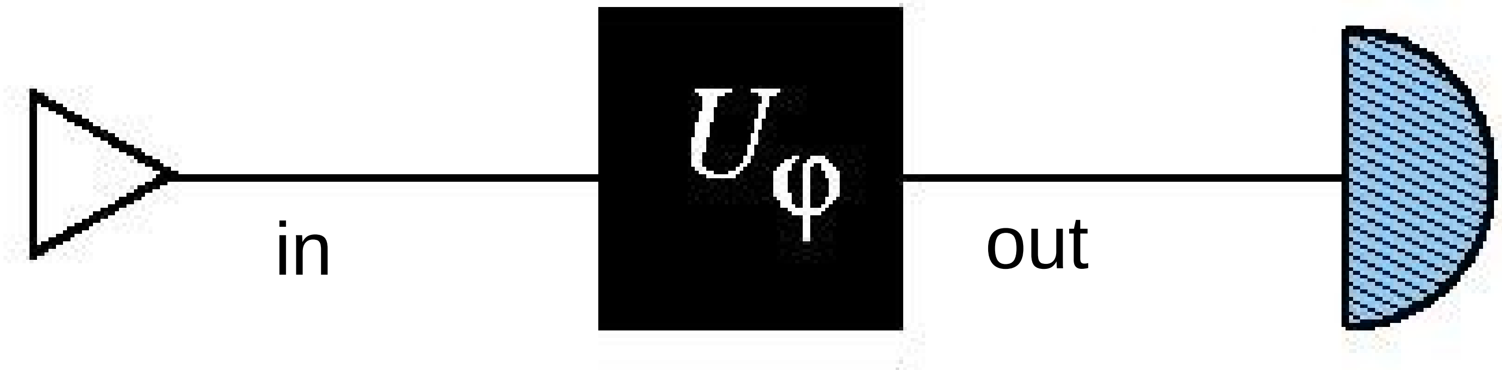


Probe preparation

interaction

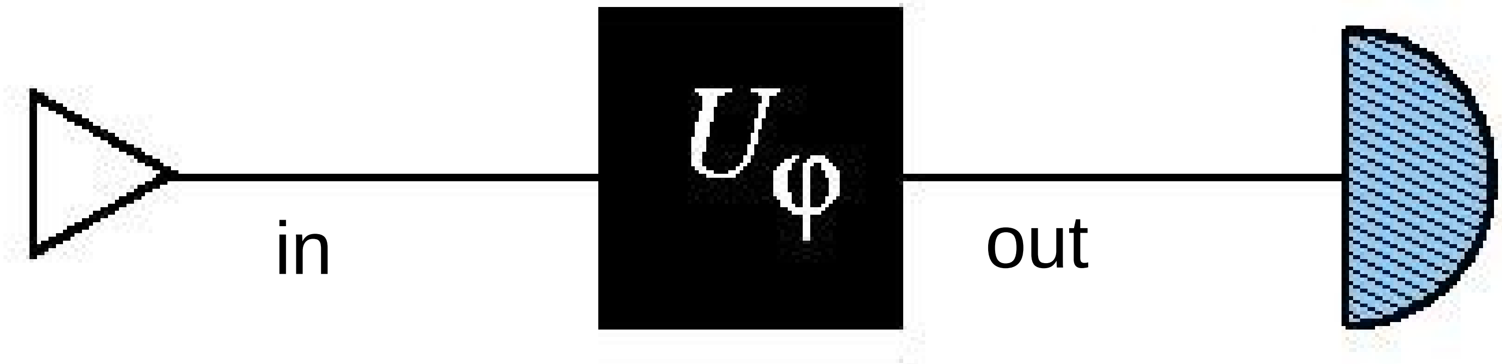
Probe measurem and data analysis

How?



Main idea: minimize the overlap between the input and the output state!

How?



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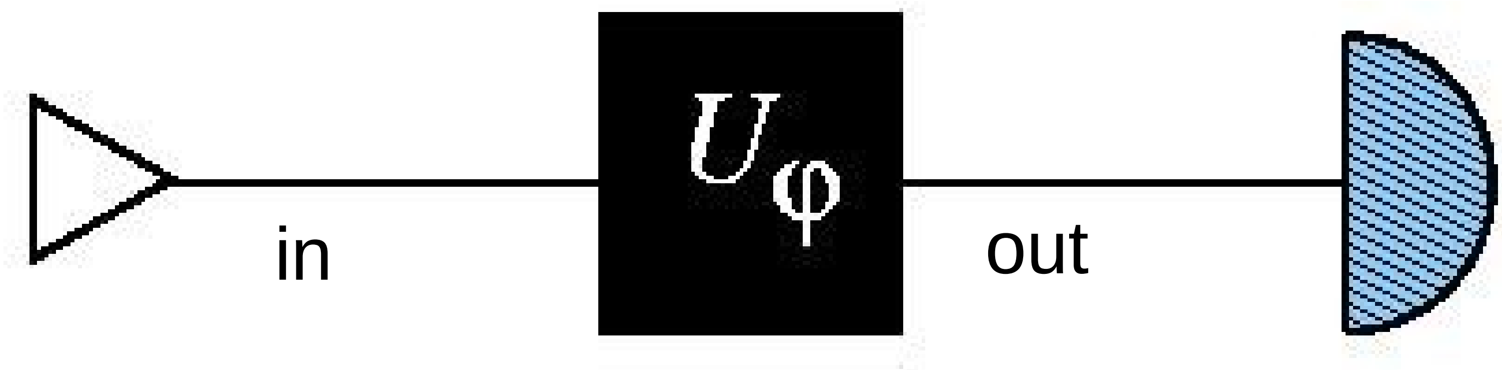
Optimize **fidelity** (overlap) → DISCRIMINATION "U_φ is present or not"

How?



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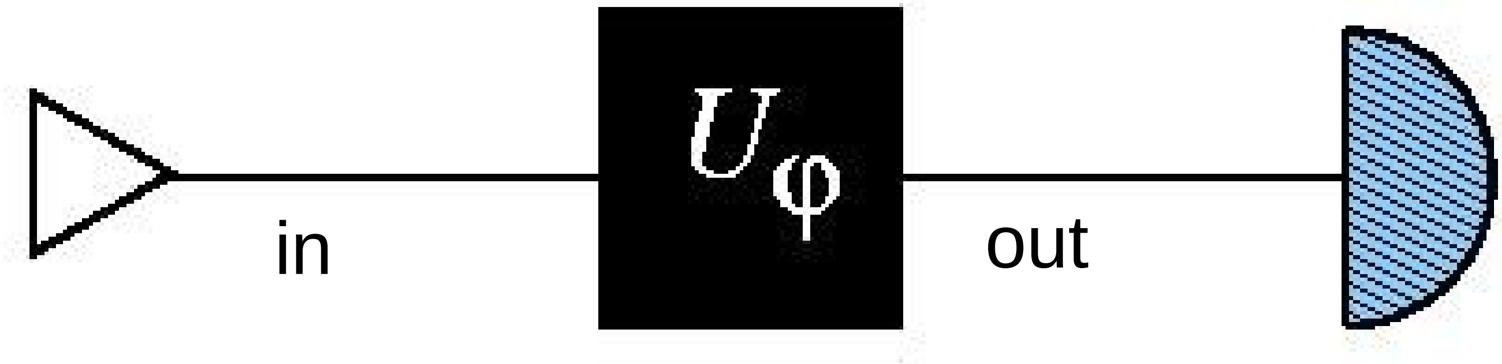
"U_φ is present or not"

Optimize **Q Fisher Information** → ESTIMATION

"what's the value of φ?"

How?



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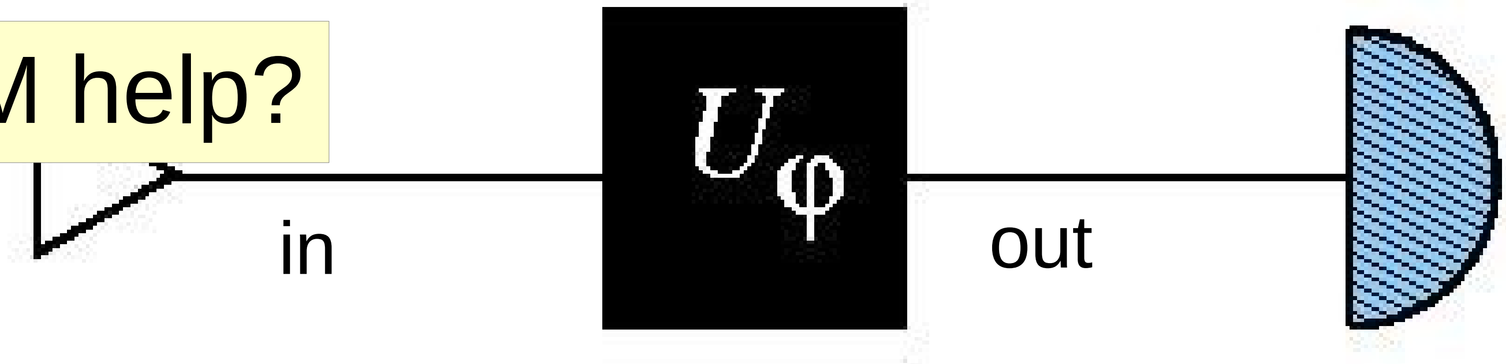
“what’s the value of φ ?”

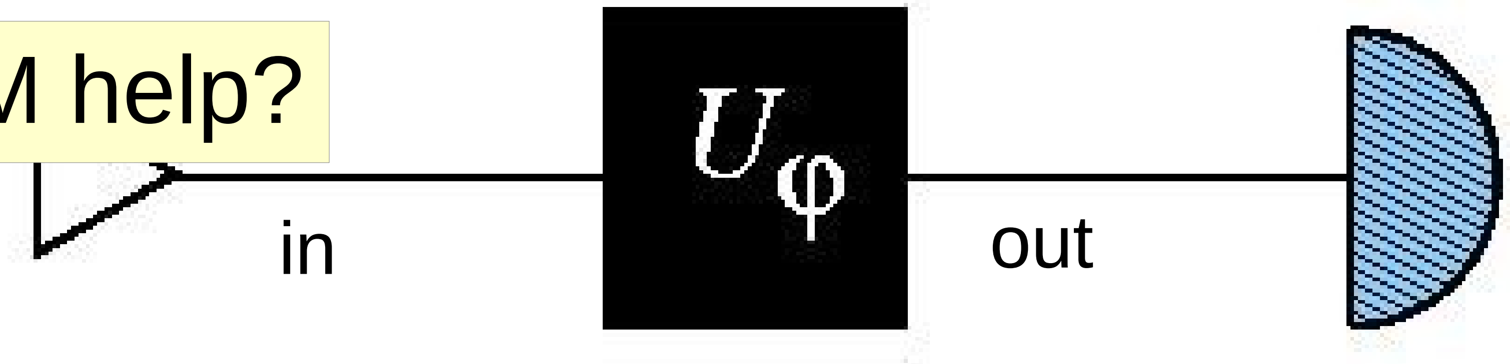
the metric in Hilbert space (to measure distances)

Does QM help?



Compare quantum strategies to the **corresponding** classical strategy

Does QM help?



Compare quantum strategies to the **corresponding** classical strategy

Same number of uses of U_φ

Same employed energy

What is quantum metrology?

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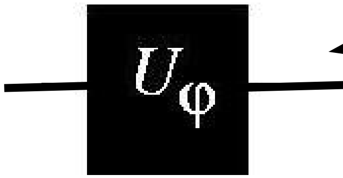
GIVEN: 

black box that implements a transformation $U_{\varphi} = e^{i\varphi H}$

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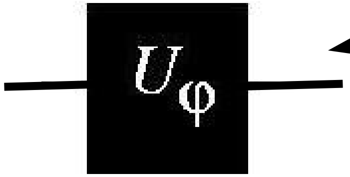

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GOAL: use it νN times and get the best estimate of φ

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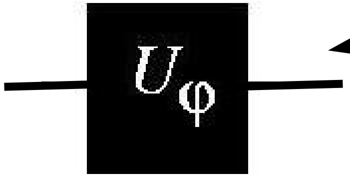

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GOAL: use it νN times and get the best estimate of φ

RESULT:

quantum strategies: $\Delta\varphi \sim \frac{1}{\sqrt{\nu}N}$

Heisenberg bound

number of times the N-experiment is repeated

What is quantum metrology?

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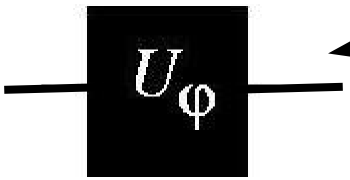

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quantum strategies: $\Delta\varphi \sim \frac{1}{\sqrt{\nu}N}$ ← Heisenberg bound

classical strategies: parallel $\Delta\varphi \sim \frac{1}{\sqrt{\nu N}}$ ← shot noise

What is quantum metrology?

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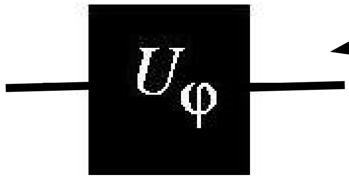

black box that implements a transformation $U_\varphi = e^{i\varphi H}$

GOAL: use it ν*N* times and get the best estimate of φ

$\sqrt{N}$ **gain of quantum metrology**

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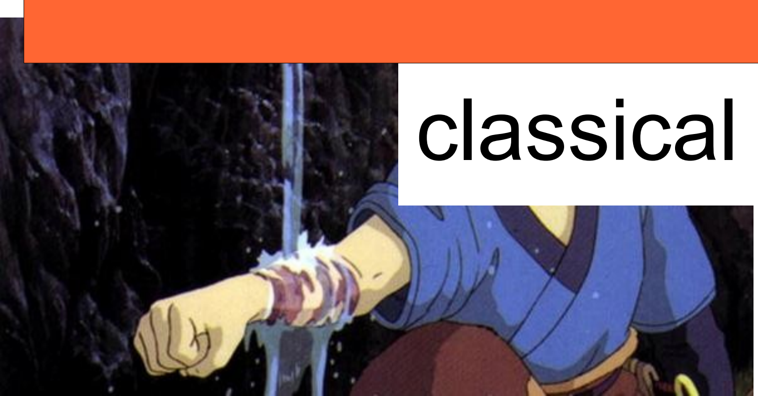
parallel $\Delta\varphi \sim \frac{1}{\sqrt{\nu N}}$ ← shot noise

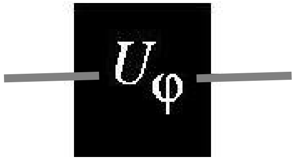
classical strategies:



Parallel strategies

PRL 96,010401

Parallel strategies

use 

in parallel:

PRL 96,010401

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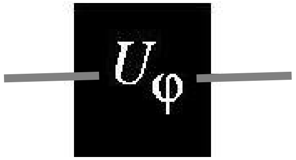
use  in parallel:

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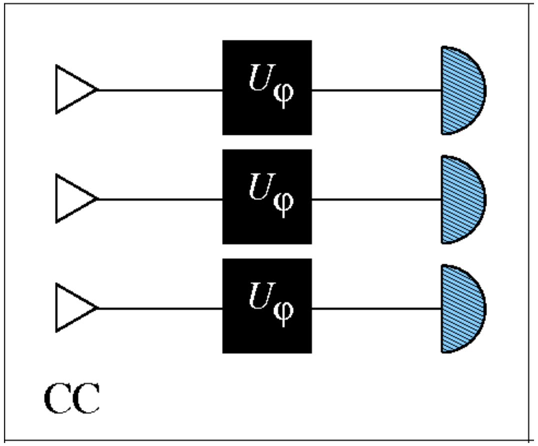

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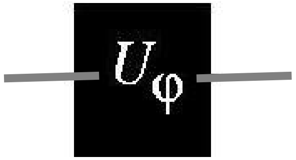
use 

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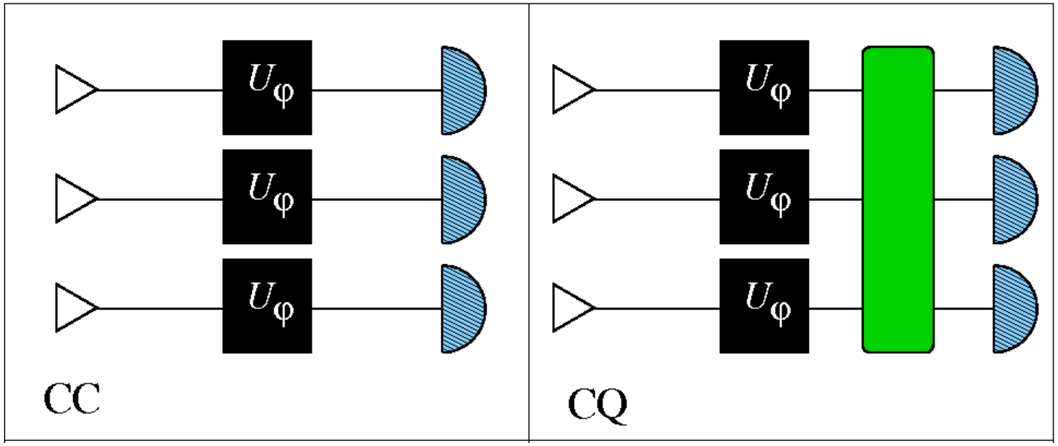

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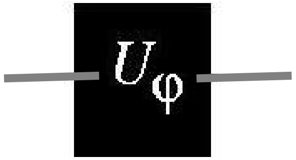
use  in parallel:

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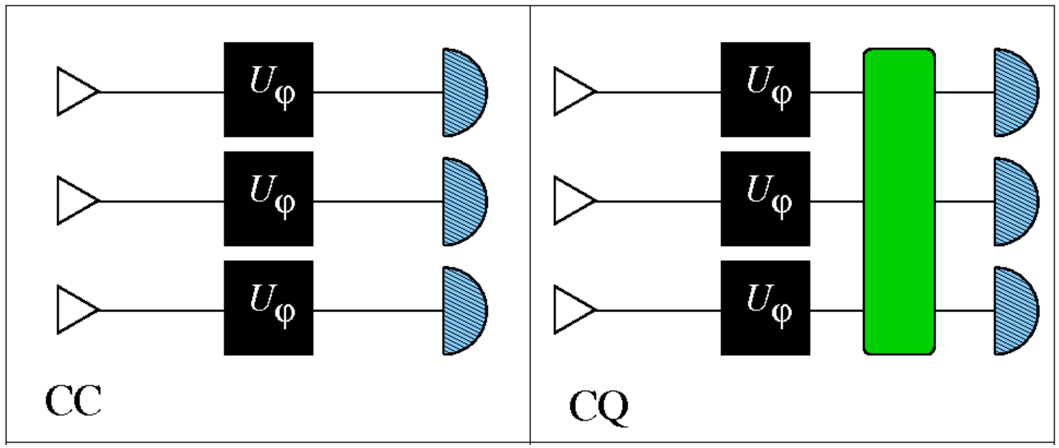

$$\Delta\varphi \propto \frac{1}{\sqrt{N}}$$

(shot noise)

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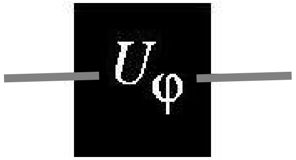
use  in parallel:

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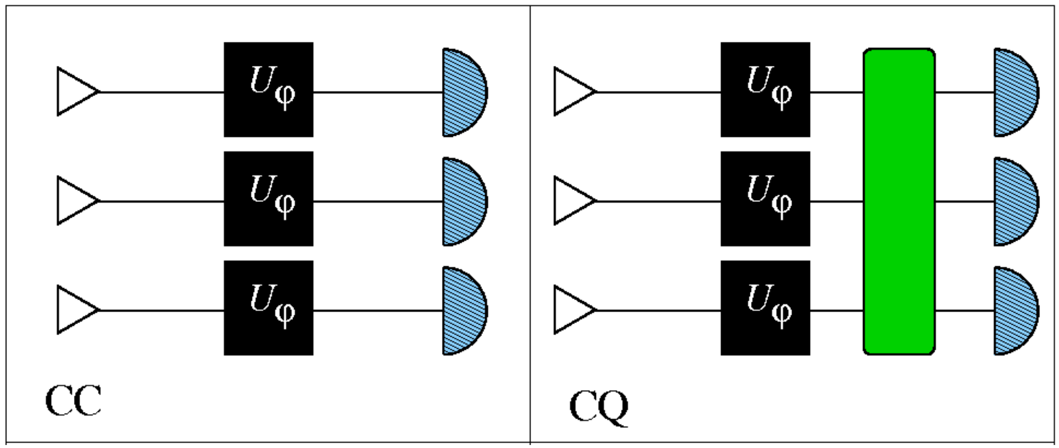

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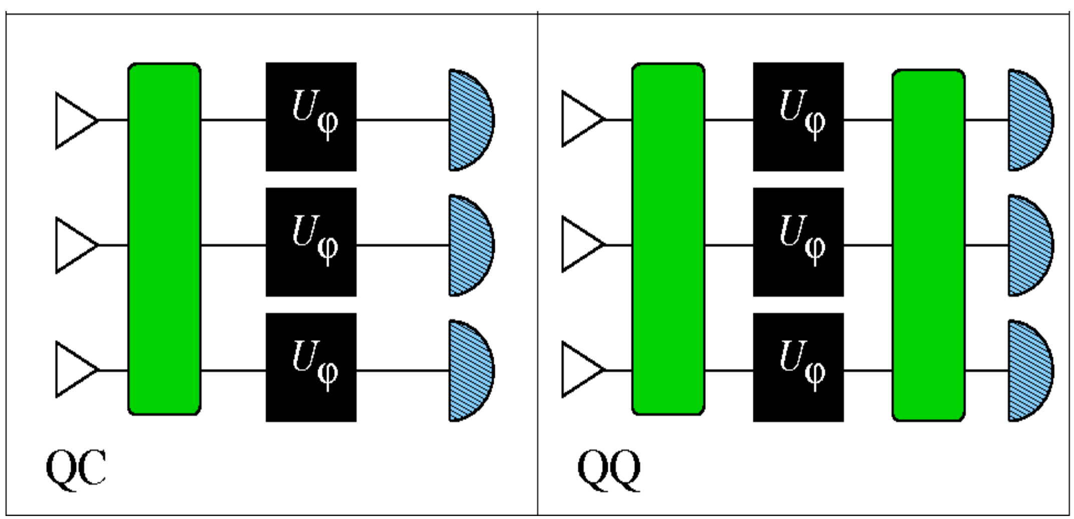

the *N* transformations act on an **entangled state**

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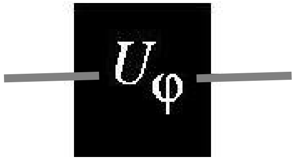
Parallel strategies

use 

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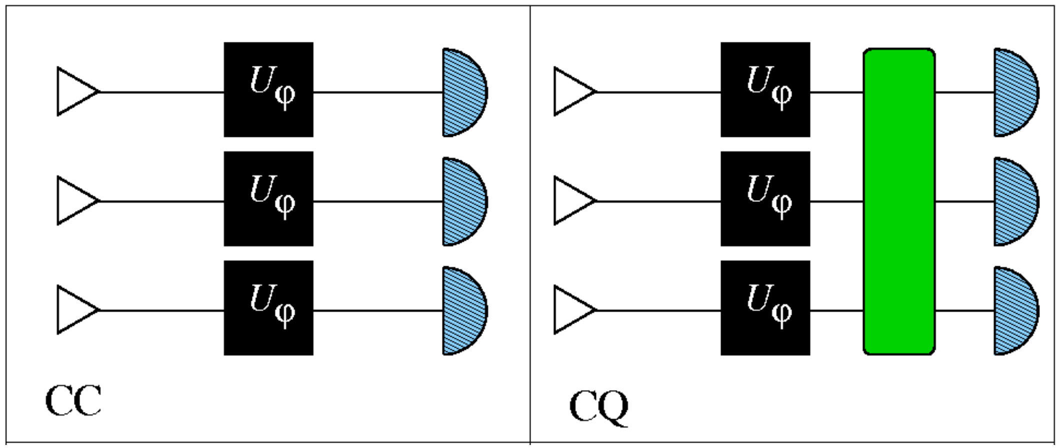

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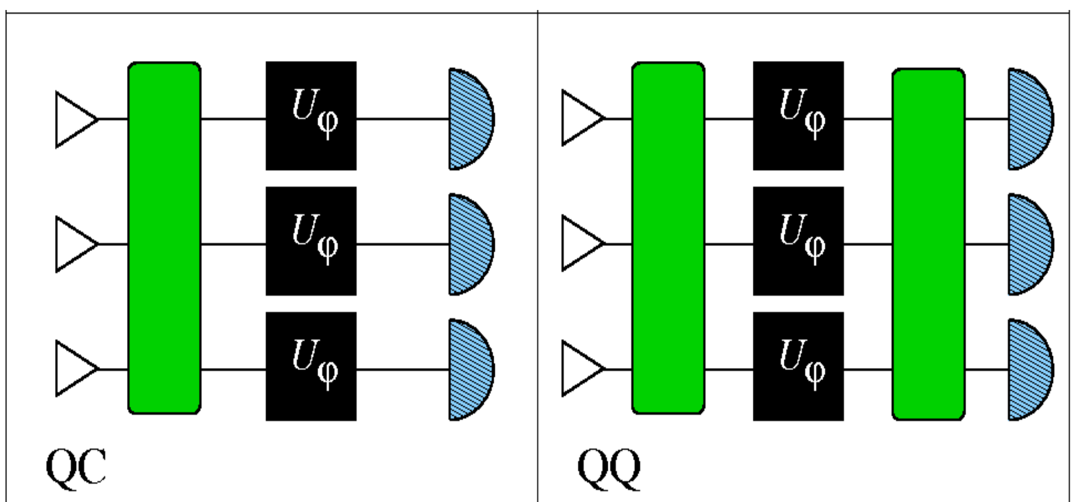

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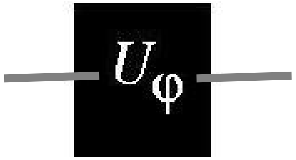
use  in parallel:

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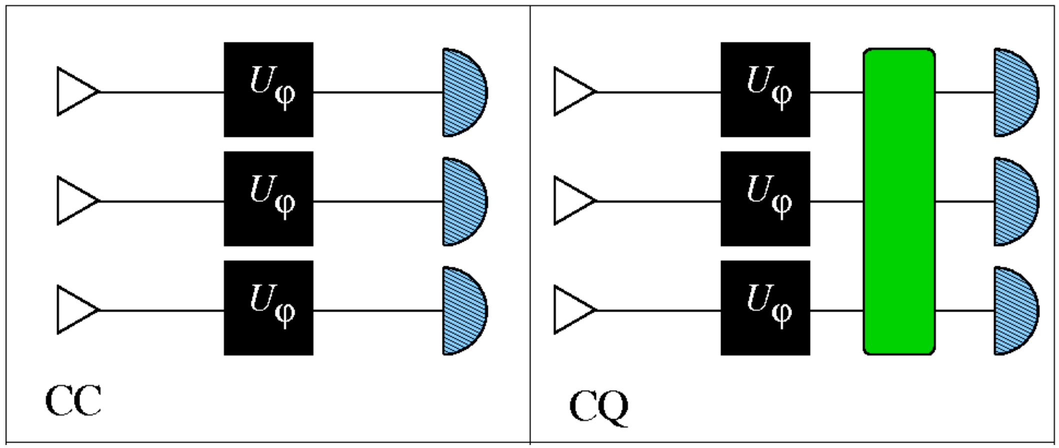

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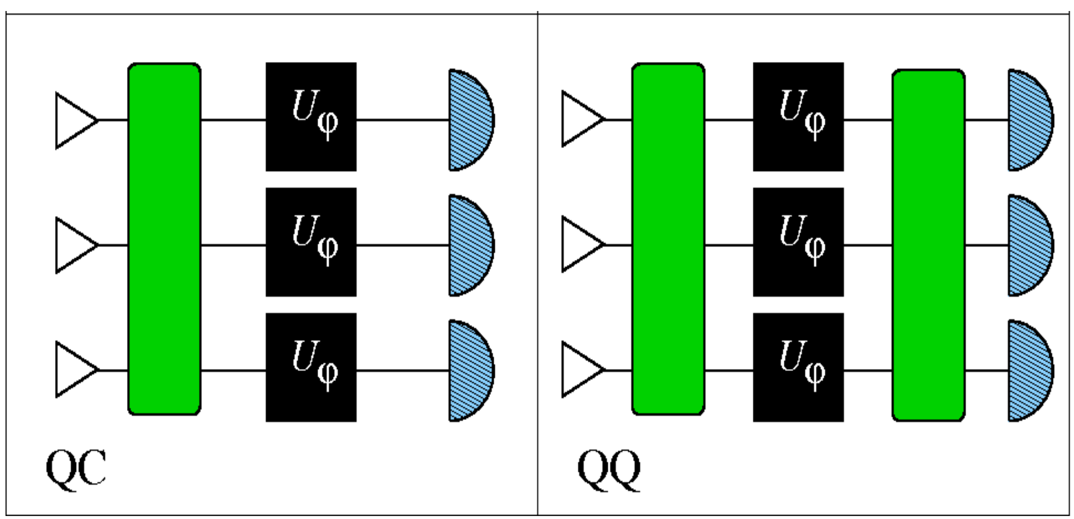

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Parallel strategies

Note: entanglement at the measurement stage is useless!

u

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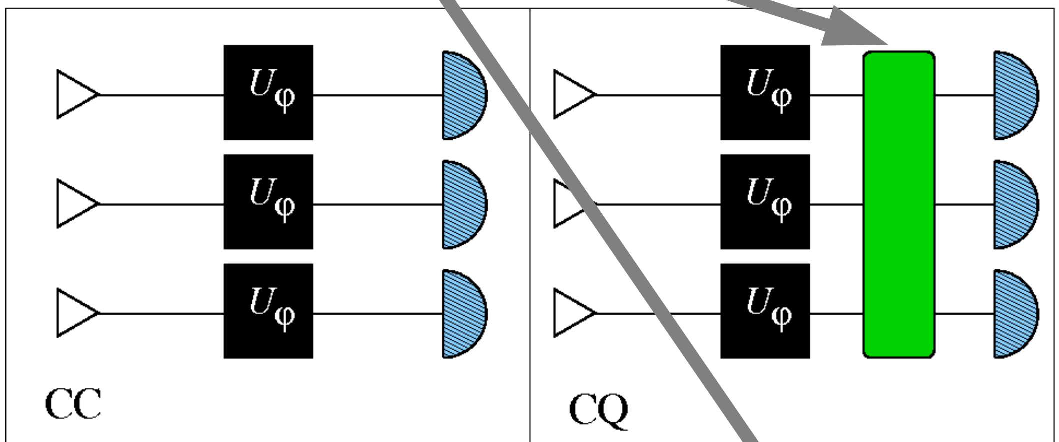

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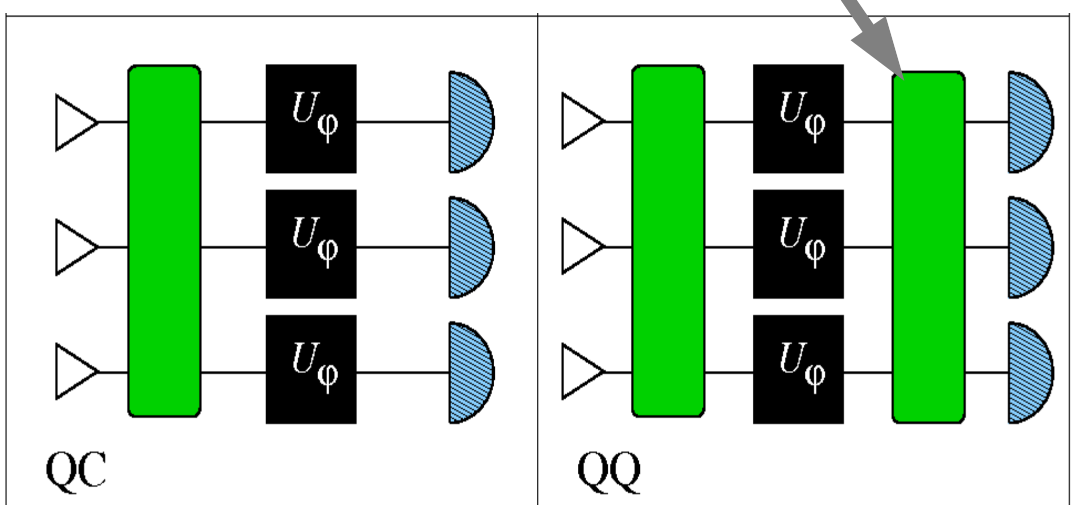

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the *N* transformations act on an **entangled state**

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Sequential (multiround) strategies

PRL 96,010401

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use —U_φ— in series and start from $|+\rangle$ or $|-\rangle$ states:

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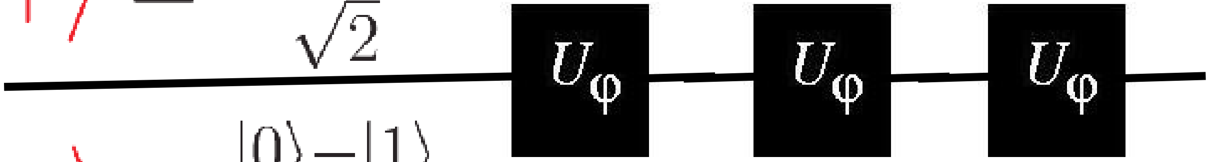
Sequential (multiround) strategies

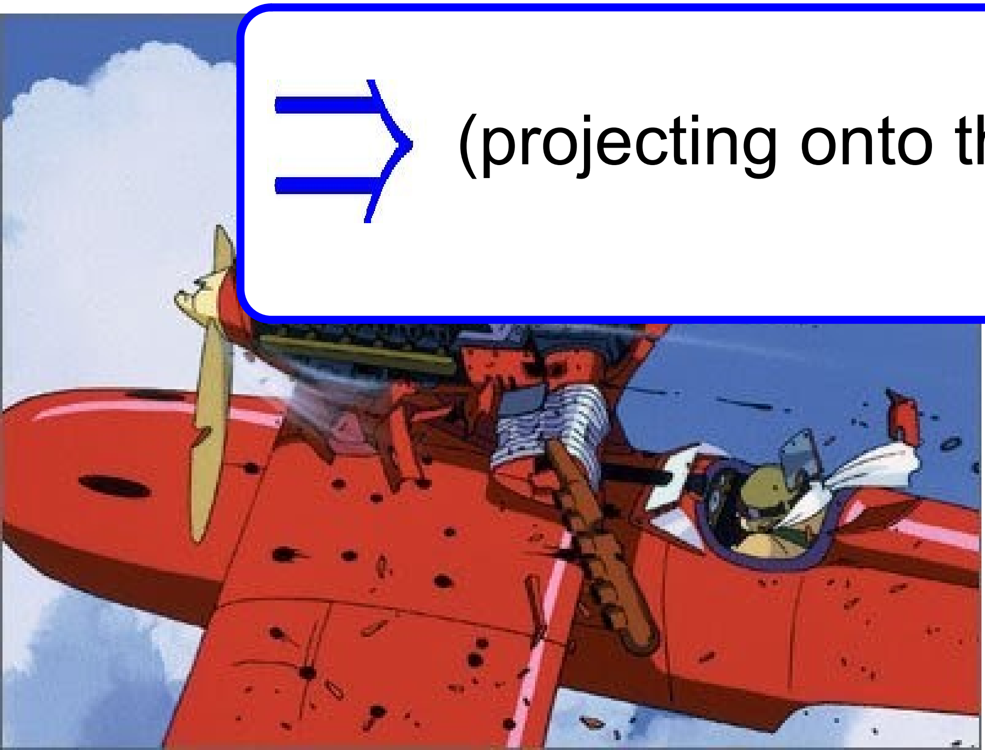
use $-\boxed{U_\varphi}-$ in series and start from $|+\rangle$ or $|-\rangle$ states:

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$$\frac{|0\rangle \pm e^{iN\varphi}|1\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$



$\Rightarrow$ (projecting onto the initial state) $\Delta\varphi \propto \dfrac{1}{N}$

"Heisenberg"-like scaling

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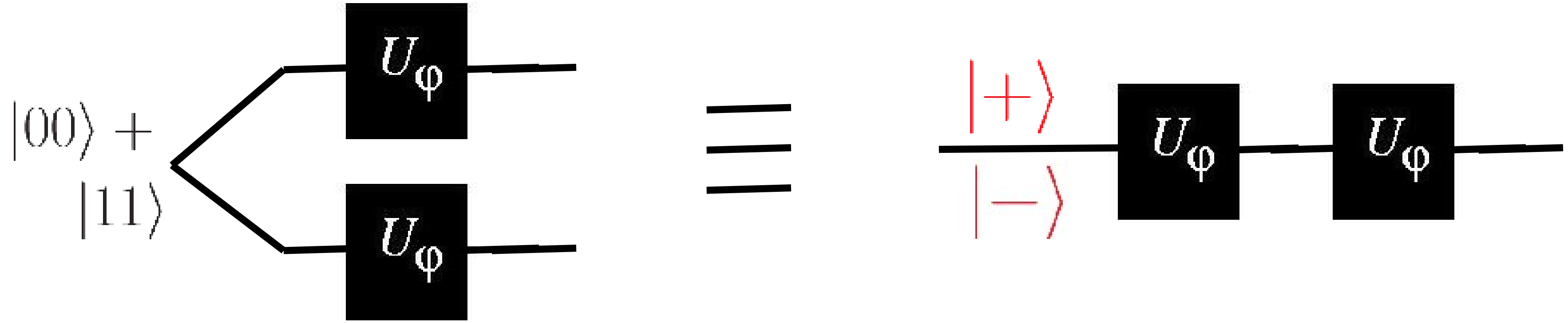
So... Why entanglement?

BECAUSE



i.e. entanglement turns a **parallel** strategy **into** a **sequential** one.

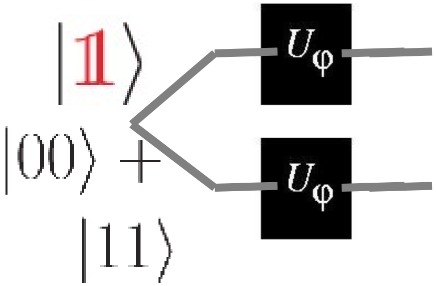
Simple proof

define a state $|C\rangle = \sum_{ij} c_{ij}|ij\rangle$ for any operator $C = \sum_{ij} c_{ij}|i\rangle\langle j|$

$$\Rightarrow \quad (A \otimes B)|C\rangle = |ACB^T\rangle$$

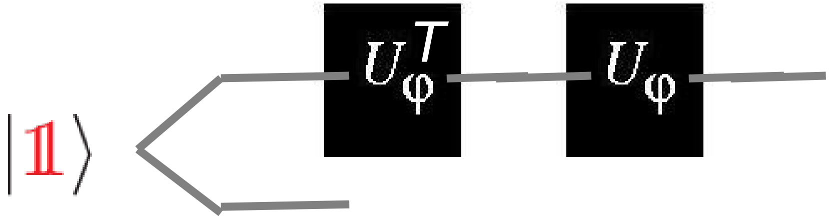
[Phys Lett A 272,32]

$|00\rangle + |11\rangle = |\mathbb{1}\rangle$ so

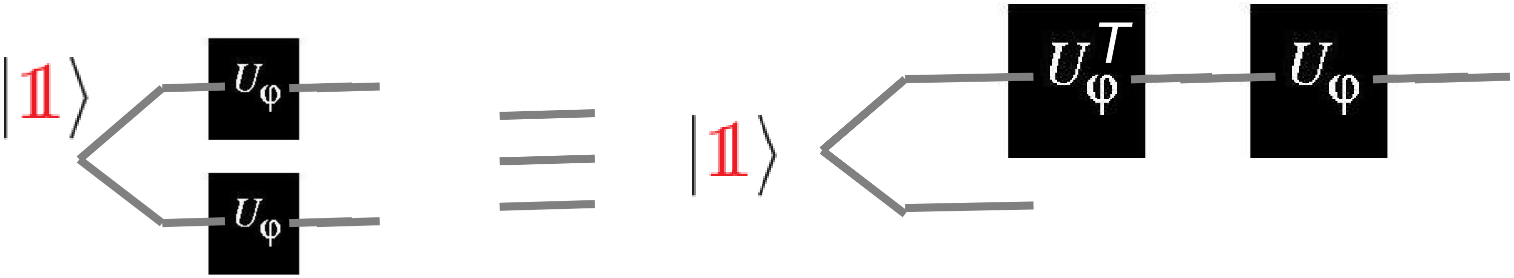


$$= (U_\varphi \otimes U_\varphi)|\mathbb{1}\rangle \doteq |U_\varphi \mathbb{1} U_\varphi^T\rangle =$$

$$= |U_\varphi U_\varphi^T \mathbb{1}\rangle \doteq (U_\varphi U_\varphi^T \otimes \mathbb{1})|\mathbb{1}\rangle =$$



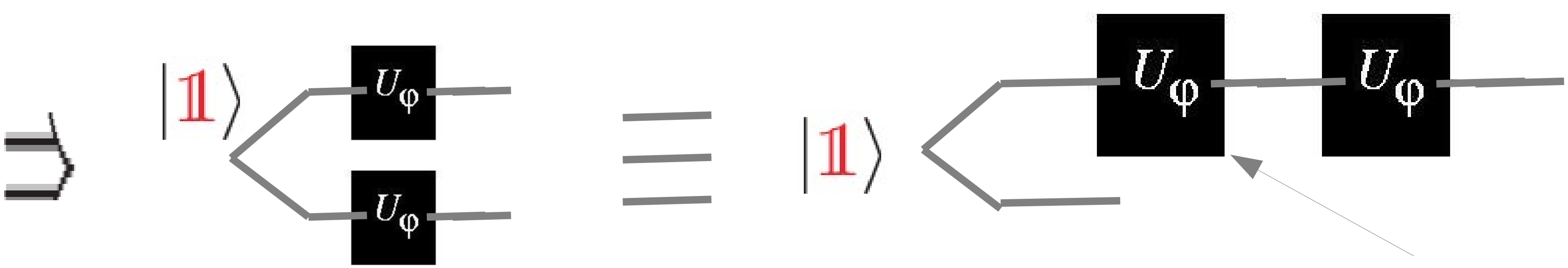
Namely,



$$|00\rangle + |11\rangle = |\mathbb{1}\rangle$$

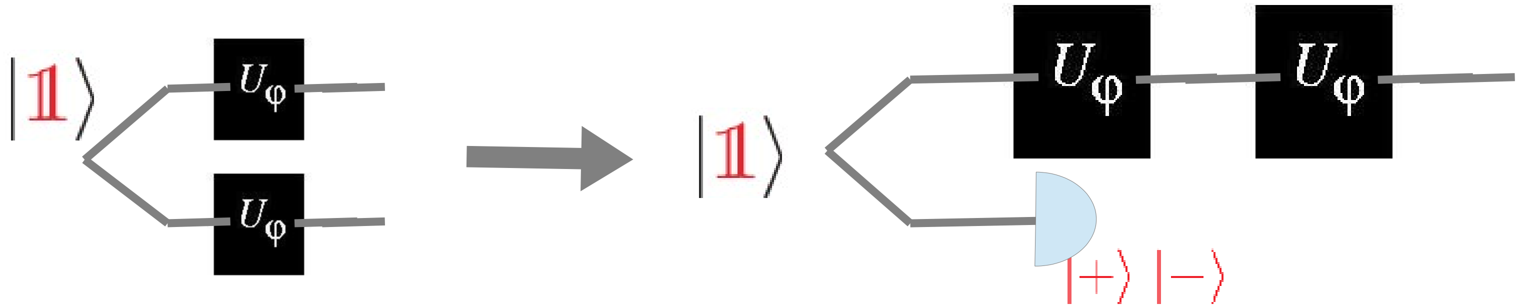
Now choose the $|0\rangle, \ |1\rangle$ basis as the U_φ basis:

$$U_\varphi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} = U_\varphi^T$$



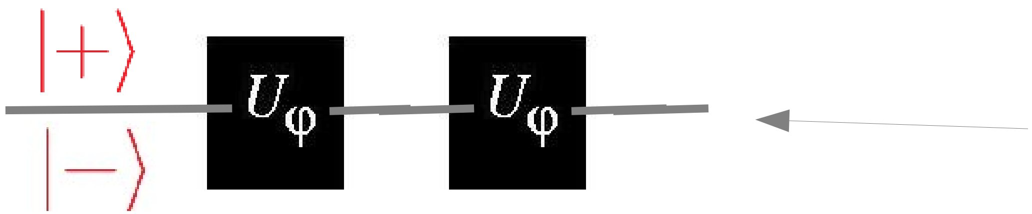
dropped the *T*

Finally, measure the 2° qubit in the $|+\rangle$ $|-\rangle$ basis:



The other qubit is collapsed on the **same** state

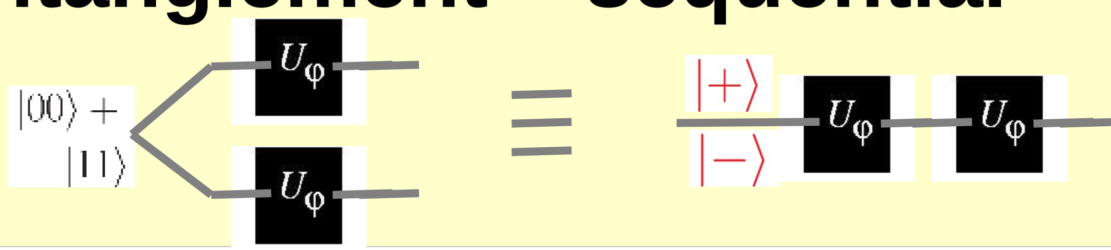
(Klyshko mechanism)



sequential strategy

We have shown that

parallel+entanglement = sequential



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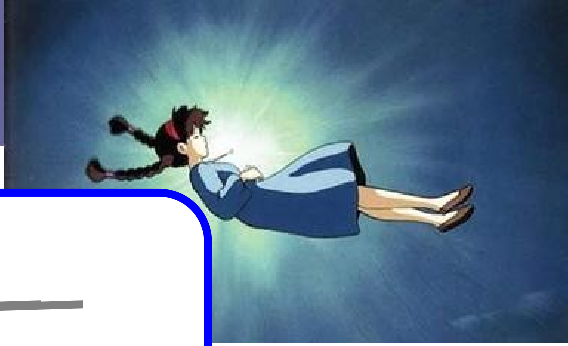
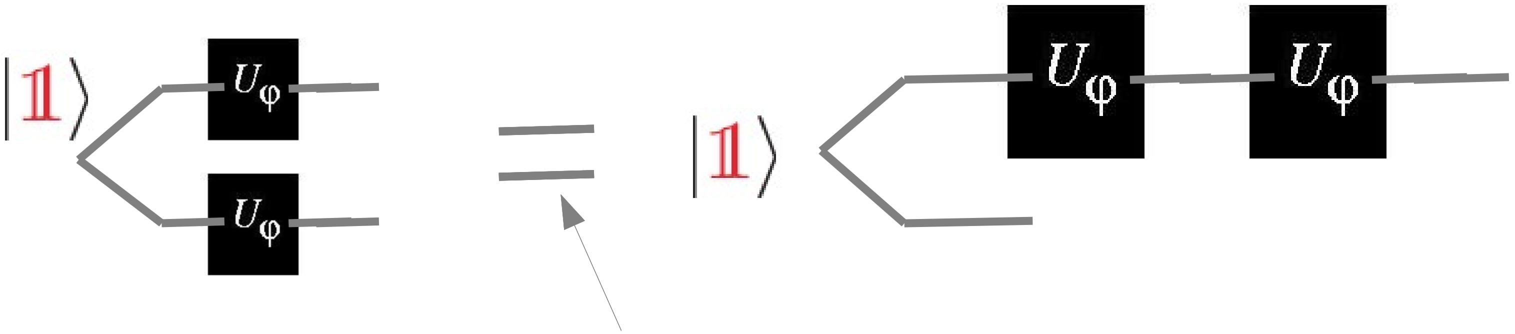
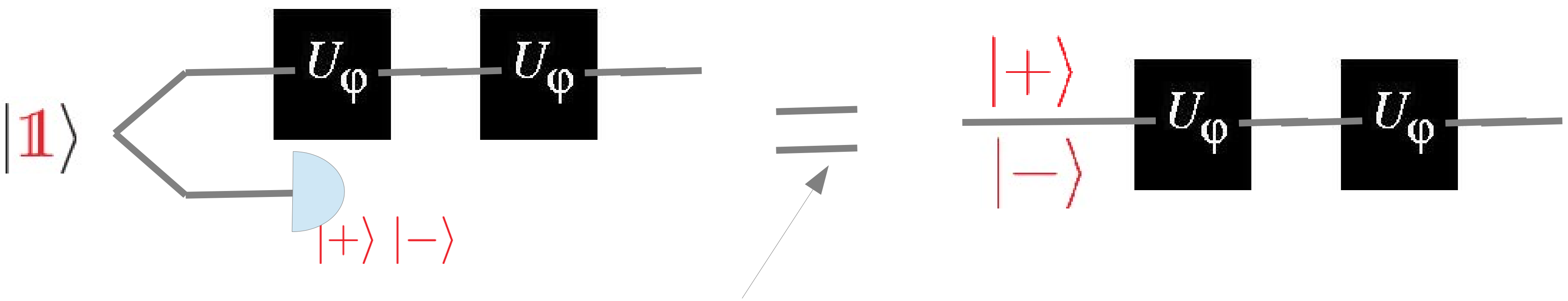




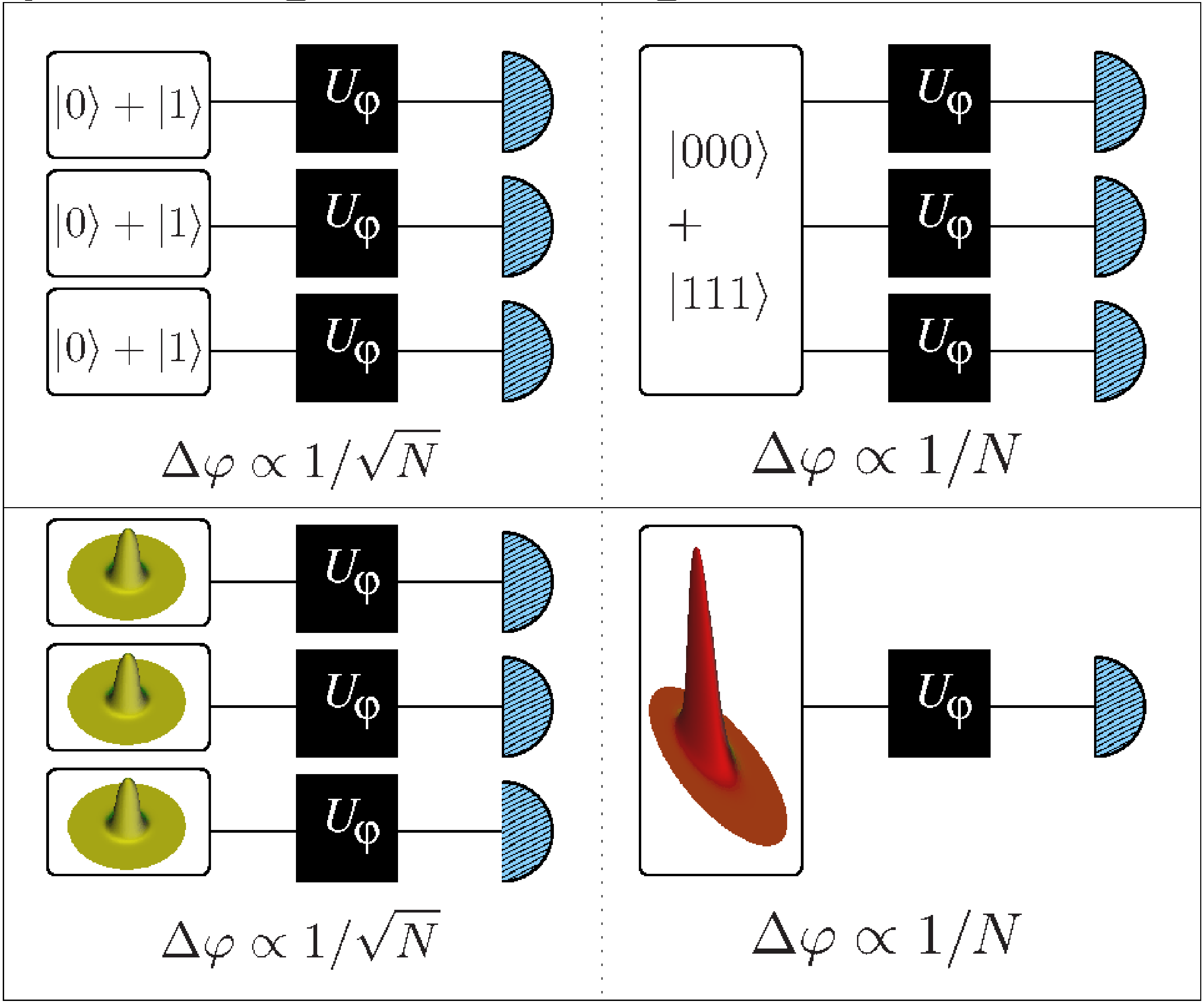
- requires correlation in the $|0\rangle, \ |1\rangle$ basis



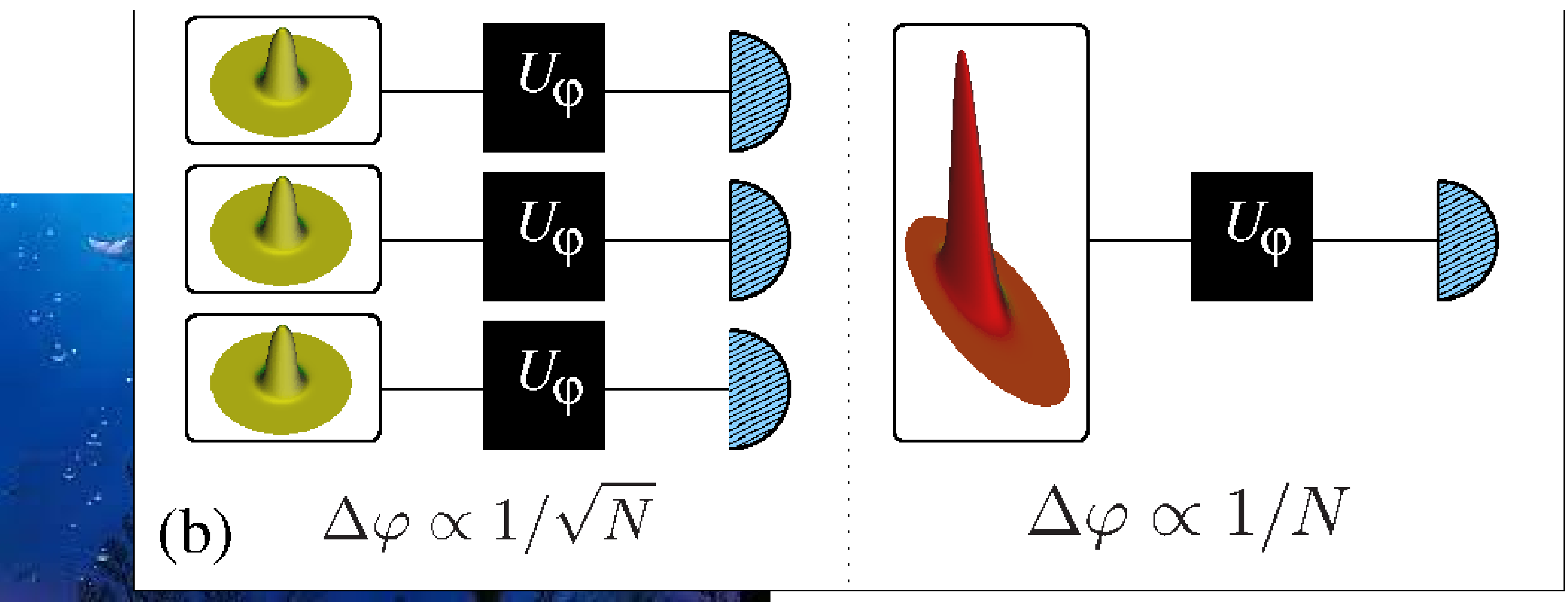
- requires correlation in the $|+\rangle \ |-\rangle$ basis

$\Rightarrow$ (they are **complementary** basis) we **need** entanglement!!

Squeezing vs entanglement



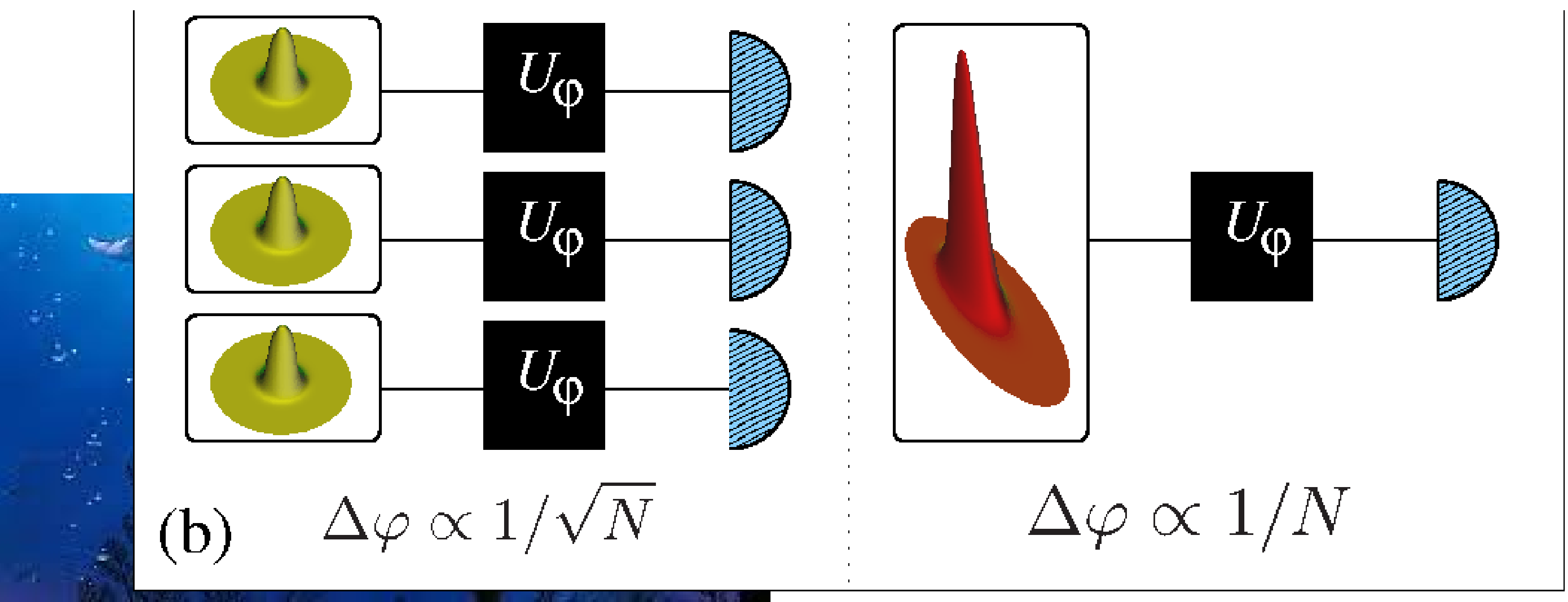
Squeezing



U_φ
U_φ
U_φ
U_φ
(b)
$\Delta\varphi \propto 1/\sqrt{N}$
$\Delta\varphi \propto 1/N$

Squeezing

- Take the energy used by N coherent (classical) probes



Squeezing

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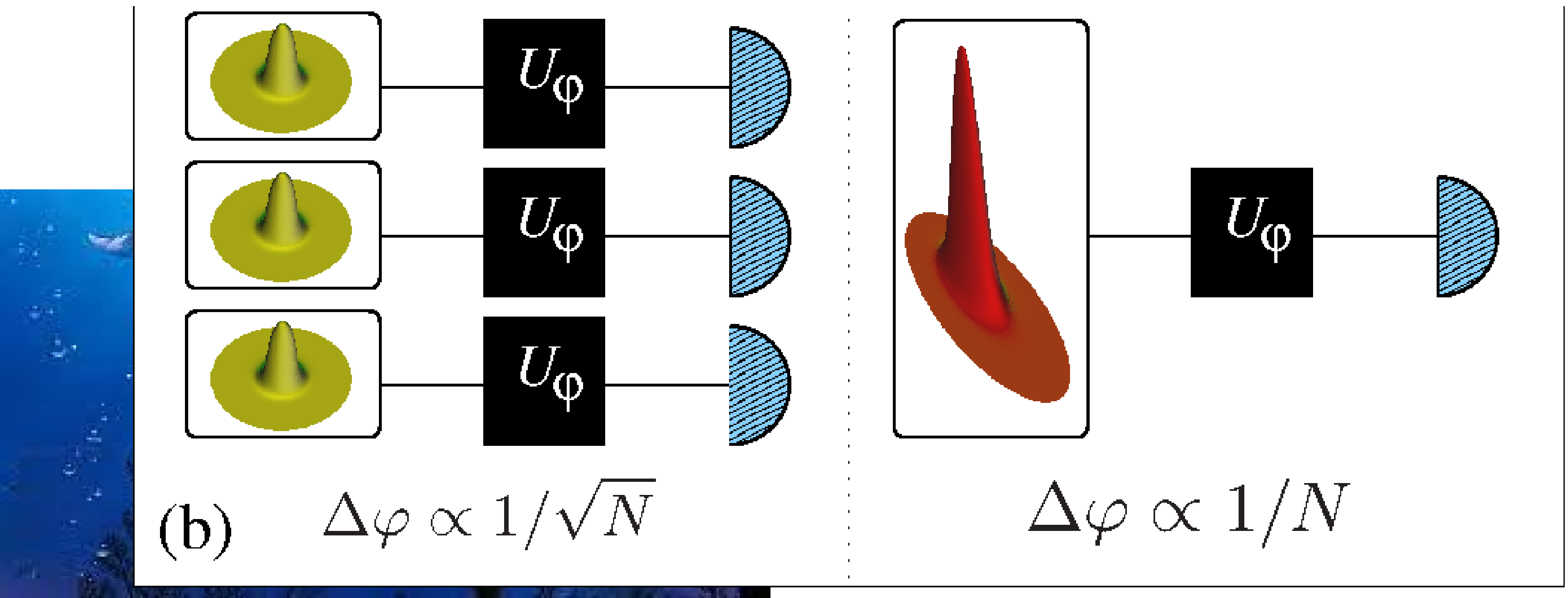
- Use it to squeeze *one* probe



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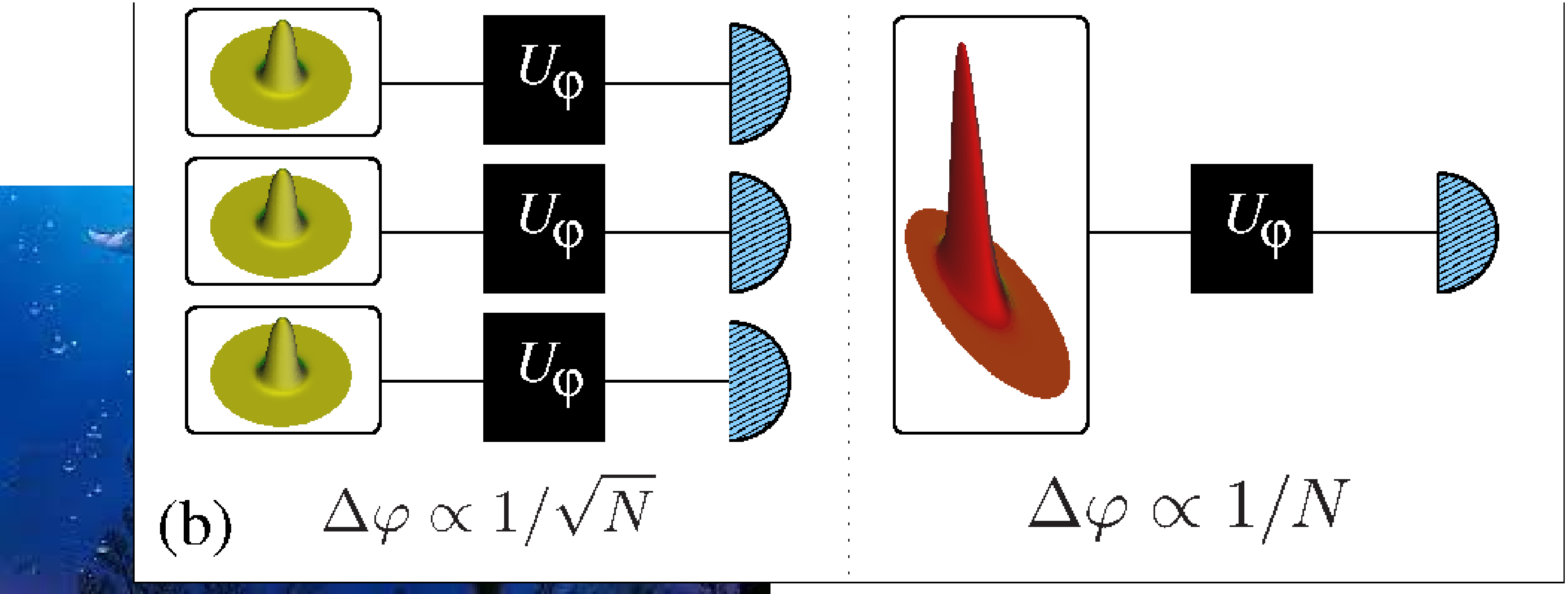
- A quadratic enhancement!!

$$\frac{1}{\sqrt{N}} \rightarrow \frac{1}{N}$$



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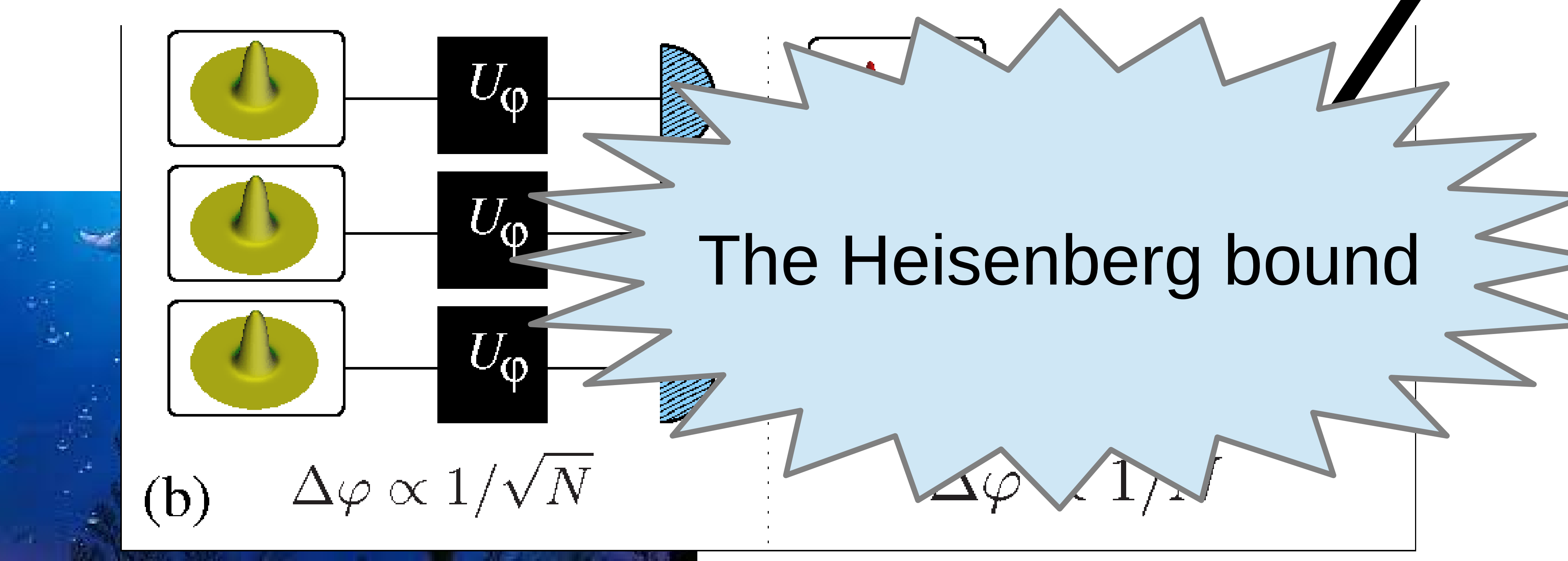

Noiseless case is simple and that's where the $\sqrt{N}$ quantum advantage is

The noisy case is a **mess**, but the noiseless case is an upper bound.

An application to HEP:

spreading channels,
useful for axion searches.

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Is the axion field present?

what is its intensity?

spreading channels!

$$U_{\alpha,\varphi} = U_\varphi^\dagger e^{i\alpha G} U_\varphi$$

with random rotation φ

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Want to estimate α, don't care about φ
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with random rotation φ

Want to estimate α, don't care about φ
(it's just noise)

NOTE: the channel rotates, not the state

Example: displacement (nonzero α
if the axion and em field interact)

$$D(\alpha,\varphi) = e^{\alpha(e^{i\varphi}a^\dagger - e^{-i\varphi}a)}$$

It was known that an optimal state is an excited Fock state (impractical). No optimal measurement was known

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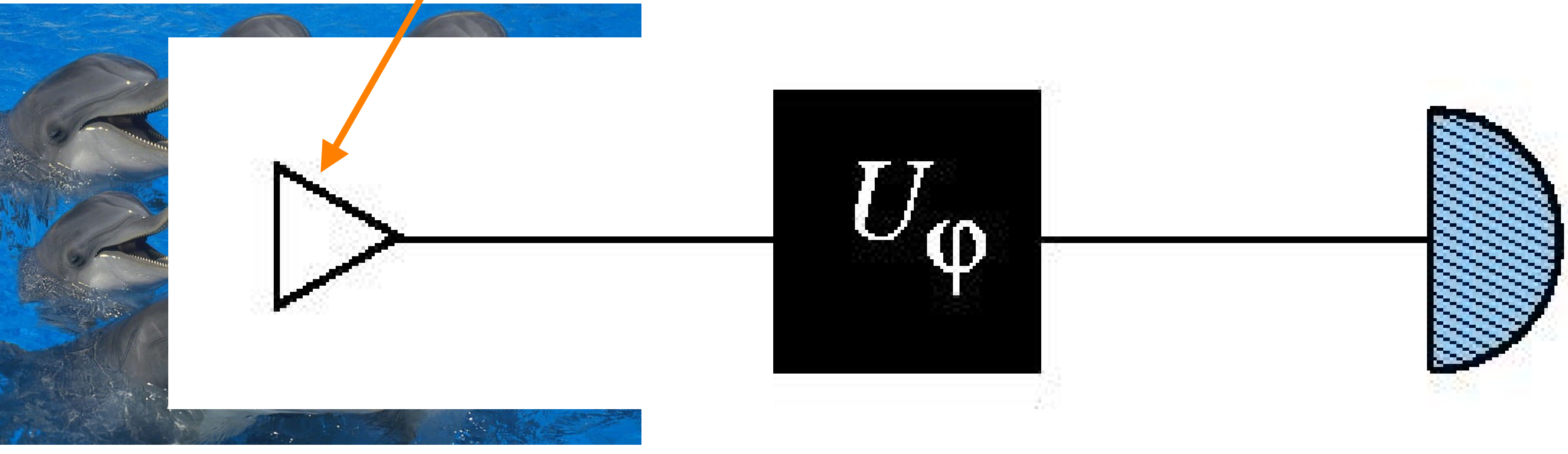
Our results:

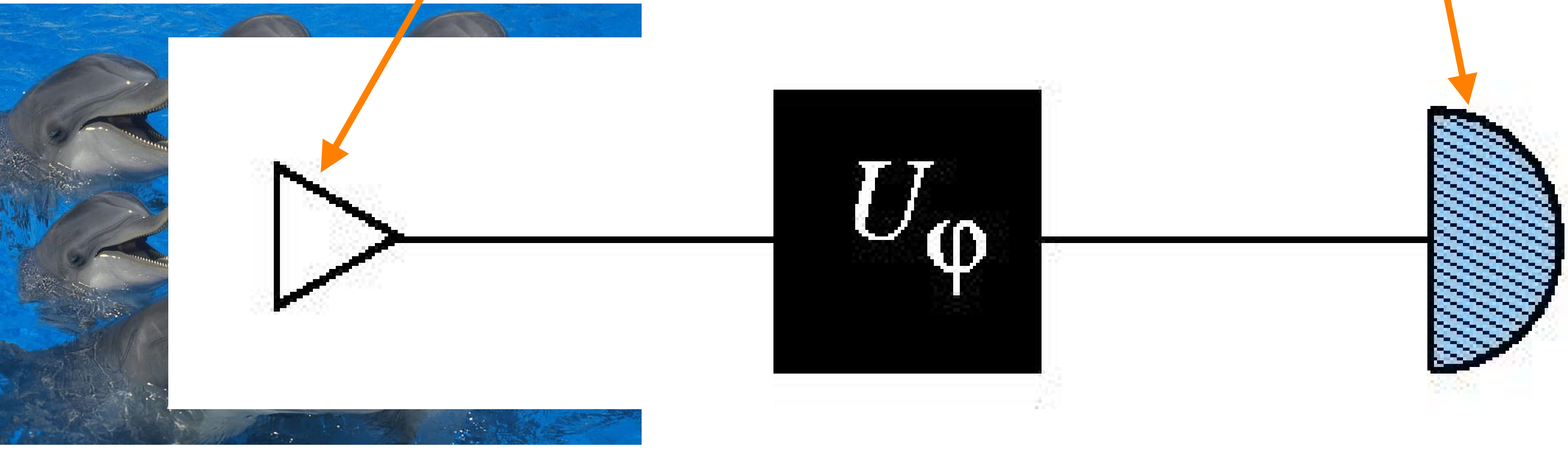
1) A squeezed state is an optimal input



It was known that an optimal state is an excited Fock state (impractical). No optimal measurement was known

Our results:

1) A squeezed state is an optimal input
2) Squeezing+photodetection is an optimal measurement



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Our results:

1) A squeezed state is an optimal input

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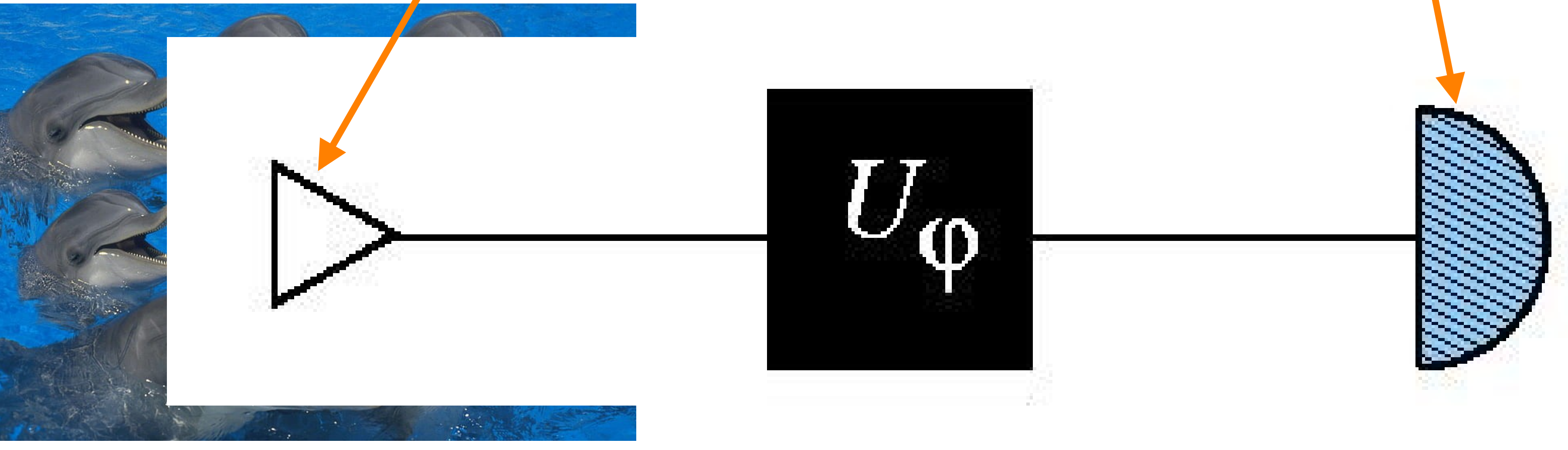
3) we retain the $\sqrt{N}$ advantage over the classical strategies! Just as the noiseless case!

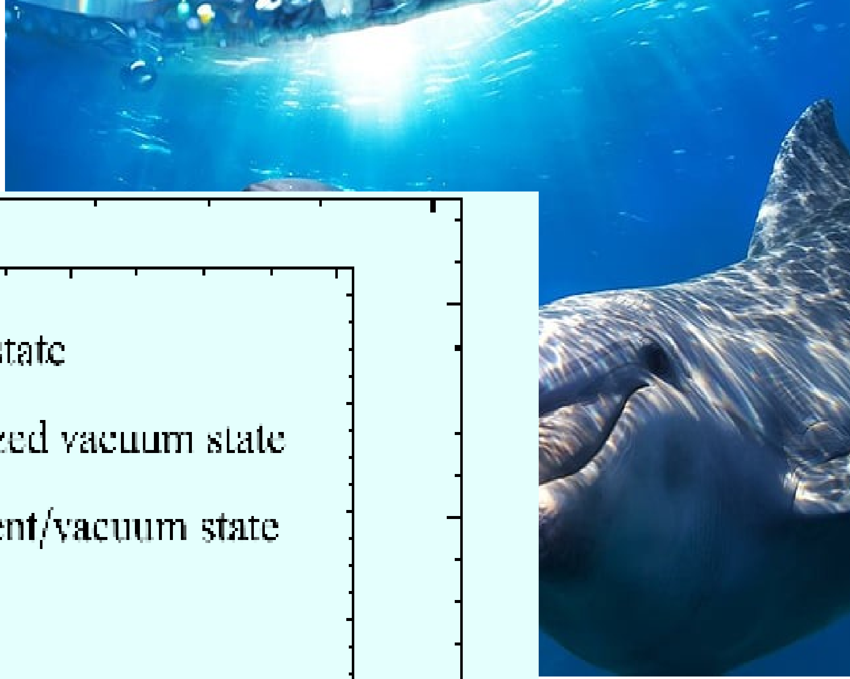


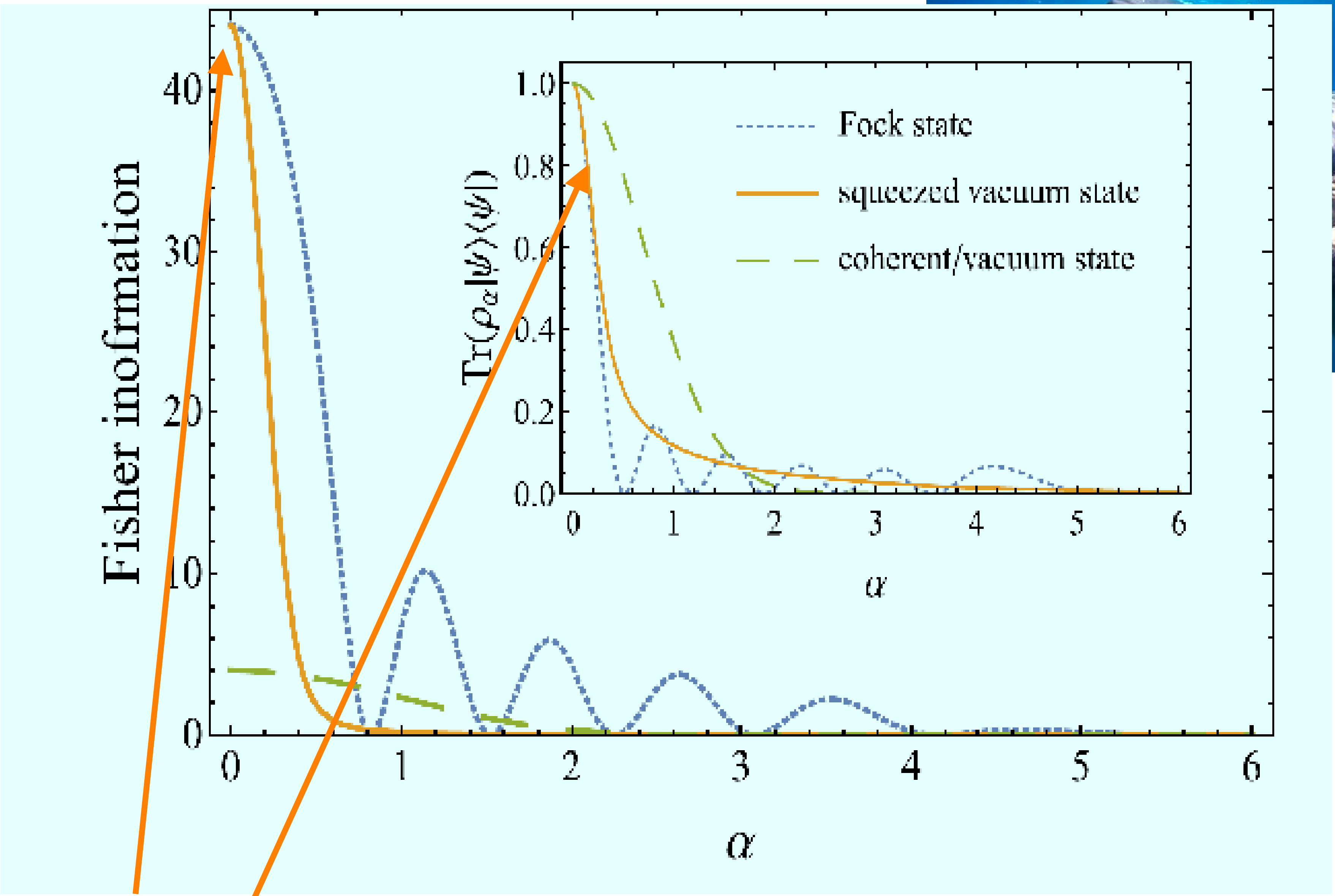
Homodyne detection does not work!

A surprising connection to Rayleigh's curse in imaging, which is a particular case of the class of channels we consider

Results

Results

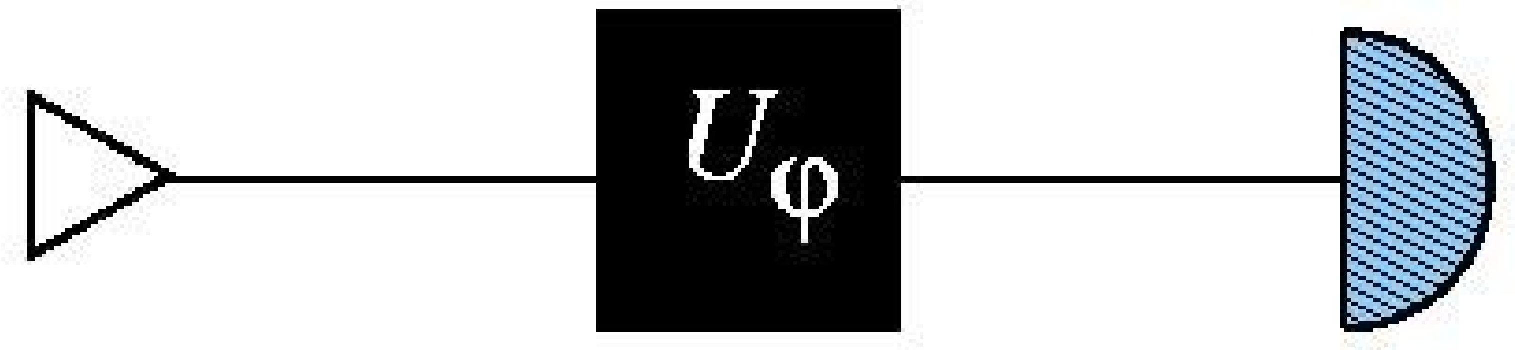




Large Fisher info=good estimation

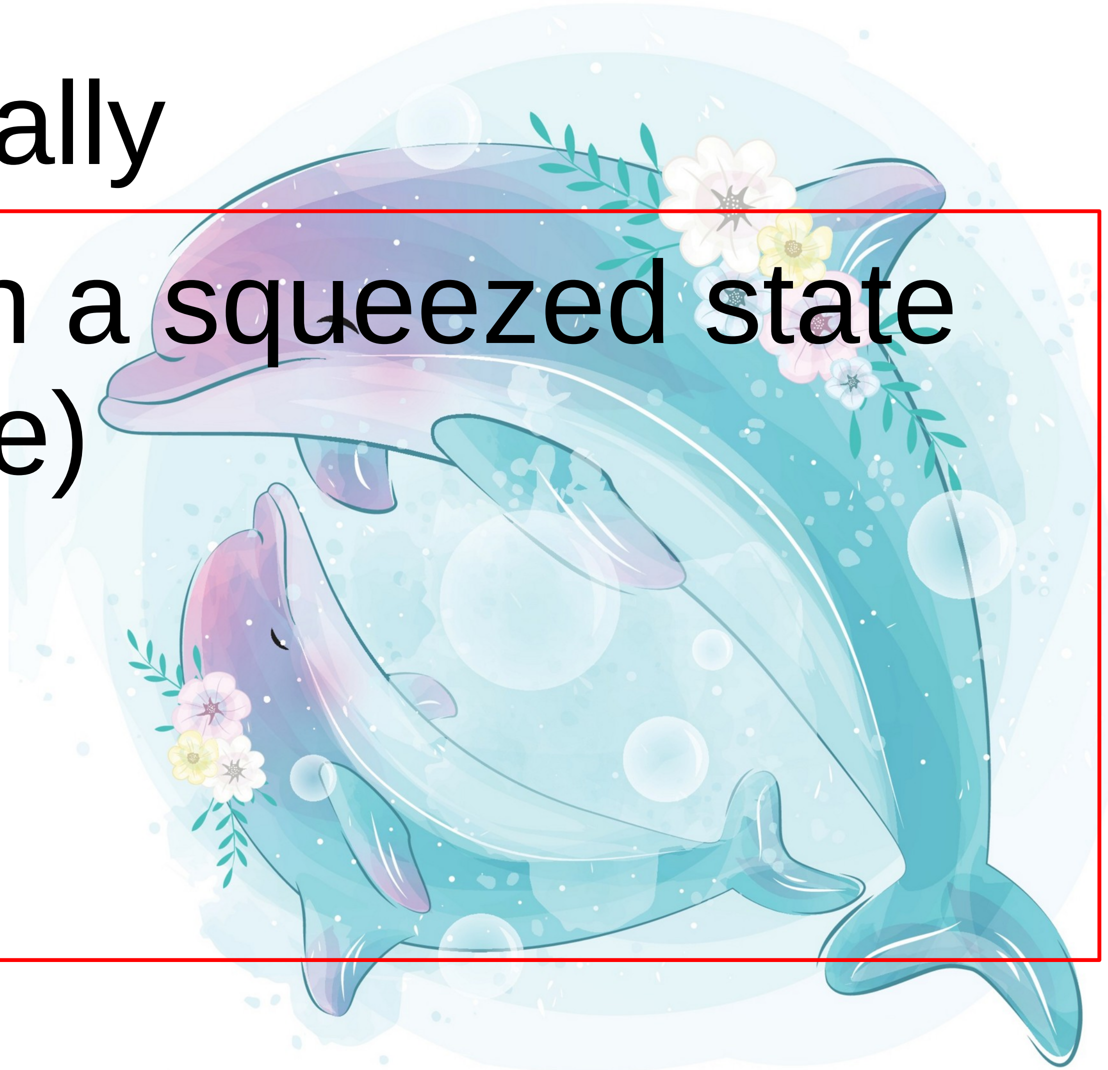
Small fidelity between initial and final state=good discrimination

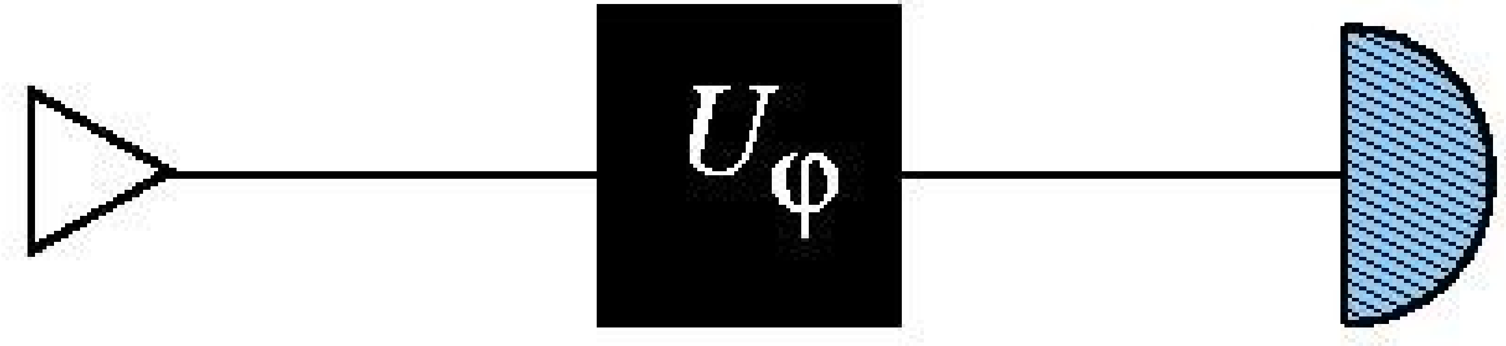
Experiment ideally



Experiment ideally

1) Prepare cavity in a squeezed state (with random phase)





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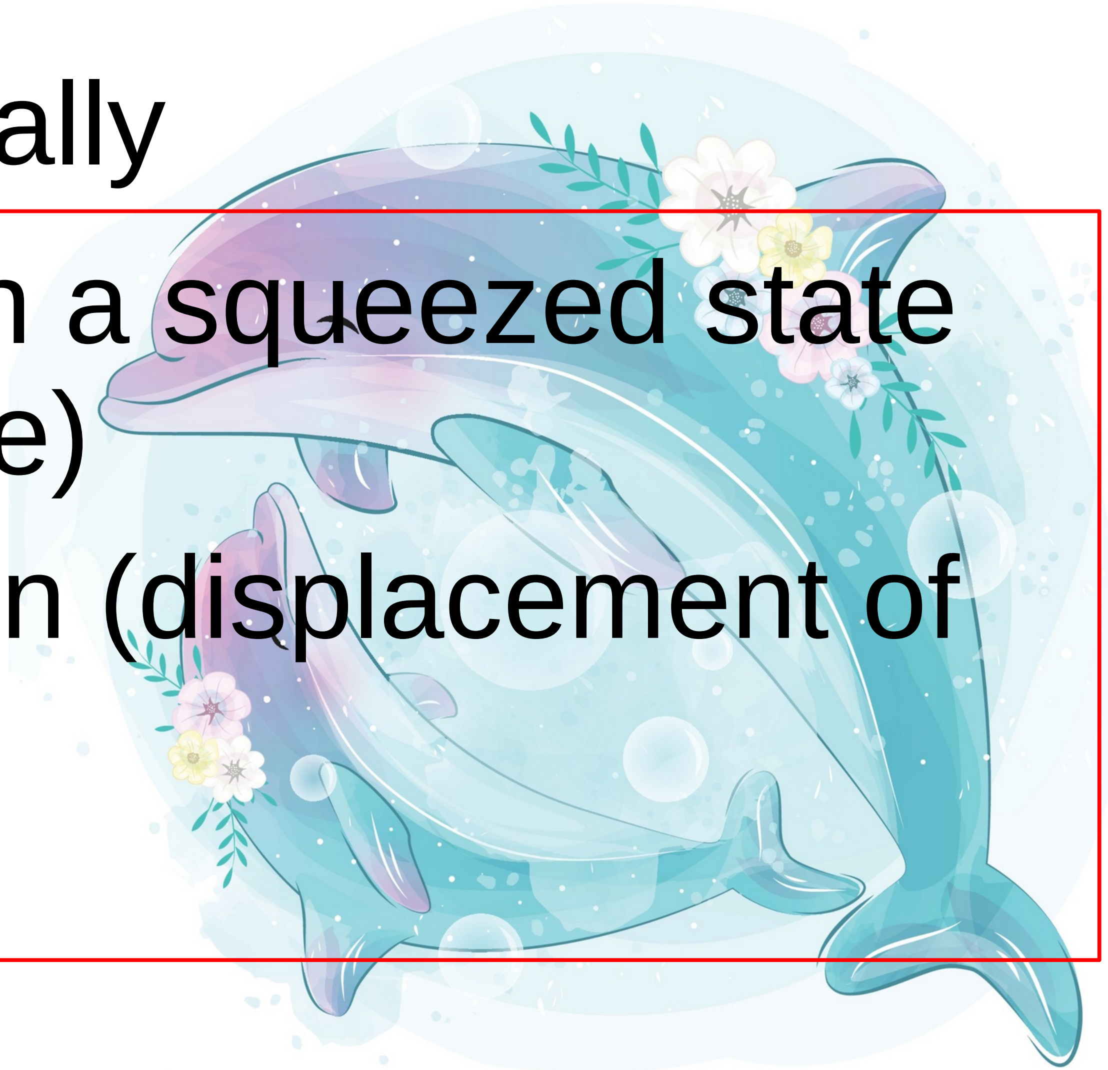
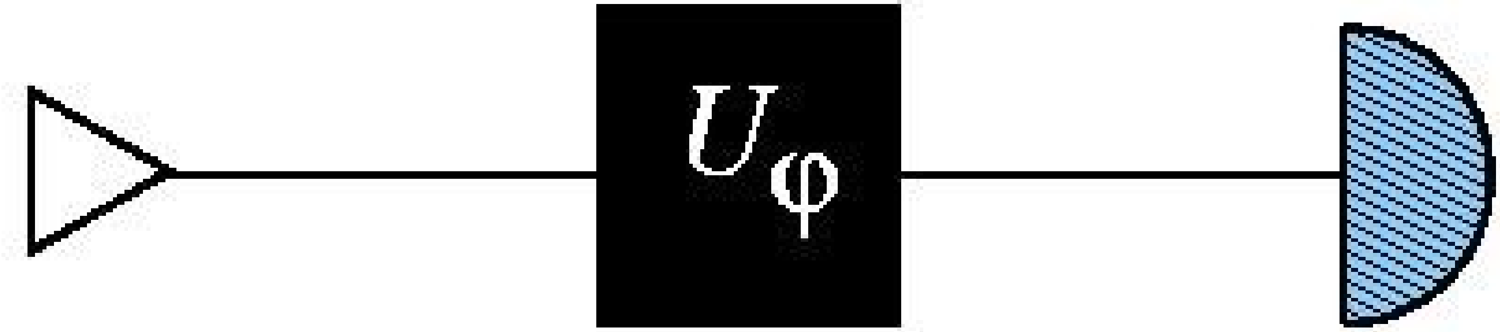


1) Prepare cavity in a squeezed state (with random phase)

2) Wait for the axion (displacement of the state)



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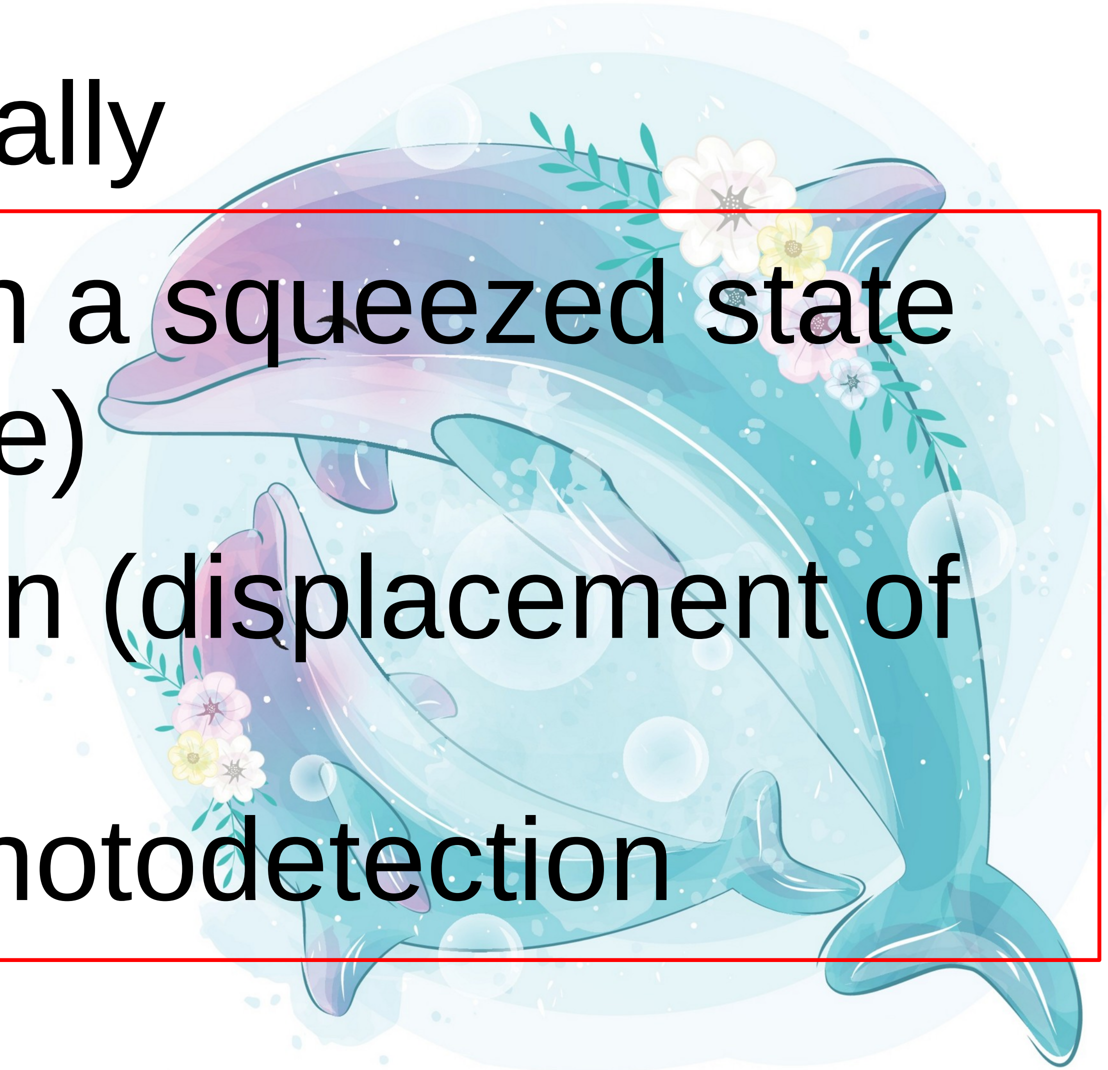
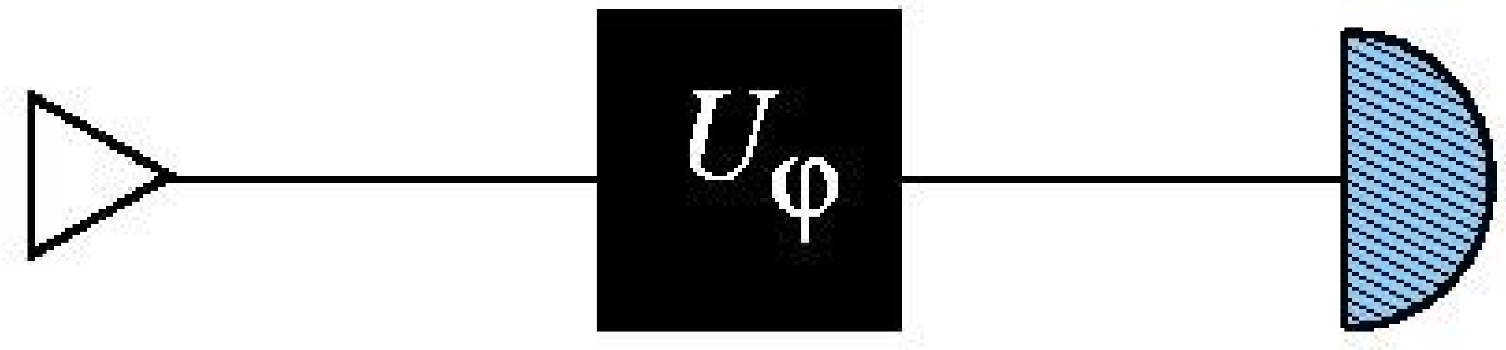


1) Prepare cavity in a squeezed state (with random phase)

2) Wait for the axion (displacement of the state)

3) Anti-squeeze+photodetection



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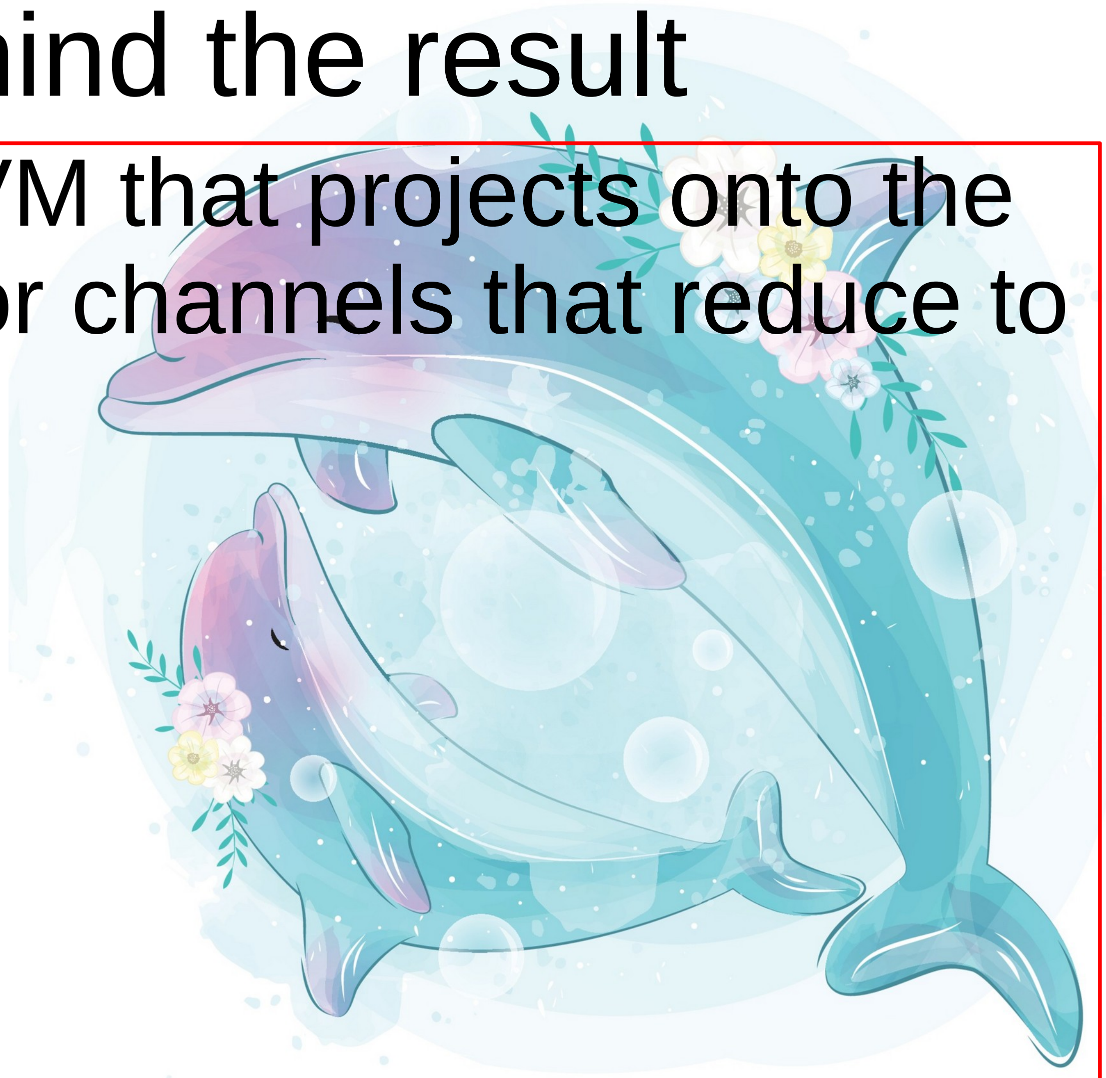
1) Conjecture: the POVM that projects onto the initial state is optimal for channels that reduce to the identity for $\alpha \to 0$



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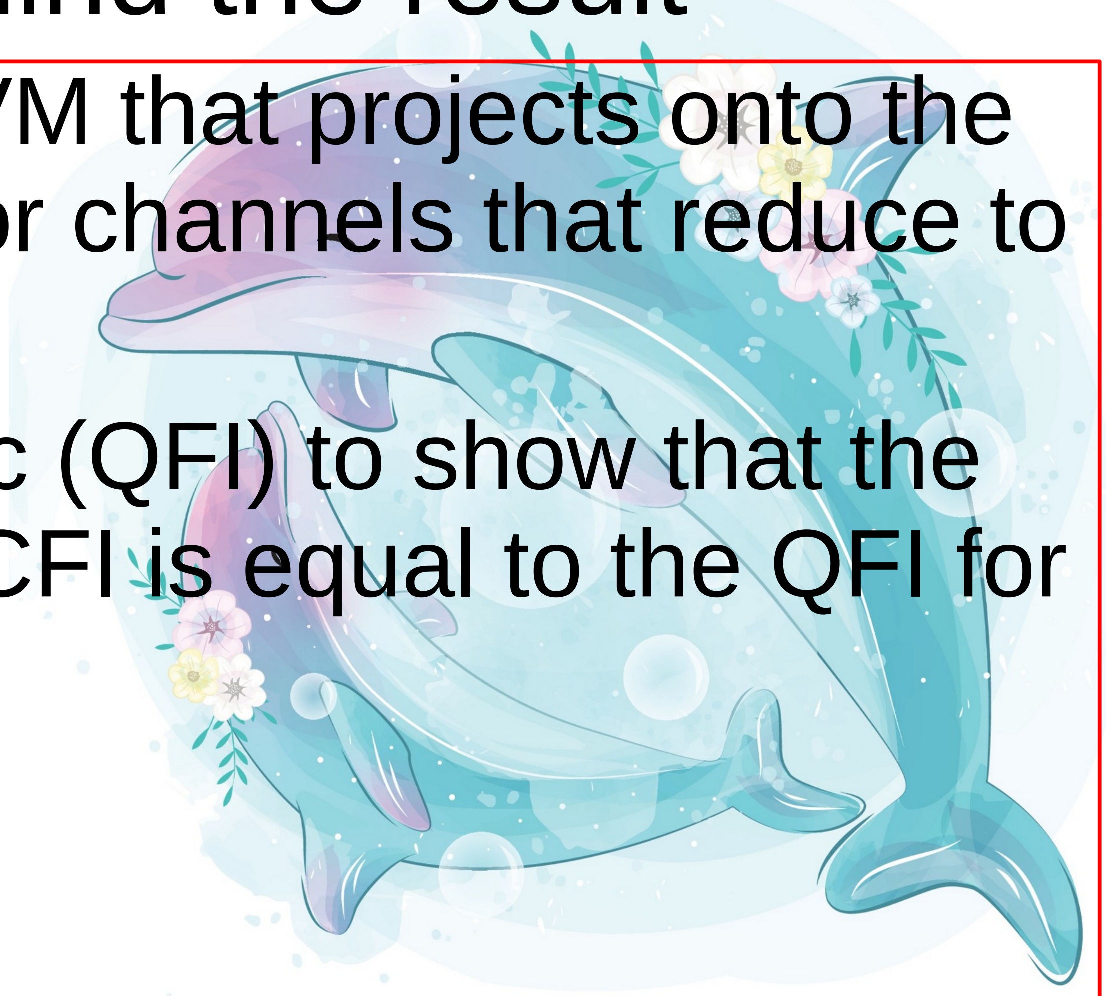
1) Conjecture: the POVM that projects onto the initial state is optimal for channels that reduce to the identity for $\alpha \rightarrow 0$

2) Use the Bures metric (QFI) to show that the conjecture is true: the CFI is equal to the QFI for small α



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3) use convexity of the QFI to show that, in the limit $\alpha \to 0$, the noise doesn't matter: the QFI is equal to the QFI averaged by the noise.

4) for optical displacements show that the QFI averaged over noise is bounded by the average energy of the state: both Fock and sq. vacuum saturate the bound

Open problems! (next steps)

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Other noises (loss?) might change the results...

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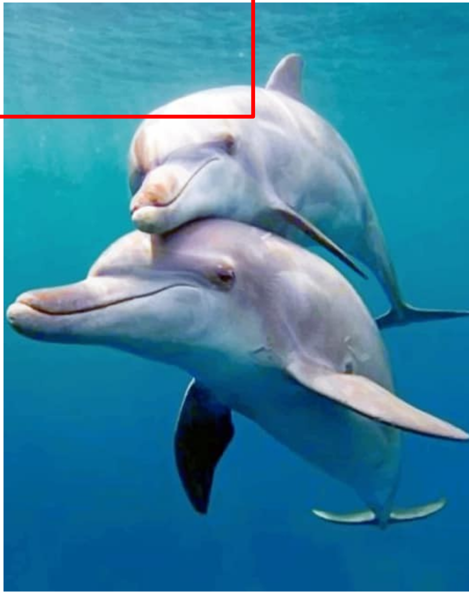

Open problems! (next steps)

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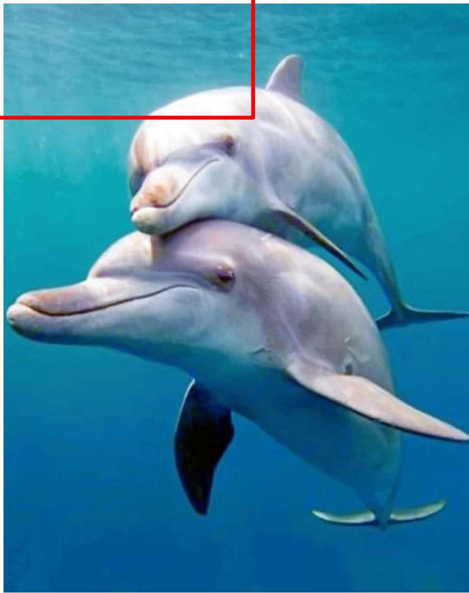
Not all required transformations can be easily implemented in the lab, the single mode analysis may not be



Open problems! (next steps)

What’s the best figure of merit?

Open problems! (next steps)

What's the best figure of merit?

Typically we compare quantum and classical strategies that use the **same resources** (energy or uses of the channel)

In practice → other figures of merit may be more relevant (scan rate!)

What did I say?

1. Quantum metrology noiseless case

$\sqrt{N}$

2. Entangled strategies (role of entanglement)

3. Squeezing.

3. QFI vs fidelity: overlap between initial and final state

4. Spreading channels: Squeezing input and antisqueezing+photodetection optimal.

Take home message

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Squeezing is optimal to estimate displacements with random (and irrelevant) phase

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