Signal formation in radiation detectors: Ramo's theorem and its application to practical cases

A. Castoldi

Politecnico di Milano & INFN, Sez. Milano

andrea.castoldi@polimi.it

Advanced School and Workshop on Nuclear Physics Signal Processing

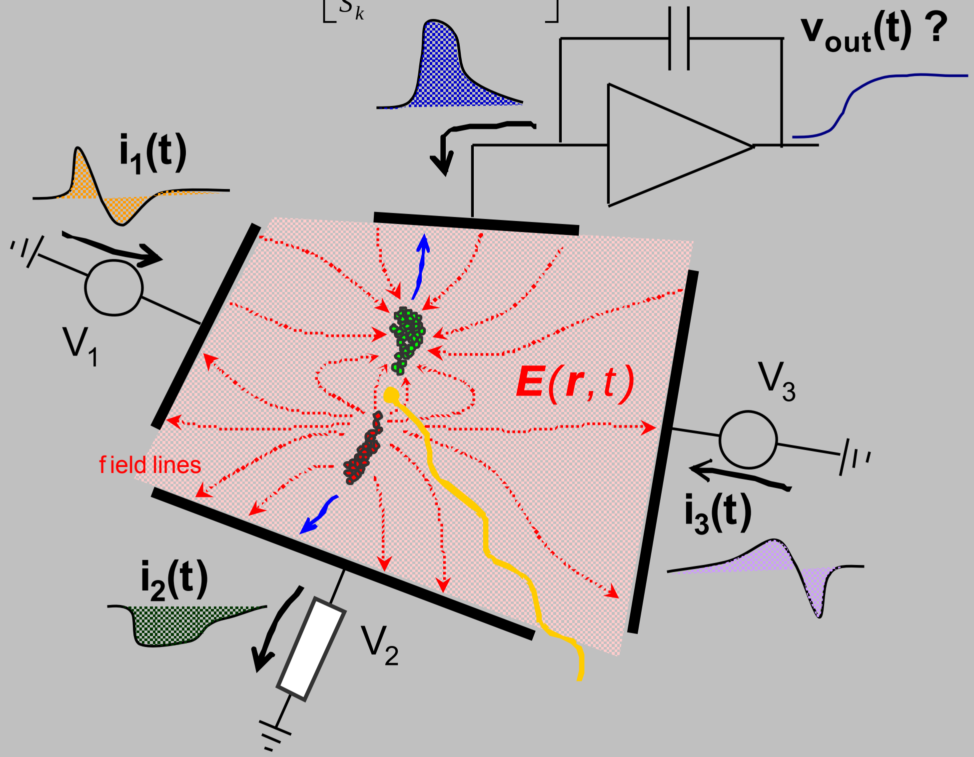
November 21-24, 2011, Acireale (CT), Italy

Outline

1. Shockley-Ramo's theorem
2. Induced current calculation
   - planar geometry
   - pn junction
   - microstrip detector
   - pixel vs strips
3. Single-polarity charge sensing
4. Application to special case studies:
   - pn detectors for particle identification
   - Multi-Linear Silicon Drift Detectors
   - Monolithic silicon detector telescopes

The problem of signal formation

$$i_k(t) = \frac{d}{dt}\left[\oint_{S_k} \varepsilon \mathbf{E}(\mathbf{r},t)\cdot d\mathbf{S}\right] = ?$$



Concept of induced current on electrode k: ***rate of change of electrostatic flux*** **on the electrode surface (not the collection of charge by the electrode!!!)**

- requires to compute the **total field E**(*x,y,z,t*) (due to bias voltages, fixed space charge and moving charges) **at every time instant**, the integral of the flux on the electrode surface, etc.

Understanding signal formation is a crucial to optimize measurement quality of time, energy, position, shape,….

- collection time, amplitude, shape depend on type and point of interaction
- detector topology and readout must be jointly optimized for the desired observables
- “tomographic” view of the event, i.e. exploitation of signals on all electrodes’

Goal: the induced current on a given electrode as a function of the instantaneous position of the moving charges within the device

A theorem for two...

S.Ramo, Proc. IRE, 27 (1939) p.584

Currents Induced by Electron Motion*

SIMON RAMO†, ASSOCIATE MEMBER, I.R.E.

Summary—*A method is given for computing the instantaneous current induced in neighboring conductors by a given specified motion of electrons. The method is based on the repeated use of a simple equation giving the current due to a single electron's movement and is believed to be simpler than methods previously described.*

Introduction

In designing vacuum tubes in which electron transit-time is relatively long, it becomes necessary to discard the low-frequency concept that the instantaneous current taken by any electrode is proportional to the number of electrons received by

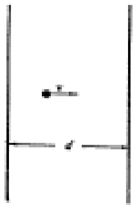


Fig. 1

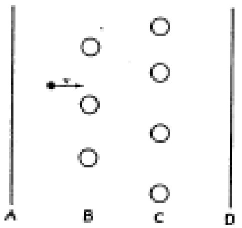


Fig. 2

it per second. Negative grids, it is known, may carry current even though they collect no electrons and current may be noted in the circuit of a collector during the time the electron is still approaching the collector. A proper concept of current to an electrode must consider the instantaneous change of electrostatic flux lines which end on the electrode and the methods given in the literature for computing induced current due to electron flow are based on this concept.

A method of computing the induced current for a specified electron motion is here explained which is believed to be more direct and simpler than methods previously described. In the more difficult cases, in which flux plots or other tedious field-determination methods must be used, only one field plot is needed by the present method while the usual methods require a large number.

Method of Computation

The method is based on the following equation whose derivation is given later:

$$i = E_v e v \quad (1)$$

where i is the instantaneous current received by the given electrode due to a single electron's motion, e is the charge on the electron, v is its instantaneous velocity, and E_v is the component in the direction v of that electric field which would exist at the electron's instantaneous position under the following circumstances: electron removed, given electrode raised to unit potential, all other conductors grounded. The equation involves the usual assumptions that induced currents due to magnetic effects are negligible and that the electrostatic field propagates instantaneously.

Simple Example

A simple example is offered in the computation of the instantaneous current due to an electron's motion between two infinite plates (Fig. 1). (The result is a starting point for the analysis of a diode, for example, when the transit-time is long.)

From (1) we obtain immediately

$$i = evE_v = \frac{ev}{d}.$$

In the literature[1] it is stated that this same result is deduced from image theory. This involves the setting up of an infinite series of image charges on each side of the plates for a given position of the electron and a consideration of the total flux crossing one of the planes due to the series of charges, a method which is lengthy and requires no little familiarity with methods of handling infinite series.

The General Case

Consider a number of electrodes, A, B, C, D, in the presence of a moving electron (Fig. 2) whose path and instantaneous velocity are known. A tedious way to find the current induced in, say, electrode

* Decimal classification: R138. Original manuscript received by the Institute, September 16, 1938.

† General Engineering Laboratory, General Electric Company, Schenectady, N. Y.

[1] D. O. North, "Analysis of the effects of space charge on grid impedance," Proc. I.R.E., vol. 24, pp. 108–158; February (1936).

W.Shockley, J. Appl. Phys. 9, 635 (1938)

Currents to Conductors Induced by a Moving Point Charge

W. Shockley
Bell Telephone Laboratories, Inc., New York, N. Y.

(Received May 14, 1938)

General expressions are derived for the currents which flow in the external circuit connecting a system of conductors when a point charge is moving among the conductors. The results are applied to obtain explicit expressions for several cases of practical interest.

In the earlier days of vacuum tube technique when the radiofrequencies in use were relatively low compared to those attained at present, it was acceptable to regard the transit of an electron across a vacuum tube as an instantaneous burst of current. At present, however, the time of transit of the electron is of comparable duration with the periods of alternating circuits and it is consequently of interest to know the instantaneous value of the current induced by the moving charge over its entire time of transit.

Before discussing what effect the moving charge has, we must introduce certain conventions as to what part of the total field is to be attributed to the charge and what part to other causes. It proves most convenient to consider that all of the conductors are grounded and to examine the currents to them through the external circuit due to the motion of the charge. If the voltages on the conductors are varying, however, charges will be induced and currents will flow as dictated by the coefficients of capacity. In keeping with the superposition principle,[1] the net current is found by adding the currents induced by the moving charge (or each moving charge if there are several) and the currents due to changing voltages.

We are thus led to consider the charges and currents induced on a system of grounded stationary conductors by the motion of a point charge. If we have a system of grounded conductors, perhaps as illustrated in Fig. 1, num-

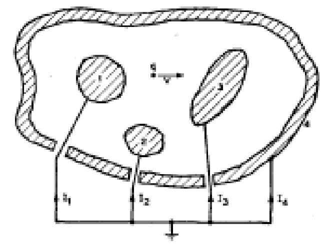


Fig. 1. Schematic representation of conductors and currents.

bered 1, 2, ⋯ n, say, then the charge q_1 induced on conductor 1 due to a unit point charge in the space is calculated as follows: Let conductor 1 be at unit potential and the others be grounded and let the space between the conductors be free of charge. The electrostatic potential produced by this situation has the value $V_1(r)$ at the arbitrary point r of space. Then, in terms of this potential distribution, the charge, q_1, induced on 1 by a unit charge at r is

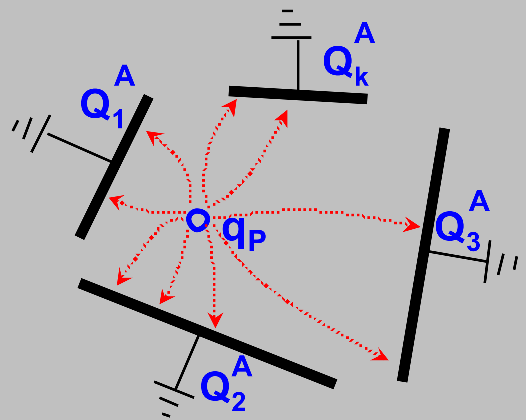
$$q_1 = -V_1(r). \quad (1)$$

[1] Jeans, *Mathematical Theory of Electricity and Magnetism*, fourth edition, p. 90.

Shockley (Bell Labs) and Ramo (General Electric) found independently (!) a more efficient method to compute induced charges and currents in vacuum tubes, it turned out valid for gas/solid state detectors

Ramo's Theorem - I

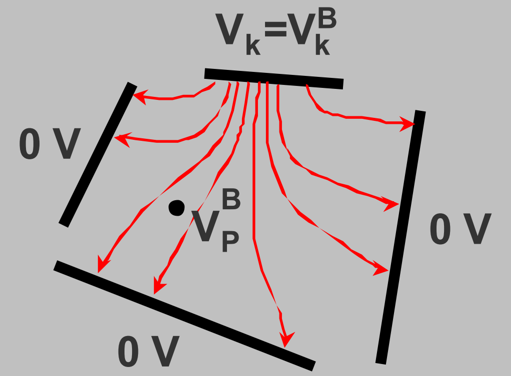
✓ ***Green's reciprocity theorem***



q_p present, $V_k^A=0$
(all electrodes grounded)

A, B are two possibile sets of {charges, potentials} of the system of conductors

$$\sum_i Q_i^A V_i^B = \sum_i Q_i^B V_i^A$$



q_p removed, $V_k=V_k^B$
(other electrodes grounded)

✓ ***Charge induced by q_P***

by reciprocity: $q_P V_P^B + Q_k^A V_k^B = 0$ $\Rightarrow$

$$\boxed{Q_k^A = -q_P \frac{V_P^B}{V_k^B} = -q_P \tilde{V}_w}$$

potential at P due to electrode *k* at 1 V

charge induced by q_P on electrode *k*

Ramo's Theorem - II

✓ ***Current induced on electrode k by the motion of charge q:***

By reciprocity:

$$Q_k = -q_P \tilde{V}_w \longrightarrow i_k(t) = \frac{dQ_k}{dt} = -\frac{d\left(q_P \tilde{V}_w\right)}{dt} = -q_P \frac{d\tilde{V}_w}{dt} \cdot \frac{d\vec{\mathbf{l}}}{d\vec{\mathbf{l}}} = -q_P \frac{d\tilde{V}_w}{d\vec{\mathbf{l}}} \cdot \frac{d\vec{\mathbf{l}}}{dt}$$

$$i_k(t) = q_P \tilde{\boldsymbol{E}}_{\boldsymbol{w}} \cdot \boldsymbol{v}(x(t), y(t), z(t))$$

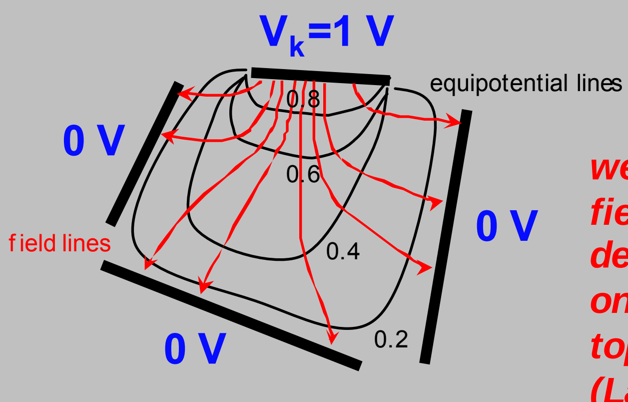
weighting field:

$$\tilde{\boldsymbol{E}}_{\boldsymbol{w}} = -grad\, \tilde{V}_w$$

<u>true</u> carrier velocity:

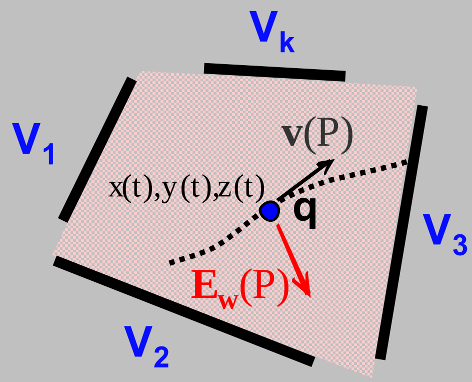
$$\boldsymbol{v} = \mu \boldsymbol{E}(x(t), y(t), z(t))$$

(here charge transport by drift is assumed)



(V_k=1 V, all others grounded)

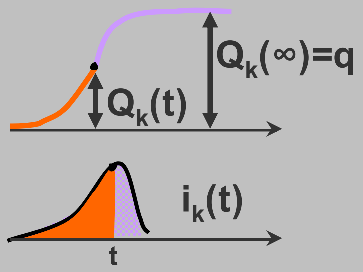
weighting field/potential depends only on device topology (Laplace eq.)



carrier trajectory computed in the <u>true</u> electric field (i.e. with bias voltages, fixed space charge, etc.)

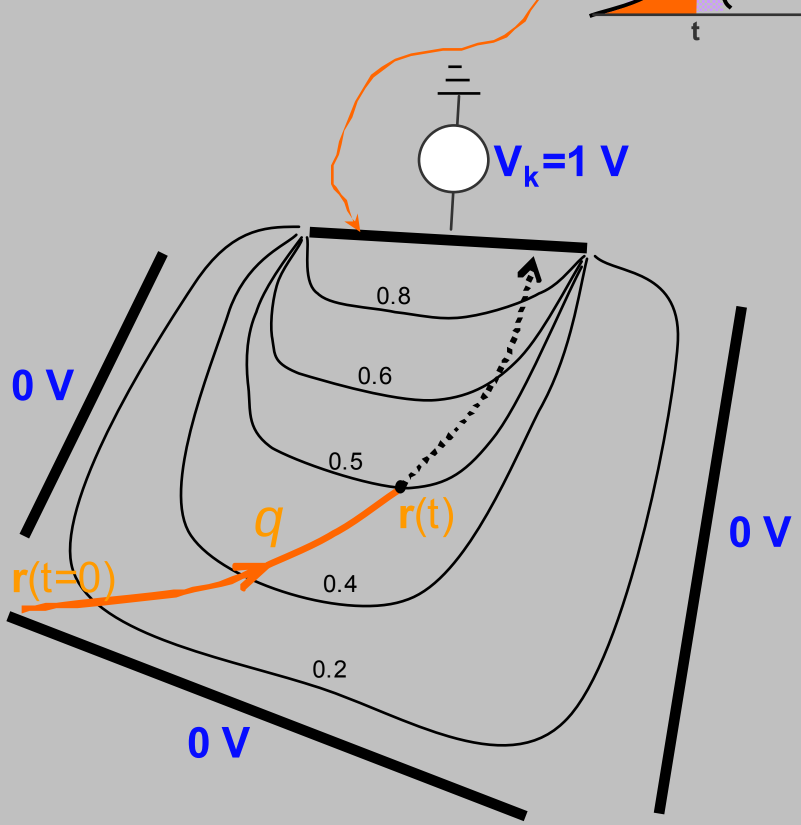
Ramo’s Theorem - III

*the **induced charge** can be computed directly on the **weighting potential map** (depends only on moving charge and topology of device)*



$$Q(t)=\int_0^t i(\tau)d\tau =$$

$$q\int_{\vec{r}_0}^{\vec{r}} \vec{E}_w \cdot d\vec{l} = -q\left[V_w(\vec{r})-V_w(\vec{r}_0)\right]$$



Limitations of Ramo’s theorem

▪assume **instantaneous field propagation** (i.e. transit time of charges longer than propagation of the field across the volume)

- q non-relativistic
  (but electron saturation velocity @RT in Si ~c/3000)
- electrode voltages must not vary too fast

Generalizations

▪multiple moving charges

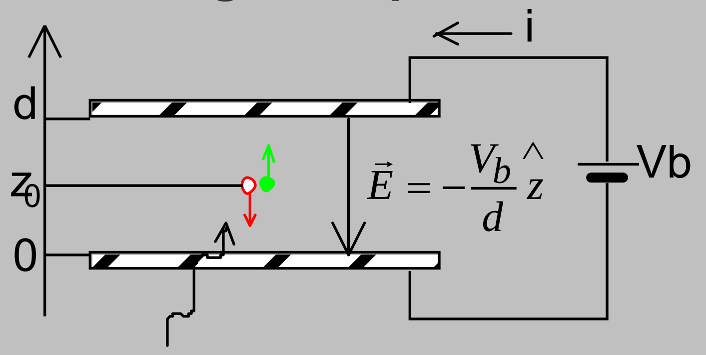
▪non-uniform or non-isotropic dielectric constant

▪other transport modalities (thermal broadening, ballistic motion, etc.) driven by continuity equation

Signal Formation and Ramo's theorem - I

✓ *Induced current (charge) in planar electrode geometry*

Single e-h pair

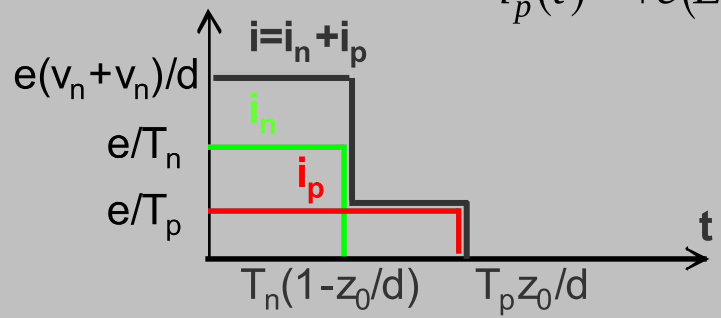


$$E_w = -\frac{1}{d}\hat{z}$$ weighting field

$$i_n(t) = -e(-E_w v_n) = e\frac{v_n}{d} = e\frac{1}{T_n} \qquad 0 \le t \le T_n(1 - z_0/d)$$

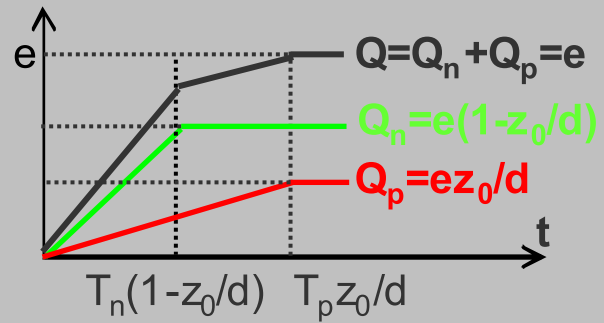
$$i_p(t) = +e(E_w v_p) = e\frac{v_p}{d} = e\frac{1}{T_p} \qquad 0 \le t \le T_p\frac{z_0}{d}$$

induced current
$\boldsymbol{i}(t)$



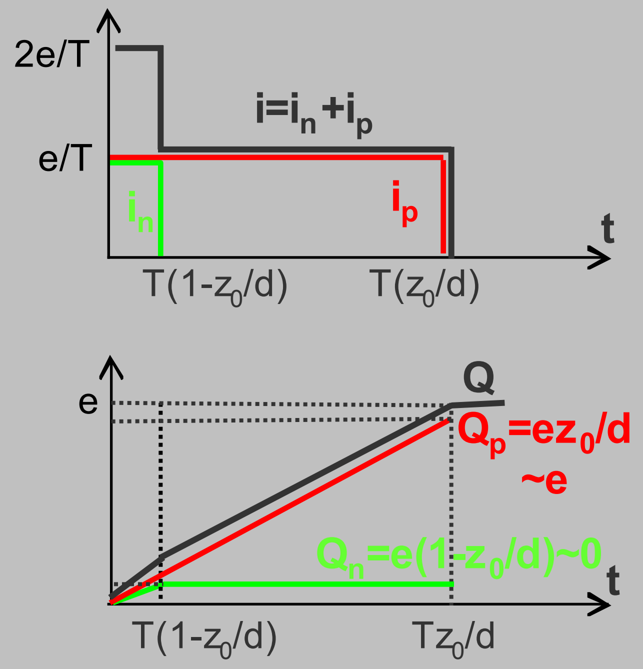
induced charge

$$Q(t) = \int \boldsymbol{i}(t)dt$$



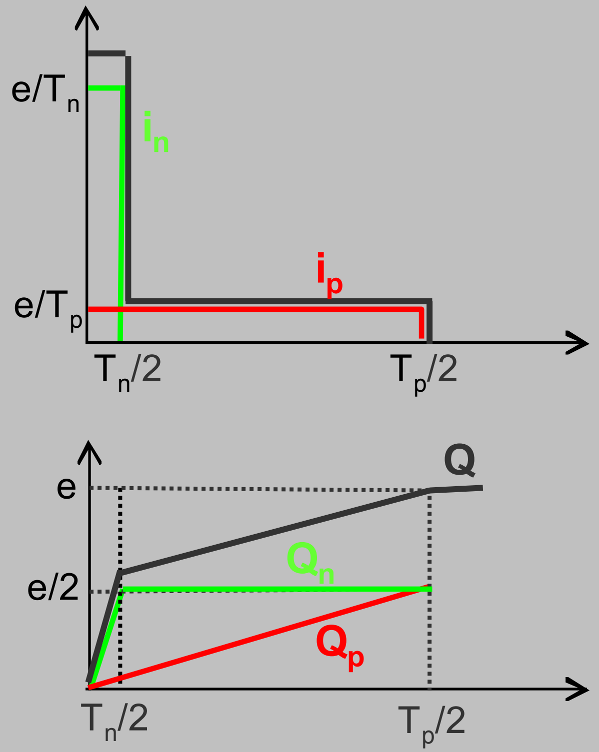
Signal Formation and Ramo's theorem - II

z_0~d (anode side)
$v_n=v_p=v$



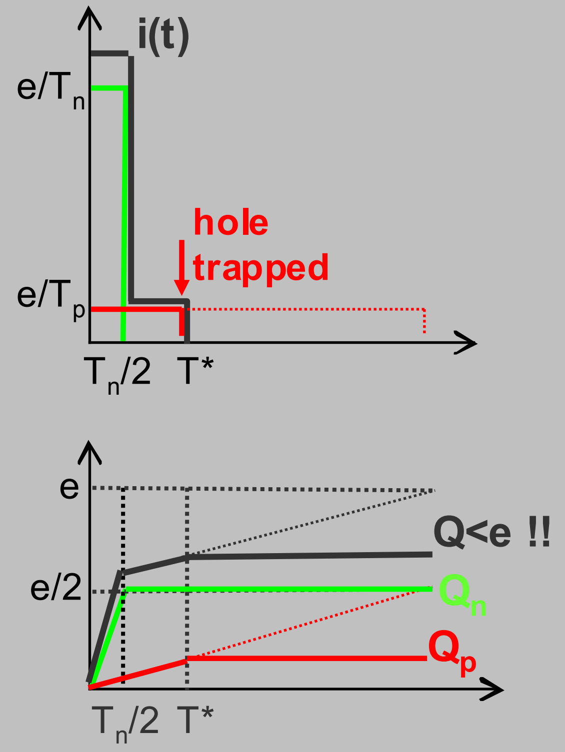
- equal currents
- hole induction dominant

z_0=0.5d
v_n>>v_p



- i_n>>i_p but equal contribution to ind. charge
- total induction after <u>slow</u> hole collection

as before, but hole trapped @T*

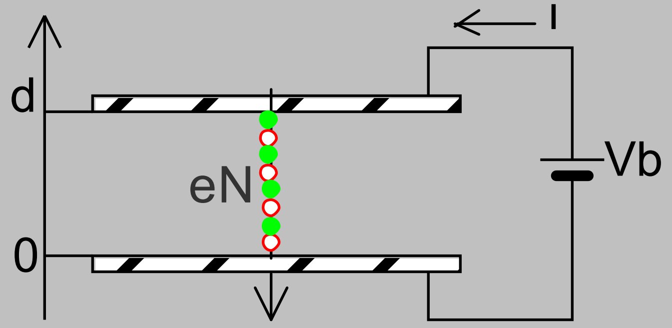


- <u>signal loss</u> !!
- total induced charge dependent on trap position

Signal Formation and Ramo's theorem - III

✓ ***Induced current (charge) in planar electrode geometry***

Continuous ionization

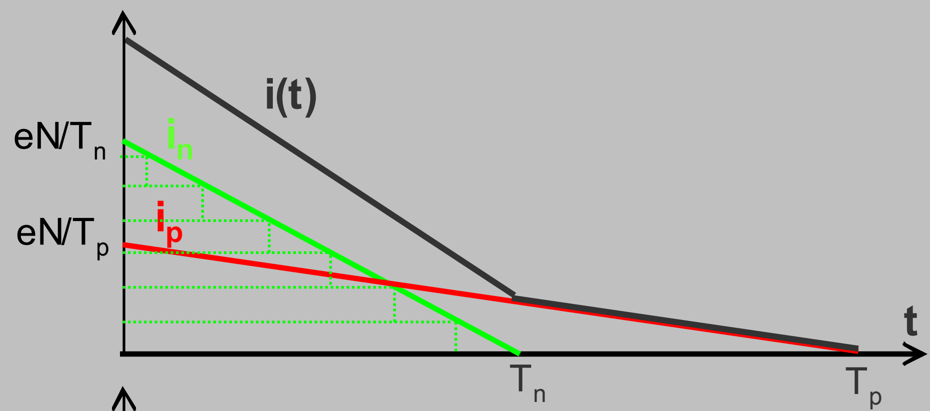


$$\boldsymbol{E}_w = -\frac{1}{d}\boldsymbol{k}$$ **weighting field**

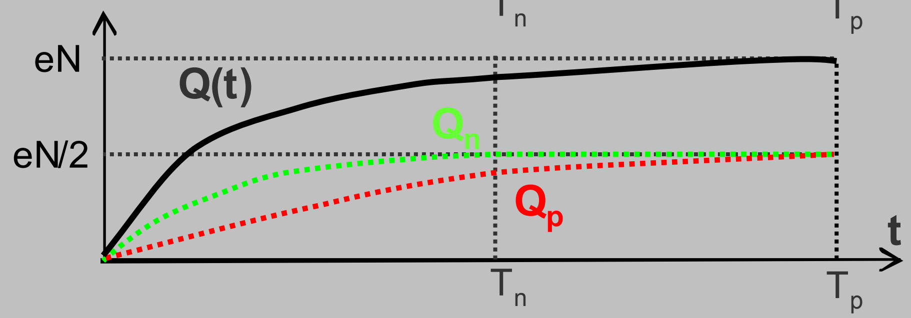
$$i_n(t) = eN\frac{v_n}{d}\left(1 - t/T_n\right) = \frac{eN}{T_n}\left(1 - t/T_n\right) \qquad 0 \le t \le T_n$$

$$i_p(t) = eN\frac{v_p}{d}\left(1 - t/T_p\right) = \frac{eN}{T_p}\left(1 - t/T_p\right) \qquad 0 \le t \le T_p$$

induced current

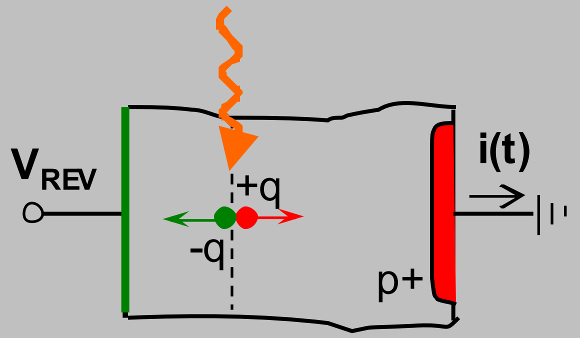


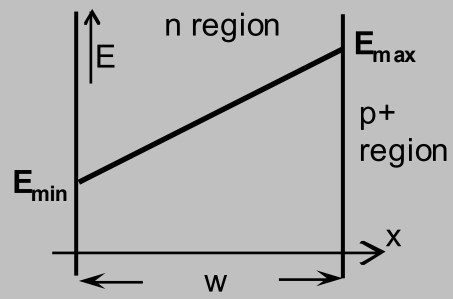
induced charge



Signal Formation and Ramo's theorem - IV

✓ ***Induced current (charge) in pn junction***

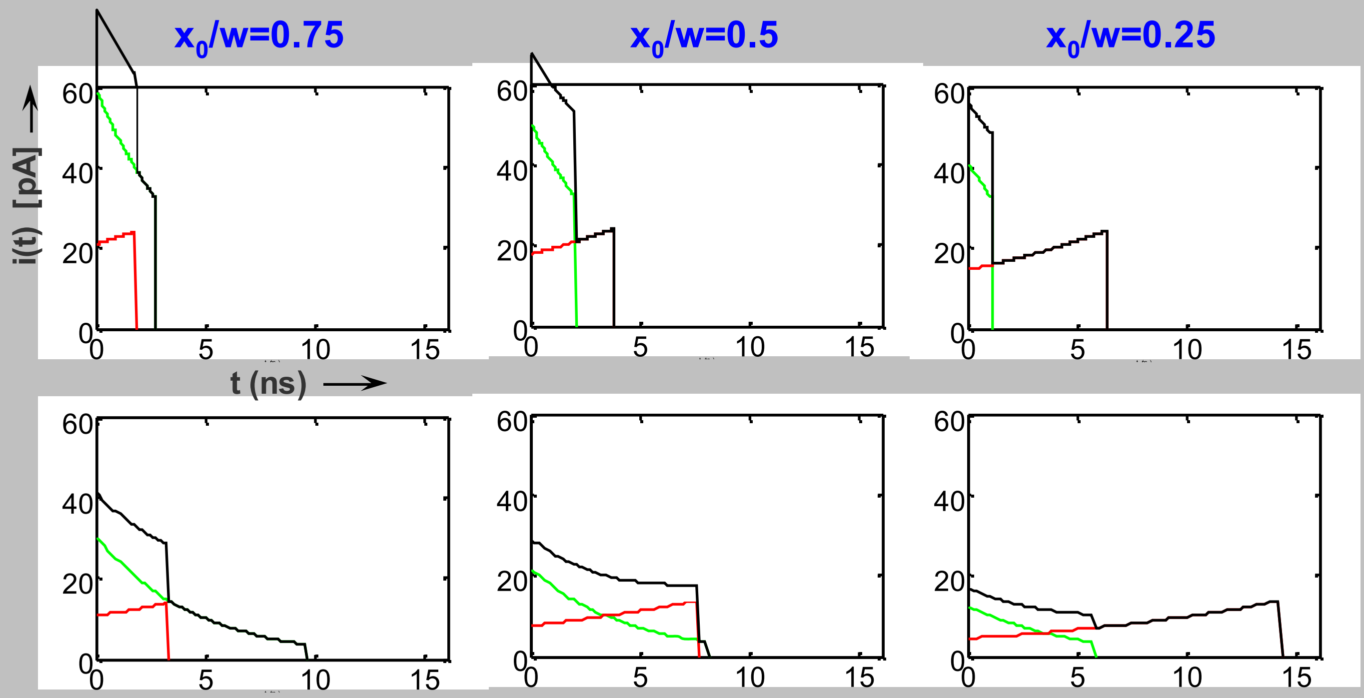




induced currents

$$i_h(t) = \frac{q}{w}\mu_h\left(E_{\min} + \frac{qN_D}{\varepsilon}x_0\right)e^{\mu_h q\frac{N_D}{\varepsilon}t} \qquad 0 \le t \le t_h$$

$$i_e(t) = \frac{q}{w}\mu_e\left(E_{\min} + \frac{qN_D}{\varepsilon}x_0\right)e^{-\mu_e q\frac{N_D}{\varepsilon}t} \qquad 0 \le t \le t_e$$



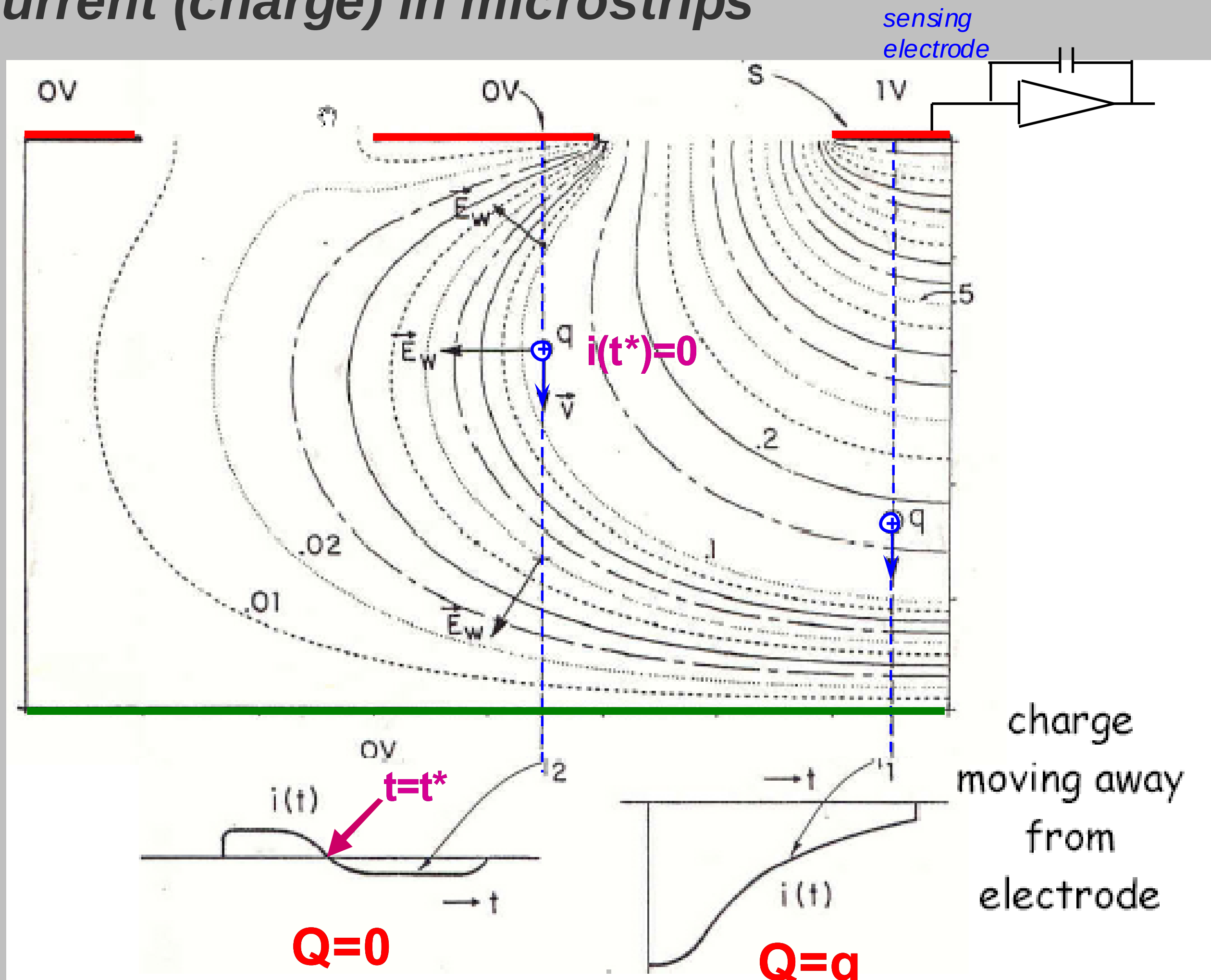
Emin / Ecrit =0.9

Emin / Ecrit =0.1

Silicon
w=300µm
$N_D=10^{12}$ cm^{-3}
$E_{crit}=qwN_D/\varepsilon$ =
=4800 V/cm

Signal Formation and Ramo's theorem - V

✓ ***Induced current (charge) in microstrips***

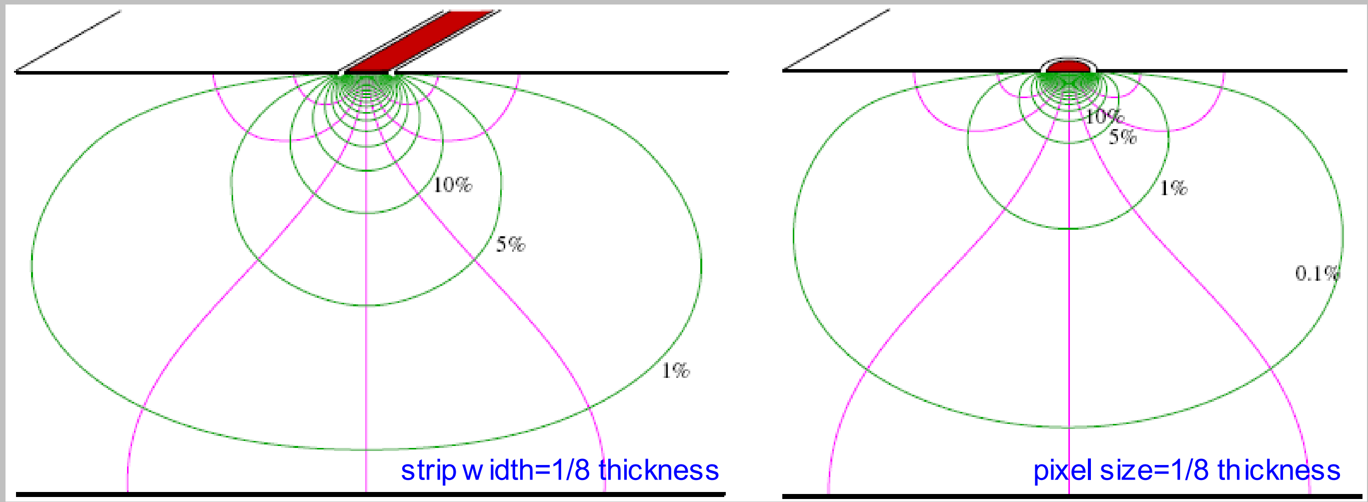


Weighting potential map of sensing electrode

if measurement time < pulse duration → Q≠0 !

Weighting field/potential – strips vs pixels

2D case (strip) **3D case (pixel)**



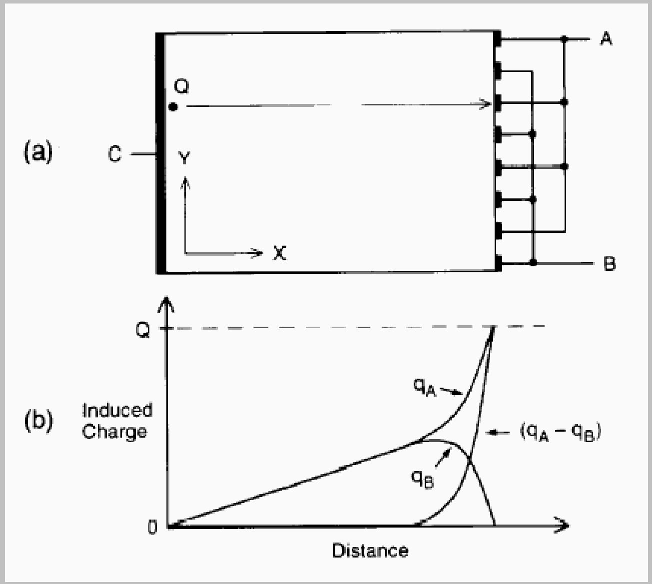
from V.Radeka

The shielding effect is proportional to the ratio of the distance between the planar electrodes and the strip width (or pixel size)

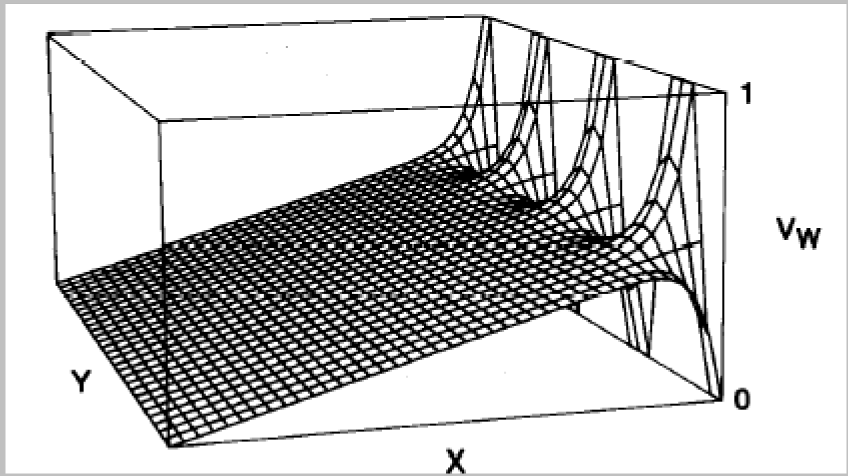
→ signal charge independent of the position of the origin of ionization for most of the volume of the detector, except near the readout electrodes (small pixel effect)

→ reduced sensitivity to the problem of *hole trapping* (i.e. pulse height dependent on depth of interaction)

Single polarity charge sensing with “coplanar grid” readout - 1



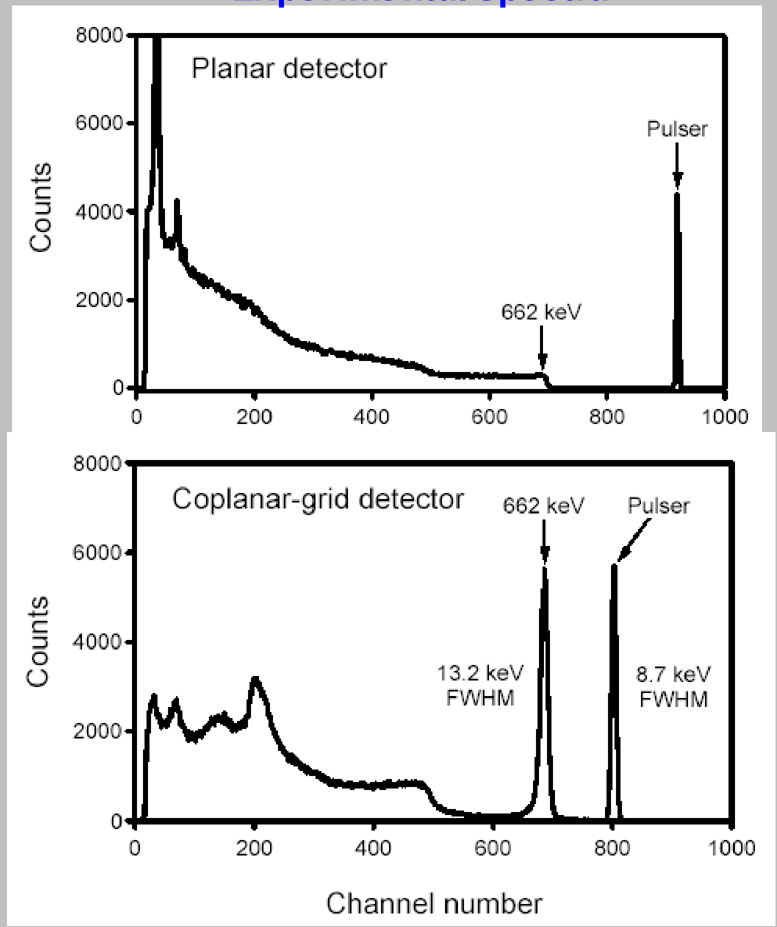
P.Luke, IEEE Trans Nucl. Sci., vol.42, no.4, Aug. 1995



- Two inter-digitated coplanar grid electrodes sense the motion of charge carriers in the detector (solid-state equivalent of the “Fritsch grid” of gas detectors)
- A small potential difference applied btw the C-grid and NC-grid to avoid charge sharing and double polarity signals
- When generated in the bulk, a charge carrier induces equal amount of charge on the 2 grids. A net difference signal is induced only when the carriers to be collected (e.g. electrons) are close to the coplanar electrodes.
- The net result is a measured charge nearly independent of the interaction depth

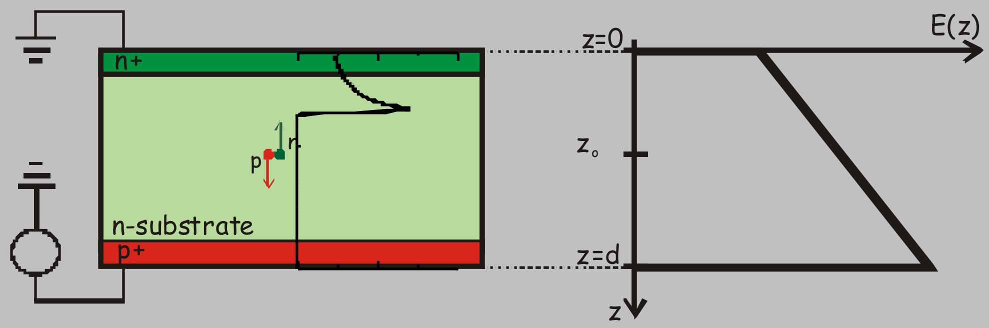
Single polarity charge sensing with “coplanar grid” method - 2

Experimental spectra



Measured charge independent of the depths of gamma-ray interactions over much of the detector volume → excellent energy resolution

Particle identification of charged ions with pn detectors



$$E(z) = \frac{qN_d}{\varepsilon} z + E_{min}$$

Ramo's Theorem
+
MonteCarlo Simulation

$$i_{p,\rho}(t) = \sum_k \rho(z_k)\, i_p(z_k, t)\Delta z$$

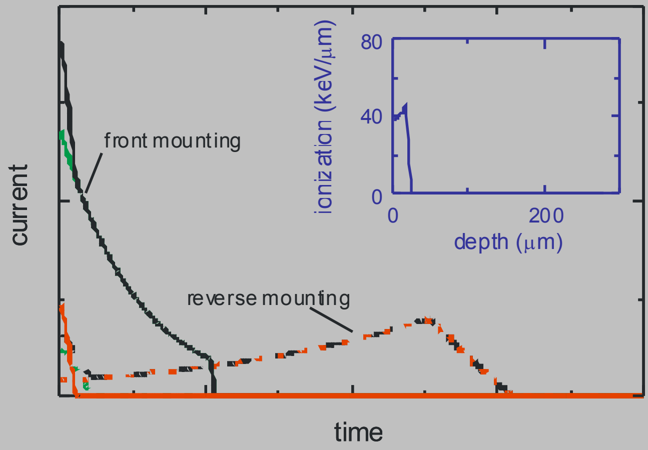
$$i_{n,\rho}(t) = \sum_k \rho(z_k)\, i_n(z_k, t)\Delta z$$

$$i_{tot}(t) = i_{p,\rho}(t) + i_{n,\rho}(t)$$

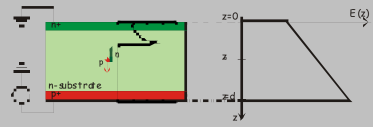
Even if neglecting the important contribution of plasma effects, this simple analytical model is a precious guide for interpretation of the experimental results and for the optimization of the detector topology and operating parameters.

Energy-RiseTime plots

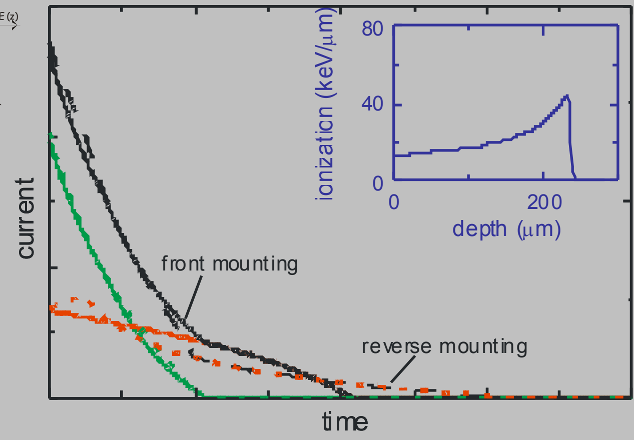
^{16}O - 30 MeV



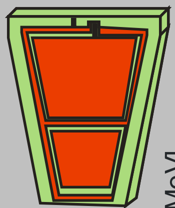
induced current



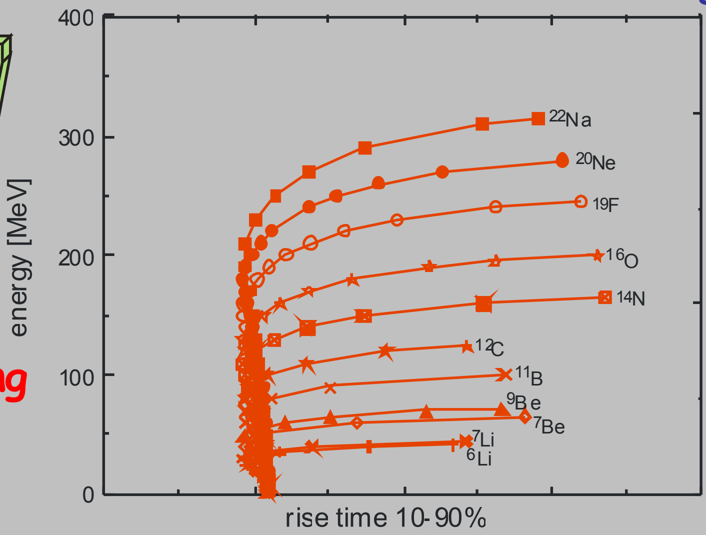
^{16}O - 100 MeV

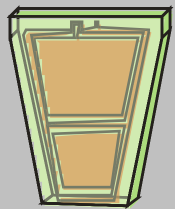


E_{Si} - RT

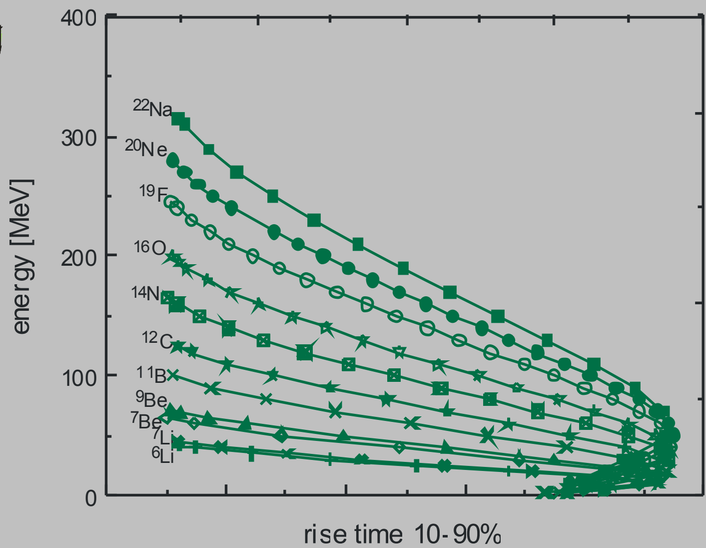


front mounting



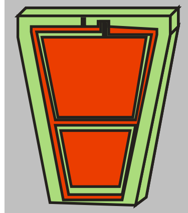


reverse mounting

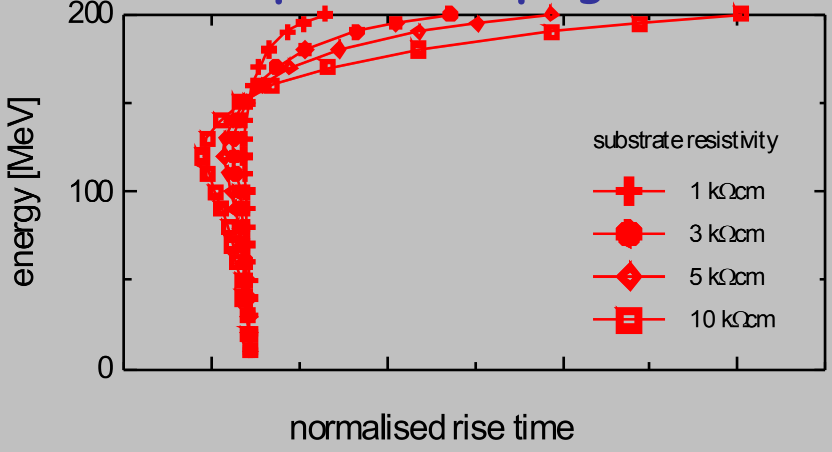


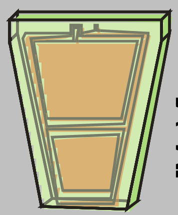
Sensitivity studies

Impact of doping level

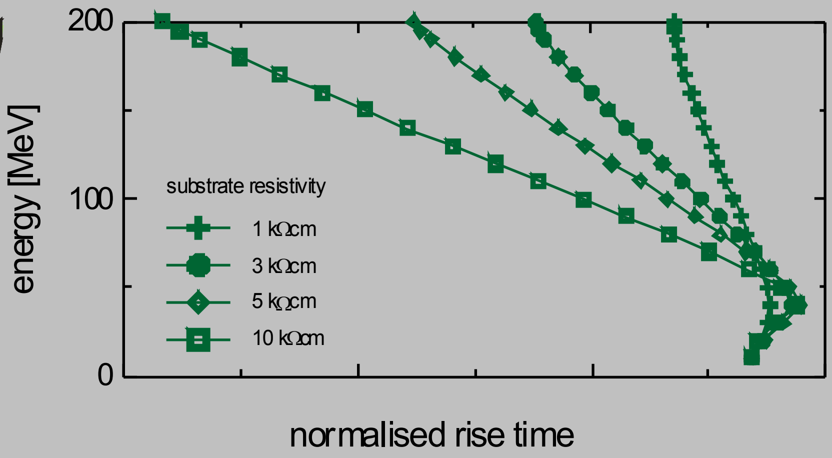


front mounting

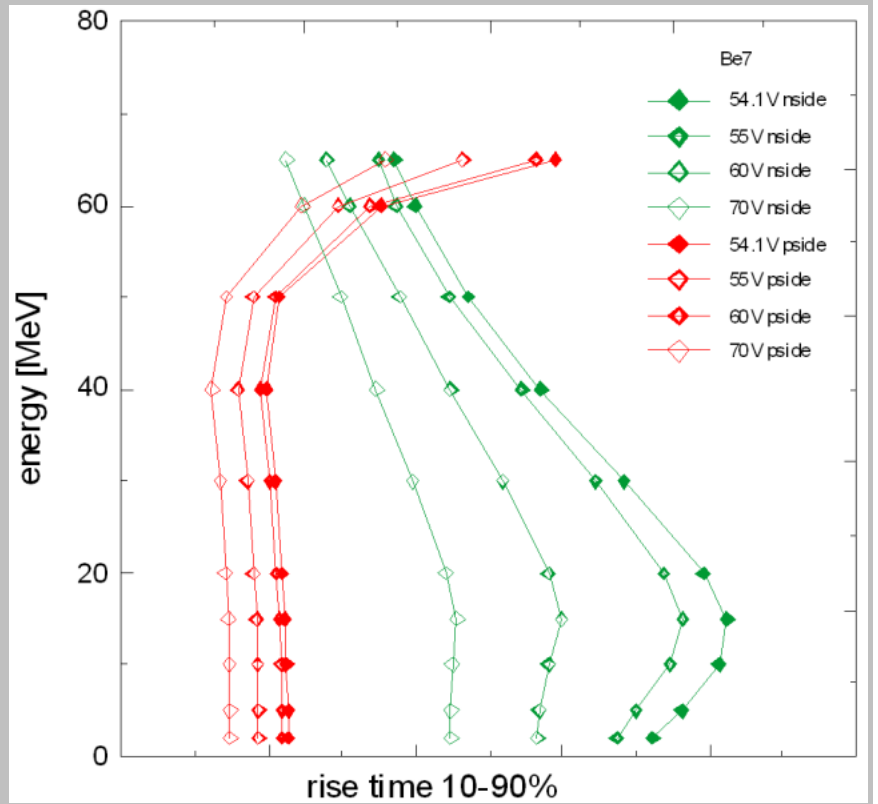




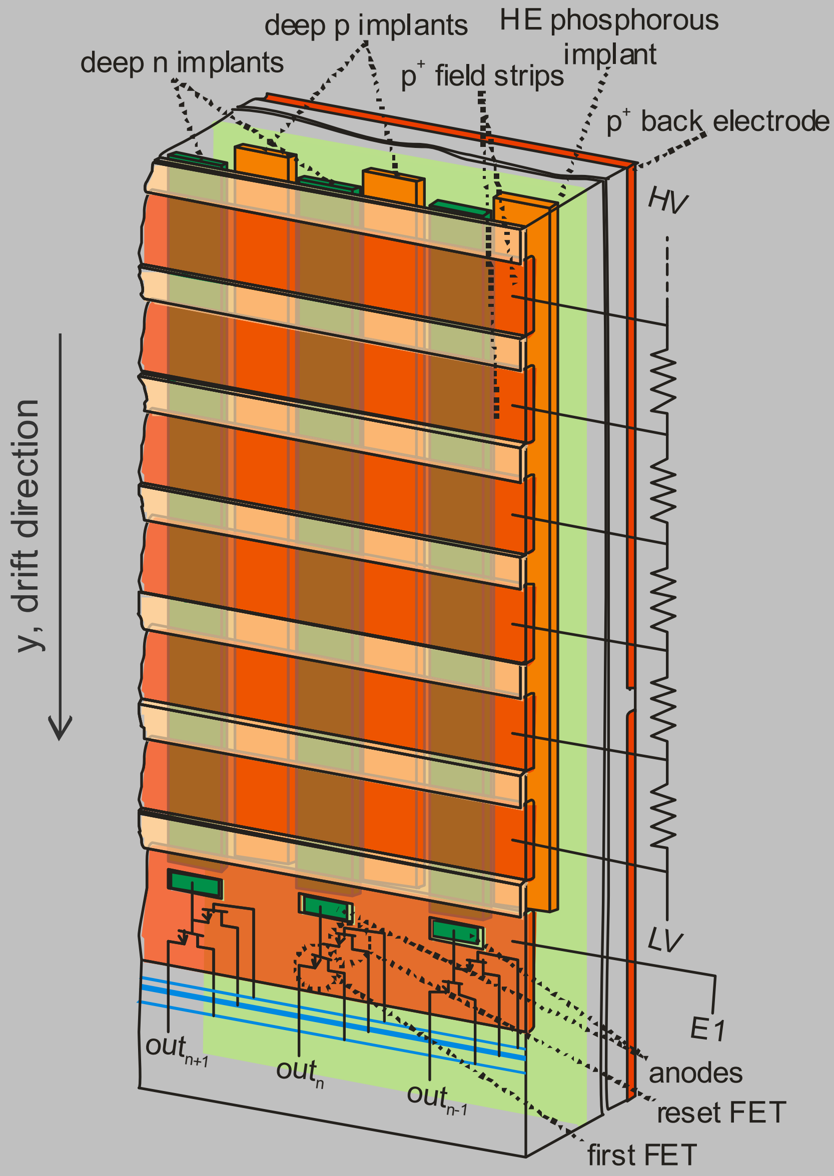
reverse mounting



Impact of bias voltage



The Multi-Linear Silicon Drift Detector



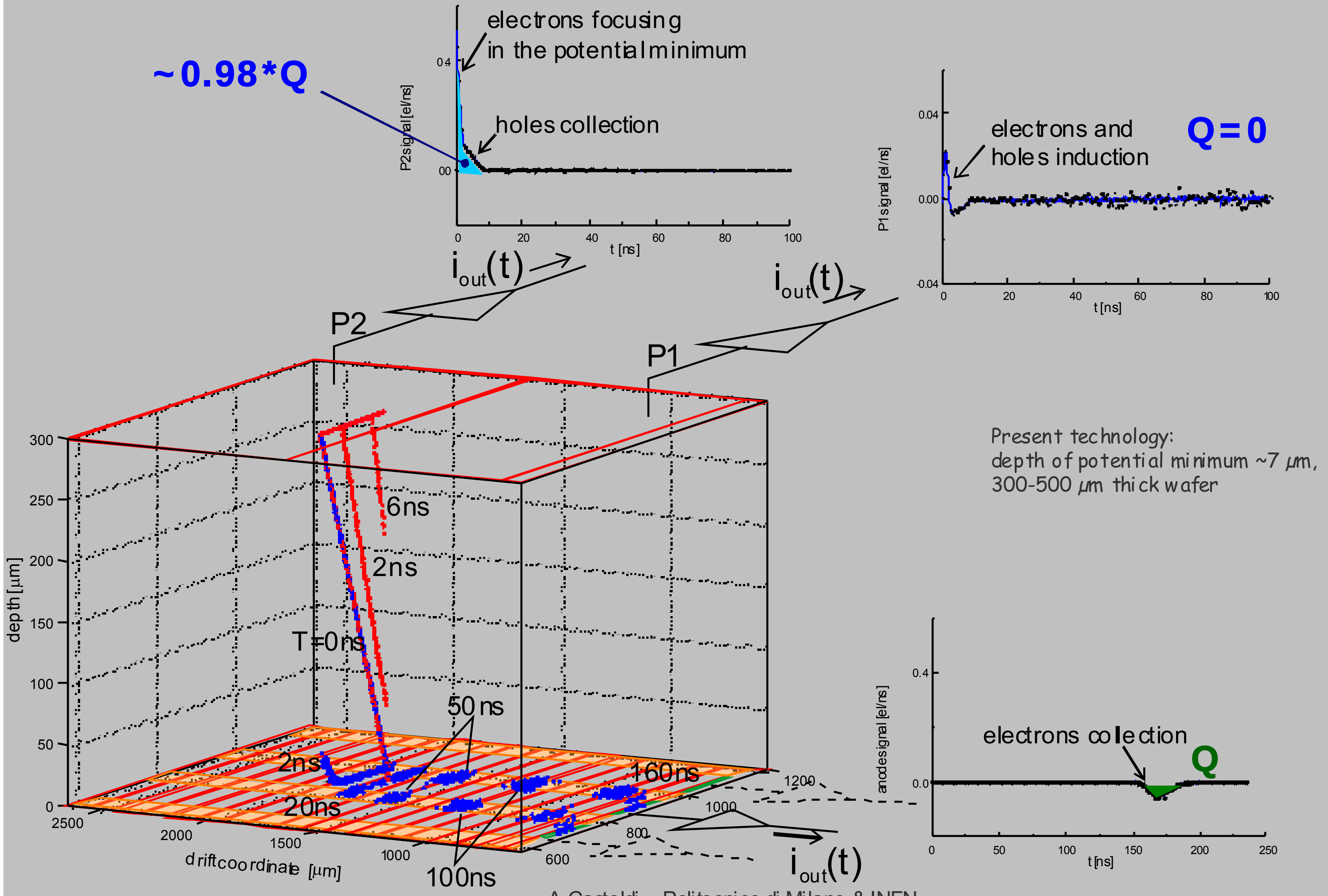
- fully depleted n-type bulk
- p+ entrance window on the back side
- implanted p+ strips on the front side
- continuous readout (drift) mode
- Rad-hard on-chip JFET for low-noise readout
- *channel-stops* (deep p-implants) for lateral confinement
- *channel-guides* (deep n-implants) for lateral confinement and drift enhancement
- fast readout (up to 1 cm/μs)

⬇

simultaneous **2D position-sensing and energy spectroscopy**

Signal Formation in Multi-Linear SDD

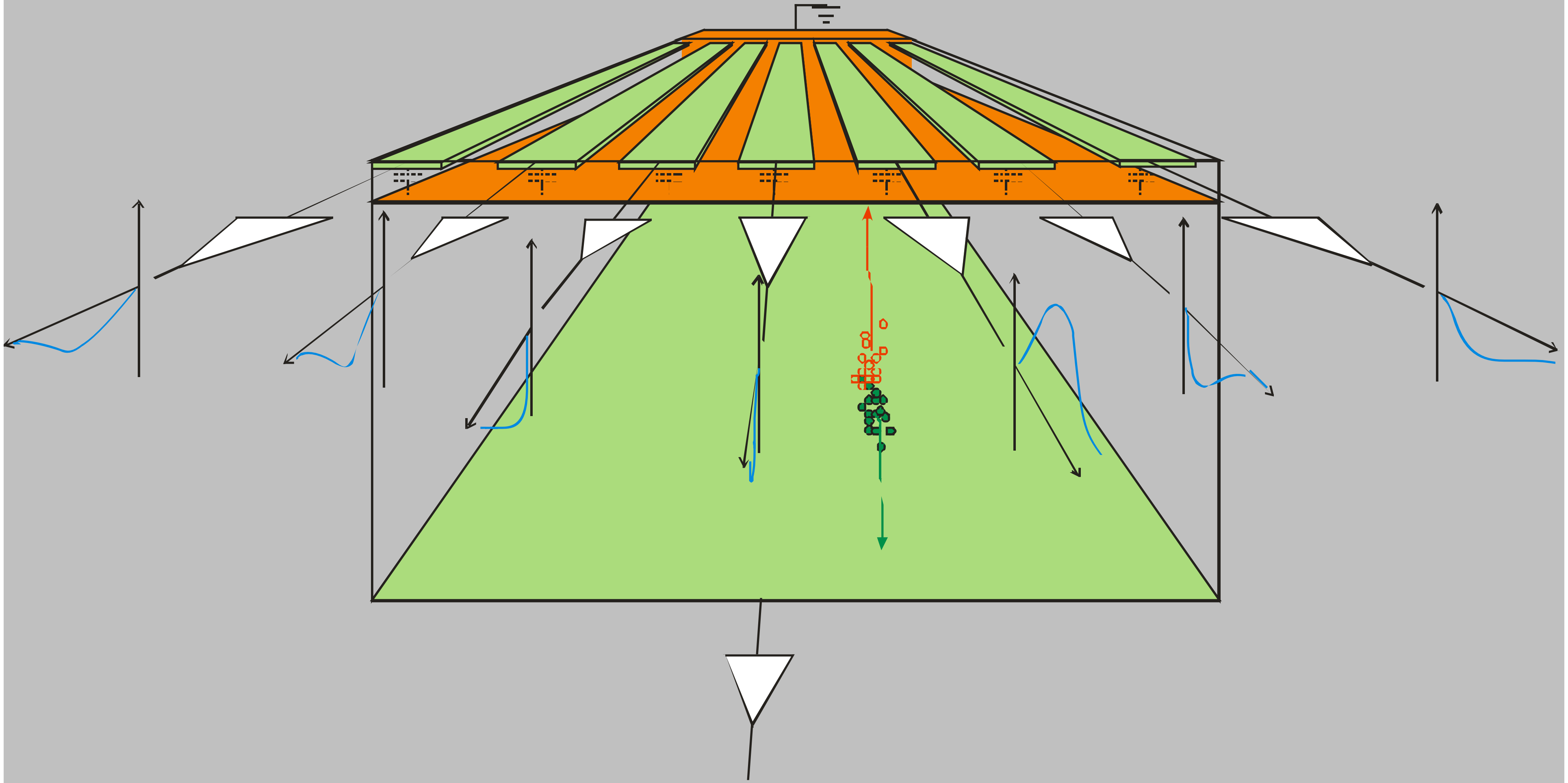
IONIZING PARTICLE INTERACTION



Present technology:
depth of potential minimum ~7 µm,
300-500 µm thick wafer

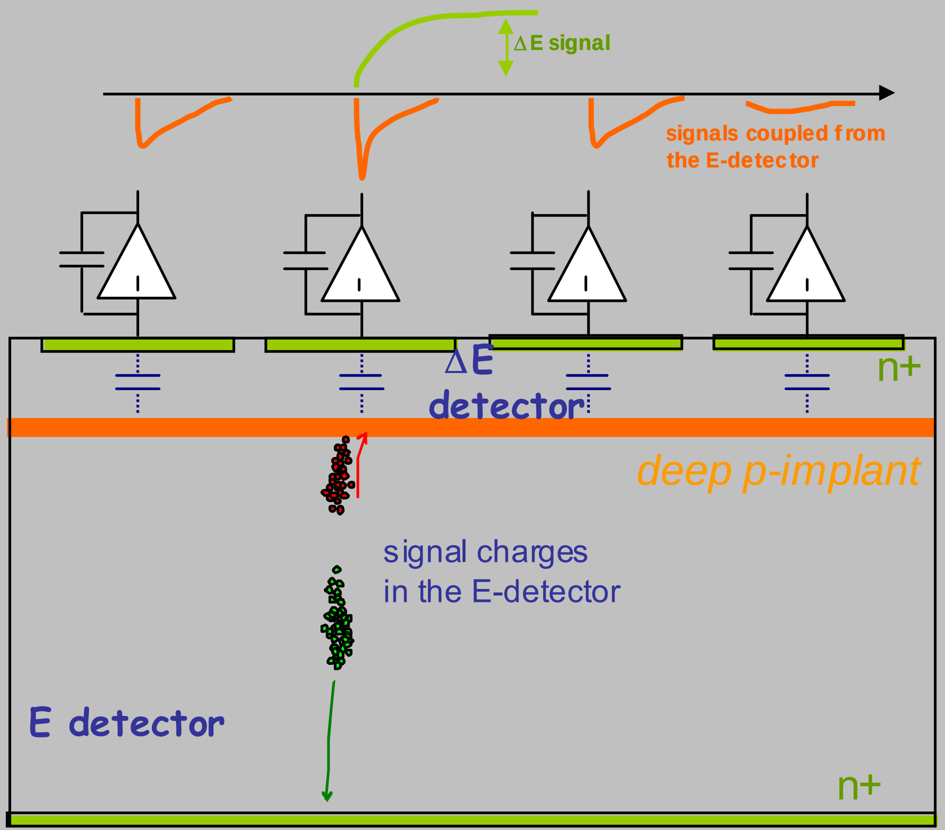
A.Castoldi – Politecnico di Milano & INFN

Monolithic Silicon detector telescopes



Signal shape interpreted on the basis of induction calculations

Physical model & interpretation of induced signals

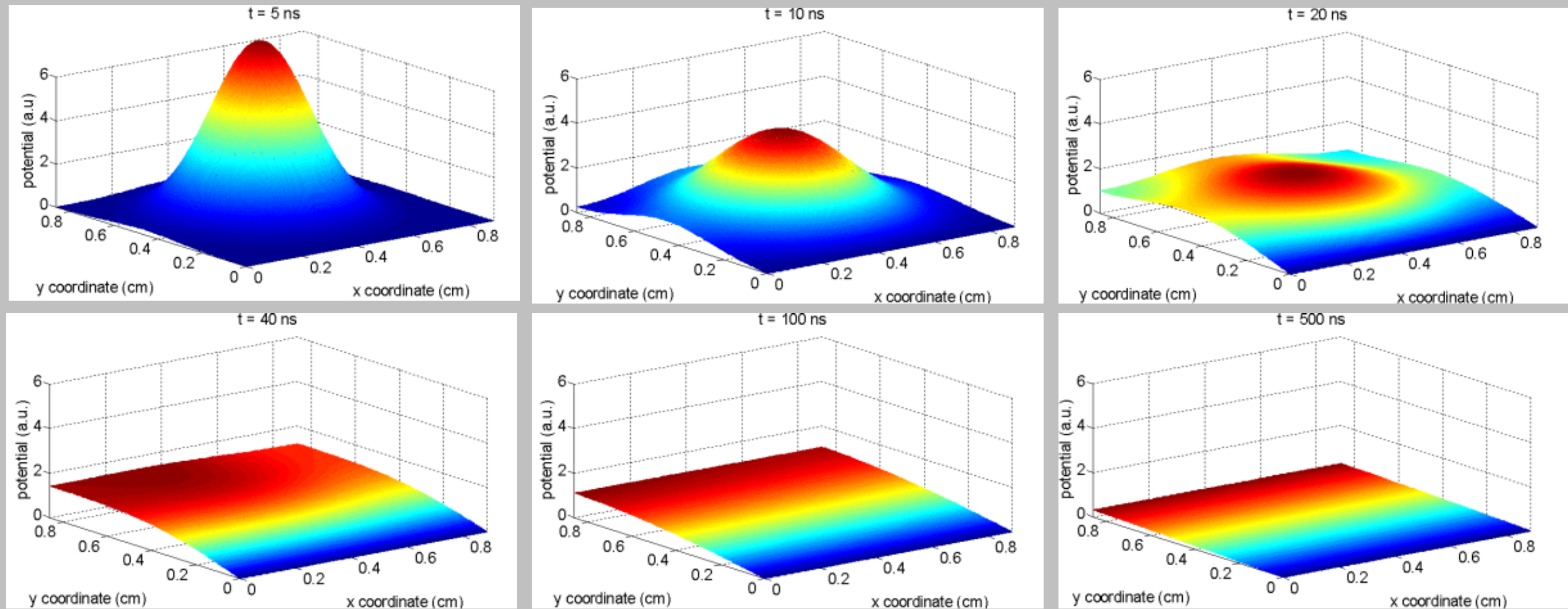


- Finite sheet resistance of the deep p-implant that forms the buried electrode
→ **local rise of the potential of the buried electrode that relaxes to equilibrium with a time constant comparable with charges's transit time (several hundreds ns)**

- Voltage signals induced on the buried electrode **AC coupled to the strips of the ΔE detector** via the relatively large ΔE capacitance

→ **appear attenuated and inverted at the ΔE preamplifier output.**

Temporal behaviour of the potential of the buried electrode - 1

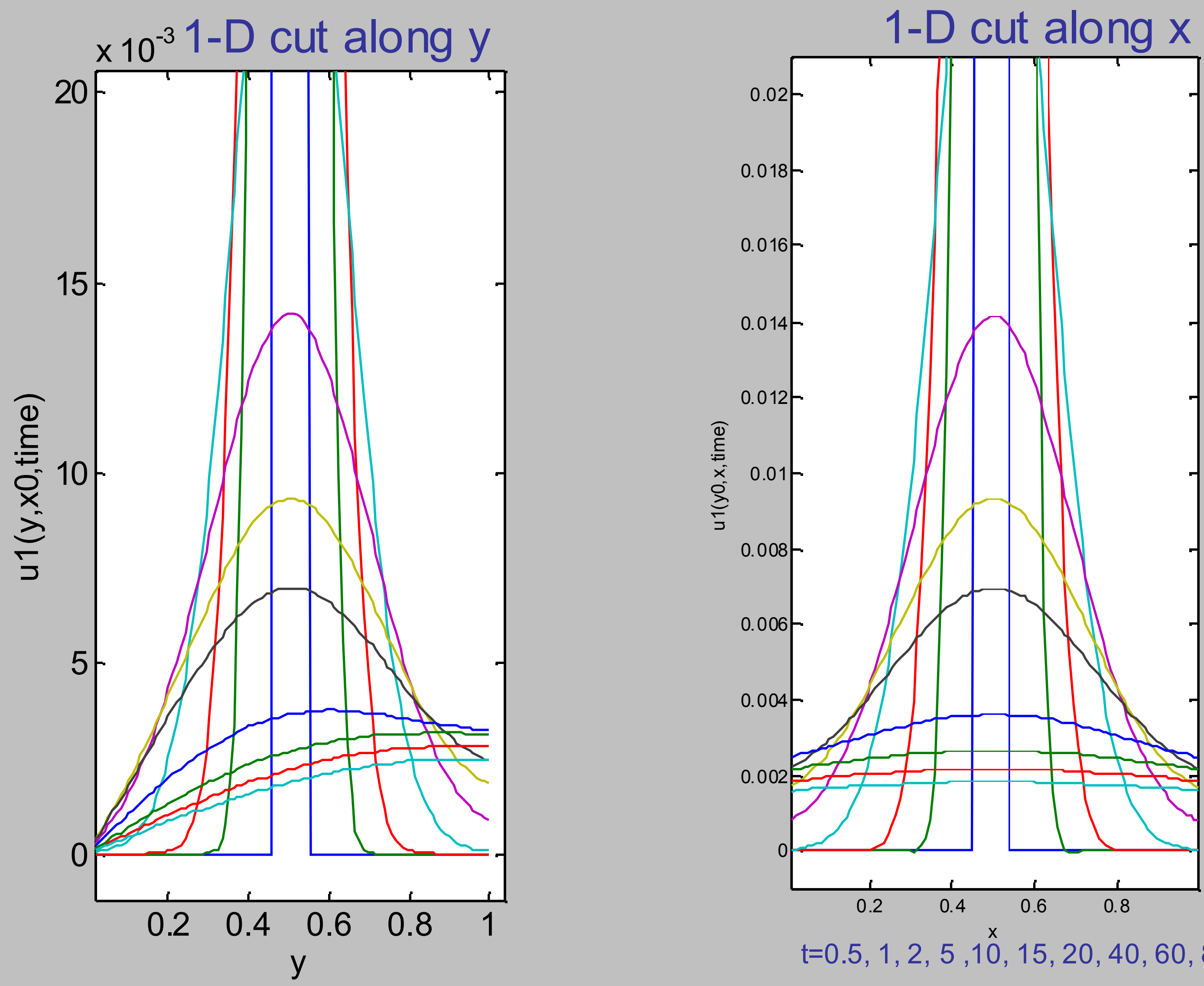


• potential distribution on the buried resistive electrode computed through the solution of the diffusion equation:

$$-D\cdot div\left(grad\ V\left(x,y,t\right)\right)+\frac{\partial V\left(x,y,t\right)}{\partial t}=0 \qquad D=\frac{1}{R_{sheet}C'}$$

• proper boundary conditions describe the behavior of the buried electrode:
- Dirichlet boundary condition on the grounded side,
- Neumann conditions at the three other sides.

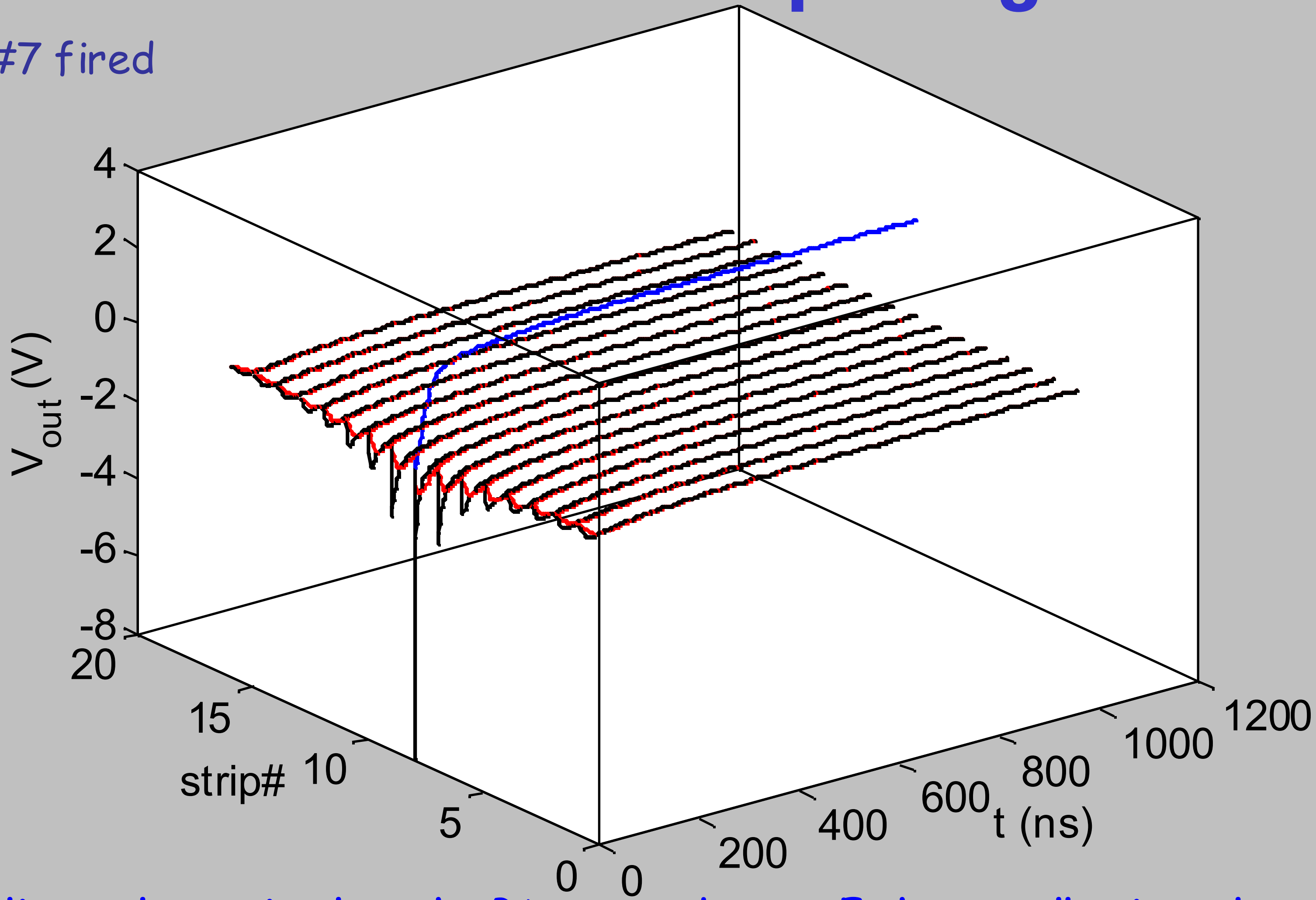
Temporal behaviour of the potential of the buried electrode -2



t=0.5, 1, 2, 5 ,10, 15, 20, 40, 60, 80, 100ns

Simulated induced output signals - 1

Strip #7 fired



Blue line: voltage signal at the PA output due to ΔE charge collection only.

Black lines: induction effect at the PA output with an ideal PA.

Red lines: induction at the PA output, taking into account PA finite bandwidth.

Simulated induced output signals - 2