g-mode Oscillations in Neutron Stars

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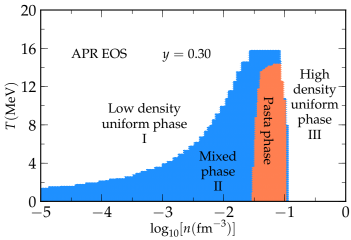
Matter in Astrophysical Phenomena

	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
Baryon Density(n_0)	$10^{-8} - 10$	$10^{-8} - 10$	$10^{-8} - 10$
Temperature(MeV)	$0 - 30$	$0 - 50$	$0 - 100$
Entropy(k_B)	$0.5 - 10$	$0 - 10$	$0 - 100$
Proton Fraction	$0.35 - 0.45$	$0.01 - 0.3$	$0.01 - 0.6$



- Phase diagram corresponding to the APR EOS at a lepton fraction of 0.3.

Supranuclear EOS

Phenomenological Approaches:

- Skyrme(-like): $\hat{V}_{NN} = \sum_{i<j} \hat{V}_{ij} + \sum_{i<j<k} \hat{V}_{ijk}$, zero-range.
  Evaluated in the Hartree-Fock approximation $\Rightarrow \mathcal{H} = \frac{\hbar^2}{2m^*}\tau + V(n)$.
- Relativistic meson exchange in the mean-field approximation (= negligible meson-field fluctuations, uniform and static system).
- Momentum-dependent interactions of the Yukawa type, borrowed from heavy-ion physics.

Microscopic Approaches:

- High-precision interactions fitted to NN scattering data
  - meson-exchange models
    e.g. Nijmegen, Paris, Juelich-Bonn
  - sums of local operators
    e.g. Urbana, Argonne
- Interactions from chiral EFT
- RG-evolved potentials

Extension of the above to bulk matter by a variety of techniques: SCGF, BHF, variational, etc.

Laboratory Equilibrium Constraints

- Near the Nuclear Equilibrium Point ($n = n_0, \alpha = 0$),

  $E(n,\alpha) \simeq E_0(n) + S_2(n)\alpha^2 + \mathcal{O}(\alpha^4)$
  $E_0(n) \simeq \mathcal{E}_0 + \frac{1}{2}K_0\left(\frac{n-n_0}{3n_0}\right)^2 + \ldots$
  $S_2(n) \simeq S_v + L\left(\frac{n-n_0}{3n_0}\right) + \ldots$

- Saturation density, $n_0 = 0.16 \pm 0.01\ \mathrm{fm}^{-3}$
  High-energy electron scattering: $r_0 \propto \pi/qR$, $n_0 = \left(\frac{4}{3}\pi r_0^3\right)^{-1}$

- Energy per particle, $\mathcal{E}_0 = -16 \pm 1$ MeV
  Fits to masses of atomic nuclei :
  $B(N,Z) = \mathcal{E}_o A - b_{surf}A^{2/3} - S_v\frac{(N-Z)^2}{A} - b_{Coul}Z^2A^{-1/3}$

- Symmetry energy, $S_v = 30 - 35$ MeV
  (fits to masses of atomic nuclei)

- Slope of S_2, $L = 40 - 70$MeV
  (variety of experiments)

- Compression modulus, $K_0 = 240 \pm 30$ MeV
  Giant monopole resonances : $E_{GMR} = \left(\frac{K_A}{m<r^2>}\right)^{1/2}$
  $K_A = K_0 + K_{surf}A^{-1/3} + K_\tau\frac{(N-Z)^2}{A^2} + K_{Coul}\frac{Z^2}{A^{4/3}}$

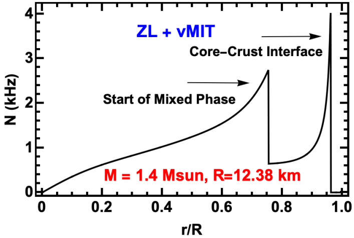
Neutron Stars

- Matter in β-equilibrium supported against gravitational collapse by neutron degeneracy.

- Structure determined by simultaneous solution of:
  - Interior mass, $m(r) = 4\pi \int_0^r \epsilon(r')r'^2 dr'$
  - Hydrostatic equilibrium, $\frac{dp}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{m(r)}\right]\left[1-\frac{2Gm(r)}{r}\right]^{-1}$
  - EOS, $p = p(\epsilon)$

- Constraints
  - Largest observed mass, (binaries) $M \simeq 2\ M_\odot$
  - Largest observed frequency, (pulsars) $\Omega = 114$ rad/s
  - Inferred radius range, (photospheric emission, thermal spectra) $10 \text{ km} \leq R \leq 14 \text{ km}$

- Relativistic binaries not in equilibrium : Gravitational wave (GW) emission leads to orbital decay.
- Early stage: only gravitational interactions, GW signal contains information for the masses of the components.
- Coalescence stage: Tidal disruption of the lower-mass star, mass transfer onto the more massive one. Mass transfer rate depends on $\mathcal{C} = M_{NS}/R_{NS}$ and reflected in GW signal. Ejected matter is very neutron-rich and can lead to heavy element formation via the r-process.
- Late stage: Black hole or hypermassive neutron star formation.
- EOS relevance
  - Tidal disruption of NS during coalesence of BH-NS binary depends on the stiffness of the EOS. GW frequency sensitive to orbital frequency at disruption.
  - r-process production rates and abundances depend on the composition of the ejecta and thus the EOS.
  - Tidal deformability, $\Lambda = \frac{2}{3} k_2 \left(\frac{R\, c^2}{G\, m}\right)^5$.
  - g-mode frequencies: $N^2 = g^2\left(\frac{1}{c_e^2} - \frac{1}{c_s^2}\right) \mathrm{e}^{\nu-\lambda}$
    $g = -\nabla[p/(\epsilon + p)]$



Motivation: Hybrid Stars (?)

- The size of nucleons (uncertain as it may be) implies that deconfined quark matter can exist in the cores of NSs.
- However, such a possibility lacks observational and theoretical support:
  - Measurements of M, R, Λ cannot differentiate normal and hybrid stars.
  - LQCD and PQCD not applicable to NS conditions.
- Possible solution: identify an observable with strong dependence on composition.
- Enter g-modes!

- Global, long-lived, nonradial fluid oscillations resulting from fluid-element perturbations in a stratified environment.
- Slow chemical equilibration generates buoyancy forces to oppose dispacement.
- In stably-stratified systems the opposing force sets up oscillations with a characteristic frequency (Brunt-Väisälä) which depends on both the equilibrium and the adiabatic sound speeds.
- g-mode oscillations couple to tidal forces; they can be excited in a NS merger and provide information on the interior composition.
- Detection remains a challenge; but within sensitivity of 3rd generation detectors.

Cowling vs. linearized GR

- In linearized GR, the calculation of g-mode frequencies, damping times, and amplitudes requires the solution of 4 coupled ODEs.

- The relativistic Cowling approximation neglects metric perturbations that must accompany matter perturbations in a GR treatment reducing complexity:

$$\frac{dU}{dr} = \frac{g}{c_{\rm ad}^2} U + \mathrm{e}^{\lambda/2} \left[\frac{l(l+1)\mathrm{e}^{\nu}}{\omega^2} - \frac{r^2}{c_{\rm ad}^2} \right] V$$

$$\frac{dV}{dr} = \mathrm{e}^{\lambda/2-\nu} \frac{\omega^2 - N^2}{r^2} U + g\Delta(c^{-2}) V$$

where $U = r^2 \mathrm{e}^{\lambda/2} \xi_r$, $V = \omega^2 r \xi_h$, $\Delta(c^{-2}) = c_{\rm eq}^{-2} - c_{\rm ad}^{-2}$,
$N^2 = g^2 \Delta(c^{-2}) \mathrm{e}^{\nu-\lambda}$, $g = -\nabla P/(\varepsilon + P)$,
and λ, ν are Schwarzchild metric functions.

- Accurate to a few % compared to GR.

- Cannot compute imaginary part of eigenfrequency (damping time).

- Nucleons: Zhao - Lattimer

$$\epsilon_B = \sum_{h=n,p} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{M_B^2 + k^2}\, dk + n_B V(u,x)$$

$$V = 4x(1-x)(a_0 u + b_0 u^\gamma) + (1-2x)^2 (a_1 u + b_1 u^{\gamma_1})$$

- Quarks: vMIT

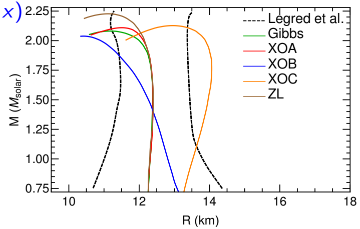
$$\mathcal{L} = \sum_{q=u,d,s} \left[\bar{\psi}_q \left(i\partial\!\!\!/ - m_q - B\right)\psi_i + \mathcal{L}_{\rm int}\right]\Theta$$

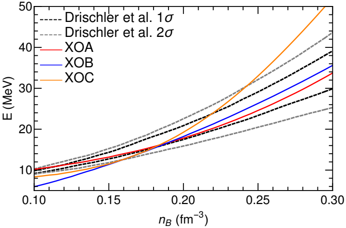
$$\mathcal{L}_{\rm int} = -G_v \sum_q \bar{\psi}\gamma_\mu V^\mu \psi + \left(m_V^2/2\right) V_\mu V^\mu$$

$$\epsilon_Q = \sum_q \epsilon_{\rm FG,q} + \frac{1}{2}\left(\frac{G_v}{m_V}\right)^2 n_Q^2 + B$$

- Leptons: noninteracting, relativistic fermions

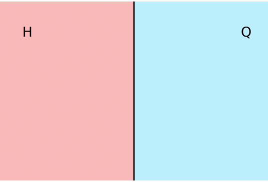
$$\epsilon_L = \sum_{l=e,\mu} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{m_L^2 + k^2}\, dk$$



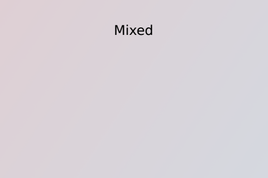


Hybrid Matter: 1st Order Transitions

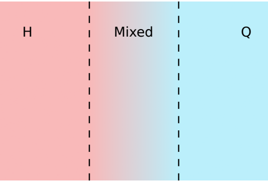
- Maxwell (“strong”, “stiff”, ...)
  - Infinite interface tension
  - No phase mixing
  - Local charge neutrality
  - $\varepsilon = f(\varepsilon_H + \varepsilon_{eH}) + (1-f)(\varepsilon_Q + \varepsilon_{eQ})$



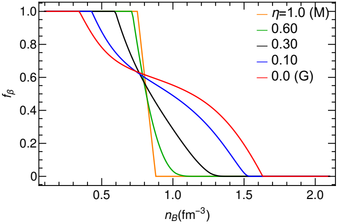
- Gibbs (“weak”, “soft”, ...)
  - Zero surface tension
  - Complete phase mixing
  - Global charge neutrality
  - $\varepsilon = f\,\varepsilon_H + (1-f)\,\varepsilon_Q + \varepsilon_{eM}$



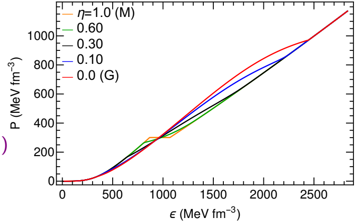
- Intermediate case
  - Some phase mixing
  - Charge neutrality is partially local and partially global
  - $\varepsilon = f(\varepsilon_H + \eta\,\varepsilon_{eH}) + (1-f)(\varepsilon_Q + \eta\,\varepsilon_{eQ}) + (1-\eta)\varepsilon_{eM}$



- Constraints
  - Baryon number conservation
    $1 = f(y_n + y_p) + (1-f)(y_u + y_d + y_s)/3$
  - Lepton number conservation
    $0 = y_e - f\eta y_{eH} - (1-f)\eta y_{eQ} - (1-\eta) y_{eM}$
  - Local charge neutrality
    $0 = (y_p - y_{eH}) = (2y_u - y_d - y_s)/3 - y_{eQ}$
  - Global charge neutrality
    $0 = f y_p + (1-f)(2y_u - y_d - y_s)/3 - y_{eM}$



- Equilibrium (= minimization of ε wrt f, y_i, η)
  - Mechanical, $P_H + \eta P_{eH} = P_Q + \eta P_{eQ}$
  - Quark weak, $\mu_d = \mu_s$
  - Neutral strong, $\mu_n = \mu_u + 2\mu_d$
  - Charged strong, $\mu_p = 2\mu_u + \mu_d - \eta(\mu_{eH} - \mu_{eQ})$
  - Beta $\mu_d = \mu_u + \eta\mu_{eQ} + (1-\eta)\mu_{eM}$
    -or- $\mu_p = \mu_n - \eta\mu_{eH} - (1-\eta)\mu_{eM}$
  - η optimization, $\varepsilon_{eM} = f\varepsilon_{eH} + (1-f)\varepsilon_{eQ}$



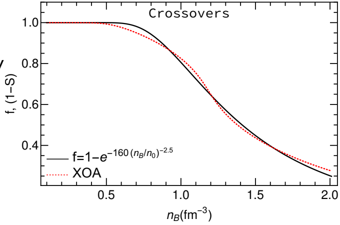
Hybrid Matter: Crossovers

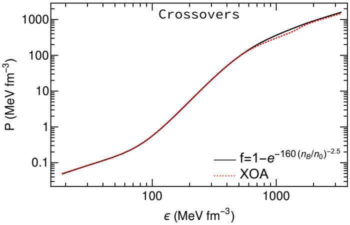
- In contrast to 1st-order transitions, crossovers involve only a single phase of matter. Thus, in the present framework, charge neutrality is achieved globally, i.e., $\eta = 0$.
- No mechanical equilibrium condition; instead choose $f = f(n_B)$, s.t. it approaches asymptotically 0 and 1 at high and low densities, respectively.
- LQCD-inspired model (Kapusta-Welle):

$$P_B = (1-S)P_H + S\ P_Q$$

$$S = \exp\left[-\left(\frac{\mu_0}{\mu}\right)^4\right]$$

$$\mu_0 \sim 2\ \mathrm{GeV}$$





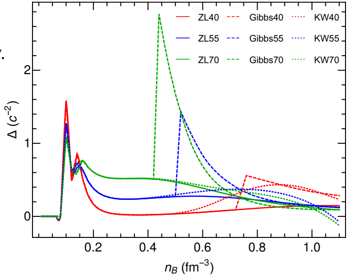
- $c_{\rm eq}^2(n_B) = \frac{dP}{d\varepsilon} = \frac{dP_\beta}{dn_{\rm B}}\left(\frac{d\varepsilon_\beta}{dn_{\rm B}}\right)^{-1}$
  mechanical equilibrium restored instantaneously.

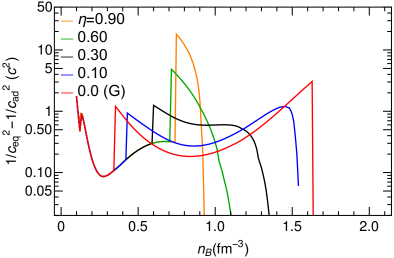
- $c_{\rm ad}^2(n_{\rm B}, x) = \left(\frac{\partial P}{\partial \varepsilon}\right)_x = \left.\frac{\partial P}{\partial n_{\rm B}}\right|_x \left(\left.\frac{\partial \varepsilon}{\partial n_{\rm B}}\right|_x\right)^{-1}$
  $c_{{\rm ad},\beta}^2(n_{\rm B}) = c_{\rm ad}^2[n_{\rm B}, x_\beta(n_{\rm B})]$
  slow restoration of chemical equilibrium because $\tau_\beta \gg \tau_{\rm oscillation}$.

- The difference $\Delta(c^{-2}) = c_{\rm eq}^{-2} - c_{\rm ad}^{-2}$ drives the restoring force for g-mode oscillations. For example, in *npe* matter

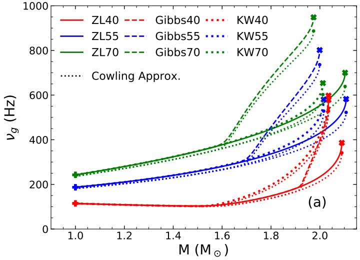
$$c_{\rm ad}^2 = c_{\rm eq}^2 + \frac{\left[n_B\left(\frac{\partial\tilde{\mu}}{\partial n_B}\right)_x\right]^2}{\mu_n\left(\frac{\partial\tilde{\mu}}{\partial x}\right)_{n_B}}$$

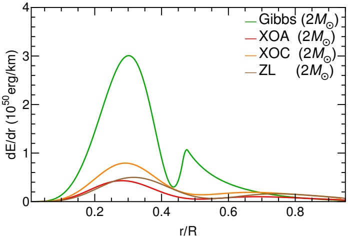
$$\tilde{\mu} = \mu_e + \mu_p - \mu_n \xrightarrow{\beta-{\rm eq.}} 0$$



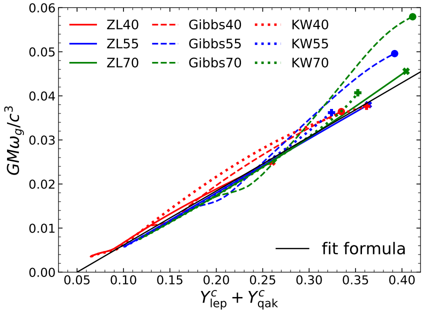


- g-modes in Gibbs hybrid matter have a larger frequency range compared to the pure-nucleon and crossover cases corresponding to the behavior of $\Delta(c^{-2})$ in the mixed phase.
- Dramatic changes in ν_g require new particle species not merely a smooth change in composition.
- The Cowling approx. is qualitatively similar to GR but underestimates ν_g by up to 10%; does better for low-mass stars.
- Energy per unit radial distance in oscillatory motion: The Gibbs energy scale is one order of magnitude larger than ZL and KW once quark matter appears ($\sim 10^{51}$ ergs/km vs. $\sim 10^{50}$ ergs/km).



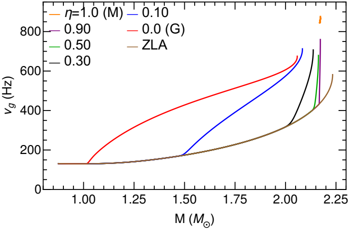


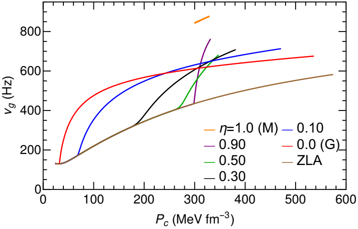
- Universal relations depend weakly on the EOS and can be used to break degeneracies and otherwise constrain difficult-to-access observables.
- Given the sensitivity of g-modes to departures from chemical equilibrium, it is likely that N and ν_g depend strongly on composition
- $\Omega_g = GM\omega_g/c^3 = 1.228(Y^c - 0.05)$



Discontinuity g-modes

- Generated by the flatness of $P(n_B)$ in a Maxwell mixed phase that leads to a density jump in the core of a hybrid star.
- Characterized by discontinuous g-mode frequencies.
- A special case of a compositional g-mode in the limit $\eta \to 1$.





Summary

- Calculation of g-mode properties for 1st-order phase transitions and for crossovers (both with the Cowling approximation as well as linearized GR).
- Construction of a thermodynamically-consistent framework for the treatment of 1st-order phase transitions intermediate to Maxwell and Gibbs.
- g-modes can detect nonnucleonic matter in the cores of NS; assuming quark matter (by some other means), g-modes can distinguish between a first-order phase transition and a crossover.
- Universal relation between Ω_g and Y^c.
- Discontinuity g-modes as a special case of compositional g-modes in the Maxwell limit.
- (Near) Future:
  - Extend 1st-order transition scheme and KW to finite T.
  - Applications to protoneutron stars (cooling, superfluidity)
  - Construct EOS that uses the same underlying description for quarks and hadrons; explore hybrid matter microscopically.
  - Other signals?