NIP Data and Computation

Projected gravitational wave constraints on primordial black hole abundance for extended mass distributions

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Overview

- We report constraints on the minimum testable PBH abundance $\mathbf{f_{PBH}}$ from **future detections of resolvable binary mergers by LISA and ET for extended PBH mass distributions**.

- We compute the expected rate of merger events using the **IMRPhenomXAS waveform** model, which applies to binaries with mass ratios up to $q = 10^3$, and are consequently able to consider **power law mass functions covering 3 orders of magnitude in PBH mass**.

- We find that for high redshifts (z = 30 - 300), the **testable mass range is generally broader** while the **minimum testable abundance is usually higher for the extended distribution case** compared to the monochromatic case.

- We also report how the minimum testable PBH abundance $\mathbf{f_{PBH}}$ varies with the **characteristic mass** and **exponent** of the assumed power law mass distribution.

Method for calculating minimum testable abundance $\mathbf{f_{PBH}}$

1) Set source and detector specifications: ET or LISA.
2) Select power law mass distribution parameters: characteristic mass $\mathbf{M_c}$ and exponent $\boldsymbol{\gamma}$.
3) Select redshift range: $z = [0.01,30)$ or $z = [30, 300]$.
4) For a given $\mathbf{f_{PBH}}$, calculate the expected merger event rate $\mathbf{N_{det}}$.
5) For a given detector, $\mathbf{M_c}$, $\boldsymbol{\gamma}$, and redshift range, check if there is a PBH abundance $\mathbf{f_{PBH}} < 1$ that satisfies $N_{det} > 1/yr$ and find the minimum value; this is our minimum testable abundance.

Power law mass distributions

We consider power law mass distributions of the form, following [1]:

$$\psi_{\rm PL}(M; M_{\rm min}, M_{\rm max}, \gamma) = \mathcal{N}_{\rm PL} M^{\gamma-1}, \quad \mathcal{N}_{\rm PL}(M_{\rm min}, M_{\rm max}, \gamma) = \frac{\gamma}{M_{\rm max}^{\gamma} - M_{\rm min}^{\gamma}}, \quad \text{for } \gamma \neq 0,$$

characterized by an exponent $\boldsymbol{\gamma}$, mass range ($\mathbf{M_{min}}$, $\mathbf{M_{max}}$), and normalization $\mathbf{N_{PL}}$.

Since we can only model GW waveforms from binaries with mass ratios up to $q = 10^3$ (using IMRPhenomXAS models), we set the mass range limits so that

$$\mathbf{q} = \mathbf{M_{max}}/\mathbf{M_{min}} = 10^3.$$

[1] N. Bellomo, J. L. Bernal, A. Raccanelli, and L. Verde, Primordial black holes as dark matter: converting constraints from monochromatic to extended mass distributions, J. Cosmol. Astropart. Phys. 2018, 004 (2018).

Power law mass distributions (cont.)

We also define two additional parameters, following [2]:

- width $\boldsymbol{\sigma} = 1/|\boldsymbol{\gamma}|$

- characteristic mass $\mathbf{M_c}$ related to $\mathbf{M_{min/max}}$ as follows:
    - for $\gamma < 0$: $\mathbf{M_{min} = M_c \exp(1/\gamma)}$
    - for $\gamma > 0$: $\mathbf{M_{max} = M_c \exp(1/\gamma)}$

For our numerical calculations, we consider:
- 10 equally-spaced values of $\boldsymbol{\sigma}$ between 0.01 and 5
- 100 equally log-spaced values of $\mathbf{M_c}$ between 10^1 to 10^8 solar masses for LISA and between 10^1 to 10^9 solar masses for ET.

[2] B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen, H. Veermäe, Primordial black hole constraints for extended mass functions, Phys. Rev. D 96, 023514 (2017).

Calculation of the expected merger event rate $\mathbf{N_{det}}$

PBH binary merger rate

$$\boxed{N_{\rm det}} = \int \mathrm{d}z \mathrm{d}M_1 \mathrm{d}M_2 \, \frac{1}{1+z} \frac{\mathrm{d}V_c(z)}{\mathrm{d}z} \boxed{\frac{\mathrm{d}^2 R_{\rm PBH}}{\mathrm{d}M_1 \mathrm{d}M_2}} \boxed{p_{\rm det}(M_1, M_2, z)}.$$

Number of merger events detected per year

Probability of binary detection
(detector sensitivity, signal-to-noise ratio)

- z: redshift
- M_1 and M_2: component masses of the binaries
- V_c : size of the comoving volume

Calculation of the expected merger event rate $\mathbf{N_{det}}$ (cont.)

For calculation of the probability of binary detection $\mathbf{p_{det}}$:

- $\mathbf{p_{det}}$ is determined by the ratio between the optimal detector signal-to-noise ratio (SNR) $\boldsymbol{\rho_{det}}$ and a preset detection threshold $\boldsymbol{\rho_{thr}}$ (we set $\boldsymbol{\rho_{thr}} = \mathbf{8}$)

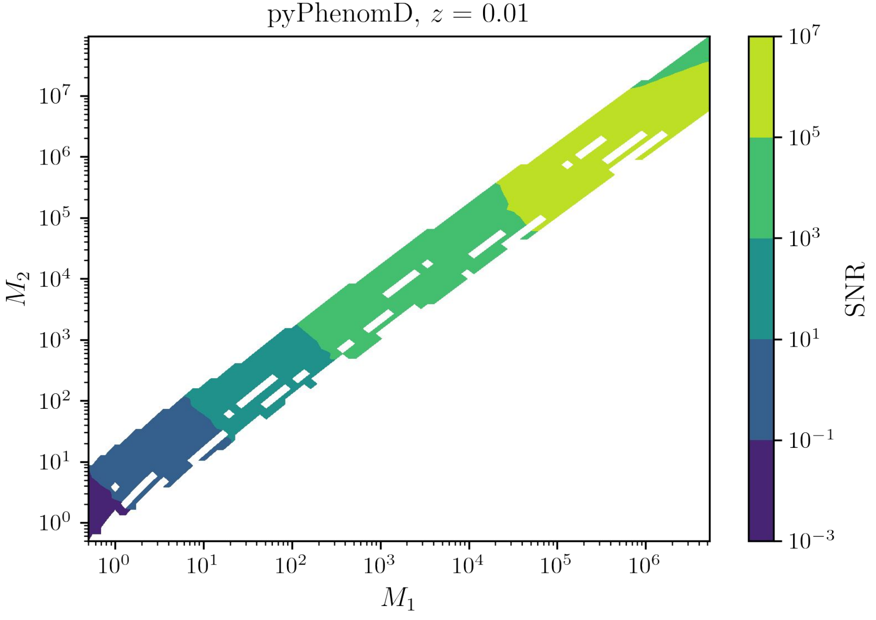
- Detector SNR $\boldsymbol{\rho_{det}}$ is a function of source parameters (binary mass components, spin components, redshift), source waveform models (i.e. IMRPhenomD, **IMRPhenomXAS** [3]), and detector sensitivity curves
  - We assume the sources are non-spinning and non-precessing.

- Simulated $\boldsymbol{\rho_{det}}$ is calculated by inputting the appropriate parameters (mass range, mass ratios, BH spins, etc.) into Python package `gwent`[4]

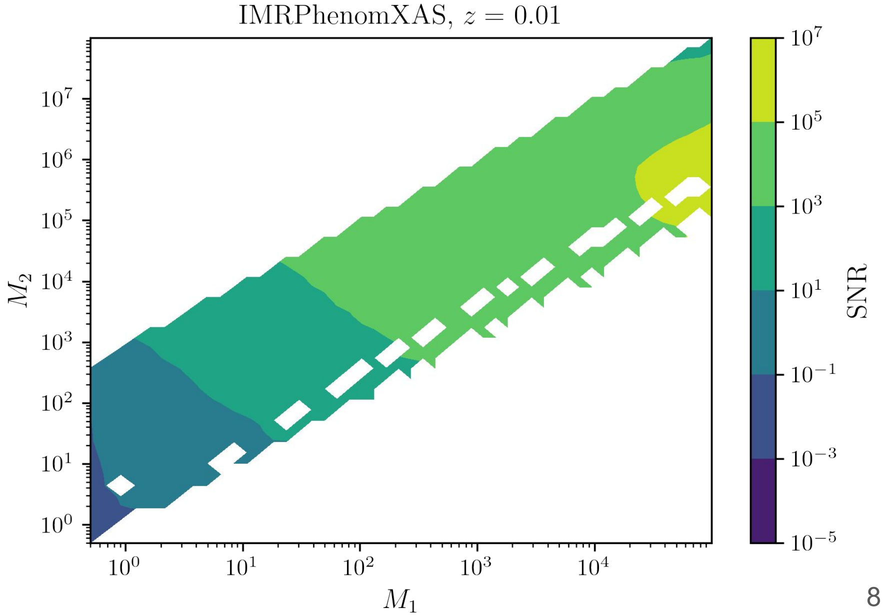
[3] - G. Pratten, S. Husa, C. Garcia-Quiros, M. Colleoni, A. Ramos-Buades, H. Estelles, R. Jaume. Setting the cornerstone for the IMRPhenomX family of models for gravitational waves from compact binaries: The dominant harmonic for non-precessing quasi-circular black holes. Phys. Rev. D 102, 064001 (2020).
[4] - A. R. Kaiser and S. T. McWilliams, Sensitivity of present and future detectors across the black-hole binary gravitational wave spectrum, Class. Quantum Gravity 38, 055009 (2021).

Comparison of projected LISA SNR $\boldsymbol{\rho}_{\mathbf{det}}$ using different waveform models

pyPhenomD*
q = 1 - 18



IMRPhenomXAS
q = 1 - 10^3



*Python implementation of IMRPhenomD

Calculation of the expected merger event rate $\mathbf{N_{det}}$ (cont.)

PBH binary merger rate

$$\boxed{N_{\mathrm{det}}} = \int \mathrm{d}z\mathrm{d}M_1\mathrm{d}M_2 \, \frac{1}{1+z} \frac{\mathrm{d}V_c(z)}{\mathrm{d}z} \boxed{\frac{\mathrm{d}^2 R_{\mathrm{PBH}}}{\mathrm{d}M_1\mathrm{d}M_2}} \boxed{p_{\mathrm{det}}(M_1, M_2, z)}.$$

Number of merger events detected per year

Probability of binary detection
(detector sensitivity, signal-to-noise ratio)

- z: redshift
- M_1 and M_2: component masses of the binaries
- V_c : size of the comoving volume

Calculation of the expected merger event rate $\mathbf{N_{det}}$ (cont.)

For calculation of the PBH binary merger rate:

PBH abundance (constrained variable)

$$\frac{d^2 R_{\rm PBH}}{dM_1 dM_2} = \frac{1.6\times10^6}{\rm Gpc^3\,yr} \boxed{f_{\rm PBH}^{53/37}} \left(\frac{(M_1+M_2)^2}{M_1 M_2}\right)^{34/37} \left(\frac{t}{t_0}\right)^{-34/37} \left(\frac{M_1+M_2}{M_\odot}\right)^{-32/37}$$
$$\times\, S(M_1, M_2, \boxed{f_{\rm PBH}}, t) \boxed{\psi(M_1)\psi(M_2)},$$

PBH mass distribution

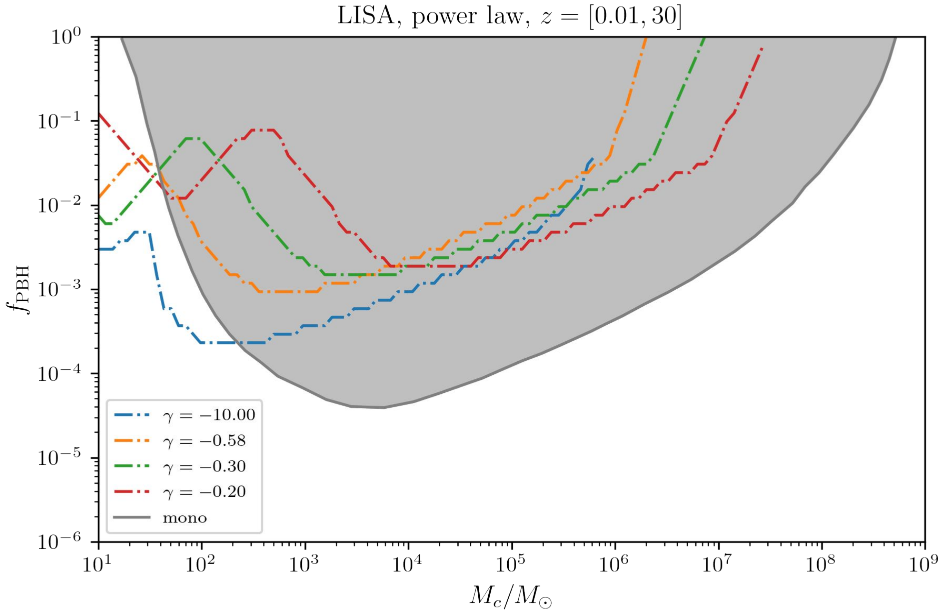
- M_1 and M_2: component masses of the binaries
- t: age of universe upon merger; t_0: current age of the universe
- S: merger suppression factor

Results

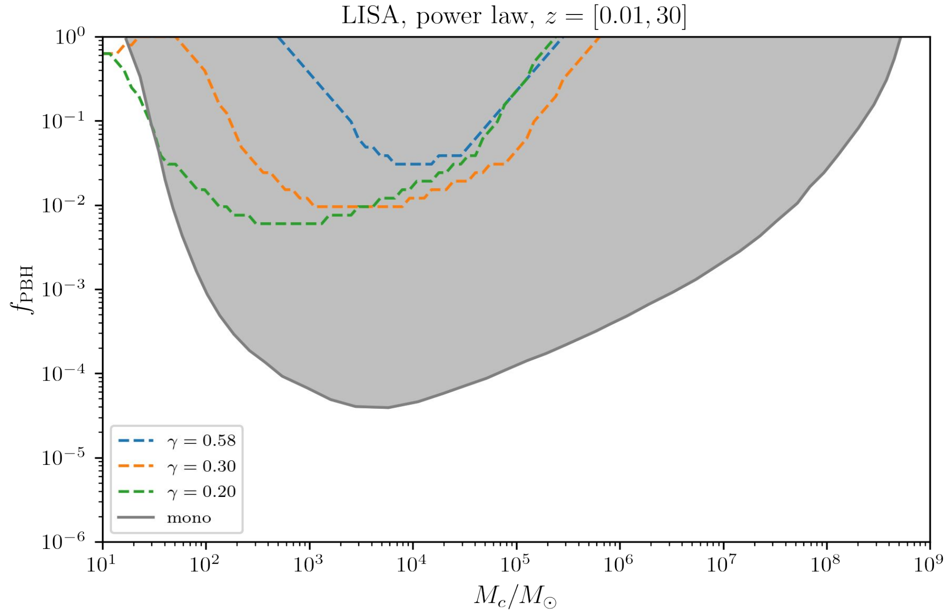
f_{PBH}-M_c constraint curves

Minimum testable f_{PBH}: low z (z < 30), LISA

γ<0



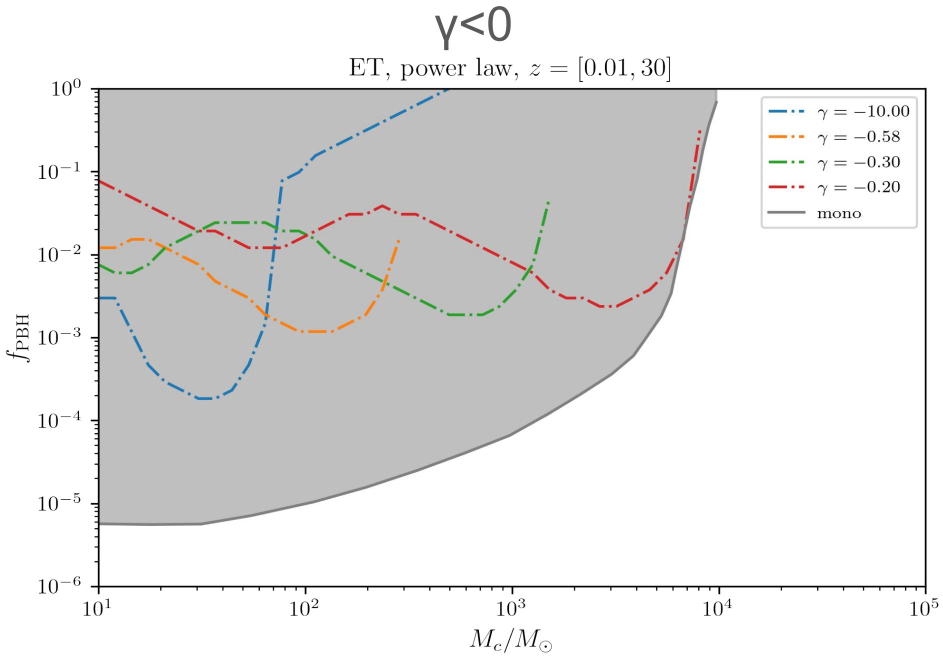
γ>0

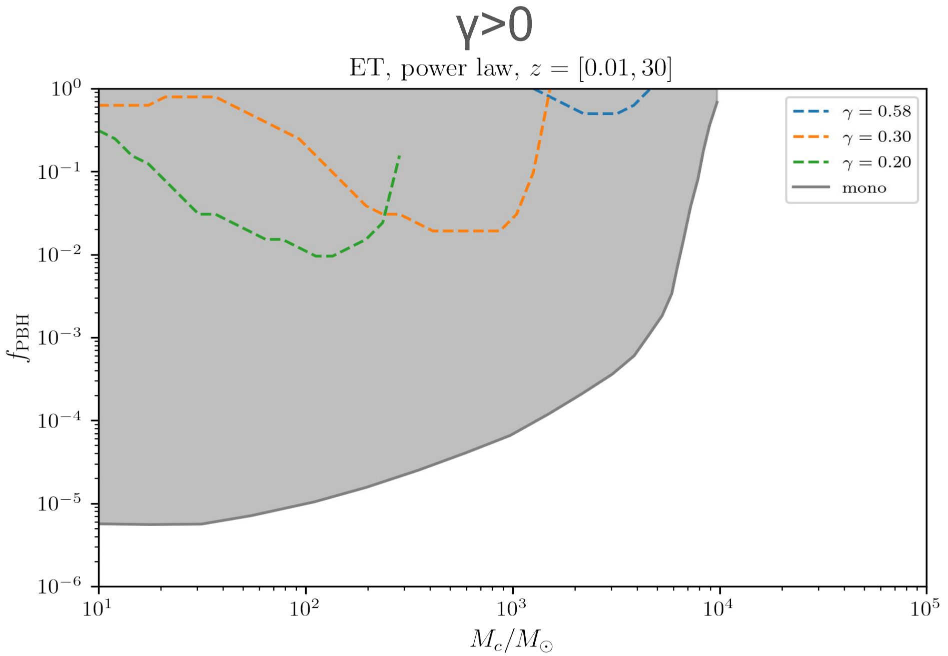


For both γ<0 and γ>0, constraints are generally less stringent for the power law distributions than the monochromatic case (grey region; taken from De Luca, et al. 2021) [5]

[5] - V. De Luca, G. Franciolini, P. Pani, and A. Riotto, The minimum testable abundance of primordial black holes at future gravitational-wave detectors, J. Cosmol. Astropart. Phys. 2021, 039 (2021).

Minimum testable f_{PBH}: low z (z < 30), ET

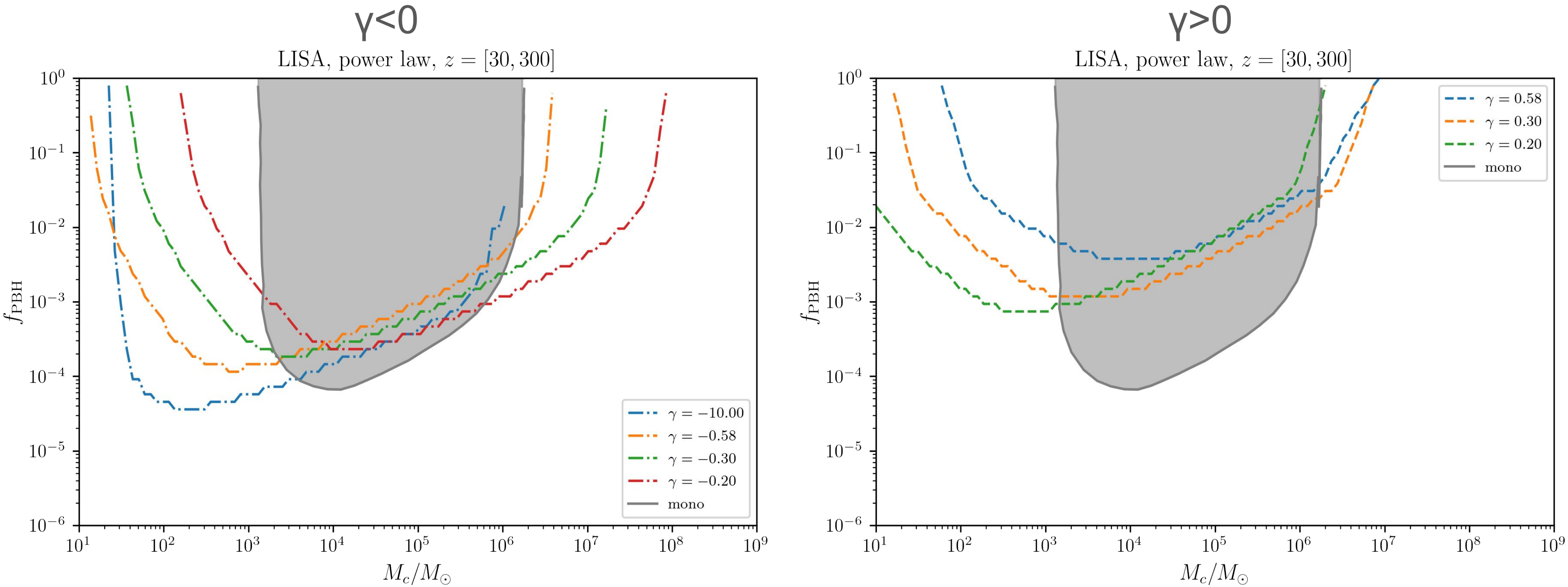




Same as for LISA, constraints are generally less stringent for the power law distributions than the monochromatic case (grey region; taken from De Luca, et al. 2021) [5]

[5] - V. De Luca, G. Franciolini, P. Pani, and A. Riotto, The minimum testable abundance of primordial black holes at future gravitational-wave detectors, J. Cosmol. Astropart. Phys. 2021, 039 (2021).

Minimum testable f_{PBH}: high z (30 < z < 300), LISA

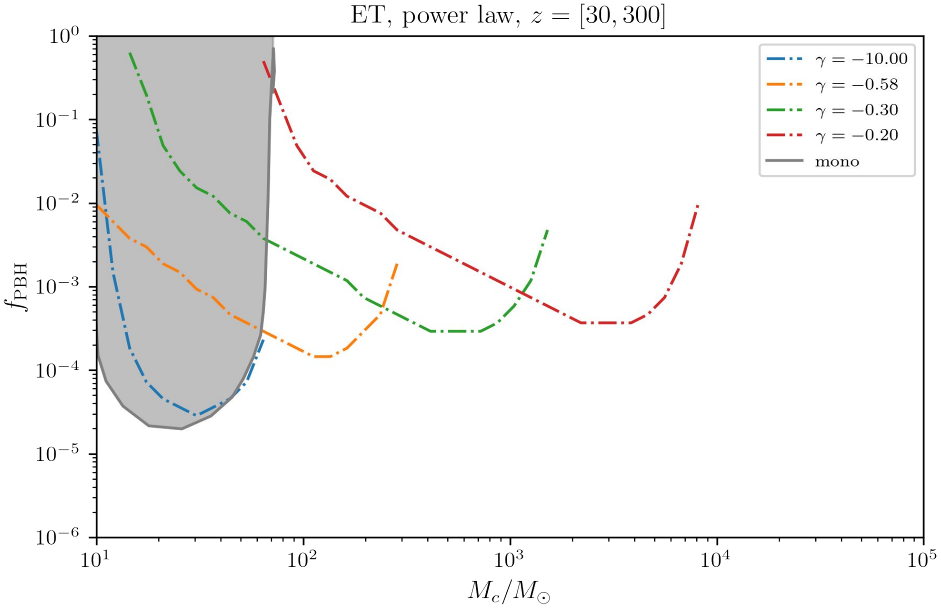


Broader testable range of characteristic masses M_c for the power law distributions than the monochromatic case (grey region; taken from De Luca, et al. 2021) [5]

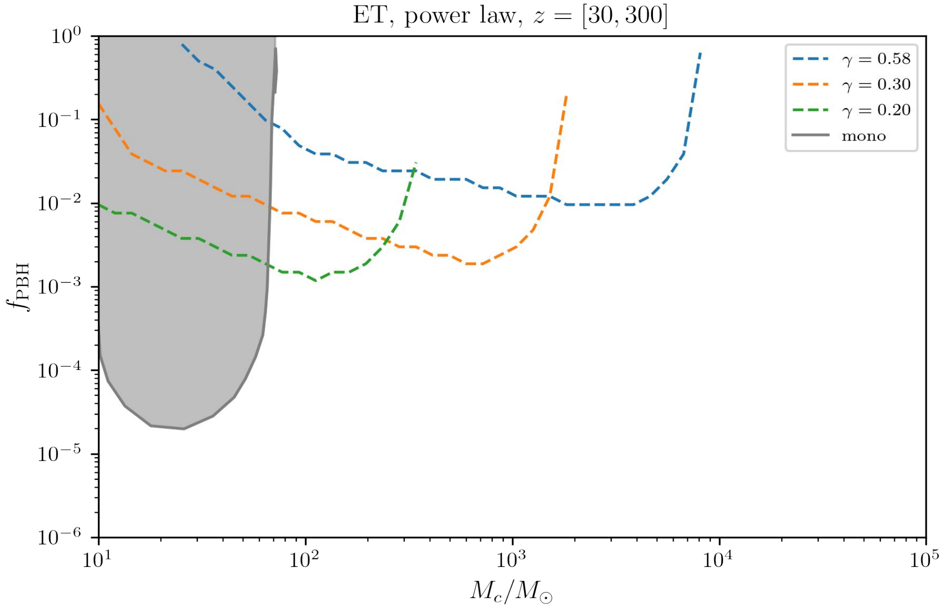
[5] - V. De Luca, G. Franciolini, P. Pani, and A. Riotto, The minimum testable abundance of primordial black holes at future gravitational-wave detectors, J. Cosmol. Astropart. Phys. 2021, 039 (2021).

Minimum testable f_{PBH}: high z (30 < z < 300), ET

γ<0



γ>0



Broader testable range of characteristic masses M_c for the power law distributions than the monochromatic case (grey region; taken from De Luca, et al. 2021) [5]

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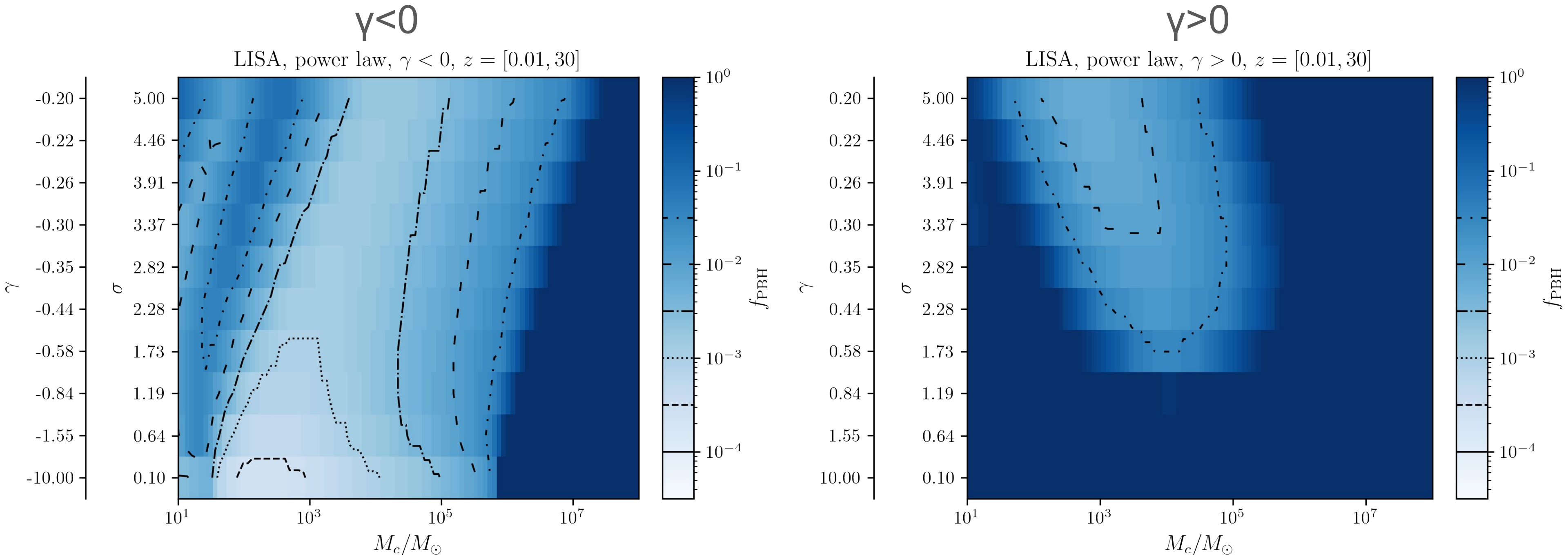
Results

f_{PBH} constraint contours in M_c-σ space

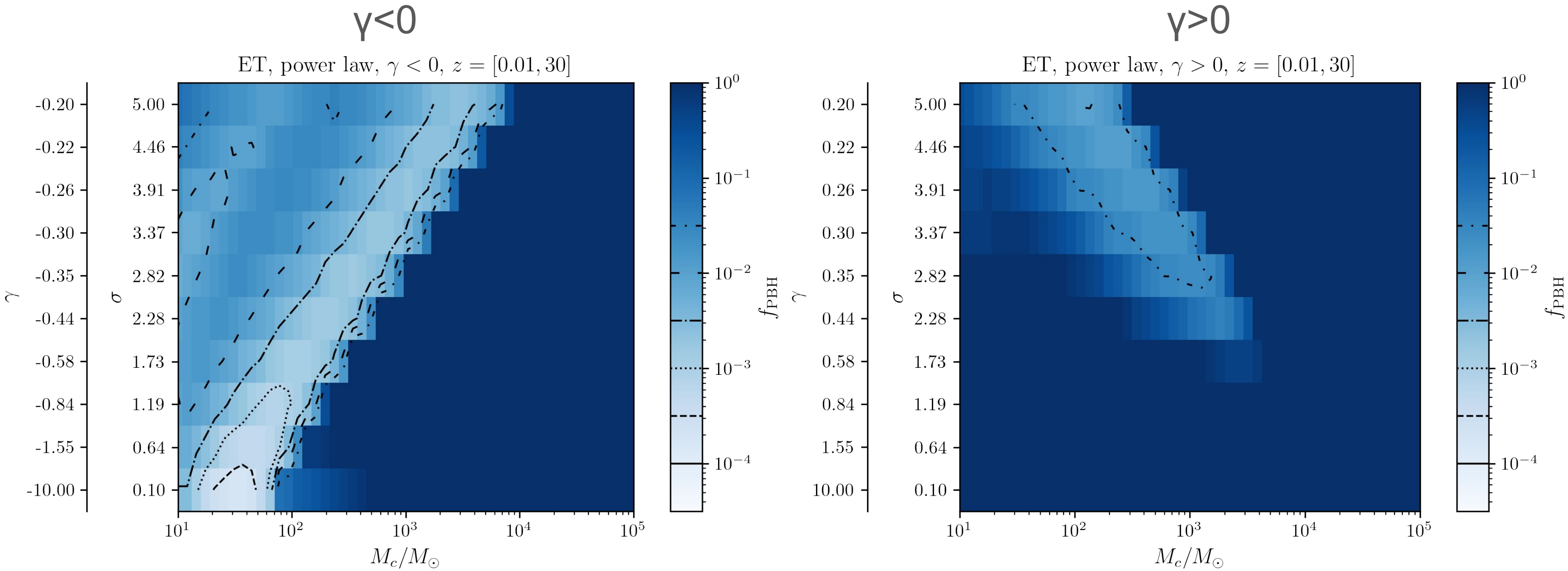
Minimum testable f_{PBH}: low z (z < 30), LISA



Contour lines represent $\log_{10}(f_{PBH})$ values at -1.5 (wide dash-dotted), -2 (wide dashed), -2.5 (dash-dotted), -3 (dotted), -3.5 (dashed), and -4 (solid line).

N.B. Regions shown as dark blue are *not* testable (N_{det} < 1/yr even with f_{PBH} = 1).

Minimum testable f_{PBH}: low z (z < 30), ET



Contour lines represent $\log_{10}(f_{PBH})$ values at -1.5 (wide dash-dotted), -2 (wide dashed), -2.5 (dash-dotted), -3 (dotted), -3.5 (dashed), and -4 (solid line).

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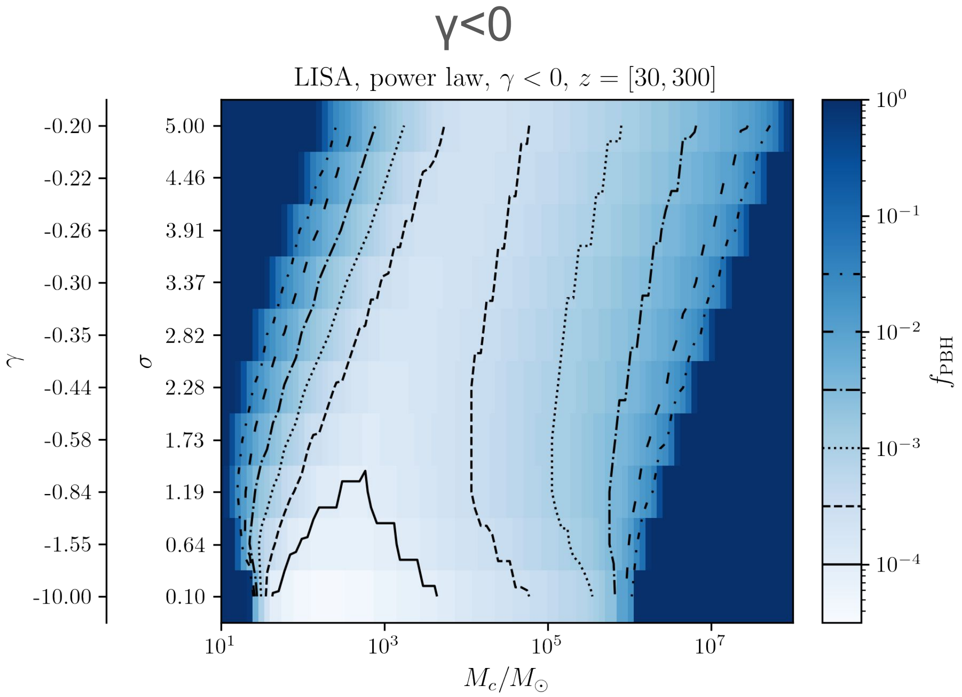
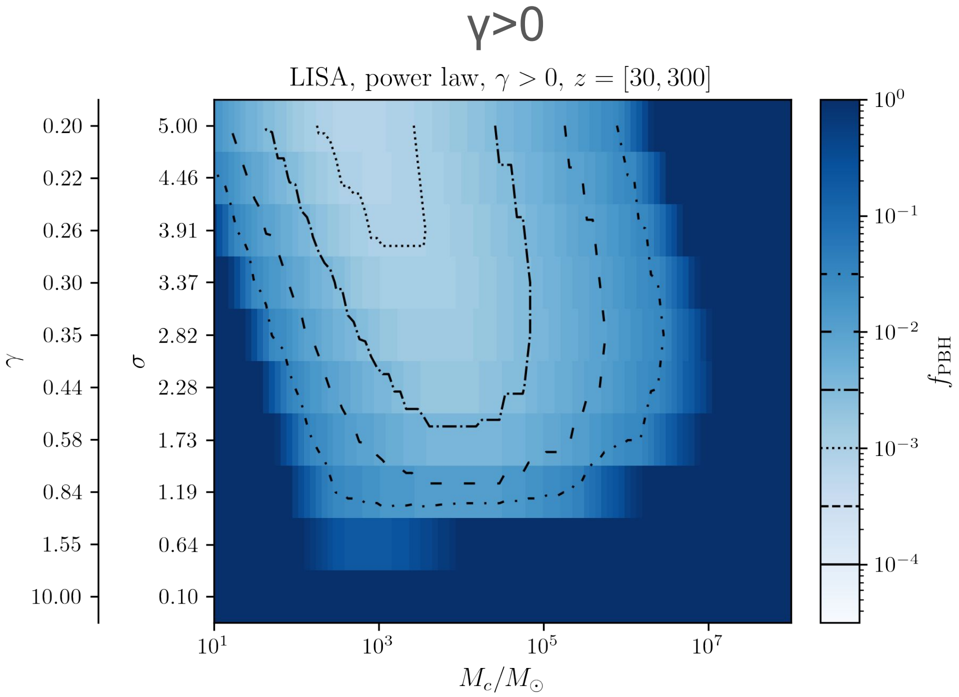
Minimum testable f_{PBH}: high z (30 < z < 300), LISA





Contour lines represent $\log_{10}(f_{PBH})$ values at -1.5 (wide dash-dotted), -2 (wide dashed), -2.5 (dash-dotted), -3 (dotted), -3.5 (dashed), and -4 (solid line).

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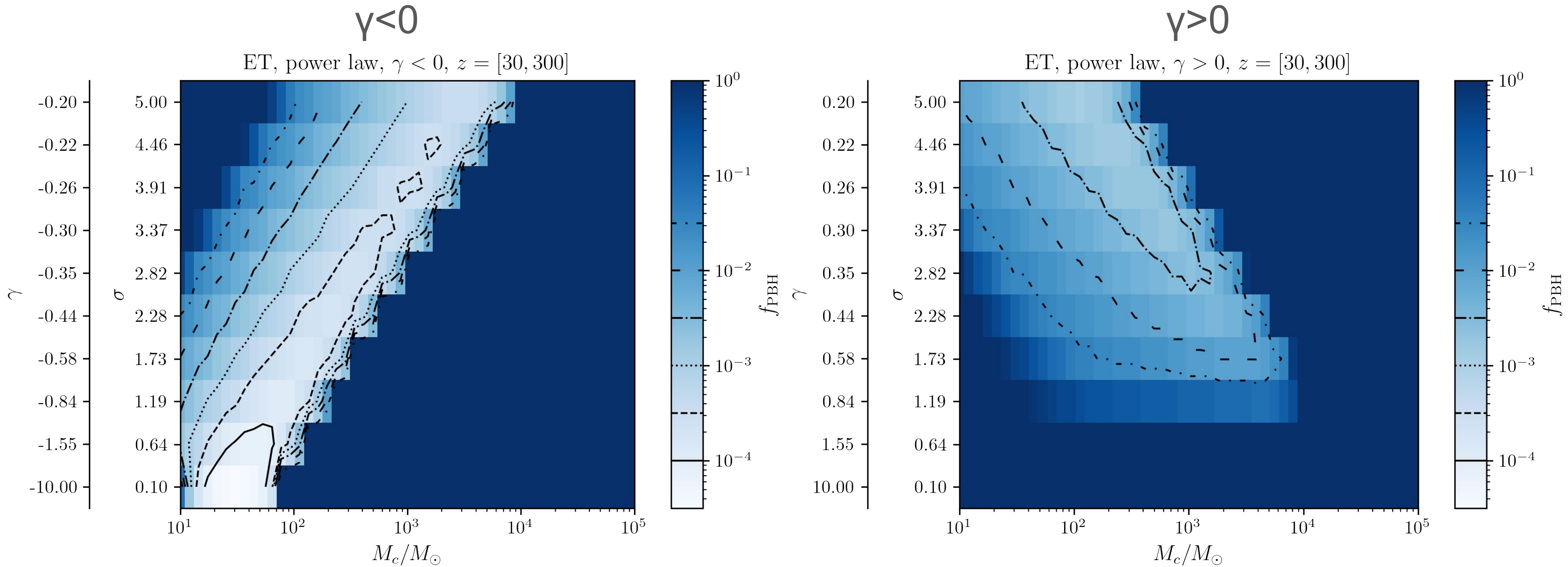
Minimum testable f_{PBH}: high z (30 < z < 300), ET



Contour lines represent $\log_{10}(f_{PBH})$ values at -1.5 (wide dash-dotted), -2 (wide dashed), -2.5 (dash-dotted), -3 (dotted), -3.5 (dashed), and -4 (solid line).

N.B. Regions shown as dark blue are *not* testable (N_{det} < 1/yr even with f_{PBH} = 1).

Results: f_{PBH} constraint contours in M_c-σ space

- For power law distributions with negative exponents $\gamma < 0$, the **testable mass range extends to larger characteristic masses M_c with smaller $|\gamma|$** (larger σ).

  This effect is more pronounced for ET than LISA detections, with the maximum testable M_c showing an increase by ~2 orders of magnitude as σ increases from 0.01 to 5.

- For power law distributions with positive exponents $\gamma > 0$, the **testable mass range becomes narrower with larger $|\gamma|$** (smaller σ) until some threshold value, above which the detectors do not provide any constraints (N_{det} < 1/yr even with $f_{PBH} = 1$).

  - LISA thresholds: $\gamma \sim 0.6$ for low redshift, $\gamma \sim 1.6$ for high redshift
  - ET thresholds: $\gamma \sim 0.6$ for low redshift, $\gamma \sim 0.8$ for high redshift

Summary

- We calculate projected constraints on the PBH abundance $\mathbf{f_{PBH}}$ from future detections of resolvable binary mergers (by LISA and ET) for power law PBH mass distributions and find that they **generally become less stringent compared to the monochromatic case** (which is the assumption used in most existing studies).

- We find that for high redshifts ($z = 30 - 300$), where astrophysical sources are expected to have lower or no contribution:

    - The **testable mass range is generally broader for the extended case** than the monochromatic case (albeit with higher minimum testable abundances).

    - Over the parameter range considered, **both LISA and ET can potentially detect PBH merger signals if $\mathbf{f_{PBH} \gtrsim 10^{-5}}$**.

Extensions (works in progress)

- Apply to stochastic gravitational wave background
- Include effects of taking into account clustering and accretion
- Explore effects of extended populations on the PBH merger rate in more detail by considering toy model distributions

Thank you.

Looking for collaborators!
Email me at gldizon@nip.upd.edu.ph if you're interested.

Overview

- We report constraints on the minimum projected PBH abundance $\mathbf{f_{PBH}}$ from future detections of resolvable binary mergers by LISA and ET for **extended PBH mass distributions**.

- We compute the expected rate of merger events using the **IMRPhenomXAS waveform** model, which applies to binaries with mass ratios up to q = 1000, and are consequently able to consider **power law mass functions covering 3 orders of magnitude in PBH mass**.

- We find that incorporating higher mass ratios into the calculation raises the required minimum abundances in order to achieve a detectable event signal for broad mass functions, resulting in minimum abundances higher by an order of magnitude or greater relative to the monochromatic case.

Power law mass distributions

We consider power law mass distributions of the form, following [1]:

$$\psi_{\rm PL}(M; M_{\rm min}, M_{\rm max}, \gamma) = \mathcal{N}_{\rm PL} M^{\gamma-1}, \quad \mathcal{N}_{\rm PL}(M_{\rm min}, M_{\rm max}, \gamma) = \frac{\gamma}{M_{\rm max}^{\gamma} - M_{\rm min}^{\gamma}}, \quad \text{for } \gamma \neq 0,$$

- γ refers to the slope of the distribution
    - Carr, et al. (2017) [1] provides a relation between slope γ and width $\sigma = 1/|\gamma|$
- initial $M_{min/max}$ bounds set as $M_{min/max} = M_c \exp(1/\gamma)$, M_{min} if $\gamma < 0$, M_{max} if $\gamma > 0$
    - M_c defined as the reference mass of the distribution
- remaining bound is set by multiplying/dividing initial bound by maximum mass ratio $q = 1000$ (i.e. $M_{max} = qM_{min}$ for $\gamma < 0$, and vice versa)

[1] - B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen, H. Veermäe, Primordial black hole constraints for extended mass functions, Phys. Rev. D 96, 023514 (2017).

Power law distribution

$$\psi_{\rm PL}(M; M_{\rm min}, M_{\rm max}, \gamma) = \mathcal{N}_{\rm PL} M^{\gamma-1}, \quad \mathcal{N}_{\rm PL}(M_{\rm min}, M_{\rm max}, \gamma) = \frac{\gamma}{M_{\rm max}^{\gamma} - M_{\rm min}^{\gamma}}, \quad \text{for } \gamma \neq 0,$$

- cases considered: $0.01 < \sigma < 5$, equally spaced into 10 bins
    - γ determined by the relation $\sigma = 1/|\gamma|$
- reference mass M_c for determining bounds $M_{min/max}$ sampled from 10 solar masses to 10^9 solar masses

Results: f_{PBH} constraint contours in M_c-σ space

- For power law distributions with negative exponents $\gamma < 0$, the **testable mass range extends to larger characteristic masses M_c with smaller $|\gamma|$** (larger σ).

  This effect is more pronounced for ET than LISA detections, with the maximum testable M_c showing an increase by ~2 orders of magnitude as σ increases from 0.01 to 5.

- For power law distributions with positive exponents $\gamma > 0$, the **testable mass range becomes narrower with larger $|\gamma|$** (smaller σ) until some threshold value, above which the detectors do not provide any constraints ($N_{det} < 1$/yr even with $f_{PBH} = 1$).

  - LISA thresholds: $\gamma \sim 1.2$ for low redshift, $\gamma \sim 0.6$ for high redshift
  - ET thresholds: $\gamma \sim 1.7$ for low redshift, $\gamma \sim 1.2$ for high redshift

Summary & Conclusions

- Power law models result in overall higher minimum abundances in comparison to the monochromatic case, indicating that the latter assumption leads to overestimation on constraints from future detectors.
- More positive power law slopes project the need for higher PBH abundances to produce resolvable signals within detection windows.
- For $\gamma > 0$, the most constrained reference mass increases with higher γ until a certain threshold, beyond which the detector cannot provide a constraint for the entire mass range.
    - LISA threshold: $\gamma \sim 1.2$ for low redshift, $\gamma \sim 0.6$ for high redshift
    - ET threshold: $\gamma \sim 1.7$ for low redshift, $\gamma \sim 1.2$ for high redshift

Summary & Conclusions

- For $\gamma < 0$, the maximum reference mass constraint decreases with more negative γ.
- Higher redshifts decrease required minimum abundances but do not go far below monochromatic projections.
- Bounds from GW observations may still change if coupling to clustering and accretion effects are accounted for.