Detection of PBH with Einstein Telescope: forecast for statistical methods to distinguish between BH populations

Based on:

MM, Scarcella, Hogg, Kavanagh, Gaggero, Fleury JCAP (2022)

Dark sirens as a measure of distance

Since the first GW detection in 2015 the number of available observations from binary mergers has been increasing steadily.

Most of the GW observations we have available do not have an EM counterpart and therefore bring no redshift information with them.

$$h(f) = A\, f^{-\frac{7}{6}} \exp\left[i(2\pi f t_0 - \frac{\pi}{4} + 2\Psi(f/2) - \varphi_{(2,0)})\right]$$

$$A = \frac{1}{d_L(z)}\sqrt{F_+^2\left[1+\cos^2\iota\right]^2 + 4F_\times^2\cos^2\iota}\sqrt{\frac{5\pi}{96}}\pi^{-7/6}\mathcal{M}^{\frac{5}{6}}$$

Analyzing these events one can extract information on the existence of a PBH population.

Modelling ABH and PBH

PBH merger events will have features that distinguish them from other events. We focus on the possibility to use their distance distribution to distinguish them from a dominant ABH population.

In order to do so we need to model the populations that are present in the catalogues.

$$p(z) = \frac{T_{\rm obs} R(z)}{\bar{N}_{\rm tot}} = \frac{R(z)}{\int_{z_{\rm min}}^{z_{\rm max}} \mathrm{d}\zeta \; R(\zeta)} \qquad R(z) = \frac{\mathcal{R}(z)}{1+z} \frac{\mathrm{d}V_{\rm c}}{\mathrm{d}z} \qquad \bar{N}_{\rm tot} = T_{\rm obs} \int_{z_{\rm min}}^{z_{\rm max}} \mathrm{d}z \; R(z)$$

Assuming cosmology as given by external surveys, we need to model the merger rate density for the two populations.

Theoretical setting: ABH progenitors for GW

To obtain the merger rate density for ABH we follow the approach of
Dvorkin et al. MNRAS (2016)

$$\mathcal{R}_{\rm ABH}(t, M_{\rm ABH}) = \mathcal{N}_{\rm ABH} \int_{\Delta t_{\rm min}}^{\Delta t_{\rm max}} {\rm d}\Delta t \, p(\Delta t) \, \mathcal{R}_{\rm birth}(t - \Delta t, M_{\rm ABH})$$

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We explore two different SFR models.

- Fiducial
- GRB

$$\psi_{\rm SFR}(z) = k \, \frac{a \, {\rm e}^{b(z - z_{\rm m})}}{a - b + b \, {\rm e}^{a(z - z_{\rm m})}}$$

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We assume a monochromatic mass for ABH (7 solar masses)
Warning: we neglect the impact of possible Pop III stars

Theoretical setting: PBH progenitor for GW

We assume PBH formation to be dominated by early-time mechanisms, no initial clustering (Poisson distributed). Following e.g. Ali-Haïmoud et al. PRD (2017)

$$\mathcal{R}_{\rm PBH}[z(t)] = \frac{1}{2} n_{\rm PBH} \int {\rm d}a\, {\rm d}j\; p(j|a) p(a)\, \delta\left[t - t_{\rm merger}(a, j)\right]$$

$$t_{\rm merger} = \frac{3}{170} \frac{c^5}{G^3\, M_{\rm PBH}^3}\, a^4 j^7$$

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PBH merger rate density extends to high redshift with mergers happening at early times. We focus on PBH with masses around 10 solar masses.

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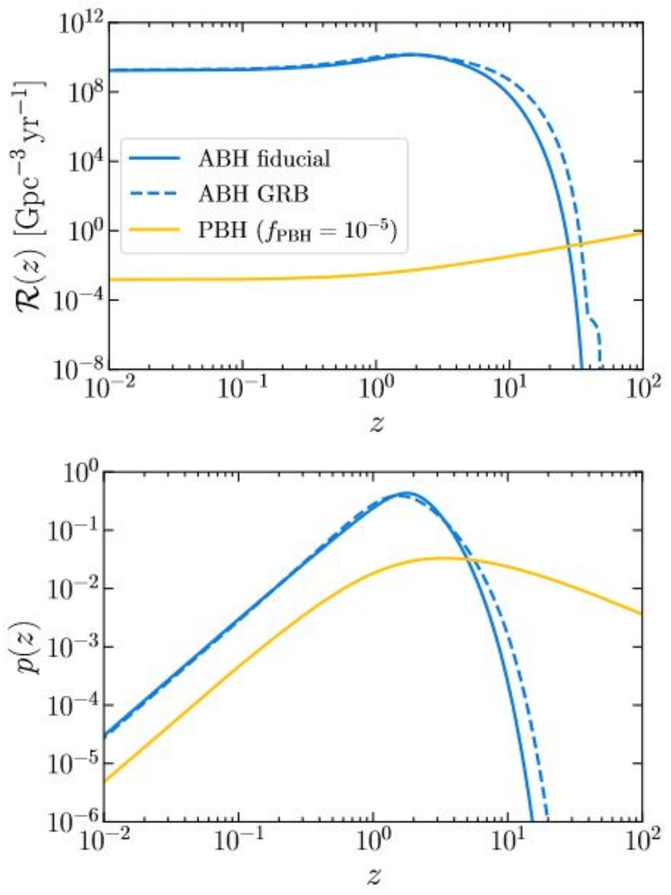
ABH and PBH events have significantly different features, in particular their redshift distribution.

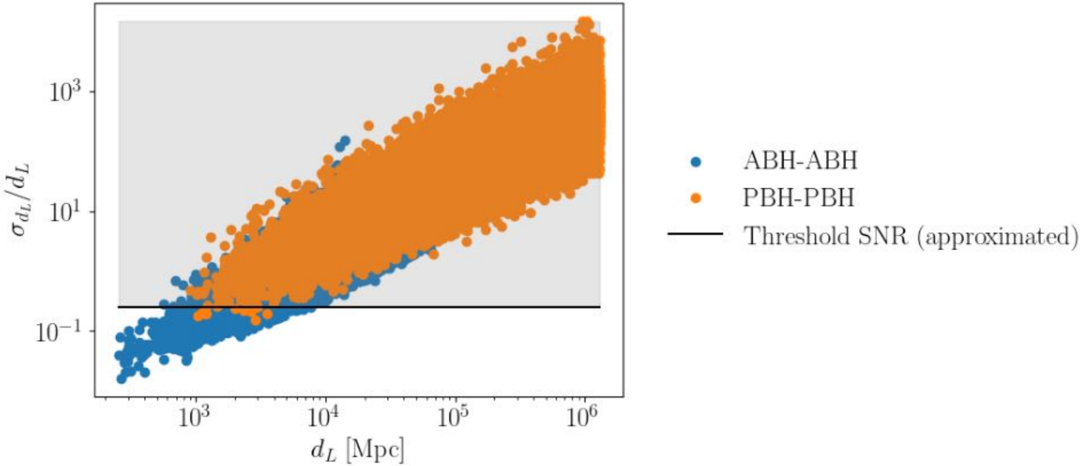
- can we find an excess at high redshift hinting for PBH?
- if we do, can we measure the quantity of DM in PBH?



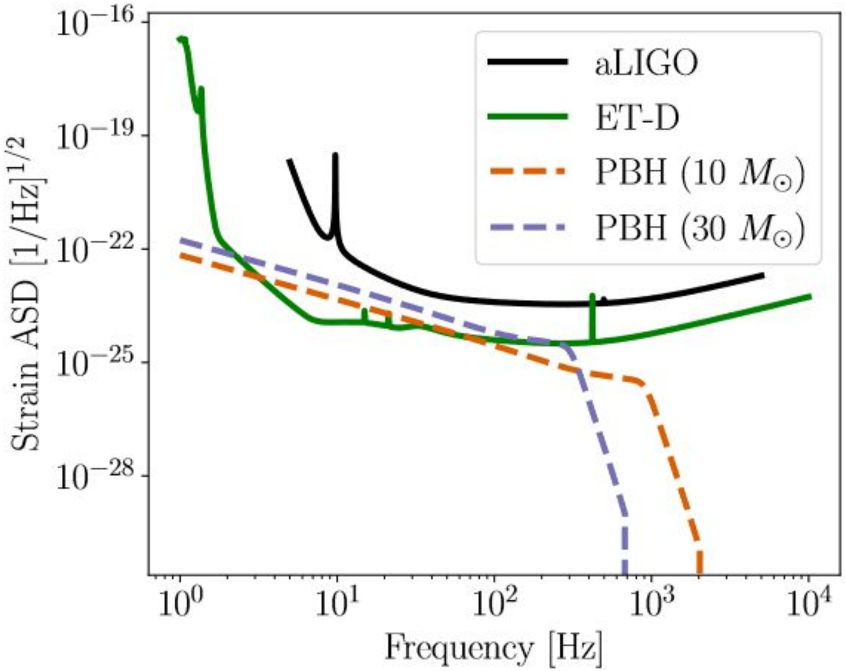
Current GW surveys

At first we thought to work with the currently available data from GWTC.

These however do not go deep enough in redshift to distinguish the PBH population, most of these events are below the SNR threshold.



The Einstein Telescope



We decided to focus on the future detector Einstein Telescope.

This will provide a great boost in sensitivity and therefore possibly allow to observe also events that are extremely distant, as those that we would expect from PBH mergers.

Simulated ET data

With our modelling of the MR, we simulate a dataset with ET specifications.

- We assume a fiducial cosmology, ABH and PBH mass and fraction of PBH;
- we normalize the merger rate so that at low redshift matches observations;
- we compute the total number of observed events given the specs;
- for each event we compute the observational error;

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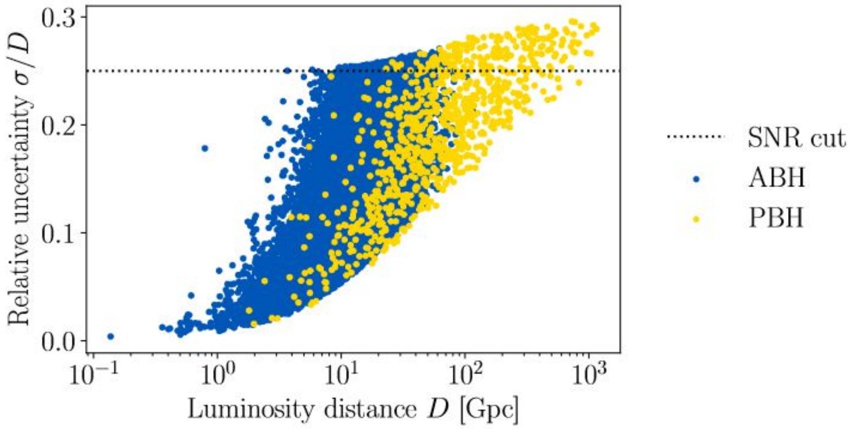

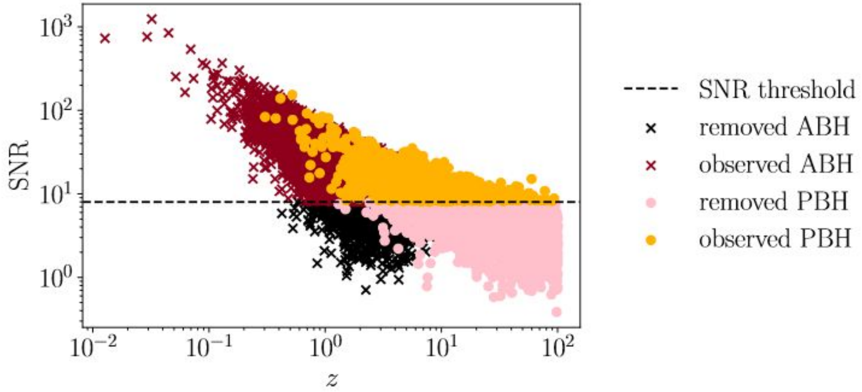

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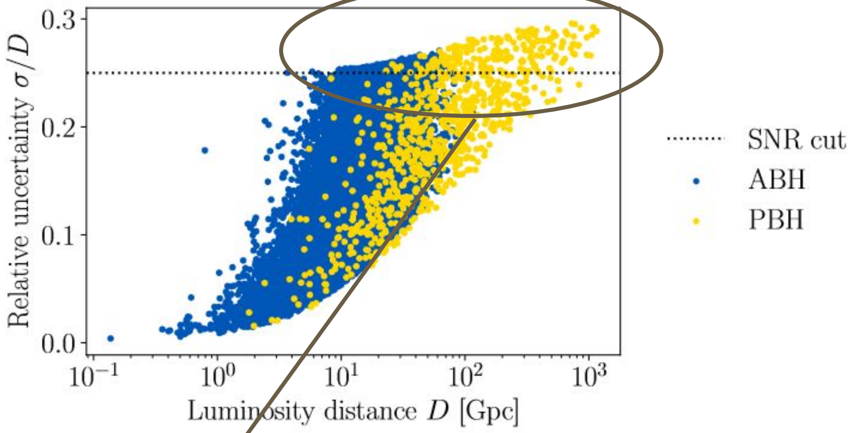

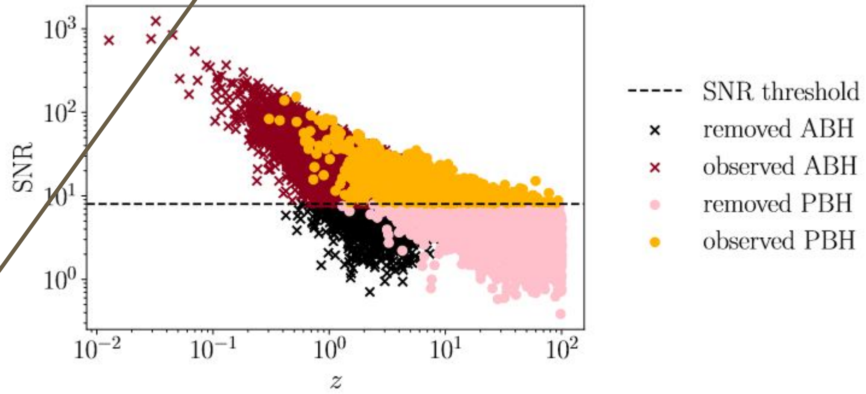

Effect of GW lensing

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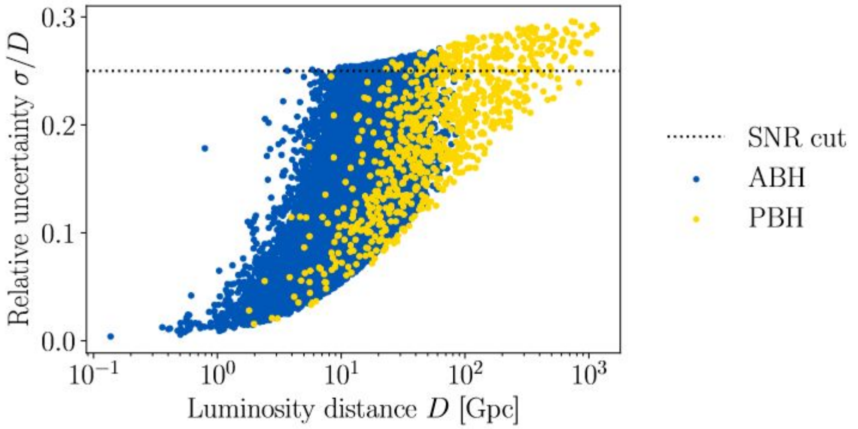
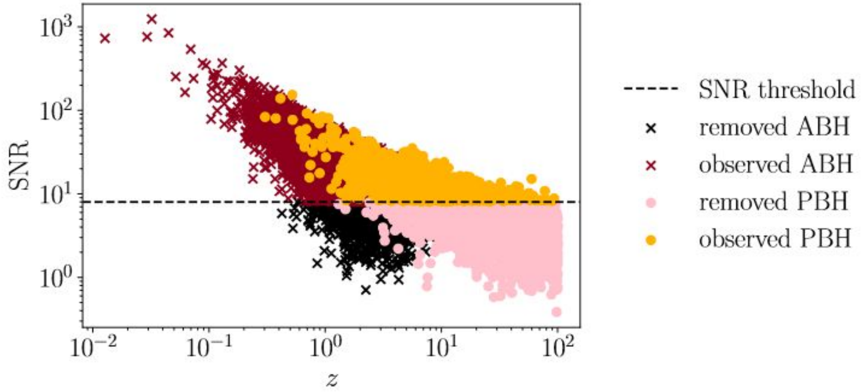




Publicly released code *darksirens*: simulation of GW observations from ABH and PBH

Can we distinguish the two populations?

The different behaviour at high redshift of the two population implies that a sufficiently deep survey would allow to distinguish ABH and PBH.

GW observation going deep enough can provide information on the presence of PBH and possibly constrain them.

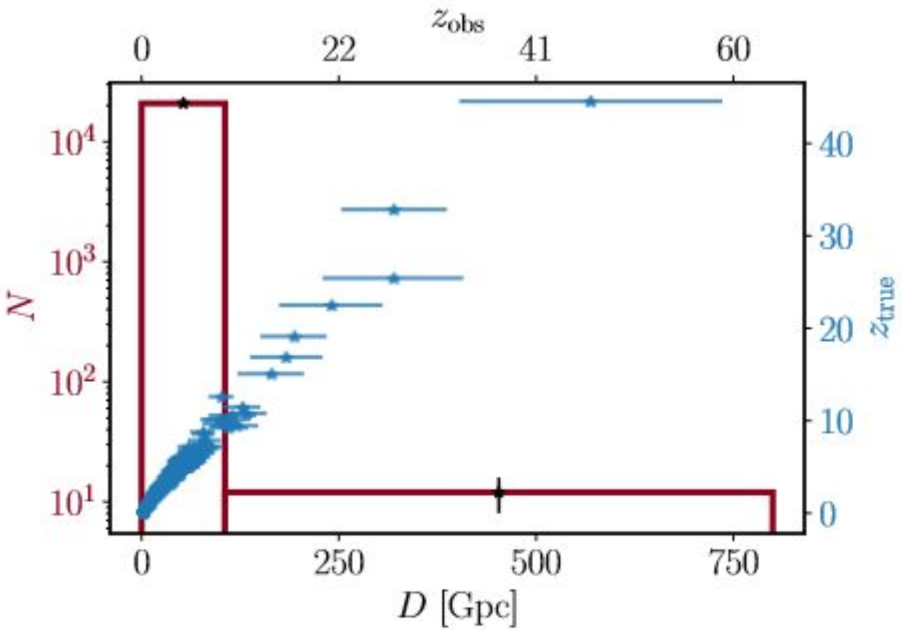
With such data we can measure the excess of events at high redshift with respect to the ABH only prediction and attempt to

- *detect* the presence of PBH
- *measure* the amount of DM made up by PBH

Detecting PBH: the cut-and-count method

We simulate data for different amounts of PBH, compute the number of events over a chosen distance threshold, and compare with the no PBH prediction.

$$\mathcal{S}(\mathcal{D}_{f_{\rm PBH}}, z_*) \equiv \frac{|N_>(\mathcal{D}_{f_{\rm PBH}}, z_*) - N_>(\mathcal{D}_0, z_*)|}{\sqrt{\sigma^2_>(\mathcal{D}_{f_{\rm PBH}}, z_*) + \sigma^2_>(\mathcal{D}_0, z_*)}}$$



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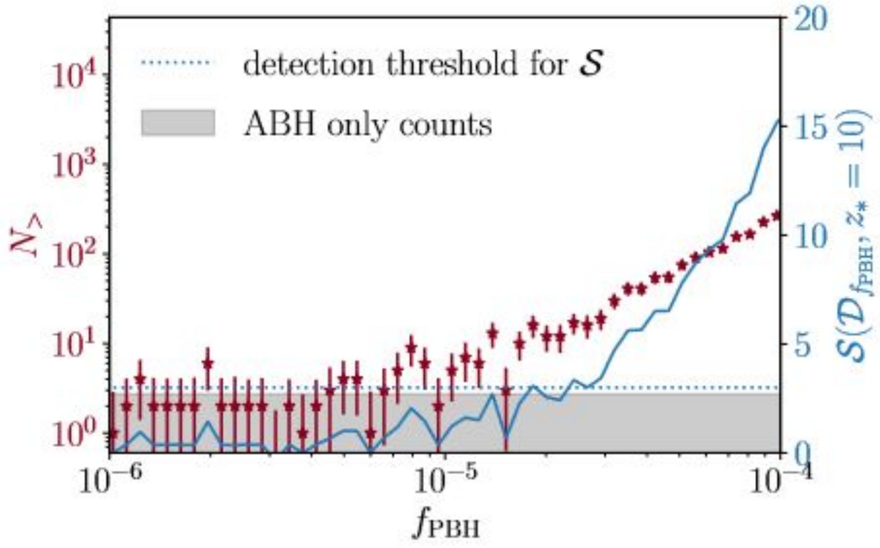

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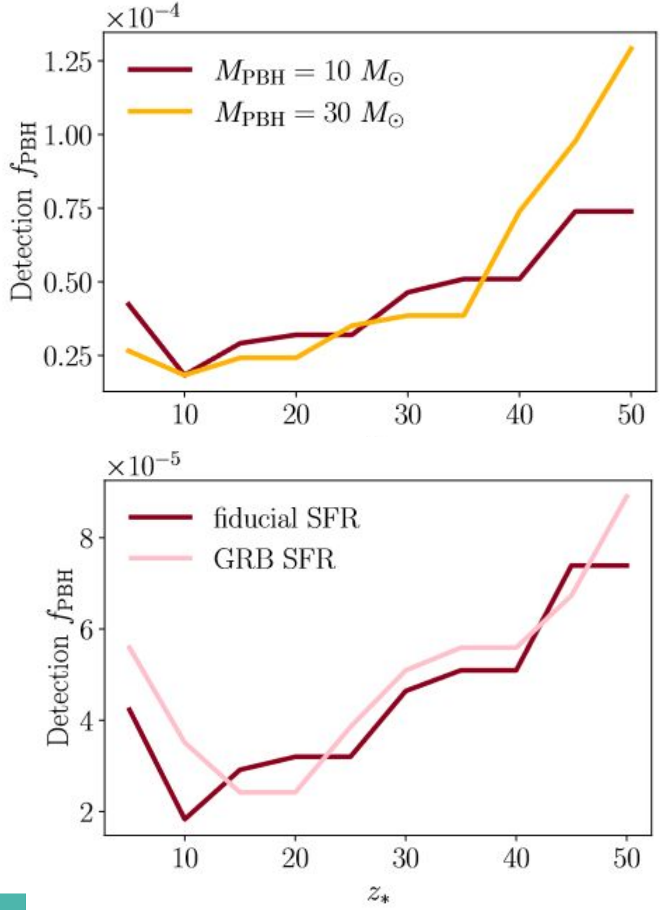

Measuring PBH: likelihood approach

The previous method only tells us if PBH are present or not.

If we want to measure the amount of PBH we need a likelihood method

$$p(f_{\rm PBH}|\mathcal{D}) \propto \mathcal{L}(\mathcal{D}|f_{\rm PBH})\Pr(f_{\rm PBH})$$

$$\mathcal{L}(\mathcal{D}|f_{\rm PBH}) = \frac{\bar{N}_{\rm obs}(f_{\rm PBH})^{N_{\rm obs}} e^{-\bar{N}_{\rm obs}(f_{\rm PBH})}}{N_{\rm obs}!} \times \prod_{i=1,N_{\rm obs}} p(D_i|f_{\rm PBH})$$

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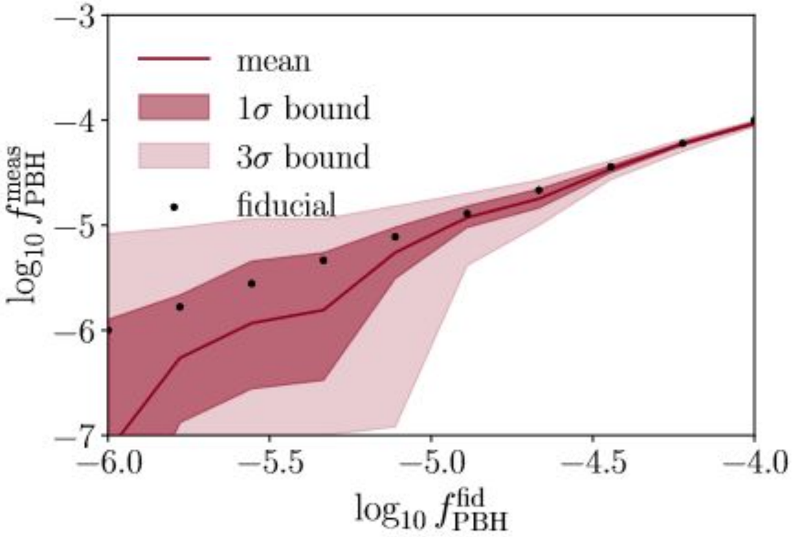

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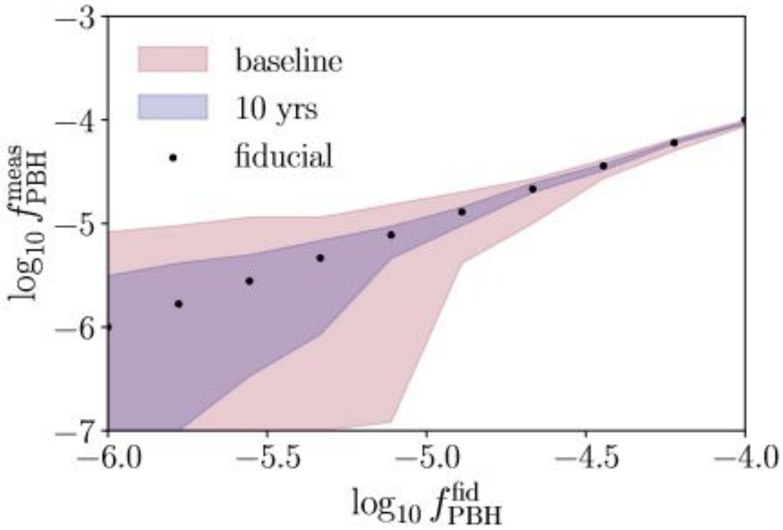

Conclusions

- GW observation are a crucial probe to detect the presence of PBH;
- If we rely on distance distribution, current data do not go deep enough to serve our purpose. ET will have the required sensitivity!
- We developed a simulation code able to provide simulated data for ET in the presence of PBH;
- Two different methods applied to such simulations highlight how this survey could potentially detect PBH and constrain their amount.

The road ahead

- Our results assume monochromatic masses for ABH and PBH. Crucial to include a mass distribution in the analysis;

- We considered limited SFR models and excluded possible Pop III stars, which will have an impact at high redshift;

- We are only looking at distance (redshift). A lot of information is neglected

- Our analysis methods are model dependent, i.e. our PBH modelling enters the analysis. We need to explore different techniques (NN classification?)

Fisher matrix for GW observations

The Fisher matrix for GW observations can be written as

$$F_{\alpha\beta} = \langle \frac{\partial h}{\partial \theta_\alpha} | \frac{\partial h}{\partial \theta_\beta} \rangle$$

with the inner product defined as

$$\langle h_1 | h_2 \rangle = 4\,\mathrm{Re} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \frac{h_1(f) h_2^*(f)}{S(f)}\, \mathrm{d}f$$

What can we obtain from this Fisher matrix?

This Fisher matrix does not give us the cosmological parameters, but rather the errors that can be estimated from observations on the quantities entering the strain.

$$h(f) = A\, f^{-7/6} \exp\left[i\left(2\pi f t_0 - \frac{\pi}{4} + 2\Psi(f/2) - \varphi_{(2,0)}\right)\right]$$

$$A = \sqrt{\frac{5\pi}{96}}\pi^{-7/6}\frac{\mathcal{M}_c^{5/6}}{d_L(z)}$$

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Considering only the distance as free

If we only consider the luminosity distance as a free quantity, our Fisher matrix will be a rank 1 matrix, with the only derivative being

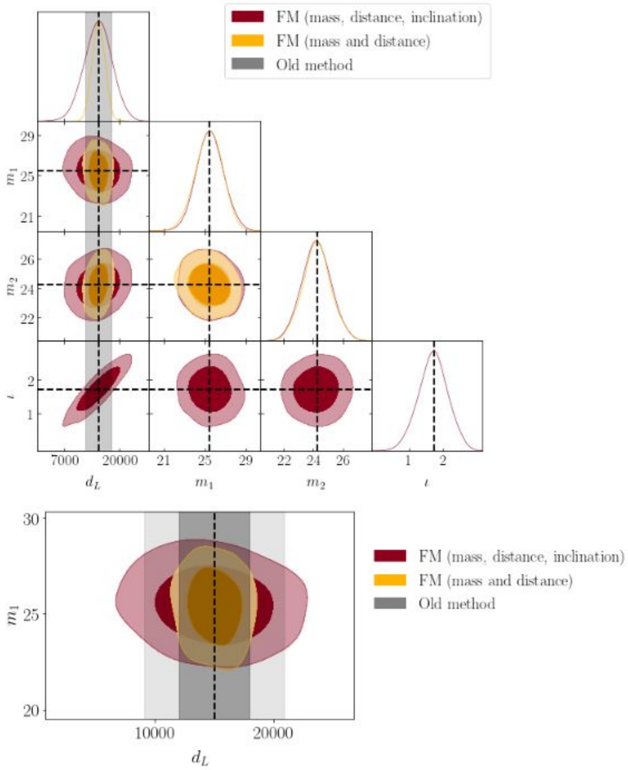
$$\frac{\partial h}{\partial d_L} = -\frac{1}{d_L} h$$

The Fisher matrix becomes

$$F_{d_L d_L} = 4 \int \frac{\frac{\partial h}{\partial d_L} \frac{\partial h^*}{\partial d_L}}{S(f)} \mathrm{d}f = \frac{1}{d_L^2} \left[4 * \int \frac{h\, h^*}{S(f)} \mathrm{d}f \right] = \frac{\langle h|h \rangle}{d_L^2} \equiv \frac{\mathrm{SNR}^2}{d_L^2}$$

$$\sigma_{d_L} = \sqrt{(F^{-1})_{d_L d_L}} = \frac{d_L}{\mathrm{SNR}}$$

Preliminary results: single event



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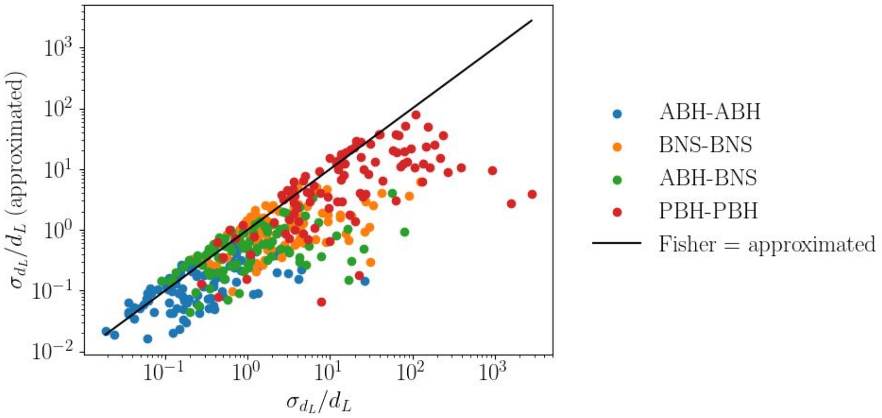

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