FUTURE PERSPECTIVES ON PRIMORDIAL BLACK HOLES

ROME 2023

ANATOMY OF SINGLE-FIELD INFLATIONARY MODELS FOR PRIMORDIAL BLACK HOLES

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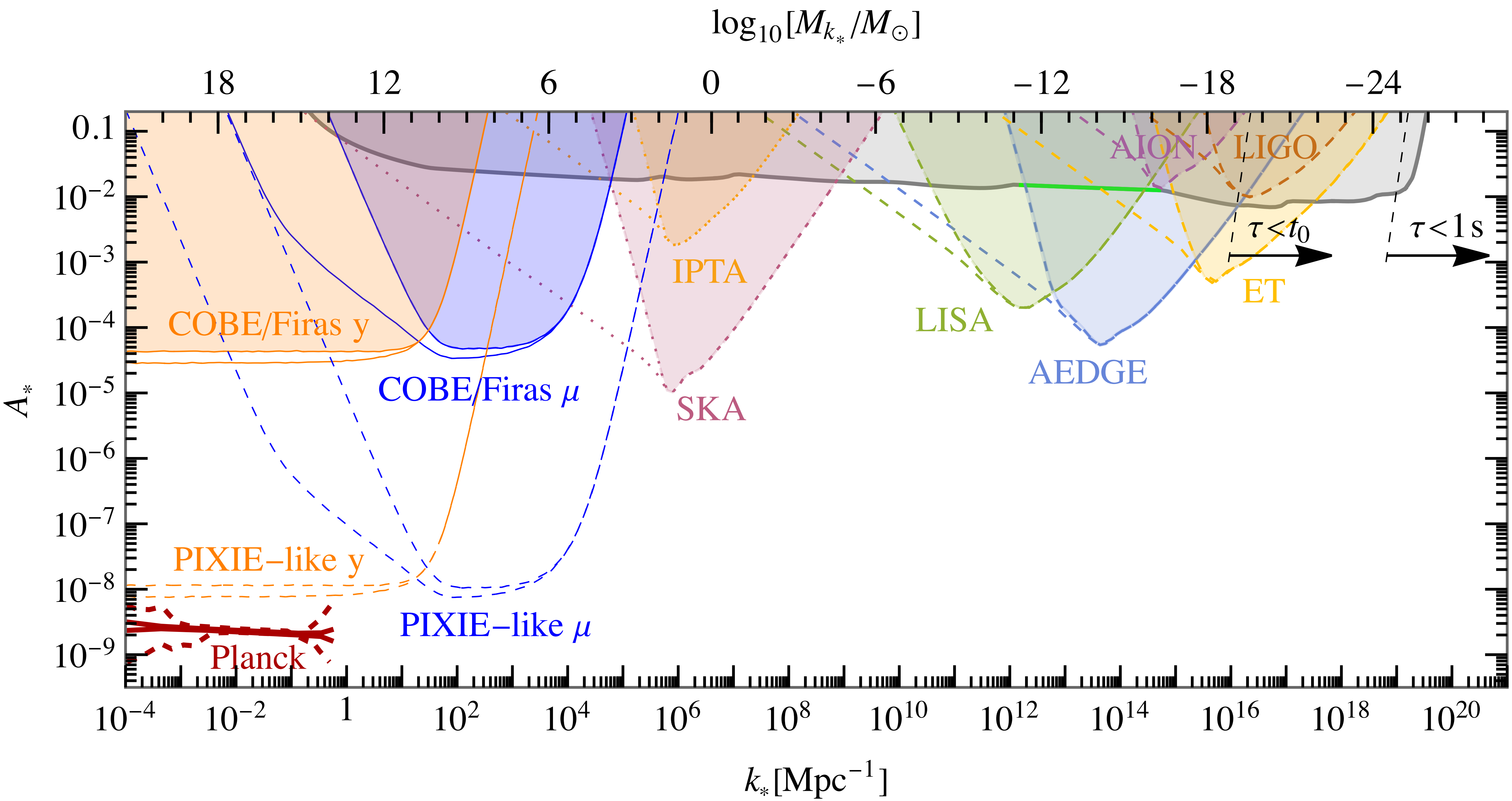
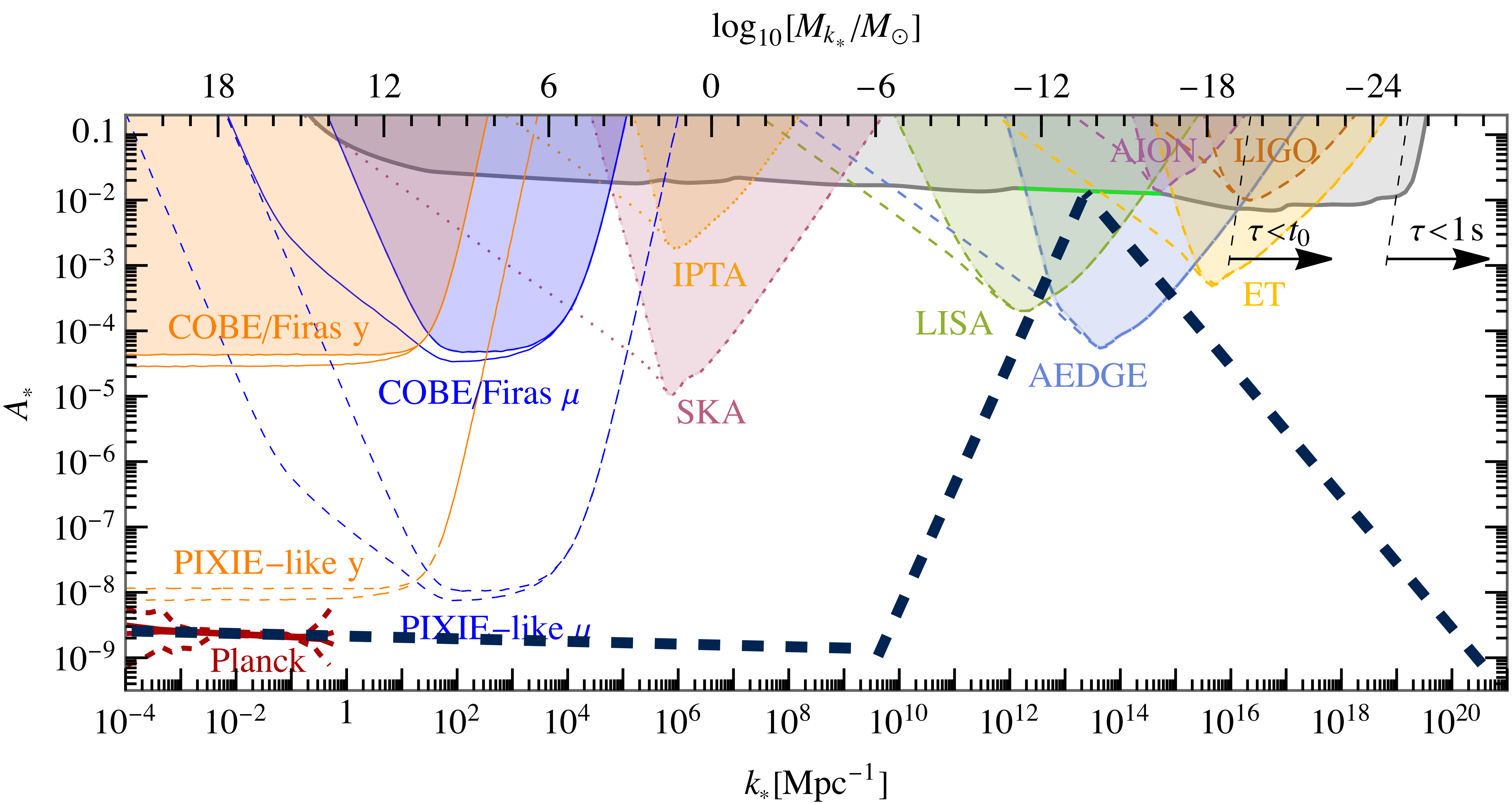
Keemilise ja Bioloogilise Füüsika Instituut

National Institute of Chemical Physics and Biophysics



[2205.13540 Karam et al]



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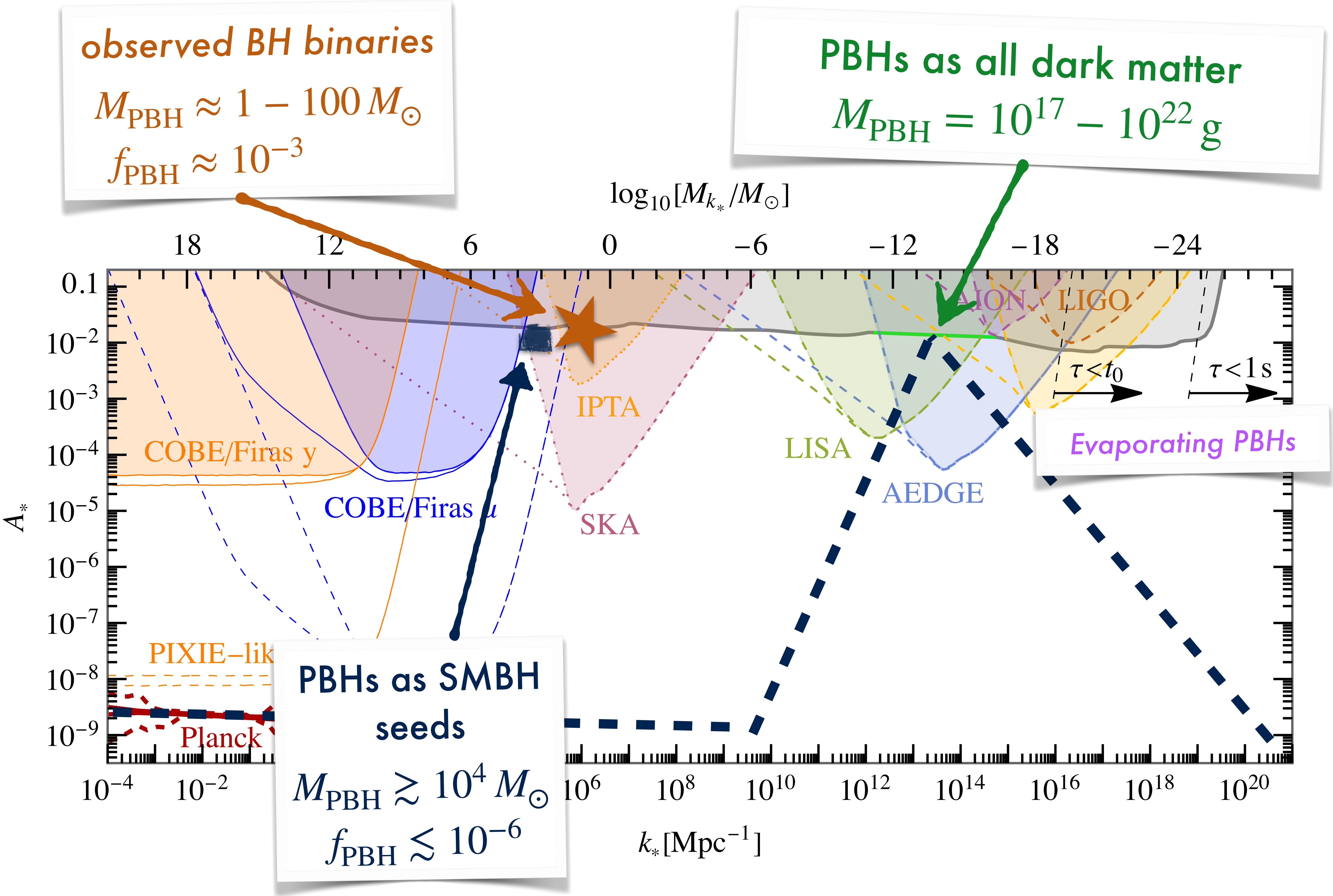

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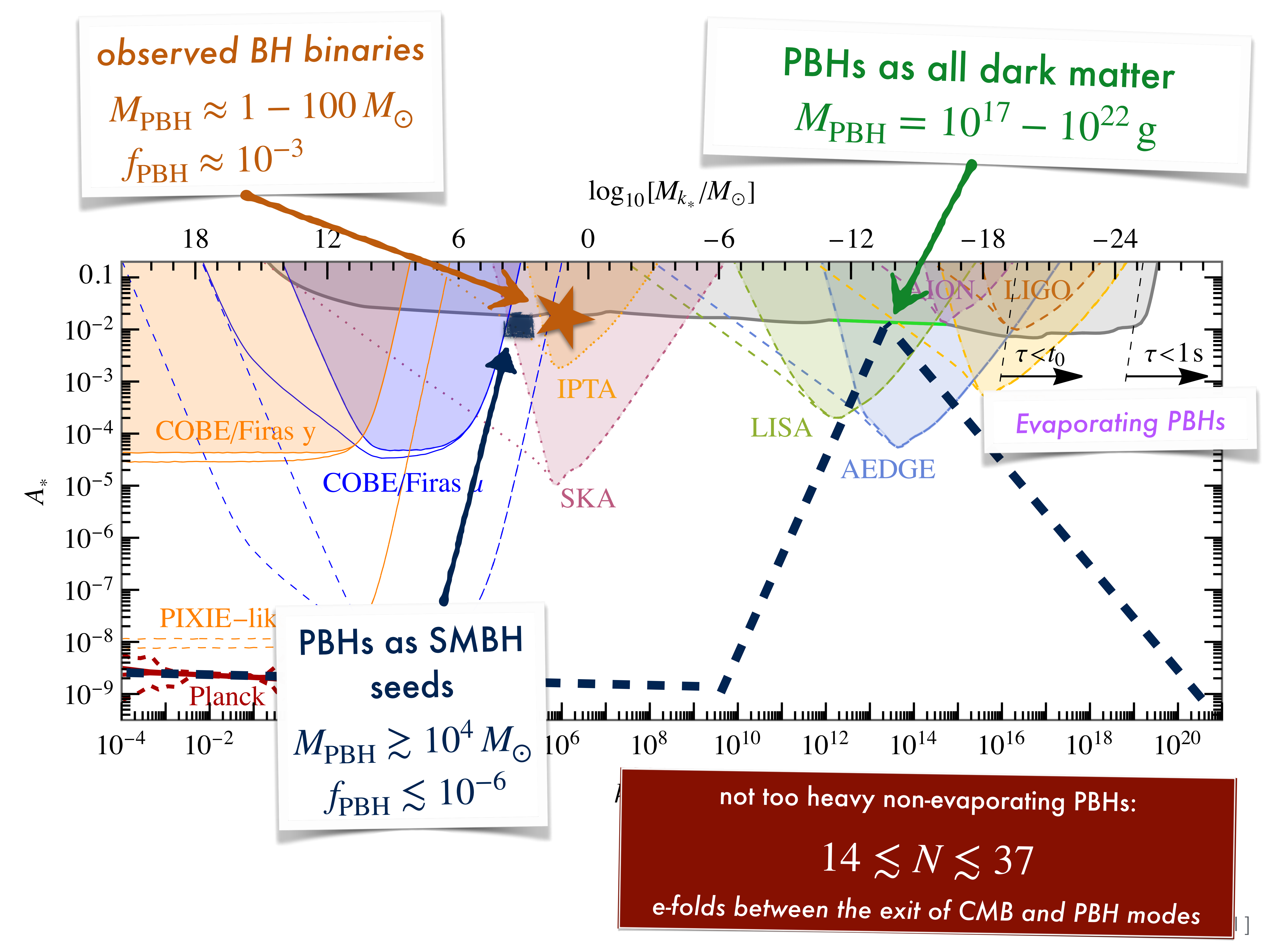
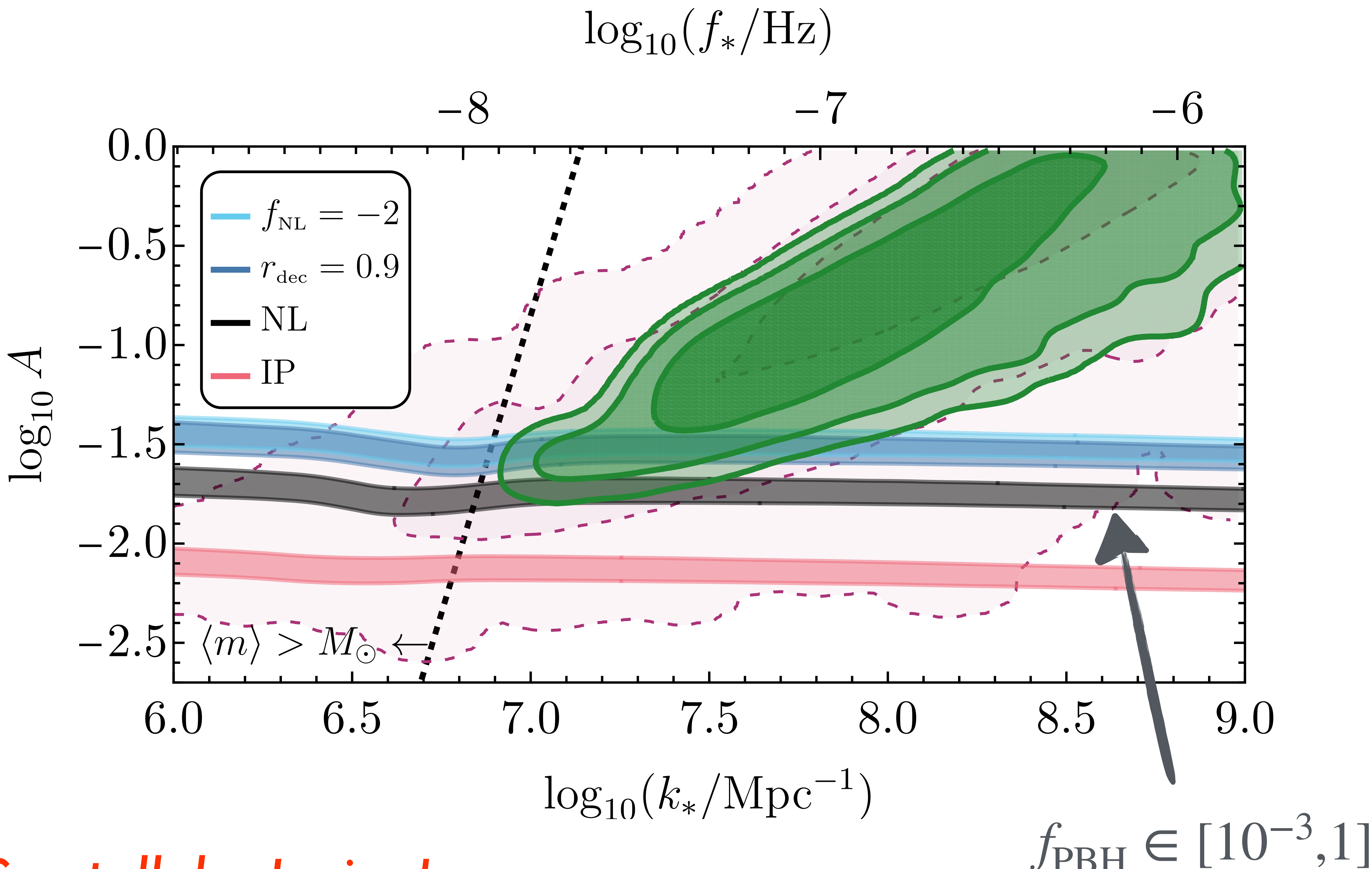


observed BH binaries
$M_{\rm PBH} \approx 1 - 100\, M_\odot$
$f_{\rm PBH} \approx 10^{-3}$
PBHs as all dark matter
$M_{\rm PBH} = 10^{17} - 10^{22}\,{\rm g}$
$\log_{10}[M_{k_*}/M_\odot]$
18
12
6
0
−6
−12
−18
−24
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
A_*
AION
LIGO
IPTA
SKA
LISA
AEDGE
COBE/Firas y
COBE/Firas μ
PIXIE–lik
Planck
$\tau < t_0$
$\tau < 1\,{\rm s}$
Evaporating PBHs
PBHs as SMBH seeds
$M_{\rm PBH} \gtrsim 10^4\, M_\odot$
$f_{\rm PBH} \lesssim 10^{-6}$
10^{-4}
10^{-2}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
10^{16}
10^{18}
10^{20}
not too heavy non-evaporating PBHs:
$14 \lesssim N \lesssim 37$
e-folds between the exit of CMB and PBH modes

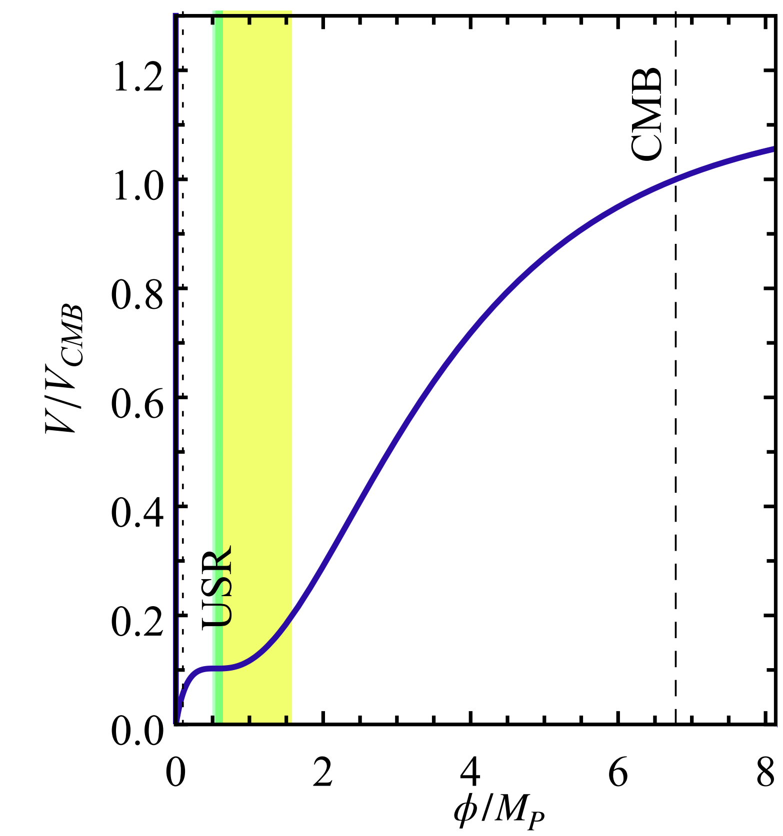
Induced GWs with PTAs

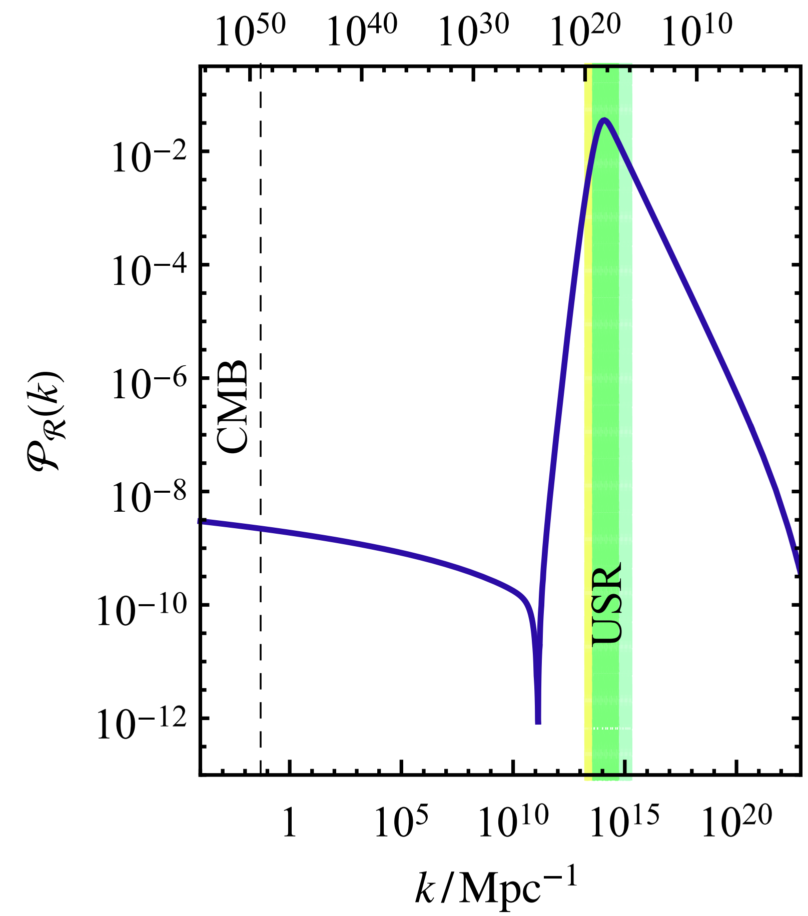


See talk by Iovino!

INFLATIONARY TIMELINE

[2205.13540 Karam et al]



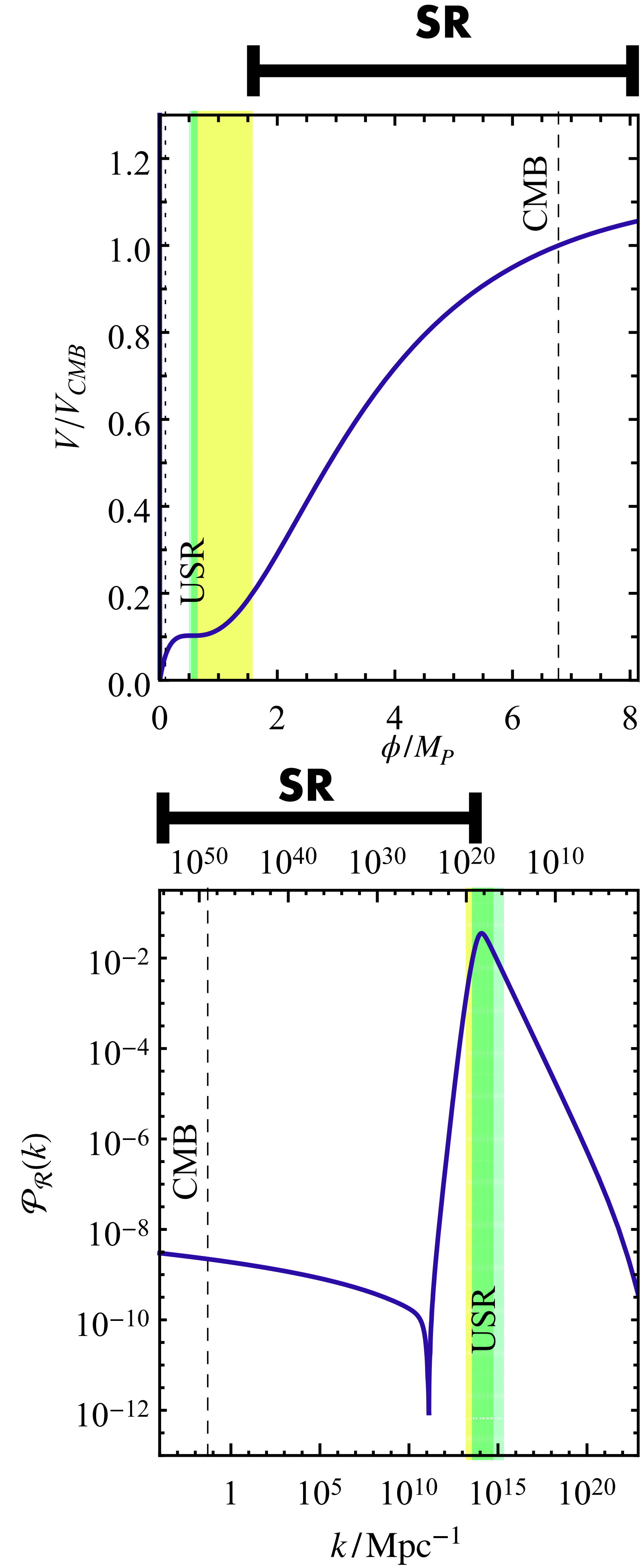


INFLATIONARY TIMELINE

[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

   lasts $\mathcal{O}(30)$ e-folds



INFLATIONARY TIMELINE

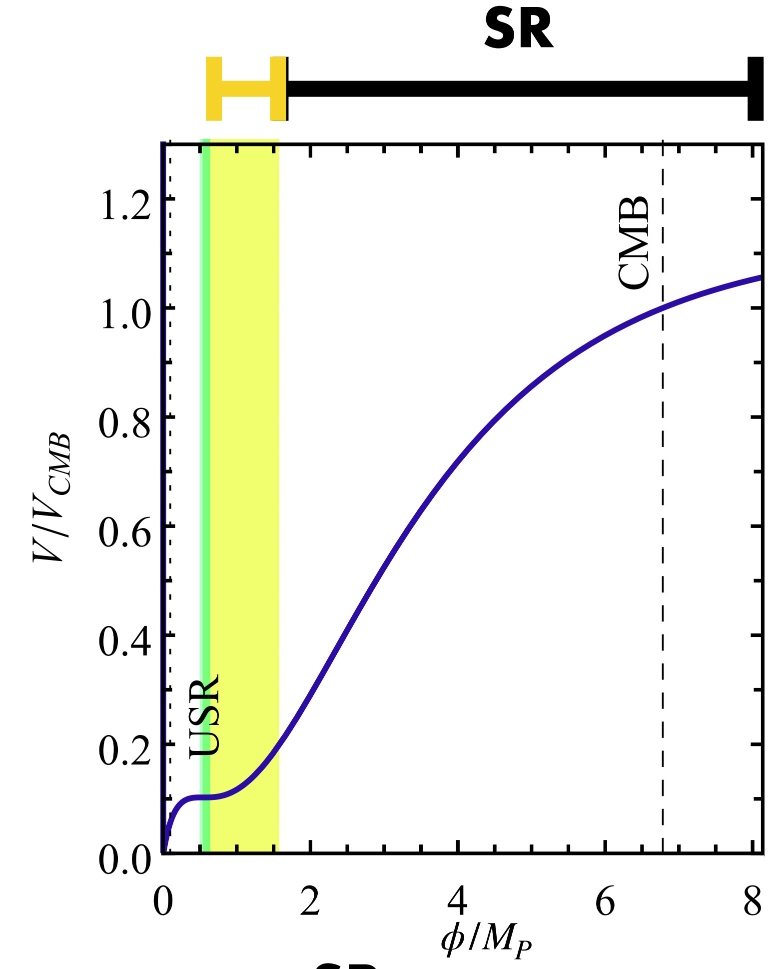
[2205.13540 Karam et al]

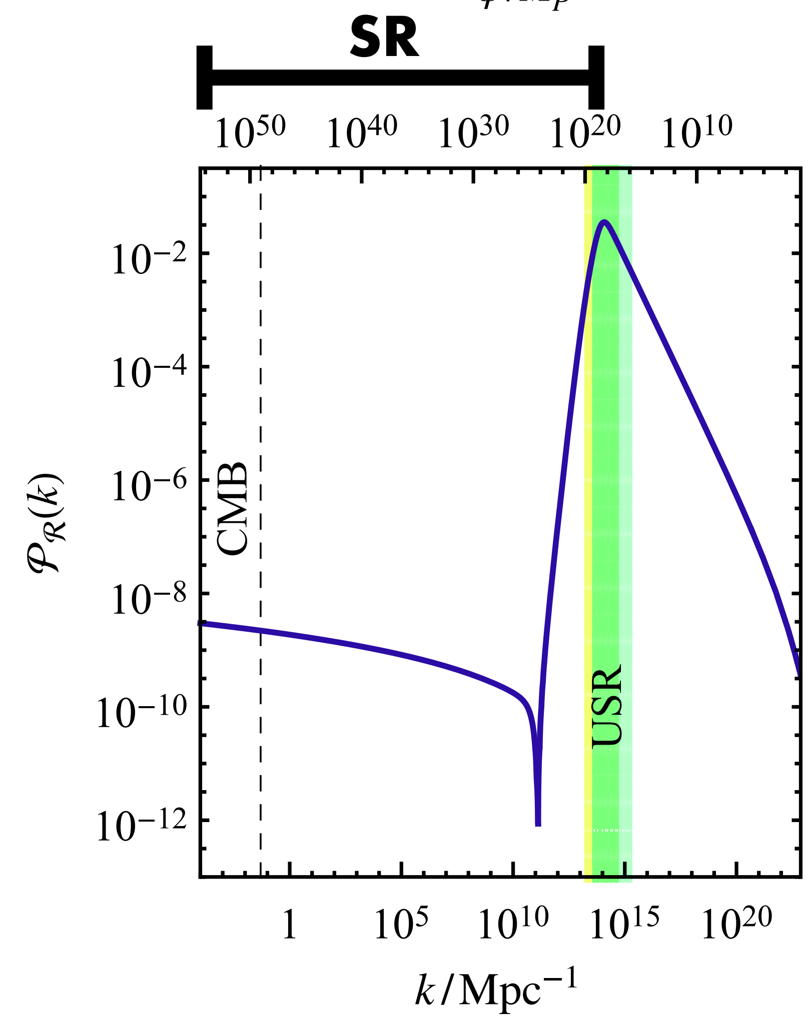
1. SLOW-ROLL (SR)

   lasts $\mathcal{O}(30)$ e-folds

2. TRANSITION from SR to USR

   lasts $\lesssim 1$ e-folds





INFLATIONARY TIMELINE

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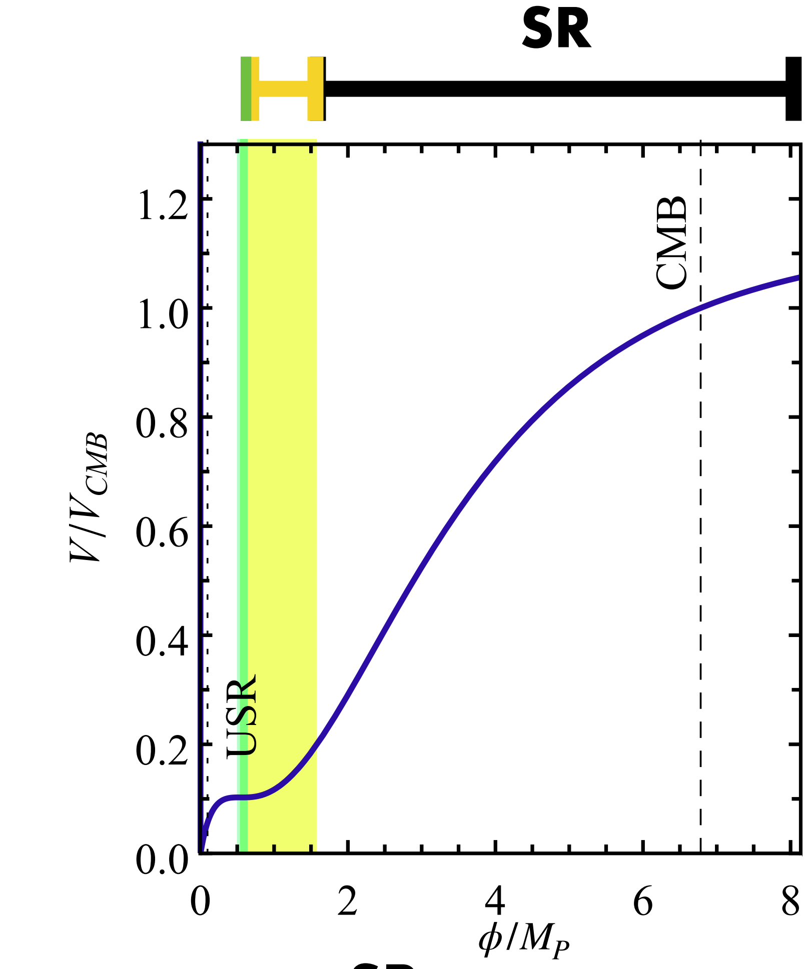
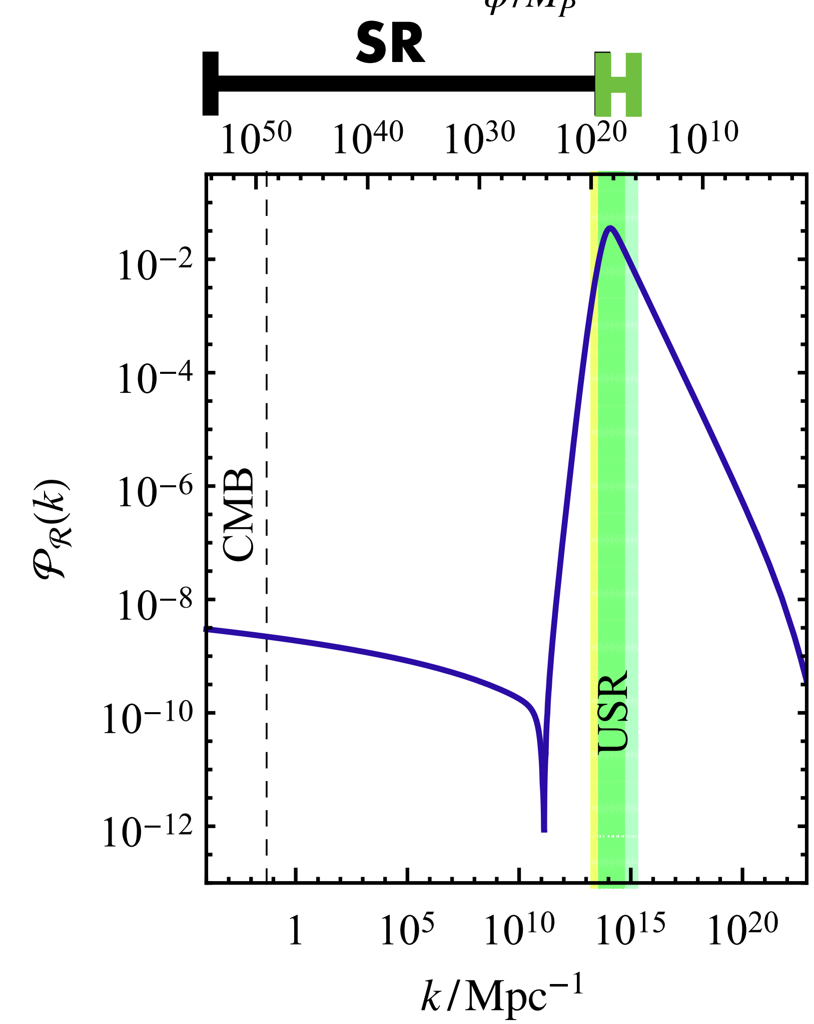
   lasts $\lesssim 1$ e-folds

3. ULTRA-SLOW-ROLL (USR)

   lasts $\mathcal{O}(3)$ e-folds





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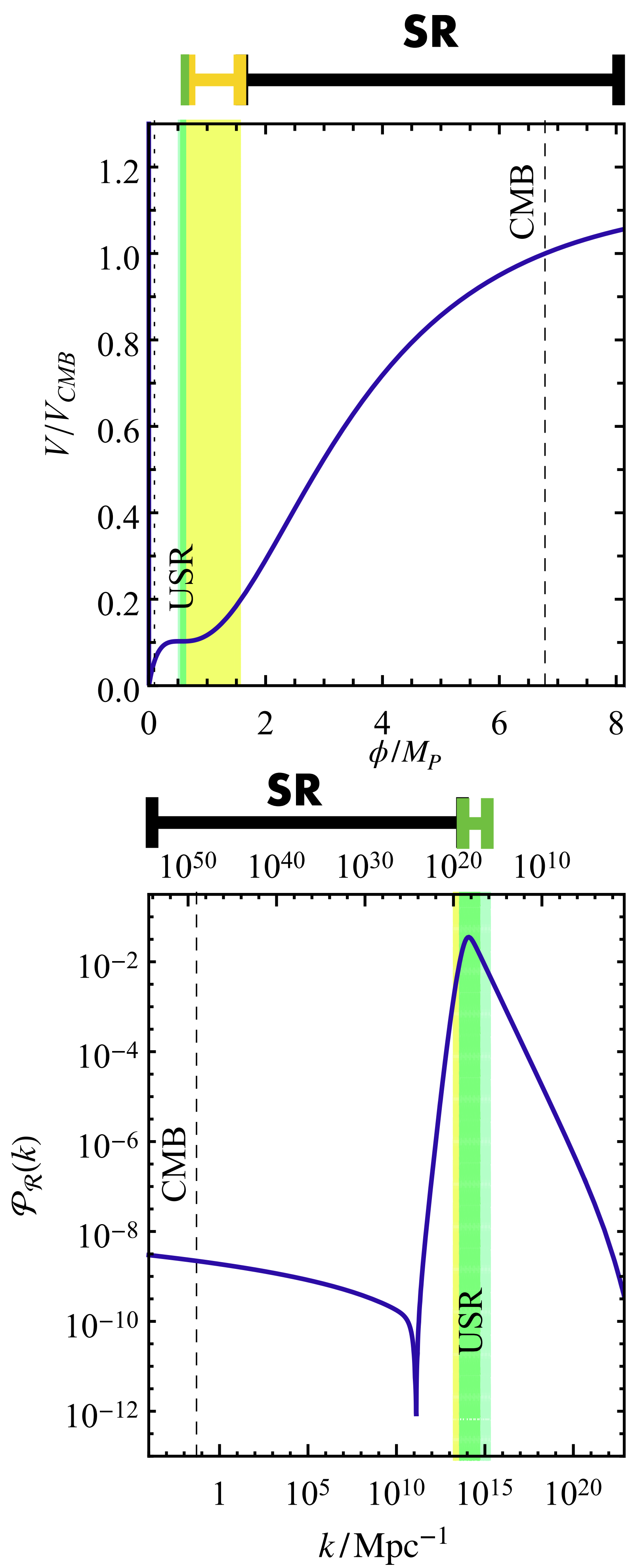
4. TRANSITION from USR to CR



INFLATIONARY TIMELINE

[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

   lasts $\mathcal{O}(30)$ e-folds

2. TRANSITION from SR to USR

   lasts $\lesssim 1$ e-folds

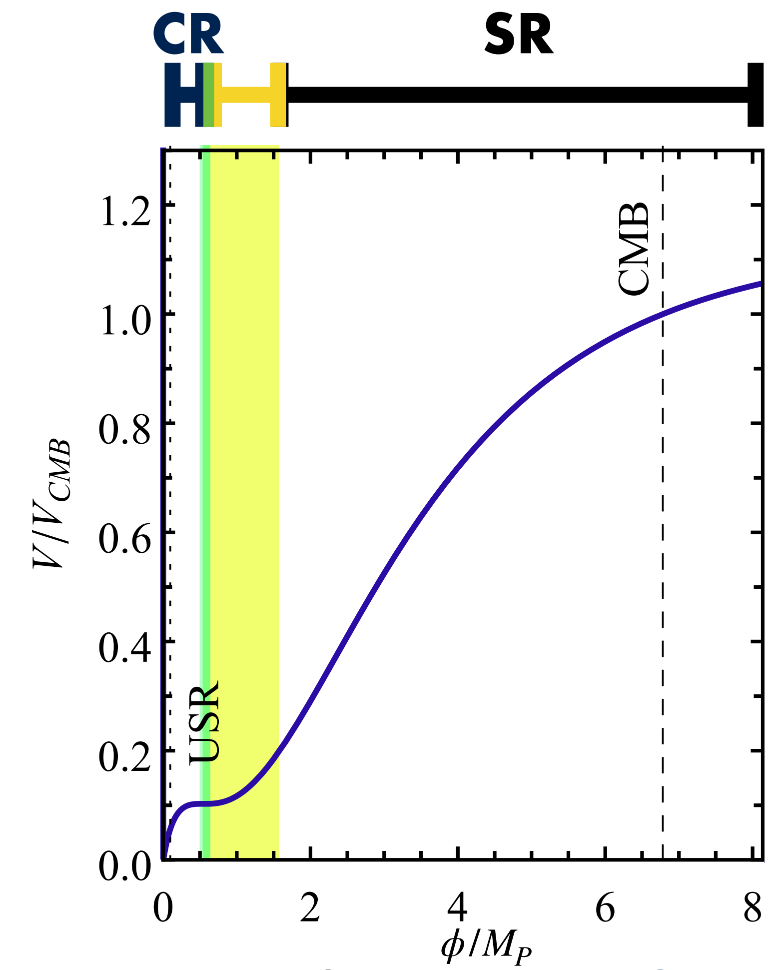
3. ULTRA-SLOW-ROLL (USR)

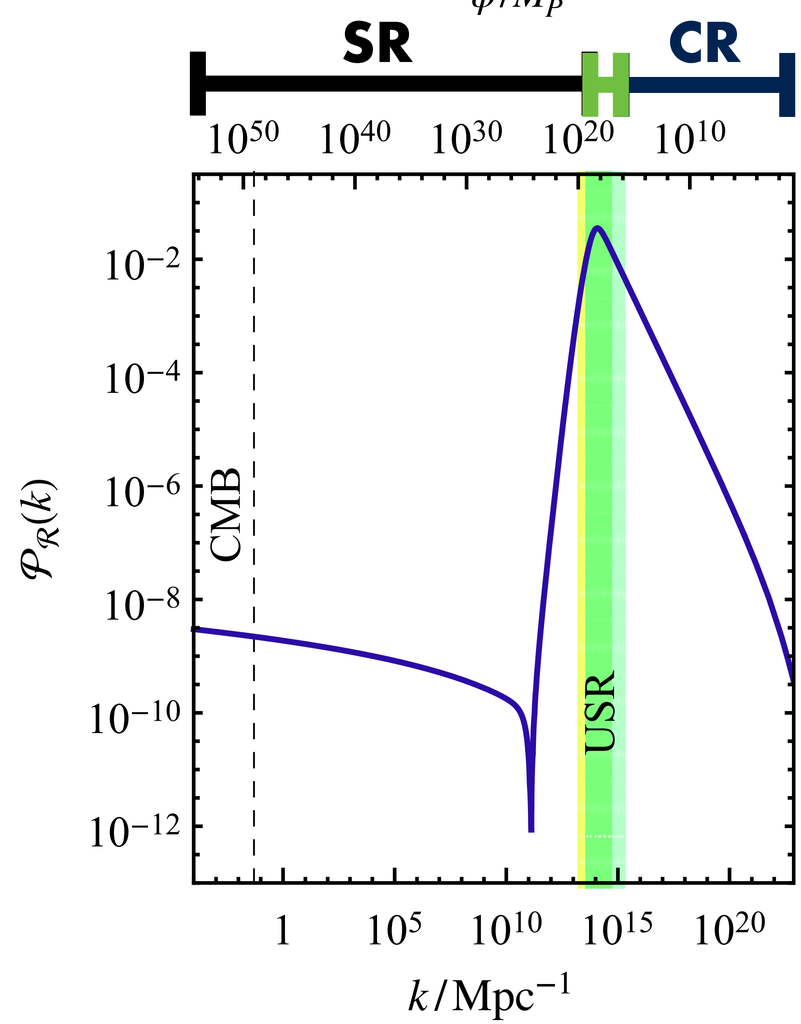
   lasts $\mathcal{O}(3)$ e-folds

4. TRANSITION from USR to CR

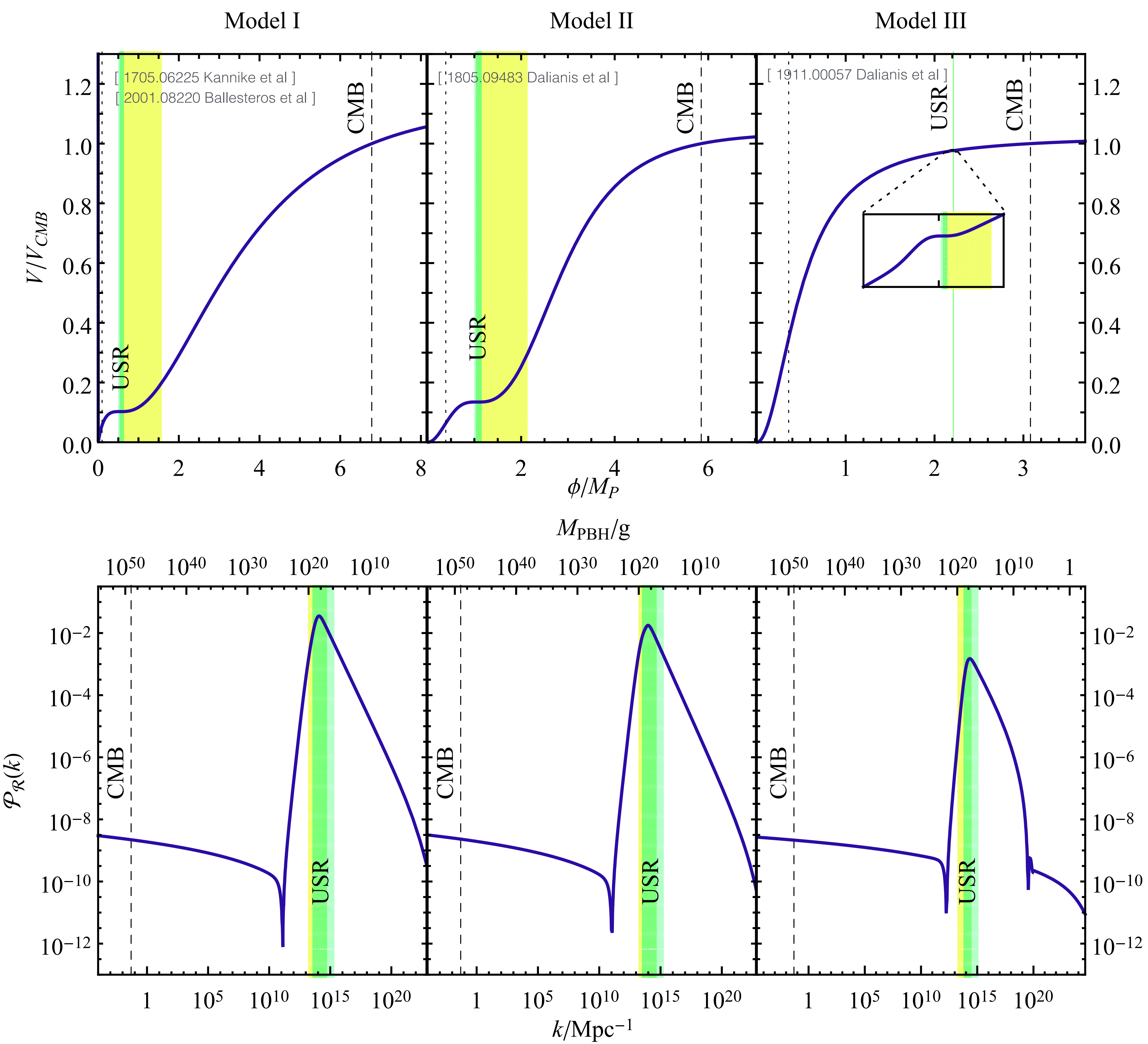
5. CONSTANT-ROLL (CR)

   ...or also SR





TYPICAL MODELS



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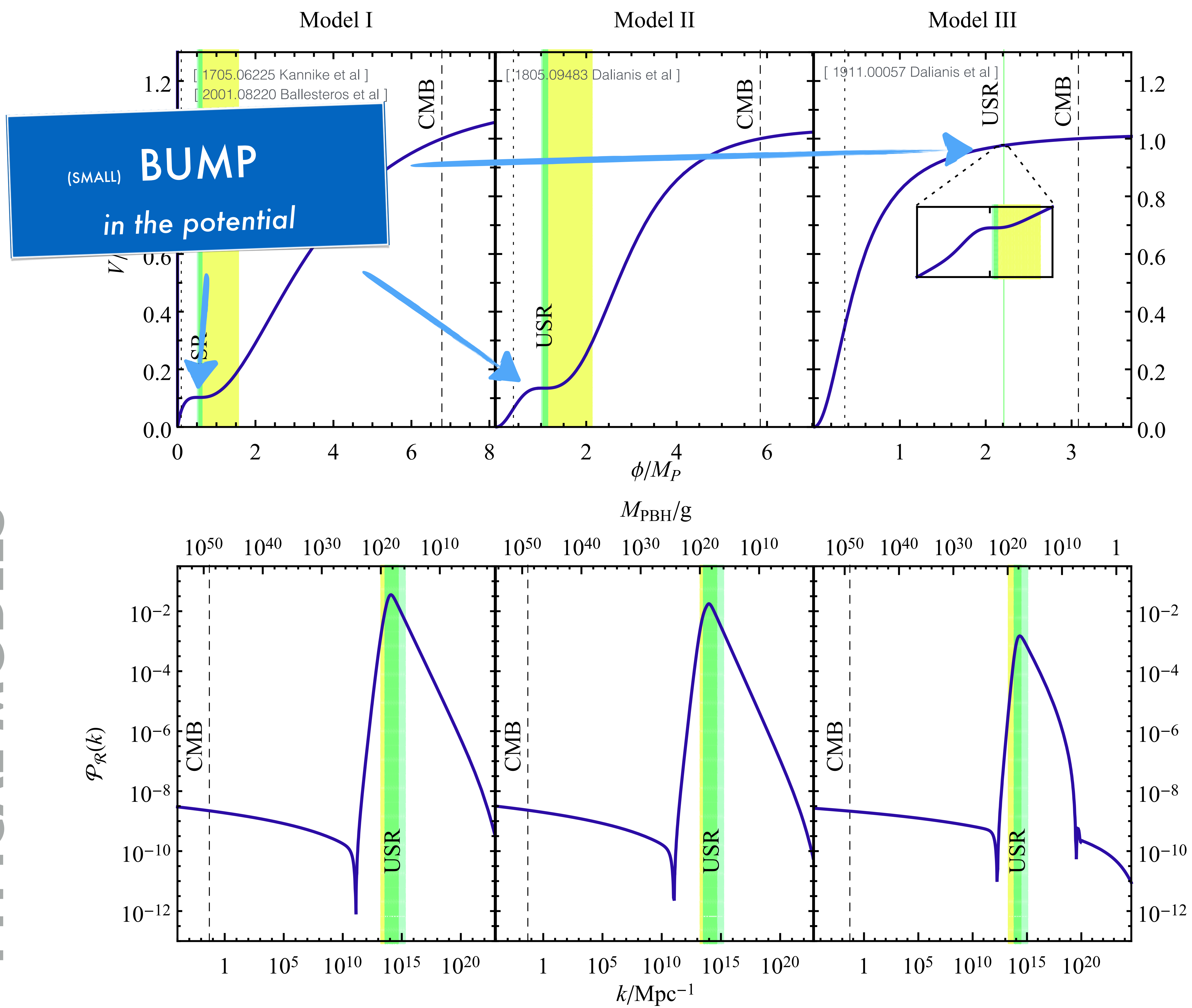

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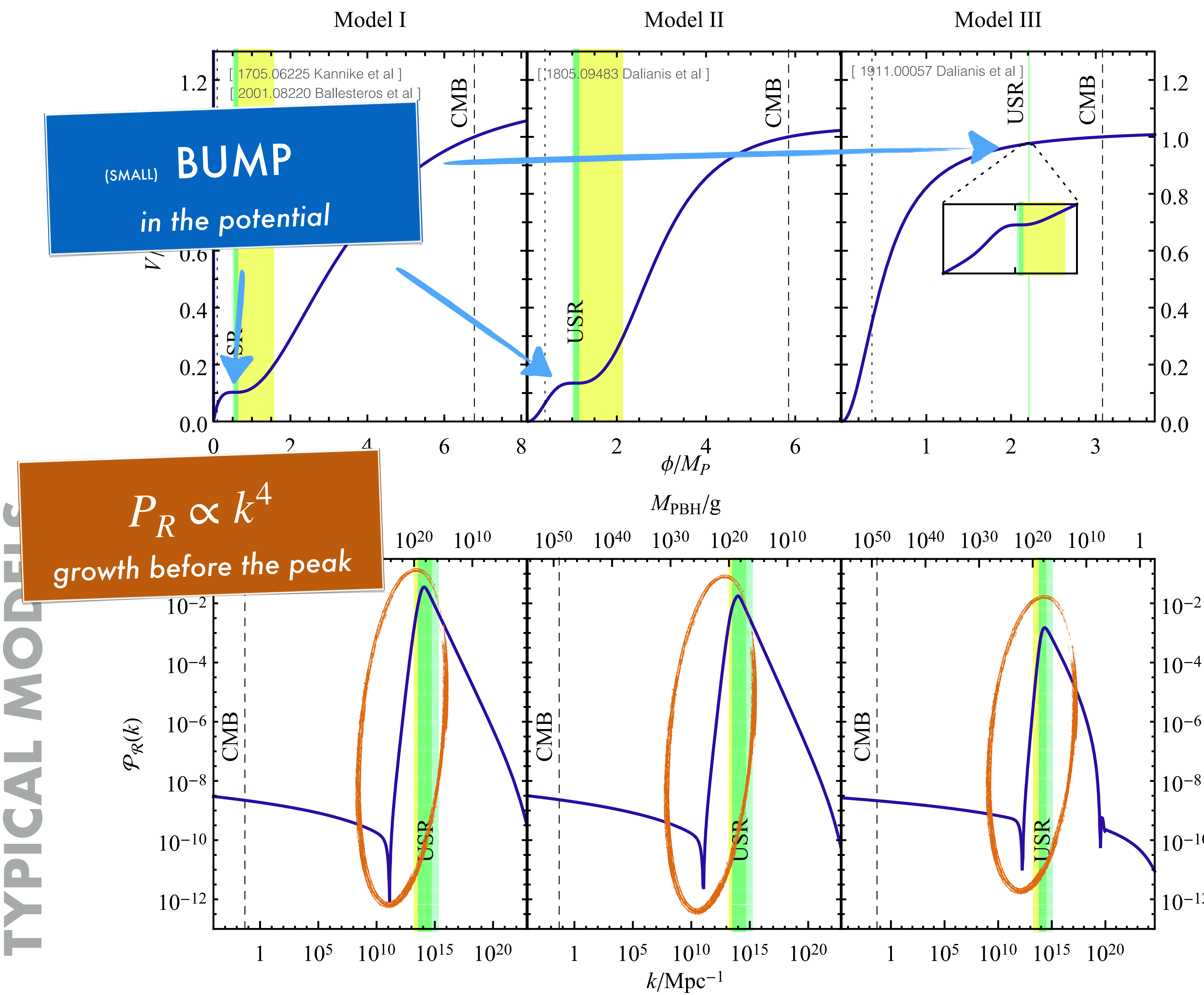
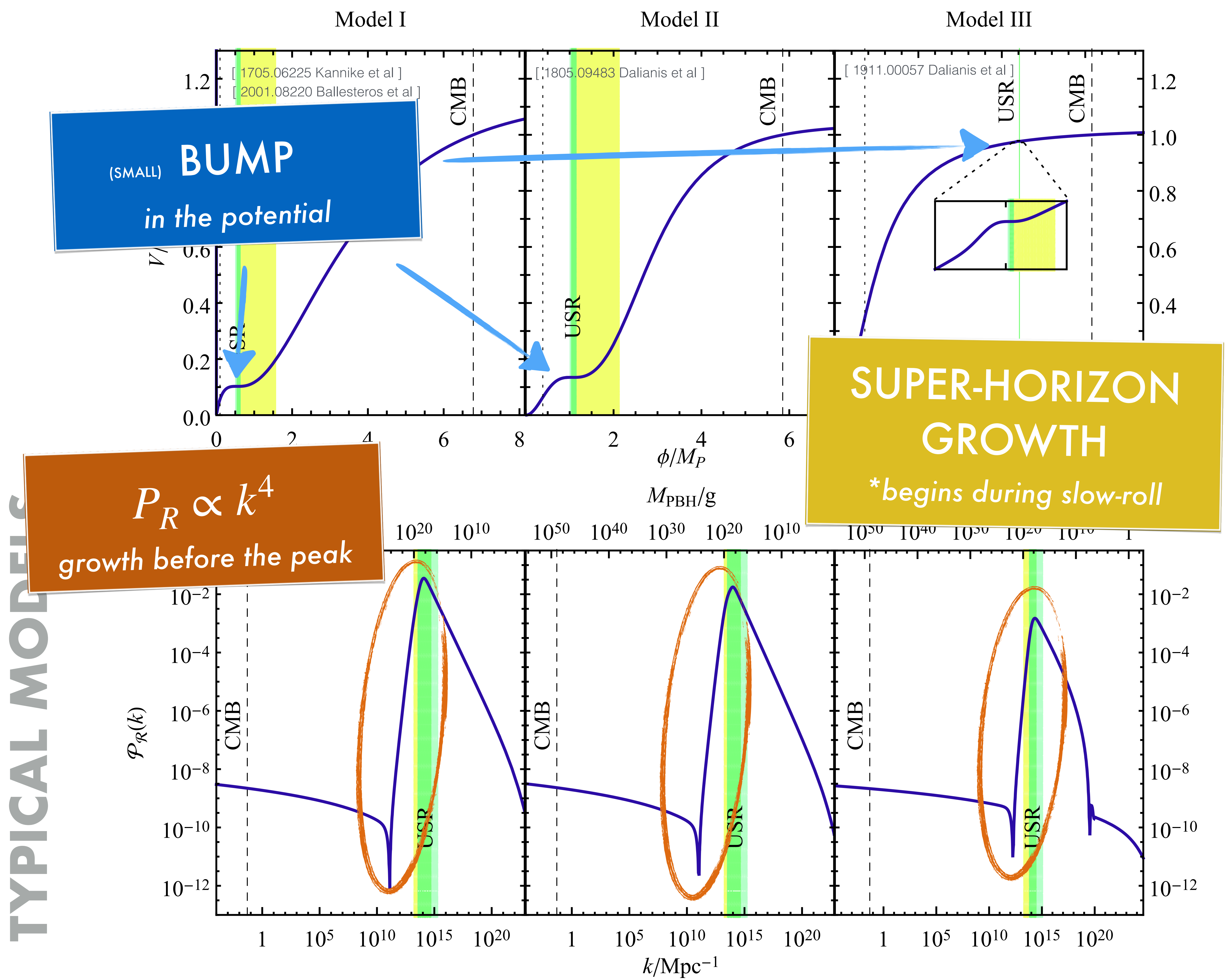
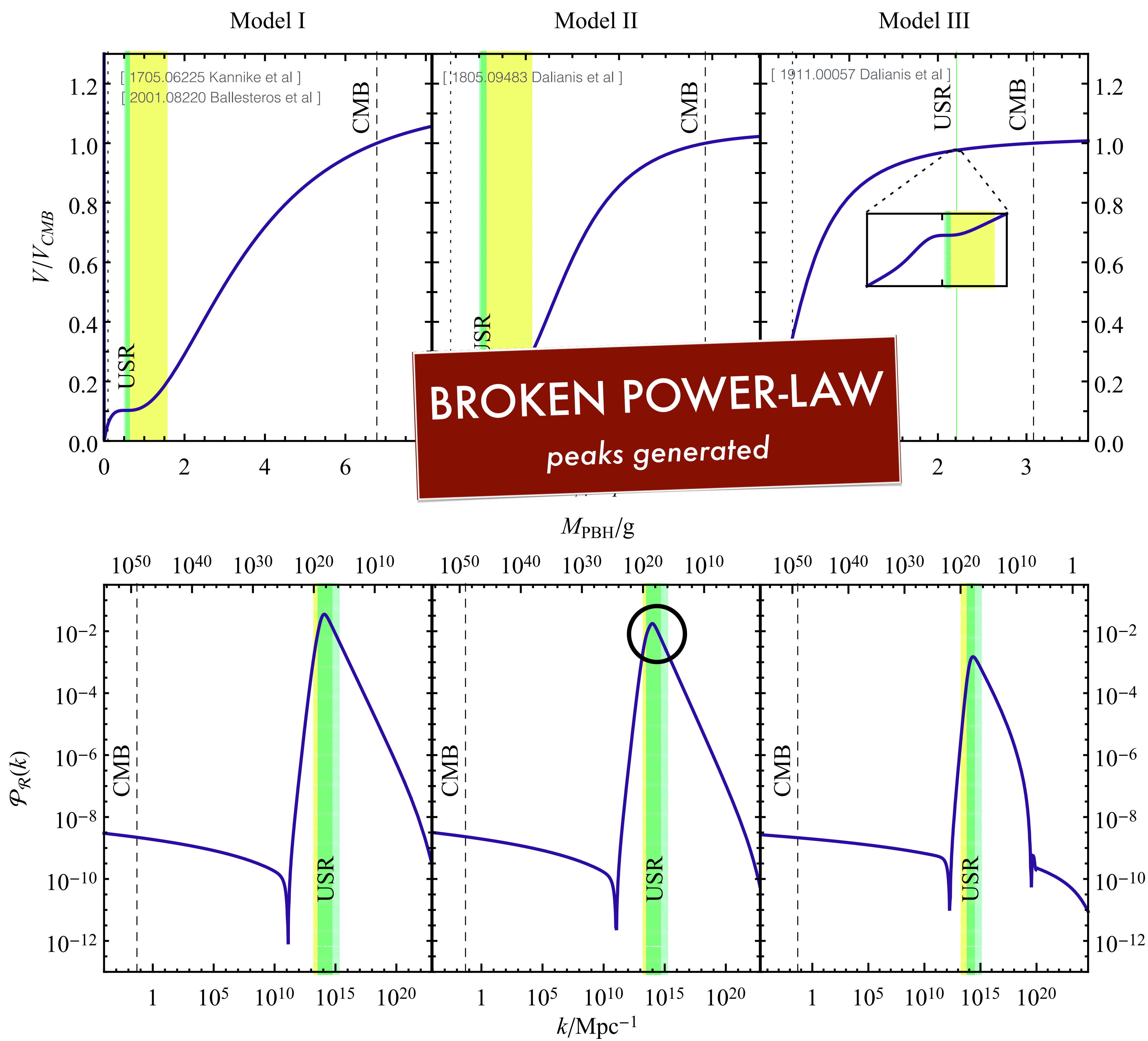




TYPICAL MODELS
Model I
Model II
Model III
[1705.06225 Kannike et al]
[2001.08220 Ballesteros et al]
[1805.09483 Dalianis et al]
[1911.00057 Dalianis et al]
(SMALL) BUMP
in the potential
SUPER-HORIZON GROWTH
*begins during slow-roll
$P_R \propto k^4$
growth before the peak
CMB
USR
ϕ/M_P
$M_{\rm PBH}$/g
$\mathcal{P}_R(k)$
$k/{\rm Mpc}^{-1}$

TYPICAL MODELS



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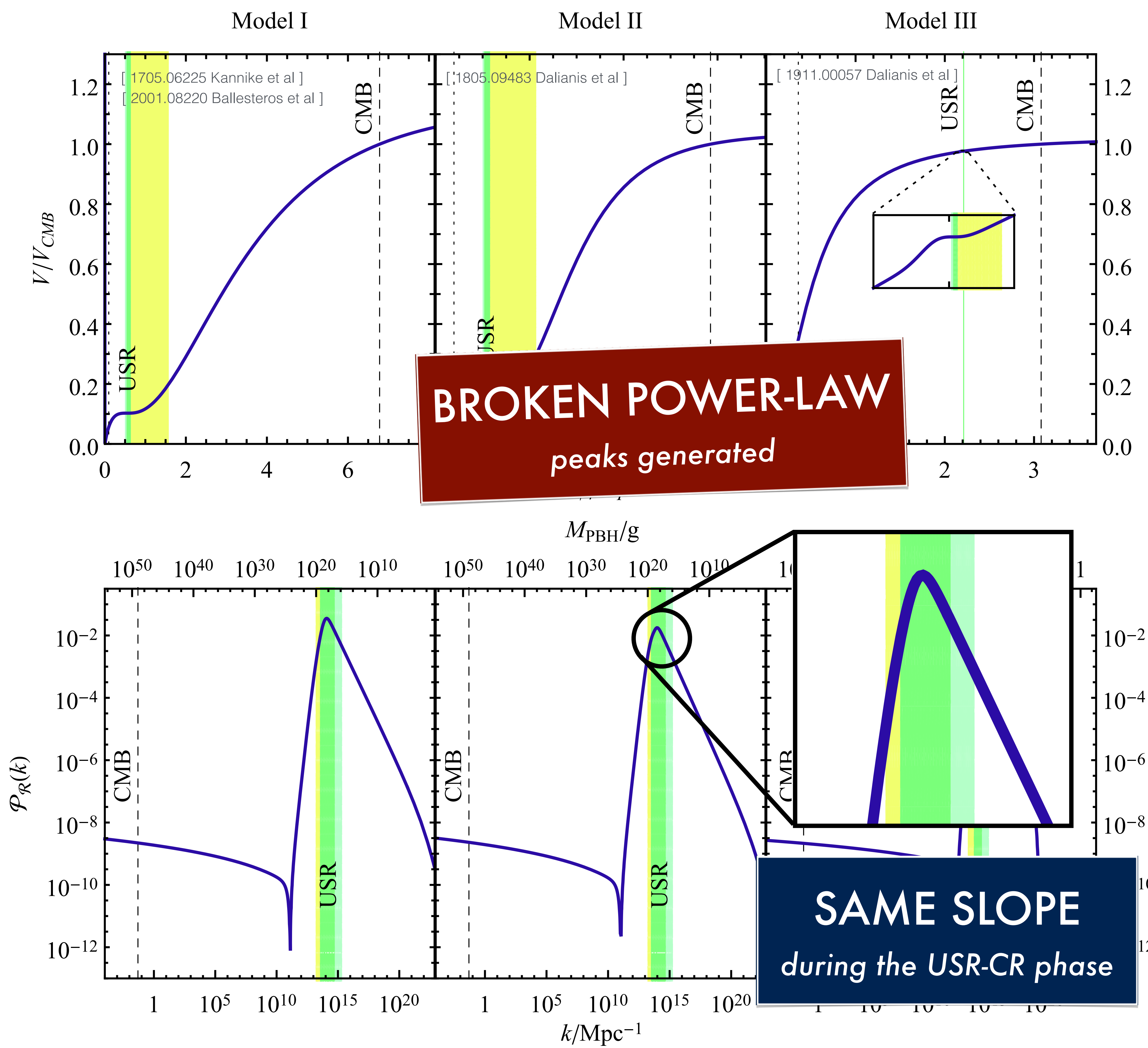
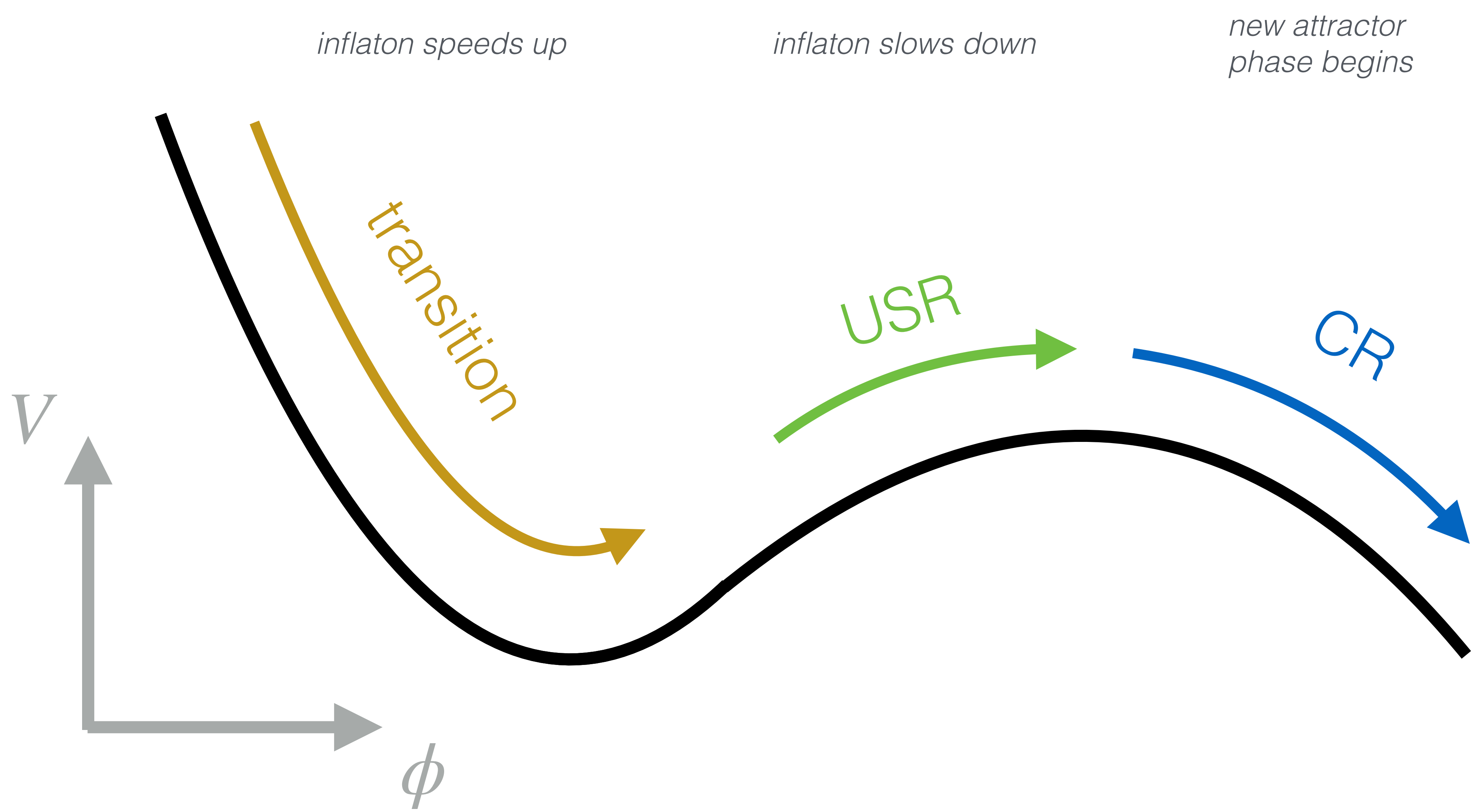


WANDS DUALITY



[2205.13540 Karam et al]

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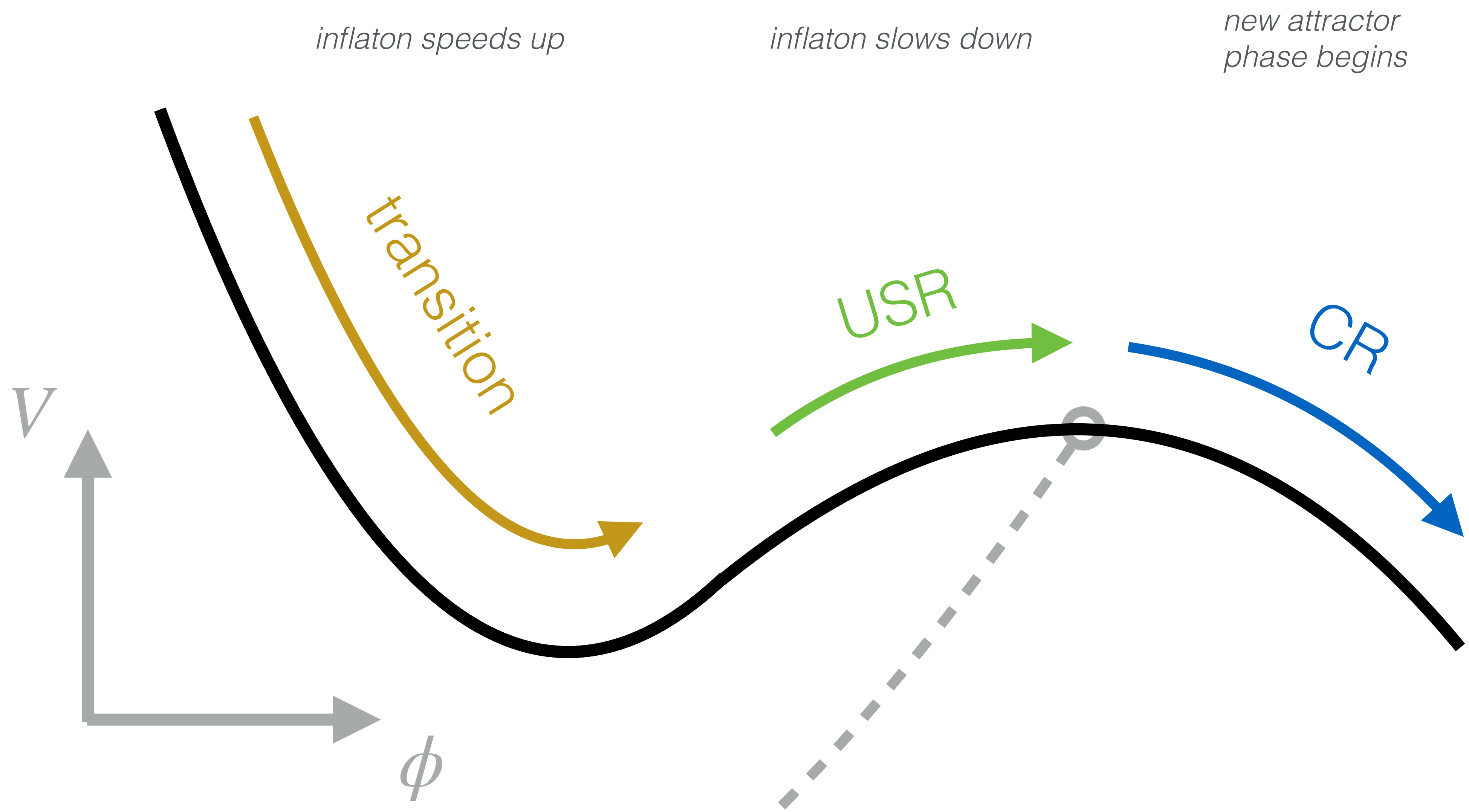


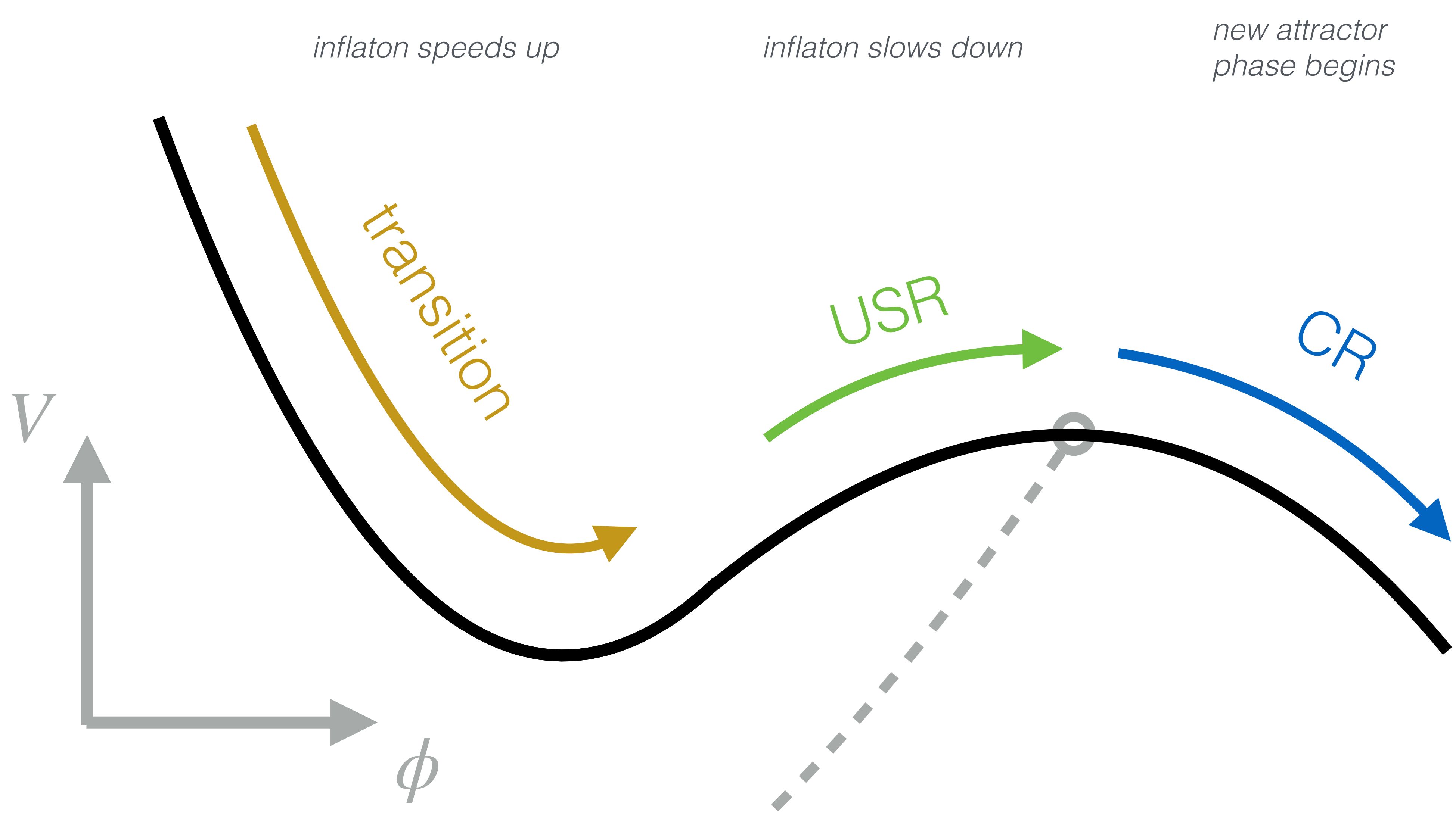
expansion around the local maximum:

$$\partial_N^2\phi + 3\partial_N\phi \approx -\eta_{V,c}\left(\phi - \phi_c\right)$$

$\eta_{V,c} \equiv V''(\phi_c)/V(\phi_c)$

[2205.13540 Karam et al]

WANDS DUALITY



expansion around the local maximum:

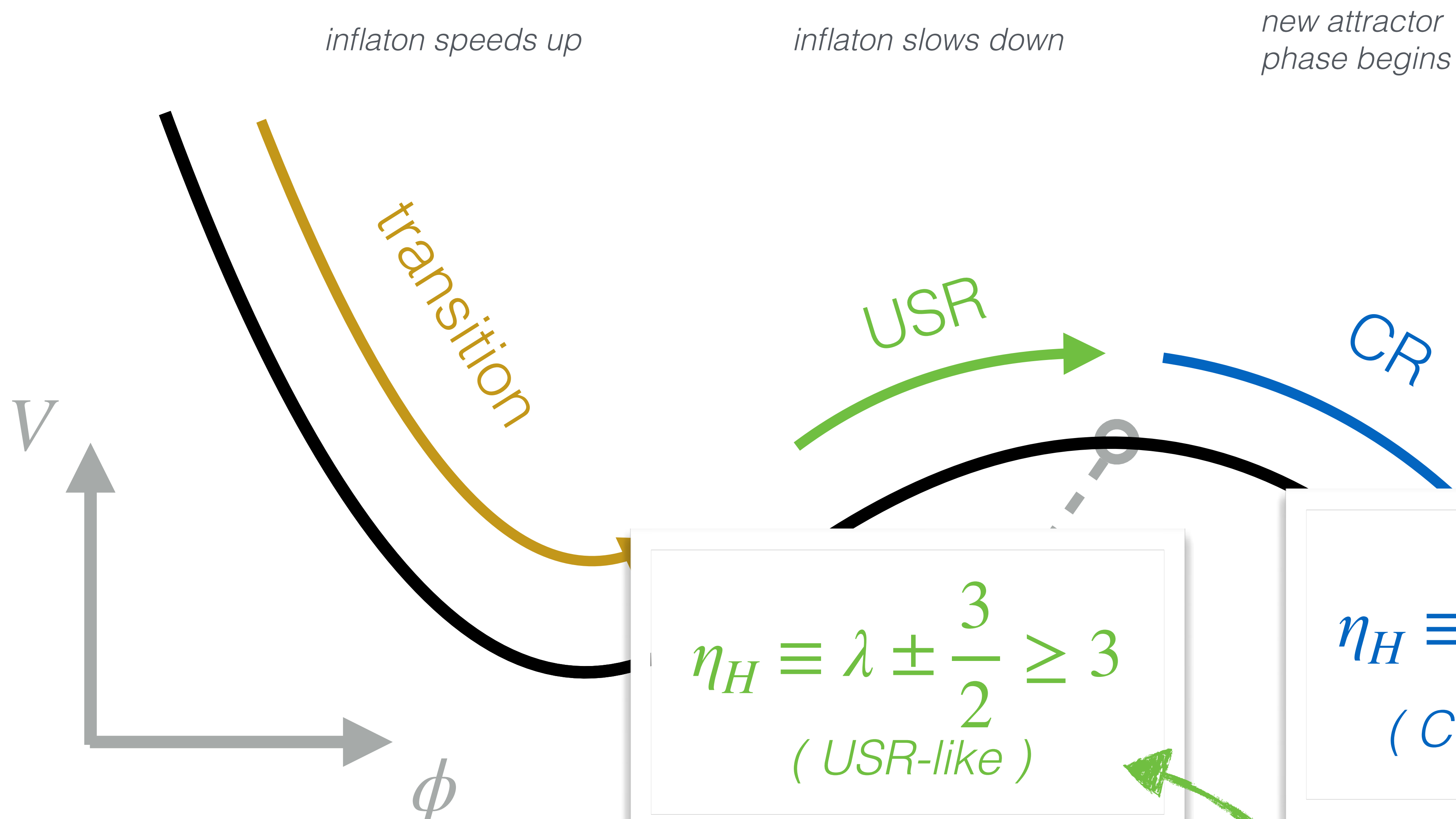
$$\partial_N^2\phi + 3\partial_N\phi \approx -\eta_{V,c}\left(\phi - \phi_c\right) \quad \Rightarrow \quad \phi - \phi_c = a^{-3/2}\left(\phi_- e^{-\lambda N} + \phi_+ e^{\lambda N}\right)$$

$$\lambda \equiv \frac{3}{2}\sqrt{1 - \frac{4}{3}\eta_{V,c}}$$

$\eta_{V,c} \equiv V''(\phi_c)/V(\phi_c)$

[2205.13540 Karam et al]

WANDS DUALITY



$$\eta_H \equiv \lambda \pm \frac{3}{2} \geq 3$$

(USR-like)

$$\eta_H \equiv \lambda - \frac{3}{2} \leq 0$$

(Constant roll)

expansion around the local maximum:

$$\partial_N^2 \phi + 3\partial_N \phi \approx -\eta_{V,c}\left(\phi - \phi_c\right) \quad \Rightarrow \quad \phi - \phi_c = a^{-3/2}\left(\phi_- e^{-\lambda N} + \phi_+ e^{\lambda N}\right)$$

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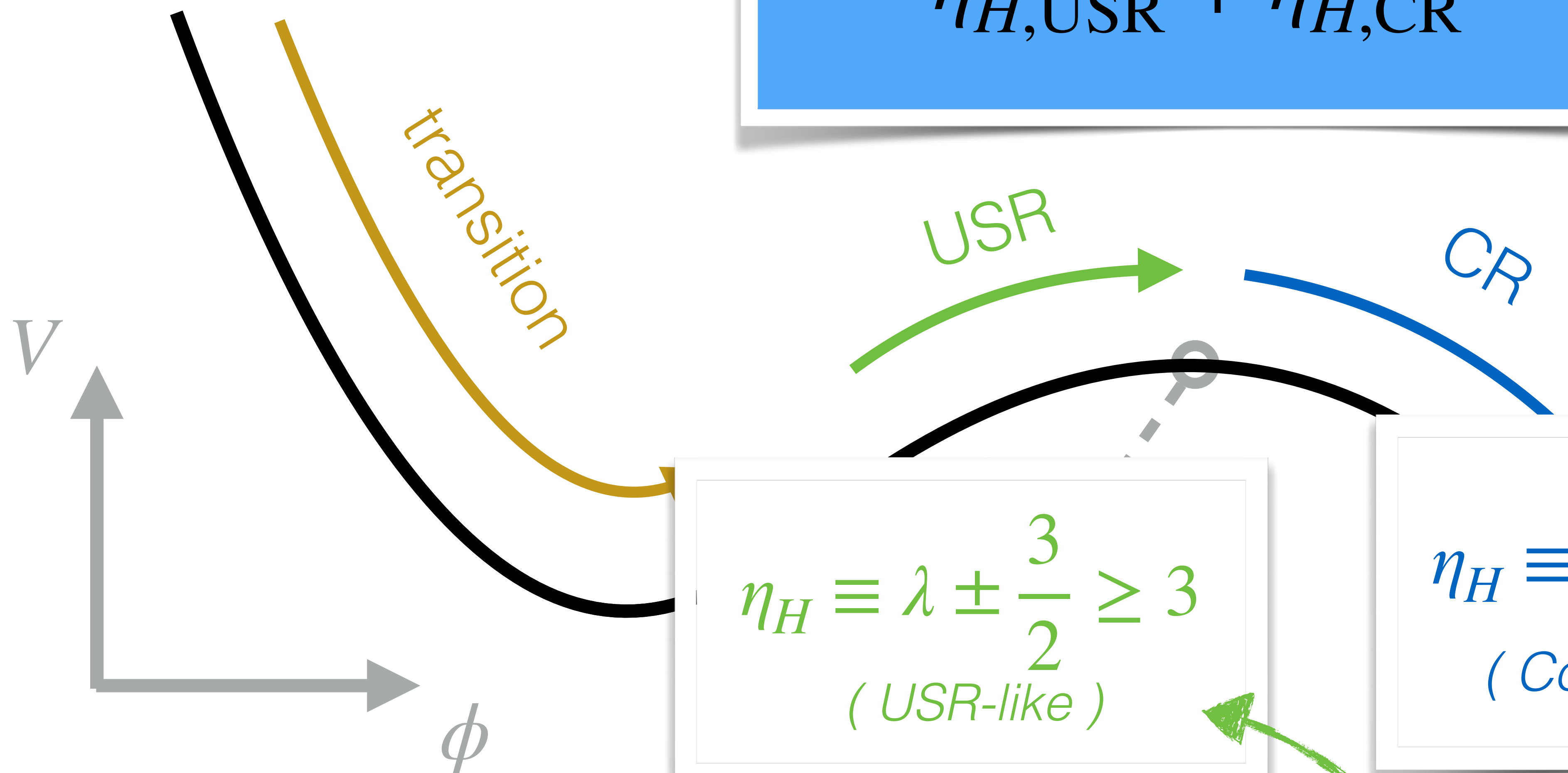
[2205.13540 Karam et al]

WANDS DUALITY

for smooth potentials

$$\eta_{H,\mathrm{USR}} + \eta_{H,\mathrm{CR}} = 3$$



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[2205.13540 Karam et al]

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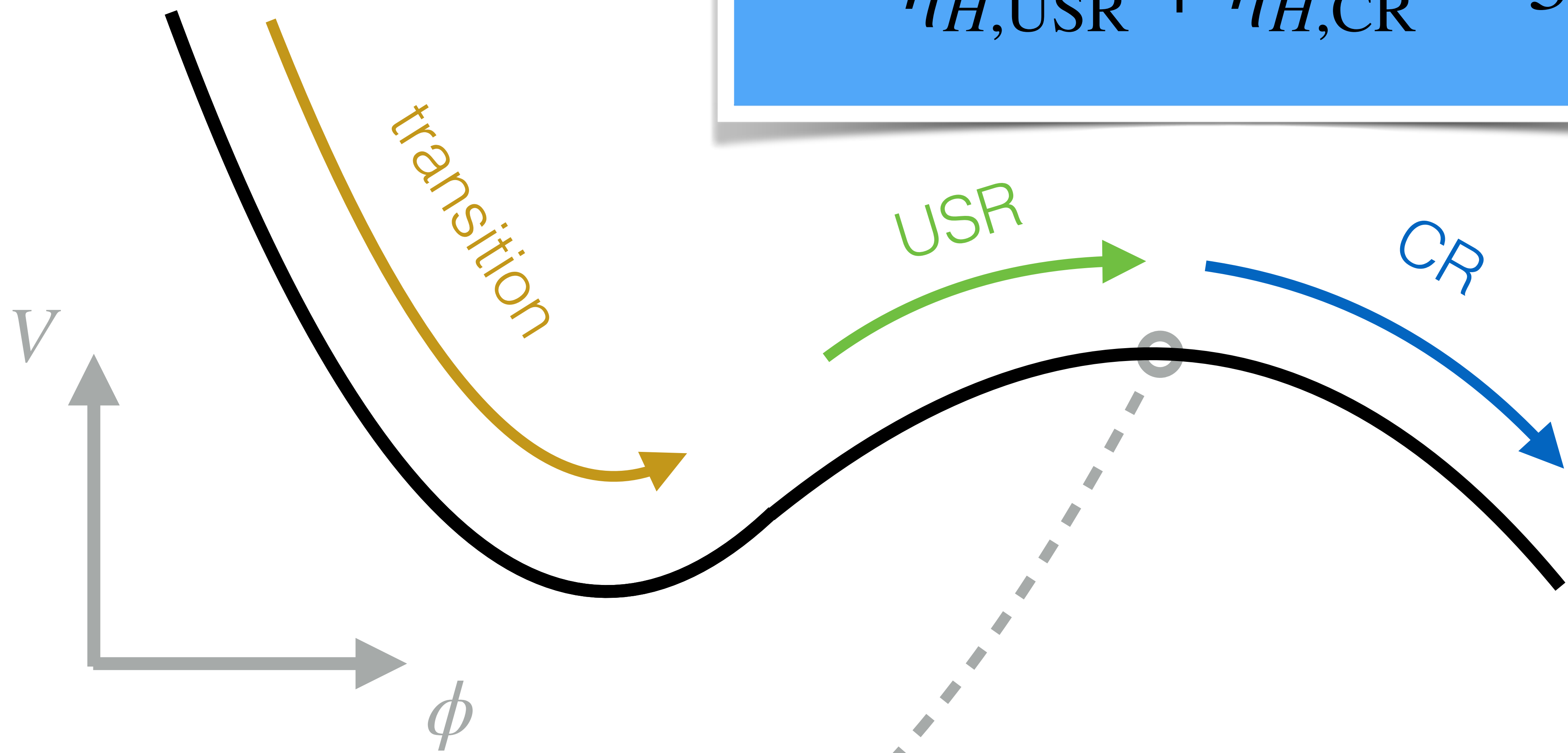


for smooth hills

$$\eta_{H,\mathrm{USR}} + \eta_{H,\mathrm{CR}} = 3$$

expansion around the local maximum:

$$\partial_N^2 \phi + 3\partial_N \phi \approx -\eta_{V,c}\left(\phi - \phi_c\right)$$

=>

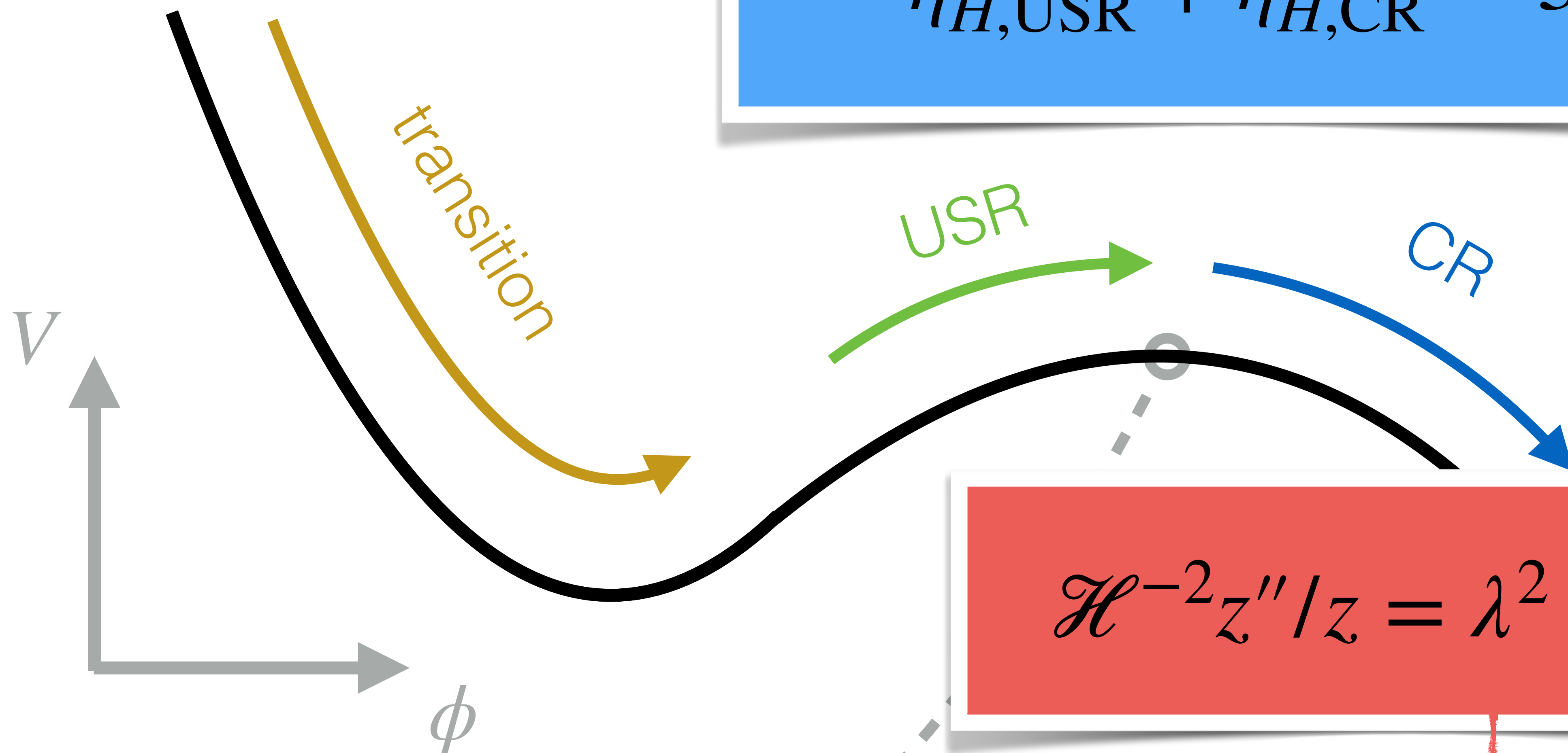
mode evolution:

$$u_k'' + \left(\frac{z''}{z} - k^2\right) u_k = 0$$

$z \equiv a\partial_N \phi = a\dot{\phi}/H$

[2205.13540 Karam et al]

WANDS DUALITY



for smooth hills

$$\eta_{H,\mathrm{USR}} + \eta_{H,\mathrm{CR}} = 3$$

$$\mathcal{H}^{-2} z''/z = \lambda^2 - 1/4$$

expansion around the local maximum:

$$\partial_N^2 \phi + 3\partial_N \phi \approx -\eta_{V,c}\left(\phi - \phi_c\right)$$

=>

mode evolution:

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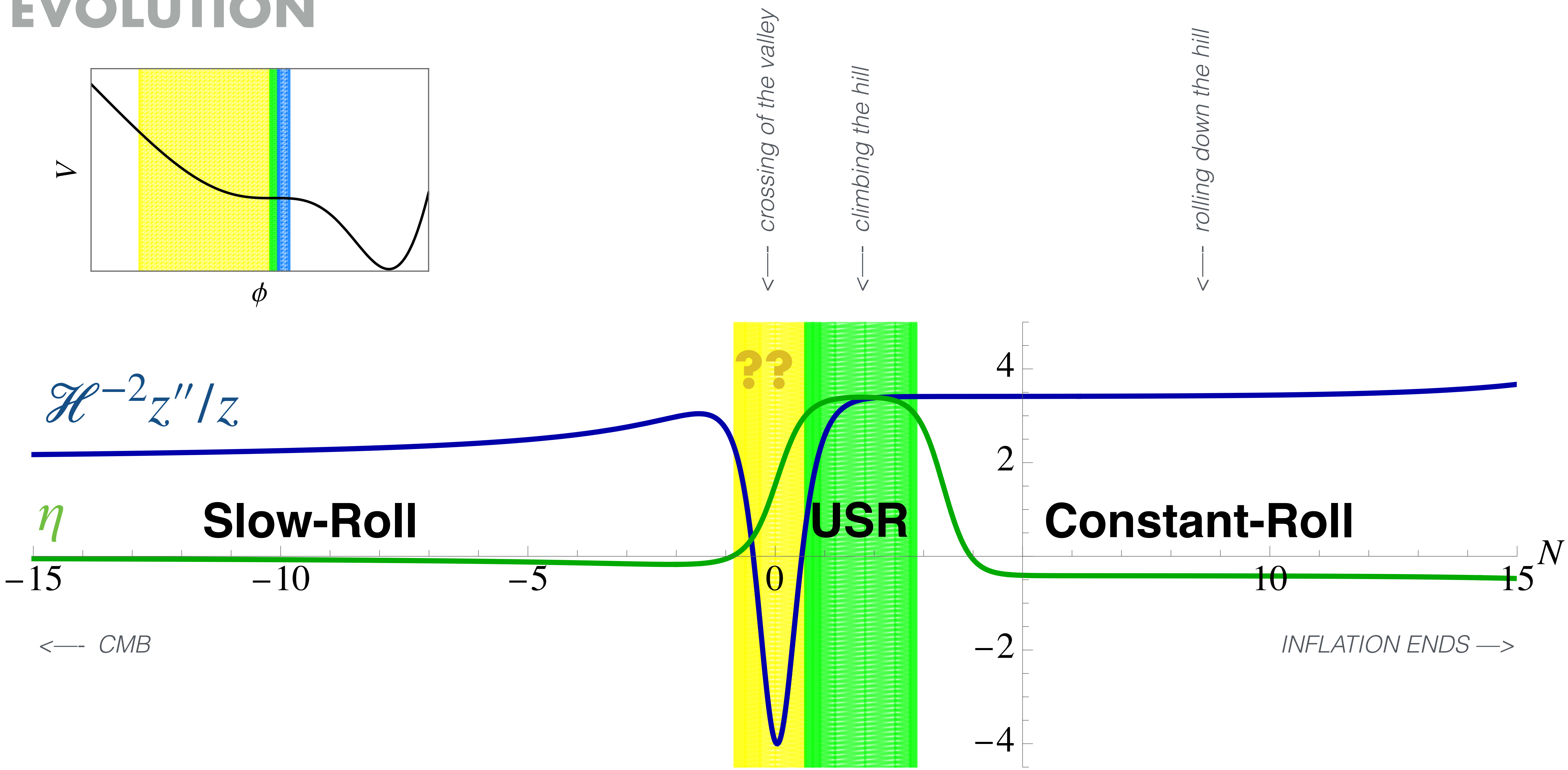
$z \equiv a\partial_N\phi = a\dot{\phi}/H$

mode evolution the same during USR and CR

[2205.13540 Karam et al]

BACKGROUND EVOLUTION



$$\mathscr{H}^{-2} z''/z \approx 2$$

$^*\lambda \approx 3/2$

$$\mathscr{H}^{-2} z''/z = \lambda^2 - 1/4$$

$^*\lambda \approx 1.91$

$$\eta \approx 0$$

$$\eta \approx \frac{3}{2} + \lambda$$

$$\eta \approx \frac{3}{2} - \lambda$$

MODE EVOLUTION

GROWING DAMPED

$$u_{k,1} = c_{1+} J_{-\lambda_1}(\kappa) + c_{1-} J_{\lambda_1}(\kappa)$$

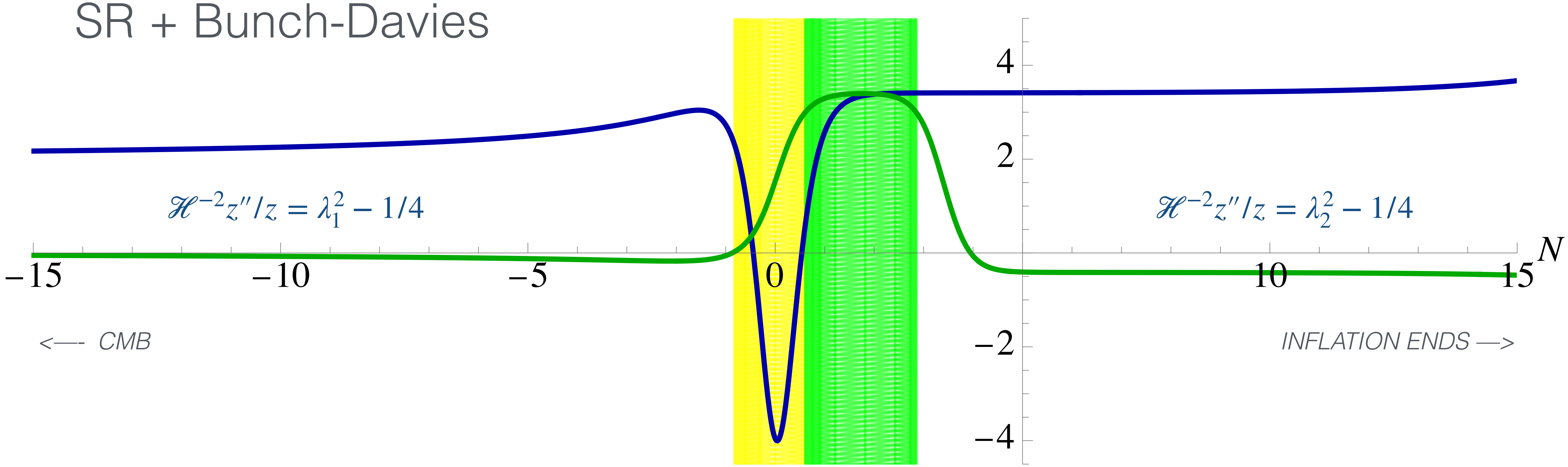
$$= \frac{E^{-i\kappa}}{\sqrt{2k}}\left(1 - \frac{i}{\kappa}\right)$$

SR + Bunch-Davies

GROWING DAMPED

$$u_{k,2} = c_{2+} J_{-\lambda_2}(\kappa) + c_{2-} J_{\lambda_2}(\kappa)$$

$\kappa \equiv k/\mathscr{H}$



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SR + Bunch-Davies

MIXING

$$\begin{pmatrix} c_{1+} \\ c_{2+} \end{pmatrix} = M_k \begin{pmatrix} c_{1-} \\ c_{2-} \end{pmatrix}$$

GROWING DAMPED

$$u_{k,2} = c_{2+} J_{-\lambda_2}(\kappa) + c_{2-} J_{\lambda_2}(\kappa)$$

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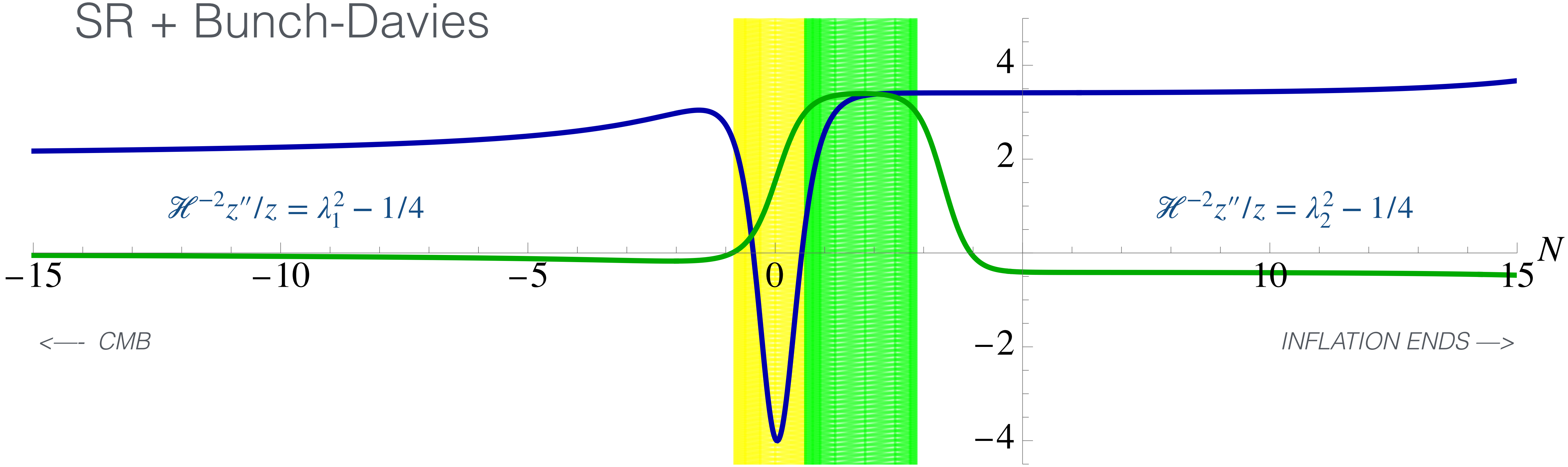

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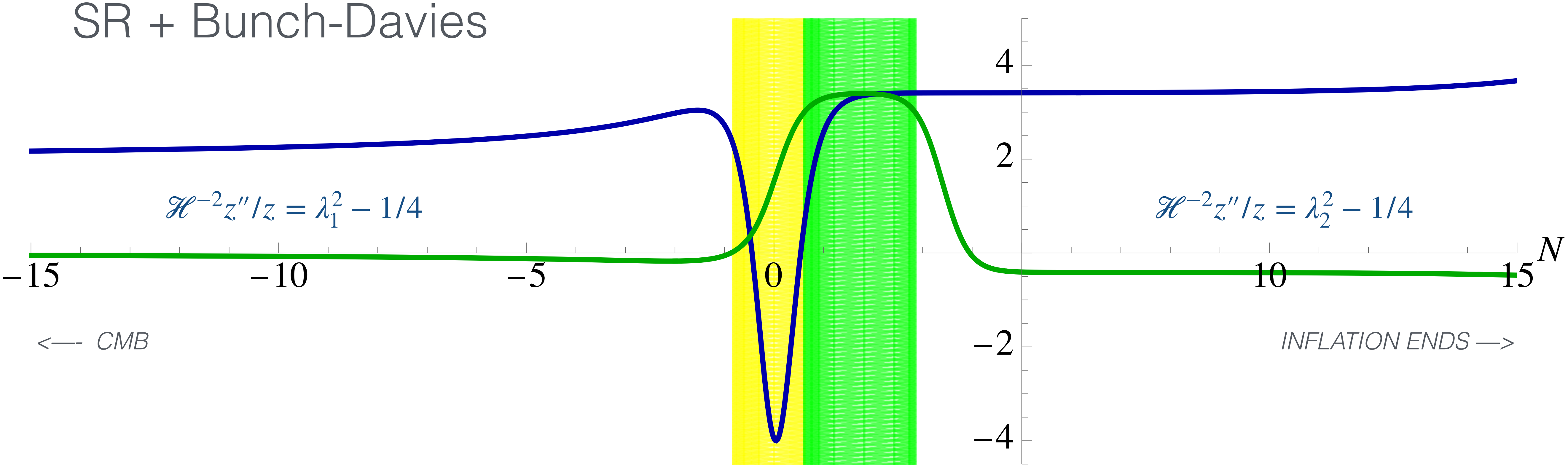
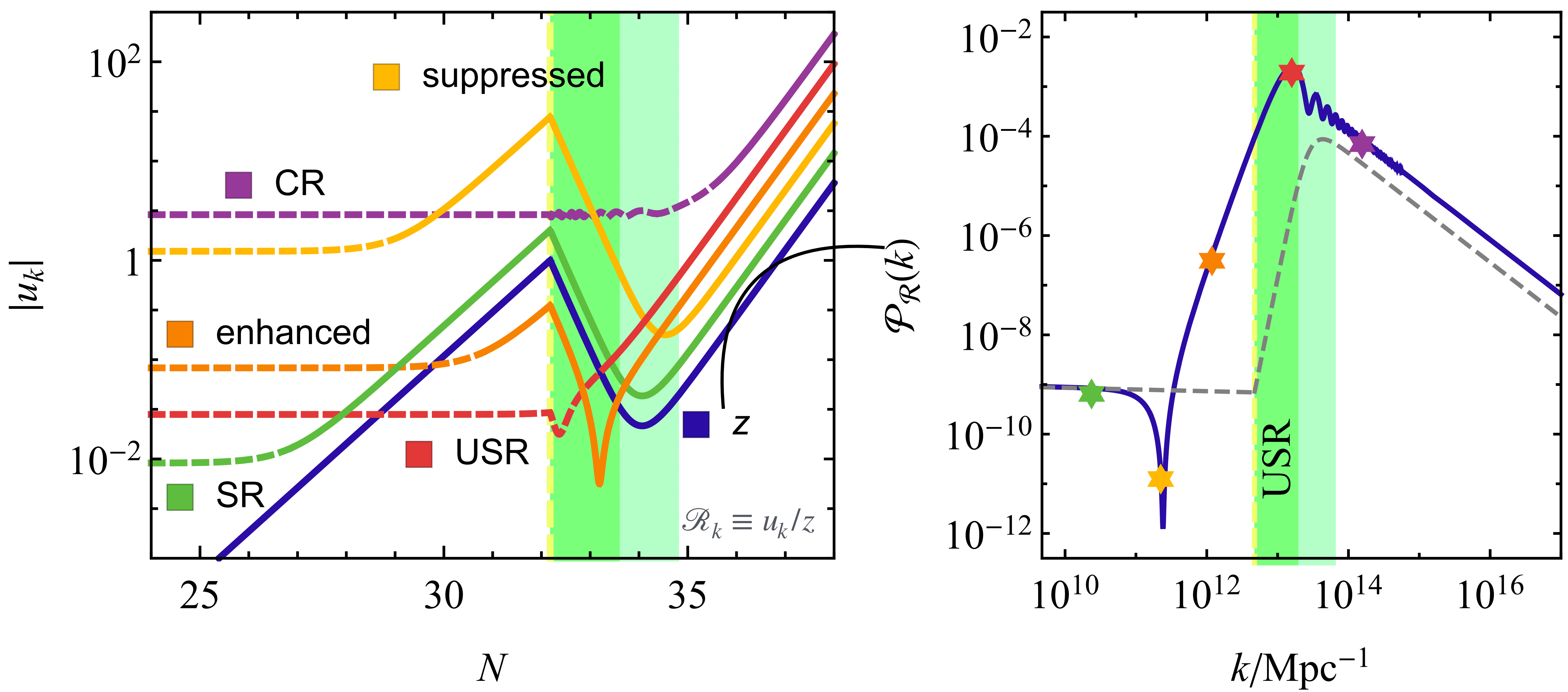


power spectrum depends on the transfer matrix:

$$P_{\mathscr{R}} = \frac{k^3}{2\pi^2}|\mathscr{R}_c|^2 = \frac{k^2\Gamma(\lambda_2)^2}{4\pi^3}\left|\frac{c_{2+}}{\zeta_2}\right|^2\left(\frac{k}{2\mathscr{H}(N_c)}\right)^{1-2\lambda_2}$$

MODE EVOLUTION



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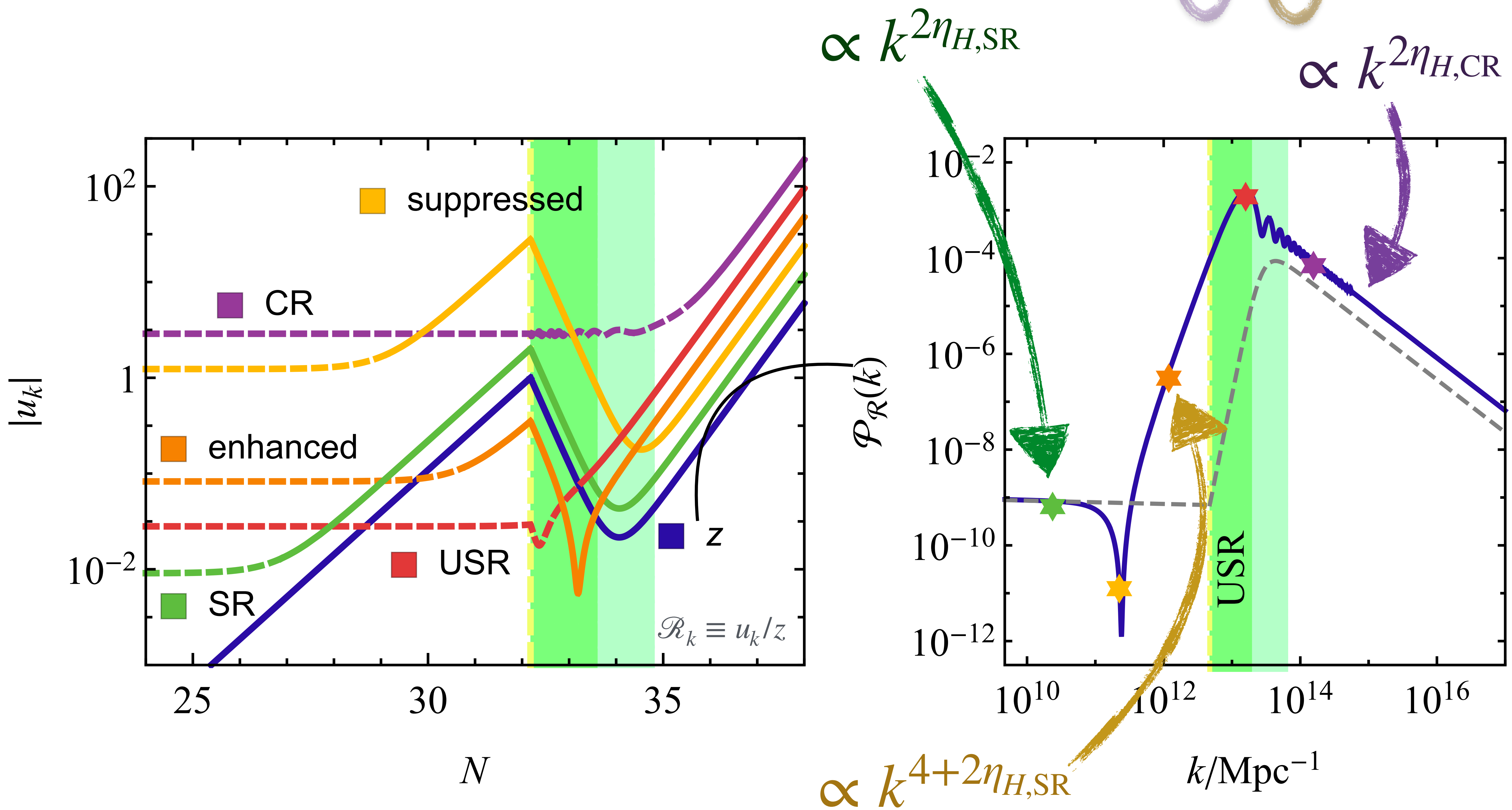

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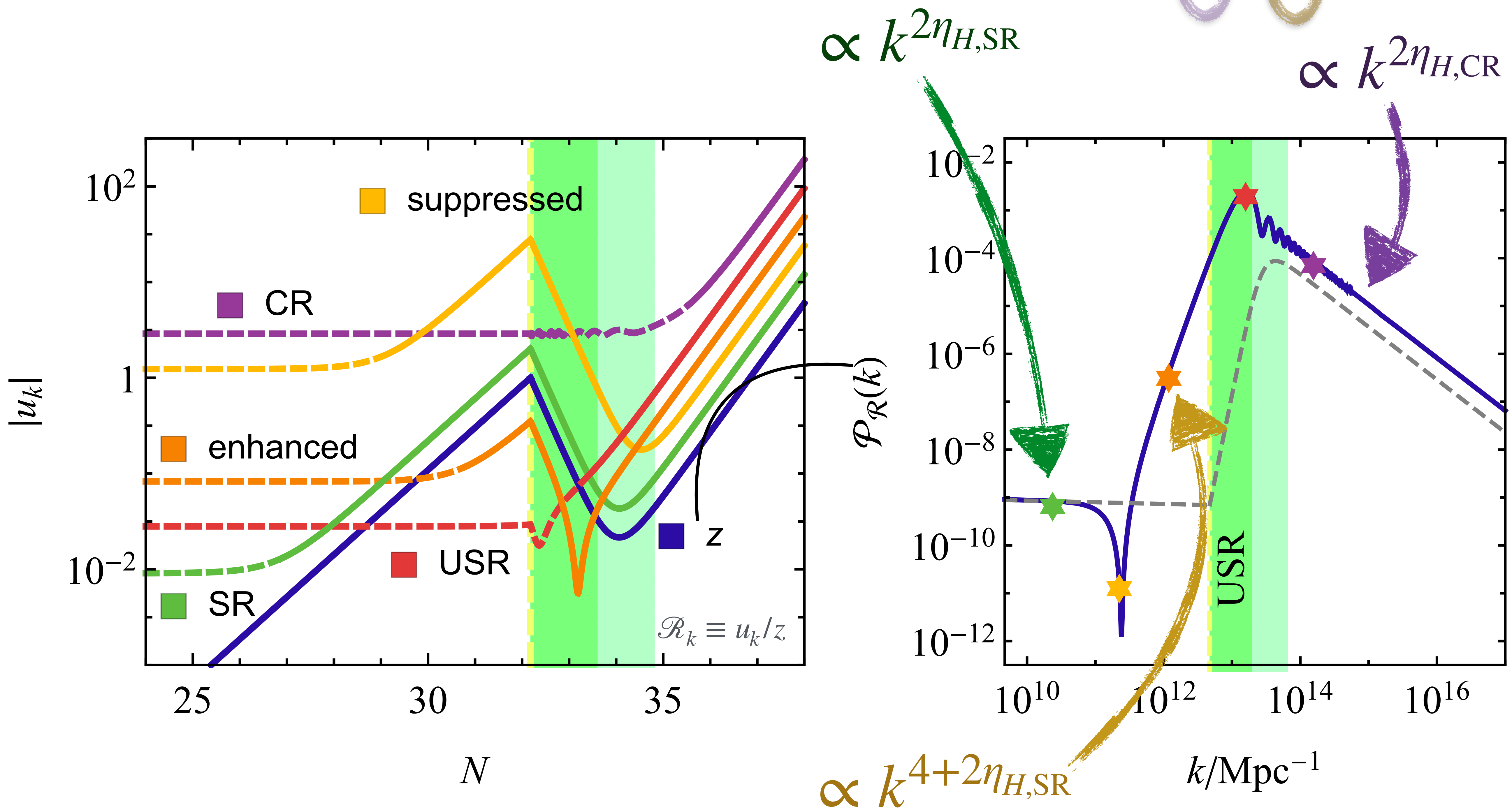


- faster than k^4 growth possible if pre-USR phase spectrum is blue tilted
- for instance, stacking growth phases can give k^8 growth [2012.02518 Tasinato]

INFLATIONARY TIMELINE

1. SLOW-ROLL (SR)
   CMB

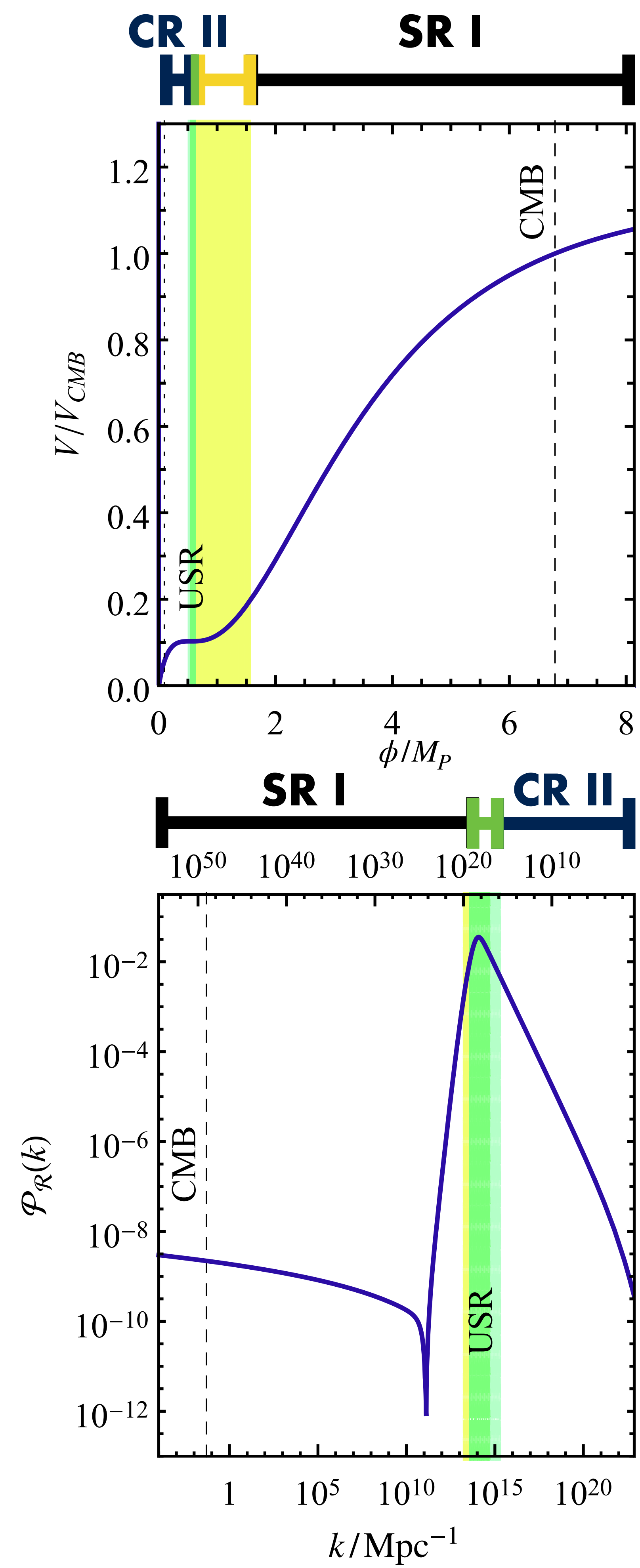
2. TRANSITION from SR to USR
   PEAK SHAPE

DUALITY {

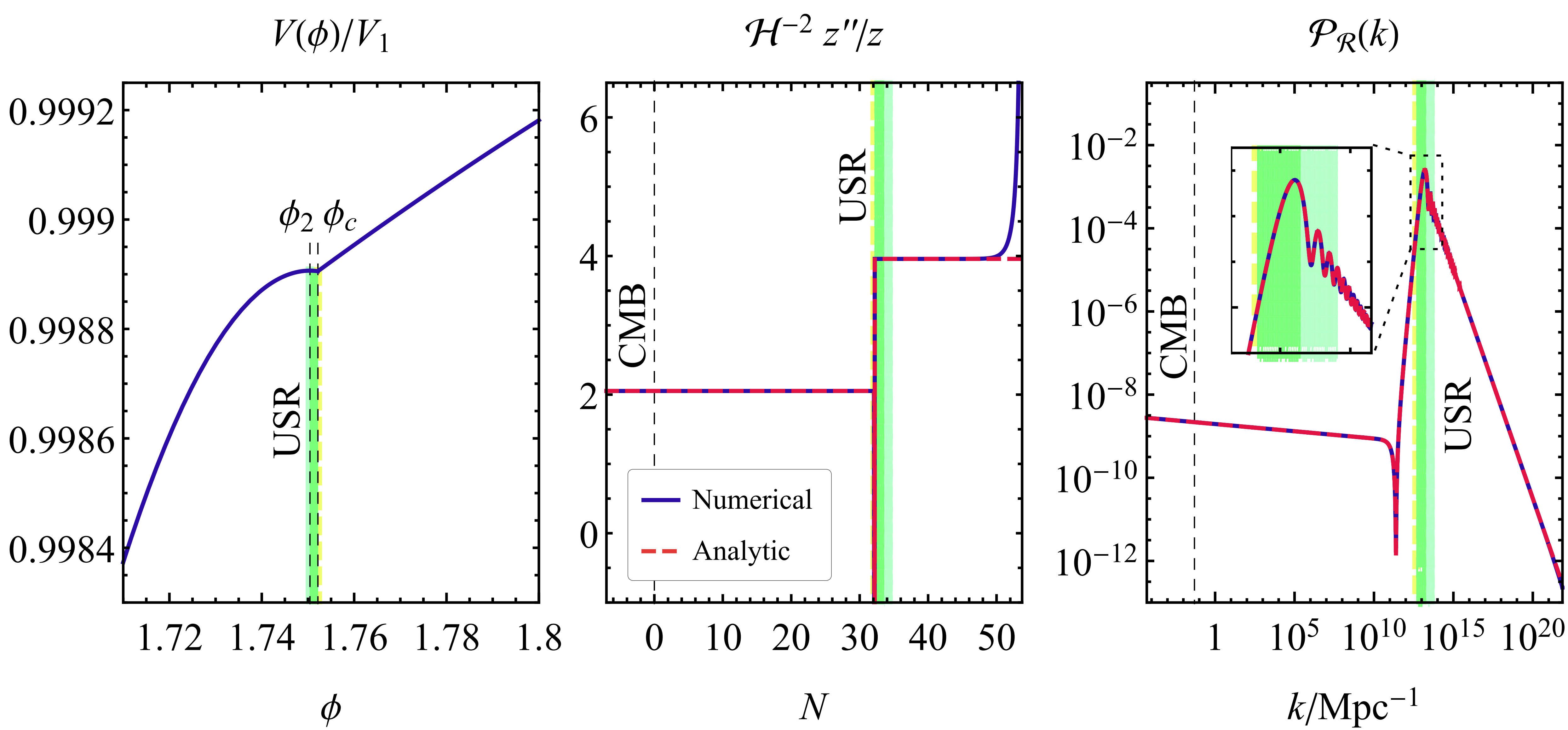
3. ULTRA-SLOW-ROLL (USR)

4. TRANSITION from USR to CR

5. CONSTANT-ROLL (CR)

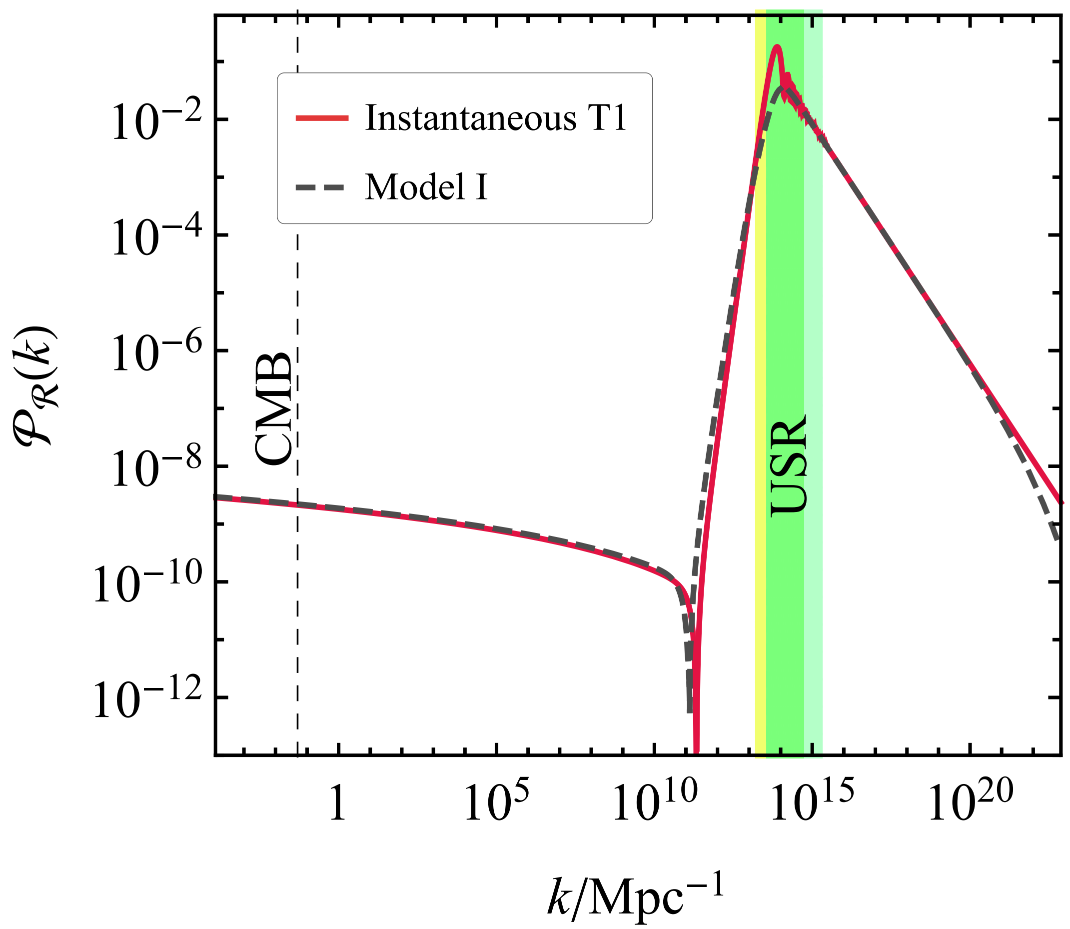


A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS*



A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS*

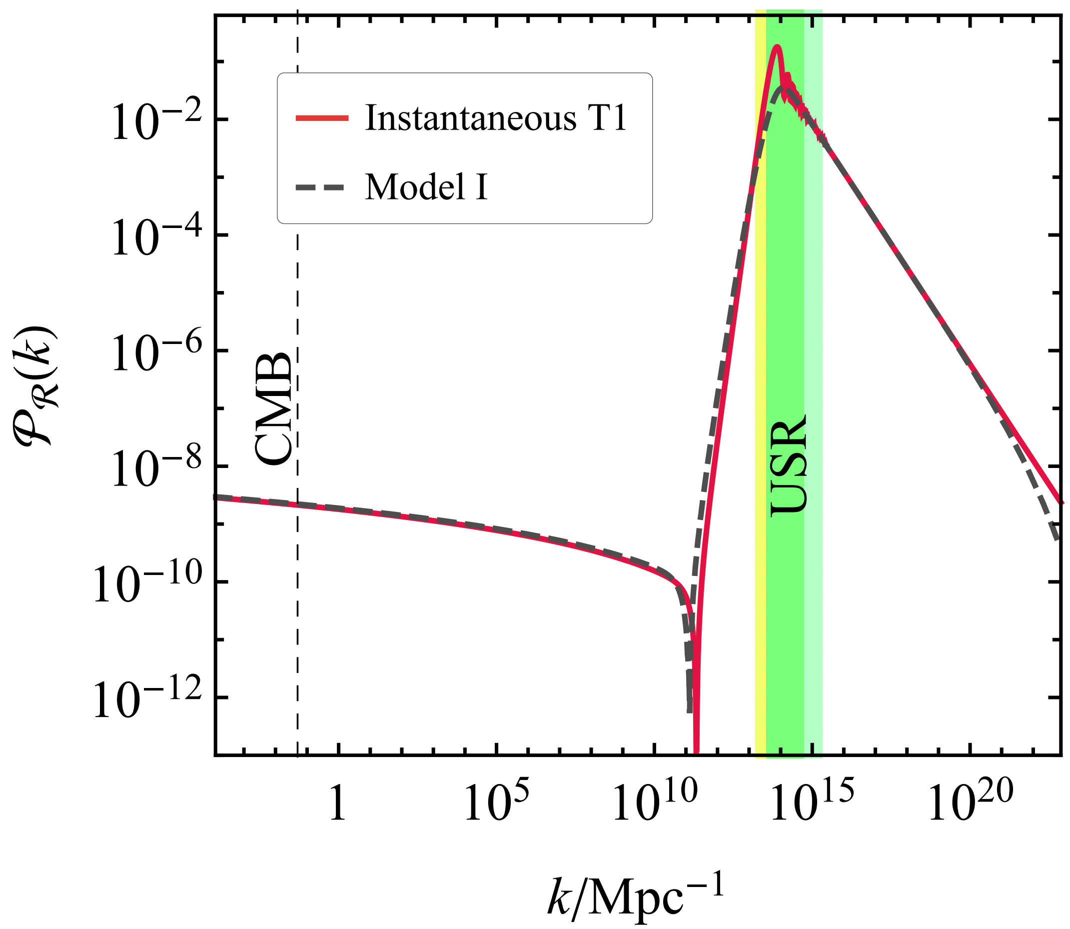
[2205.13540 Karam et al]



CURVATURE POWER SPECTRUM

A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS*

[2205.13540 Karam et al]

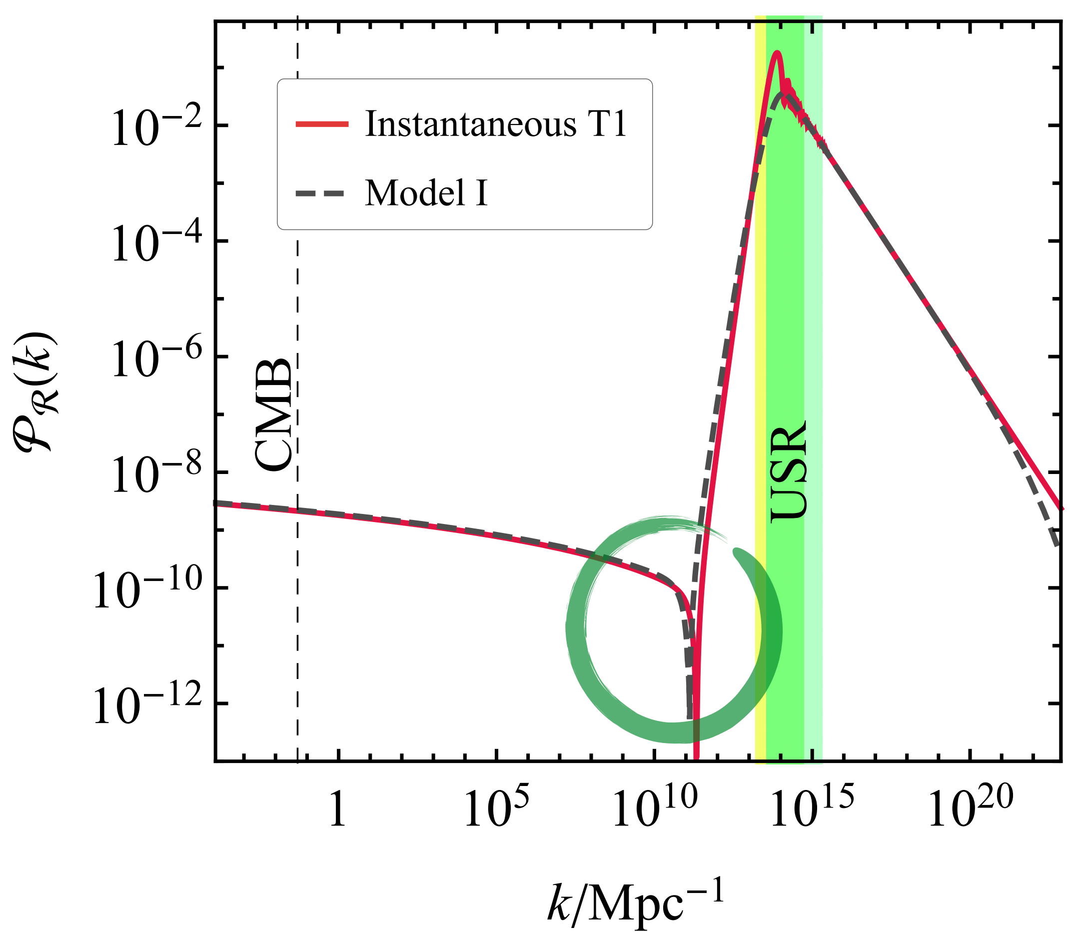


CURVATURE POWER SPECTRUM

1. can be found analytically
2. contains power spectra viable for both PBHs and CMB
3. approximates quasi-inflection point models quite well

A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS*

[2205.13540 Karam et al]



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4. expanis the dip

A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS*

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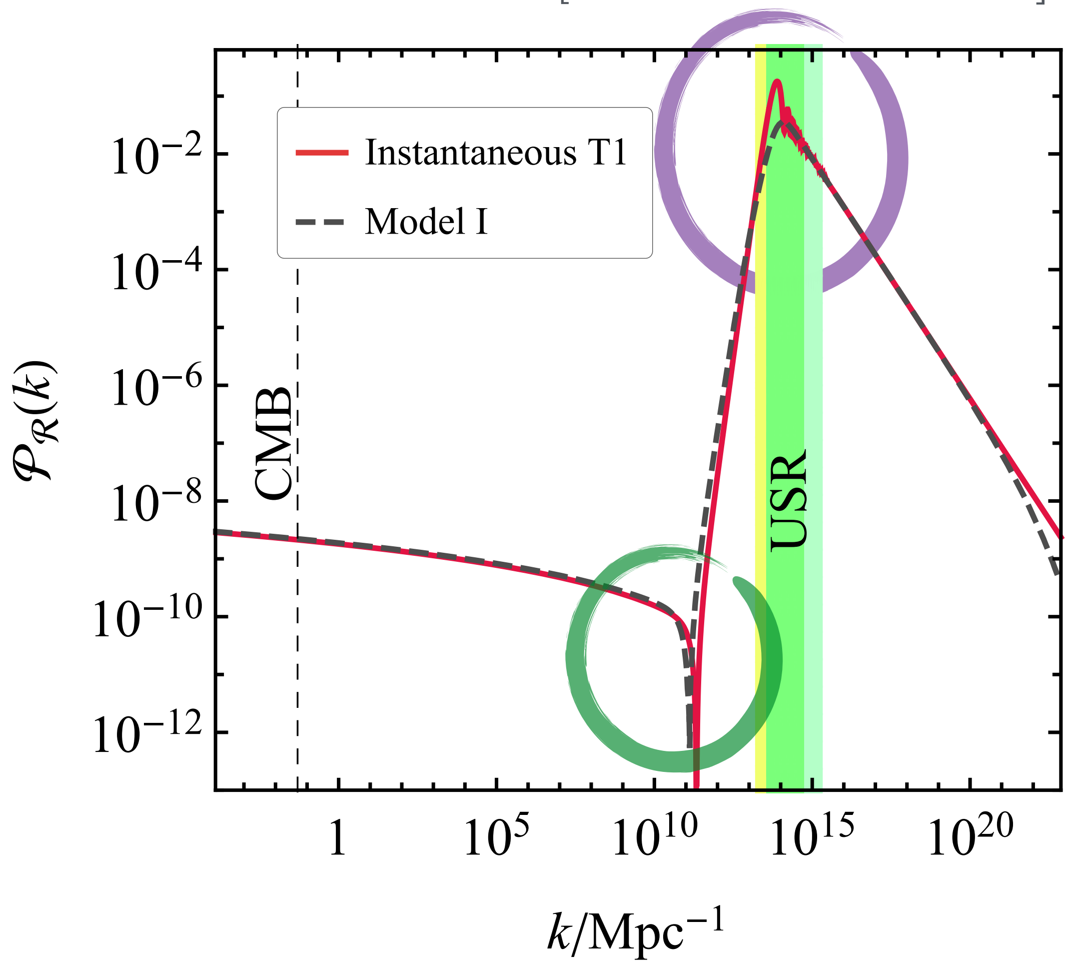

CURVATURE POWER SPECTRUM

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4. expanis the dip
5. oscillatory features

A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS*

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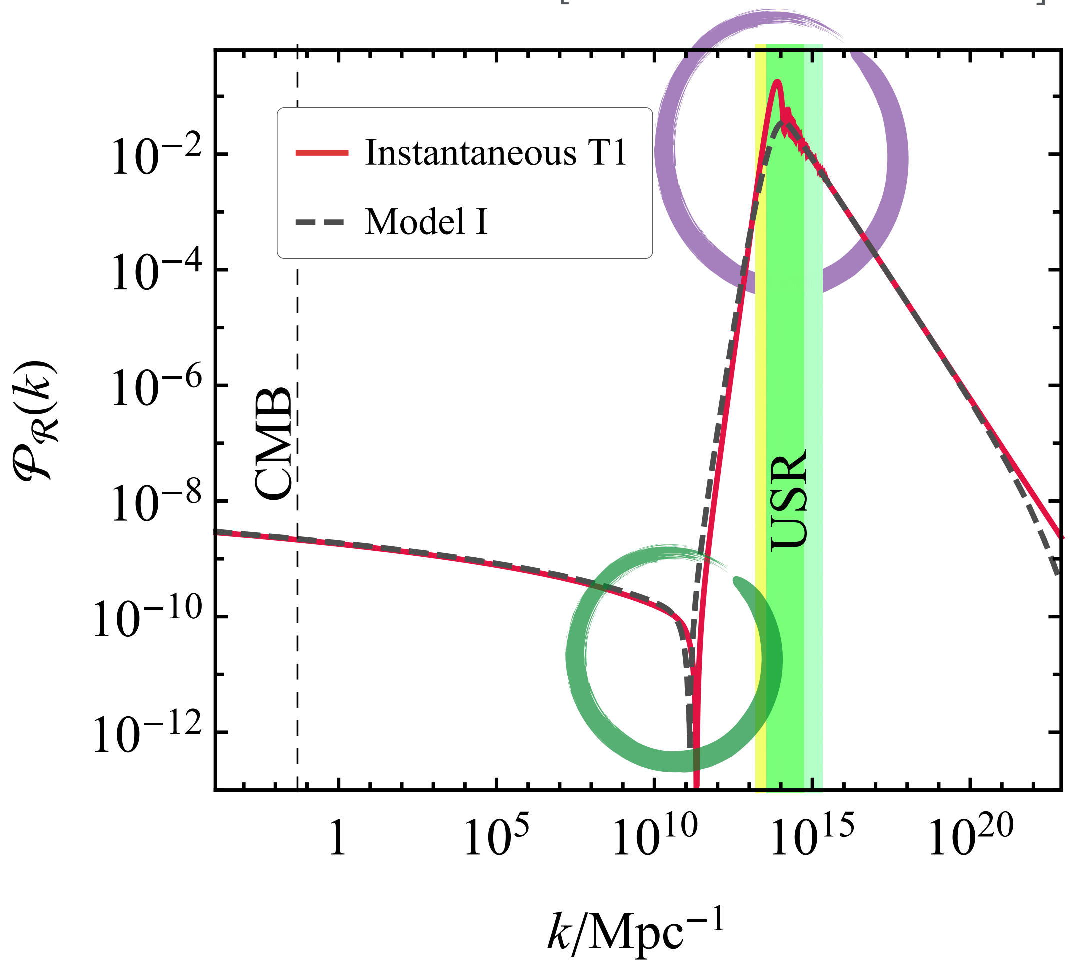


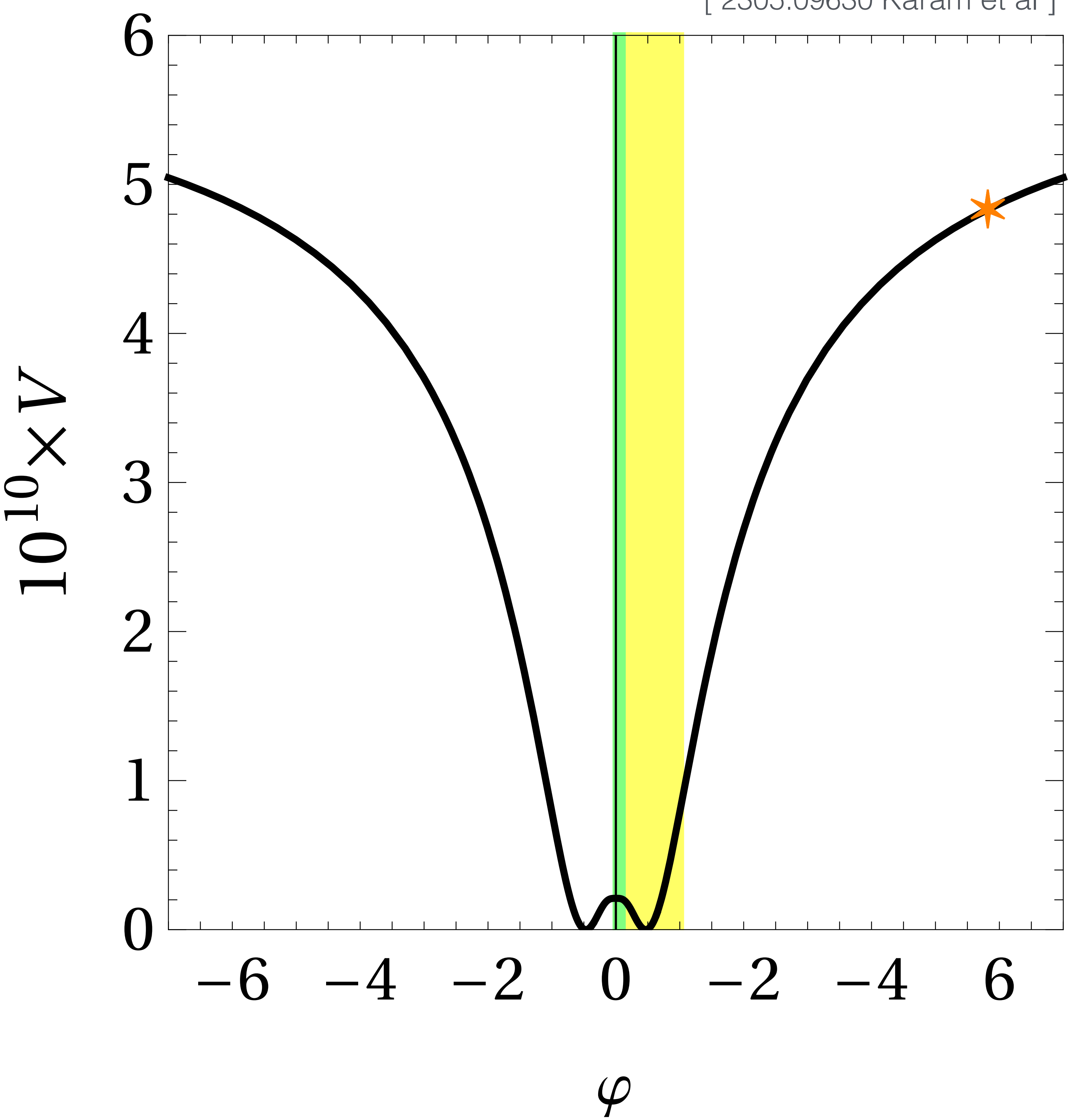
CURVATURE POWER SPECTRUM

1. can be found analytically
2. contains power spectra viable for both PBHs and CMB
3. approximates quasi-inflection point models quite well
4. expanis the dip
5. oscillatory features

=> PEAK SHAPE DEPENDS ON THE SR to USR TRANSITION

DOUBLE-WELL POTENTIALS

[2305.09630 Karam et al]



* *same inflationary timeline (SR to USR to SR)*

a candidate model:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\Omega(\phi)R - \frac{1}{2}K(\phi)(\partial\phi)^2 - U(\phi)\right]$$

with

$$\Omega(\phi) = 1 + \xi\phi^2$$

$$K(\phi) = \left(1 - \frac{\phi^2}{6\alpha}\right)^{-n}$$

$$U(\phi) = \frac{\lambda}{4}\left[\frac{1}{2}\phi^4\left(\ln\frac{\phi^2}{v^2} - \frac{1}{2}\right) + \frac{v^4}{4}\right]$$

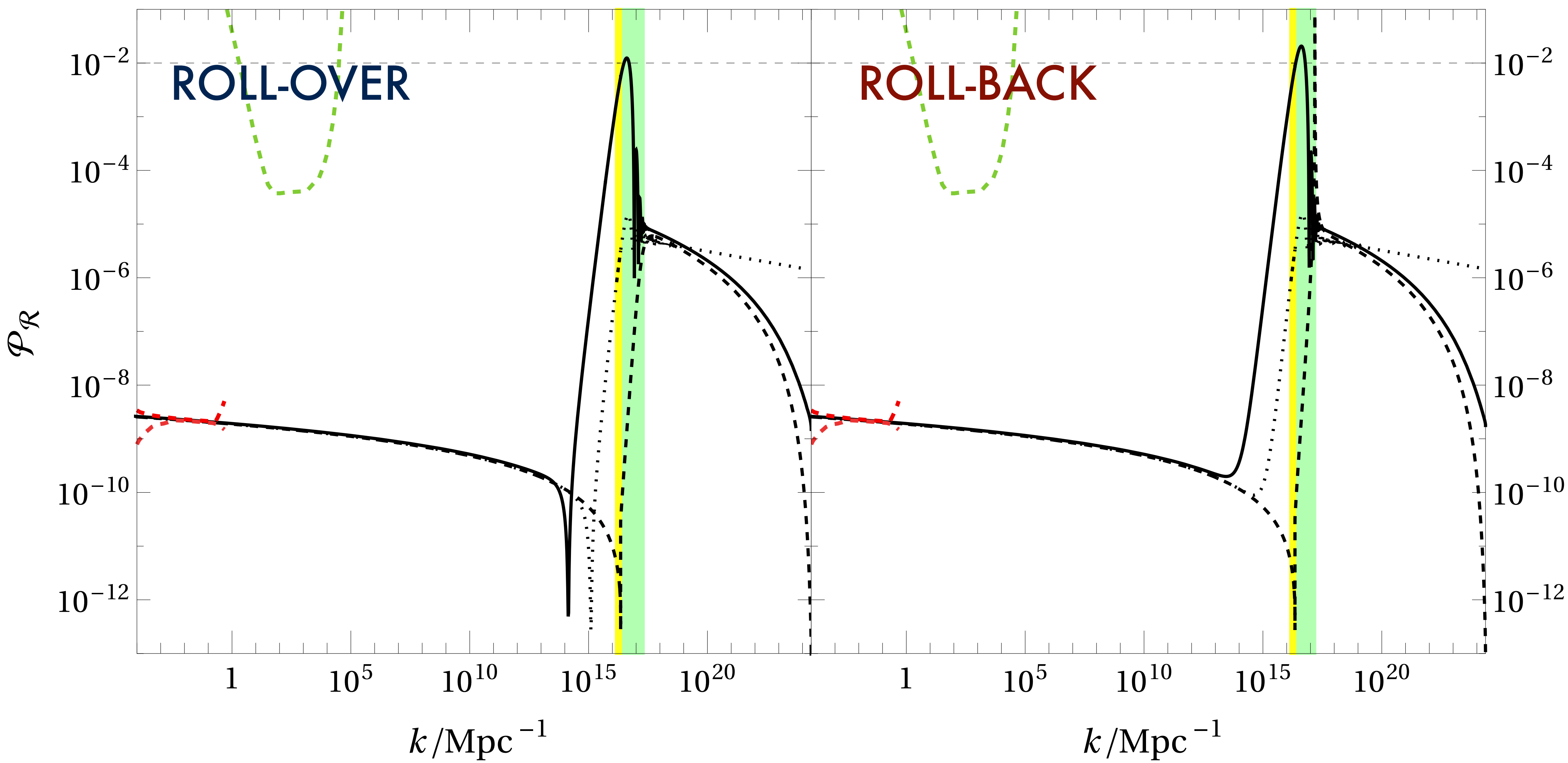
**** the CR phase***

$$\eta_{H,c} \approx \left.\frac{V''}{V}\right|_{\phi=0} = -\frac{m^2}{U_0} - 4\xi$$

$|\eta_{H,c}|$ can't be too large!!

DOUBLE-WELL POTENTIALS: *POWER SPECTRUM*

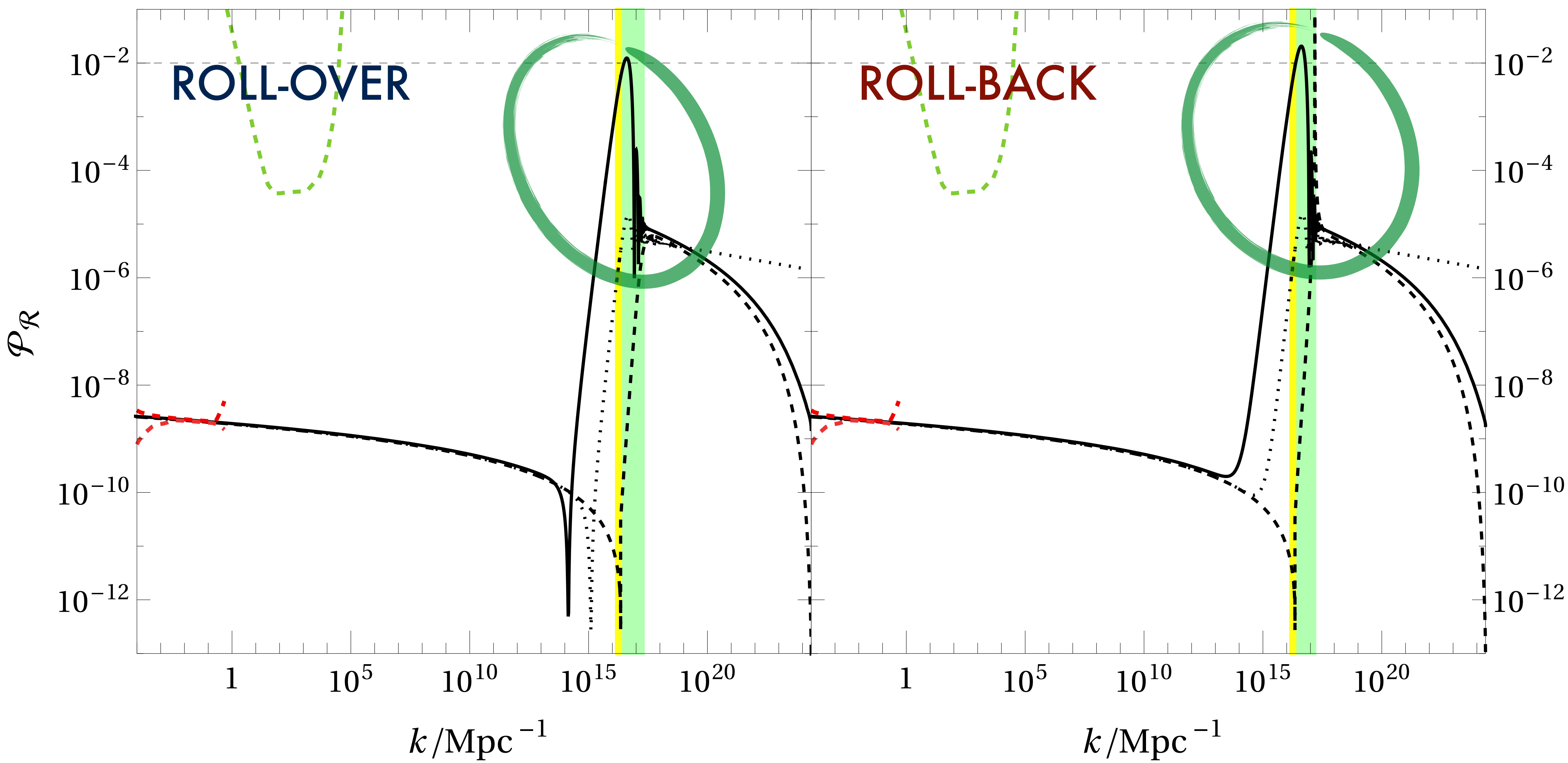
[2305.09630 Karam et al]



DOUBLE-WELL POTENTIALS: *POWER SPECTRUM*

enhanced oscillatory features

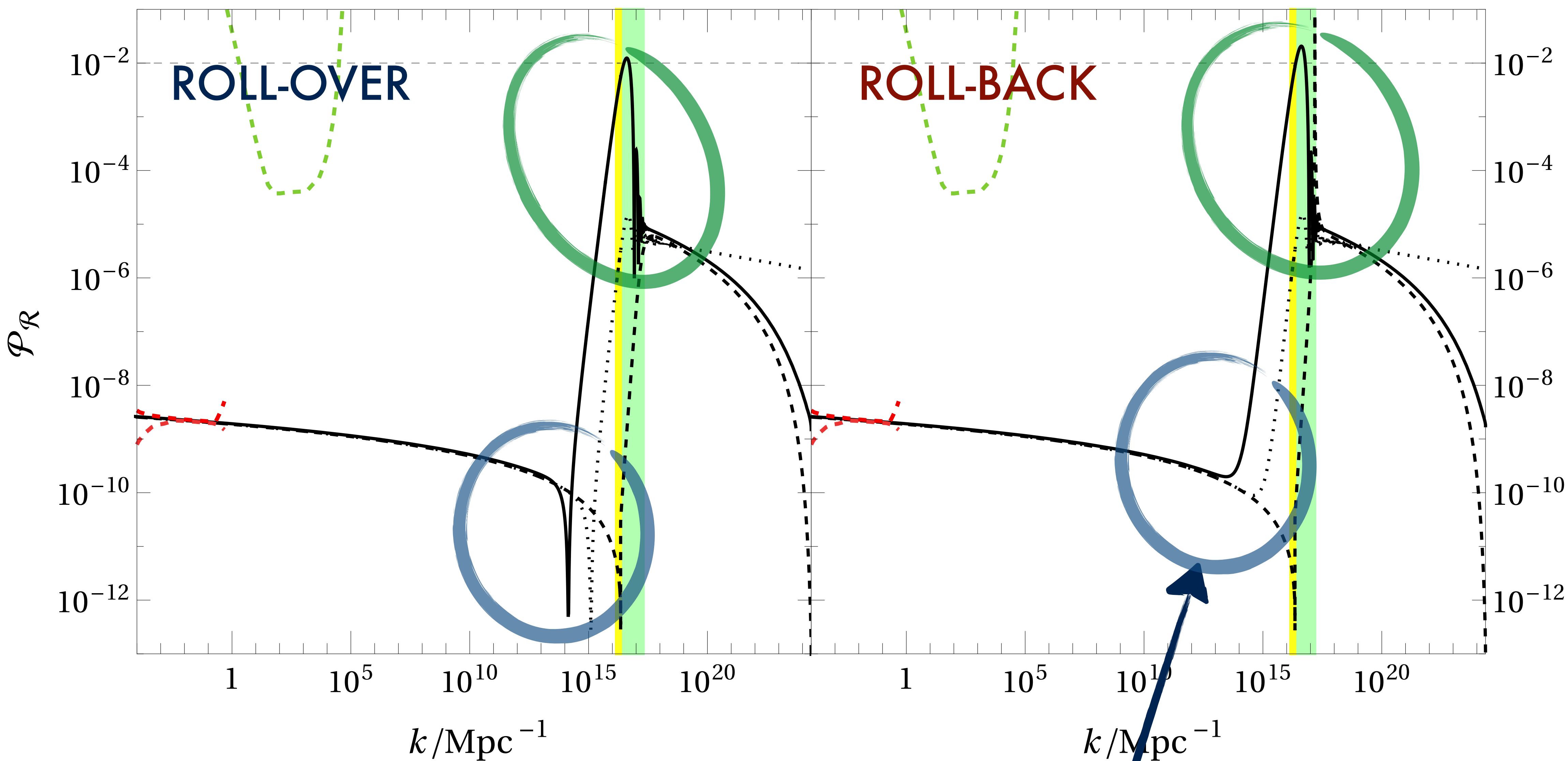
[2305.09630 Karam et al]



DOUBLE-WELL POTENTIALS: *POWER SPECTRUM*

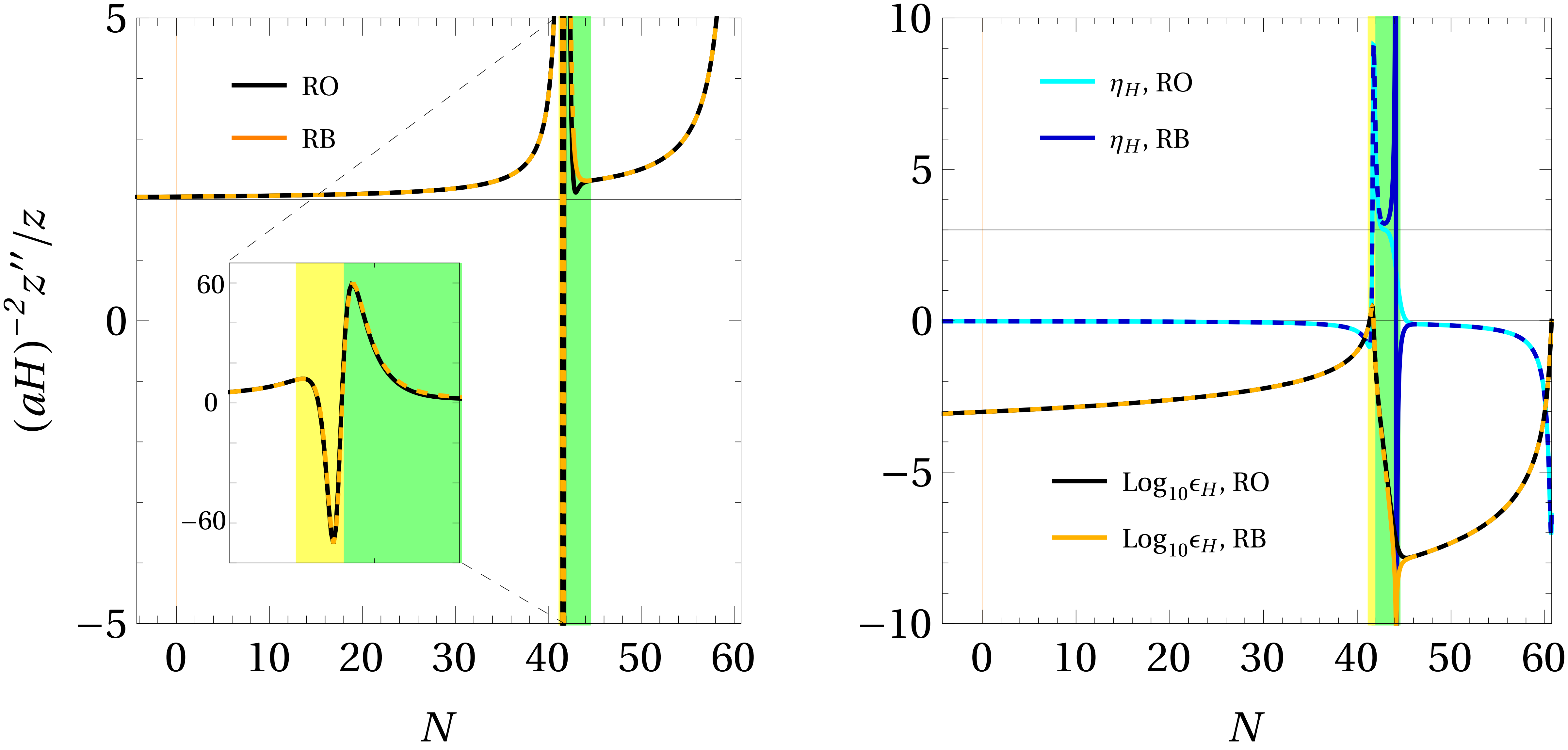
enhanced oscillatory features

[2305.09630 Karam et al]



dip dissapears

DOUBLE-WELL POTENTIALS: *SPECTRAL OSCILLATIONS*



STRONG TIME DEPENDENCE IN MODE EVOLUTION

DOUBLE-WELL POTENTIALS: *THE DIP*

FIRST PHASE

SECOND PHASE

$$P_{\mathscr{R}}(k) \approx \left| \theta(\mathscr{H}_c - k)\left[\frac{H^2}{2\pi\dot{\varphi}}\right]_{\mathscr{H}=k} + \frac{k\, p(k/\mathscr{H}_c, \lambda_2)}{k_{\rm CR}\,|\,p(k_{\rm CR}/k_c, \lambda_2)\,|}\left[\frac{\Gamma(\lambda_2)}{2^{3/2-\lambda_2}\pi^{3/2}}\frac{H^2}{\dot{\varphi}}\right]_{k=k_{\rm CR}} \right|^2$$

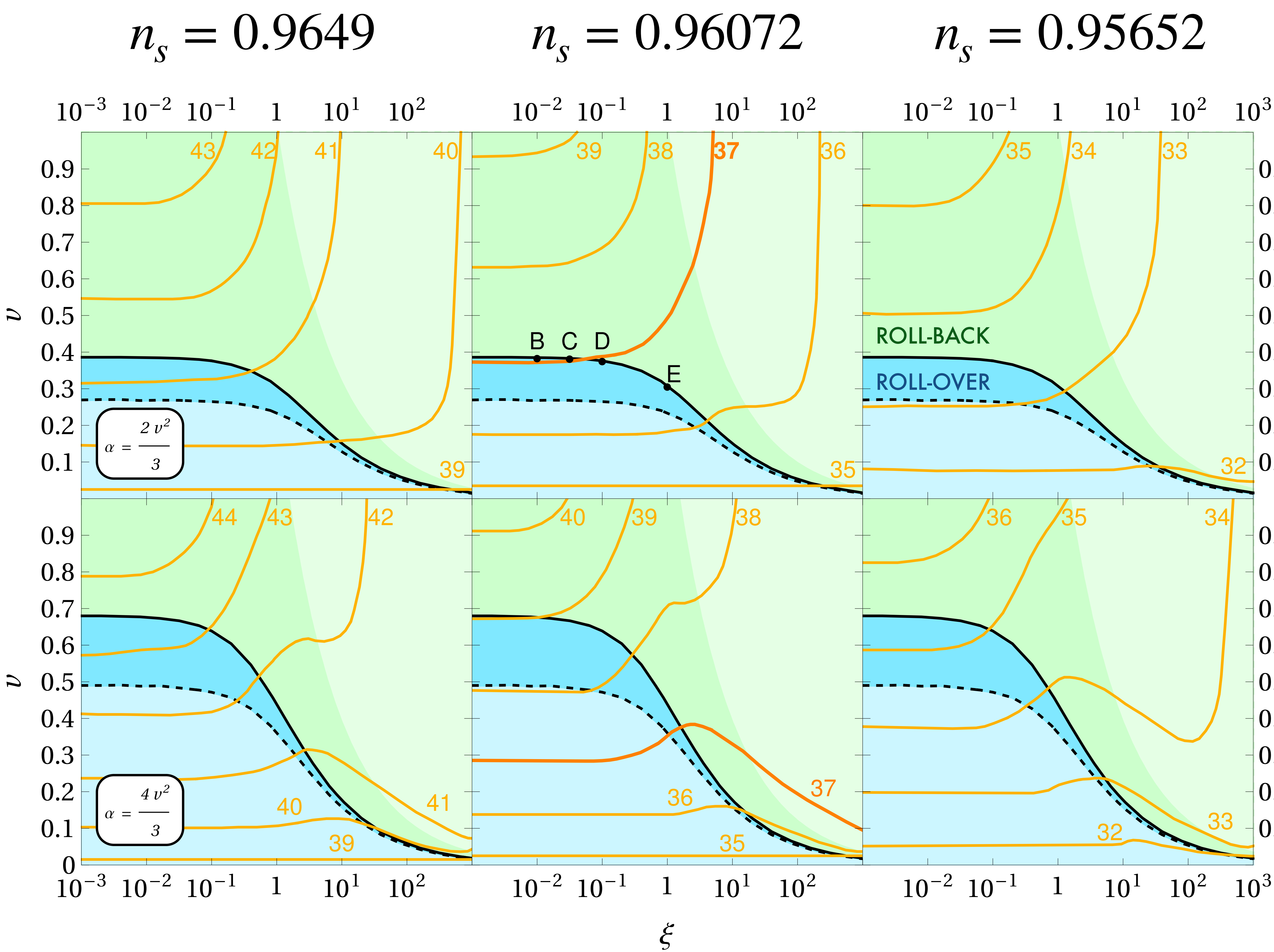
during SR

shape function

during CR

DIP CAN BE REMOVED VIA VELOCITY REVERSAL

DOUBLE-WELL POTENTIALS: *TUNING for CMB*

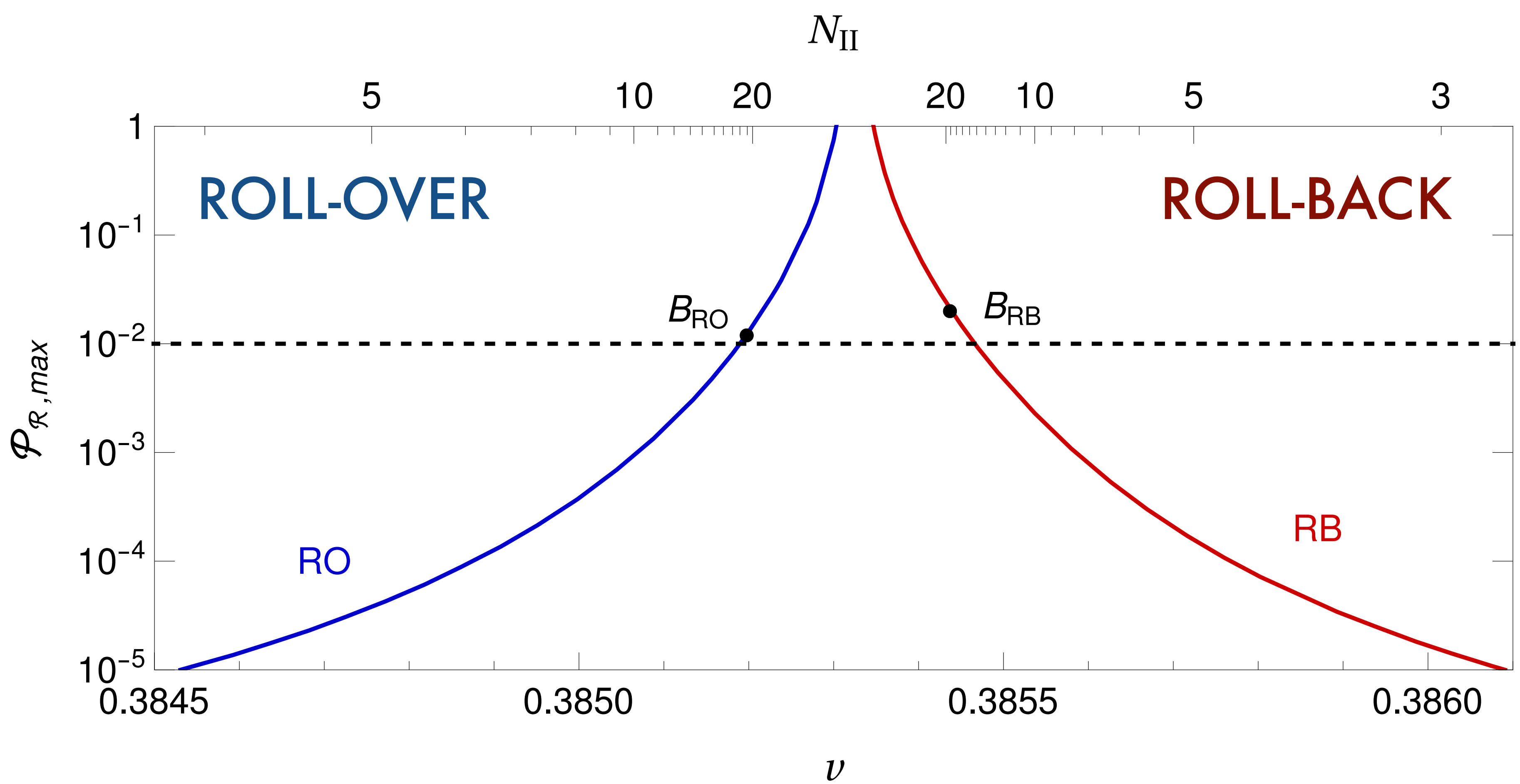


FINE TUNING of the potential

NESSECITY for TUNING:

1. abundance *exponentially* sensitive to peak height
2. peak height polynomially sensitive to parameters of the model

$$\mathscr{P}_{\mathscr{R},\max} \approx 5 \times 10^{-16} \,|\, v/v_c - 1 \,|^{-3.9}$$



[2305.09630 Karam et al, 2304.01997 Byrnes et al]

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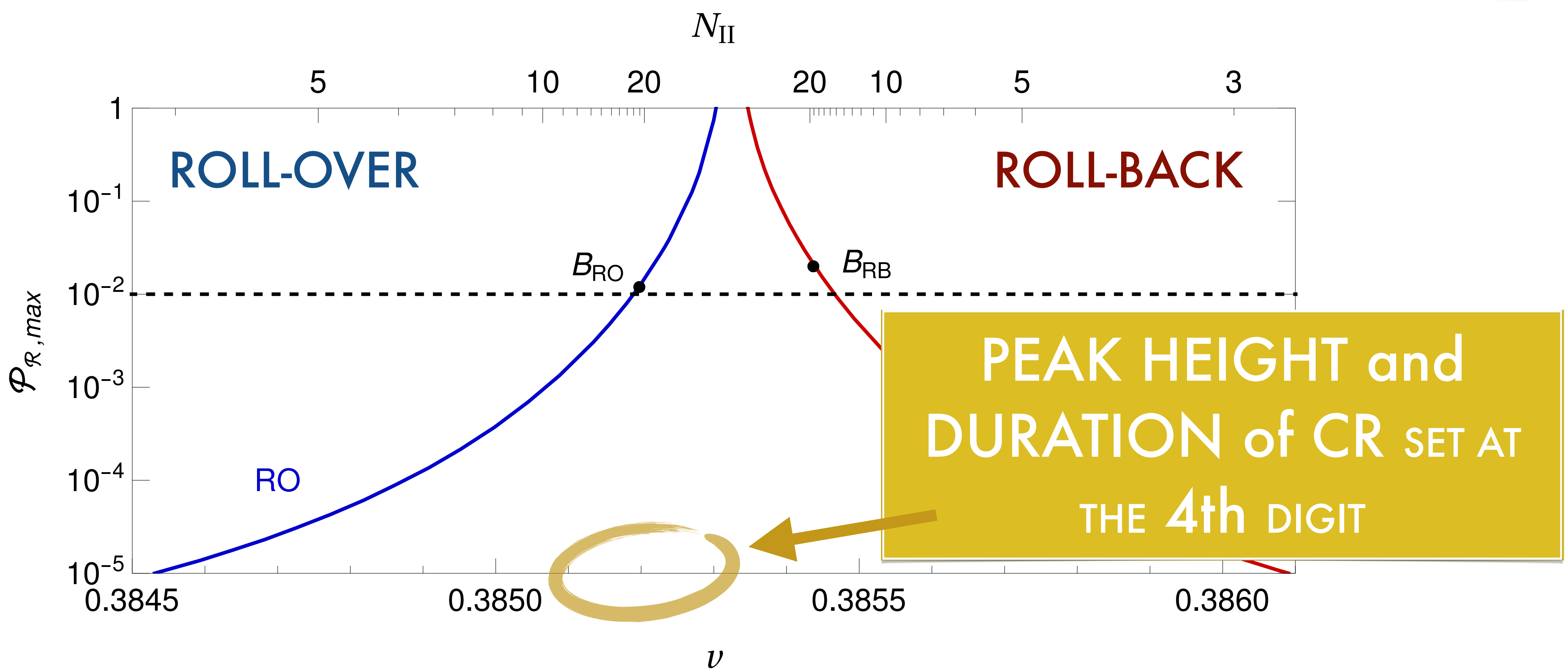
$$\mathcal{P}_{\mathcal{R},\max} \approx 5 \times 10^{-16} |v/v_c - 1|^{-3.9}$$



[2305.09630 Karam et al, 2304.01997 Byrnes et al]

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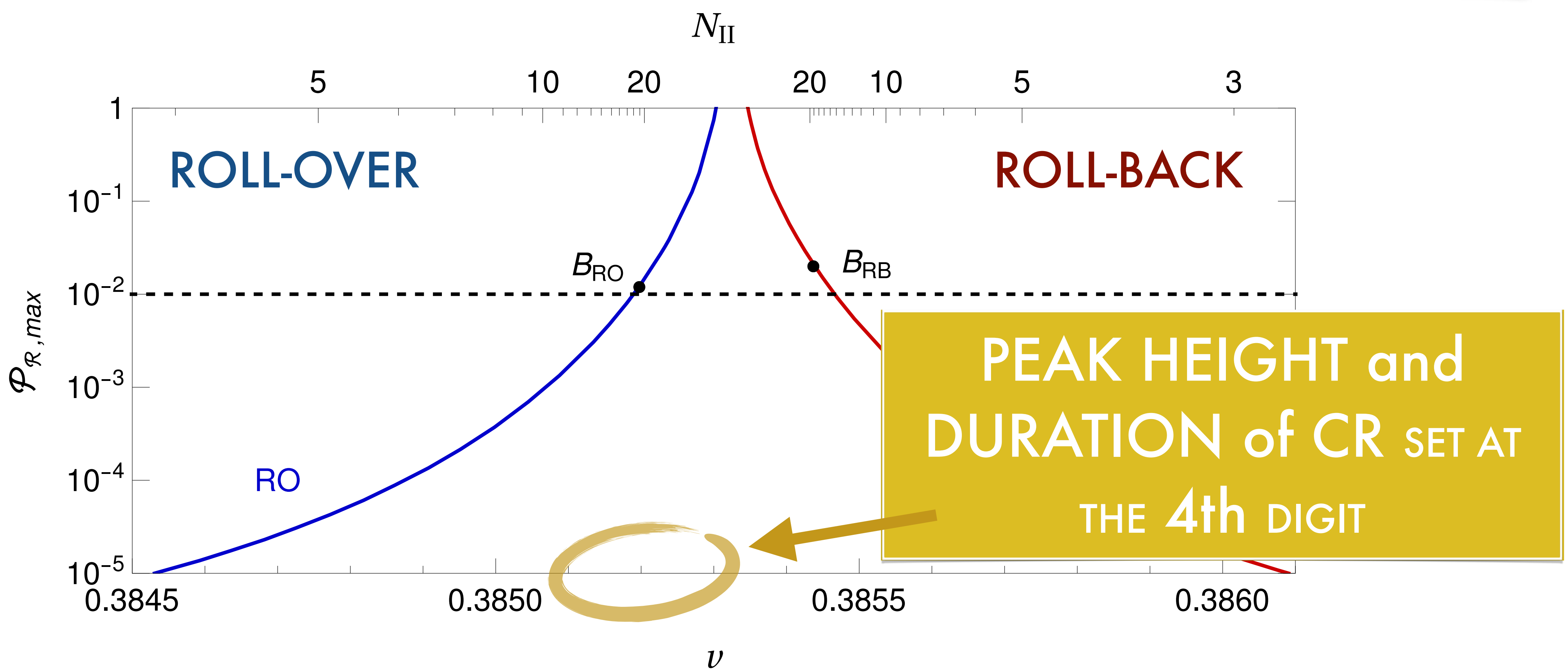

Less tuning with sizeable spectral oscillations

[2305.09630 Karam et al, 2304.01997 Byrnes et al]

In single field inflation...

- the **length of the USR-like** phase **does not** determine the enhancement of the power specreum *(strictly speaking, USR is uncommon)*
- the **peak shape** is deterined by the SR to USR transition which is **the most model dependent feature** of the spectrum *(non-Gaussianities?)*
- **no steepest growth** without additional assumptions