Primordial black holes: A (fine) tuned opportunity for probing inflation

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Motivation: Probe initial conditions and contents of the universe!

- **Black holes are cold and dark**
- **Primordial (unlike astrophysical) BHs are a DM candidate**
- **Decaying BHs could probe quantum gravity**

PBHs could explain

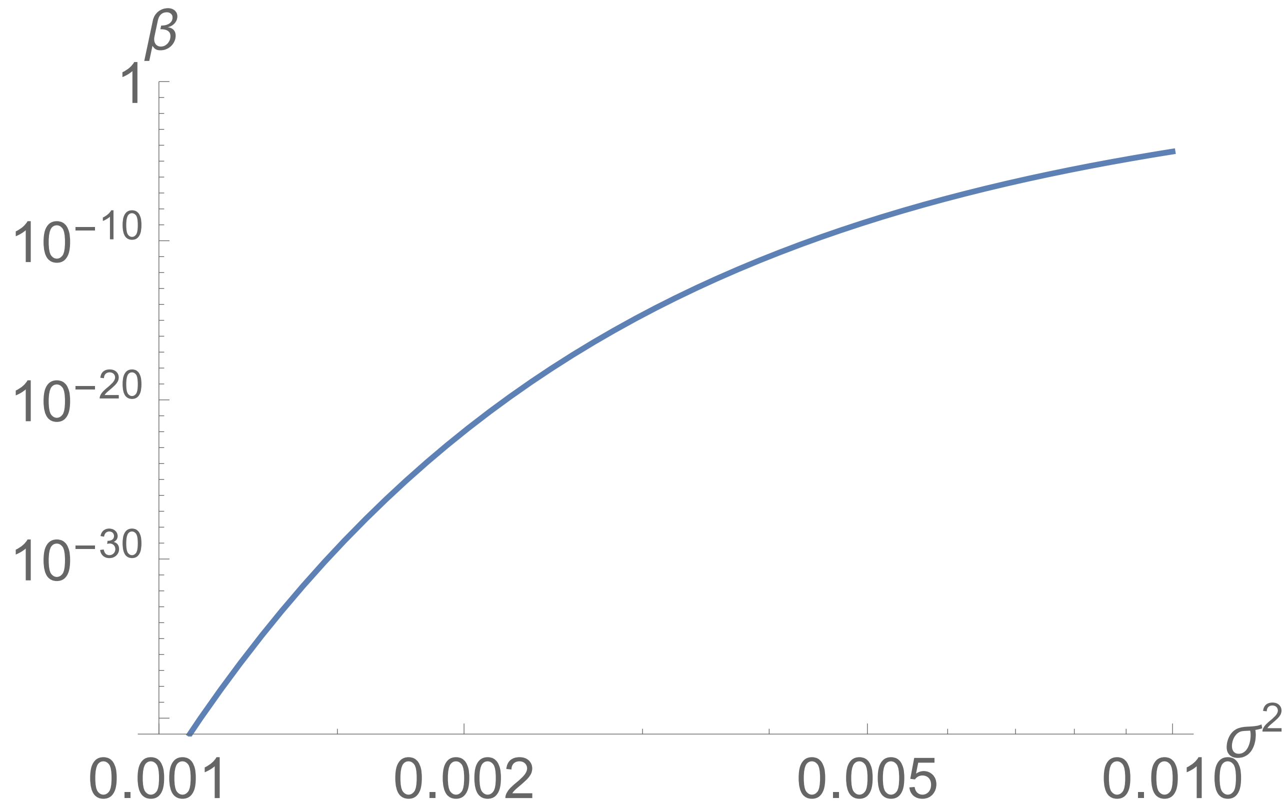
- CDM, microlensing events, unexpected properties of the LIGO-Virgo-KAGRA detections, etc
- We don't need a new particle for DM, but we do need new initial conditions
- What does inflation need to do to generate the right initial conditions?
- Focus on the collapse of large amplitude overdensities shortly after horizon entry

PBHs fine-tuning

- It's well known that for Gaussian perturbations in a radiation era that

$$\beta \sim e^{-\frac{\delta_c^2}{2\sigma^2}} \sim e^{-0.1/\sigma^2}$$

- Exponential sensitivity to the power spectrum amplitude



Fine-tuning questions

- This exponential sensitivity is well known and (fairly) model independent
- Is that as bad as it gets?
- Is generating an observable amplitude of secondary induced GWs (SIGWs) much more natural? (no exponential sensitivity)

Inflationary questions

- Aren't inflationary perturbations meant to be quasi scale invariant? We normally say such CMB observations are evidence of inflation.
- We need the power spectrum amplitude to grow from 10^{-9} to $\sim 10^{-2}$ such that PBHs form a tiny (but non-zero) fraction
- What is required from inflation?

Start with single-field

- We study single-field inflation with an inflection point (ultra slow roll).
- Arguably the simplest model
- *Cole, Gow, CB, Patil 2304.01997*

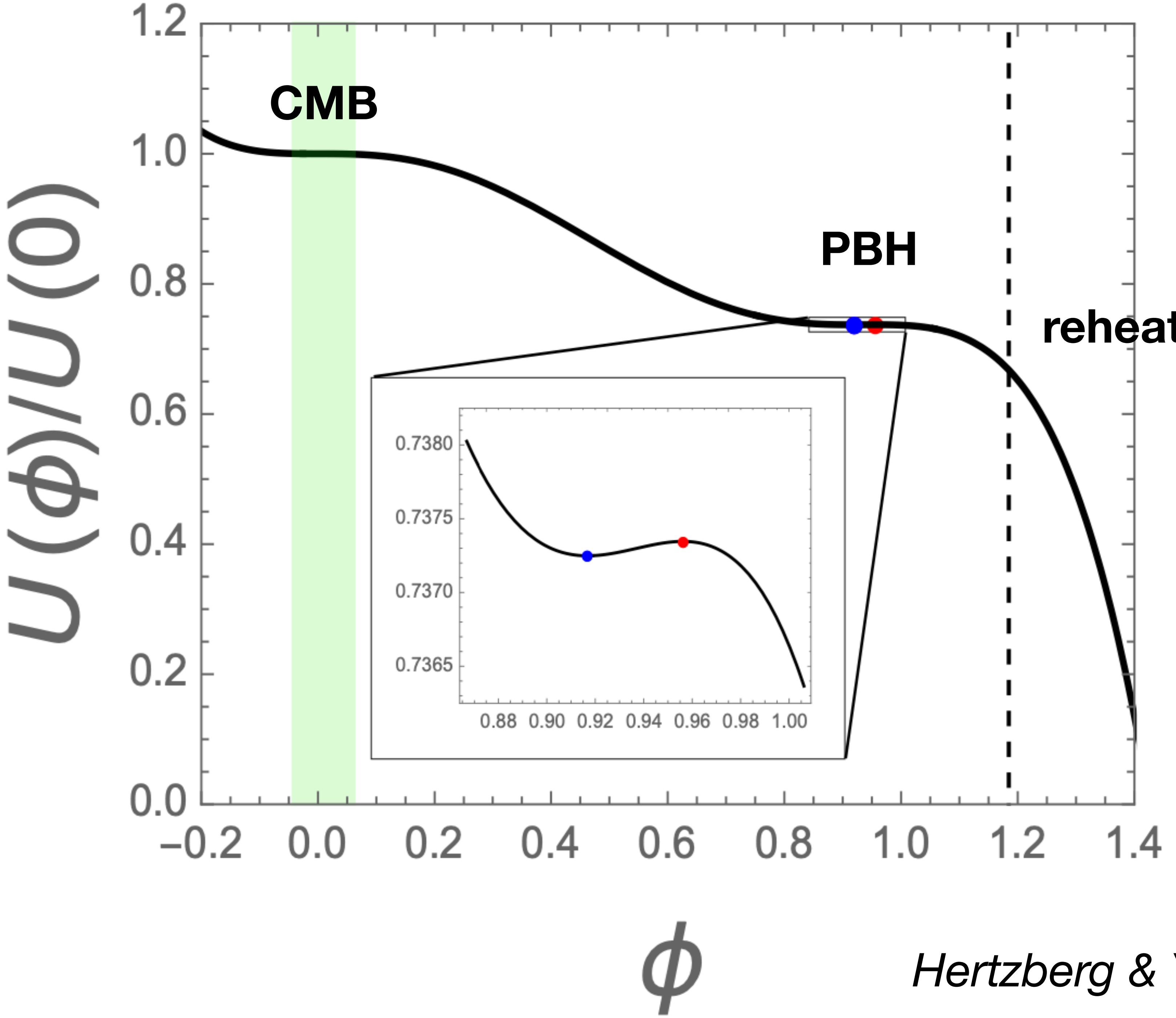
Piecewise step

- Exactly flat potential implies $KE \propto a^{-6}$
- Ultra-slow roll inflation occurs with exponential growth
- The field can roll a maximum distance - a longer flat part spells disaster (also tunnelling e.g. *Animali & Vennin '22*)
- A shorter flat part is safe. How boosted is the power spectrum?
- If the flat step is 90% of the maximum, the boost is only 100
- Need the flat step to be 99.9% of maximum to boost by 10^6

Smooth potentials

- A piecewise step is just a toy model
- Does a smooth potential require less tuning?

A polynomial potential



Hertzberg & Yamada 2017

A polynomial potential

$$V(\phi) = c_0 + \frac{c_1}{\Lambda}\phi + \frac{c_2}{2\Lambda^2}\phi^2 + \frac{c_3}{3!\Lambda^3}\phi^3 + \frac{c_4}{4!\Lambda^4}\phi^4 + \frac{c_5}{5!\Lambda^5}\phi^5$$

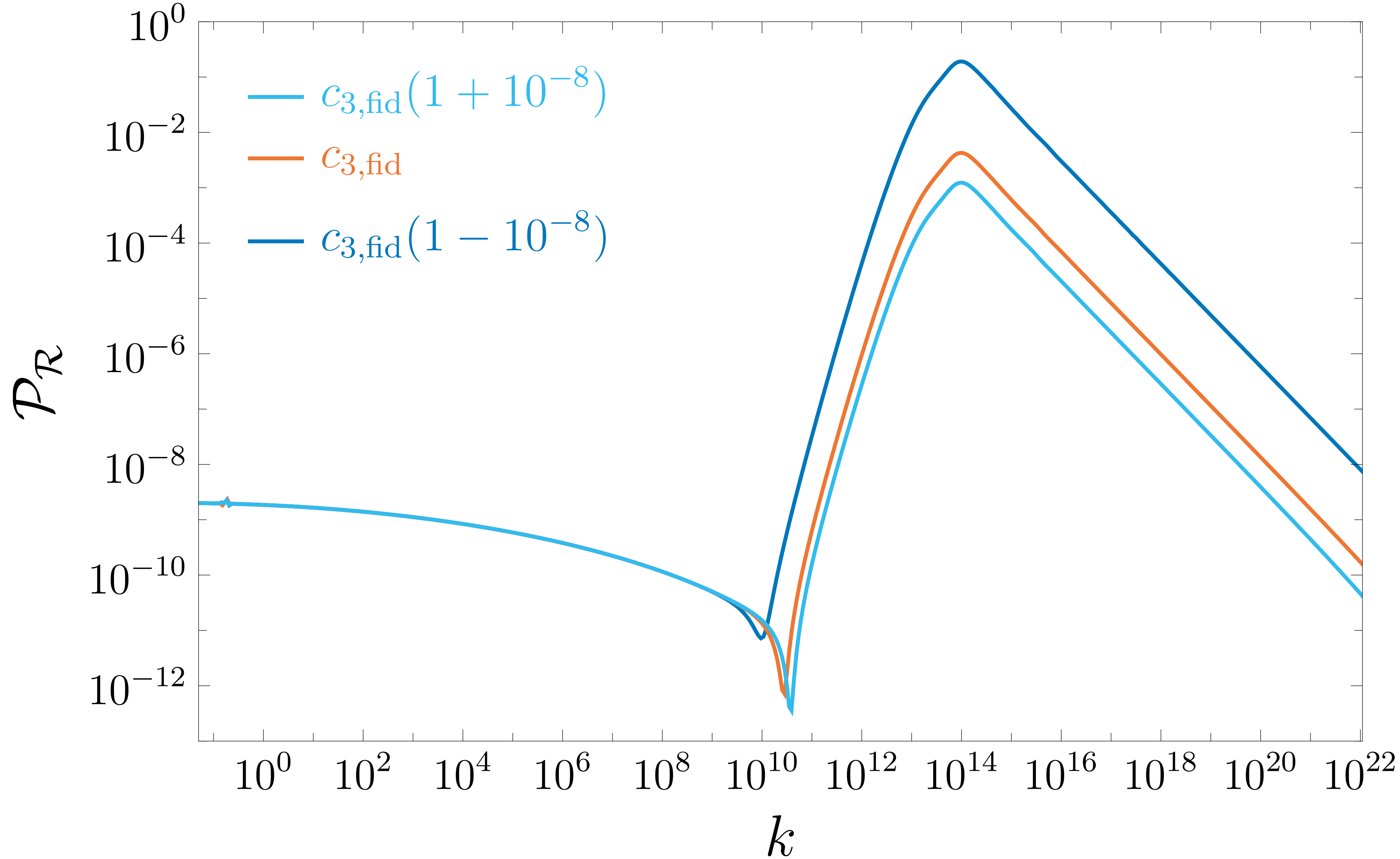
	Fiducial
ϕ_0	$-\lvert\Lambda^3/c_3\rvert$
ϕ_{CMB}	5.51674×10^{-4}
Λ	0.3
c_0	1
c_1	$(2\pi^2\Lambda^4)/(4225c_3)$
c_2	0
c_3	-0.52
c_4	1
c_5	-0.640725043

Clearly c_5 has been tuned to get the desired PBH abundance

Is this the only tuning required?

Hertzberg & Yamada 2017
but we use slightly different parameter values

A polynomial potential



Hertzberg & Yamada 2017

Polynomial problems

- A cubic potential is enough for an inflection point, why go further?

- One needs an inflection point for ultra-slow-roll inflation.

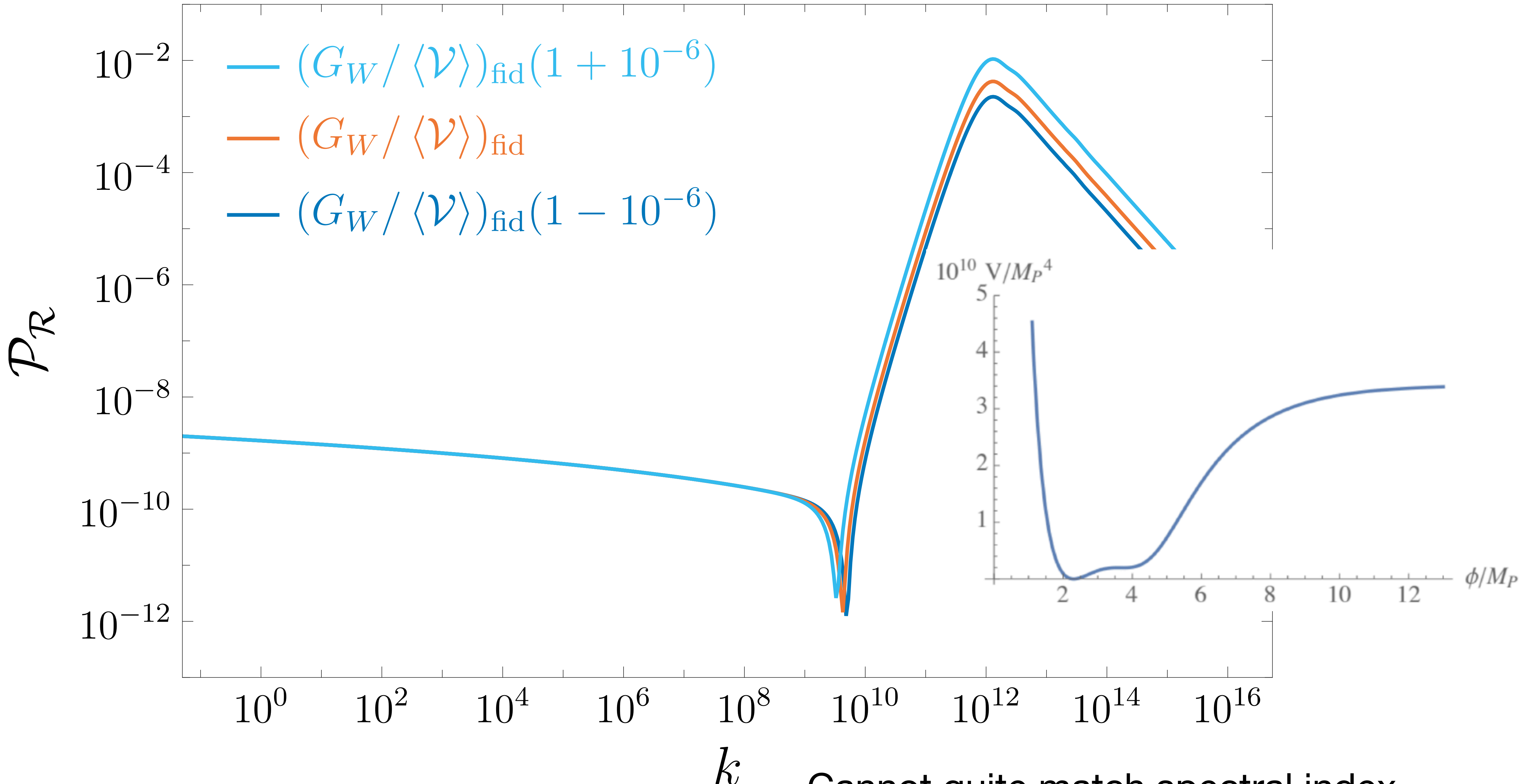
- Need a second “nearly” inflection point for the CMB perturbations, to reduce tensor perturbations

- Also need to fine-tune initial velocity to stop overshooting the CMB flat part

- I had no idea it would be this hard

Hertzberg & Yamada 2017

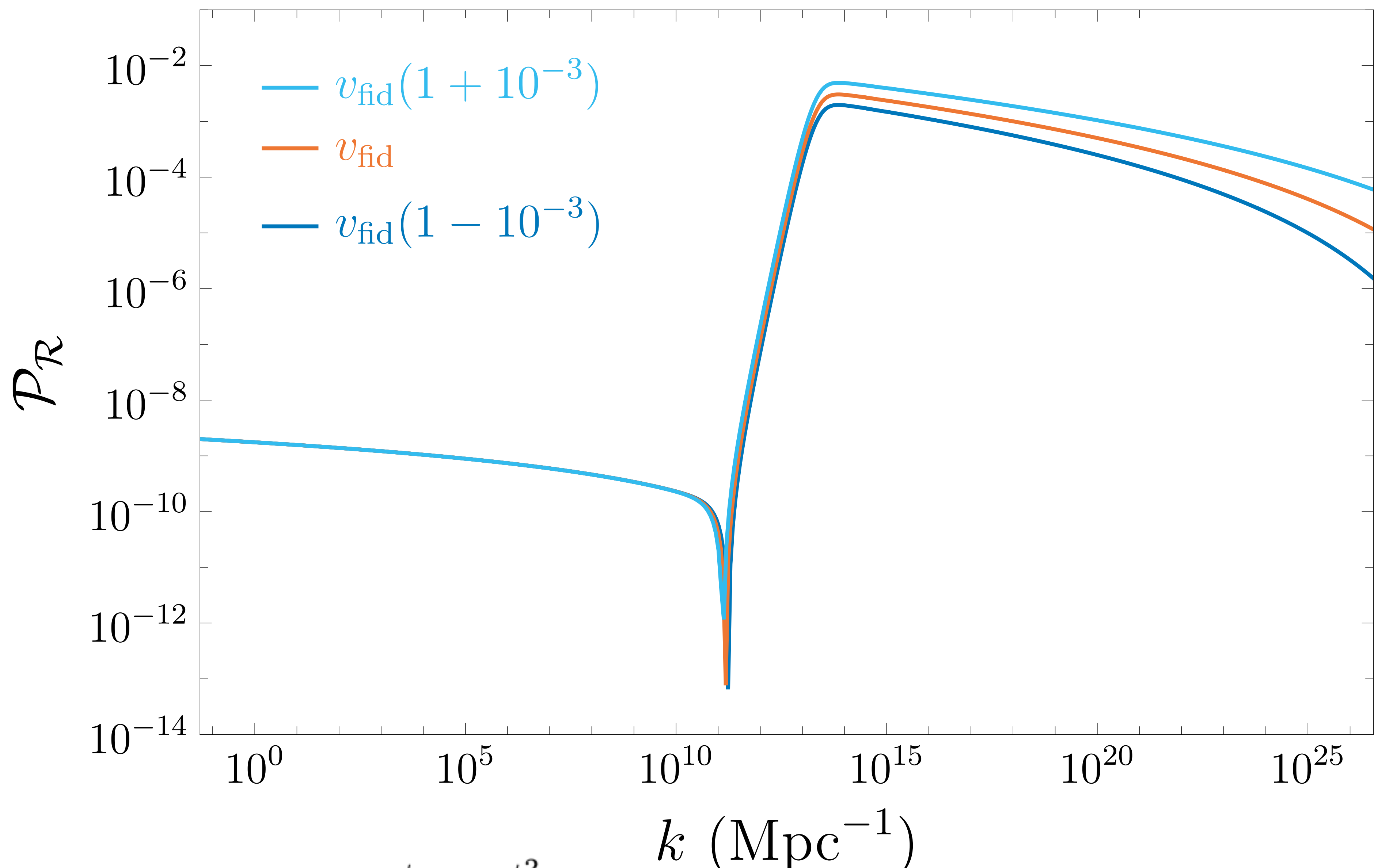
String inflation model



Cannot quite match spectral index

Cicoli, Diaz & Pedro 2018

Less tuned model

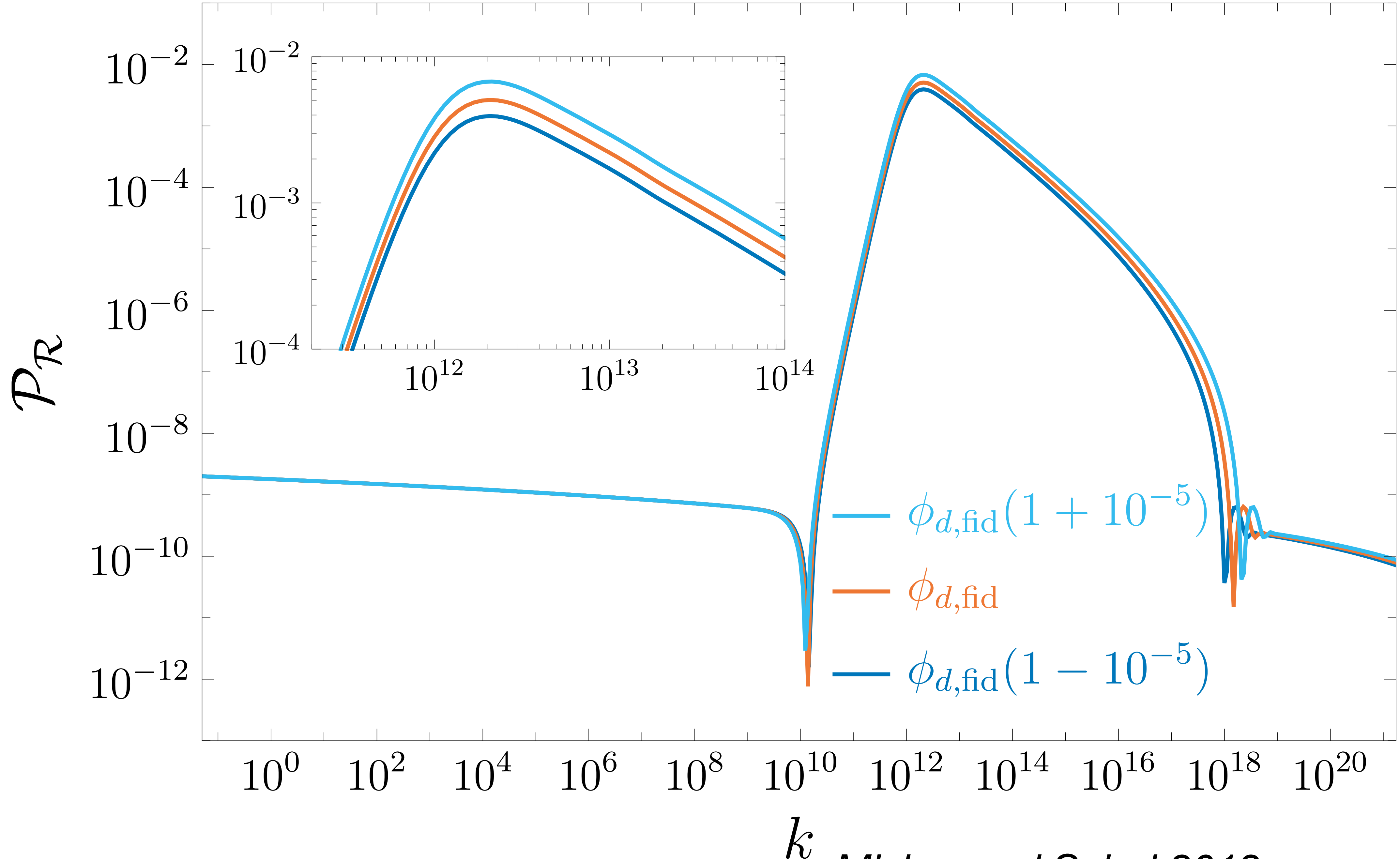


$$V(\phi) = \frac{\lambda}{12}\phi^2 v^2 \frac{6 - 4a\frac{\phi}{v} + 3\frac{\phi^2}{v^2}}{\left(1 + b\frac{\phi^2}{v^2}\right)^2}$$

Cannot quite match spectral index
Germani & Prokopec 2017

Designer potential

V=V*(1+bump)



Mishra and Sahni 2019

General thoughts

- Forming light PBHs is “easier” since the PBH and CMB scales are more widely separated
- A discontinuous potential separates scales but is hard to motivate
- Matching CMB observations to an inflection point smoothly is hard
- You can say nature just picks parameter values. But then how can you interpret model comparisons or penalise contrived/overfitted models?

Quantifying fine-tuning

- $\epsilon = \dfrac{\partial \ln(\text{observable})}{\partial \ln(\text{parameter})}$

- How sensitive are observables to model parameters - PBH context *Azhar & Loeb 2018; Nakama & Wang 2019 (Franciolini et al ‘18* non-G context*)*

- A value ~1 against all parameters implies the observable is robust

- A value of 100 means that the observable changes 100 times more quickly than the model parameter value

- Let’s start with the power spectrum amplitude

Relating PS amplitude to f_{PBH}

$$\beta \sim e^{-\frac{\delta_c^2}{2A(p)}}$$

$$\epsilon_{AL} = \frac{d\ln(\beta)}{d\ln(p)} = \frac{2\delta_c^2}{A(p)} \frac{d\ln(A(p))}{d\ln(p)} . \qquad \frac{2\delta_c^2}{A(p)} \simeq 10 - 100$$

- f_{PBH} values are fine-tuned by 1-2 orders of magnitude more than the power spectrum amplitude - only
- We have confirmed this numerically
- SGWB and PBH production both require tuning

Comparing the two tunings

- Generating the large amplitude peak is "typically" a much bigger tuning than going from peak to PBH production

- SGWB and PBH production both require tuning

- Makes a Bayesian model comparison disfavour either

- For the least tuned model we studied (rho is the ratio of the tunings)

	Fiducial	$\epsilon_{\mathcal{P}_{\rm peak}}$	$\epsilon_{f_{\rm PBH}}$	$\boldsymbol{\rho}$
$\boldsymbol{a}$	$1/\sqrt{2}$	-6.0×10^2	-2.2×10^4	37
$\boldsymbol{v}$	0.19669	4.4×10^2	1.6×10^4	37

Alternatives within the inflationary scenario

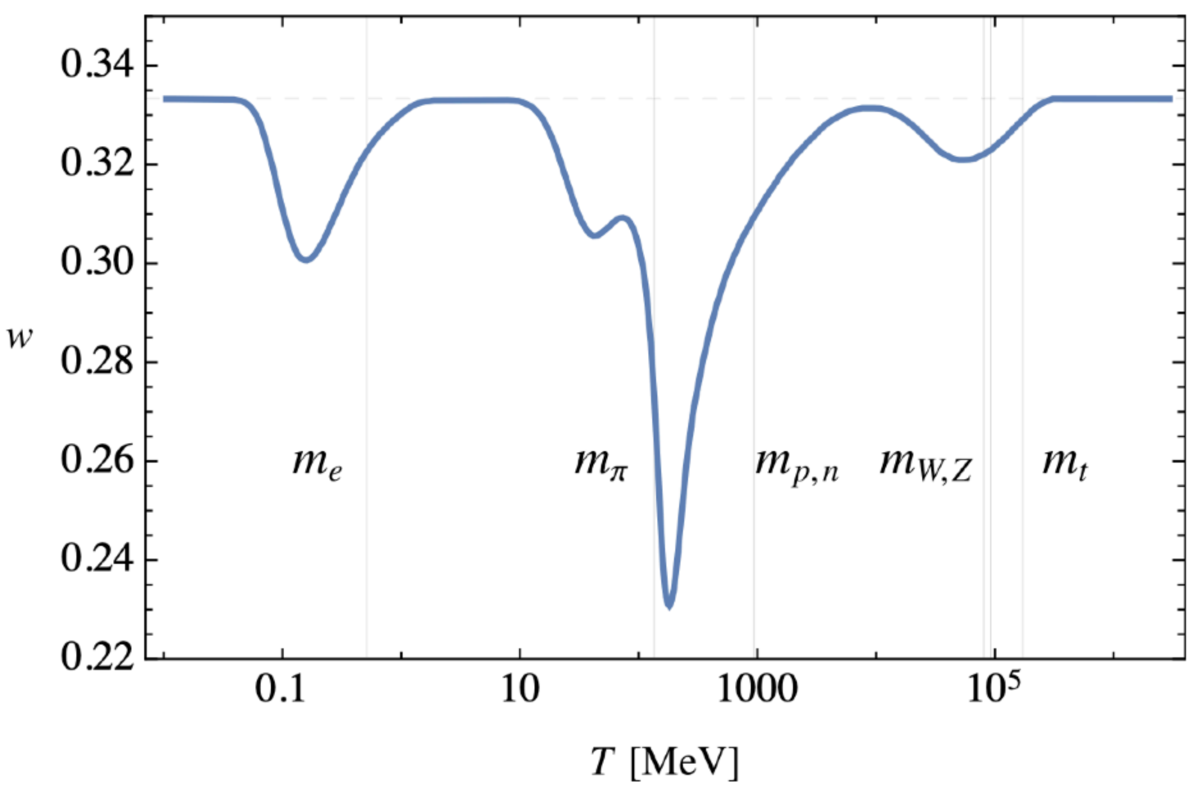
- We have empirically found the fine-tuning scaling

$$\epsilon \propto A_{\rm peak}^{1/2}$$

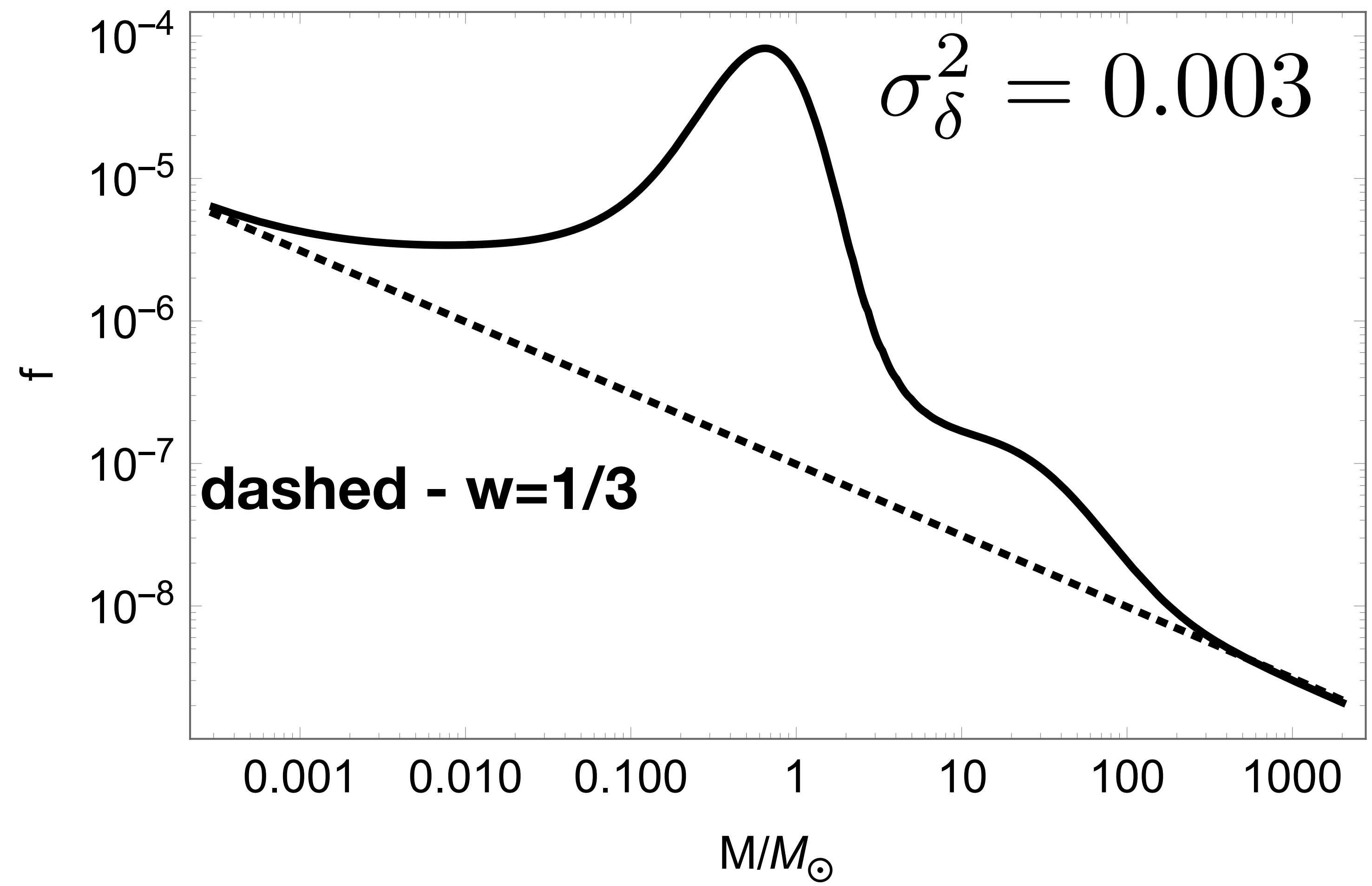
Phase transitions

$$f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma_\delta^2}}$$

QCD bump



Carr et al 2019



CB, Hindmarsh, Young & Hawkins 2018

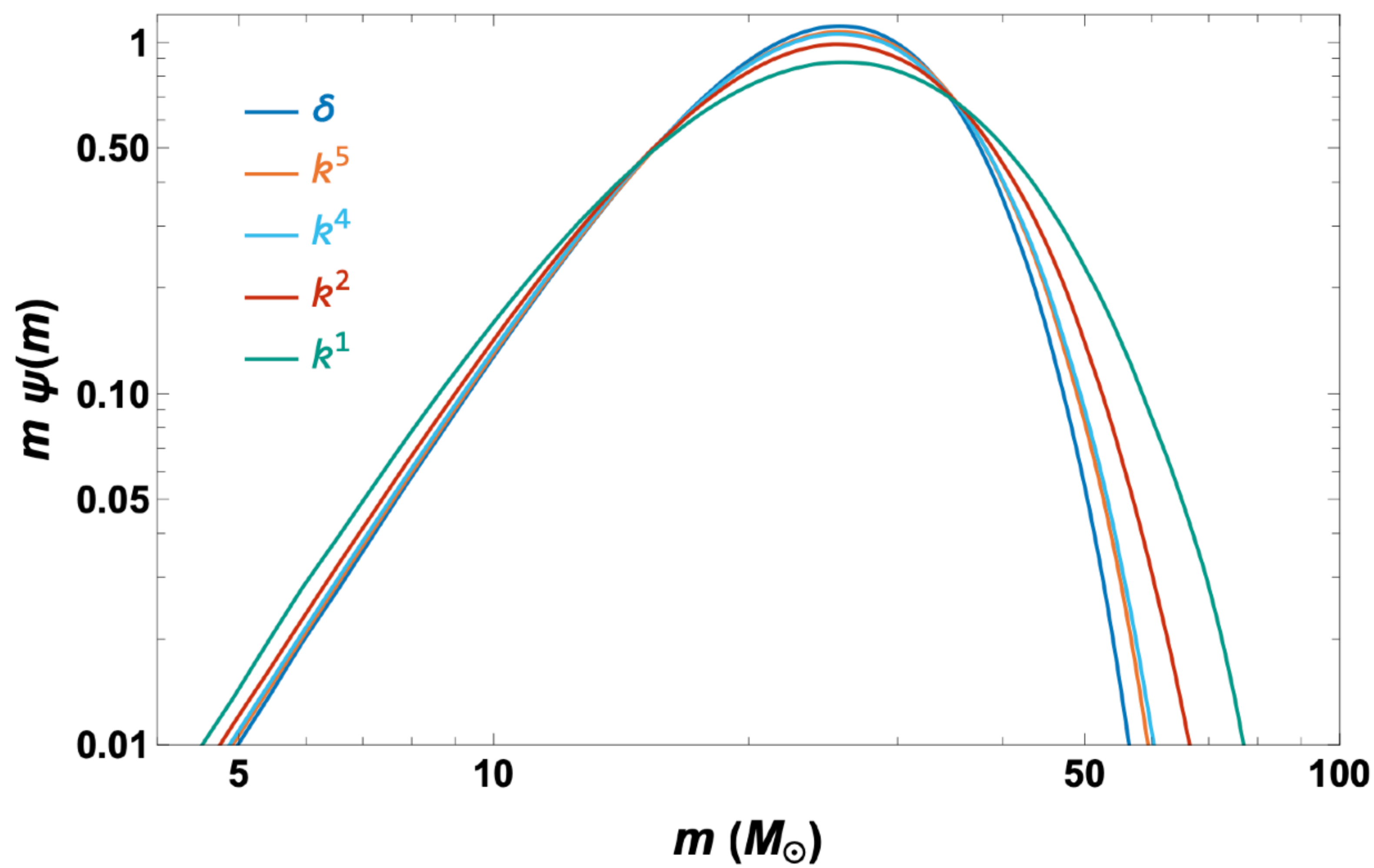
A phase transition motivates particular mass scales, especially the solar mass via QCD. However, it only reduces required peak amplitude by factor 2
QCD details: *Franciolini et al; Escriva et al 22; Musco et al 23*

Multifield inflation

- Numerous studies suggesting less tuning - e.g. *Papanikolaou et al '22; Braglia et al '22; Qin et al '22; Stamou & Clesse '23* +++
- Degeneracy directions do not imply less overall tuning
- One should check the tuning against all possible parameters (including initial conditions)
- Tuning against "background" parameters may be comparable to "feature" parameters
- Ideally quantify the overall volume of parameter space with a given peak amplitude - numerically expensive

Single vs multifield

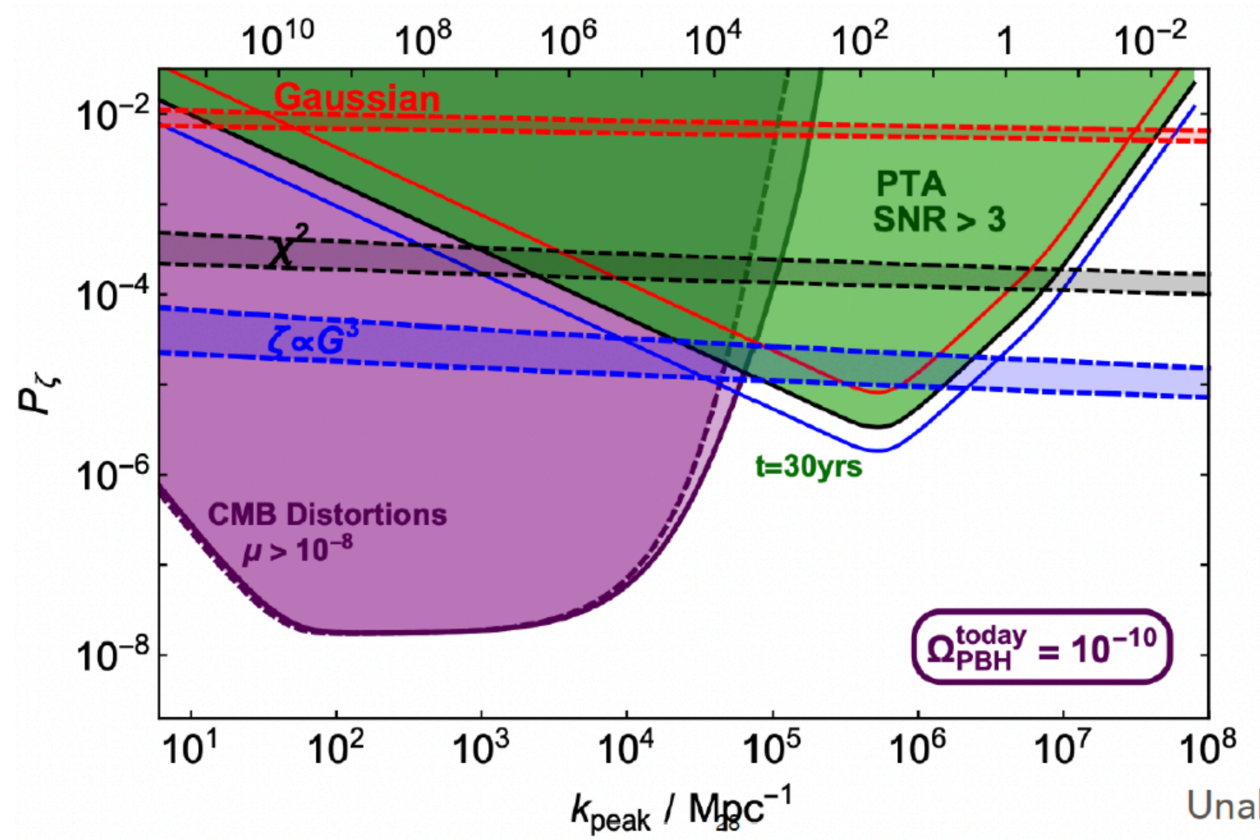
- A characteristic single-field signature is the steepest k^4 growth
- Negligible impact on PBH mass function, but SGWB could test this
- Diffusion/non-Gaussian signature should help discriminate but still uncertain - e.g. *Tomberg '23, Firouzjahi & Riotto '23*
- I leave the loops to other talks…



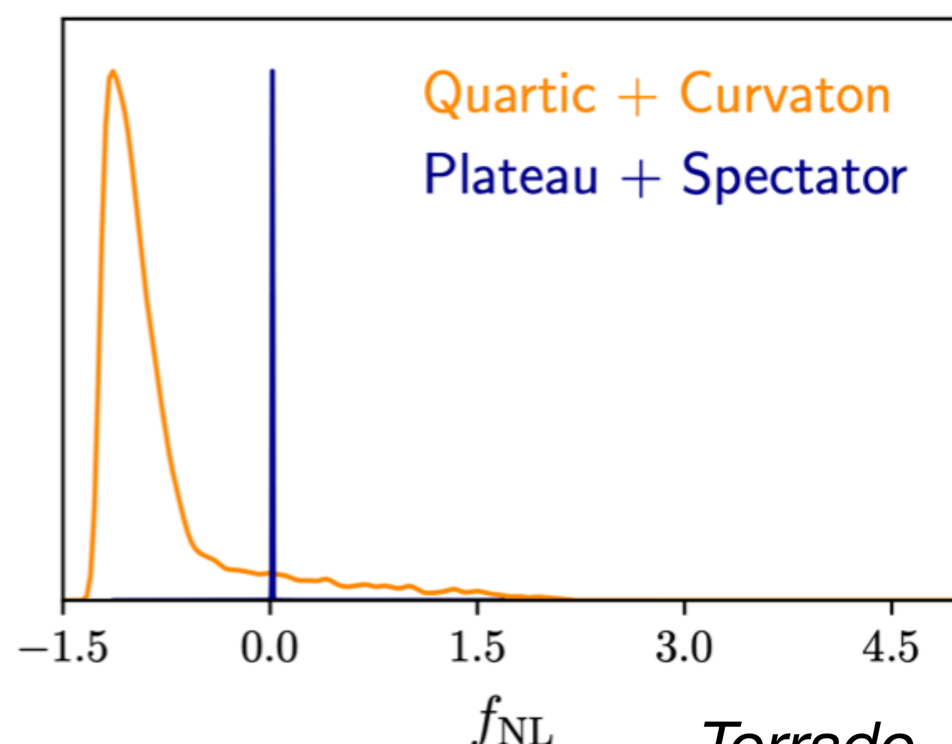
Cole, Gow, CB, Patil 2022

Non-Gaussianity

- Positive skewness does reduce required peak amplitude
- But need to non-perturbative non-G
- This itself is perhaps tuned
- Local non-G can couple to CMB scales and generate an unacceptable PBH CDM isocurvature perturbation - *van Laak & Young 2023*



Unal et al. 2021



Torrado, CB, et al '17

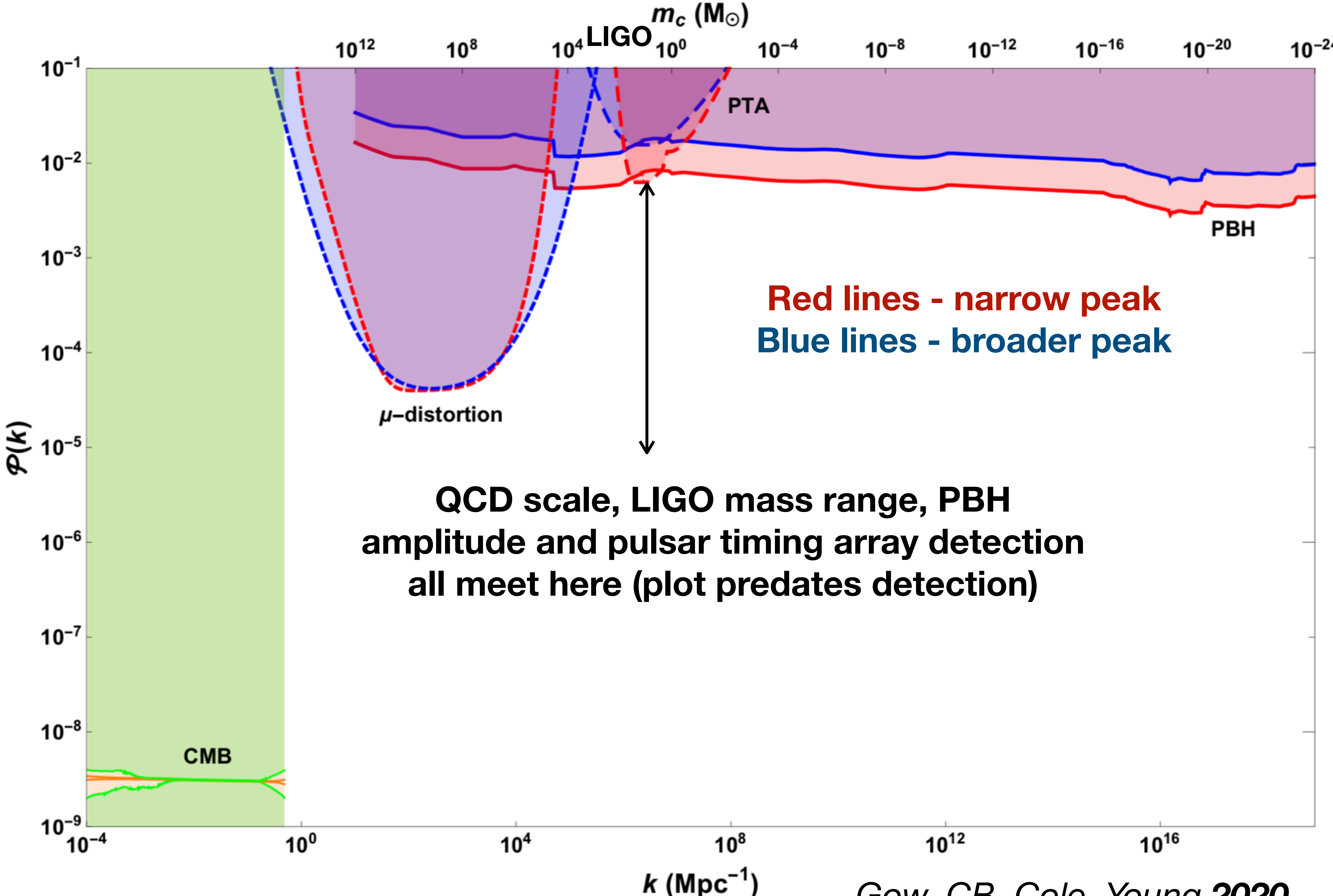
More alternatives

- An **early matter dominated epoch** helps, if it lasts long enough
- PBH formation is not spherical

$$\beta \propto \mathcal{P}_{\rm peak}^{5/4} \qquad\qquad \frac{\mathrm{d}\ln\beta}{\mathrm{d}\ln\mathcal{P}_{\rm peak}} = \frac{5}{4}$$

- Preheating generation on subhorizon scales - *Martin et al 2019*
- Other PBH formation mechanisms…

Exciting times

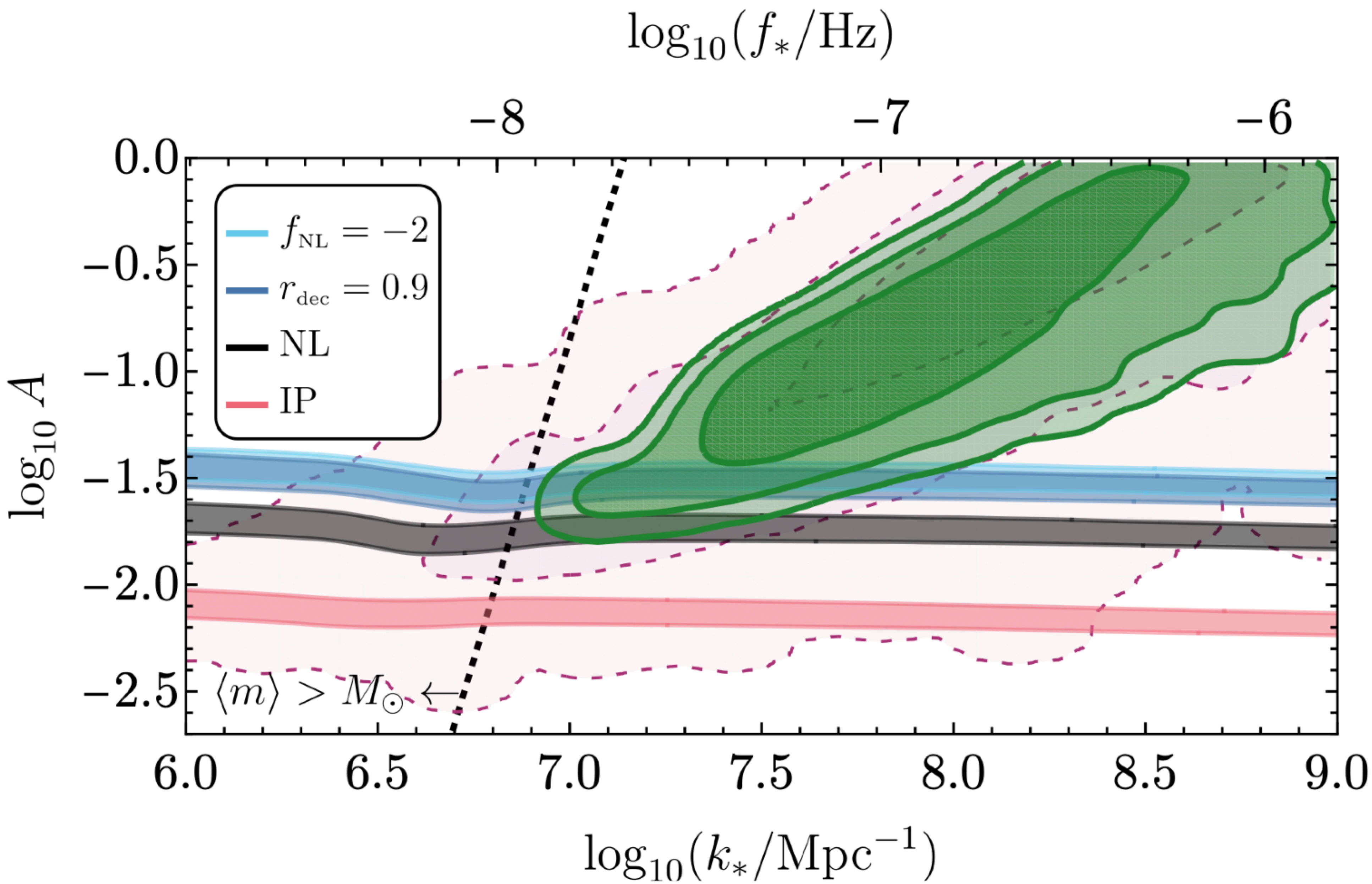


Gow, CB, Cole, Young ***2020***

Explaining PTA + PBHs via SIGW

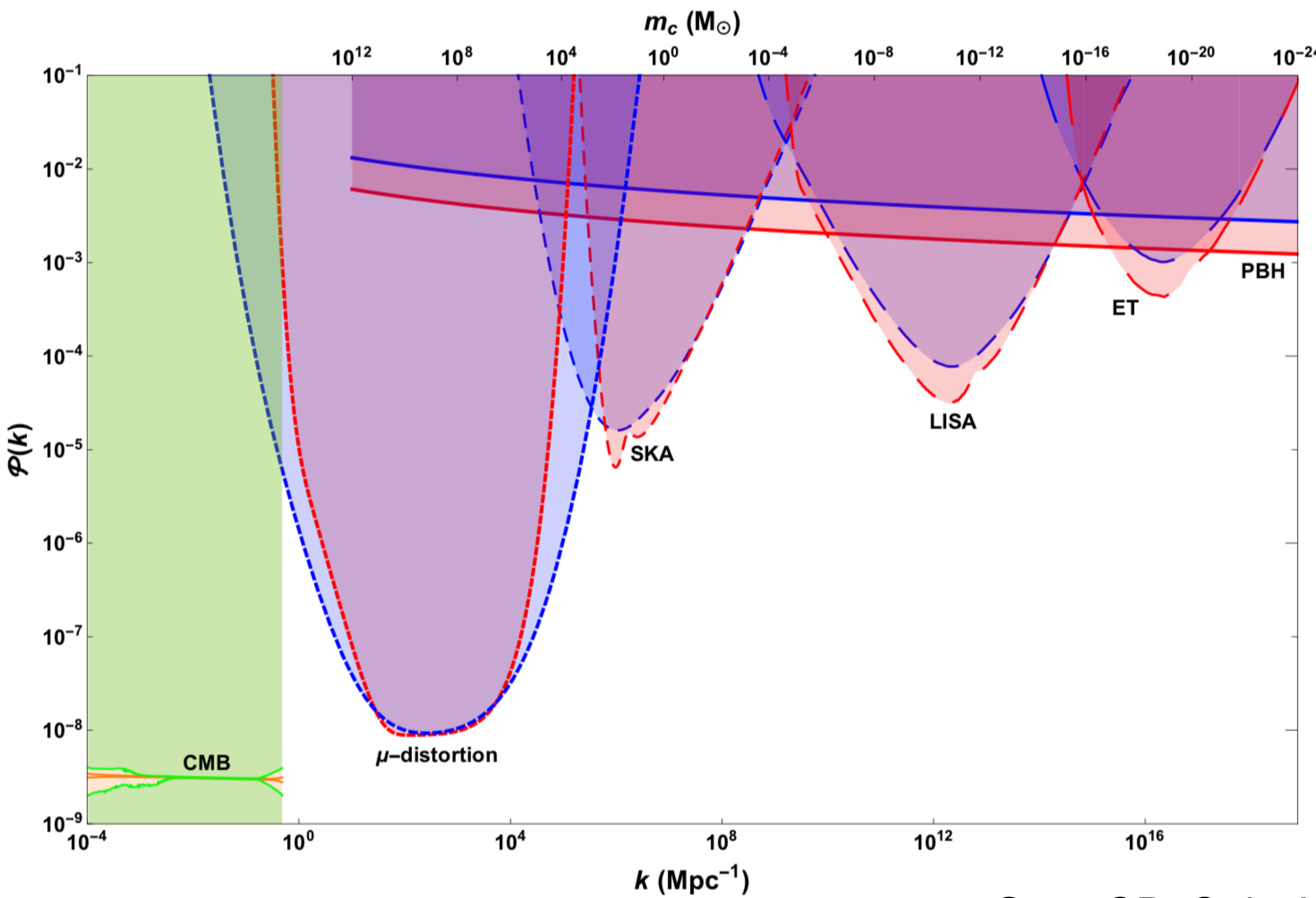
NANOGrav amplitude arguably too high for Gaussian PBH limit - invoke negative skewness

Premature to reach strong conclusions, amplitudes amazingly close and calculations not 100% watertight - *de Luca et al 2023*



Franciolini, Iovini, Vaskonen & Veermäe '23

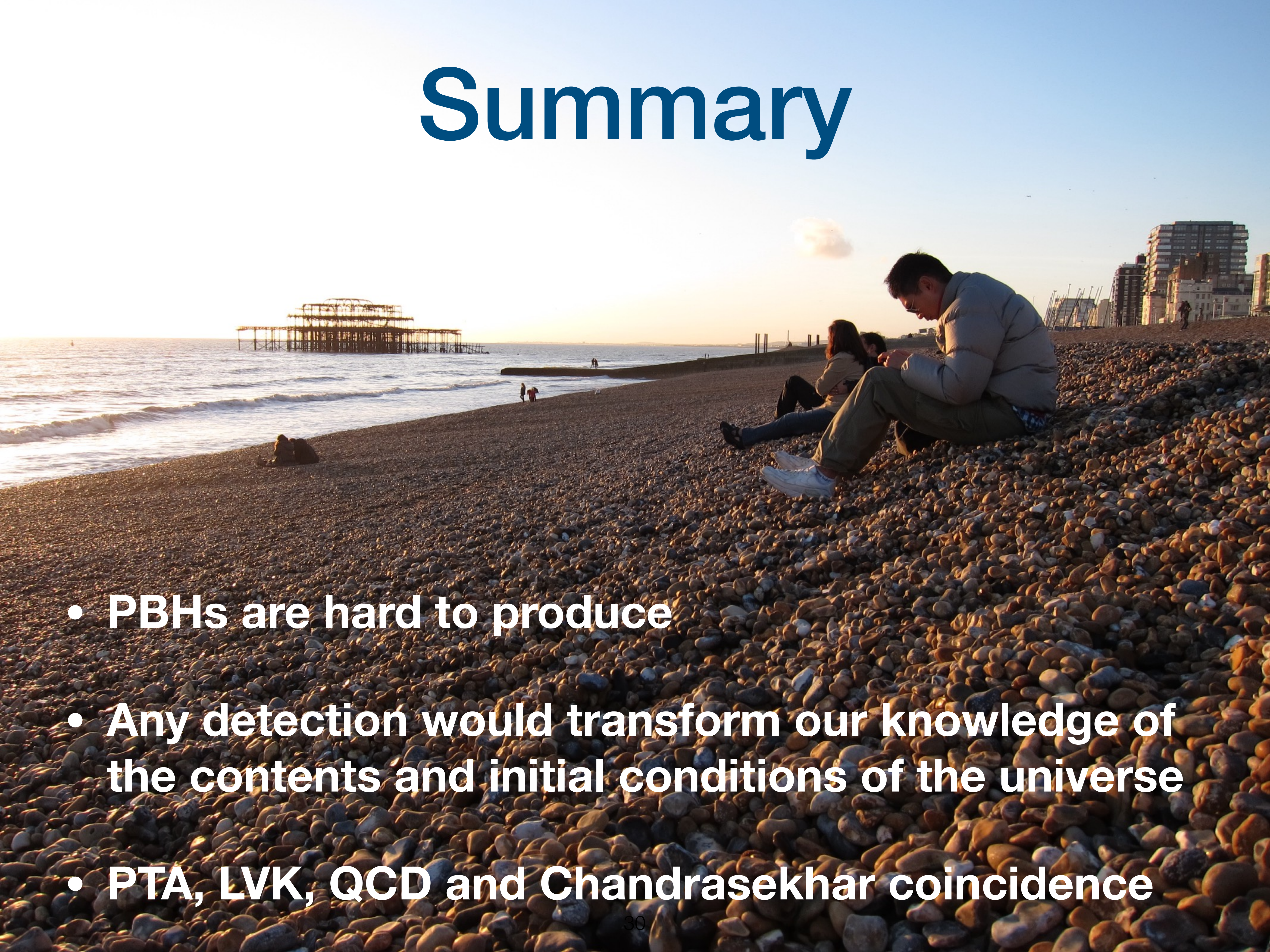
A bright future (forecast)



The PBH lines correspond to zero PBHs
Cole & CB ‘17

Gow, CB, Cole, Young 2020

Summary



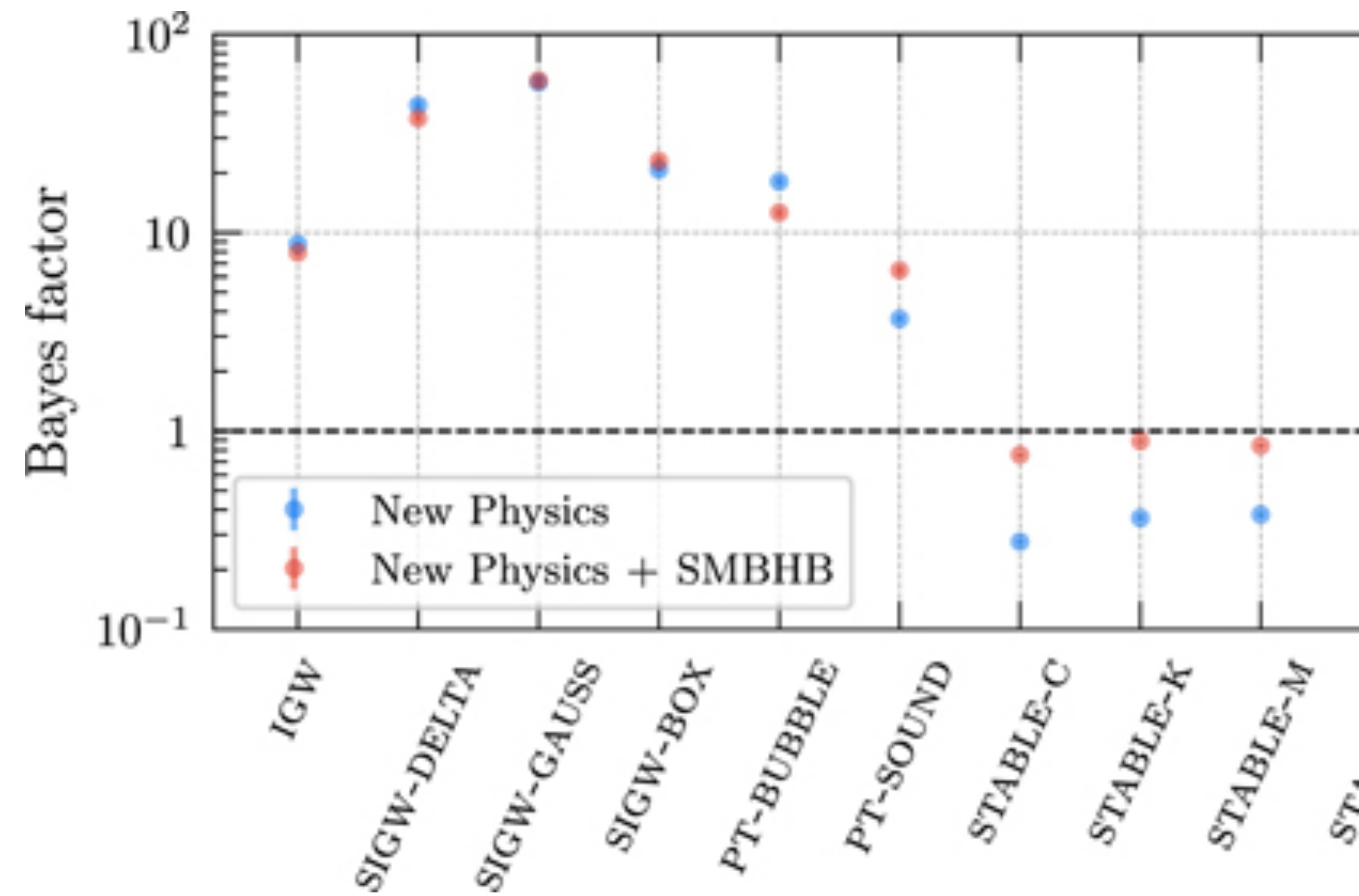
- PBHs are hard to produce
- Any detection would transform our knowledge of the contents and initial conditions of the universe
- PTA, LVK, QCD and Chandrasekhar coincidence

Pulsar timing arrays

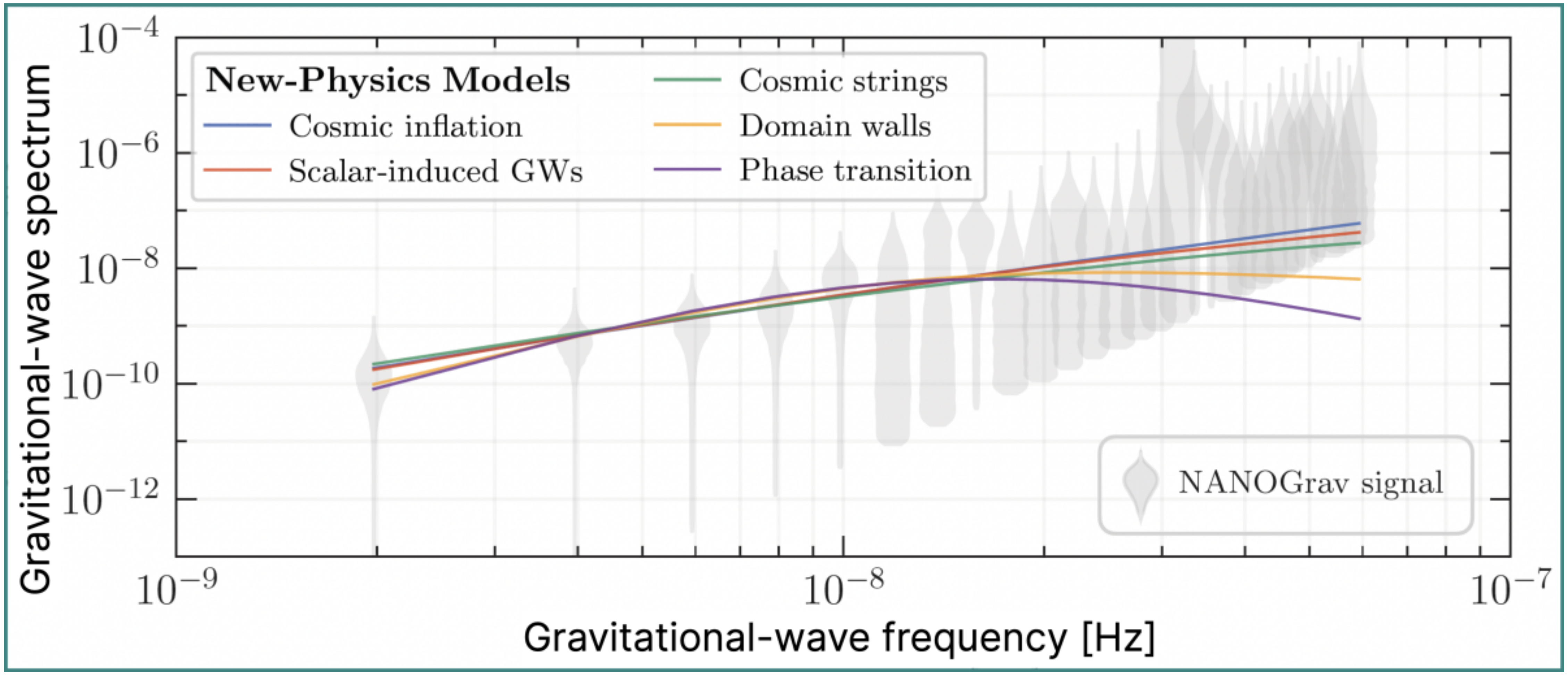
- First detection of a stochastic GW background - 28 June 2023
- Expected supermassive BH inspiral is not a perfect fit
- Huge flurry of papers about alternative GW sources
- At second order, scalar and tensor perturbations couple - *Review Domenech 2022*

$$\Omega_{\rm GW}^{\rm induced} \sim \frac{1}{12}\Omega_{r,0}\mathscr{P}_{\mathscr{R}}^2 \sim 10^{-6}\mathscr{P}_{\mathscr{R}}^2(k \gg k_{\rm CMB})$$

NANOGrav



SIGW=stochastic induced GW



- Note the rising data points means the peak must be to the right of the data => larger k and smaller PBH masses

- Peak amplitude also large. Does PBH overproduction rule out the SIGW scenario?

https://nanograv.org/15yr/Summary/NewPhysics

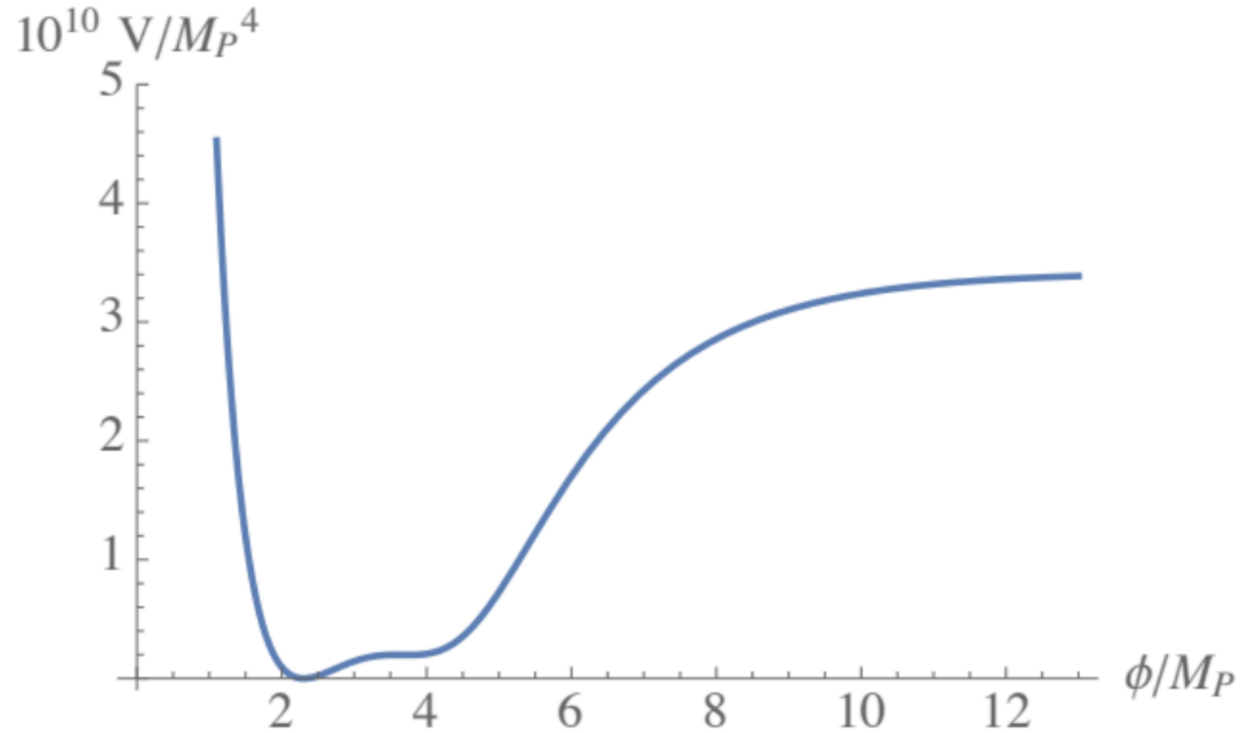
String inflation model

$$V(\phi) = V_0 \left[C_1 - e^{-\frac{1}{\sqrt{3}}\hat{\phi}} \left(1 - \frac{C_6}{1 - C_7 e^{-\frac{1}{\sqrt{3}}\hat{\phi}}} \right) + C_8 e^{\frac{2}{\sqrt{3}}\hat{\phi}} \left(1 - \frac{C_9}{1 + C_{10} e^{\sqrt{3}\hat{\phi}}} \right) \right]$$

$$C_6 = \frac{A_W}{C_W} \qquad C_7 = \frac{B_W}{\langle \tau_{K_3} \rangle^1 / 2} \qquad C_8 = 0$$

$$C_8 C_9 = \frac{G_W}{\langle \mathcal{V} \rangle} \frac{\langle \tau_{K_3} \rangle^3 / 2}{C_W} \qquad C_{10} = \frac{R_W}{\langle \mathcal{V} \rangle} \langle \tau_{K_3} \rangle^3 / 2$$

	Fiducial
ϕ_0	12
ϕ_{CMB}	9.33731
A_W	2/100
B_W	1
C_W	4/100
$\langle \tau_{K_3} \rangle$	14.30
$G_W / \langle \mathcal{V} \rangle$	3.08054×10^{-5}
$R_W / \langle \mathcal{V} \rangle$	7.071067×10^{-4}



Plot with their parameters
Cicoli, Diaz & Pedro 2018

Designer potential

$$V(\phi) = V_0 \frac{\phi^n}{\phi^n + M^n} \left(1 + A \exp\left[-\frac{(\phi - \phi_d)^2}{2\sigma^2}\right]\right)$$

	Fiducial
ϕ_0	4
$\phi_{\rm CMB}$	3
n	2
M	$M_{\rm Pl}/2$
A	1.17×10^{-3}
σ	1.59×10^{-2}
ϕ_d	2.18812

phi$_d$, sigma and *A* set the details of the bump which are added by hand

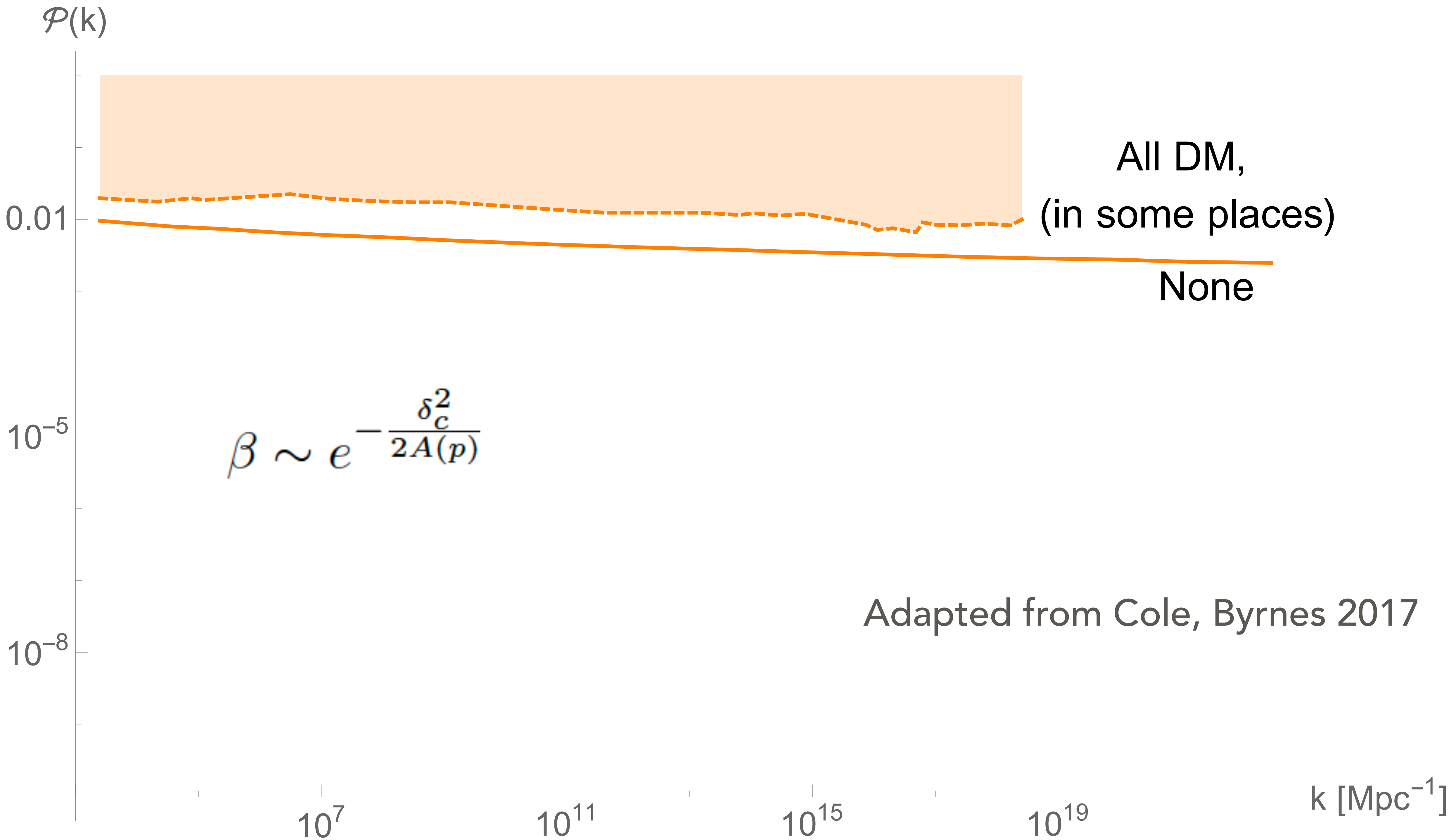
This allows the bump to be added to any model which otherwise matches observations

Does the fine tuning only depend on the amplitude of the bump?

Mishra and Sahni 2019

Required peak amplitude?

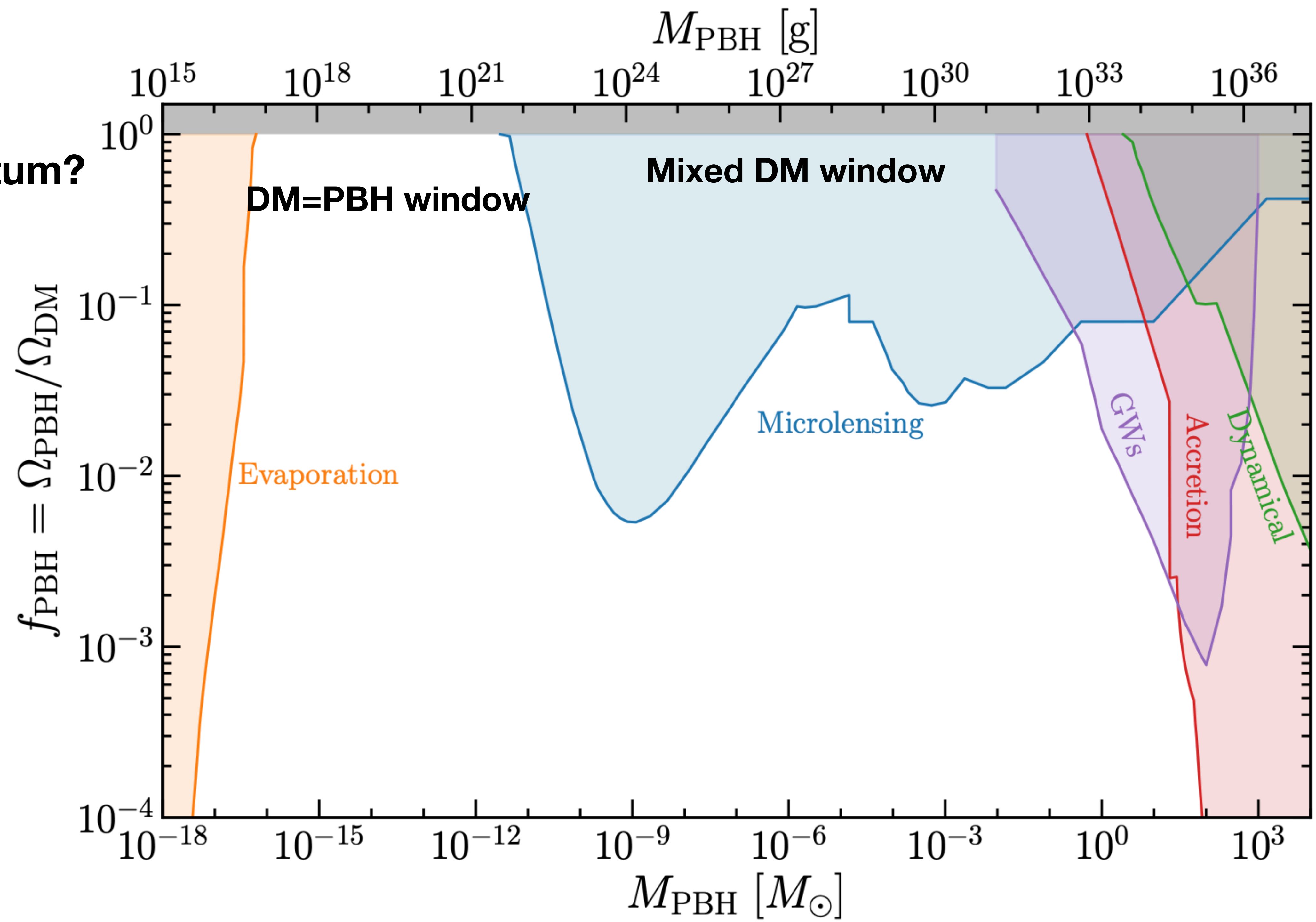
Hardly varies for any mass/f_{PBH}



Is DM a new particle?

quantum?

PBHs=All DM



PBHs=1% DM

Caveats apply: *Green and Kavanagh review* *https://arxiv.org/pdf/2007.10722.pdf*

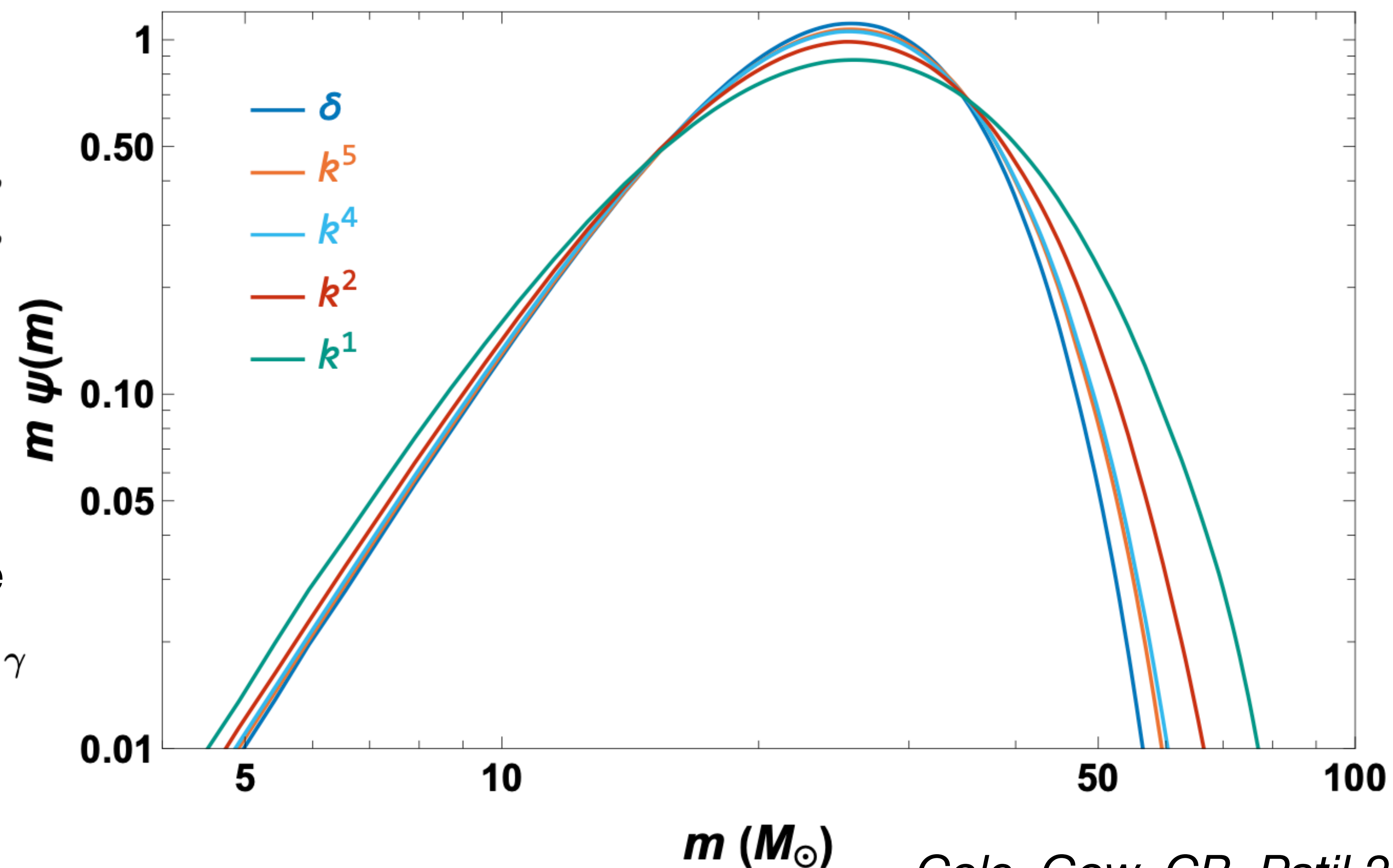
Monochromatic mass function

- This is not realistic
- Not even when the power spectrum is monochromatic, and that itself is not realistic

$$\mathcal{P}_{\mathcal{R}}(k) = A \begin{cases} \left(\frac{k}{k_p}\right)^n & k \leqslant k_p \\ \left(\frac{k}{k_p}\right)^{-n} & k > k_p \end{cases}$$

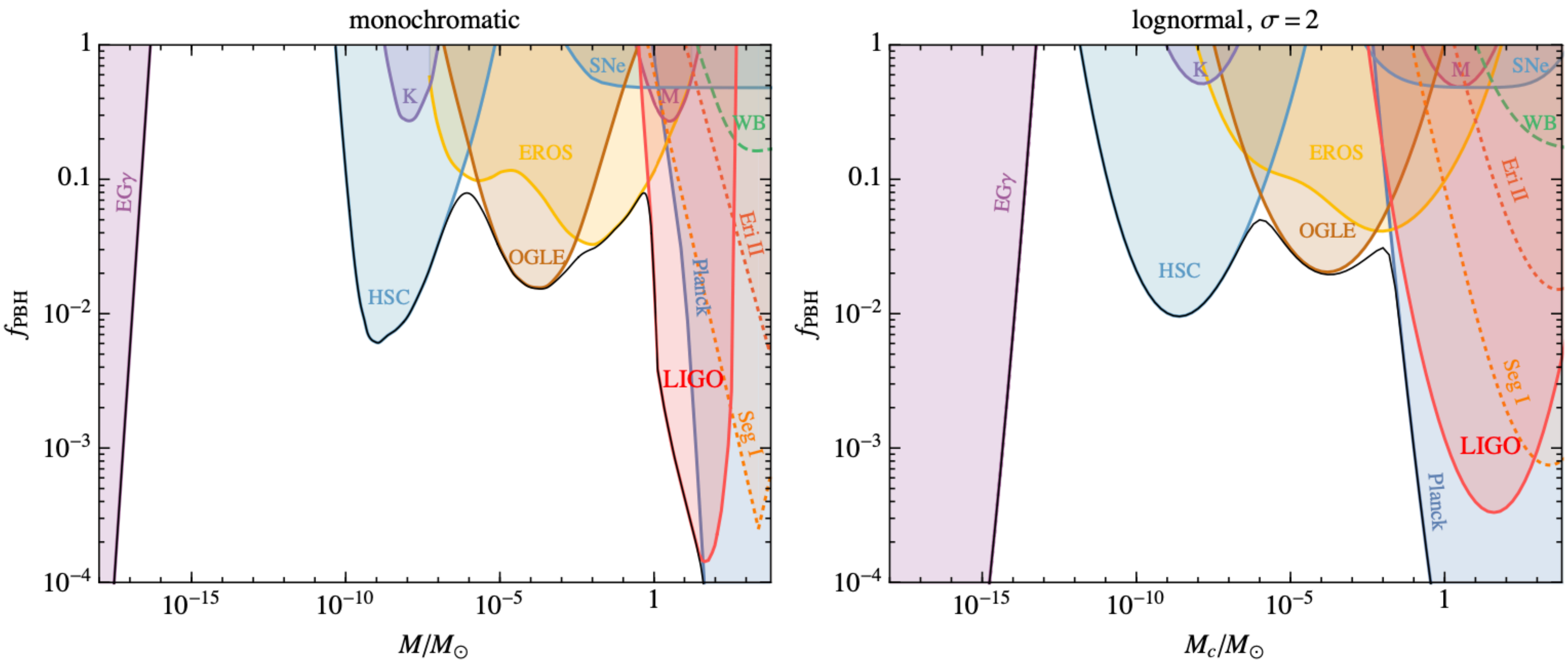
Critical collapse

$$M = k M_{\rm H}(\delta - \delta_c)^\gamma$$

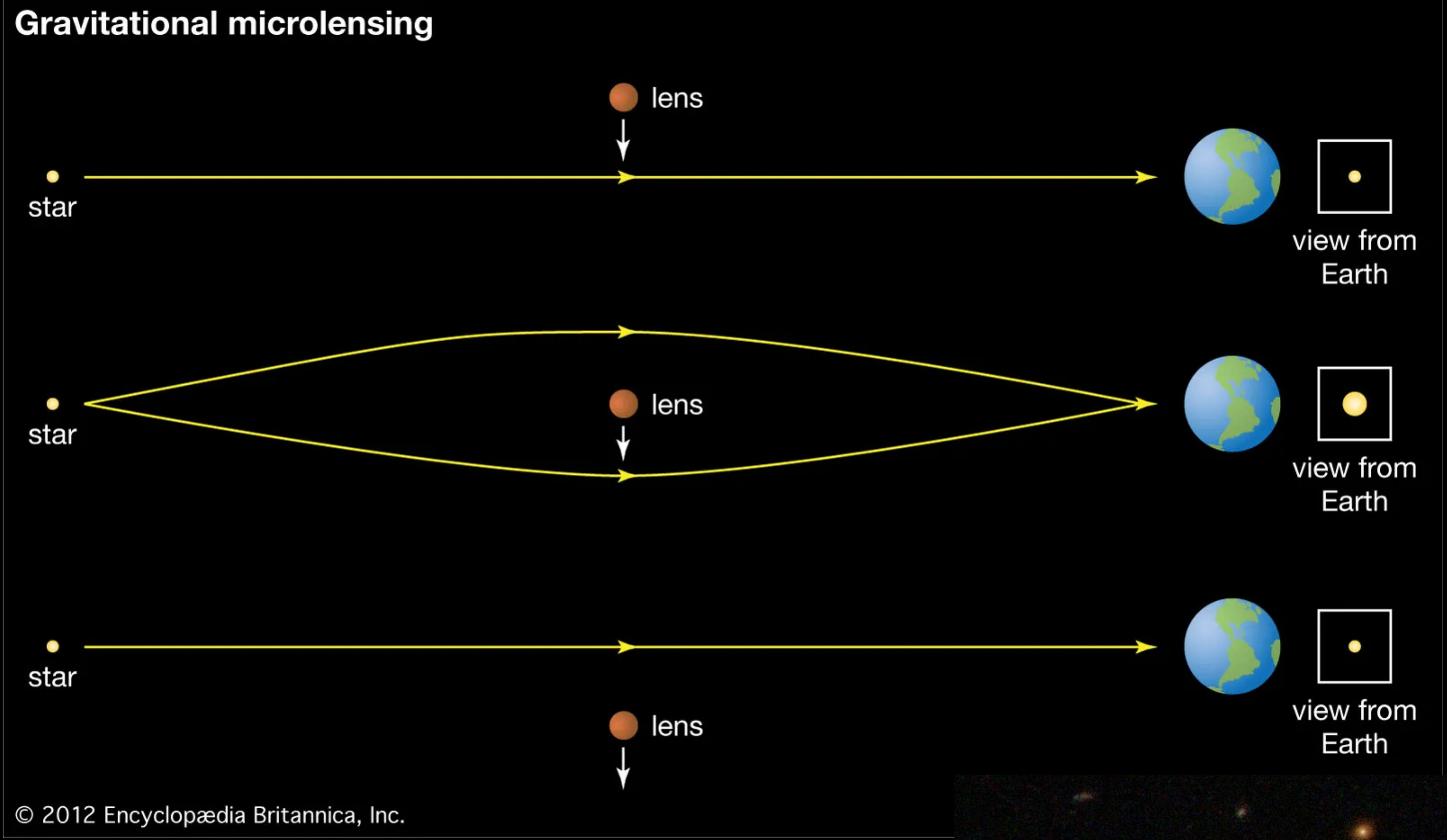


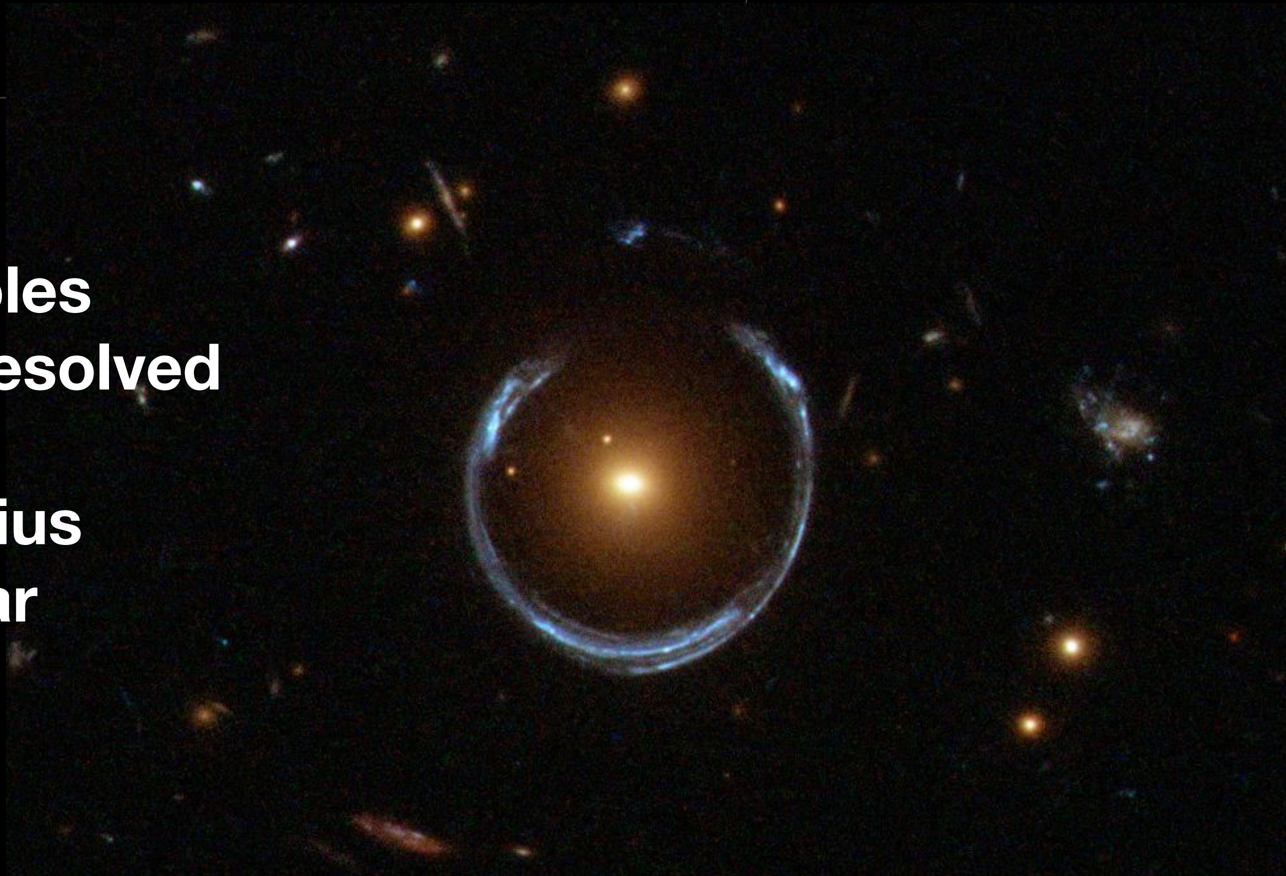
Cole, Gow, CB, Patil 2022

The PBH DM window shrinks



Carr, Kohri, et al 2021 based on Carr, Raidal et al 2017

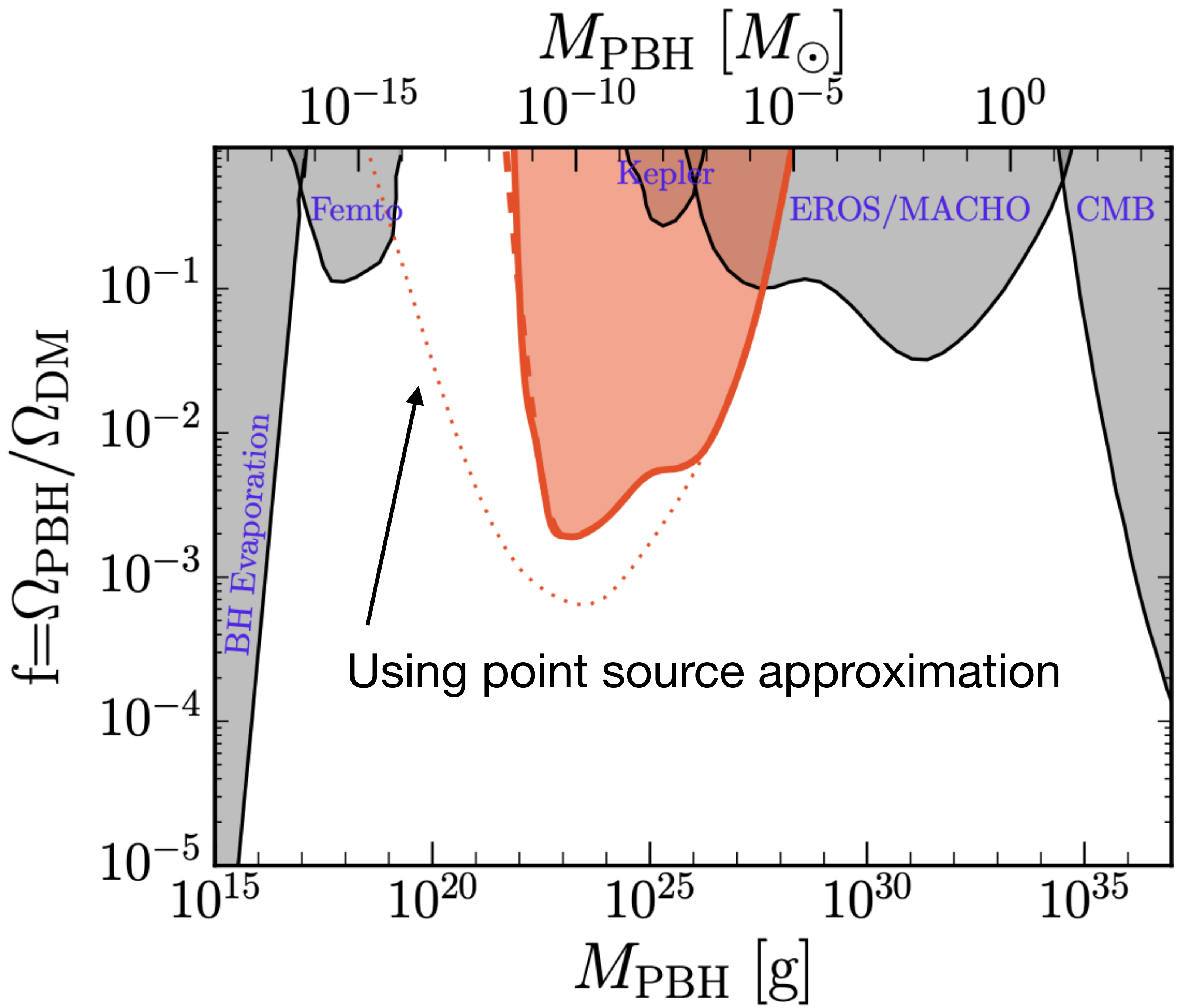




Mass gap BH has atomic sized black holes
Tiny masses mean Einstein ring cannot be resolved

Little brightening when the Einstein radius becomes smaller then background star

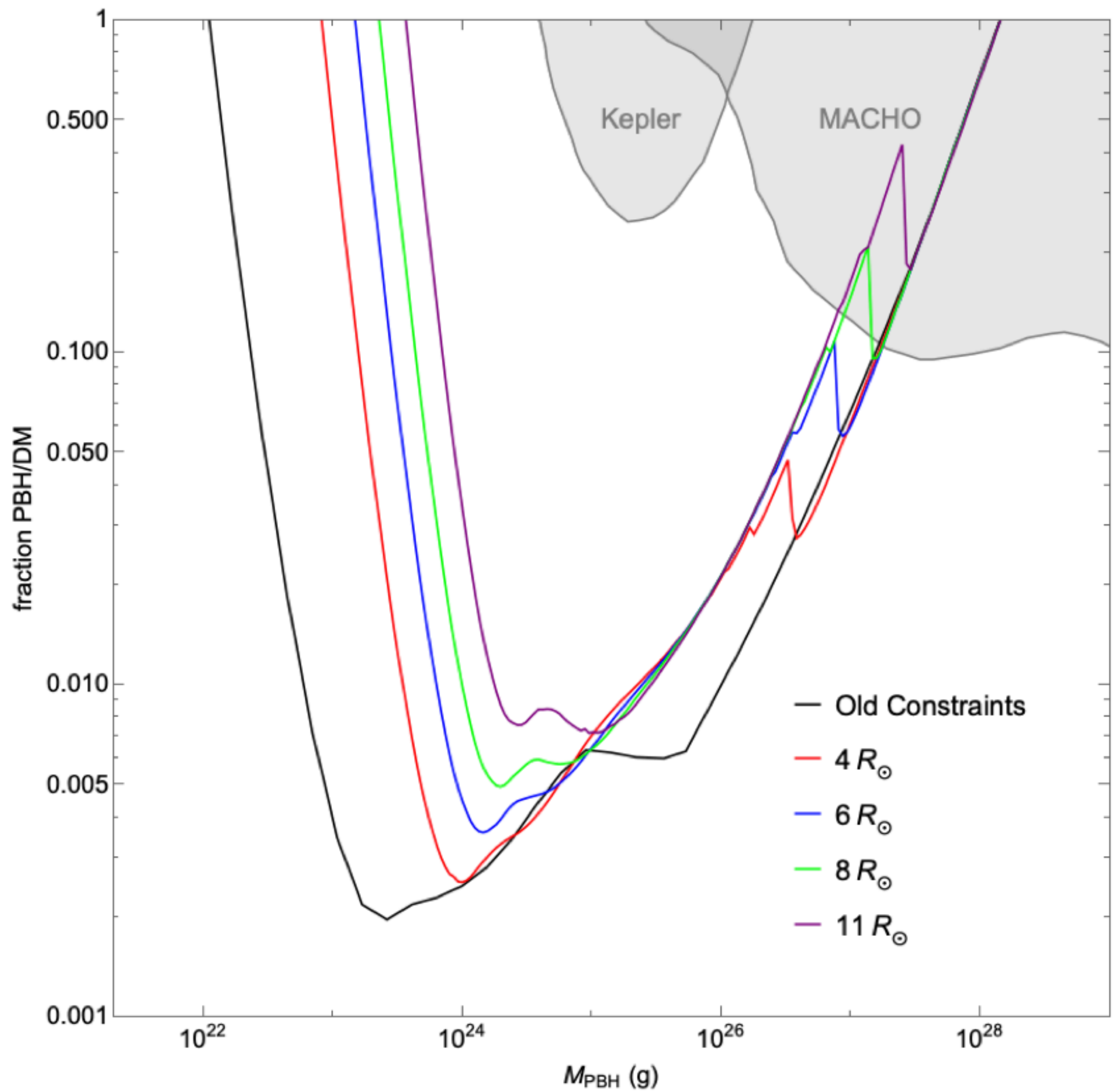
The window had closed



Niikura et al, Subaru/HSC observations 2017

The PBH DM window ~~shrinks~~

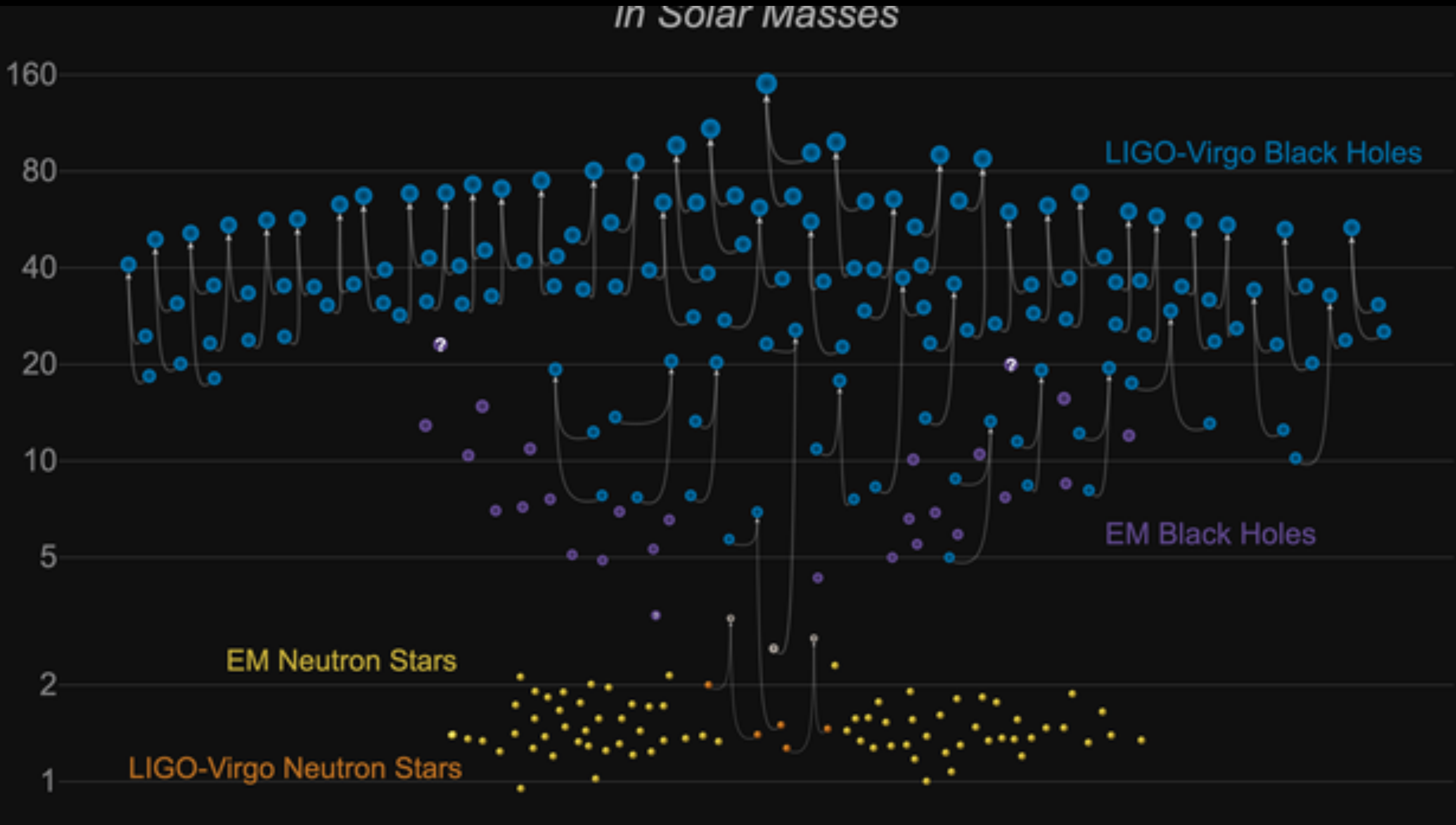
Previous paper assumed the average Andromeda star was solar sized, but HSC mainly resolves bigger ones



Smyth et al 2019

The LIGO-Virgo-Kagra events

- Unlikely that more than 1% of the dark matter can be made out of LIGO mass PBHs
- But could the LIGO BHs be primordial?
- Black holes have no hair, so how can we know?
- Most detected events have low (effective) spin



LIGO & Virgo collaboration

Fine opportunities

- More LIGO-Virgo-KAGRA data coming
- Current detections must be mainly or purely astrophysical compact objects - *Hall, Gow, CB, '20; Hutsi et al 2021; de Luca et al 2022; Franciolini 2022*
- Opportunity to observe sub-Chandrasekhar mass object
- QCD and PTA connections to follow