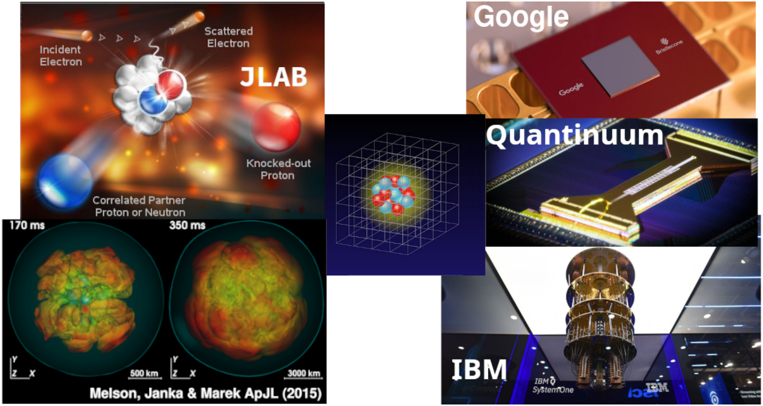
Quantum Simulation of Nuclear Many Body Systems

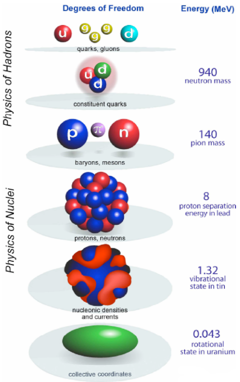
Alessandro Roggero



MONSTRE meeting - Milano

12 May, 2023

The nuclear many-body problem



Bertsch, Dean, Nazarewicz (2007)

$$\mathcal{L}_{QCD} = \sum_f \overline{\Psi}_f \left(i\gamma^\mu D_\mu - m_f\right) \Psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

- in **principle** can derive everything from here

Effective theory for nuclear systems

$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2}\sum_{i,j} V_{ij} + \frac{1}{6}\sum_{i,j,k} W_{ijk} + \cdots$$

- easier to deal with than the QCD lagrangian
- describes low energy physics correctly
- non-perturbative $\rightarrow$ still very challenging

Monte Carlo Calculations of the Ground State of Three- and Four-Body Nuclei

(Received July 2, 1962)

Kalos, Phys. Rev (1962)

Structure of the Lightest Tin Isotopes

(Received 21 September 2017)

Morris et al, PRL (2018)

The need for ab-initio many-body dynamics in NP

- ν scattering for supernovae explosion and NS cooling
- capture reactions for crust heating and nucleosynthesis
- cross sections for dark-matter discovery and neutrino physics
- transport properties of neutron star matter for X-ray emission

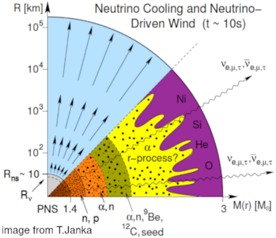
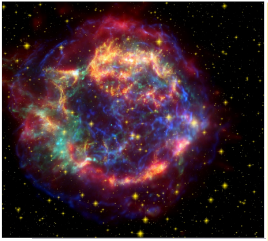


image from T.Janka



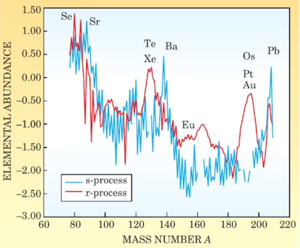
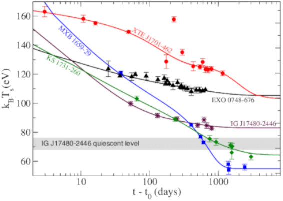


image from Cowan & Thielemann (2004)



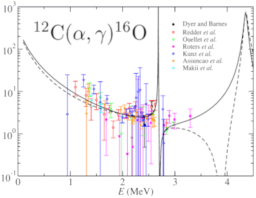
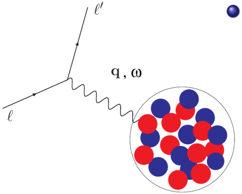


image from Brune & Davis (2015)

Inclusive cross section and the response function

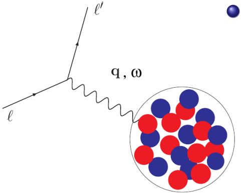


- cross section determined by the response function

$$R_O(\omega) = \sum_f \left|\langle f|\hat{O}|\Psi_0\rangle\right|^2 \delta\left(\omega - E_f + E_0\right)$$

  - excitation operator $\hat{O}$ specifies the vertex

Inclusive cross section and the response function



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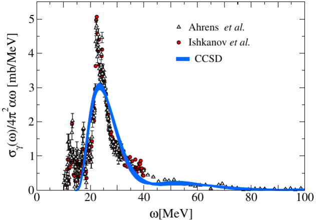
- excitation operator $\hat{O}$ specifies the vertex

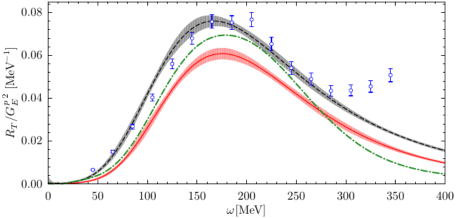
Extremely challenging classically for strongly correlated quantum systems

- dipole response of ^{16}O



Bacca et al. PRL(2013) **LIT+CC**

- quasi-elastic EM response of ^{12}C

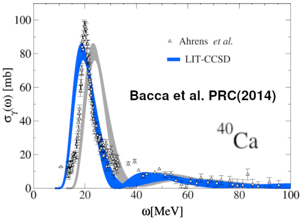


Lovato et al. PRL(2016) **GFMC**

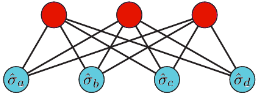
Prospects for classical simulations of nuclear dynamics

Quantum MC + Laplace/STA

- useful for quasi-elastic regime
- not yet accurate enough to go beyond $A = 12$ (sign-problem)



Machine Learning ideas could help



Coupled Cluster + Lorentz/Gauss

- useful for low energy regime
- accuracy limited by inversion

Self Consistent Green's Functions?

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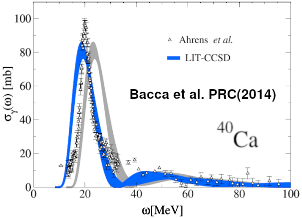

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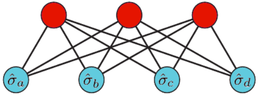

Coupled Cluster + Lorentz/Gauss

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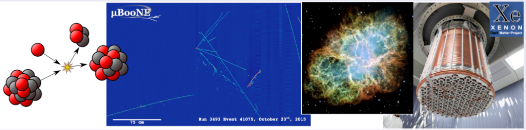
Self Consistent Green's Functions?

Some problems will still remain out of reach

- large open-shell nuclei
- exclusive cross-sections
- out of equilibrium



Quantum Computing and Quantum Simulations

R.Feynman(1982) we can use a controllable quantum system to simulate the behaviour of another quantum system

Quantum System
we have control over

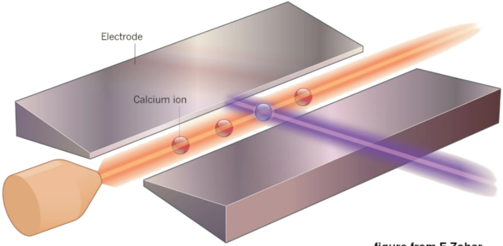


figure from E.Zohar

Quantum System
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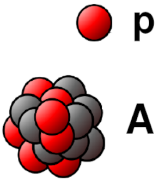

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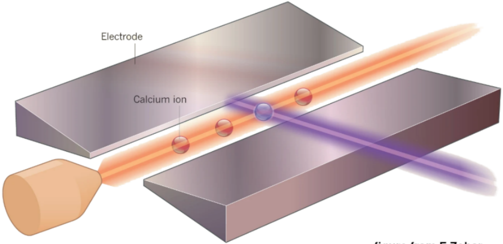

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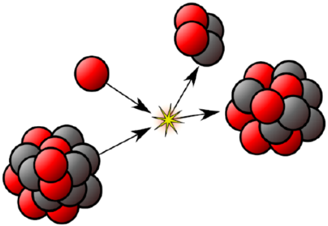

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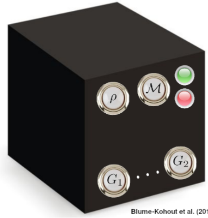
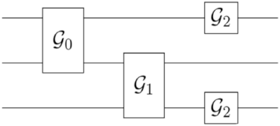


Blume-Kohout et al. (2013)



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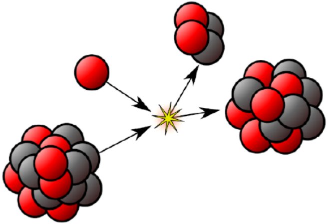
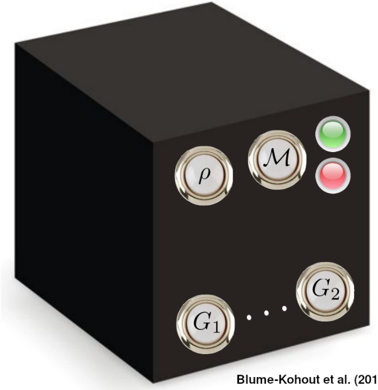


Black box model for a quantum computer

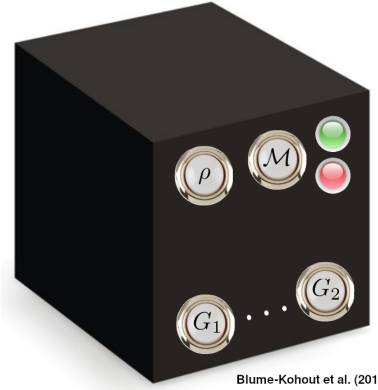


Blume-Kohout et al. (2013)

Box contains N qubits (2-level sys.) together with a set of buttons

- initial state preparation ρ
- projective measurement $\mathcal{M}$
- quantum operations G_k

Black box model for a quantum computer



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Solovay–Kitaev Theorem

We can build a **universal** black box with only a **finite number** of buttons

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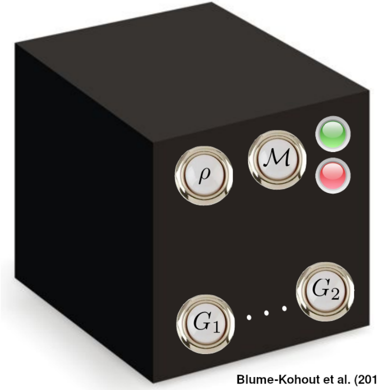

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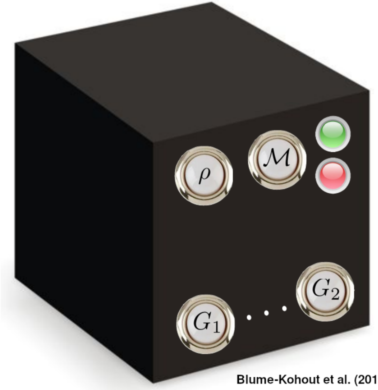

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1. discretize the physical problem

$$|\Psi(0)\rangle \rightarrow |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

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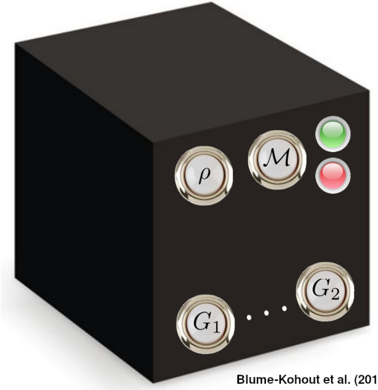

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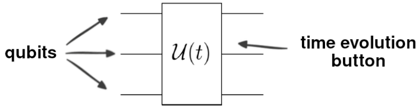

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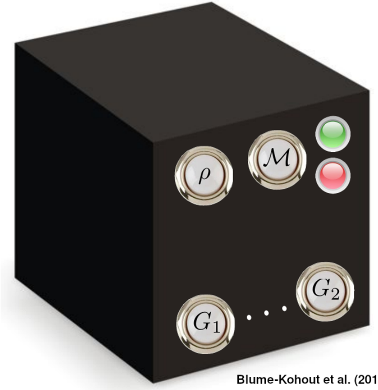

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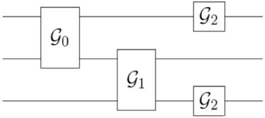
1. discretize the physical problem
2. map physical states to bb states
3. push correct button sequence

$$|\Psi(0)\rangle \rightarrow |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

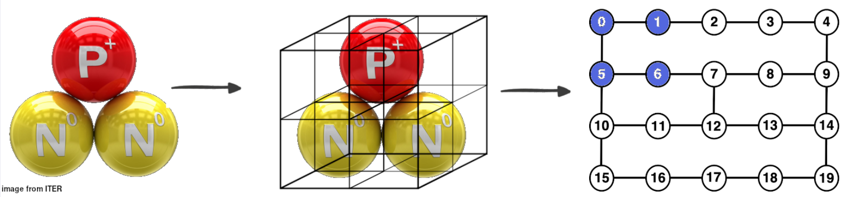


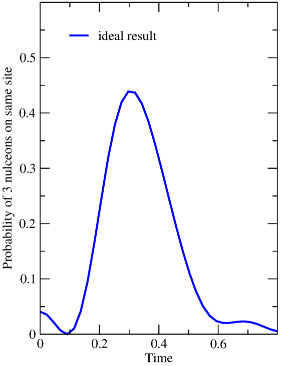
First programmable quantum devices are here

some figures from M.Savage

Real time dynamics on current generation devices

AR, Li, Carlson, Gupta, Perdue PRD(2020)





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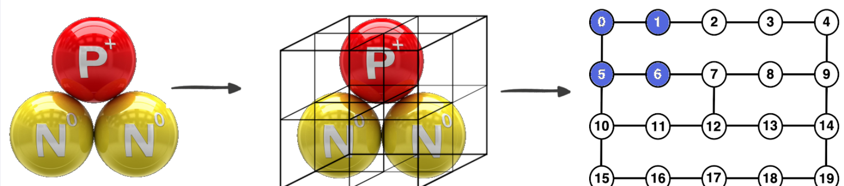
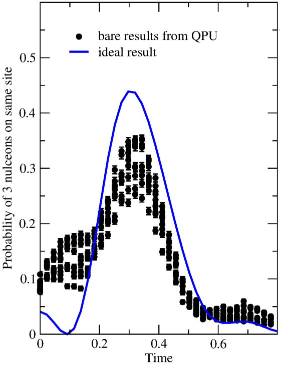


image from ITER



Error sources

- decoherence (environment)
- imperfect calibration

Blume-Kohout et al. (2013)

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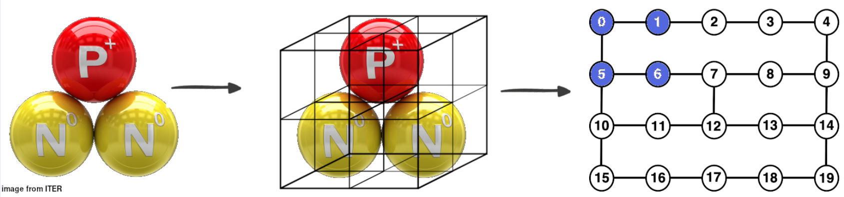
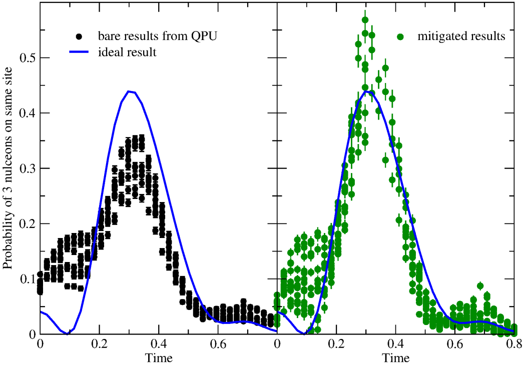
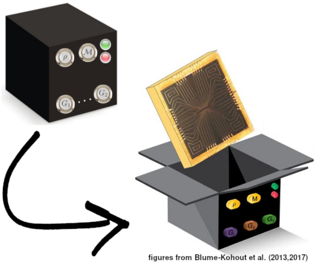
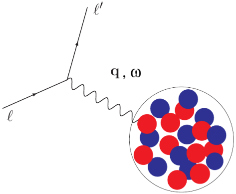




- Error mitigation is crucial



Towards exclusive scattering using quantum computing



- response $R(\omega) \Leftrightarrow$ probability for events at fixed ω
- exclusive x-sec $\rightarrow$ events with specific final states

IDEA: prepare the following state on QC

$$|\Phi\rangle = \sum_\omega \sqrt{R(\omega)}\,|\omega\rangle \otimes |\psi_\omega\rangle$$

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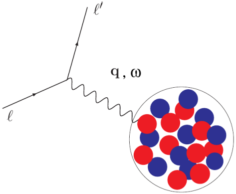

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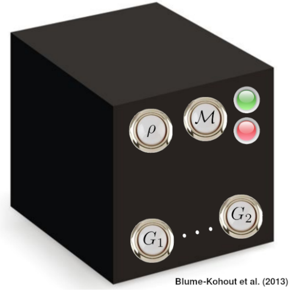
$$|\Phi\rangle = \sum_\omega \sqrt{R(\omega)}\, |\omega\rangle \otimes |\psi_\omega\rangle$$

- measurement of first register returns ω with probability $R(\omega)$
- after measurement, the second register contains final states at ω!



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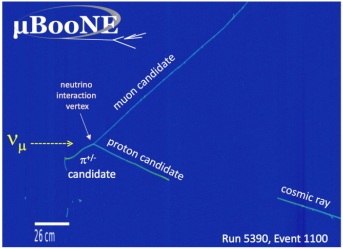

AR & Carlson PRC(2019)

Prospects of impact of QC on Nuclear Physics

AR, Li, Carlson, Gupta, Perdue PRD(2020)

Cost estimates for realistic response in medium mass nuclei

We need ≈ 4000 qubits and push the gate buttons $\approx 10^6 - 10^8$ times

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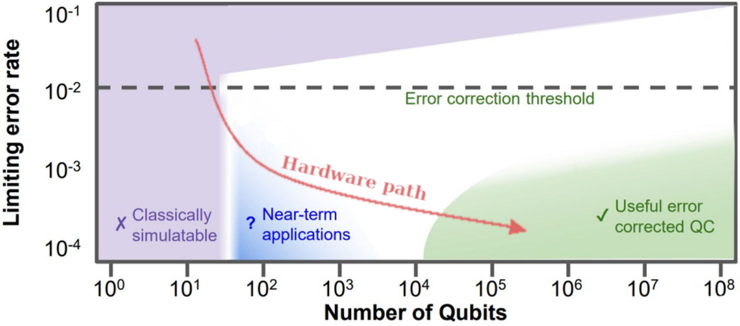
image adapted from Google AI



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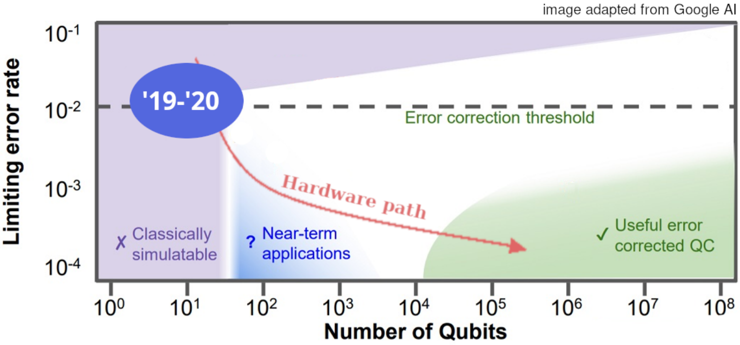

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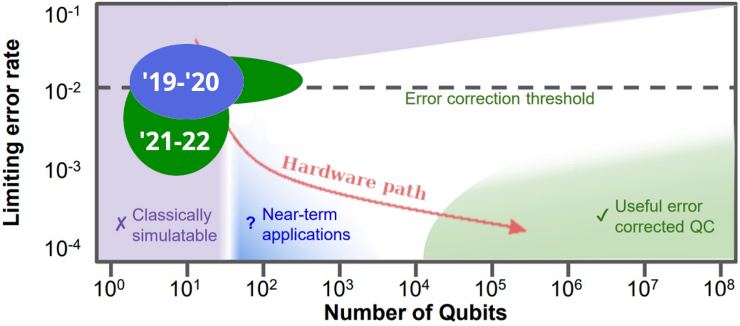

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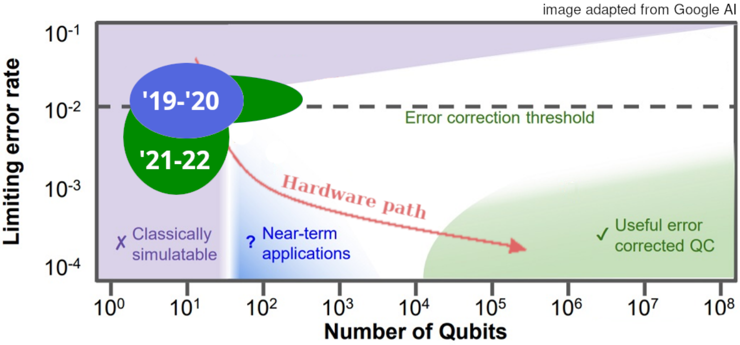


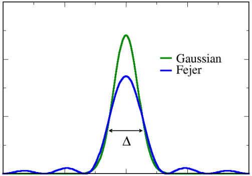
- Still possible to optimize further (other encodings need ≈ 500 qubits)
- Insights for classical methods could come before we have a large QC!

Nuclear dynamics with quantum (inspired) computing?

We can prepare the following state

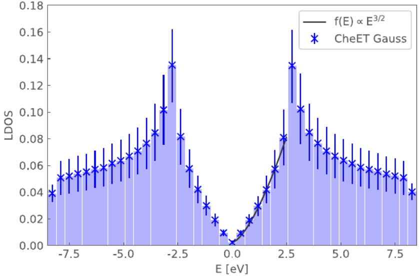
$$|\Phi_\Delta\rangle = \sum_\omega \sqrt{R_\Delta(\omega)}\,|\omega\rangle \otimes |\psi_\omega\rangle$$

with R_Δ is an integral transform of the response with energy resolution Δ

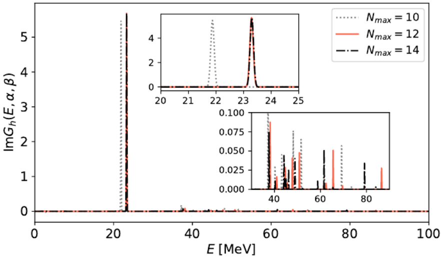


AR & Carlson PRC(2019), **AR** PRA(2020)

- Gaussian approach uses the fact that Chebyshev polynomials can be evaluated efficiently on quantum computers (Berry, Childs, Low, Chuang, . . .)



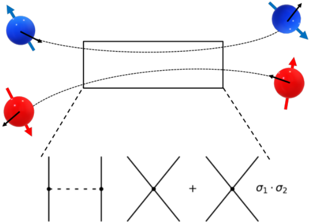
Sobczyk, **AR** PRE(2022)

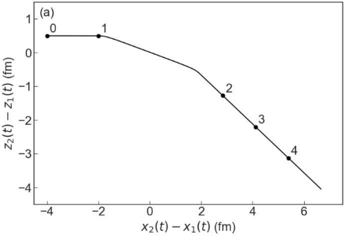


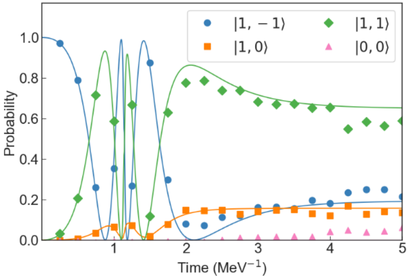
Sobczyk, Bacca, Hagen, Papenbrock (2022)

Nuclear reactions in a semiclassical approach

Turro, Chistolini, Hashim, King, Livingston, Wendt, Dubois, **Pederiva**, Quaglioni, Santiago, Siddiqi (2023)







Neutrino oscillations in astrophysical environments

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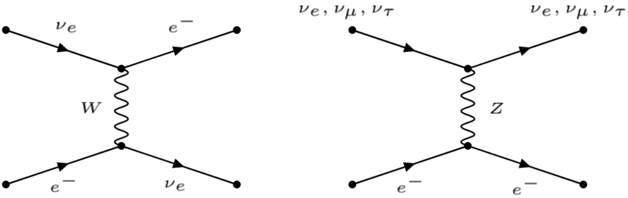
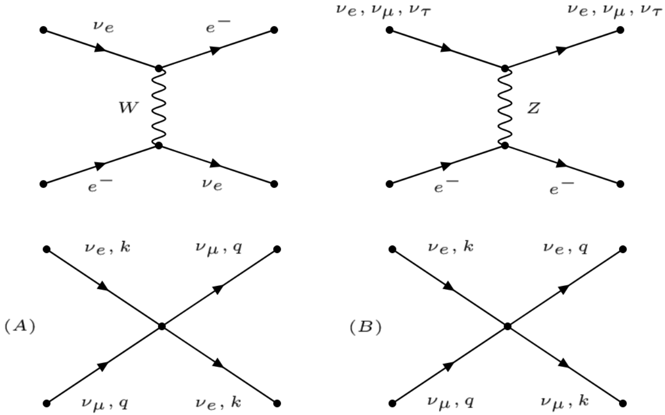


Neutrino oscillations in astrophysical environments

- energy deposition behind shock and in the wind proceeds through charge-current reactions (large differences in $\nu_e - \nu_{\mu/\tau}$)
- neutrino oscillation rates can get enhanced through elastic forward scattering with external matter (MSW effect) or **neutrinos**



Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, . . .

Two-flavor approximation and the iso-spin Hamiltonian

Consider two active flavors (ν_e,ν_x) and encode flavor amplitudes for a neutrino with momentum p_i into an $SU(2)$ iso-spin:

$$|\Phi_i\rangle = \cos(\eta_i)|\nu_e\rangle + \sin(\eta_i)|\nu_x\rangle \equiv \cos(\eta_i)|\uparrow\rangle + \sin(\eta_i)|\downarrow\rangle$$

A system of N interacting neutrinos is then described by the Hamiltonian

$$H = \sum_i \frac{\Delta m^2}{4E_i}\vec{B}\cdot\vec{\sigma}_i + \lambda\sum_i \sigma_i^z + \frac{\mu}{2N}\sum_{i<j}\left(1-\cos(\phi_{ij})\right)\vec{\sigma}_i\cdot\vec{\sigma}_j$$

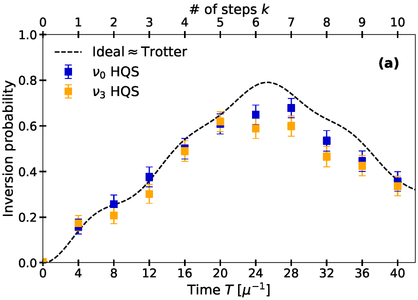
- vacuum oscillations: $\vec{B} = (\sin(2\theta_{mix}), 0, -\cos(2\theta_{mix}))$
- interaction with matter: $\lambda = \sqrt{2}G_F\rho_e$
- neutrino-neutrino interaction: $\mu = \sqrt{2}G_F\rho_\nu$
  - dependence on momentum direction: $\cos(\phi_{ij}) = \frac{\vec{p}_i}{\|\vec{p}_i\|}\cdot\frac{\vec{p}_j}{\|\vec{p}_j\|}$

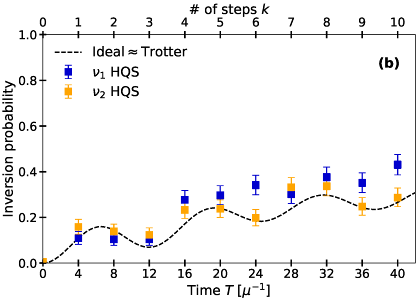
for a full derivation, see e.g. Pehlivan et al. PRD(2011)

Recent results of flavors dynamics with trapped ions

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, PRD (2023)

$N = 4$ neutrinos, multiple time steps





Last two points required: ≈ 350 two-qubit gates over 8 qubits

Summary & Conclusions

- Advances in theory and computing are opening the way to ab-initio calculation of equilibrium properties in the medium-mass region
- New ideas are needed to study nuclear dynamics in large open-shell nuclei, out-of-equilibrium processes and QCD at finite μ
- Quantum Computing has the potential to bridge this gap and increasingly better experimental test-beds are being built
- Error mitigation techniques will be critical to make the best use of these noisy near-term devices
- Early impact of QC on nuclear physics might come as insights into classical many-body methods and the role of entanglement

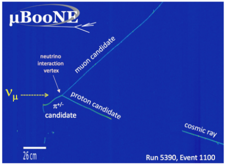


image from Brune & Davis (2015)

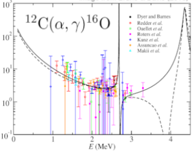
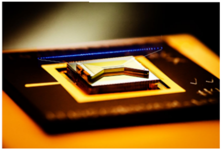


image from Chandra collab.

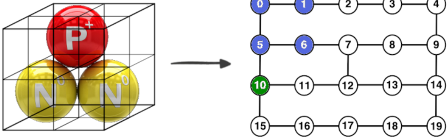


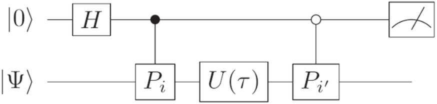
Real time correlators on current generation devices

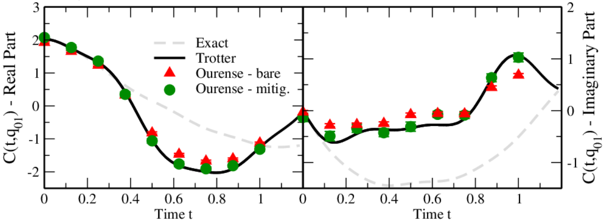
- First steps toward nuclear response: real-time correlators

$$R(\omega) = \int dt e^{i\omega t} C(t) \quad \text{with} \quad C(t) = \langle \Psi_0 | O(t) O(0) | \Psi_0 \rangle$$

- Can be done "easily" using one additional qubit (Somma et al. (2001))







Baroni, Carlson, Gupta, Li, Perdue, AR PRD(2022)

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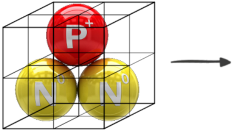

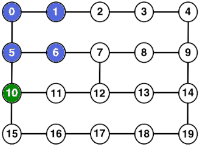

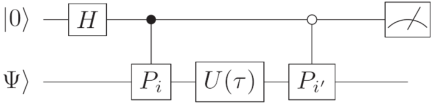

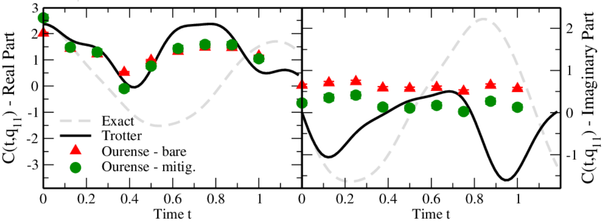

Baroni, Carlson, Gupta, Li, Perdue, AR PRD(2022)

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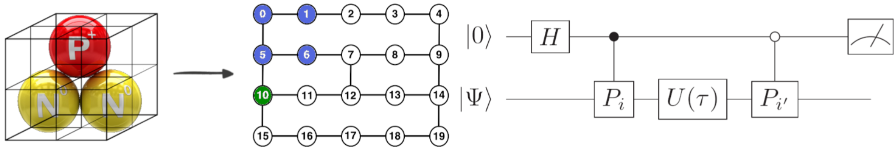

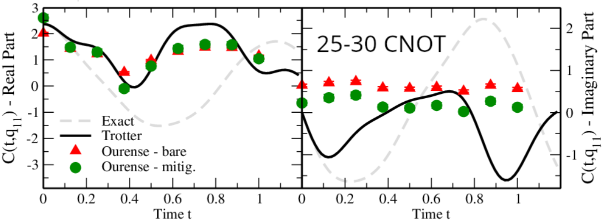

Baroni, Carlson, Gupta, Li, Perdue, AR PRD(2022)

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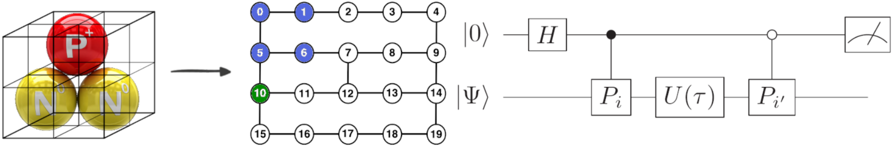
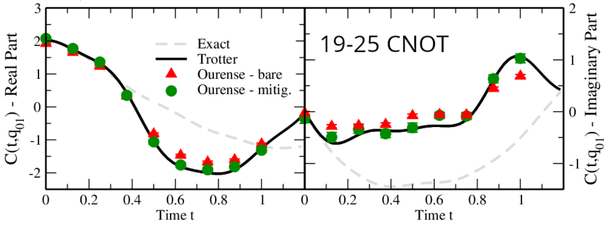
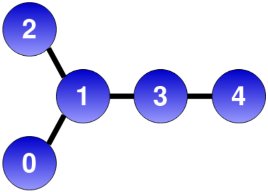




Baroni, Carlson, Gupta, Li, Perdue, AR PRD(2022)

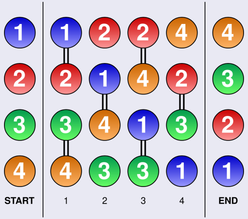
Quantum simulation of collective neutrino oscillations

$$H_\nu = \sum_i \omega_i \vec{B} \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$



- with only 2 flavors direct map to spin 1/2 degrees of freedom (qubits)
- only one- and two-body interactions $\Rightarrow$ only $\mathcal{O}(N^2)$ terms
- all-to-all interactions are difficult with reduced connectivity

SWAP network



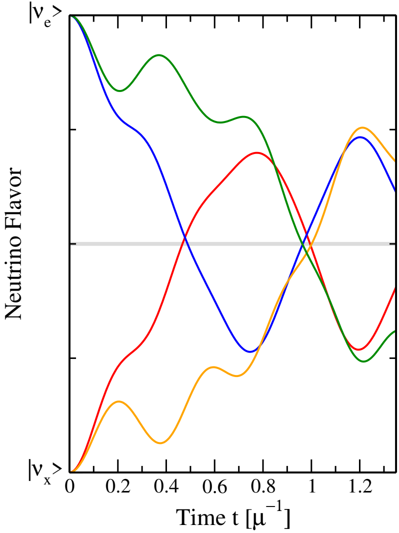
- SWAP qubits every time we apply time-evolution for neighboring terms
- in N steps we perform full evolution using only $\binom{N}{2}$ two qubit gates
  - NOTE: final order will be reversed

Kivlichan et al. PRL (2018)

B.Hall, **AR**, A.Baroni, J.Carlson PRD(2021), **AR** PRD(2021)

Accuracy in flavor evolution

How's the current (≈Fall 2020) accuracy in predicting flavor evolution?



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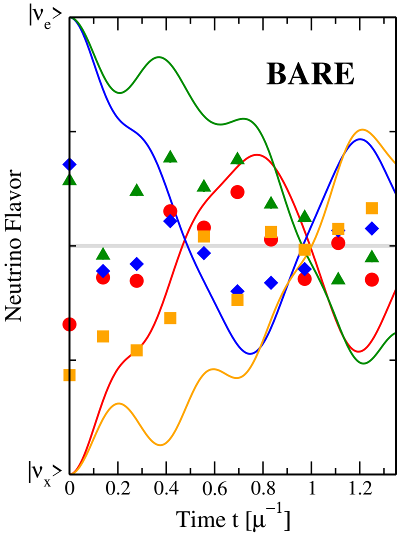

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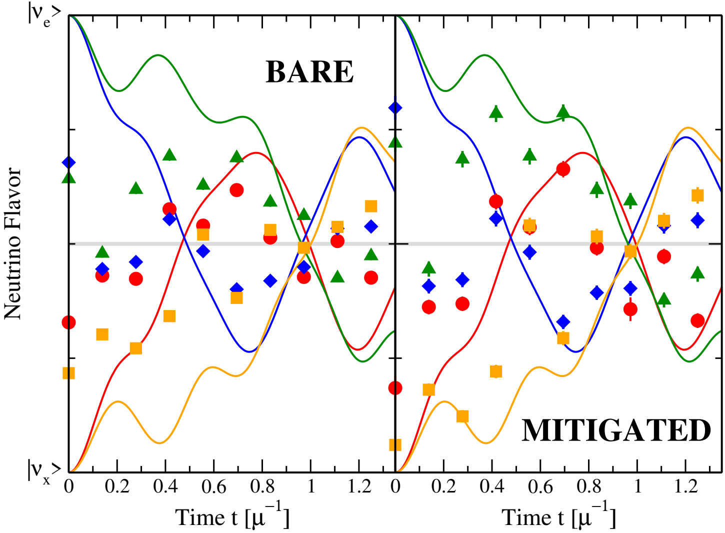

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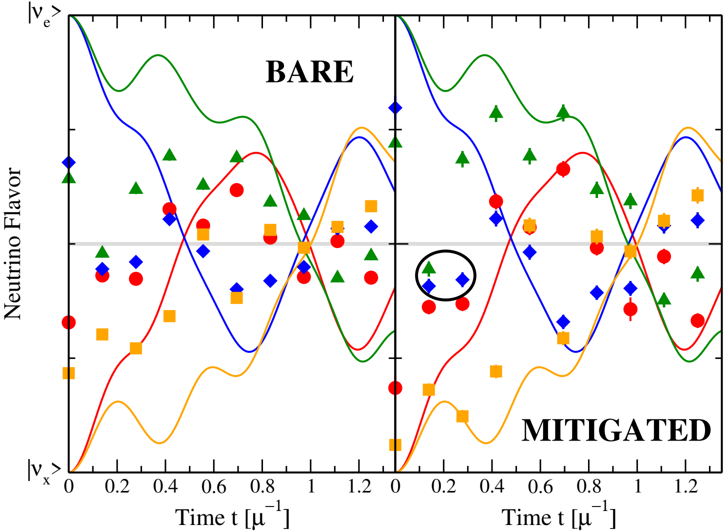

Recent progress in porting the scheme to trapped ions

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, PRD (2023)

$N = 4$ neutrinos, one time step