Study of heavy ion collisions of stable and exotic nuclei within a semiclassical model

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In semiclassical models it is assumed that the motion of the two nuclei can be described according to the classical mechanics. This is true when the De Broglie wave length is small with respect to the distance of closest approach.

$$\lambda = \frac{h}{\mu v} \ll d = \frac{Z_A Z_B e^2}{\mu v^2}$$

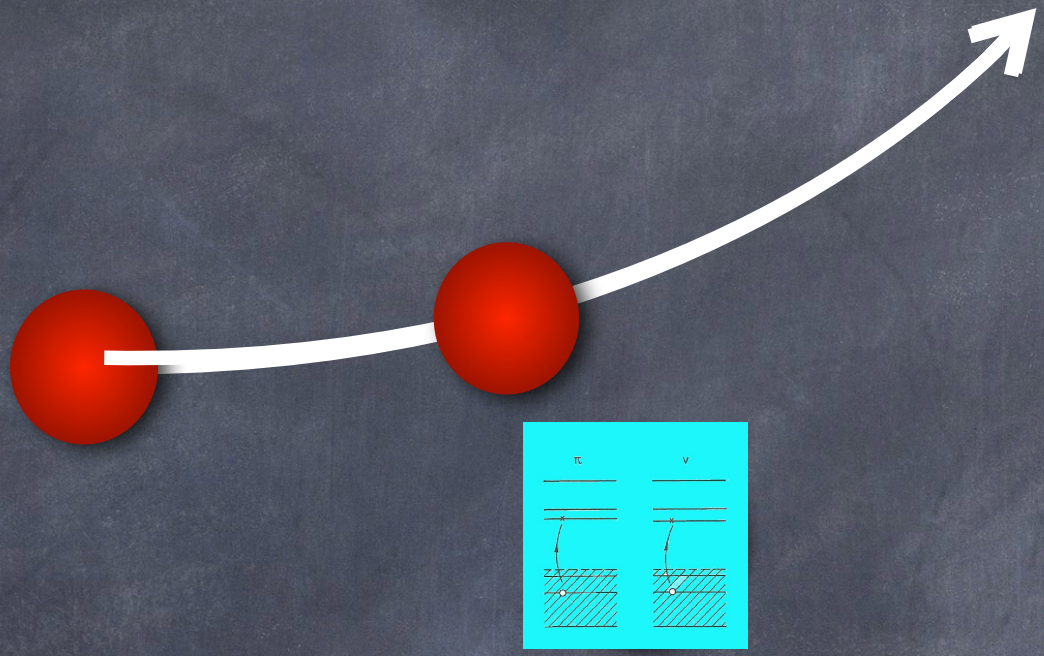
In the coulomb case this condition is satisfied when the Sommerfeld parameter is much bigger than one.

$$\eta = \frac{Z_A Z_B e^2}{\hbar v} >> 1$$

Furthermore, the energy loss has be be small compared to the incident energy, and the angular momentum loss has to be small compared to the total angular momentum of relative motion

$$\frac{\Delta E}{E} << 1 \; ; \quad \frac{\Delta L}{L} << 1$$

Time dependent Semiclassical Approximation to the Coupled Channel method



The two colliding nuclei move according to a classical trajectory determined by the Coulomb plus nuclear fields, while the inelastic excitations are described according to quantum mechanics.

This is realized by building a set of coupled first order differential equations for the time dependent coefficients $C(t)$ of the channels wave functions

Semiclassical Model

The two nuclei move according to a classical trajectory while quantum mechanics is used to describe the internal degrees of freedom

$$H = H_A + H_B$$

where

$$H_A = H_A^0 + W_A(t)$$

$$H_A^0 = \sum_i \epsilon_i a_i^\dagger a_i + \frac{1}{4}\sum_{ijlk} V_{ijlk} a_i^\dagger a_j^\dagger a_l a_k$$

$$W_A(t) = \sum_{ij} < i|U_B(\vec{R}(t))|\, j > a_i^+ \, a_j + h.c.$$

t-dependence through R(t)

$$W = W^{00} + \sum_\nu W_\nu^{10} Q_\nu^\dagger + h.c.$$

The term W^{00} represents the interaction of the two colliding nuclei in their ground state; in the present case it has also an imaginary part that describes the absorption due to the nonelastic channels.

The term W^{10} connect states differing by one phonon

The wave function of the system

$$|\Psi(t)>=|\psi_A(t)>|\psi_B(t)>$$

To solve the Schrödinger equation

$$i\hbar\frac{\partial|\psi_A(t)>}{\partial t}=H_A|\psi_A(t)>$$

Calling $|\Phi_\alpha>$ the eigenstates of the internal Hamiltonian

$$H^0|\Phi_\alpha>=E_\alpha|\Phi_\alpha>$$

The time dependent state is

$$|\psi(t)>=\sum_\alpha C_\alpha(t)e^{-\frac{i}{\hbar}E_\alpha t}|\Phi_\alpha>$$

The Schrödinger equation can be cast into a set of linear differential equations

$$\dot{C}_\alpha(t)=-\frac{i}{\hbar}\sum_{\alpha'}e^{\frac{i}{\hbar}(E_\alpha-E_{\alpha'})t}<\Phi_\alpha|W(t)|\Phi_{\alpha'}>C_{\alpha'}(t)$$

SEMICLASSICAL COUPLED CHANNEL EQUATIONS

The semiclassical coupled channel equations have to be solved for each impact parameter, then $C(b,\alpha,t)$

Probability to excite the state Φ_α

b impact parameter

$$P_\alpha(b) = |C_\alpha(b, +\infty)|^2$$

its cross section is

$$\sigma_\alpha = 2\pi \int_0^{+\infty} P_\alpha(b) T(b) b \, db.$$

$T(b)$: transmission coefficient taking into account process not explicitly included in the space model.

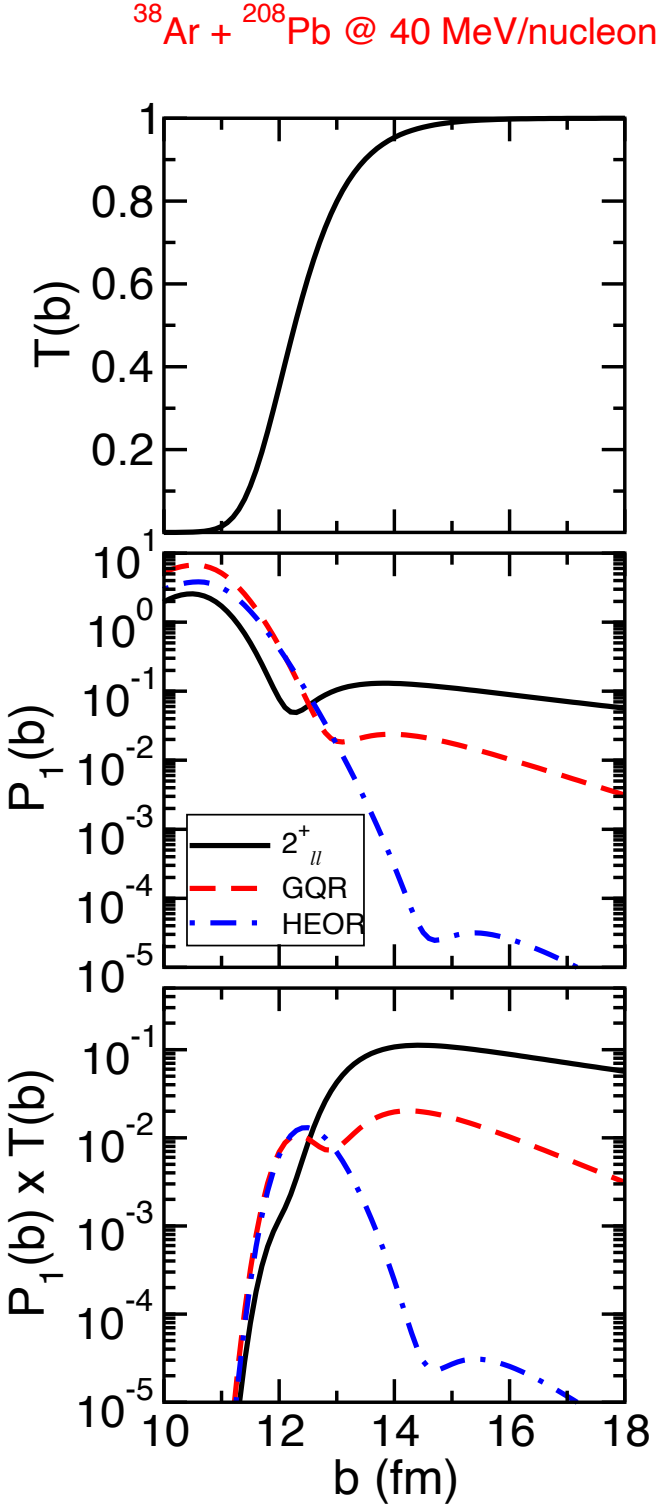
It falls to zero as the overlap between the two nuclei increases.

The effect of the absorption

The transmission coefficient $T(b)$ is taken as a depletion factor that falls to zero as the overlap between the reacting nuclei increases. A standard practice is to construct it from an integral along the classical trajectory

$$T(b) = \exp\left\{ -\frac{2}{\hbar} \int_{-\infty}^{+\infty} W(r(t'))\, dt' \right\}$$

where W is the imaginary part of the optical potential. If it is not deduced from elastic scattering fit, then it is usually chosen as $W=0.5\, U_{Real}$



To solve the coupled channel equations we need to build up the matrix elements containing all the information about the physical problem

$$\dot{C}_\alpha(t) = -\frac{i}{\hbar}\sum_{\alpha'} e^{\frac{i}{\hbar}(E_\alpha - E_{\alpha'})t} < \Phi_\alpha|W(t)|\Phi_{\alpha'} > C_{\alpha'}(t)$$

The calculations have to be performed for few selected states

A HF+RPA calculations (or similar) provide the states involved in the excitation process. We choose the ones having an EWSR% above 5%, and the low lying dipole states. Then we bunch together states close in energy. The resulting state has as energy the average energy of the states belonging to the group with the condition that the EWSR must be preserved.

^{132}Sn

State	E (MeV)	EWSR%
GMR	16,3	85
Pygmy	9,3	1,1
GDR	13,9	56
1^-_{hl}	18,3	25
2^+	5	11
GQR	13,5	77
3^-	5,9	27

The others fundamental ingredients for the calculation of the excitation process are the optical potential and the radial form factor. Both of them are calculated within the double folding procedure.

$$F_\lambda^C \approx \frac{[B(E\lambda)]}{r^{\lambda+1}}$$

Coulomb vs. Nuclear for $r > R_A + R_B$

$$F_\lambda^N \approx \frac{dU_N}{dr}$$ for $\lambda \geq 2$

range

Long Ranged: large impact parameter interval

Short Ranged: very small impact parameter interval

Multipole

Strong λ dependence: for high multipolarities the form factor reduces by a factor $\{R_A / (R_A + R_B)\}^{\lambda+1}$.
Dipole favourite, monopole inhibit.

Weak λ dependence: the r dependence is the same for all λ values.
Note that the nuclear excitation does not scale with the B(Eλ).

Q-dependence

Smooth variation with time: low probability to excite high energy states. τ_{coll}^C is large, then high probability to excite low-lying states because $\Delta E \Delta t \leq \hbar$.

Smaller collision time:
For the nuclear case
$$\tau_{coll}^N < \tau_{coll}^C$$
Therefore, for the same incident energy the nuclear interaction can excite higher energy states.

The two contribution may interfere in a positive or negative way depending on the transition density relative sign.

For non collective states, it is important to calculate them in such a way that the detailed information of the states can be brought inside the form factor.

Double Folding procedure

The nucleon nucleon interaction depends on the isospin

$$v_{12} = v_0(r_{12}) + v_1(r_{12})\tau_1 \cdot \tau_2$$

where τ_i are the isospin of the nucleons. This implies

$$v_{nn} = v_{pp} = v_0 + v_1 \; ; \quad v_{np} = v_0 - v_1 \, .$$

The double folding potential has two terms

$$\boxed{U_0(\vec{r}_\alpha) = \iint \rho_A(\vec{r}_1)\, v_0(r_{12})\, \rho_a(\vec{r}_2)\, d\vec{r}_1 d\vec{r}_2}$$

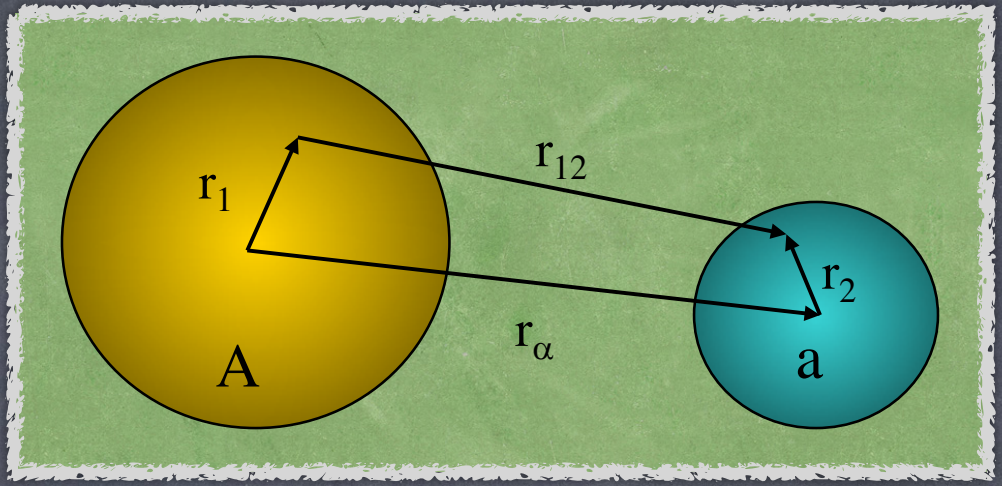
$$\boxed{\begin{aligned} U_1(\vec{r}_\alpha) = \iint & \left[\rho_{An}(\vec{r}_1) - \rho_{Ap}(\vec{r}_1)\right] \times \\ & \times v_1(r_{12}) \left[\rho_{an}(\vec{r}_2) - \rho_{ap}(\vec{r}_2)\right] d\vec{r}_1 d\vec{r}_2 \end{aligned}}$$

where $r_{12} = |\vec{r}_\alpha + \vec{r}_2 - \vec{r}_1|$

In the case $\rho_n = N/A\, \rho$; $\rho_p = N/A\, \rho$

$$U_1(\vec{r}_\alpha) = \left(\frac{N_a - Z_a}{a}\right)\left(\frac{N_A - Z_A}{A}\right) \times \int\int \rho_A(\vec{r}_1) v_1(r_{12}) \rho_a(\vec{r}_2) \quad d\vec{r}_1 d\vec{r}_2 .$$

In the case $N=Z$, for one of the nuclei, then U_1 is zero.



The nuclear form factors can be calculated within the same procedure

$$\begin{aligned} F_0(r_\alpha) = \iint & \left[\delta\rho_{An}(\vec{r}_1) + \delta\rho_{Ap}(\vec{r}_1)\right] \times \\ & \times v_0(r_{12}) \left[\rho_{an}(\vec{r}_2) + \rho_{ap}(\vec{r}_2)\right] r_1^2 dr_1 r_2^2 dr_2 \end{aligned}$$

$$\begin{aligned} F_1(r_\alpha) = \iint & \left[\delta\rho_{An}(\vec{r}_1) - \delta\rho_{Ap}(\vec{r}_1)\right] \times \\ & \times v_1(r_{12}) \left[\rho_{an}(\vec{r}_2) - \rho_{ap}(\vec{r}_2)\right] r_1^2 dr_1 r_2^2 dr_2 \end{aligned}$$

As before, in the case one of the two nuclei has $N=Z$ then F_1 is zero.

We use the M3Y nucleon nucleon potential of the Raid type: no density dependent and no exchange term.

When different density dependence and exchange terms are taken into account there are some differences among the corresponding potential obtained.

(Dao T. Khoa et al. PRC 49 (1994) 1652; PRC 56 (1997) 954.)

However, these differences are in the interior part of the nucleus while at the surface the potentials are almost identical.

Therefore, since we explore the peripheral region of the two colliding nuclei the use of the M3Y, as we employ it, seems to be well justified.

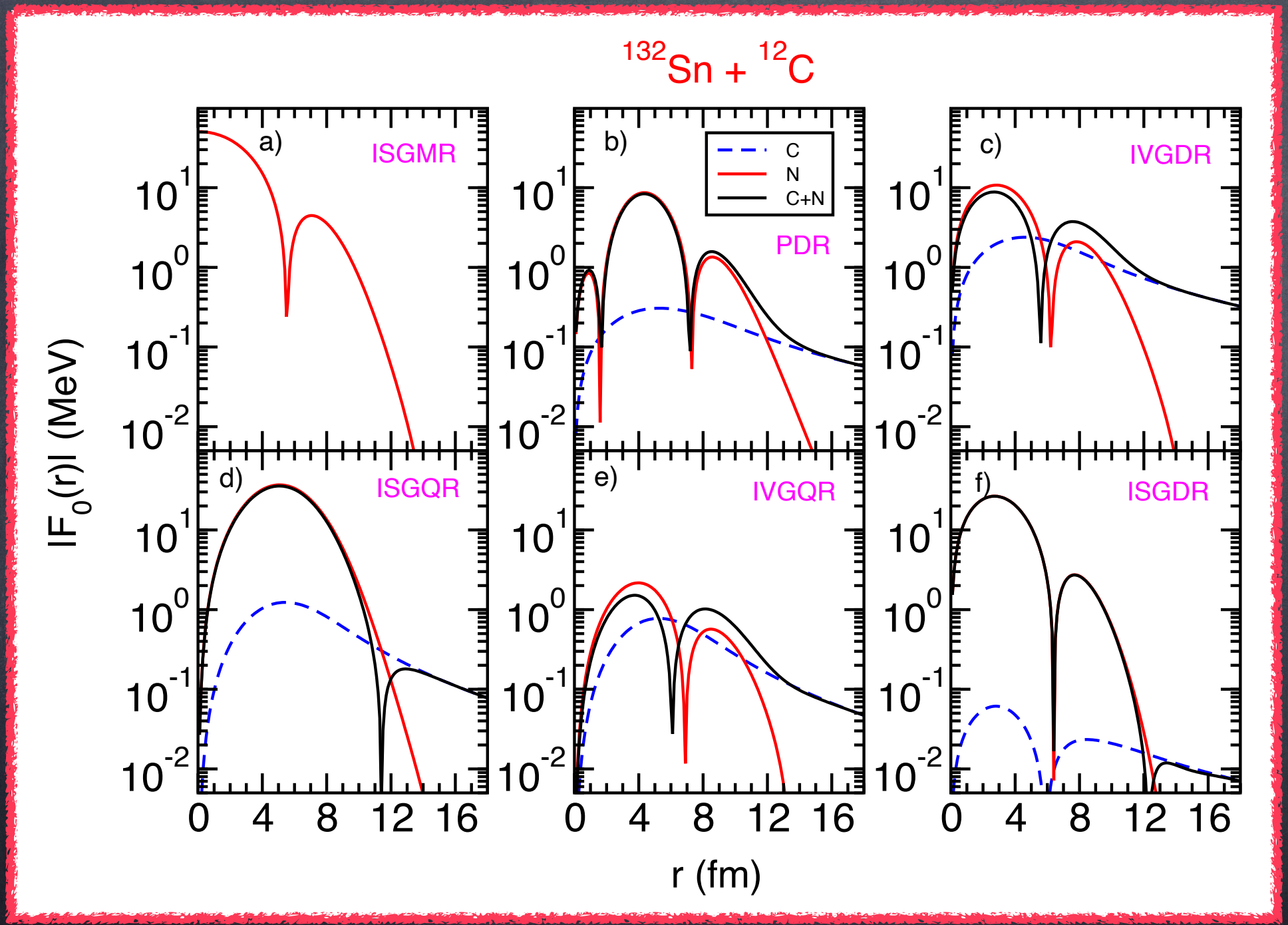
$$v_0(r) = \left[7999\frac{e^{-4r}}{4r} - 2134\frac{e^{-2.5r}}{2.5r}\right] - 262\delta(r)$$

$$v_1(r) = -\left[4886\frac{e^{-4r}}{4r} - 1176\frac{e^{-2.5r}}{2.5r}\right] + 217\delta(r)$$

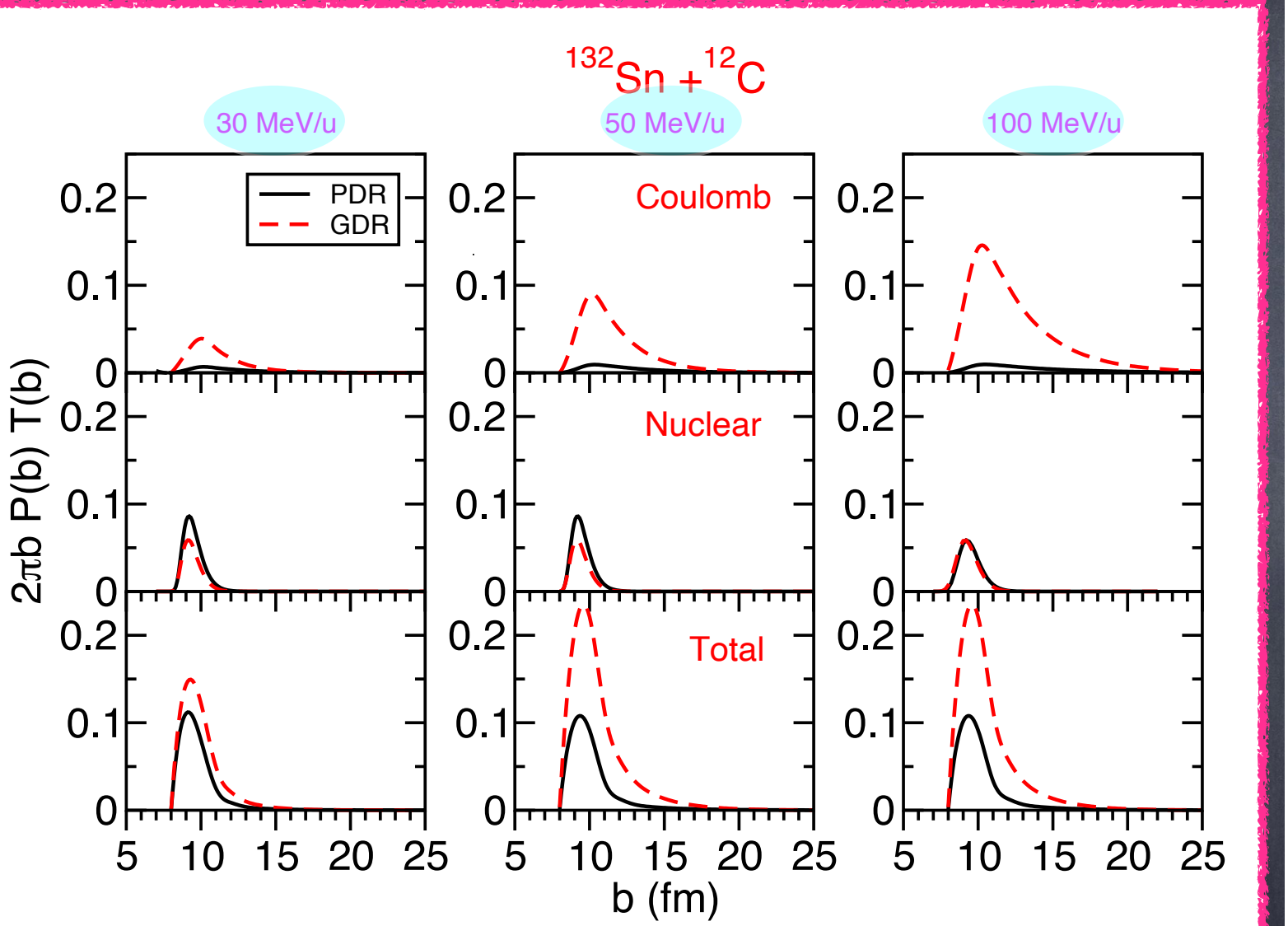
Parametrization of the M3Y interaction

Calculation done in the momentum-space making use of the Fourier transform because the calculation is fast and convenient

(G. R. Satchler and W. G. Love, Phys. Rep. 55 (1979) 183)


^{132}Sn + ^{12}C
a)
ISGMR
b)
PDR
C
N
C+N
c)
IVGDR
d)
ISGQR
e)
IVGQR
f)
ISGDR
$|F_0(r)|$ (MeV)
r (fm)

Partial waves cross section as function of the impact parameter a limited range of impact parameter

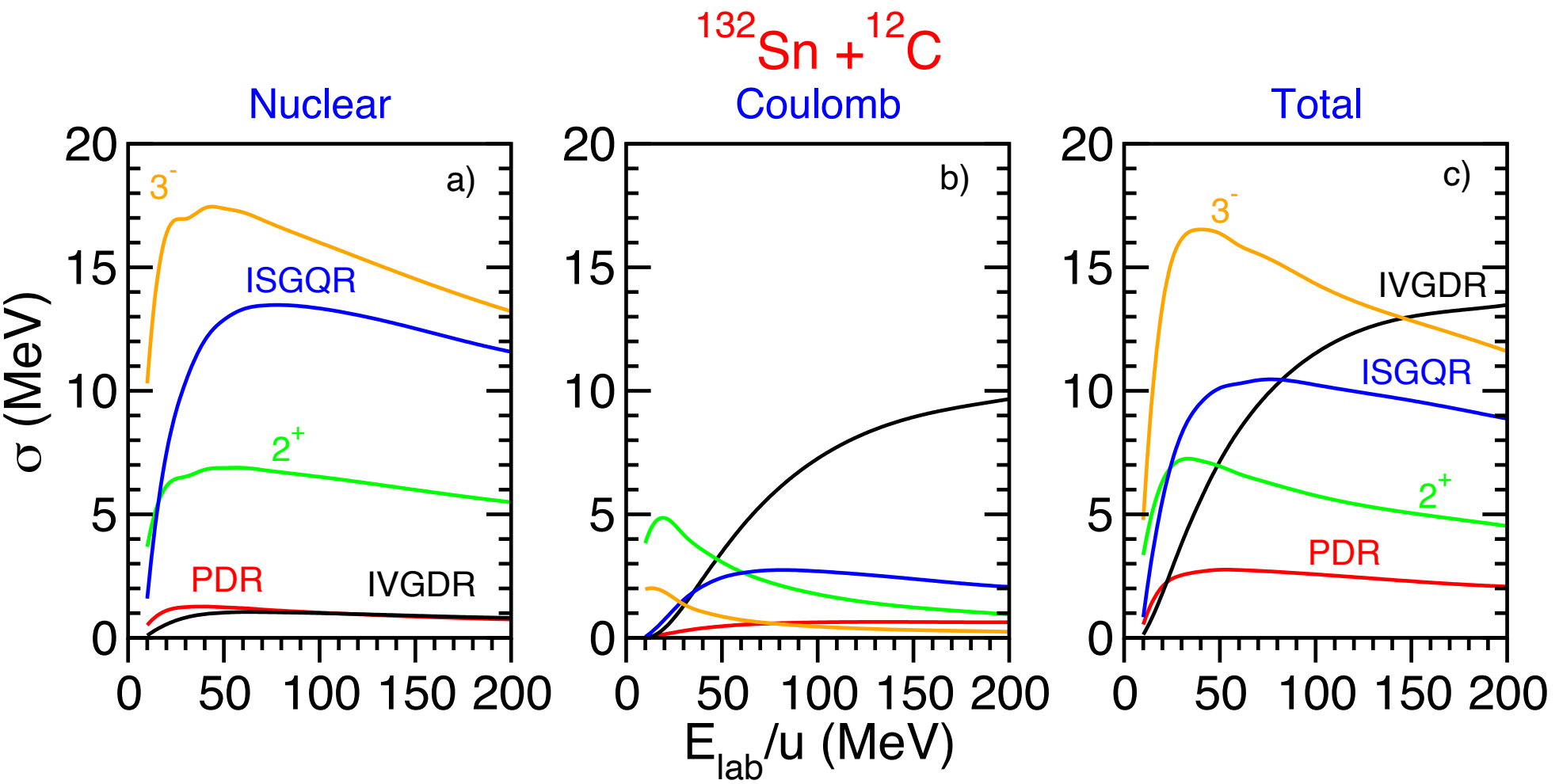


Only a limited range of impact parameters give contribution to the nuclear part; for the Coulomb part the range of b is much larger.

As the incident energy goes down, the range of impact parameters participating to the Coulomb excitation process is decreasing.

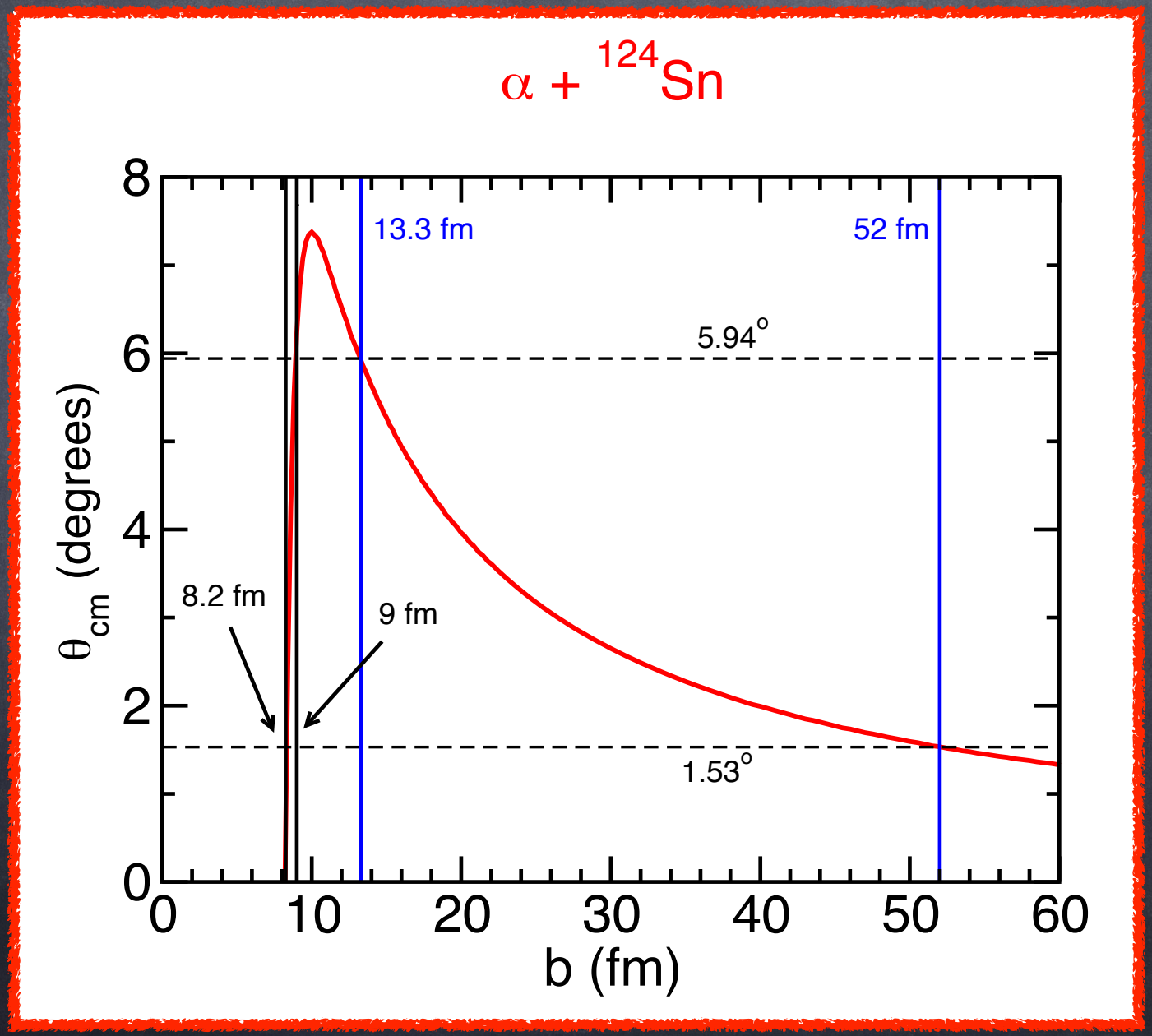
$$\sigma_\alpha = 2\pi \int_0^{+\infty} P_\alpha(b) T(b) b \, db.$$

Total cross section



Experimental data are usually taken at a finite angle range.
From the deflection function one can deduce the corresponding impact parameter range

$^{124}Sn(\alpha,\alpha'\gamma)^{124}Sn$ @ 136 MeV



The model have been successfully employed in several physical problem involving heavy ion collisions.

Calculation of polarisation potential

Multiphonons excitation in heavy ion collisions

Isoscalar excitation of low-lying dipole (PDR) states in exotic and stable nuclei

Strong collaboration with several experimental groups for the studies of

Isoscalar excitation of low-lying dipole (PDR) states in exotic, stable and deformed nuclei

Summary

Semiclassical model have been usefully used for calculations and interpretations of various nuclear phenomena.

The use of the semi-classical coupled-channel equations is more convenient than the quantum CC because the calculations can be guided by a physical insight and the number of channels included in the calculations can be orders of magnitude larger.

Combined reactions processes involving the Coulomb and nuclear interactions can provide a clue to reveal characteristic features of some particular states.

Good comparison of our calculations with experimental results.

Thank you
for your attention