Quartet structure of $N = Z$ nuclei

M. Sambataro

Istituto Nazionale di Fisica Nucleare - Sezione di Catania, Italy

MONSTRE Meeting

11-12 Maggio 2023, Milano

Outline

- An exact treatment of the isovector pairing Hamiltonian: the role of quartets
- The phenomenon of quartet condensation in the ground and excited states of a proton-neutron pairing Hamiltonian
- Spectra of $N = Z$ nuclei in a formalism of quartets
- Future perspectives

in collaboration with N. Sandulescu (NIPNE, Bucharest)

The isovector pairing Hamiltonian

$$H^{(iv)} = \sum_{i=1}^{\Omega} \epsilon_i \mathcal{N}_i - g \sum_{i,i'=1}^{\Omega} \sum_{M_T=-1}^{1} P_{iM_T}^{\dagger} P_{i'M_T}$$

$$\mathcal{N}_i = \sum_{\sigma,\tau} a_{i\sigma\tau}^{\dagger} a_{i\sigma\tau}, \quad P_{iM_T}^{\dagger} = [a_{i+}^{\dagger} a_{i-}^{\dagger}]_{M_T}^{T=1}, \quad (P_{iM_T}^{\dagger})^{\dagger} = P_{iM_T}$$

$$\left(P_{iM_T}^{\dagger} : \quad M_T = -1 \ (pp), \quad M_T = 0 \ (pn), \quad M_T = +1 \ (nn) \right)$$

We confine our analysis to $T = 0$ seniority-zero eigenstates.

The Hilbert space of the model is spanned by the states

$$P_{i_1 M_{T_1}}^{\dagger} P_{i_2 M_{T_2}}^{\dagger} \cdots P_{i_N M_{T_N}}^{\dagger} |0\rangle$$

subject to the condition

$$M_{T_1} + M_{T_2} + \cdots + M_{T_N} = 0$$

The isovector pairing Hamiltonian

$$H^{(iv)} = \sum_{i=1}^{\Omega} \epsilon_i \mathcal{N}_i - g \sum_{i,i'=1}^{\Omega} \sum_{M_T=-1}^{1} P_{iM_T}^{\dagger} P_{i'M_T}$$

$$\mathcal{N}_i = \sum_{\sigma,\tau} a_{i\sigma\tau}^{\dagger} a_{i\sigma\tau}, \quad P_{iM_T}^{\dagger} = [a_{i+}^{\dagger} a_{i-}^{\dagger}]_{M_T}^{T=1}, \quad (P_{iM_T}^{\dagger})^{\dagger} = P_{iM_T}$$

$$\left(P_{iM_T}^{\dagger} : \quad M_T = -1 \ (pp), \quad M_T = 0 \ (pn), \quad M_T = +1 \ (nn) \right)$$

We confine our analysis to $T = 0$ seniority-zero eigenstates.

The Hilbert space of the model is spanned by the states

$$P_{i_1 M_{T_1}}^{\dagger} P_{i_2 M_{T_2}}^{\dagger} \cdots P_{i_N M_{T_N}}^{\dagger} |0\rangle$$

subject to the condition

$$M_{T_1} + M_{T_2} + \cdots + M_{T_N} = 0$$

First case: 2 protons and 2 neutrons

Building blocks:

$$B_{\nu\tau}^{\dagger} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P_{k\tau}^{\dagger} \qquad (E_\nu \equiv \text{``pair energy''})$$

Ansatz for the eigenstates:

$$|\Psi\rangle = [B_1^{\dagger} B_2^{\dagger}]^{T=0}|0\rangle \qquad (\Rightarrow \text{quartet})$$

One finds:

$$\begin{aligned}
H^{(iv)}|\Psi\rangle &= (E_1 + E_2)|\Psi\rangle \\
&+ \left(1 - g\sum_k \frac{1}{2\epsilon_k - E_1} - \frac{g}{E_2 - E_1}\right)[P^{\dagger} B_2^{\dagger}]^{T=0}|0\rangle \\
&+ \left(1 - g\sum_k \frac{1}{2\epsilon_k - E_2} - \frac{g}{E_1 - E_2}\right)[P^{\dagger} B_1^{\dagger}]^{T=0}|0\rangle
\end{aligned}$$

being $P_\tau^{\dagger} = \sum_k P_{k,\tau}^{\dagger}$.

First case: 2 protons and 2 neutrons

Building blocks:

$$B_{\nu\tau}^{\dagger} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_{\nu}} P_{k\tau}^{\dagger} \qquad (E_{\nu} \equiv \text{``pair energy''})$$

Ansatz for the eigenstates:

$$|\Psi\rangle = [B_1^{\dagger} B_2^{\dagger}]^{T=0}|0\rangle \qquad (\Rightarrow \text{quartet})$$

One finds:

$$\begin{aligned}
H^{(iv)}|\Psi\rangle &= (E_1 + E_2)|\Psi\rangle \\
&+ \left(1 - g \sum_k \frac{1}{2\epsilon_k - E_1} - \frac{g}{E_2 - E_1}\right)[P^{\dagger} B_2^{\dagger}]^{T=0}|0\rangle \\
&+ \left(1 - g \sum_k \frac{1}{2\epsilon_k - E_2} - \frac{g}{E_1 - E_2}\right)[P^{\dagger} B_1^{\dagger}]^{T=0}|0\rangle
\end{aligned}$$

being $P_{\tau}^{\dagger} = \sum_k P_{k,\tau}^{\dagger}$.

First case: 2 protons and 2 neutrons

Building blocks:

$$B_{\nu\tau}^{\dagger} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_{\nu}} P_{k\tau}^{\dagger} \qquad (E_{\nu} \equiv \text{``pair energy''})$$

Ansatz for the eigenstates:

$$|\Psi\rangle = [B_1^{\dagger} B_2^{\dagger}]^{T=0}|0\rangle \qquad (\Rightarrow \text{quartet})$$

One finds:

$$
\begin{aligned}
H^{(i\nu)}|\Psi\rangle &= (E_1 + E_2)|\Psi\rangle \\
&+ \Big(1 - g\sum_k \frac{1}{2\epsilon_k - E_1} - \frac{g}{E_2 - E_1}\Big)[P^{\dagger}B_2^{\dagger}]^{T=0}|0\rangle \\
&+ \Big(1 - g\sum_k \frac{1}{2\epsilon_k - E_2} - \frac{g}{E_1 - E_2}\Big)[P^{\dagger}B_1^{\dagger}]^{T=0}|0\rangle
\end{aligned}
$$

being $P_{\tau}^{\dagger} = \sum_k P_{k,\tau}^{\dagger}$.

Second case: 4 protons and 4 neutrons

Ansatz:

$$|\Psi\rangle = d_1[B_1^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle + d_2[B_1^\dagger B_3^\dagger]^0[B_2^\dagger B_4^\dagger]^0|0\rangle + d_3[B_1^\dagger B_4^\dagger]^0[B_2^\dagger B_3^\dagger]^0|0\rangle$$

One finds:

$$H^{(iv)}|\Psi\rangle = (E_1 + E_2 + E_3 + E_4)|\Psi\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_1} - \frac{g \cdot d_{123}}{E_2 - E_1} - \frac{g \cdot d_{12}}{E_1 - E_4} - \frac{g \cdot d_{13}}{E_1 - E_3}\right)[P^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_2} - \frac{g \cdot d_{123}}{E_1 - E_2} - \frac{g \cdot d_{12}}{E_2 - E_3} - \frac{g \cdot d_{13}}{E_2 - E_4}\right)[P^\dagger B_1^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_3} - \frac{g \cdot d_{123}}{E_4 - E_3} - \frac{g \cdot d_{12}}{E_3 - E_2} - \frac{g \cdot d_{13}}{E_3 - E_1}\right)[P^\dagger B_4^\dagger]^0[B_1^\dagger B_2^\dagger]^0|0\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_4} - \frac{g \cdot d_{123}}{E_3 - E_4} - \frac{g \cdot d_{12}}{E_4 - E_1} - \frac{g \cdot d_{13}}{E_4 - E_2}\right)[P^\dagger B_3^\dagger]^0[B_1^\dagger B_2^\dagger]^0|0\rangle$$

$$+ \cdots$$

being: $\quad d_{ij} \equiv d_i + d_j, \quad d_{123} \equiv d_1 + d_2 + d_3$

Second case: 4 protons and 4 neutrons

Ansatz:

$$|\Psi\rangle = d_1[B_1^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle + d_2[B_1^\dagger B_3^\dagger]^0[B_2^\dagger B_4^\dagger]^0|0\rangle + d_3[B_1^\dagger B_4^\dagger]^0[B_2^\dagger B_3^\dagger]^0|0\rangle$$

One finds:

$$H^{(iv)}|\Psi\rangle = (E_1 + E_2 + E_3 + E_4)|\Psi\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_1} - \frac{g \cdot d_{123}}{E_2 - E_1} - \frac{g \cdot d_{12}}{E_1 - E_4} - \frac{g \cdot d_{13}}{E_1 - E_3} \right)[P^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_2} - \frac{g \cdot d_{123}}{E_1 - E_2} - \frac{g \cdot d_{12}}{E_2 - E_3} - \frac{g \cdot d_{13}}{E_2 - E_4} \right)[P^\dagger B_1^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_3} - \frac{g \cdot d_{123}}{E_4 - E_3} - \frac{g \cdot d_{12}}{E_3 - E_2} - \frac{g \cdot d_{13}}{E_3 - E_1} \right)[P^\dagger B_4^\dagger]^0[B_1^\dagger B_2^\dagger]^0|0\rangle$$

$$+ \left(d_1 - \sum_k \frac{g \cdot d_1}{2\epsilon_k - E_4} - \frac{g \cdot d_{123}}{E_3 - E_4} - \frac{g \cdot d_{12}}{E_4 - E_1} - \frac{g \cdot d_{13}}{E_4 - E_2} \right)[P^\dagger B_3^\dagger]^0[B_1^\dagger B_2^\dagger]^0|0\rangle$$

$$+ \cdots$$

being: $d_{ij} \equiv d_i + d_j, \quad d_{123} \equiv d_1 + d_2 + d_3$

The general recipe for an even-even $N = Z$ system

- Adopt the collective pairs

$$B^{\dagger}_{\nu\tau} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P^{\dagger}_{k\tau}$$

as building blocks.

- Construct the states $|s\rangle$, product of $B^{\dagger}_{\nu}$'s arranged into $T = 0$ quartets,

$$|s\rangle = \prod_{q=1}^{N_q} [B^{\dagger}_{\nu(1,q,s)} B^{\dagger}_{\nu(2,q,s)}]^{T=0} |0\rangle,$$

such that the space $\{|s\rangle\}$ be invariant under the interchange of any two pairs.

- Expand $|\Psi\rangle$ into this basis: $\quad |\Psi\rangle = \sum_{s=1}^{N_s} d_s |s\rangle$
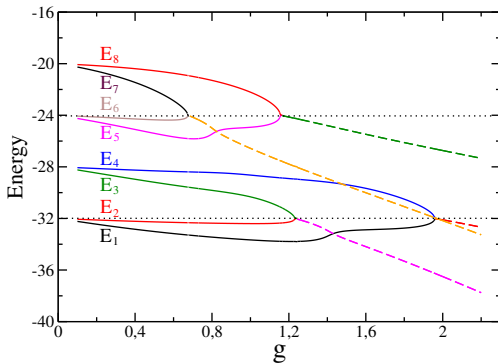
- Solve the set of equations

$$\frac{d_s}{g} - \sum_{k=1}^{\Omega} \frac{d_s}{2\epsilon_k - E_\nu} - \sum_{\nu' \neq \nu}^{(1,2N_q)} \frac{S_{\nu'\nu}(s)}{E_{\nu'} - E_\nu} = 0, \quad S_{\nu'\nu}(s) = \sum_t I(t, \nu', \nu, s) d_t$$

This guarantees that

$$H^{(iv)} |\Psi\rangle = (\sum_{\nu} E_\nu) |\Psi\rangle$$

An example of numerical results

Pair energies for the ground state of a system of
8 protons and 8 neutrons over 8 equispaced levels

M.S. and N. Sandulescu, J. Phys. G: Nucl. Phys. 47 (2020) 045112

Conclusions (1st part)

► The exact $T = 0$ seniority-zero eigenstates of a constant-strength isovector pairing Hamiltonian are linear superpositions of products of quartets.

► Quartets are the distictive features of these eigenstates.

► The isovector pairing Hamiltonian favours the formation of α-like structures in $N = Z$ nuclei.

Previous exact treatments of the isovector pairing Hamiltonian

- R.W. Richardson, Phys. Rev. **144**, 874 (1966)
  H.-T. Chen and R.W. Richardson, Phys. Lett. B **34**, 271 (1971)
  H.-T. Chen and R.W. Richardson, Nucl. Phys. A **212**, 317 (1973)

- Feng Pan and J.P. Draayer, Phys. Rev. C **66**, 044314 (2002)

- J. Links, H.-Q. Zhou, M.D. Gould, and R.H. McKenzie, J. Phys. A **35**, 6459 (2002)

- J. Dukelsky, V.G. Gueorguiev, P. Van Isacker, S. Dimitrova, B. Errea, and S. Lerma H., Phys. Rev. Lett. **96**, 072503 (2006)

Quartet condensation in $N = Z$ nuclei

The Quartet Condensation Model (QCM) assumes that

$$|\Psi_{gs}\rangle = (Q^+)^{n_q}|0\rangle$$

For the isovector pairing Hamiltonian discussed above
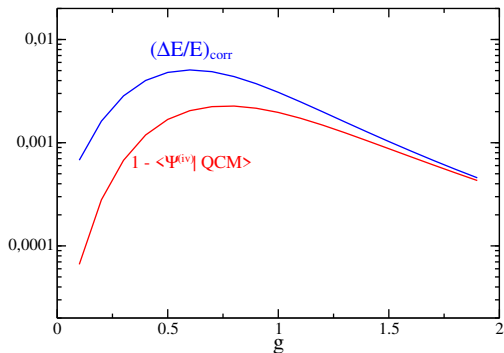
$$Q^+ = \sum_{ij} x_{ij}[P_i^\dagger P_j^\dagger]^{T=0}$$

with

$$P_{iM_T}^\dagger = [a_{i+}^\dagger a_{i-}^\dagger]_{M_T}^{T=1}$$

N. Sandulescu et al., PRC 85 (2012) 061303(R)

Validity of the QCM approximation for the IV pairing Hamiltonian

System of 6 protons and 6 neutrons over 6 equispaced levels

QCM based excited states of the IV pairing Hamiltonian: formalism

Excited states of the isovector pairing Hamiltonian are built on the QCM ground state as follows:
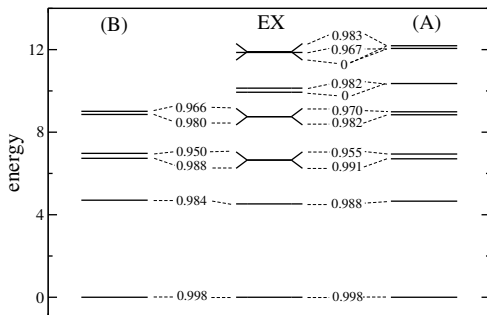
$$|\Phi_\nu\rangle = \tilde{Q}_\nu^+ (Q^+)^{n_q-1}|0\rangle$$

with

$$\tilde{Q}_\nu^+ = \sum_{ij} y_{ij}^{(\nu)} [P_i^+ P_j^+]^{T=0}$$

QCM based excited states of the IV pairing Hamiltonian: application

Spectrum of the isovector pairing Hamiltonian for a system
of 6 protons and 6 neutrons over 6 equispaced levels
$(g = 1)$

The case of a general isovector-isoscalar pairing Hamiltonian

$$H = \sum_i \epsilon_i N_i + \sum_{i,j} V_{J=0}^{T=1}(i,j) \sum_{T_z} P_{i,T_z}^+ P_{j,T_z} + \sum_{i \leq j, k \leq l} V_{J=1}^{T=0}(ij,kl) \sum_{J_z} D_{ij,J_z}^+ D_{kl,J_z}$$

$$P_{i,T_z}^+ = \sqrt{\frac{2j_i+1}{2}}[a_i^+ a_i^+]_{T_z}^{T=1,J=0}, \qquad D_{j_1 j_2 J_z}^+ = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}}[a_{j_1}^+ a_{j_2}^+]_{J_z}^{J=1,T=0}$$

▶ The QCM ground state:

$$|\Psi_{gs}\rangle = (Q_{ivs}^+)^{n_q}|0\rangle, \qquad Q_{ivs}^+ = Q_{iv}^+ + Q_{is}^+$$

$$Q_{iv}^+ = \sum_{ij} x_{ij}[P_i^+ P_j^+]^{T=0}, \qquad Q_{is}^+ = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4}[D_{j_1 j_2}^+ D_{j_3 j_4}^+]^{J=0}$$

▶ The excited states built on the QCM ground state

$$|\Phi_{\nu,JJ_z}\rangle = \tilde{Q}_{\nu,JJ_z}(Q_{ivs}^+)^{n_q-1}|0\rangle$$

$$\tilde{Q}_{\nu,JJ_z}^+ = \sum_{T'} \sum_{J_1(i_1 j_1)} \sum_{J_2(i_2 j_2)} Y_{JJ_z}^{(\nu)}(T', J_1(i_1 j_1), J_2(i_2 j_2))[P_{J_1,T'}^+(i_1,j_1)P_{J_2,T'}^+(i_2,j_2)]_{J_z}^{J,T=0}$$

The case of a general isovector-isoscalar pairing Hamiltonian

$$H = \sum_i \epsilon_i N_i + \sum_{i,j} V_{J=0}^{T=1}(i,j) \sum_{T_z} P_{i,T_z}^+ P_{j,T_z} + \sum_{i \leq j, k \leq l} V_{J=1}^{T=0}(ij,kl) \sum_{J_z} D_{ij,J_z}^+ D_{kl,J_z}$$

$$P_{i,T_z}^+ = \sqrt{\frac{2j_i+1}{2}} [a_i^+ a_i^+]_{T_z}^{T=1,J=0}, \qquad D_{j_1 j_2 J_z}^+ = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} [a_{j_1}^+ a_{j_2}^+]_{J_z}^{J=1,T=0}$$

▶ The QCM ground state:

$$|\Psi_{gs}\rangle = (Q_{ivs}^+)^{n_q}|0\rangle, \qquad Q_{ivs}^+ = Q_{iv}^+ + Q_{is}^+$$

$$Q_{iv}^+ = \sum_{ij} x_{ij} [P_i^+ P_j^+]^{T=0}, \qquad Q_{is}^+ = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4} [D_{j_1 j_2}^+ D_{j_3 j_4}^+]^{J=0}$$

▶ The excited states built on the QCM ground state

$$|\Phi_{\nu,JJ_z}\rangle = \tilde{Q}_{\nu,JJ_z}(Q_{ivs}^+)^{n_q-1}|0\rangle$$

$$\tilde{Q}_{\nu,JJ_z}^+ = \sum_{T'} \sum_{J_1(i_1 j_1)} \sum_{J_2(i_2 j_2)} Y_{JJ_z}^{(\nu)}(T', J_1(i_1 j_1), J_2(i_2 j_2)) [P_{J_1,T'}^+(i_1,j_1) P_{J_2,T'}^+(i_2,j_2)]_{J_z}^{J,T=0}$$

The case of a general isovector-isoscalar pairing Hamiltonian

$$H = \sum_i \epsilon_i N_i + \sum_{i,j} V_{J=0}^{T=1}(i,j) \sum_{T_z} P_{i,T_z}^+ P_{j,T_z} + \sum_{i \leq j, k \leq l} V_{J=1}^{T=0}(ij,kl) \sum_{J_z} D_{ij,J_z}^+ D_{kl,J_z}$$

$$P_{i,T_z}^+ = \sqrt{\frac{2j_i+1}{2}} [a_i^+ a_i^+]_{T_z}^{T=1,J=0}, \qquad D_{j_1 j_2 J_z}^+ = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} [a_{j_1}^+ a_{j_2}^+]_{J_z}^{J=1,T=0}$$

▶ The QCM ground state:

$$|\Psi_{gs}\rangle = (Q_{ivs}^+)^{n_q}|0\rangle, \qquad Q_{ivs}^+ = Q_{iv}^+ + Q_{is}^+$$

$$Q_{iv}^+ = \sum_{ij} x_{ij} [P_i^+ P_j^+]^{T=0}, \qquad Q_{is}^+ = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4} [D_{j_1 j_2}^+ D_{j_3 j_4}^+]^{J=0}$$
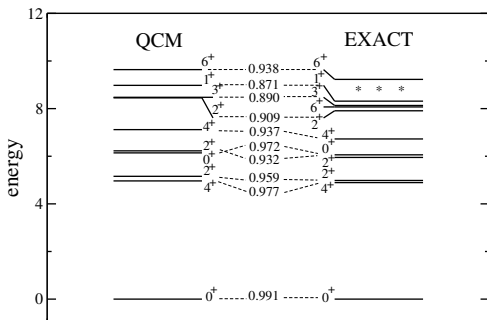
▶ The excited states built on the QCM ground state

$$|\Phi_{\nu,JJ_z}\rangle = \tilde{Q}_{\nu,JJ_z} (Q_{ivs}^+)^{n_q-1}|0\rangle$$

$$\tilde{Q}_{\nu,JJ_z}^+ = \sum_{T'} \sum_{J_1(i_1 j_1)} \sum_{J_2(i_2 j_2)} Y_{JJ_z}^{(\nu)}(T', J_1(i_1 j_1), J_2(i_2 j_2)) [P_{J_1,T'}^+(i_1,j_1) P_{J_2,T'}^+(i_2,j_2)]_{J_z}^{J,T=0}$$

QCM based approximation for a general IV + IS pairing Hamiltonian

Spectrum of the isovector + isoscalar pairing Hamiltonian
for a system of 6 protons and 6 neutrons with
s.p.e.'s and $V_{J=0}^{T=1}(i,j)$, $V_{J=1}^{T=0}(i,j)$ from USDB

Conclusions (2nd part)

- The ground state of an isovector plus isoscalar pairing Hamiltonian in $N = Z$ systems can be well described as a condensate of a $T = 0, J = 0$ quartet built by isovector plus isoscalar pairs.

- The low-lying excited states of this Hamiltonian can be constructed by promoting one of the quartets of the condensate to an excited ($T = 0$) configuration.

- The proton-neutron pairing favours the formation of α-like quartet condensates in $N = Z$ systems.

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M$$

2) How to fix them

▶ "statically": from the nearest $T = 0$ one-quartet system
(M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)

▶ "dynamically": from intrinsic states

3) What to do with them

▶ Configuration interaction calculations in the space of quartets

▶ Projecting the intrinsic states

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M$$

2) How to fix them

▶ "statically": from the nearest $T = 0$ one-quartet system
(M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)

▶ "dynamically": from intrinsic states

3) What to do with them

▶ Configuration interaction calculations in the space of quartets

▶ Projecting the intrinsic states

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M$$

2) How to fix them

▶ "statically": from the nearest $T = 0$ one-quartet system
(M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)

▶ "dynamically": from intrinsic states

3) What to do with them

▶ Configuration interaction calculations in the space of quartets

▶ Projecting the intrinsic states

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q_{JM}^+ = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a_{i_1}^+ a_{j_1}^+]^{J_1 T'} [a_{i_2}^+ a_{j_2}^+]^{J_2 T'}]_M^{JT=0}$$

2) How to fix them
- "statically": from the nearest $T = 0$ one-quartet system
  (M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)
- "dynamically": from intrinsic states

3) What to do with them
- Configuration interaction calculations in the space of quartets
- Projecting the intrinsic states

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M$$

2) How to fix them
  ▶ "statically": from the nearest $T = 0$ one-quartet system
     (M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)
  ▶ "dynamically": from intrinsic states

3) What to do with them
  ▶ Configuration interaction calculations in the space of quartets
  ▶ Projecting the intrinsic states

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q_{JM}^+ = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a_{i_1}^+ a_{j_1}^+]^{J_1 T'} [a_{i_2}^+ a_{j_2}^+]^{J_2 T'}]_M^{JT=0}$$

2) How to fix them
   - ▶ "statically": from the nearest $T = 0$ one-quartet system
     (M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)
   - ▶ "dynamically": from intrinsic states

3) What to do with them
   - ▶ Configuration interaction calculations in the space of quartets
   - ▶ Projecting the intrinsic states

Intrinsic states of deformed $N = Z$ nuclei in a formalism of quartets

▶ "ground" intrinsic state

$$|\Theta_g\rangle = \mathcal{N}_g (Q_g^+)^n |0\rangle, \qquad Q_g^+ = \sum_J \alpha_{g,J} (q_g^+)_{J0}$$

$$(q_g^+)_{J0} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'}^{(g)} [[a_{i_1}^+ a_{j_1}^+]^{J_1 T'} [a_{i_2}^+ a_{j_2}^+]^{J_2 T'}]_0^{JT=0}$$

▶ "excited" intrinsic states

$$|\Theta_k\rangle = \mathcal{N}_k Q_k^\dagger (Q_g^\dagger)^{(n-1)} |0\rangle, \qquad Q_k^\dagger = \sum_J \alpha_{k,J} (q_k^\dagger)_{Jk}$$

$$(q_k^+)_{Jk} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'}^{(k)} [[a_{i_1}^+ a_{j_1}^+]^{J_1 T'} [a_{i_2}^+ a_{j_2}^+]^{J_2 T'}]_k^{JT=0}$$

$$k = 0 \to \text{"}\beta\text{"}, \ k = 2 \to \text{"}\gamma\text{"}, \cdots$$

Intrinsic states of deformed $N = Z$ nuclei in a formalism of quartets

▶ "ground" intrinsic state

$$|\Theta_g\rangle = \mathcal{N}_g (Q_g^+)^n |0\rangle, \qquad Q_g^+ = \sum_J \alpha_{g,J} (q_g^+)_{J0}$$

$$(q_g^+)_{J0} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'}^{(g)} [[a_{i_1}^+ a_{j_1}^+]^{J_1 T'} [a_{i_2}^+ a_{j_2}^+]^{J_2 T'}]_0^{JT=0}$$

▶ "excited" intrinsic states

$$|\Theta_k\rangle = \mathcal{N}_k Q_k^\dagger (Q_g^\dagger)^{(n-1)} |0\rangle, \qquad Q_k^\dagger = \sum_J \alpha_{k,J} (q_k^\dagger)_{Jk}$$

$$(q_k^+)_{Jk} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'}^{(k)} [[a_{i_1}^+ a_{j_1}^+]^{J_1 T'} [a_{i_2}^+ a_{j_2}^+]^{J_2 T'}]_k^{JT=0}$$

$$k = 0 \to \text{``}\beta\text{''}, \ k = 2 \to \text{``}\gamma\text{''}, \cdots$$

Generating spectra of $N = Z$ nuclei

Two ways of proceedings:

- ▶ Via configuration interaction calculations in the space of quartets q_{JM}^+

$$|\Psi_{\overline{M}}^{(n)}, \{N_{JM}\}\rangle = \prod_{J\in(0,J_{max}); M\in(-J,J)} (q_{JM}^+)^{N_{JM}}|0\rangle$$

$$\sum_{JM} N_{JM} = n, \qquad \sum_{JM} MN_{JM} = \overline{M}$$

M. S. and N. Sandulescu, PLB 827 (2022) 136987

- ▶ Via projection
  - ▶ 1) projecting states of good angular momentum from the intrinsic states:

$$\hat{P}_J|\Theta_g\rangle$$

$$\hat{P}_J|\Theta_k\rangle, \quad k = 0, 2, \cdots$$

  - ▶ 2) diagonalizing the Hamiltonian in the space of the projected states:

$$\{\hat{P}_J|\Theta_g\rangle, \hat{P}_J|\Theta_k\rangle\}$$

M. S. and N. Sandulescu, EPJA (2023), in press

Generating spectra of $N = Z$ nuclei

Two ways of proceedings:

- Via configuration interaction calculations in the space of quartets q^+_{JM}

$$|\Psi^{(n)}_{\overline{M}}, \{N_{JM}\}\rangle = \prod_{J \in (0, J_{max}); M \in (-J, J)} (q^+_{JM})^{N_{JM}} |0\rangle$$

$$\sum_{JM} N_{JM} = n, \qquad \sum_{JM} M N_{JM} = \overline{M}$$

M. S. and N. Sandulescu, PLB 827 (2022) 136987

- Via projection
  - 1) projecting states of good angular momentum from the intrinsic states:
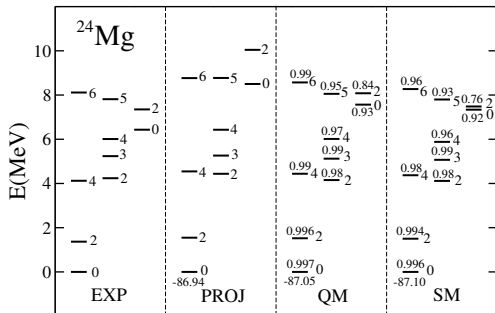
    $$\hat{P}_J |\Theta_g\rangle$$

    $$\hat{P}_J |\Theta_k\rangle, \quad k = 0, 2, \cdots$$

  - 2) diagonalizing the Hamiltonian in the space of the projected states:
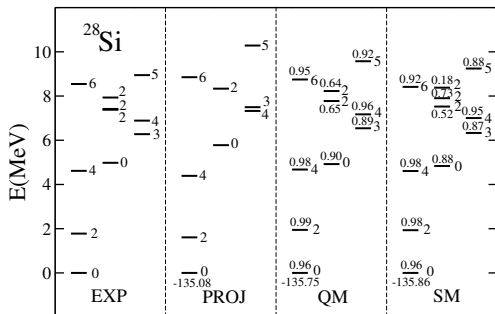
    $$\{\hat{P}_J |\Theta_g\rangle, \hat{P}_J |\Theta_k\rangle\}$$

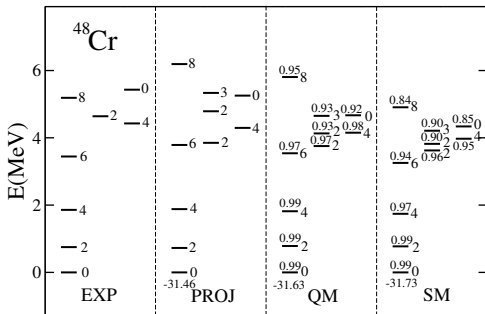   M. S. and N. Sandulescu, EPJA (2023), in press
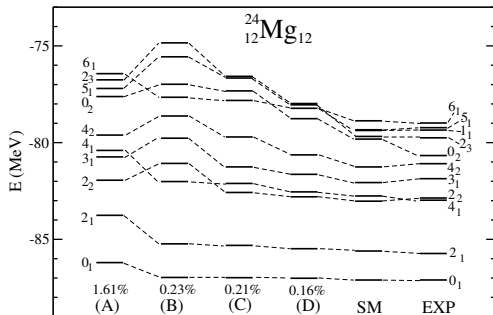
QM vs projection from intrinsic states: ^{24}Mg

Conclusions (3rd part)

- Quartet-based intrinsic states provide a framework to understand in a conceptually simple and intuitive manner the emergence of band-like structures in $N = Z$ nuclei.
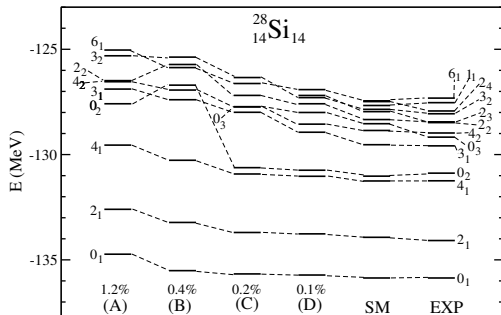
Future perspectives

- anomalous behaviour of elastic α-scattering on $N = Z$ nuclei

- superallowed α-decay in heavy $N = Z$ nuclei

QM results: ^{24}Mg

(A)$\rightarrow$ *static* : $J = 0, 2, 4$ (B)$\rightarrow |\Theta_g\rangle : J = 0, 2, 4$
(C)$\rightarrow |\Theta_2\rangle : J = 2, 3, 4$ (D)$\rightarrow |\Theta_0\rangle : J = 0, 2, 4$

(A)→ *static* : $J = 0, 2, 4$ (B)→ $|\Theta_g\rangle$: $J = 0, 2, 4$
(C)→ $|\Theta_0\rangle$: $J = 0, 2, 4$ (D)→ $|\Theta_3\rangle$: $J = 3, 4$

(A)$\rightarrow$ *static* : $J = 0, 2, 4, 6$ (B)$\rightarrow |\Theta_g\rangle : J = 0, 2, 4, 6$
(C)$\rightarrow |\Theta_2\rangle : J = 2, 3, 4$ (D)$\rightarrow |\Theta_0\rangle : J = 0, 2, 4$

QCM results in the sd shell

Using the USDB Hamiltonian:

	E_{corr}(SM)	E_{corr}(QCM)	$\langle SM \vert QCM \rangle$
^{20}Ne	24.77	24.77 (-)	1
^{24}Mg	55.70	53.04 (4.77%)	0.85
^{28}Si	88.75	86.52 (2.52%)	0.86
^{32}S	122.51	122.02 (0.40%)	0.98

M.S. and N. Sandulescu, EPJA 53, 47 (2017)