HALO, WEAKLY BOUND NUCLEI AND CLUSTERING

Summary of scientific activity of Padova unit and collaborators

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MONSTRE

INFN PADOVA

- Disclaimer: I will summarize the scientific activity in nuclear structure and reactions in Padova, relevant to the MONSTRE initiative, focussing on the title topic, but our unit expresses certainly more than what is represented here : I won’t talk about development of shell model interactions, shell evolution, mirror energy differences and the like, even though they form a good part of our research.

TOPICS:

- 29F and 31F halos, island of inversion and neutron correlations
- Algebraic cluster model and nuclear reactions (discrete symmetries)
- Reactions involving 12C and 16O
- Gamma-rays polarization experiments

Unsolved

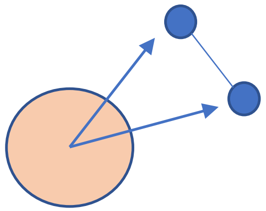
One of the big Unanswered question / Unsolved problem in nuclear physics:

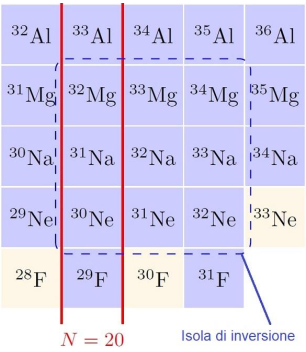
- How are cluster d.o.f. emerging from microscopic single-particle theories?
- How are molecular normal modes vanishing with larger A, leaving space for collective modes?
- How are pairing correlations affecting structure and reaction properties?

These questions are most certainly not new, but they are today still largely unanswered. A great deal of information is known, but even ab initio models still miss some crucial ingredient in order to grasp the physics of clusters.

Fluorine-29 stands on the coast of the island of inversion

COMMUNICATIONS PHYSICS





PERSPECTIVE

https://doi.org/10.1038/s42005-020-00402-5

OPEN

The ^{29}F nucleus as a lighthouse on the coast of the island of inversion

L. Fortunato [1,2]✉, J. Casal [1,2], W. Horiuchi [3], Jagjit Singh [4] & A. Vitturi[1,2]

Fluorine-29 stands on the coast of the island of inversion

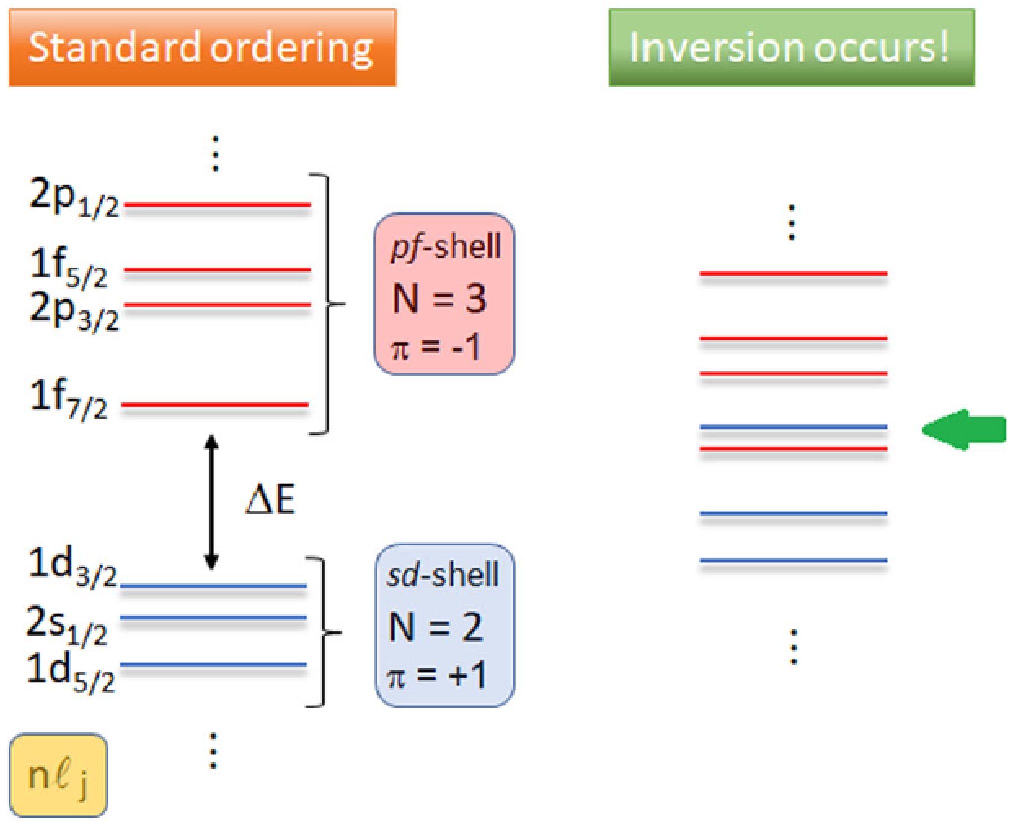


Fig. 1 Standard ordering of shell-model energy levels and typical inversion mechanism. The $N = 2$ *sd*-shell and the $N = 3$ *pf*-shell with positive and negative parity π, respectively, are shown on the left in the standard ordering (states are labeled by the standard set of quantum number $n\ell_j$). Inversion occur (right) when the shell gap, ΔE, associated with the filling of 20 neutrons, disappears and one level (or more) of the $N = 3$ *pf*-shell gets lower than one (or more) of the levels of the $N = 2$ *sd*-shell.

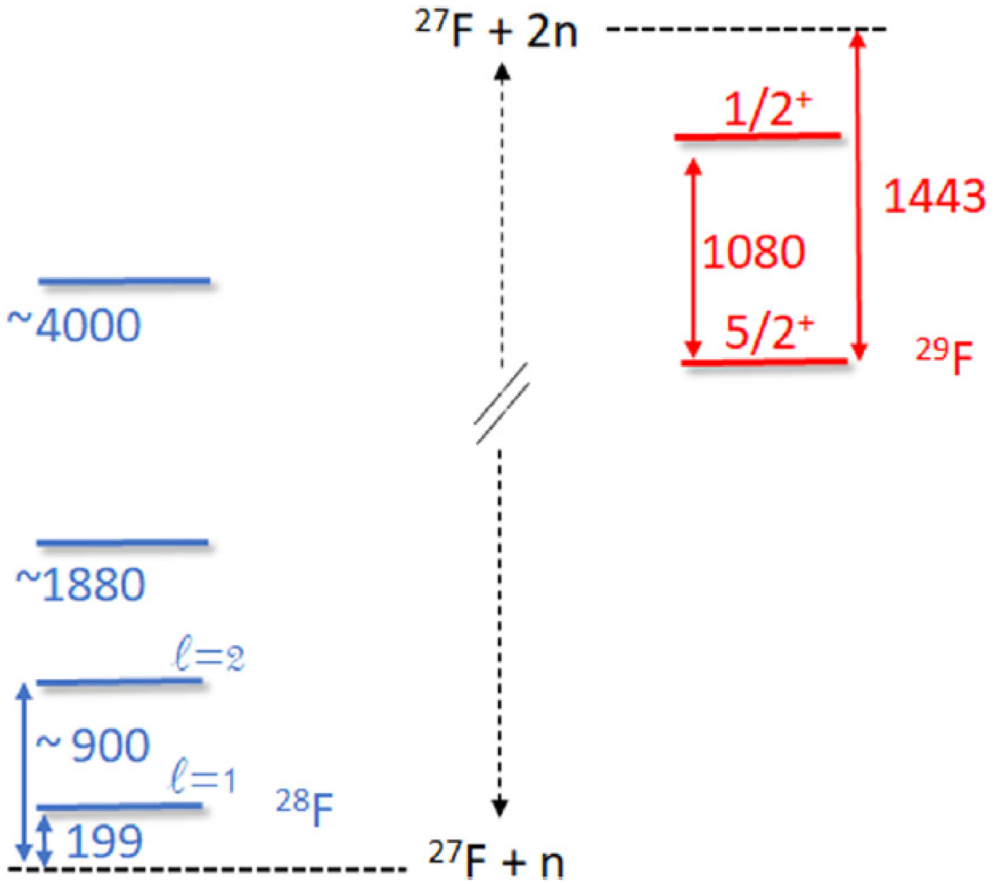
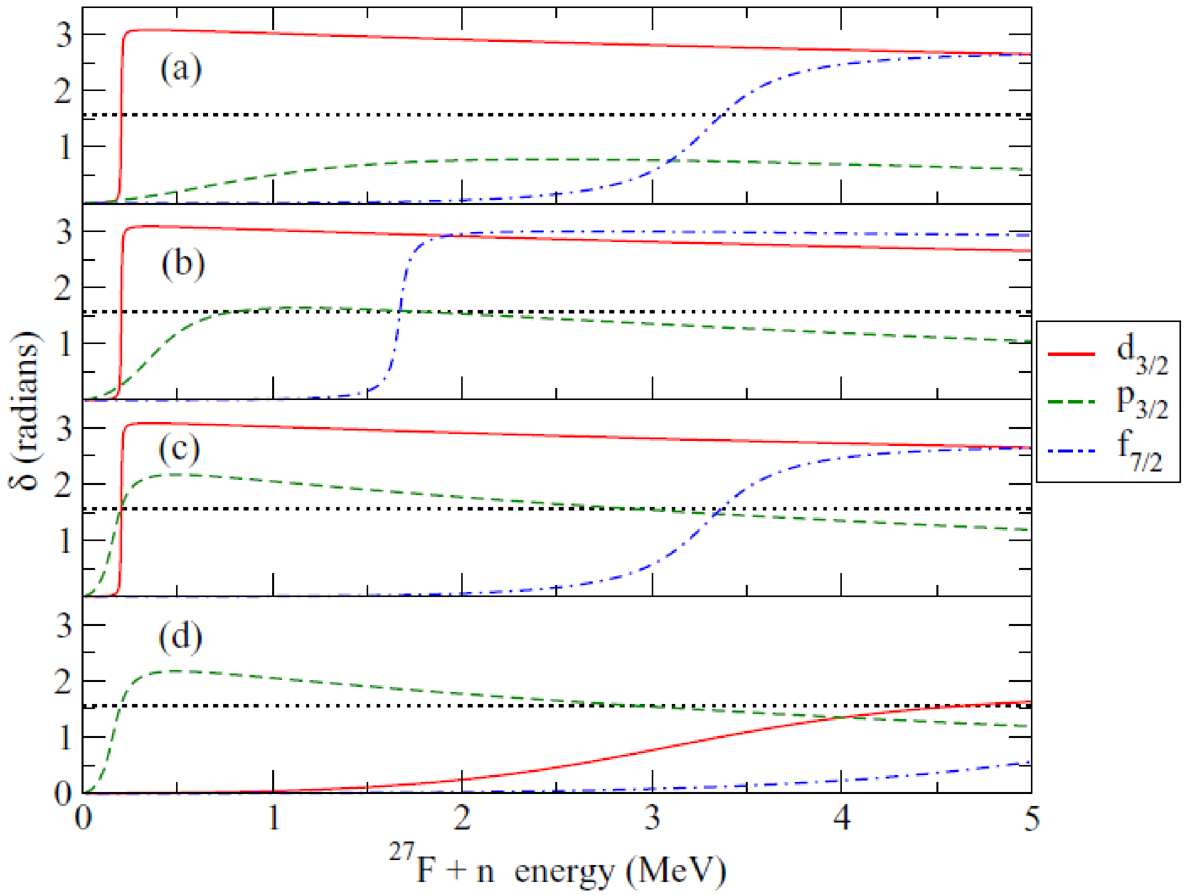


Fig. 2 Synopsis of known experimental data on ^{28,29}F. All energies are in keV (not to scale) from refs.[4–6,11]. States in red are labelled by the J^{π} quantum numbers and energies are referred to the ^{27}F + 2n threshold. States in blue are inferred from the ^{29}F(−1n) column of Fig. 2 of ref.[6], and correspond only to the states decaying to the ground state of ^{27}F. They are labelled by the orbital angular momentum quantum number, ℓ, when available. Energies are referred to the ^{27}F + n threshold.

Fluorine-29 stands on the coast of the island of inversion

We had previously PRC **101**, 024310 (2020) proposed 4 scenarios for the structure of the very exotic nucleus 29F, called A,B,C,D , based on the three-body hyperspherical formalism by J.Casal



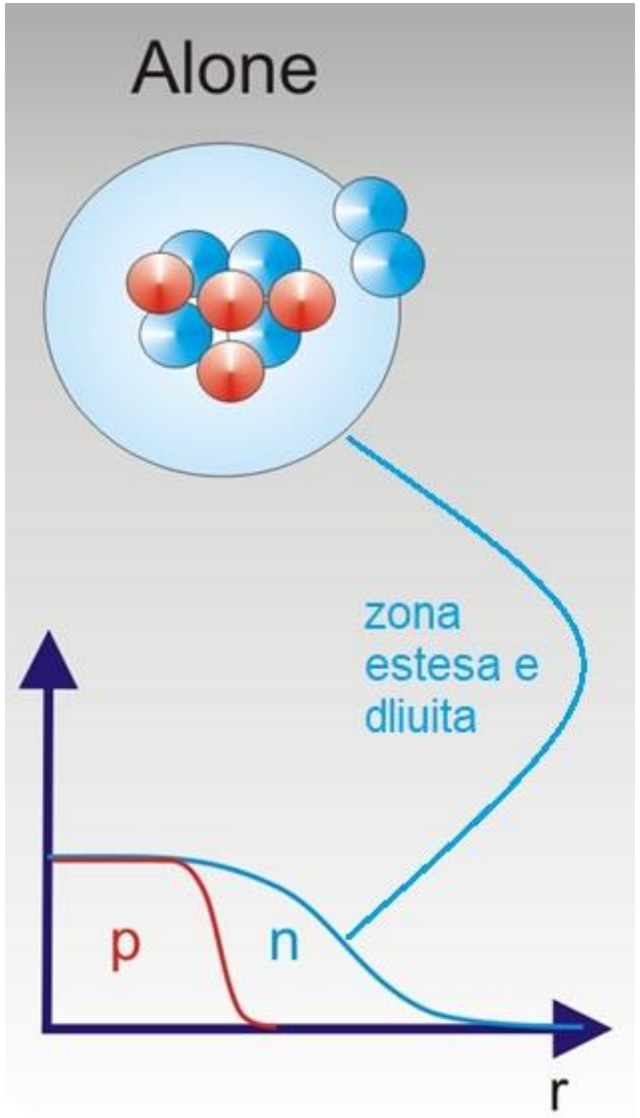
Standard

Intruder

Degenerated

Inverted

FIG. 1. $^{27}F + n$ phase shifts for $d_{3/2}$, $p_{3/2}$, and $f_{7/2}$ states, corresponding to different sets (A–D). The dotted black line corresponds to $\pi/2$.



Fluorine-29 stands on the coast of the island of inversion

New experiments by

1) Revel, A. et al. "Extending the southern shore of the island of inversion to 28F" PRL 124, 152502 (2020)

and then

2) Bagchi, S. et al. "Two-neutron halo is unveiled in 29F" PRL 124, 222504 (2020)

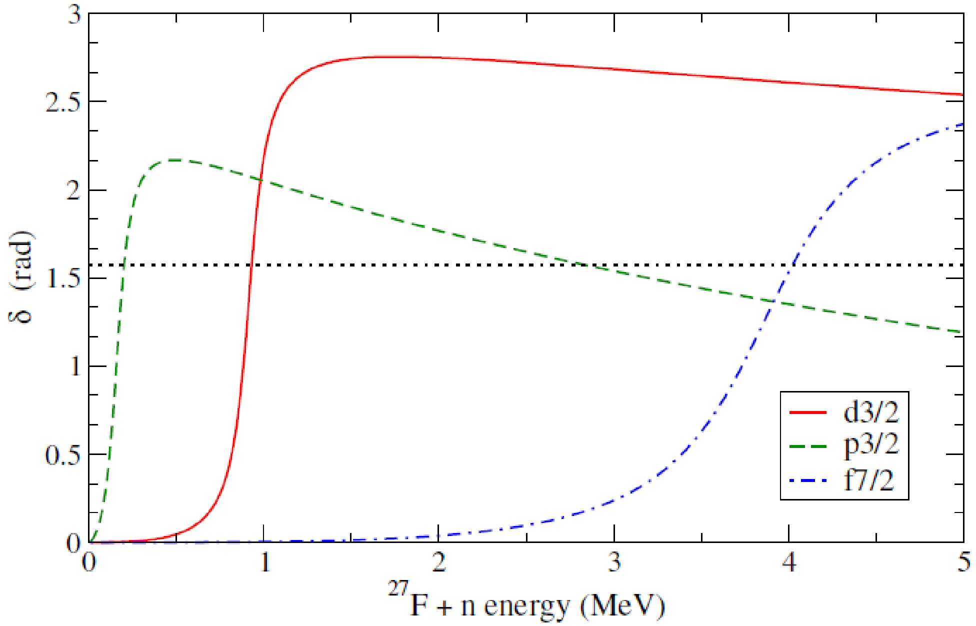
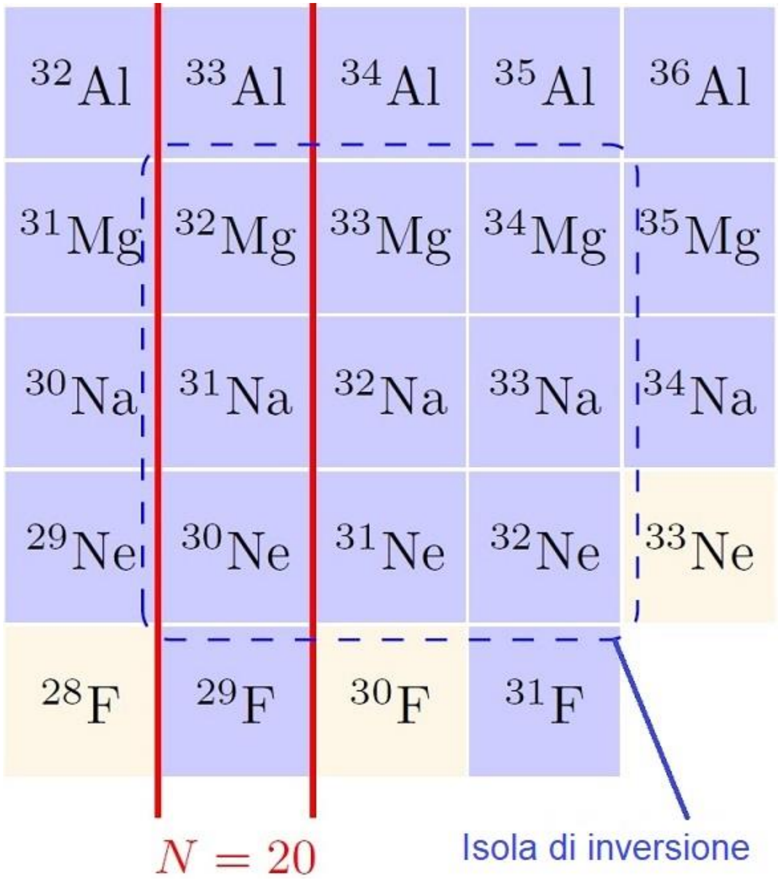
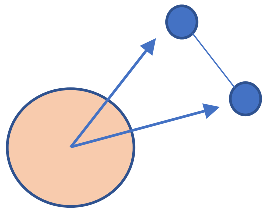


Fig. 3 Phase-shifts, δ, for the $^{27}F + n$ system in the D$^\flat$ scenario as a function of the neutron-core relative energy. Adjusting the red curve to reproduce the *d*-resonance at about 0.9 MeV, also gives the *f*-wave state (blue curve) at about 4 MeV.



Final picture … Scenario D ♭

Fluorine-29 stands on the coast of the island of inversion



$$\frac{d\sigma}{d\varepsilon} = (\alpha Z_2)^2 \sum_{\pi\lambda\mu} \left(\frac{\Delta E}{\hbar c}\right)^{2\lambda-2} \frac{dB}{d\varepsilon}(\pi\lambda; \mathrm{gs} \to \mathrm{Cont.}) |G_{\pi\lambda\mu}(\beta^{-1})|^2 g_\mu(\xi)$$

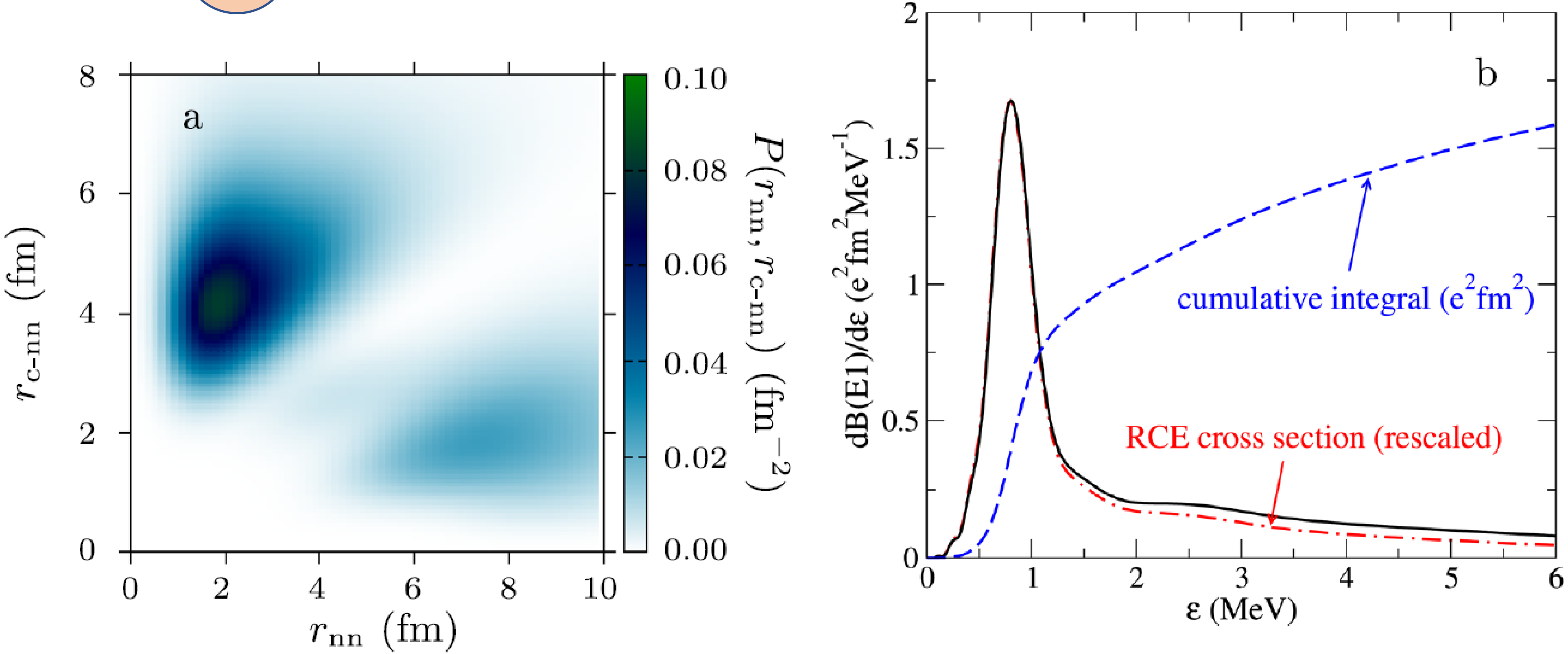


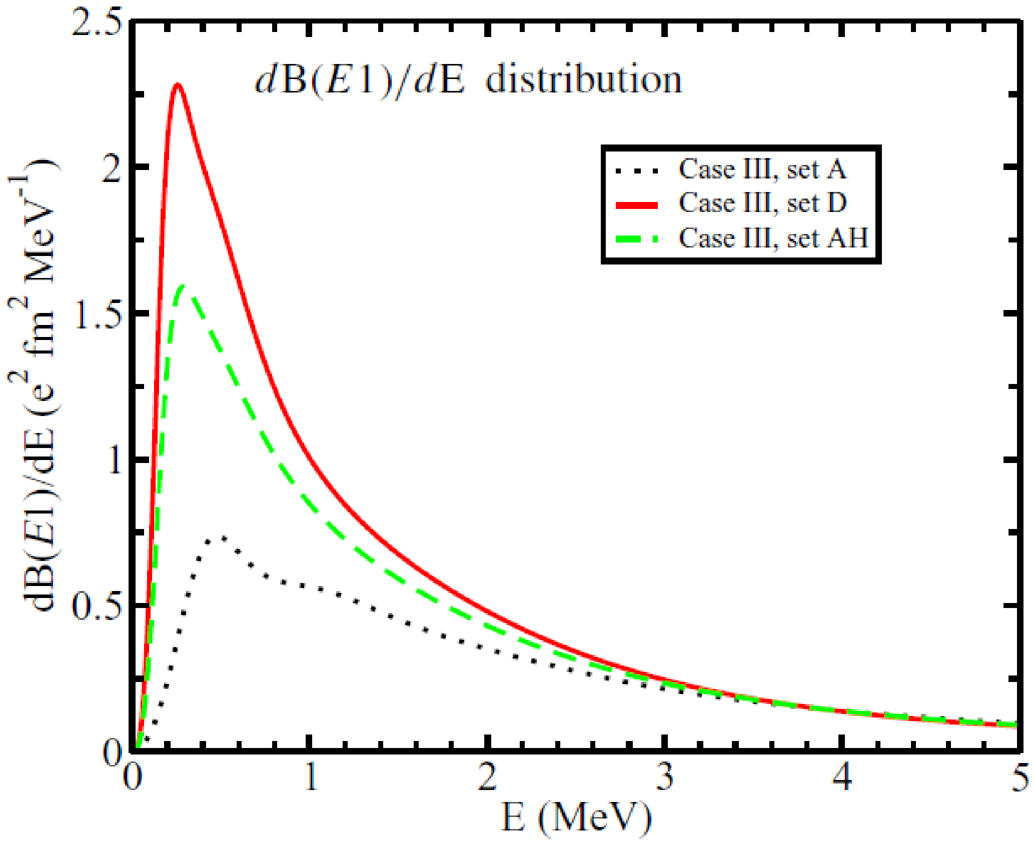
Fig. 4 Results on ^{29}F within the new $D^\flat$ scenario. a Ground-state probability density of ^{29}F as a function of the distance between the two valence neutrons (r_{nn}) and that between their center of mass and the core ($r_{\text{c-nn}}$). The maximum probability density corresponds to the dineutron configuration. **b** Electric dipole (*E*1) strength function from the ground state to continuum as a function of the ^{27}F + n + n energy. The dashed line indicates the cumulative integral. The dash-dotted line is the corresponding Relativistic Coulomb Excitation (RCE) cross-section, scaled to the same maximum to illustrate the decreasing proportionality with the energy.

Next in line : Fluorine-31

PHYSICAL REVIEW C **105**, 014328 (2022)

Exploring the halo character and dipole response in the dripline nucleus ^{31}F

G. Singh[1,2,*] Jagjit Singh[3,4,†] J. Casal[5,‡] and L. Fortunato[1,2,§]



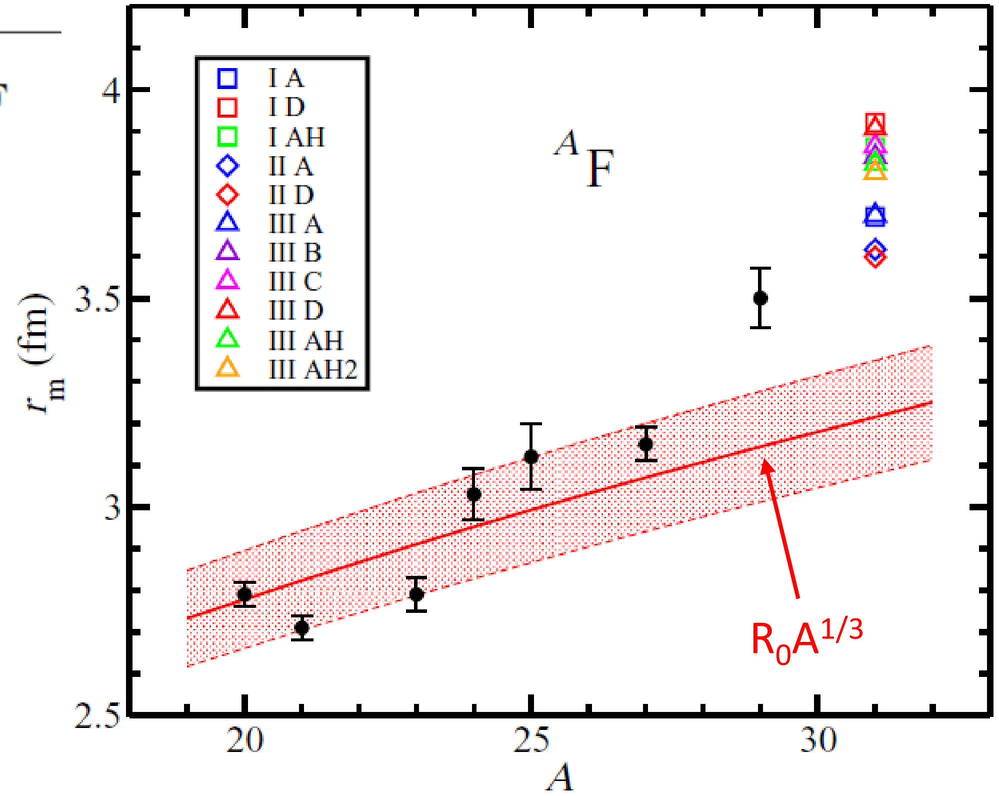
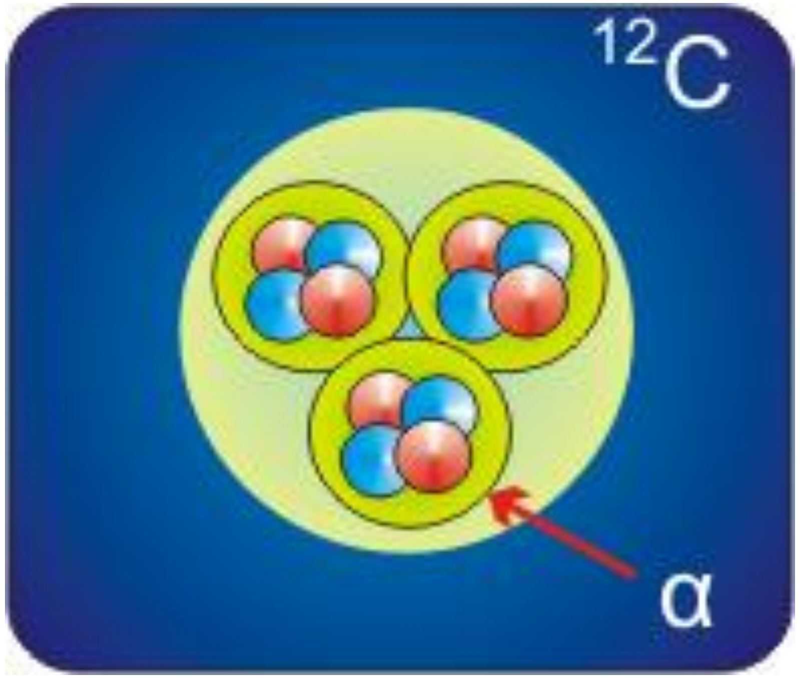


FIG. 5. Variation of matter radius of fluorine isotopes with mass

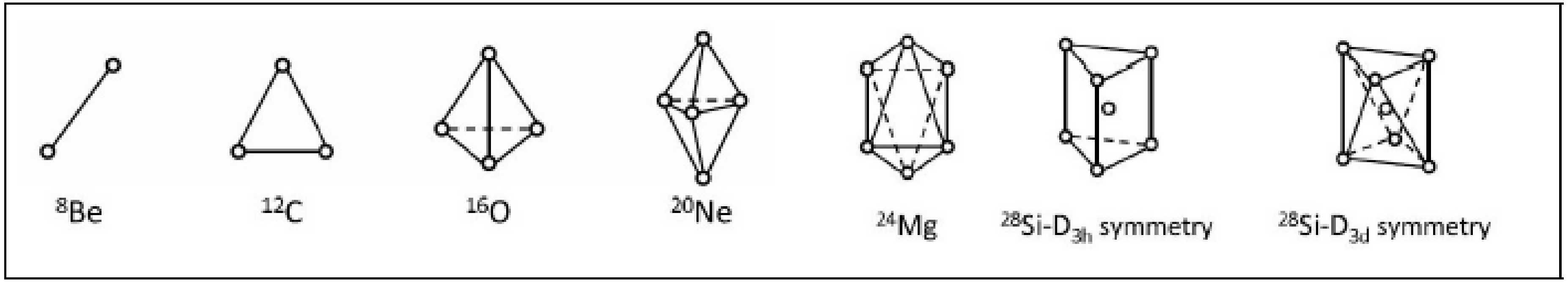
Large spatial extention for 31F (due to p3/2 components) and large B(E1) strength, indicate the probable formation of a halo.

Molecular models of the nucleus



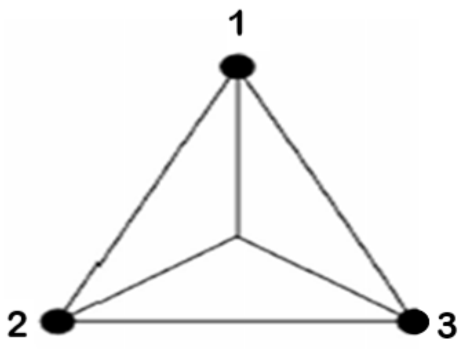
The common idea behind many cluster models is that even-even alpha-conjugate nuclei are seen as **molecular structures** made up of point-like objects that can perform **vibrations and rotations** around geometrical **equilibrium configurations**. These equilibrium shapes can be found minimizing the total PES built from alpha-alpha potentials and should be **understood in a dynamical sense**: unlike ions in ordinary rigid molecules that stay close to the equilibrium configurations, the alpha particles inside the nucleus have **large fluctuations** (translational and rotational motions). This is an intrinsic feature due to dimensions and energies involved in the nuclear domain. The point-group symmetry that is relevant in the case of 3 alpha particles at the vertexes of an equilater triangle is:

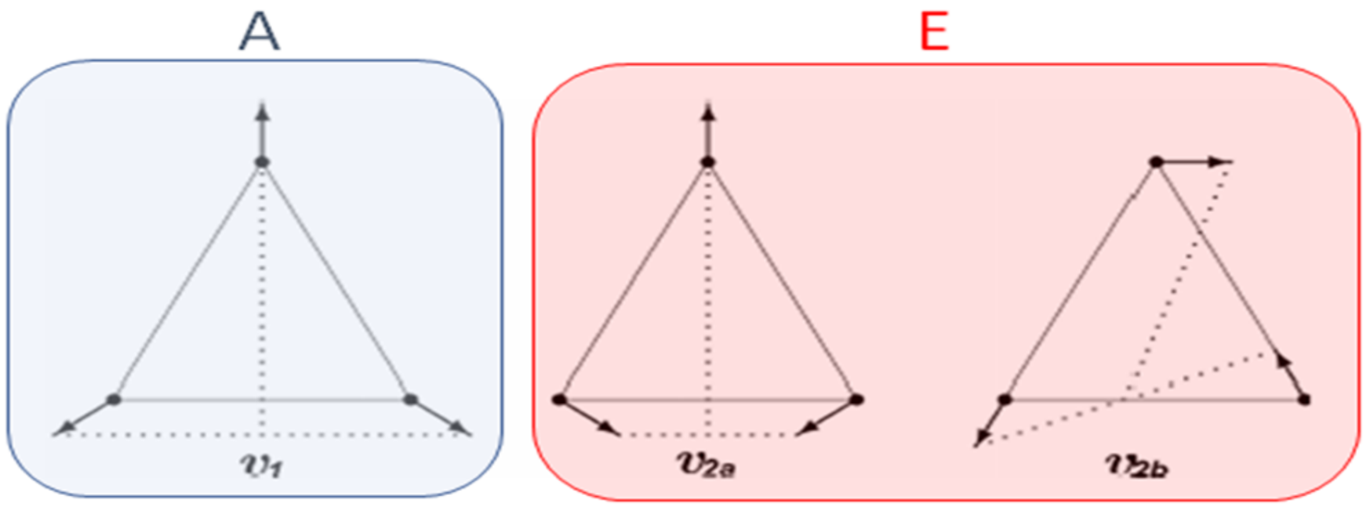
D_{3h}



Algebraic cluster model

R.Bijker and F.Iachello have demonstrated the occurence of alpha cluster states in 12C and 16O based on the discrete point-group symmetries and the Algebraic Cluster Model, where the language of representations of discrete groups is used to classify states (bands).





Vibrations of an oblate top.

Quantum numbers → vibrations and rotations

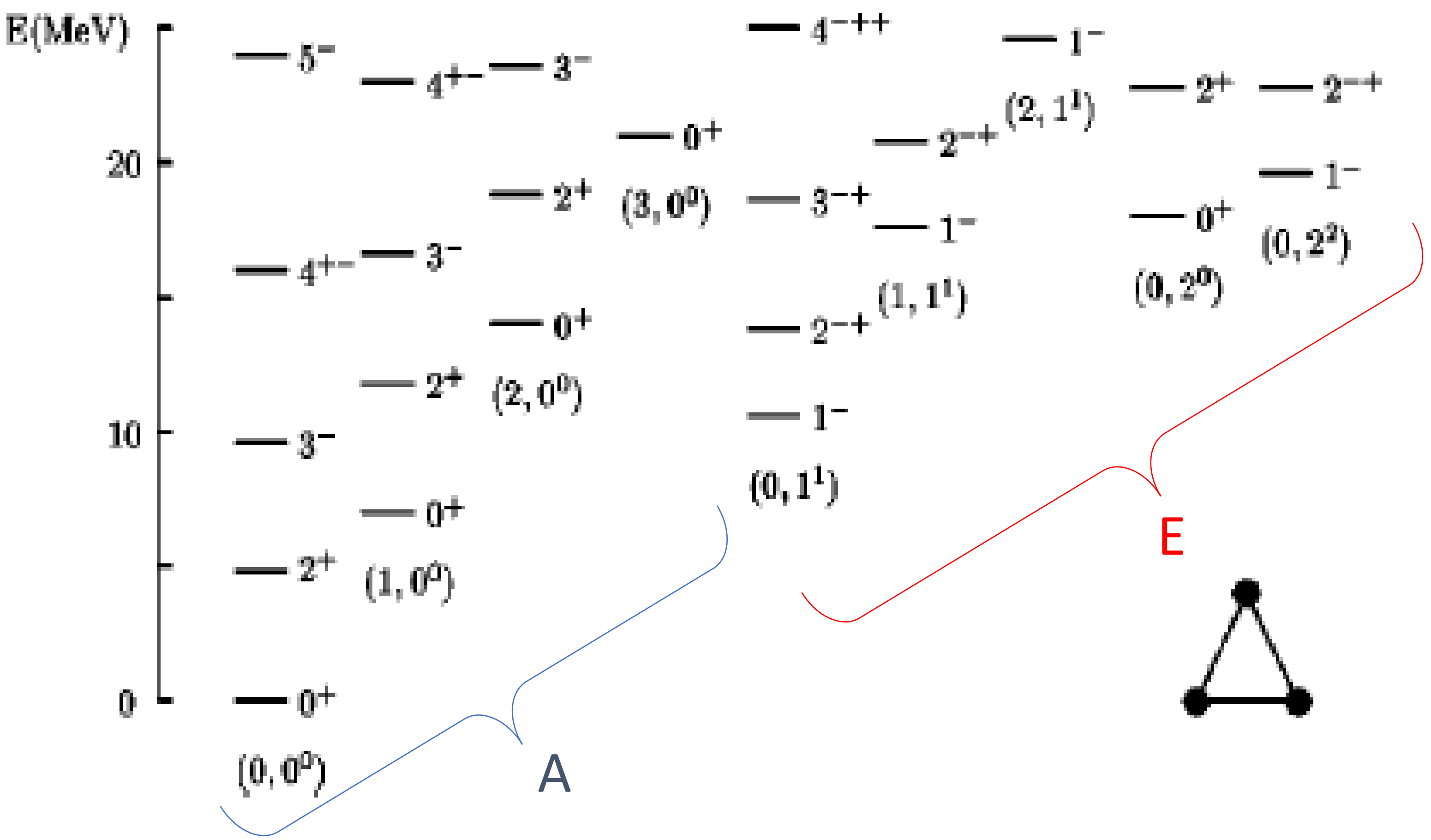
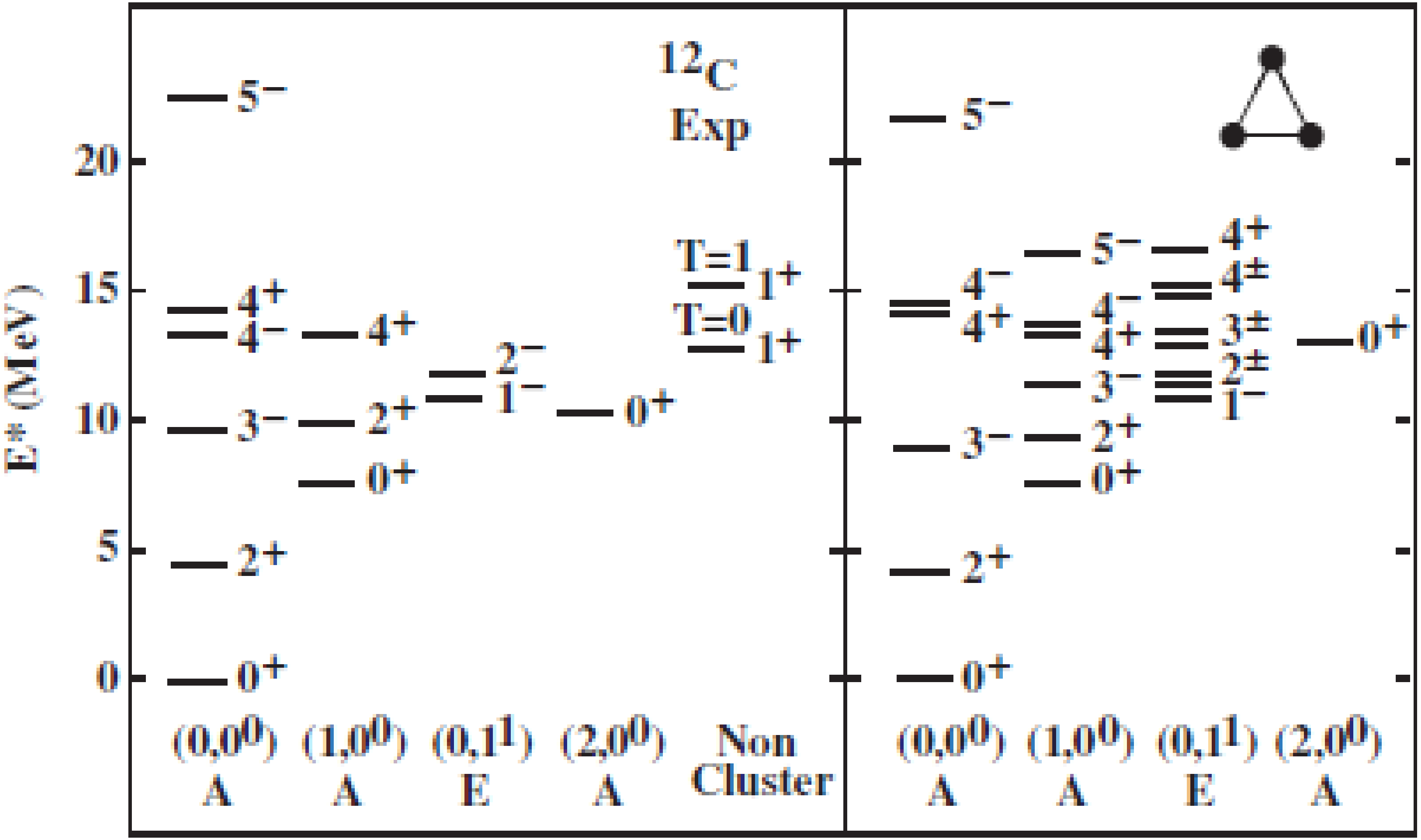


FIG. 1. Spectrum of an equilateral triangle configuration

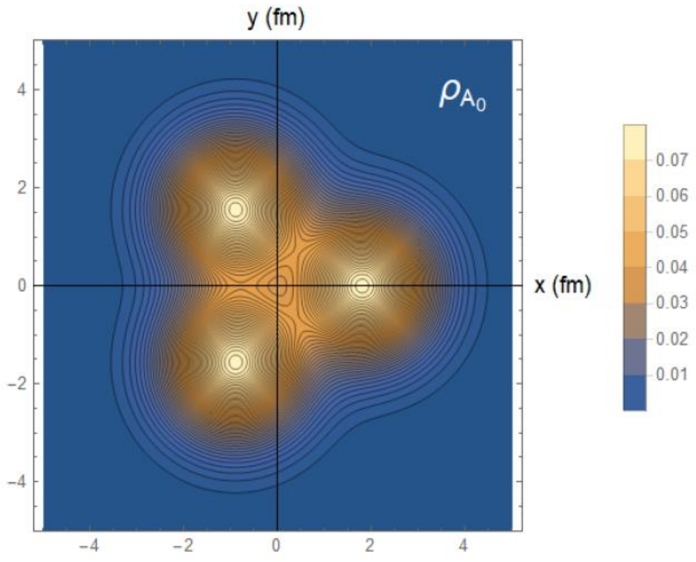
Notice the ‘apparently strange’ quantum numbers. They have a perfectly clear interpretation in the theory of point-groups !

L. Fortunato

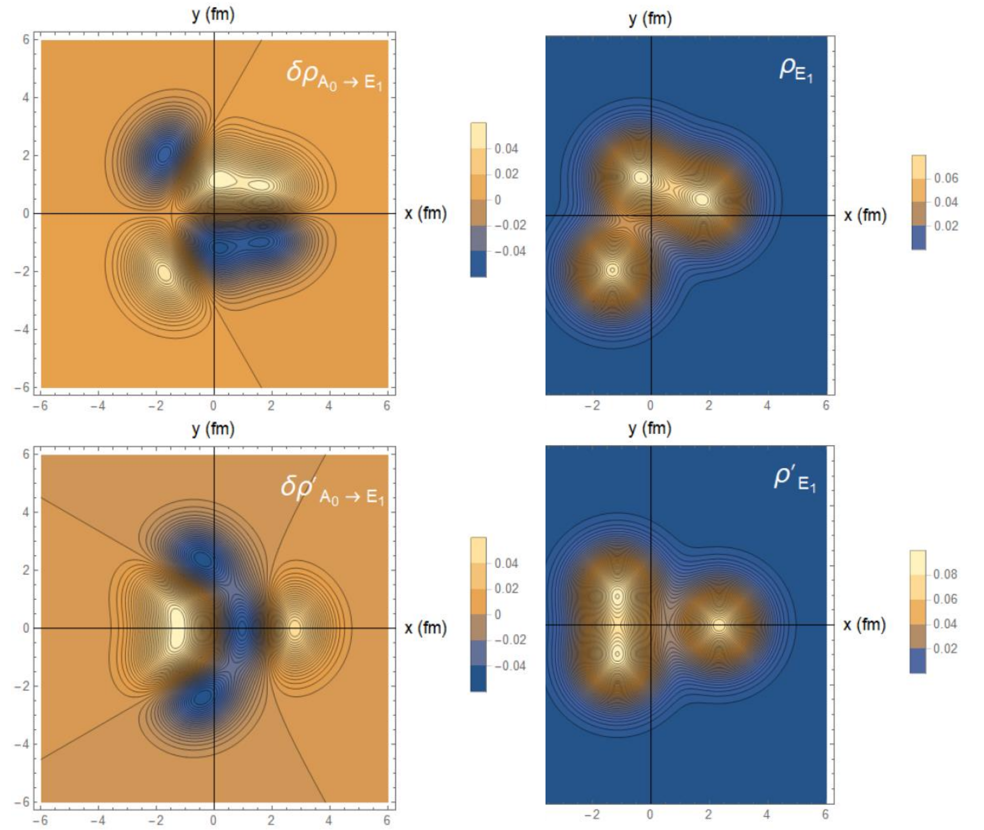
Bijker and Iachello have clearly demonstrated the succesfull application of the ACM, or algebraic cluster model, to the vibrational-rotational spectrum of alpha-alpha conjugate nuclei like 12C and 16O.



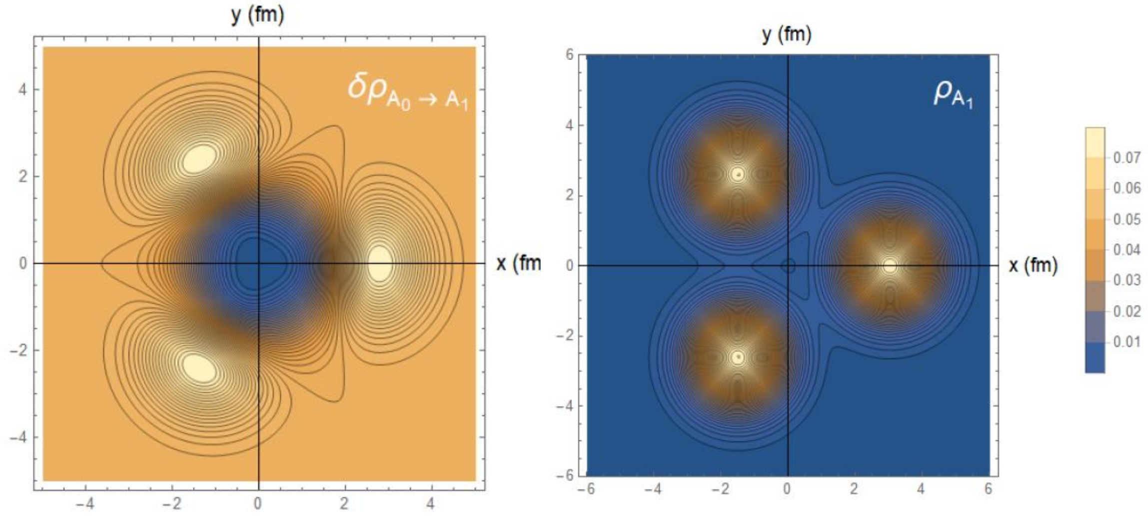
Transition densities in the cluster model of 12C



Ground state density



E-type vibration



A-type vibration (Hoyle)

From
Vitturi, Casal, Fortunato, Lanza,
PRC **101**, 014315 (2020)

L. Fortunato

Transition densities -> Form Factors -> Coupled Channels -> Cross-sections

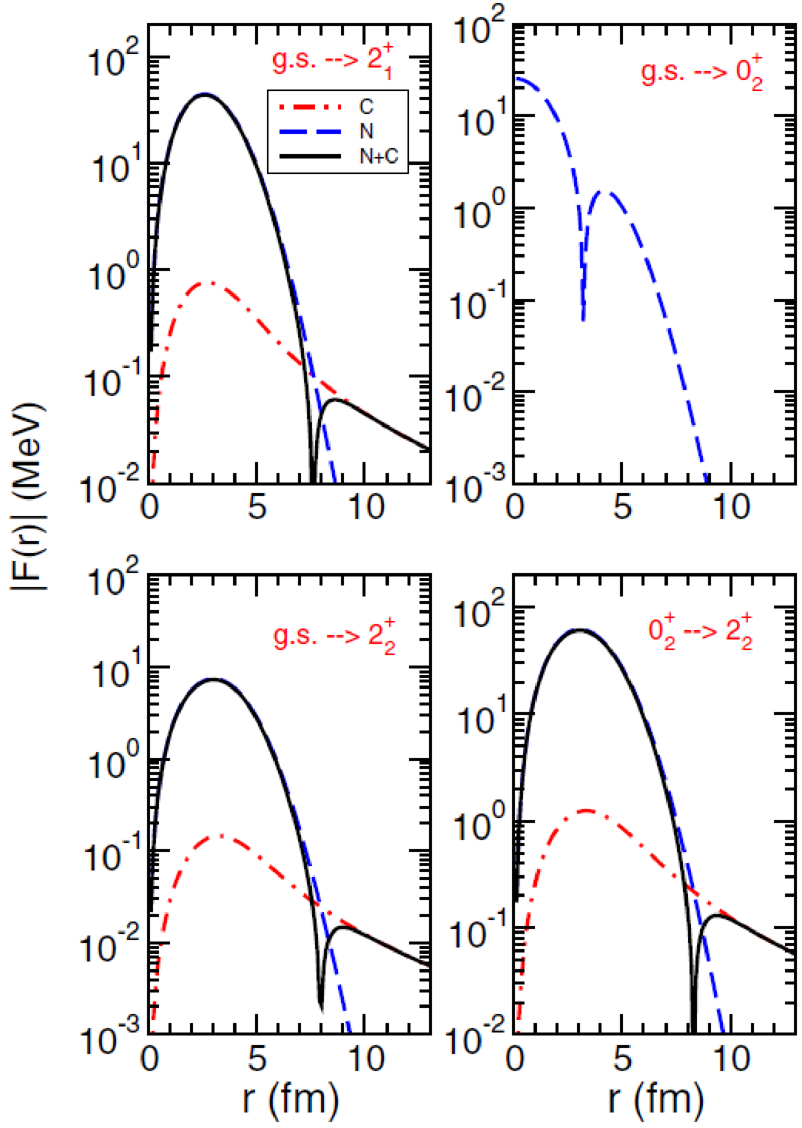


FIG. 15. Form factors in logarithmic scale for a few inelastic excitation processes of interest. We show the nuclear, Coulomb, and total form factors.

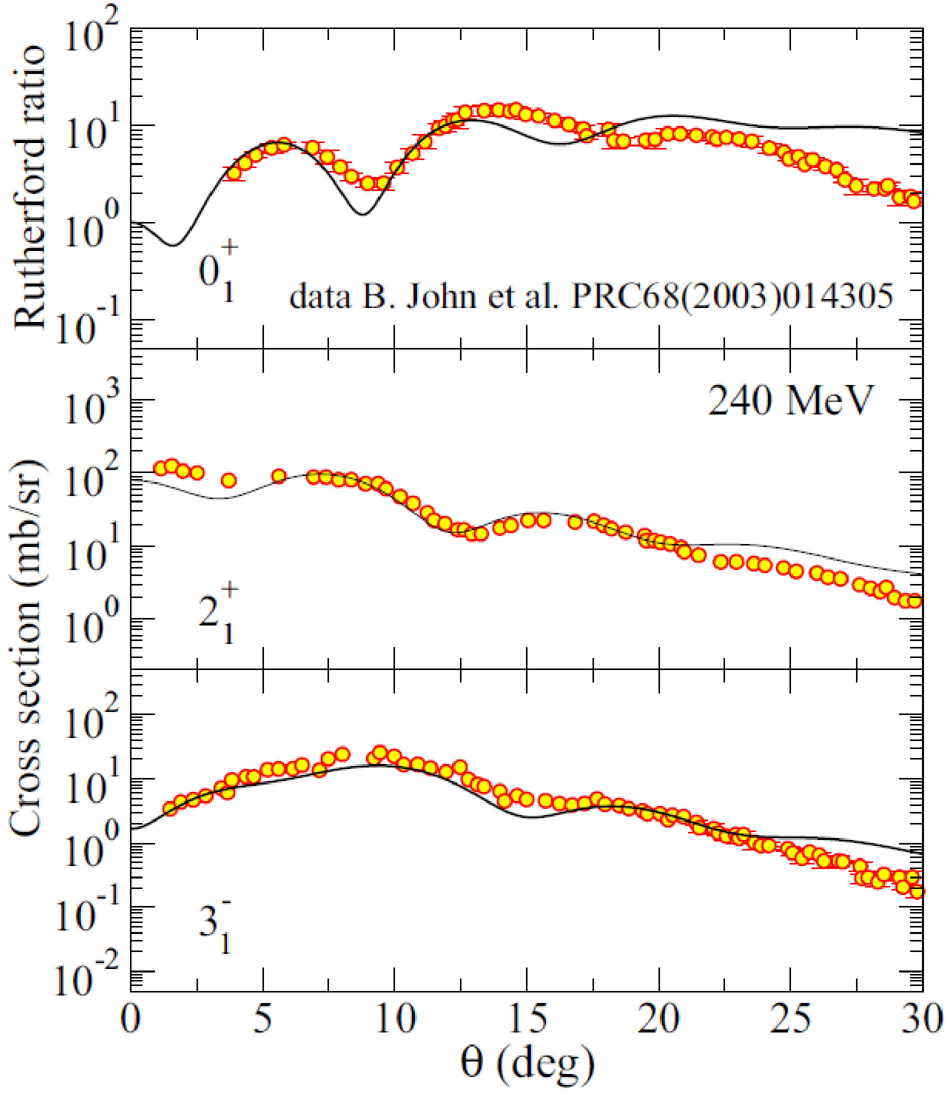


FIG. 16. Differential cross section for the elastic scattering and the transitions $0_1^+ \to 2_1^+$ and $0_1^+ \to 3_1^+$ at 240-MeV bombarding energy. Data are from Ref. [41] (retrieved through EXFOR).

Lots of results that I dont'have time to discuss in details.
They confirm that with just a simple triangular model one catches all the gross features, not only of the nuclear structure, but also of reaction dynamics of 12C.

Importance of the imaginary part of the ion-ion potential

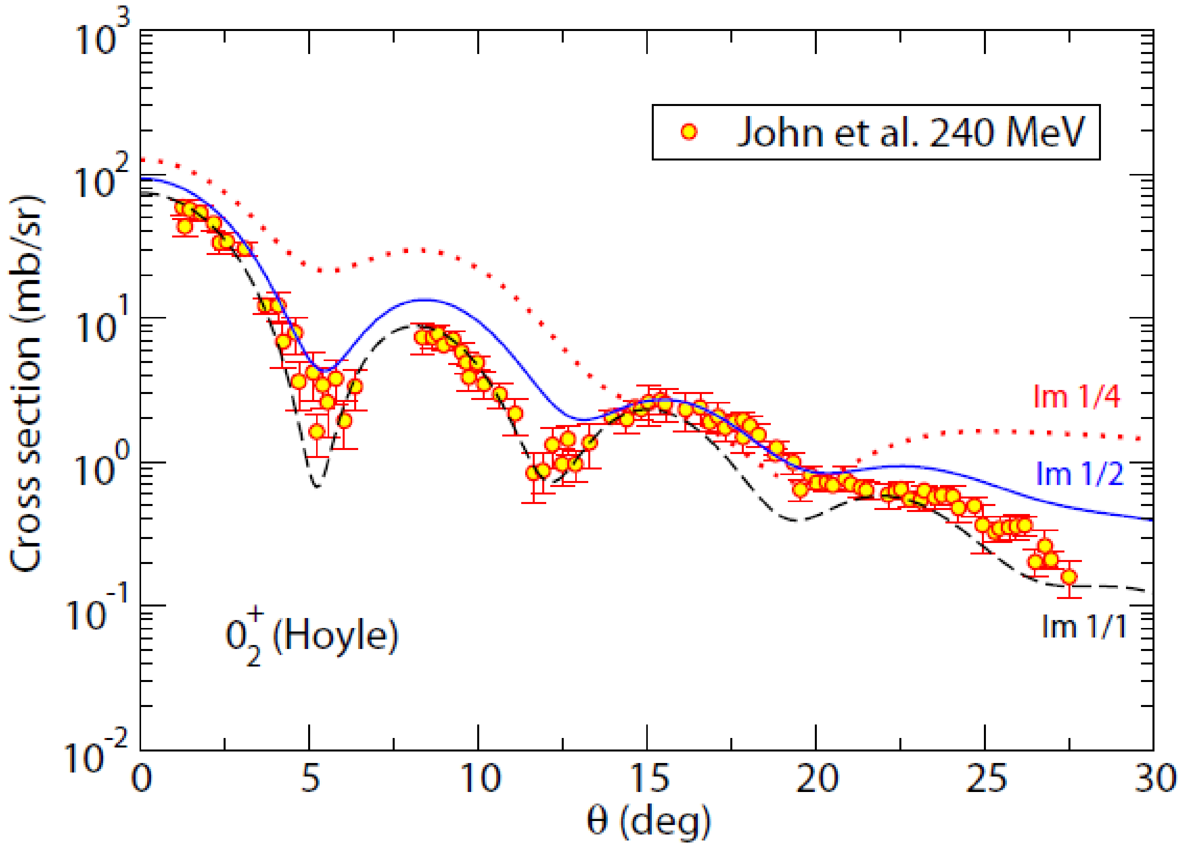
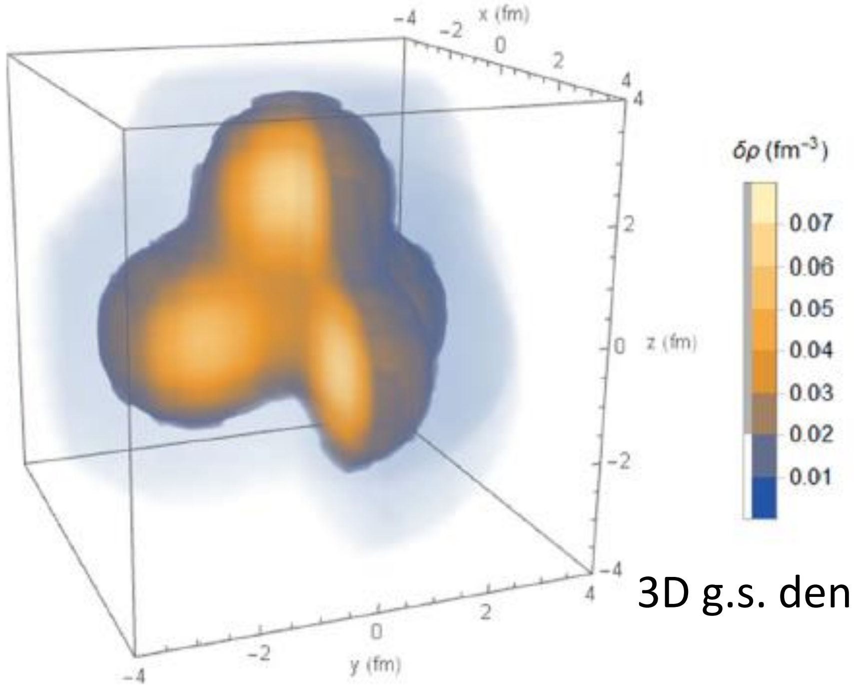


FIG. 17. Differential cross section for the transition $0_1^+ \rightarrow 0_2^+$ at 240-MeV bombarding energy. Data are from Ref. [41] (retrieved through EXFOR) and the three curves have different factors for the depth of the imaginary part as indicated in the figure.

$V(r) + iW(r)$

Extended to 16O in a tetrahedral arrangement

Eur. Phys. J. A (2021) 57:33



3D g.s. density plot

Eur. Phys. J. A (2021) 57:33
https://doi.org/10.1140/epja/s10050-021-00347-5

THE EUROPEAN PHYSICAL JOURNAL A

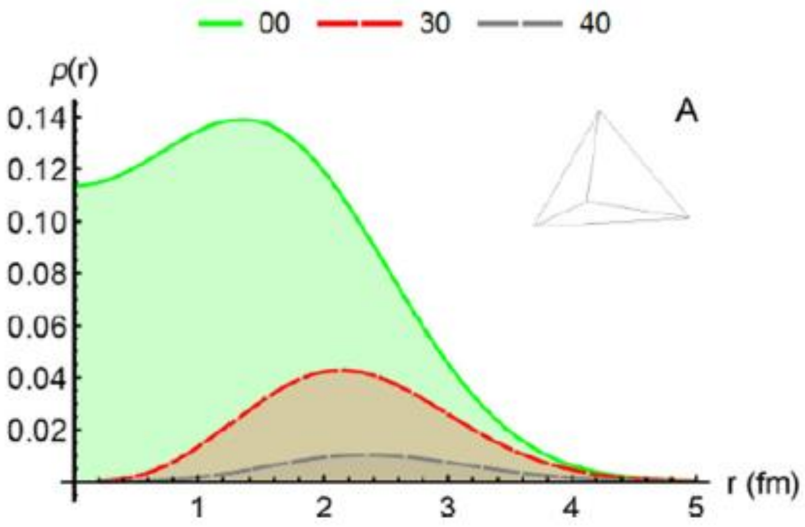
Regular Article - Theoretical Physics

Alpha-induced inelastic scattering and alpha-transfer reactions in ^{12}C and ^{16}O within the Algebraic Cluster Model

Jesus Casal[1,2], Lorenzo Fortunato[1,2], Edoardo G. Lanza[3,4], Andrea Vitturi[1,2,a]

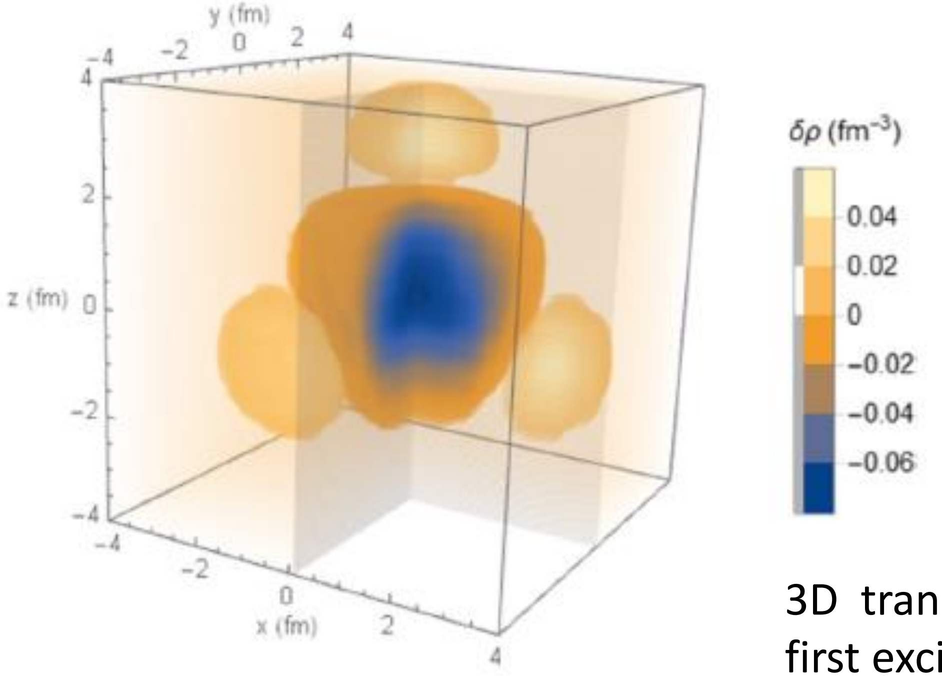
The tetrahedral group Td allows for singly- , doubly- and triply-degenerate representations

→ one can see all of these excitation modes in the spectrum of 16O !

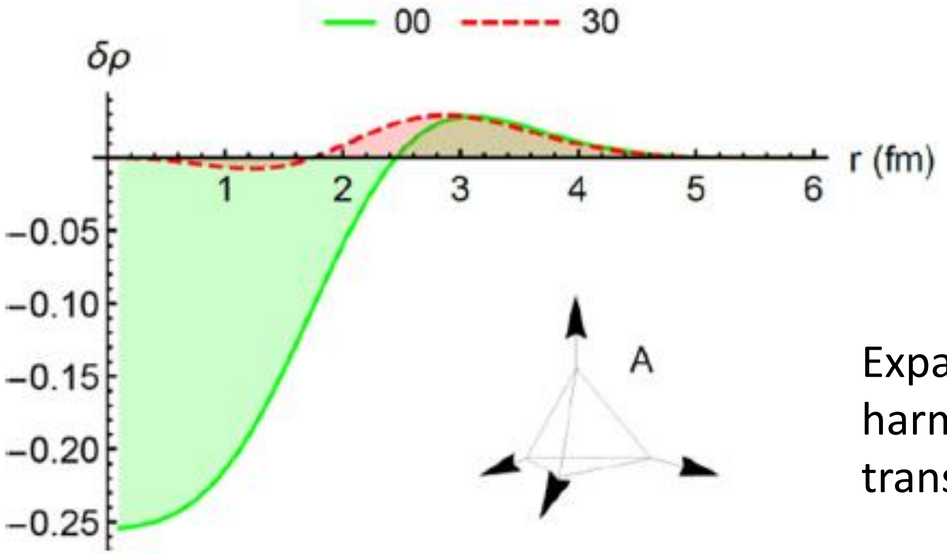


Expansion in spherical harmonics

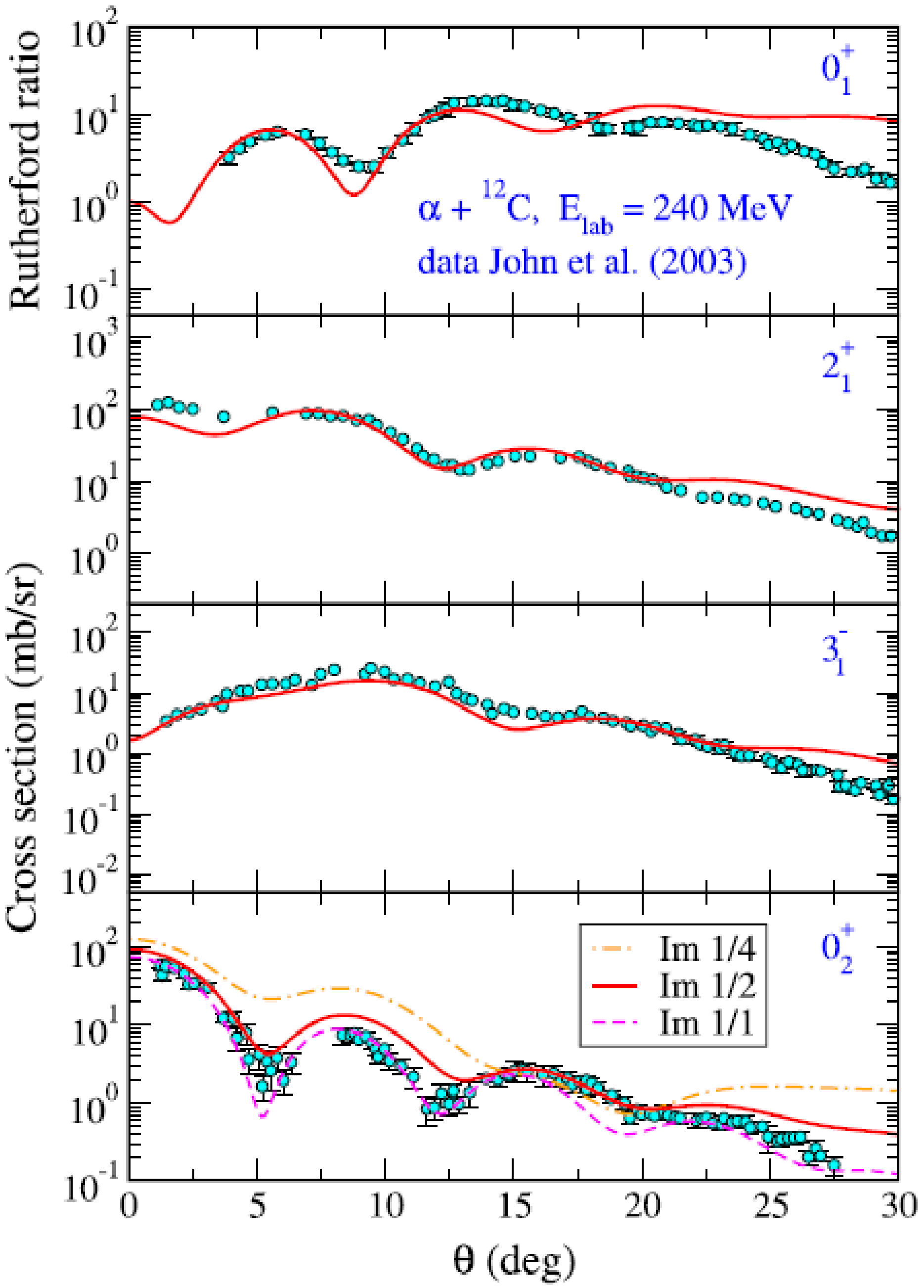
Extended to 16O in a tetrahedral arrangement -> 12C(α,γ)16O



3D transition density to first excited A band (0+)



Expansion in spherical harmonics of the transition densities



Discrete symmetries and polarized gamma-rays in 12C

PHYSICAL REVIEW C **99**, 031302(R) (2019)

Rapid Communications Editors' Suggestion

Establishing the geometry of α clusters in ^{12}C through patterns of polarized γ rays

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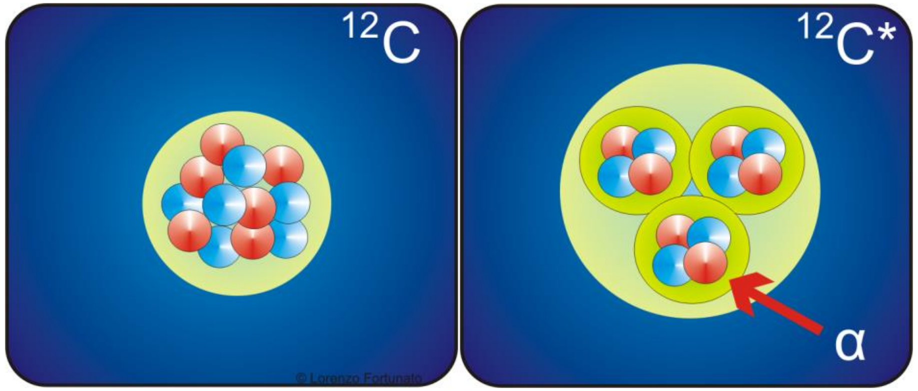
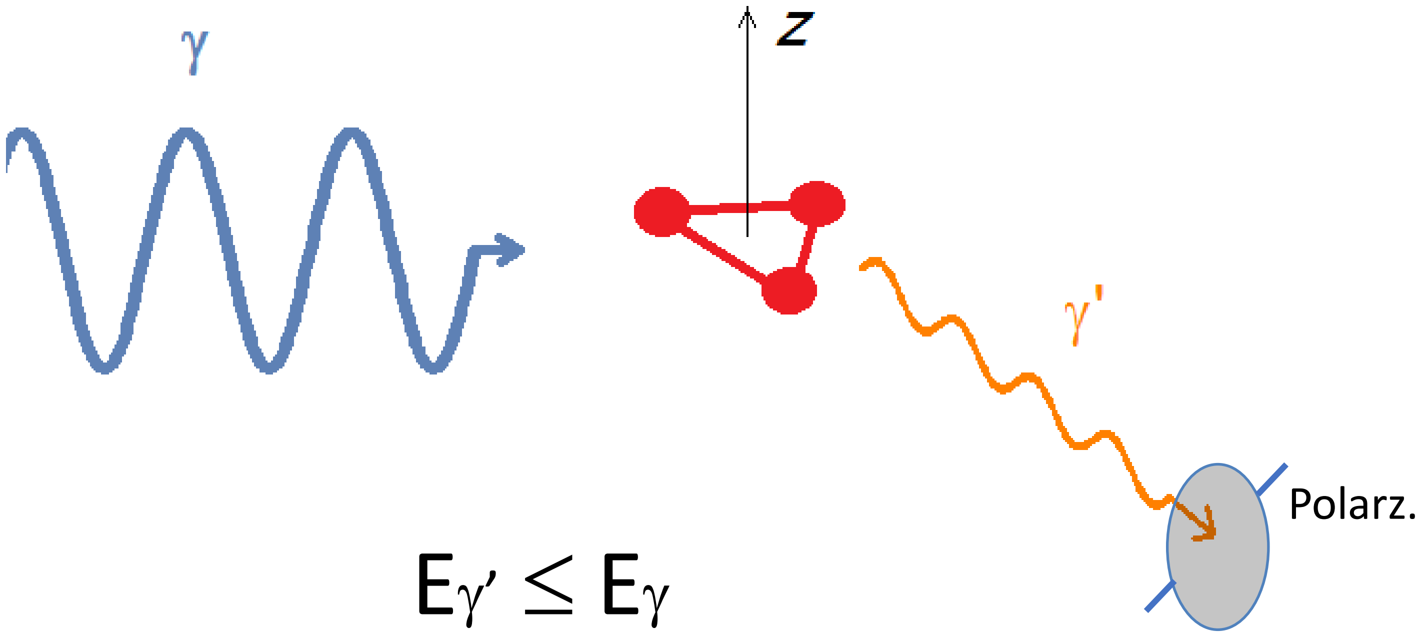
Dipartimento di Fisica e Astronomia "G.Galilei"-Università di Padova, via Marzolo 8, I-35131 Padova, Italy and INFN-Sez.di Padova, via Marzolo 8, I-35131 Padova, Italy

(Received 13 December 2018; published 11 March 2019)



One can shoot linearly polarized gamma rays (Electric field oscillating in a given direction constant in time) of appropriate energy (tuned to match the resonances of interest) and observe the outcoming gammas of the same or different energies with a polarizer/analyzer. If the nucleus has a definite geometrical symmetry (i.e. if there is an underlying discrete group structure), very strict selection rules apply. Experimentally the polarization can be measured with another inverse Compton scattering.



$$E_{\gamma'} \leq E_{\gamma}$$

Depolarization ratio

One can measure the so-called depolarization ratio between intensities, by turning the analyzer/polarizer of 90 degrees, i.e.:

$$\rho = \frac{I_\perp}{I_\parallel}$$

as a tool to determine which modes are totally symmetric modes. In fact from the theory of Raman scattering

$0 \leq \rho \leq \frac{3}{4}$ for polarized bands (symmetric modes)

$\rho = \frac{3}{4}$ for depolarized bands (non-symmetric modes)

even with a randomly oriented sample.

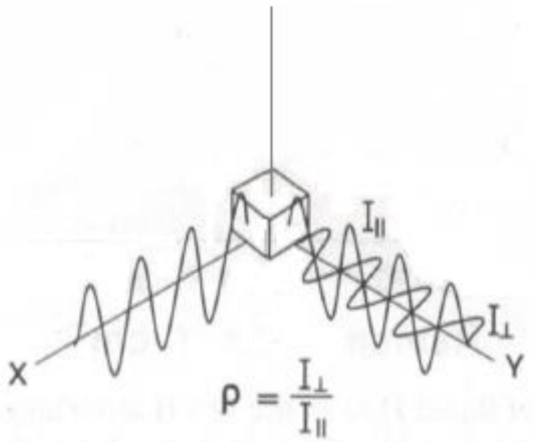


Figure 8.6. Parallel and perpendicular Raman scattering.

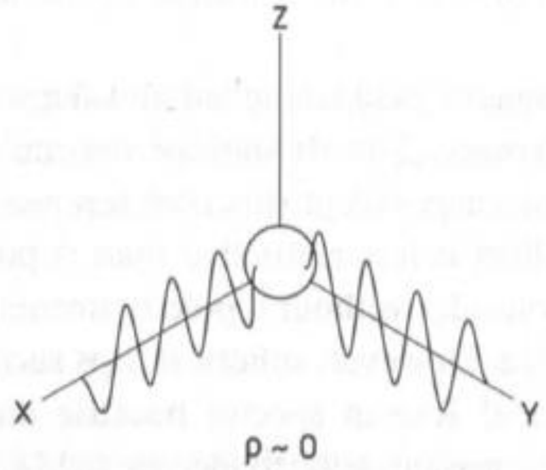


Figure 8.8. Polarized light scattering by a sphere.

Figures from book by P.Bernath

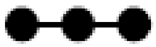
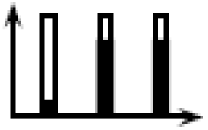
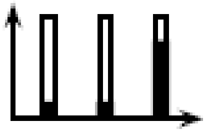
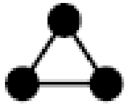
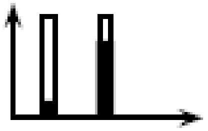
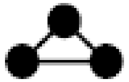
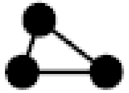
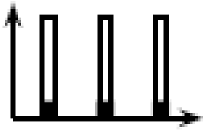
Working plan to implement this idea

Different geometric arrangements of equilibrium points → Discrete symmetry group → Precise selection rules

Work plan:

- Decide arrangement of N particles
- This means 3N-6 d.o.f (or 3N-5 d.o.f. for linear arrangement)
- Identify the underlying discrete group structure
- Find the character under transformations of the group Γ_{3N}
- Subtract translations and rotations to single out character of vibrational modes Γ_{vib}
- Identify patterns of totally symmetric modes
- Check models against measures of intensities $\rightarrow$ Eureka !!

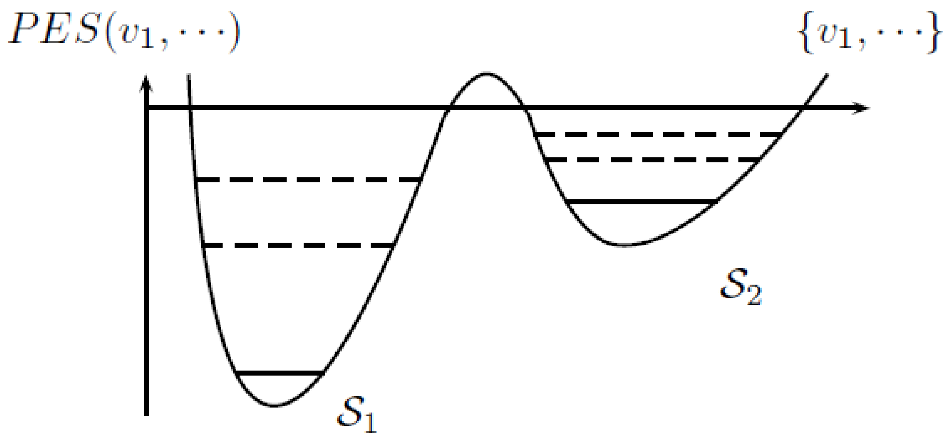
Tables in PRC 99 (2019) paper: 3 equal clusters

name	shape	group	Γ_{vib}	Patterns
linear $=$		$\mathcal{D}_{\infty h}$	$A_{1g} + A_{1u} + E_{1u}$	
linear $\neq$		$\mathcal{C}_{\infty \nu}$	$2A_1 + E_1$	
equilateral		$\mathcal{D}_{3h}$	$A_1' + E'$	
isosceles		$\mathcal{C}_{2\nu}$	$2A_1 + B_1$	
scalene		$\mathcal{C}_s$	$3A'$	

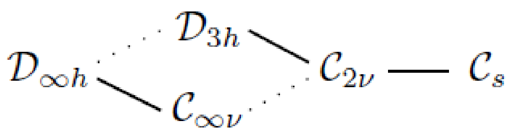
The number of totally symmetric peaks over total is different in each case, therefore one can disentagle the various possibilities

Tables in PRC 99 (2019) paper

There might be more than just one configuration! The picture complicates a little, but not too much ! One can invoke the concept of descent in symmetry ans still apply some of the rules.



Group chains



$\mathcal{D}_{\infty h}$	$A_{1g}+$	$A_{1u}+$	E_{1u}
	$\downarrow$	$\downarrow$	$\downarrow$
$\mathcal{C}_{\infty \nu}$	A_1+	A_1+	E_1
	$\downarrow$	$\downarrow$	$\downarrow$
$\mathcal{C}_{2\nu}$	A_1+	A_1+	$\overbrace{B_1 + B_2}$
	$\downarrow$	$\downarrow$	$\downarrow$
$\mathcal{C}_s$	$A'+$	$A'+$	A'

$\mathcal{D}_{\infty h}$	$A_{1g}+$	$A_{1u}+$	E_{1u}
	$\downarrow$	$\downarrow$	$\downarrow$
$\mathcal{D}_{3h}$	A'_1+	A''_2+	E'
	$\downarrow$		$\downarrow$
$\mathcal{C}_{2\nu}$	A_1+		$\overbrace{A_1 + B_1}$
	$\downarrow$		$\downarrow \quad \downarrow$
$\mathcal{C}_s$	$A'+$		$A' + A'$

FIG. 4. Descent in symmetry restricted to representations of the groups that are relevant to all possible configurations of three identical particles.

PROJECTS:

2023-2025?

- Continue with application of symmetries and molecular models to light nuclei
- Try to build a generic code for calculation of normal modes of all possible cluster structures (already advanced stage)
- Merging this with ab initio shell model approaches

- Explore other cluster transfer reaction
- Reevaluate old data at the light of cluster models (8Li, 24O, 20Ne, 24Mg, heavier neutron rich nuclei)
- Explore the description of high lying excited states (still missing in many approaches, good results on 6Li)