Clustering and two-body correlations within extended density functional approaches

Workshop MONSTRE

Milano - 11^{th} May, 2023

Speaker: S. Burrello

INFN - Laboratori Nazionali del Sud

Outline of the presentation

1. **Theoretical approaches for nuclear many-body problem**
   - Ab-initio vs phenomenological models based on energy density functionals (EDF)
   - Effective interaction and nuclear matter (NM) Equation of State (EoS)

2. **Extended EDF-based models: recent developments and results**
   - ✏ **Bridging ab-initio with phenomenological EDF approaches**
     - Benchmark on microscopic pseudo-data for low-density neutron matter
     - Power counting analysis based on many-body perturbative expansion
   - ✏ **Beyond mean-field: many-body correlations and clustering phenomena**
     - Neutron star (NS) crust modelization for a global and unified EoS
     - Embedding short-range correlations within relativistic approaches

3. **Summary and perspectives within MONSTRE**

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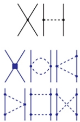
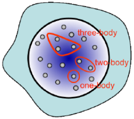
Theoretical models for EoS and finite nuclei

- **Ab-initio** approaches based on **many-body** expansion
  - Realistic or **effective field theory** (EFT) interactions
    $\Rightarrow$ Diagrammatic hierarchy (**power counting**)

LO $(Q/\Lambda_\chi)^0$

NLO $(Q/\Lambda_\chi)^2$





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  - Self-consistent mean-field (MF) approximation
  - Fit of **parameters** to reproduce various data

- Energy Density Functional (**EDF**) theory

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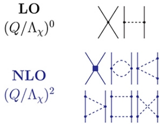

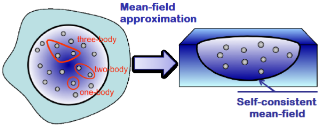

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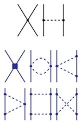

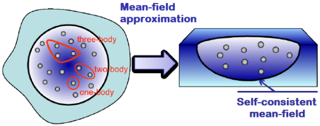

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$$|\Psi\rangle \equiv \text{independent many-particle state}$$

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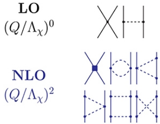

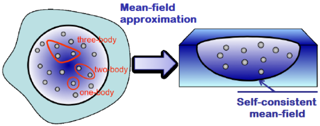

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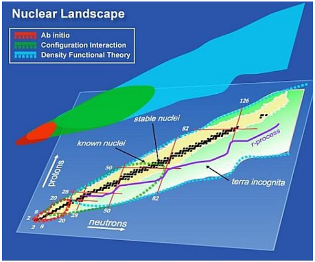

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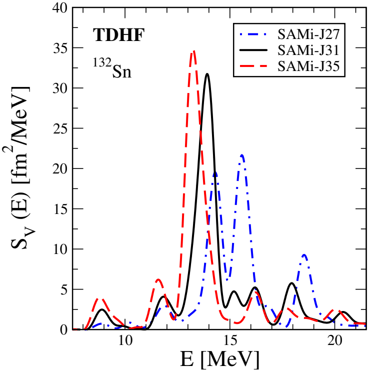
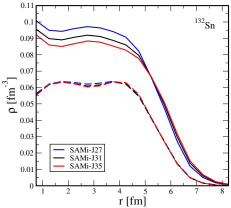
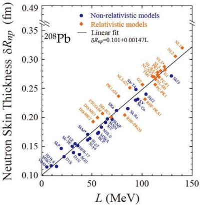
$\Rightarrow$ Description of **ground state** and **excitations**

Nuclear structure: neutron skin and pygmy resonance

- Non-relativistic **Skyrme-like** EDF
- Structure of **neutron-rich** nuclei
  [Zheng et al., PRC 94(1), 014313 (2016)]
  [S. Burrello et al., PRC 99(5), 054314 (2019)]
  - Neutron **skin** thickness $\Delta r_{np} \Leftrightarrow L$



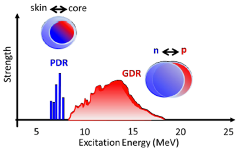


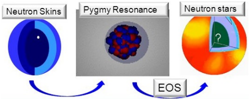


- Time-Dependent-Hartree-Fock **(TDHF)**

$$i\hbar\dot{\hat{\rho}}(t) + \left[\hat{\rho}, \hat{\mathcal{H}}_{eff}[\rho]\right] = 0$$

- **Isovector dipole** (collective) excitations:
  - **Pygmy** Dipole Resonance (**PDR**)





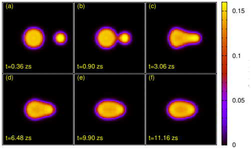
Merging nuclear structure and reaction studies

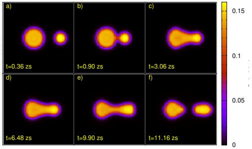
- **Pre-equilibrium** in **charge-asymmetric reactions**

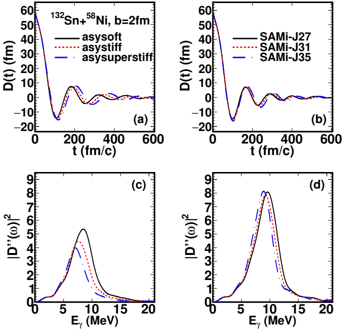
  [H. Zheng, S. Burrello, M. Colonna, V. Baran, PLB 769 (2017)]

- Interplay between **fusion** and **quasi-fission** processes
  $\Rightarrow$ formation of **super-heavy elements**

[H. Zheng, S. Burrello, M. Colonna, D. Lacroix, G. Scamps, PRC 98 (2018)]







- Same **framework** as for nuclear **structure** $\Rightarrow$ Merging with **reaction** studies
- Role of **different terms** of effective **interaction** (and EoS) on **final outcomes**
  - Importance of **momentum** dependent + **surface** terms (+ **symmetry** energy)
- Heavy ion collisions are reliable **tools** to extract **information** of EoS!

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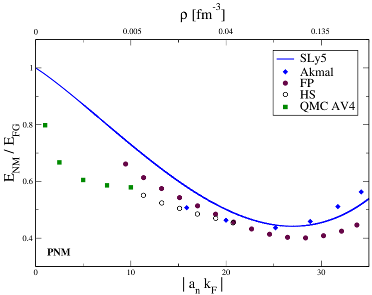
Pure neutron matter (PNM) low-density expansion

- **Dilute** PNM (a_s = -18.9 fm) $\Rightarrow$ close to **unitary** limit of interacting **Fermi** gas
- **Lee-Yang** expansion in $(a_s k_F)$ from **EFT** ($\nu_i = 2, 4$ for PNM, symmetric NM)

$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + (\nu_i - 1)\frac{2}{3\pi}(k_F a_s) + (\nu_i - 1)\frac{4}{35\pi^2}(11 - 2\ln 2)(k_F a_s)^2 + \ldots\right]$$

- New **class** of **EDFs** inspired by EFT



✓ **Application** to drops & nuclei ⇒ **surface**
[S. Burrello et al., PRC 103(6), 064317 (2021)]

✗ Improving neutron **effective mass** prediction

✗ Implementation in **dynamical** models

✓ Finite **temperature** (T) ⇒ ✗ impact on **NS modelization** (**"pasta"** formation)
[S. Burrello & M. Grasso, EPJA 58(2), 22 (2022)]

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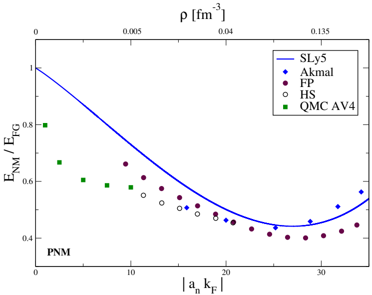

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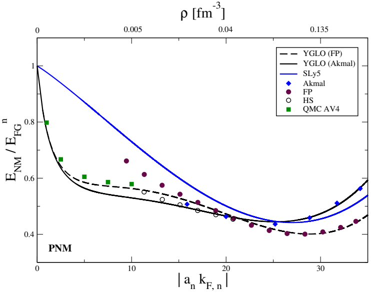

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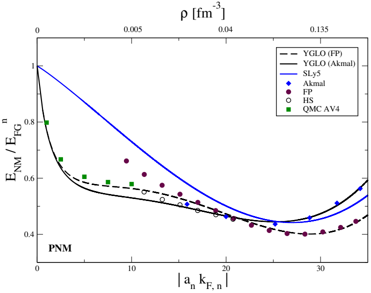

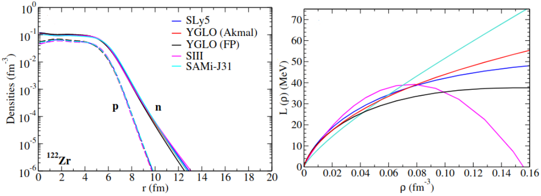

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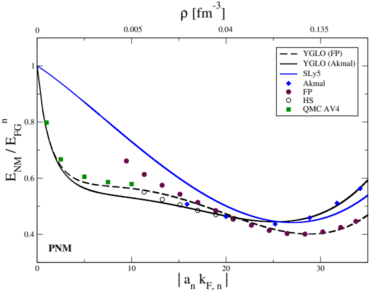

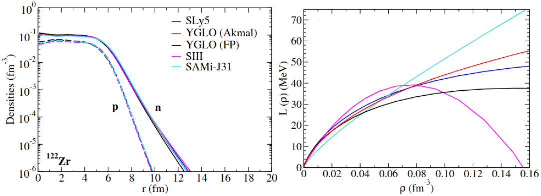

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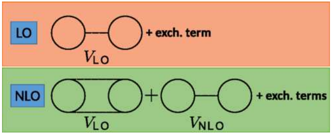
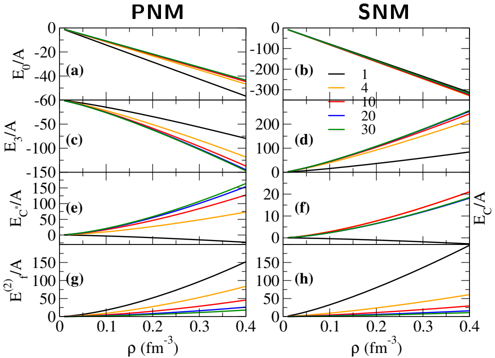
Beyond MF: towards a power counting in EDF

- Beyond MF (BMF) ⇒ **correlations** taken into account (double-counting)
  - **Hierarchy** of interaction (and EoS) contributions ⇒ **power counting** in EDF
- **EoSs** at next-to-leading order (**NLO**) for symmetric NM (SNM) and PNM





- (t_0, t_3) Skyrme-like V_{LO}
- **Renormalizability** analysis
  - ✓ perturbative scheme
- **Next-to-NLO** (EFT-analysis):
  - ✗ Expansion parameter
  - ✗ Breakdown scale

[S. Burrello, C.J. Yang, M. Grasso, PLB 811, 13593 (2020)]

✓ BMF study of **closed-shell** nuclei [C.J. Yang et al., PRC 106 (1), L011305 (2022)]

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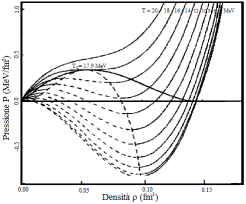
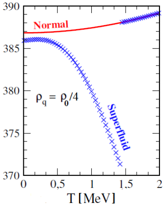
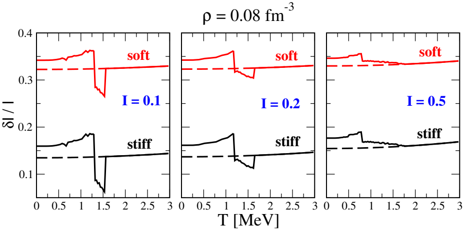
Pairing correlations and nuclear superfluidity

- **Pairing** effects on mechanical (**spinodal**) instability in low-density **nuclear matter**
  $\Rightarrow$ variation on **compressibility** and **isotopic content** of the **clusterized** matter

  [S. Burrello, M. Colonna, F. Matera, PRC 89 (2014)]

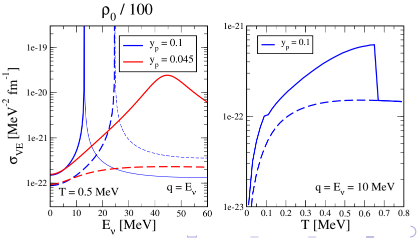






- Homogenous **stellar matter**:
  impact of **superfluidity** on **ν-scattering**
  $\Rightarrow$ **cooling** process in **proto-NS** (PNS)
  or pre-bounce of **supernova** explosions

  [S. Burrello, M. Colonna, F. Matera, PRC 94 (2016)]



Clustering phenomena and neutron star crust

- Many-body (**short-range**) correlations (SRCs) below ρ_0
  - Formation of **bound** state of nucleons (**clustering**)
- **Phenomenological** models with **clusters**
  - **Dilute** matter as a **mixture** of nucleons and nuclei
    $\Rightarrow$ Nuclear statistical equilibrium (**NSE**) model

    [A. R. Raduta, F. Gulminelli, PRC 82, 065801 (2010)]
  - **Unified** description of NS EoS & **crust-core** transition
    ✓ **Composition** and **heat capacity** of NS inner crust
    [S. Burrello et al., PRC 92, 055804 (2015)]

    ✗ **Transport** properties (ν-emissivity), **vortices**, ...

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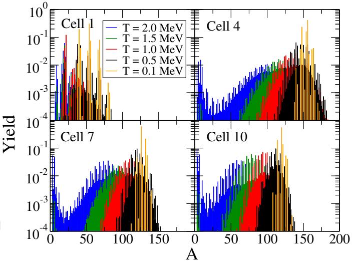

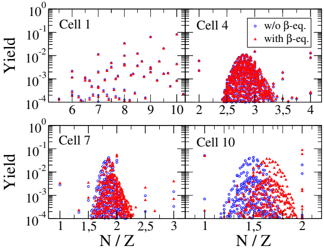

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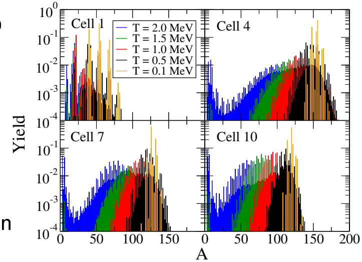

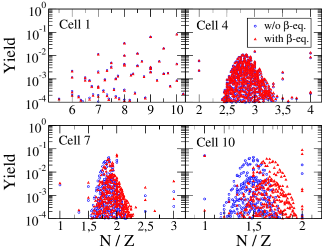

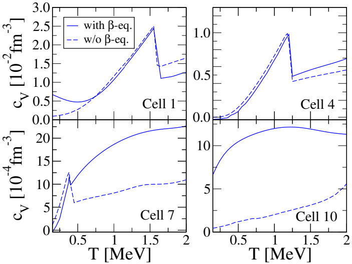

In-medium effects and cluster dissolution

- Cluster **dissolution** approaching saturation from below
  $\Rightarrow$ **Mott effect** ruled by Pauli blocking
  - Geometrical **excluded-volume** mechanism
  - **Microscopic** in-medium effects
- Generalized relativistic density functional (**GRDF**)
  $\Rightarrow$ Meson exchange with **density dependent** couplings

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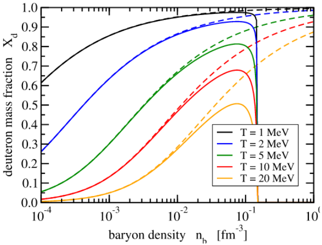

- **Mass-shift** obtained by solving the **in-medium** many-body **Schrödinger equation**
  - **Parameterization** as function of density (n_b), isospin asymmetry (β), T
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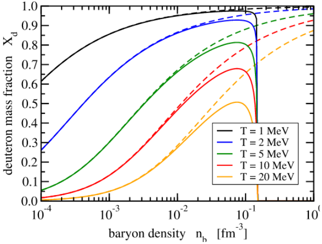

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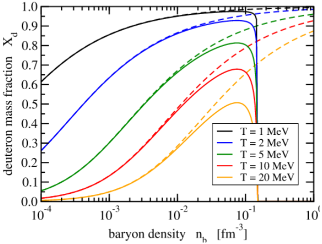

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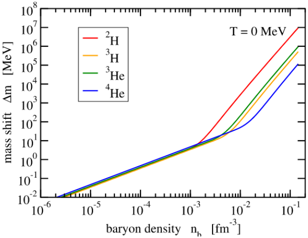

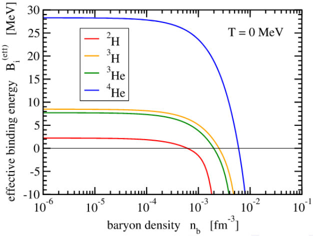

Short-range correlations and EoS at high-density

- Nucleon knock-out in **inelastic electron scattering**
  [O. Hen et al. (CLAS coll.), Science 346, 614 (2014)]
  - SRCs from **tensor** components or **repulsive** core
  - **Smearing** of Fermi surface (high-**k** tail at T=0)
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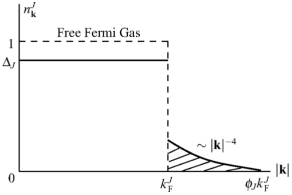

✓ Embedding SRCs in relativistic MF with quasi-deuterons ⇒ EoS at **high-density**
[S. Burrello & S. Typel, EPJA 58, 120 (2022)]

✗ Extension to **finite T** (+ momentum of clusters with respect to medium)

✗ Inclusion of quasi-deuterons within a **kinetic approach** [coll. with R. Wang + INFN CT]

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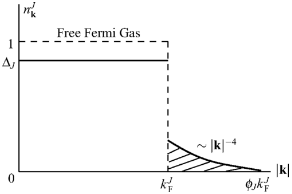

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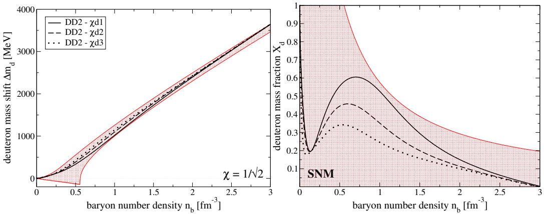

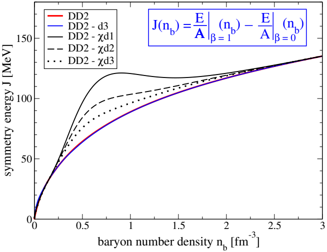

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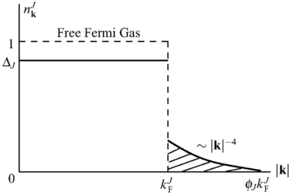

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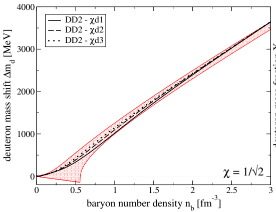

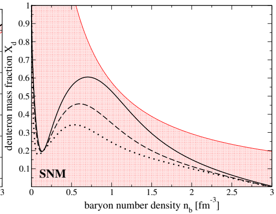

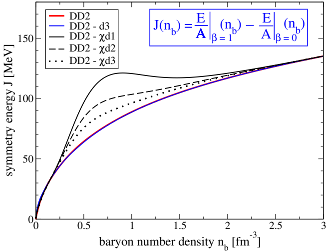

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- Single-nucleon **momentum distribution** ⇒ in-medium 2B **wave function** (wf)
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  - (existence of negative-norm "**ghost**" states)
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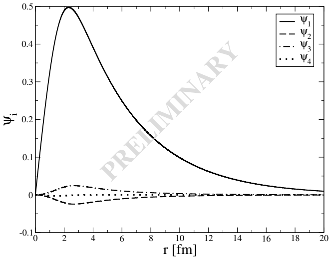

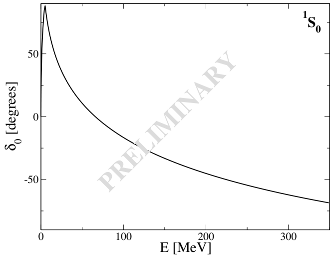

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Outline of the presentation

1. Theoretical approaches for nuclear many-body problem
   - Ab-initio vs phenomenological models based on energy density functionals (EDF)
   - Effective interaction and nuclear matter (NM) Equation of State (EoS)

2. Extended EDF-based models: recent developments and results
   - Bridging ab-initio with phenomenological EDF approaches
   - Beyond mean-field: many-body correlations and clustering phenomena

3. **Summary and perspectives within MONSTRE**

Final remarks and conclusions

Main topic

- Bridging **ab-initio** with **phenomenological** EDF approaches
- Beyond **mean-field** extension: many-body **correlations** and **clustering**

Main results

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- NLO **perturbativity** of **renormalized** scheme compatible with **power counting**
- Neutron star **crust composition** and effects of clusters on **cooling process**
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THANK YOU FOR YOUR ATTENTION!

Back-up slides

1. **Extended EDF-based models: recent developments and results**
   - **Bridging ab-initio with phenomenological EDF approaches**
     - Benchmark on microscopic pseudo-data for low-density neutron matter
     - Power counting analysis based on many-body perturbative expansion
   - **Beyond mean-field: many-body correlations and clustering phenomena**
     - Neutron star (NS) crust modelization for a global and unified EoS
     - Embedding short-range correlations within relativistic approaches

Lee-Yang-based EDFs: YGLO and ELYO

- **Dilute** PNM (a_s = -18.9 fm) $\Rightarrow$ close to **unitary** limit of interacting **Fermi** gas
- **Lee-Yang** (LY) expansion in $(a_s k_F)$ from **EFT** ($\nu_i = 2, 4$ for PNM, SNM)

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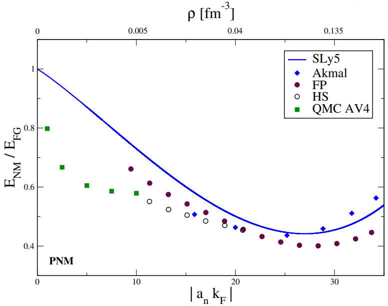

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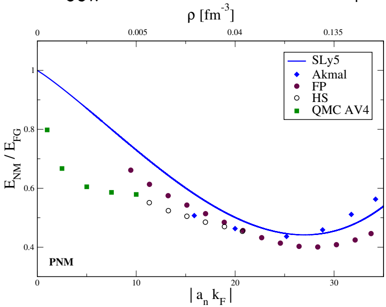

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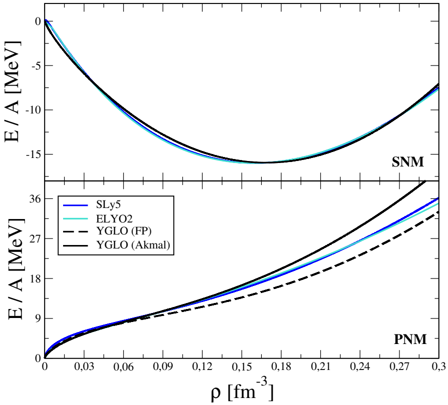

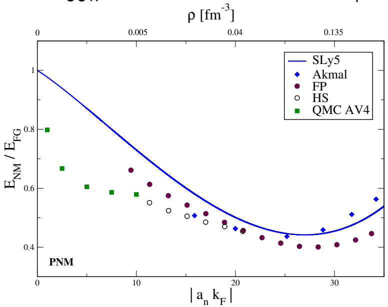

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Bridging with EFT: benchmark on ab-initio results

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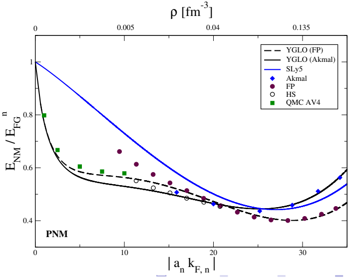

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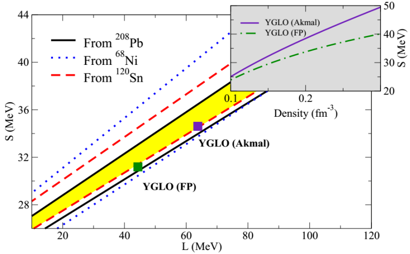

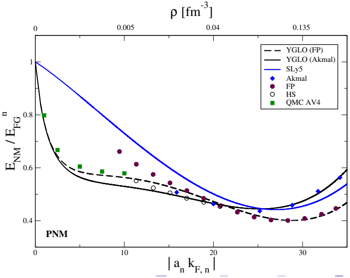

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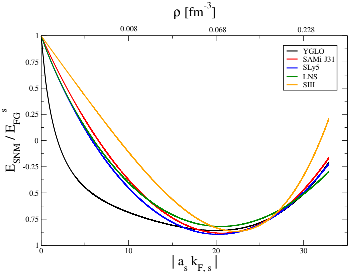

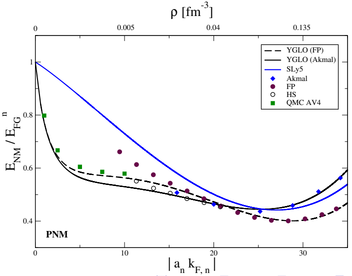

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- **Mapping** with **Skyrme** functional $\mathcal{E}_{Sk} = \mathcal{E}_0 + \mathcal{E}_3 + \mathcal{E}_{\text{eff}}$ (except for $\mathcal{E}_0 \leftrightarrow Y_i$)
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Mapping with Skyrme and splitting parameter

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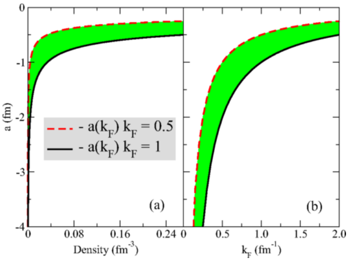
Density-dependent scattering length and p-wave

- **ELYO**: Density-dependent scattering length
  - Tuned by **low-density** condition $|a_s(k_F)k_F| = 1$
- Including LY p-wave contributions
- Mapping with s-wave Skyrme-like EDF
- Applications: finite systems and stellar matter



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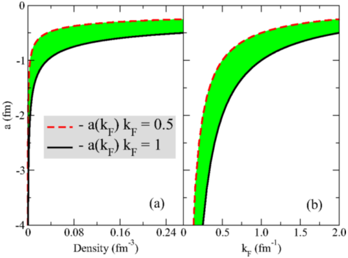

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[J. Bonnard, M. Grasso, D. Lacroix, PRC 101, 064319 (2020)]

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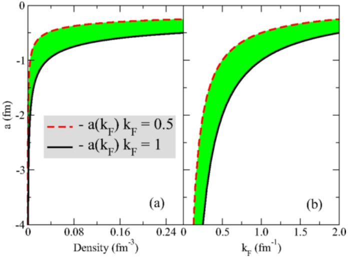

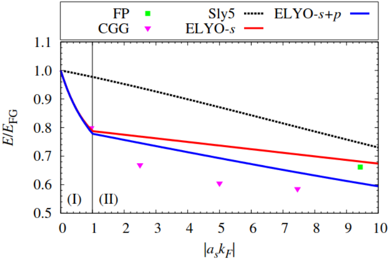

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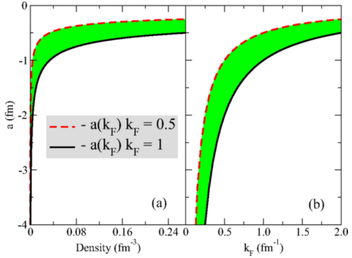

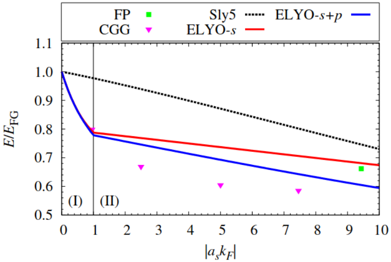

Energy of neutron drops and effective mass

- **Application** of YGLO and ELYO on finite systems ⇒ **neutron drops**

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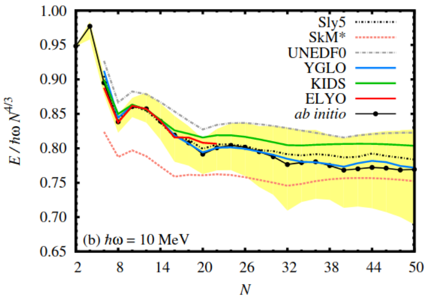

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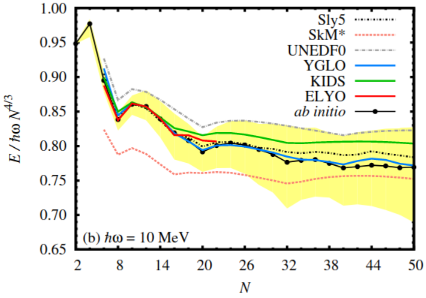

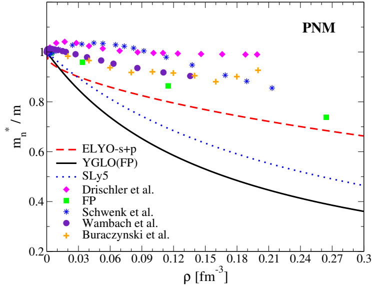

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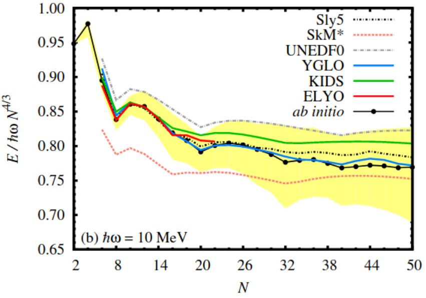

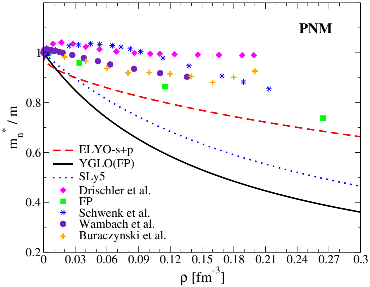

Exploring neutron dripline: isotopic chain

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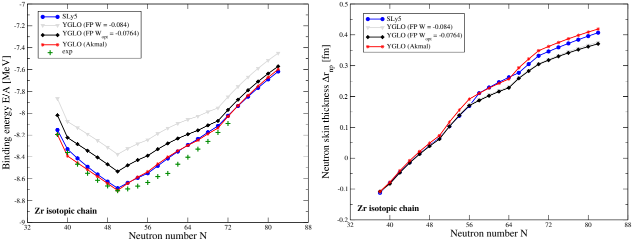

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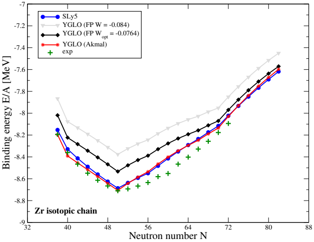

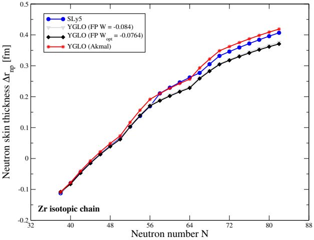

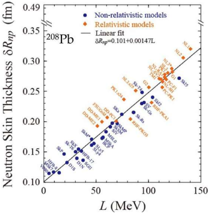

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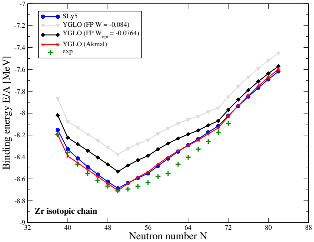

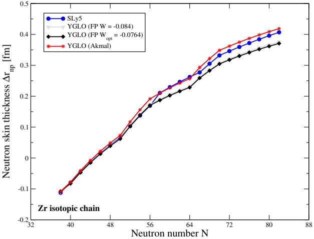

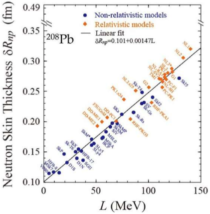

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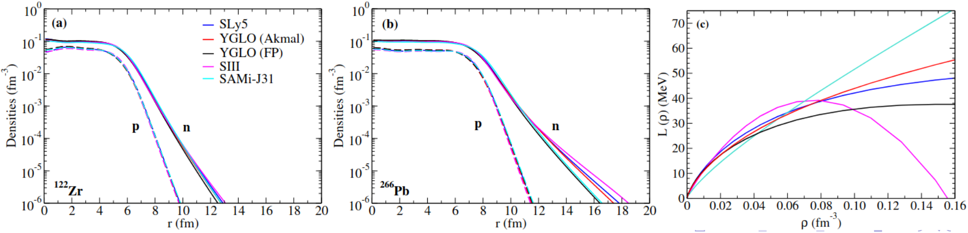

NS modeling: neutron gas energy in inner crust

- Crucial role of low-density **constraints** for pasta-phases formation in NS crust

  [H. Dinh Thi, T. Carreau, A. F. Fantina, F. Gulminelli, A&A 654, A114 (2021)]

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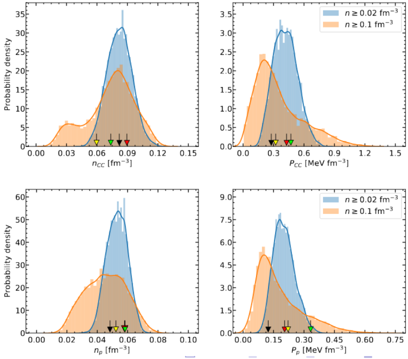

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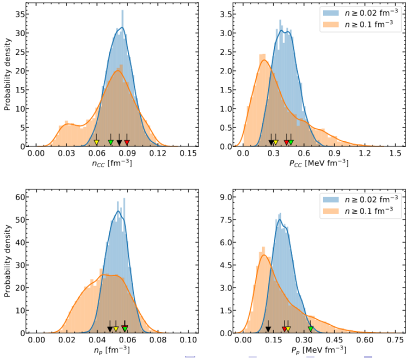

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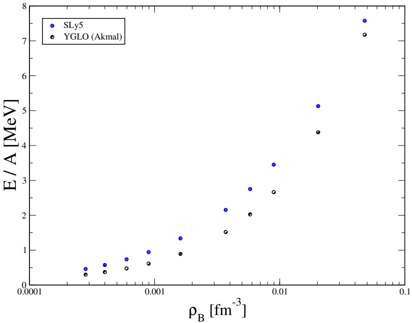

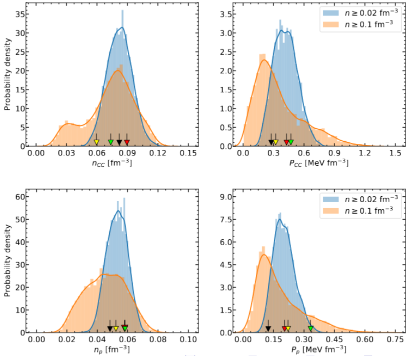

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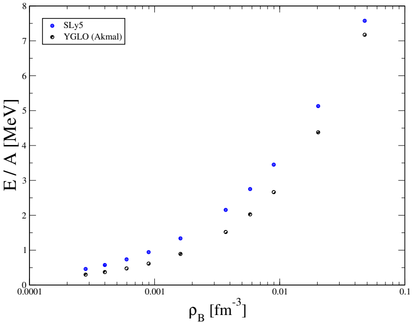

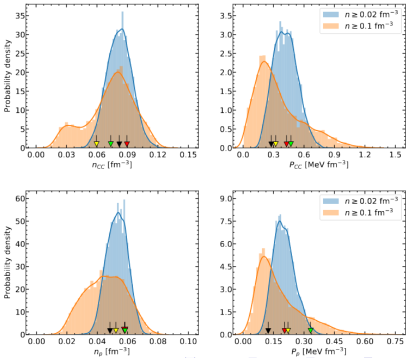

Back-up slides

1. **Extended EDF-based models: recent developments and results**
   - ✏ **Bridging ab-initio with phenomenological EDF approaches**
     - Benchmark on microscopic pseudo-data for low-density neutron matter
     - Power counting analysis based on many-body perturbative expansion
   - ✏ **Beyond mean-field: many-body correlations and clustering phenomena**
     - Neutron star (NS) crust modelization for a global and unified EoS
     - Embedding short-range correlations within relativistic approaches

Second order EoS with Skyrme interaction

- Calculations based on (**zero-range**) **Skyrme interaction** revealed very successfull
  - It may exist an **EFT-like expansion** based on **contact-type terms**
  - MF results may represent the **leading order** (LO) of such expansion
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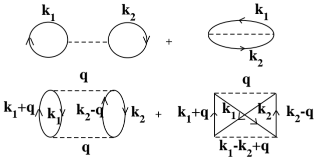

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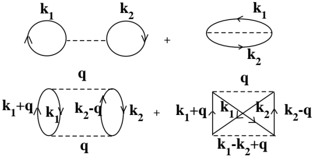

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BMF renormalizability with Skyrme interaction

- EoSs at LO for SNM and **pure neutron matter** (PNM)

$$\frac{E^{(\mathrm{LO})}_{SNM}}{A} = \frac{3}{10}\frac{\hbar k_F^2}{m} + \frac{\hbar k_F^3}{4\pi^2}t_0 + \frac{\hbar k_F^{3+3\alpha}}{4\pi^2}T_3, \quad T_3 = \left(\frac{2}{3\pi^2}\right)^{\alpha}\frac{t_3}{6}$$

$$\frac{E^{(\mathrm{LO})}_{PNM}}{N} = \frac{3}{10}\frac{\hbar k_{F,n}^2}{m} + \frac{\hbar k_{F,n}^3}{12\pi^2}t_0(1-x_0) + \frac{\hbar k_{F,n}^{3+3\alpha}}{12\pi^2}\frac{T_3}{2^{\alpha}}(1-x_3)$$

- **Renormalizability** for SNM by **constraining** interaction parameters
[C. J. Yang, M. Grasso, K. Moghrabi and U. Van Kolck, PRC 95, 054325 (2017)]

- **Renormalization group** (RG) analysis
⇒ introduction of **counter terms** (C,C*)
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BMF renormalizability with Skyrme interaction

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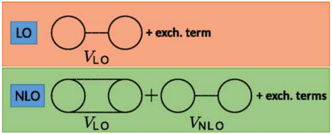
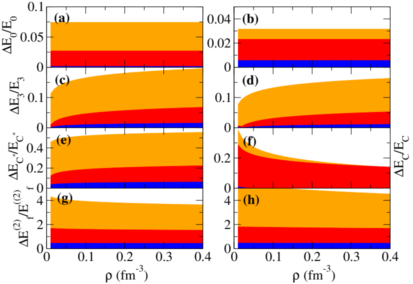
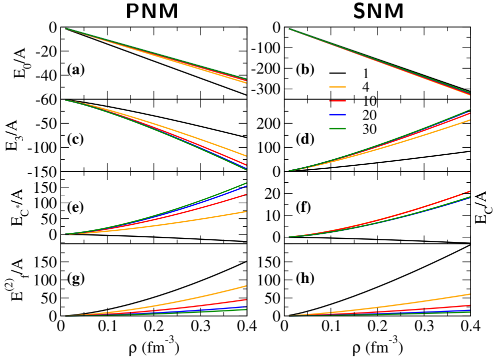


Perturbativity and power counting in EDF theories

- **Convergence** of energy contributions
- **Rate** depends on **power of k_F**





- Regularization of **Λ-divergencies**

$$\Rightarrow \frac{E_f^{(2)}(k_F)}{A} \xrightarrow[\Lambda\to\infty]{} 0$$

[S. Burrello, C.J. Yang, M. Grasso, PLB 811, 13593 (2020)]

- 2^{nd} order finite part progressively **suppressed** ⇒ **perturbative** problem
- **Expansion parameter** and uncertainties analysis ⇒ Next-to-NLO (**NNLO**)
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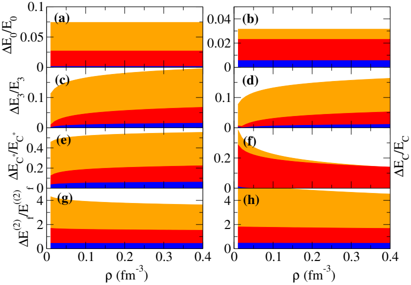

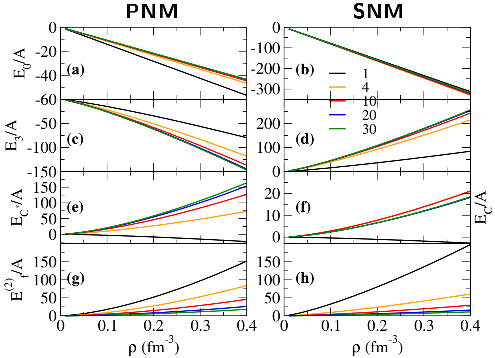

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Back-up slides

1. **Extended EDF-based models: recent developments and results**
   - Bridging ab initio with phenomenological EDF approaches
   - **Beyond mean-field: many-body correlations and clustering phenomena**
     - Neutron star (NS) crust modelization for a global and unified EoS
     - Embedding short-range correlations within relativistic approaches

Quasi-deuterons as surrogate for SRCs in GRDF

- Effective **resonances** (**quasi-clusters**) for treatment of SRCs at **supra-saturation**
  - Embedded in **GRDF** model through **in-medium modifications** of $\Delta m_d^{(\mathrm{high})}$
- **Two-body** correlations in np $^3S_1$ channel $\Rightarrow$ **quasi-deuteron**
- $T = 0 \Rightarrow$ **boson condensate** of deuterons under chemical potentials **equilibrium**

- $m^*_{\mathrm{nuc}} \geq 0 \Rightarrow 0 \leq X_d \leq \min\left\{X_d^{(\max)}, 1 - |\beta|\right\}$, $X_d^{(\max)} = \dfrac{m_{\mathrm{nuc}}}{\chi_d C_\sigma n_b} \xrightarrow[n_b \to \infty]{} 0$
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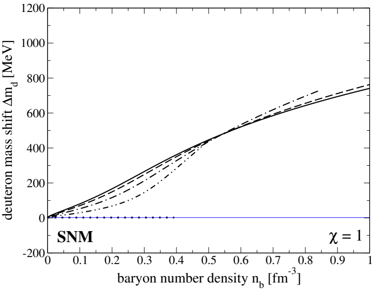

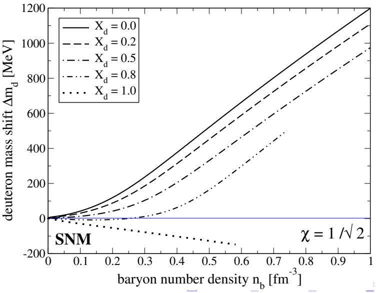

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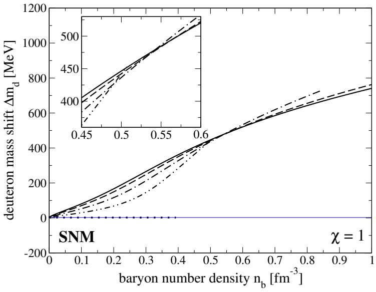

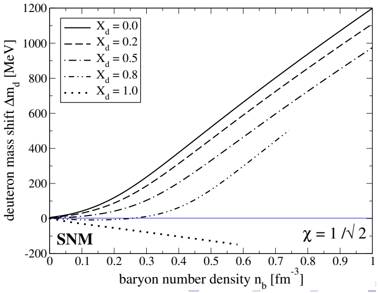

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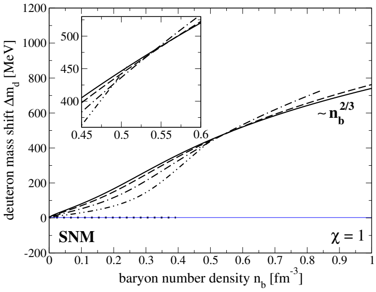

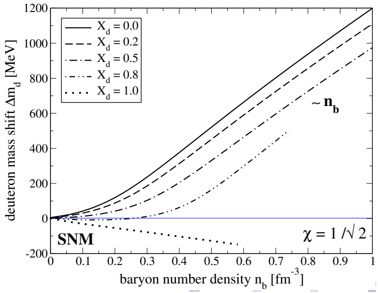

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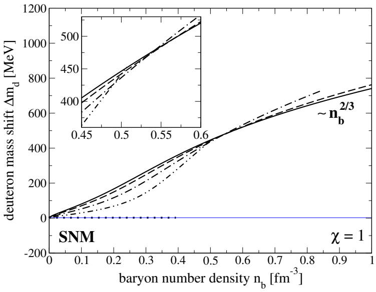

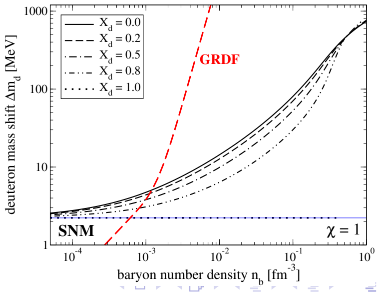

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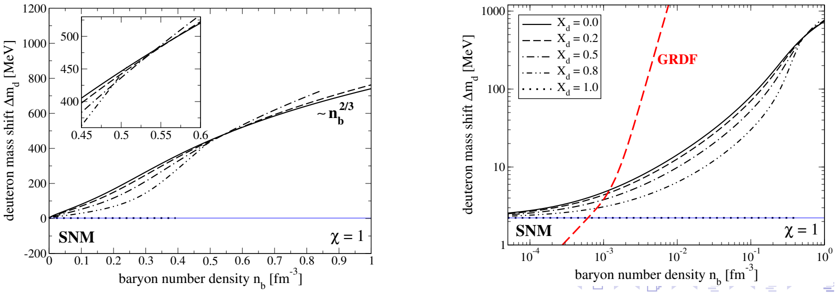
  - $\chi < 1 \Rightarrow$ **in-medium effects** and description of chemical **equilibrium constant**

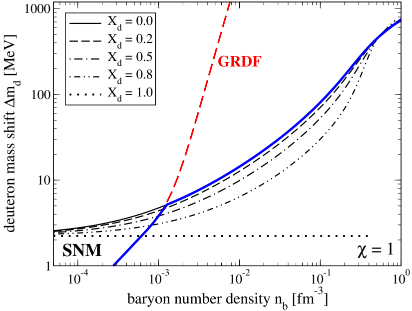
  [L. Qin et al., PRL 108, 172701 (2012); R. Bougault et al., J. Phys. G 47, 025103 (2020)]
- $1/\sqrt{2} < \chi_s = (0.85 \pm 0.05)$ **universal** scaling factor [H. Pais et al., PRC 97, 045805 (2018)]
  - No crossing $\Rightarrow \Delta m_d(n_b, X_d)$ **invertible** function for any density n_b
- $\Delta m_d^{(\mathrm{high})} \ll \Delta m_d^{\mathrm{GRDF}} \Rightarrow$ Large change beyond Mott density for **extended GRDF**

  [S. Typel, EPJ Special Topics 229, 3433–3444 (2020)]]
- **Interpolation** of low-(**Pauli blocking**) and high-(**condensate model**) density limit



Piecewise interpolation and saturation constraints

- **Piecewise** parameterization: $\Delta m_d(n_b, X_d) = \min\left\{\Delta m_d^{(\mathrm{low})}(n_b), \Delta m_d^{(\mathrm{high})}(n_b, X_d)\right\}$



× $\Delta m_d(n_b)$ no **smooth** function

× $X_d^{(\mathrm{high})} = \text{const.} \underset{n_b \to \infty}{\not\longrightarrow} 0$

✓ Zero-density limit (one half ^{2}H binding)

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- **Constraints** on NM at saturation (n_0)

  (E/A, m^*_{nuc}, pressure, symmetry energy)
- **Experimental** results of **SRCs** in nuclei

  $\Rightarrow X_{d,0} = 0.2$ (pairs $\approx$ 20% of density)

	χ	$\Gamma_{\sigma,0}$	$\Gamma_{\omega,0}$	$\Gamma_{\rho,0}$
	1	10.580042	13.217226	3.556424
	$1/\sqrt{2}$	10.919963	13.719324	3.400187
DD2	—	10.686681	13.342362	3.626940

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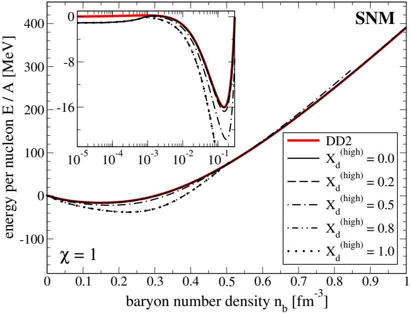

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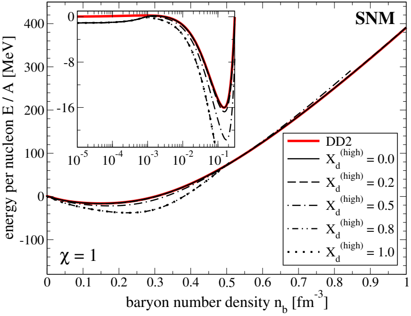

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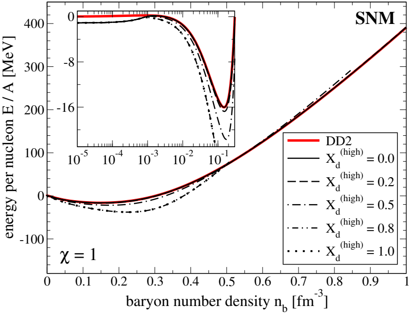

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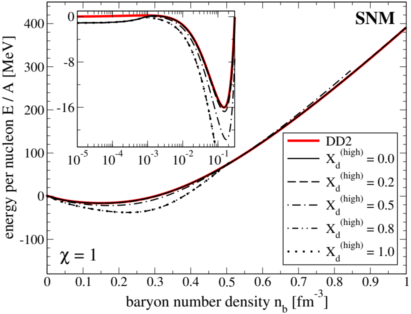

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Deuteron mass-shift parametrization

- **Unified** mass-shift **parameterization** $(\gamma = 1)$ [S. Burrello, S. Typel, EPJA 58, 120 (2022)]

$$\Delta m_d(x) = \frac{ax}{1+bx} + cx^{\eta+1}\left[1 - \tanh(x)\right] + fx^{\gamma}\tanh(gx), \qquad x = \frac{n_b}{n_0}$$

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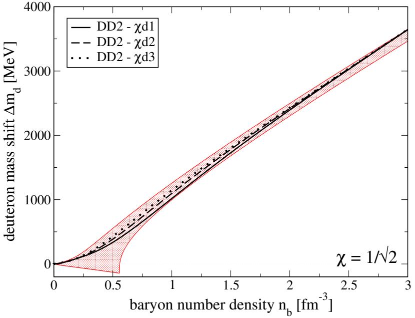

	a	b	c	η	f	g
DD2 - χd1	541.726060	243.472387	99.677247	1.656159	181.113975	0.18
DD2 - χd2	541.726060	243.472387	70.476986	1.230947	181.113975	0.22
DD2 - χd3	541.726060	243.472387	41.777908	0.257252	181.113975	0.26

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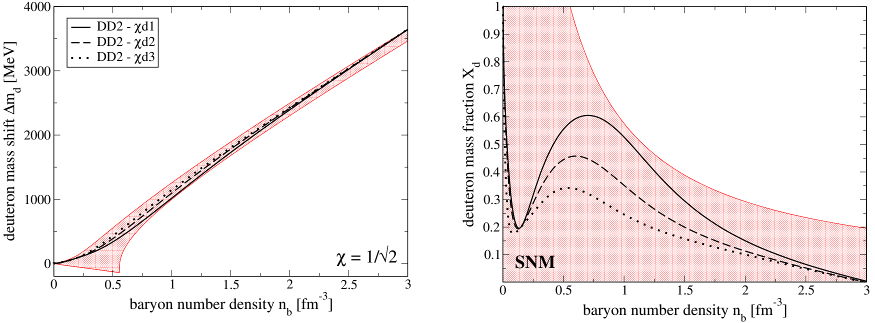

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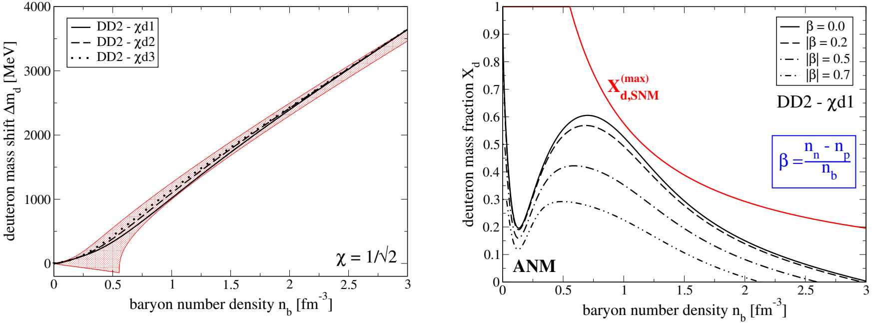

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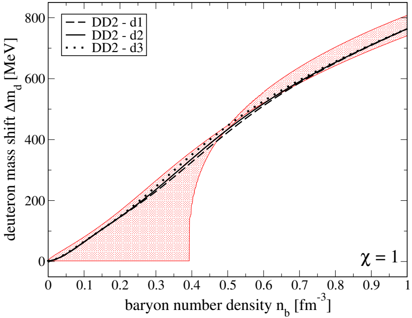
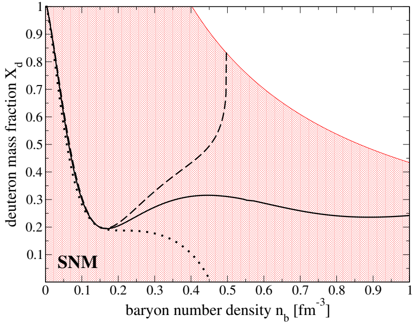
Deuteron mass-shift parametrization: $\chi = 1$

- **Unified** mass-shift **parameterization** $(\gamma = 2/3)$ [S. Burrello, S. Typel, EPJA 58, 120 (2022)]

$$\Delta m_d(x) = \frac{ax}{1+bx} + cx^{\eta+1}\left[1-\tanh(x)\right] + fx^{\gamma}\tanh(gx), \qquad x = \frac{n_b}{n_0}$$



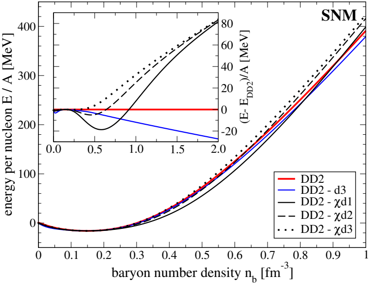


	a	b	c	η	f	g
DD2 - d1	541.726060	243.472387	−83.230901	3.491787	214.368137	0.65
DD2 - d2	541.726060	243.472387	−98.923123	3.200967	214.368137	0.67632
DD2 - d3	541.726060	243.472387	−140.309501	2.715545	214.368137	0.75

SNM: impact on EoS and matter incompressibility

- **Attraction** in presence of **quasi-deuterons** $\Longleftrightarrow$ attraction/**repulsion** for Γ_i-**refit**

[S. Burrello, S. Typel, EPJA 58, 120 (2022)]



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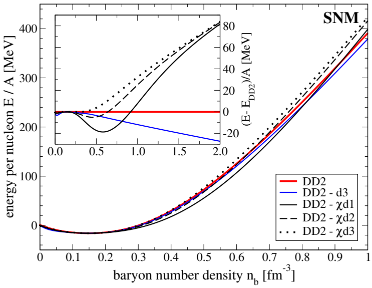

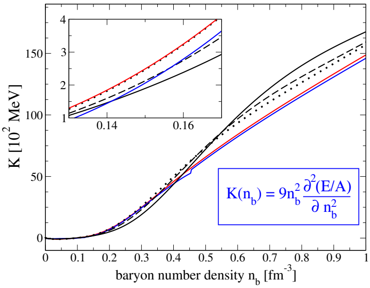

	DD2	DD2-d3	DD2-χd1	DD2-χd2	DD2-χd3
K_0 [MeV]	242.7	199.6	185.3	207.3	240.3

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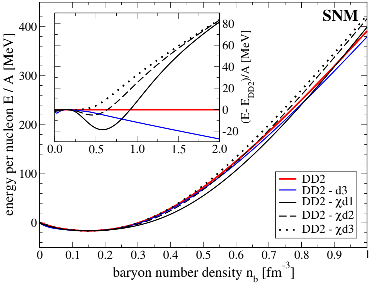
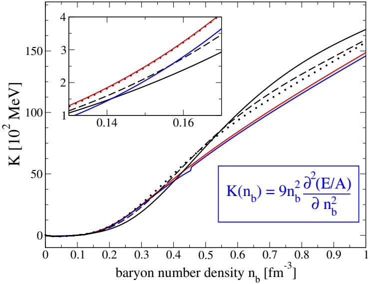


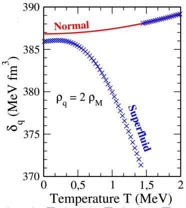


	DD2	DD2-d3	DD2-χd1	DD2-χd2	DD2-χd3
K_0 [MeV]	242.7	199.6	185.3	207.3	240.3

- Discontinuity $\Rightarrow$ 2^{nd} order phase transition?

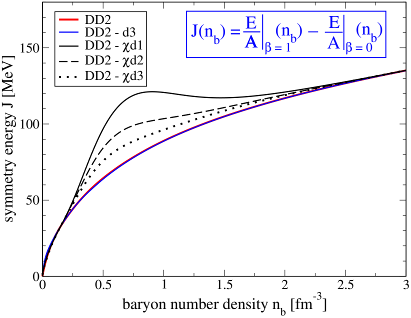
[S. Burrello, M. Colonna, F. Matera, PRC 94, 012801(R) (2016)]

[S. Burrello, M. Colonna, F. Matera, PRC 89, 057604 (2014)]



Effect on symmetry energy and its slope

[S. Burrello, S. Typel, EPJA 58, 120 (2022)]



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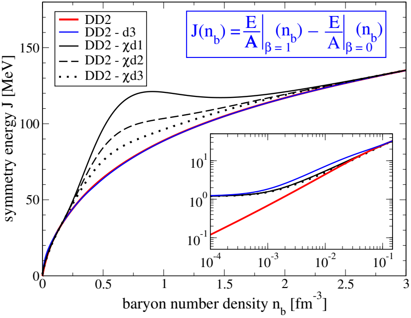

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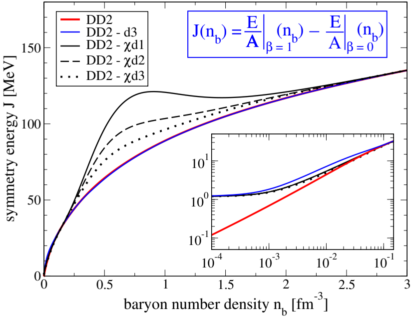

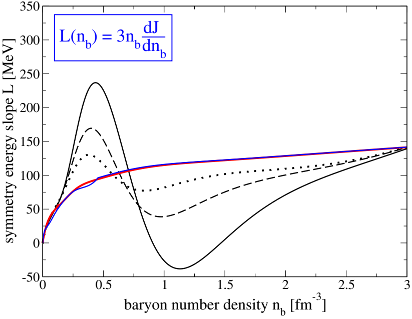

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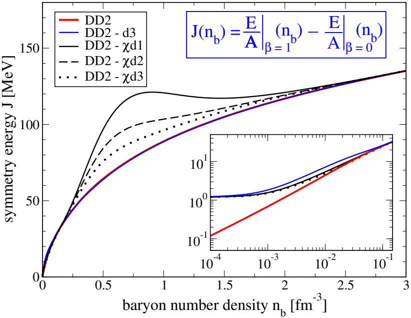
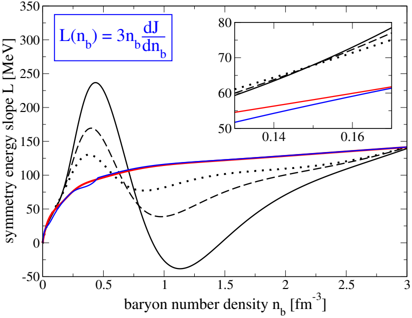
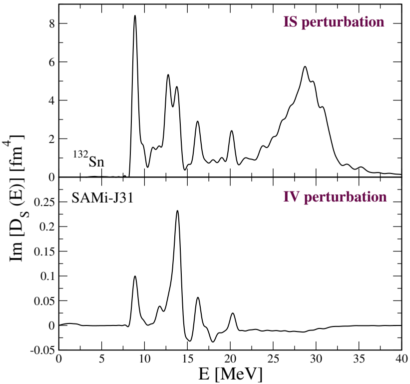




	DD2	DD2-d3	DD2-χd1	DD2-χd2	DD2-χd3
L_0 [MeV]	57.94	56.49	67.50	67.50	67.50

Coupling between IS and IV modes

- Symmetric nuclear matter: **IS** and **IV** modes are **decoupled**
- Neutron-rich systems: n and p oscillate with **different amplitudes** ⇒ coupling



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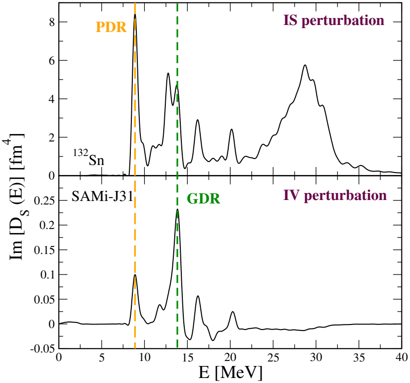

IS response

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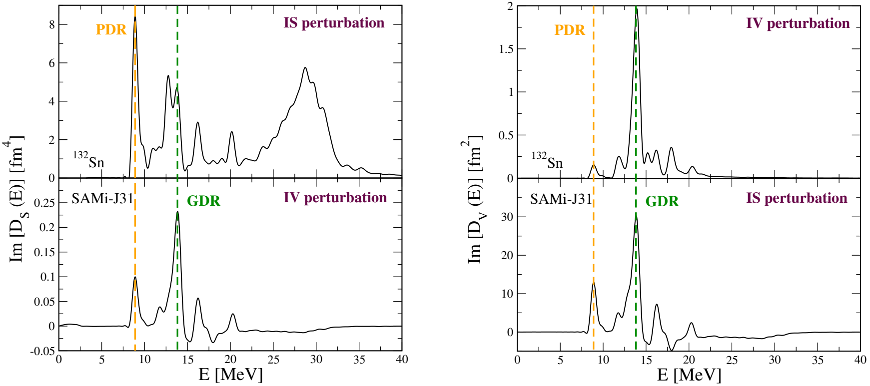
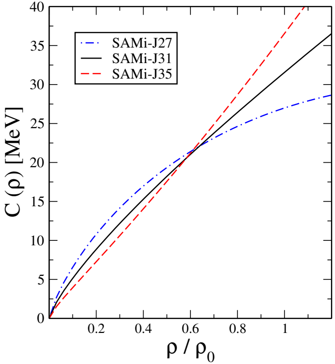


IS response **IV response**

Influence of the effective interaction

- **SAMi-J** interactions
  [X. Roca-Maza et al., PRC87, (2013)]
  ⇒ isolate influence of **IV channel**
  $E_{sym}(\rho) = C(\rho) I^2$



- **Sensitivity** of E_{IV-GDR} to E_{sym} at **crossing**
- Role of symmetry energy **slope**:
  - IV **PDR**
- **Agreement** with **Vlasov** results

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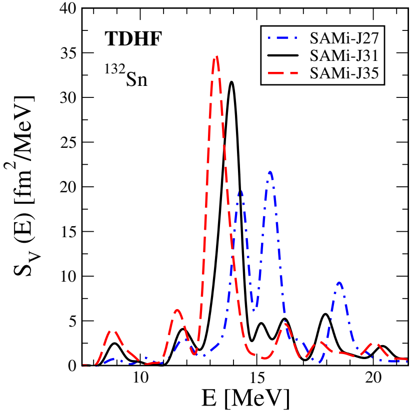

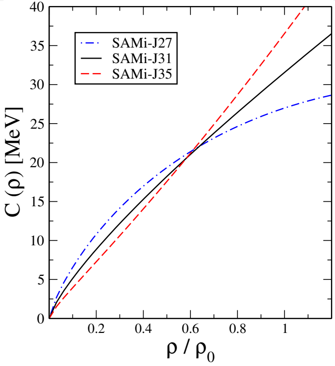

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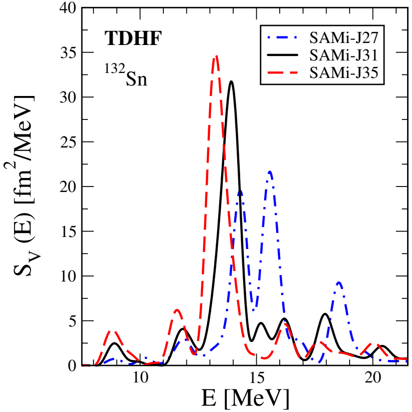

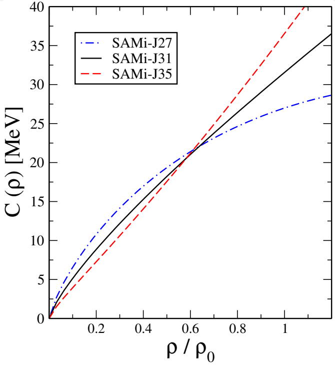

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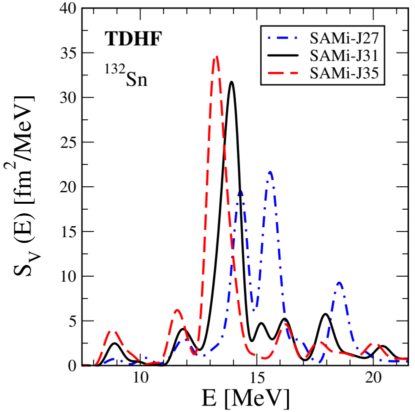

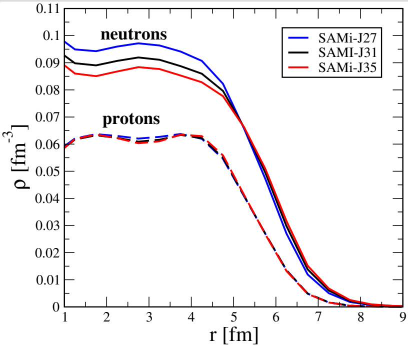

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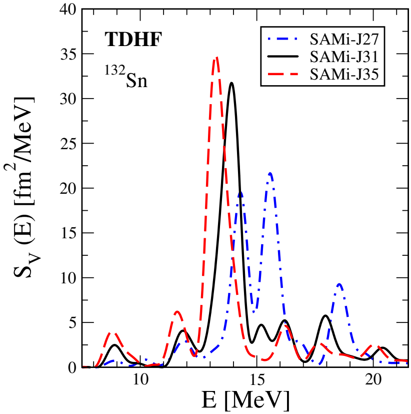
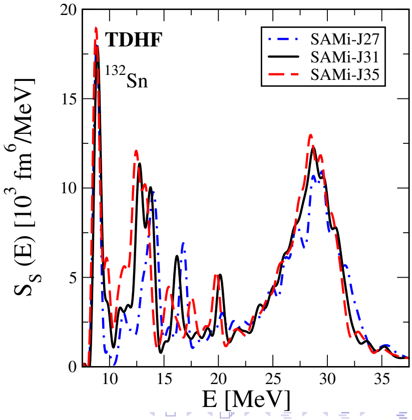
  - IV **PDR** ⇔ **neutron skin** thickness
- **Agreement** with **Vlasov** results

[Zheng, H. et al., PRC 94, (2016)]

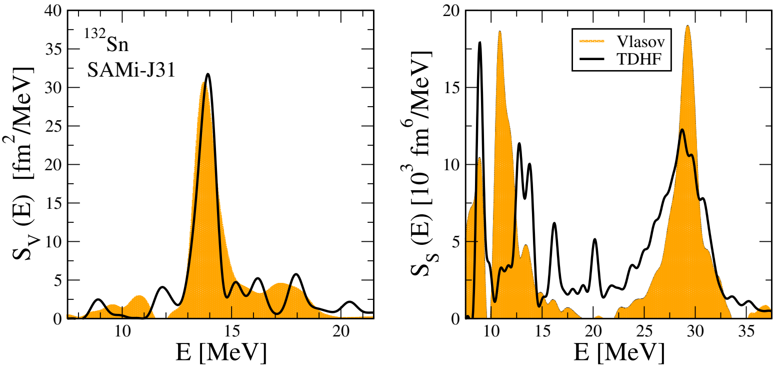
Influence of the effective interaction

- **SAMi-J** interactions
  [X. Roca-Maza et al., PRC87, (2013)]
  ⇒ isolate influence of **IV channel**
  $E_{\text{sym}}(\rho) = C(\rho) I^2$





Comparison between Vlasov and TDHF model



- Good **reproduction** of IV GDR and IS GDR
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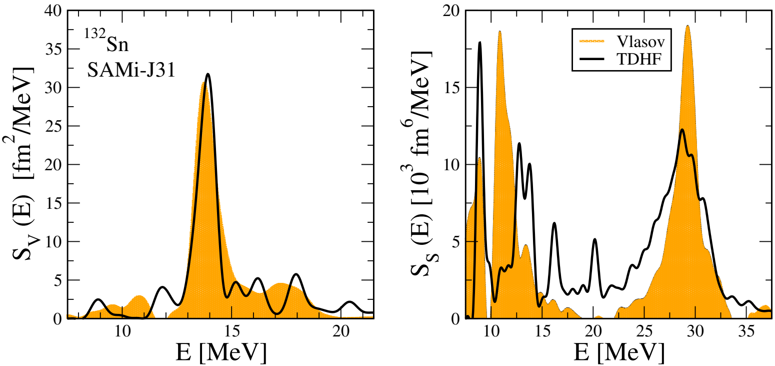

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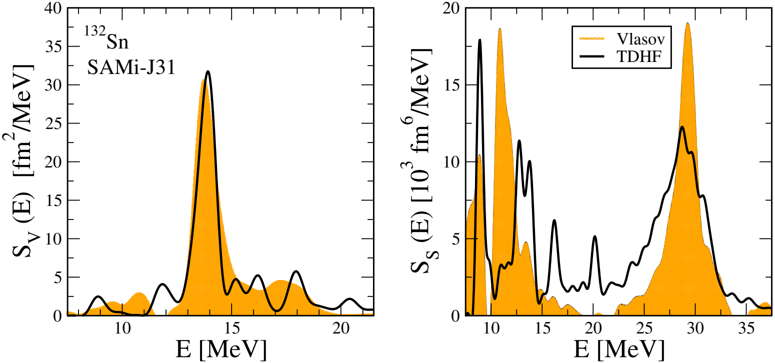

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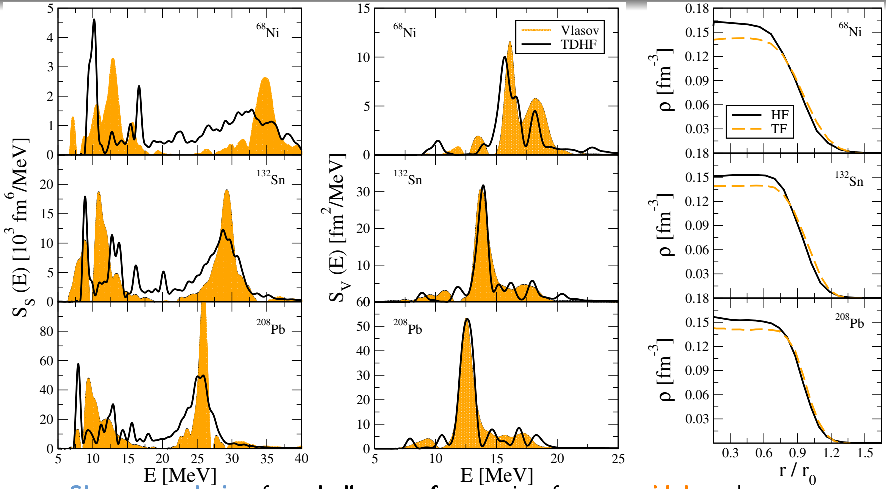
Link between nuclear response and density profiles



- **Sharper evolution** from **bulk** to **surface** region favor **toroidal** mode

  Smoother density profile leads to **robust PDR** oscillations

[S. Burrello et al., Phys. Rev. C 99, 054314 (2019)]

Sn isotope chain: N/Z evolution of PDR

- Dipole response evolution with the **neutron/proton content** $\Rightarrow$ Sn isotopes chain
- **Question**: Why **IV** PDR fraction of EWSR does not grow from N=70 to N=82?

  [S. Ebata, T. Nakatsukasa, T. Inakura, Phys. Rev. C 90, 024303 (2014)]
- **Explanation**: it reflects the **decrease** in the IS fraction and IS dipole strength

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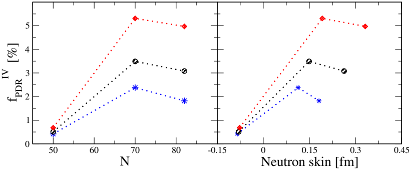

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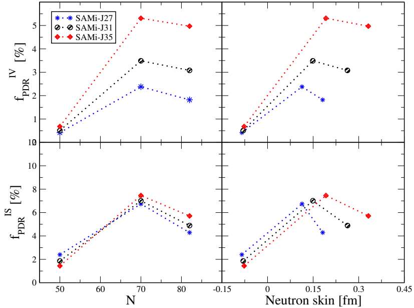

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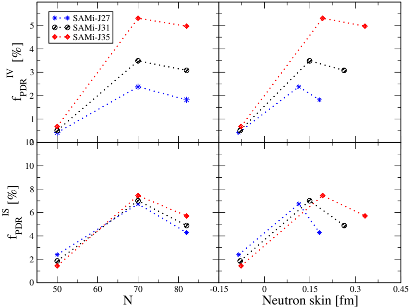

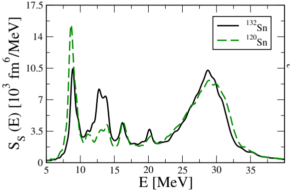

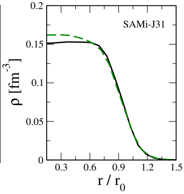

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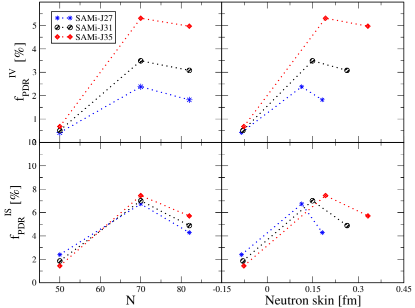

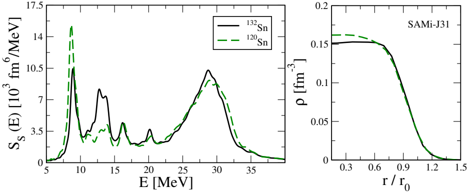

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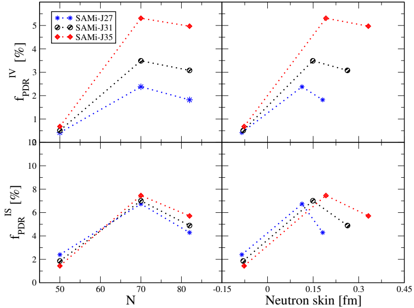

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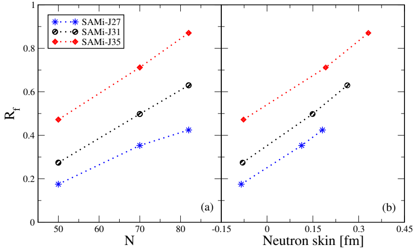

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Comparison between TDHF and RPA

- **TDHF** and **RPA** **equivalent** in zero-amplitude limit, despite **technical procedures**
- Question: which **numerical parameters** ensure the best **agreement**?
- Dependence on **box size** (i.e. **discretization** of **continuum** single-particle states)

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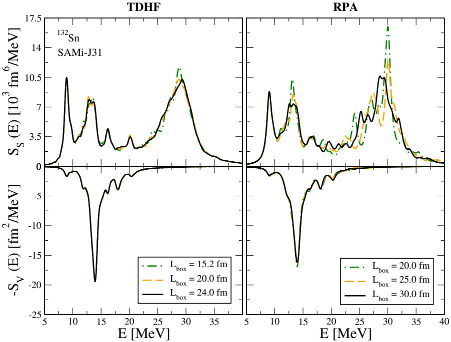

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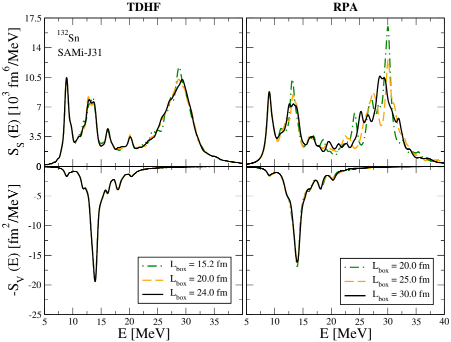

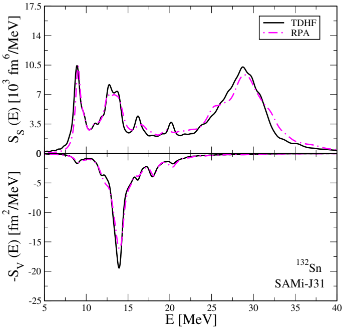

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