Achievements and future perspectives in microscopic optical potentials

MONSTRE Meeting *May 11, 2023*

Paolo Finelli

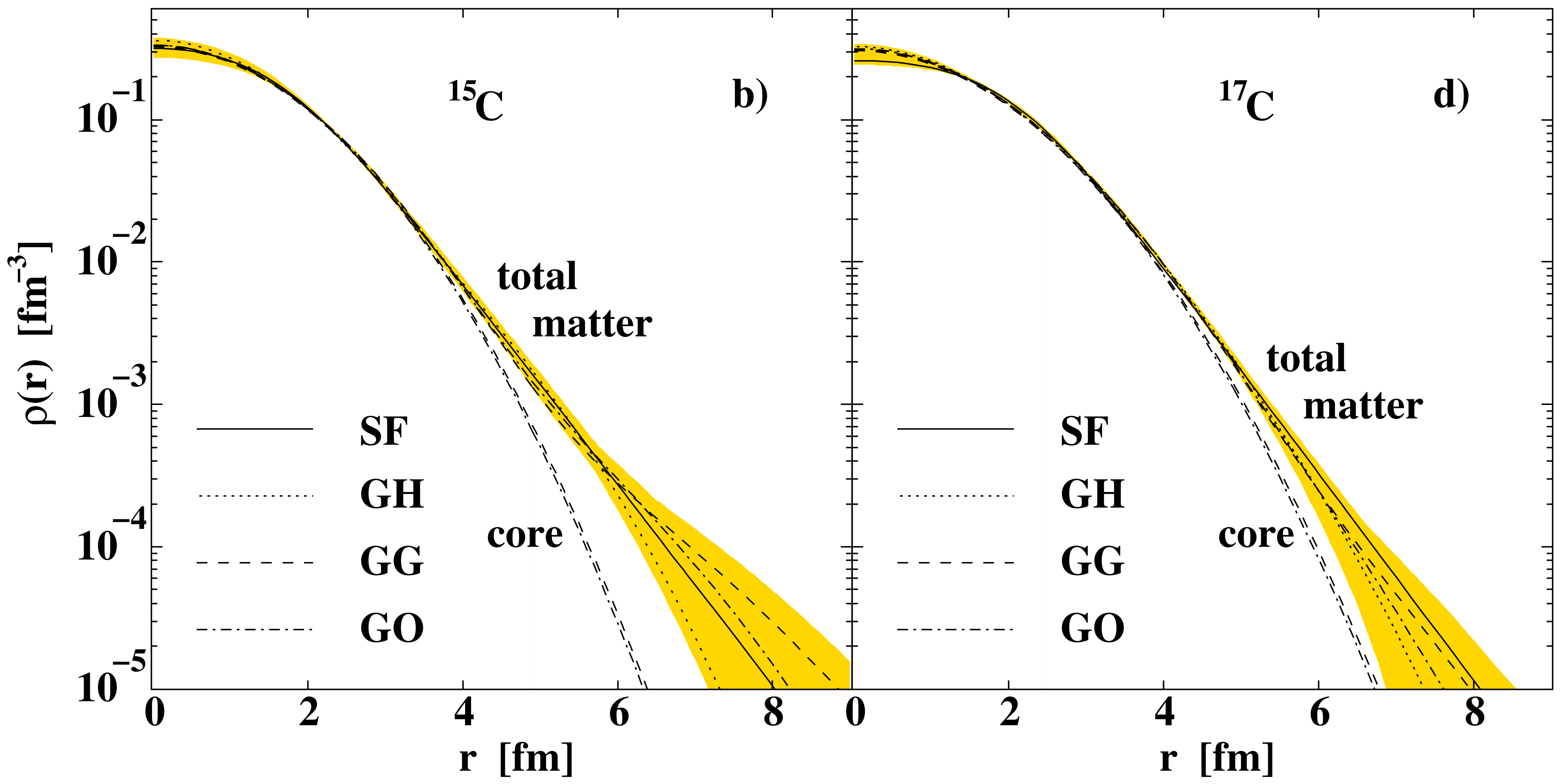
University of Bologna and INFN

Motivations

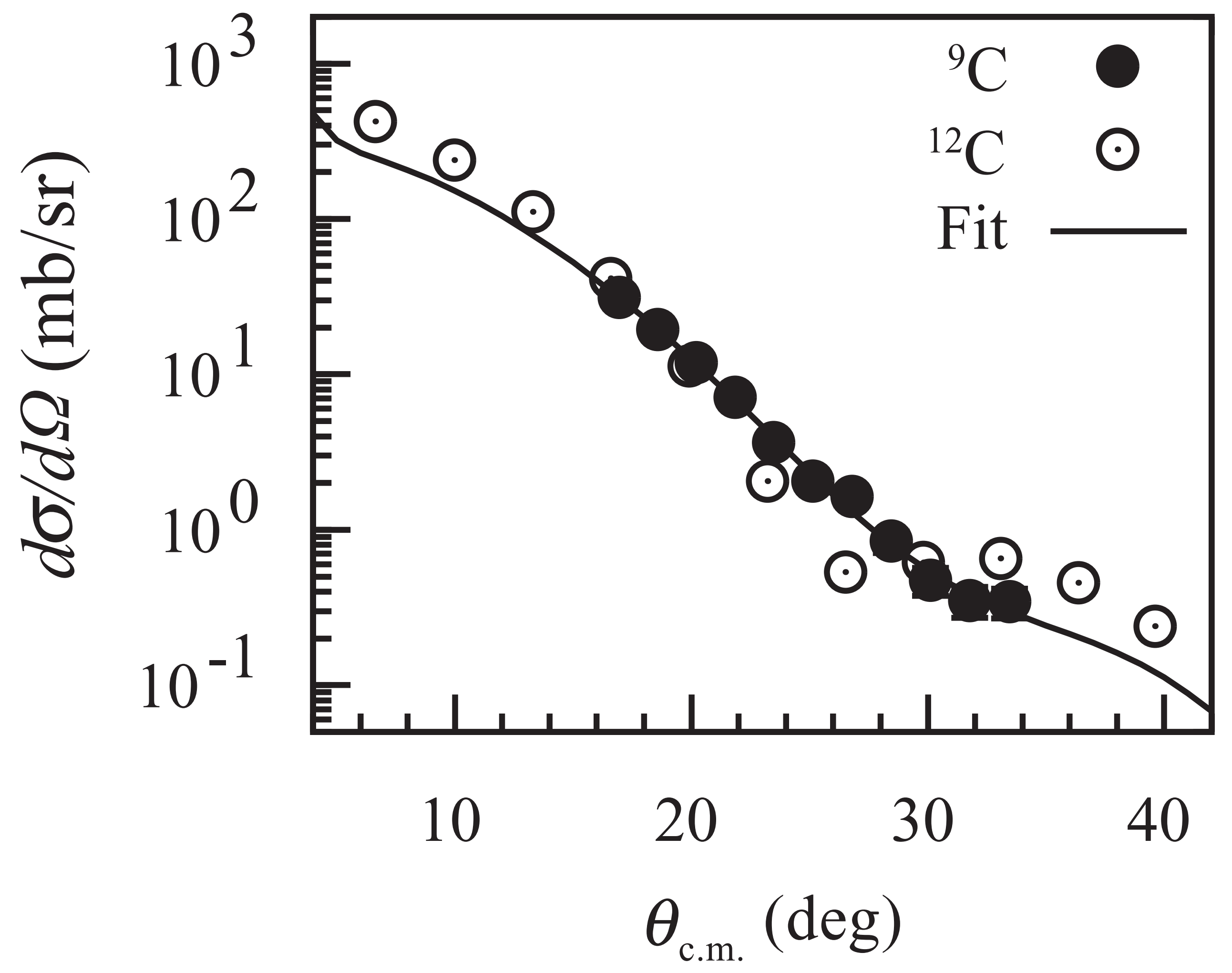
- Increasing experimental efforts to develop the technologies necessary to study the elastic proton scattering in inverse kinematics
- Attempts to use such experiments to determine the matter distribution of nuclear systems at intermediate energies

[Sakaguchi, Zenihiro, PPNP 97 (2017) 1–52]

  - Measurements are not free from sizeable uncertainties
  - The Glauber model is used to analyse the data
  - An essential step in the data analysis is the subtraction of contributions from the inelastic scattering



[Dobrovolsky et al., NPA 1008 (2021) 122154]



[Matsuda et al., PRC **87**, 034614 (2013)]

Develop a microscopic approach to make reliable predictions for elastic and inelastic scattering

Optical potentials

Phenomenological

Unfortunately, current used optical potentials for low-energy reactions are phenomenological and primarily constrained by elastic scattering data.

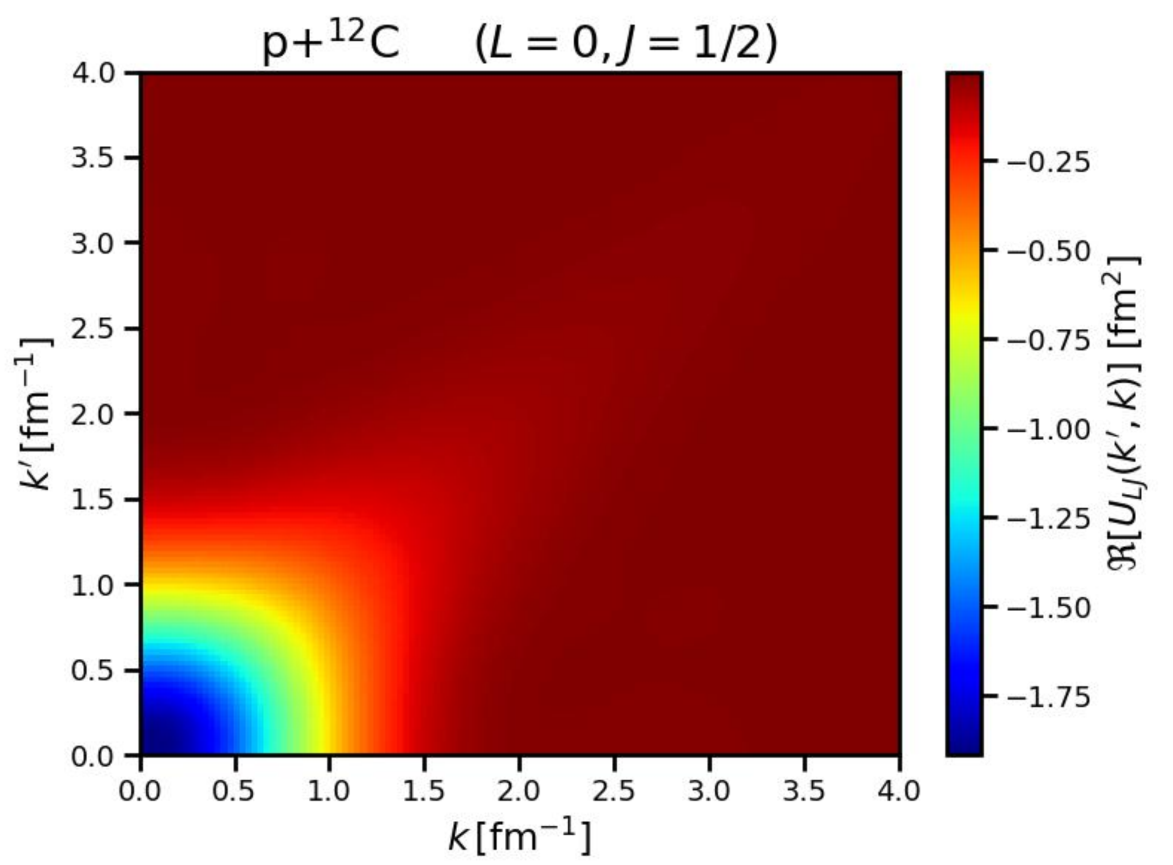
Unreliable when extrapolated beyond their fitted range in energy and nuclei

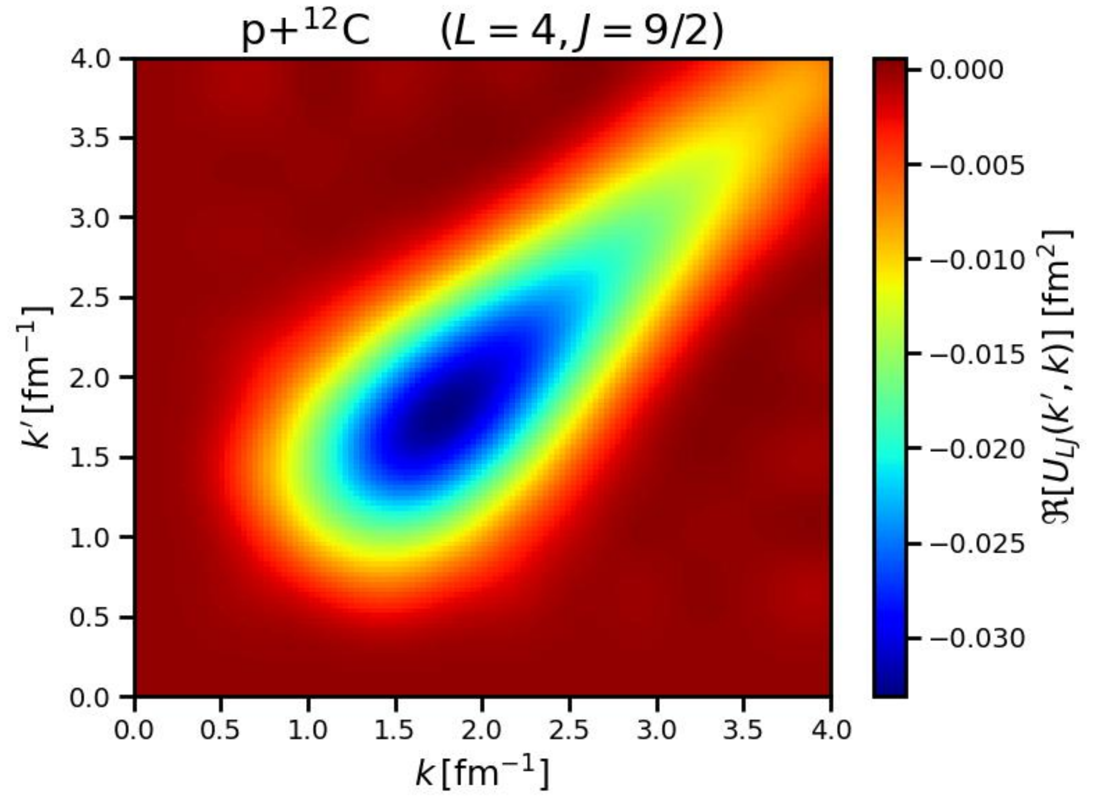
$$
\begin{aligned}
V(r) &= -V_R f_R(r) - iW_V f_V(r) \\
&+ 4a_{VD} V_D \frac{d}{dr} f_{VD}(r) + 4ia_{WD} \frac{d}{dr} f_{WD}(r) \\
&+ \frac{\lambda_\pi^2}{r} \left[V_{SO} \frac{d}{dr} f_{VSO}(r) + iW_{SO} \frac{d}{dr} f_{WSO}(r) \right] \vec{\sigma} \cdot \vec{l}
\end{aligned}
$$

Microscopic

Existing microscopic optical potentials can be developed in a low- (Feshbach theory) or high-energy regime (Watson multiple scattering theory).
Calculations are more difficult.

No fit to experimental data





The first-order optical potential

Free two-body scattering matrix

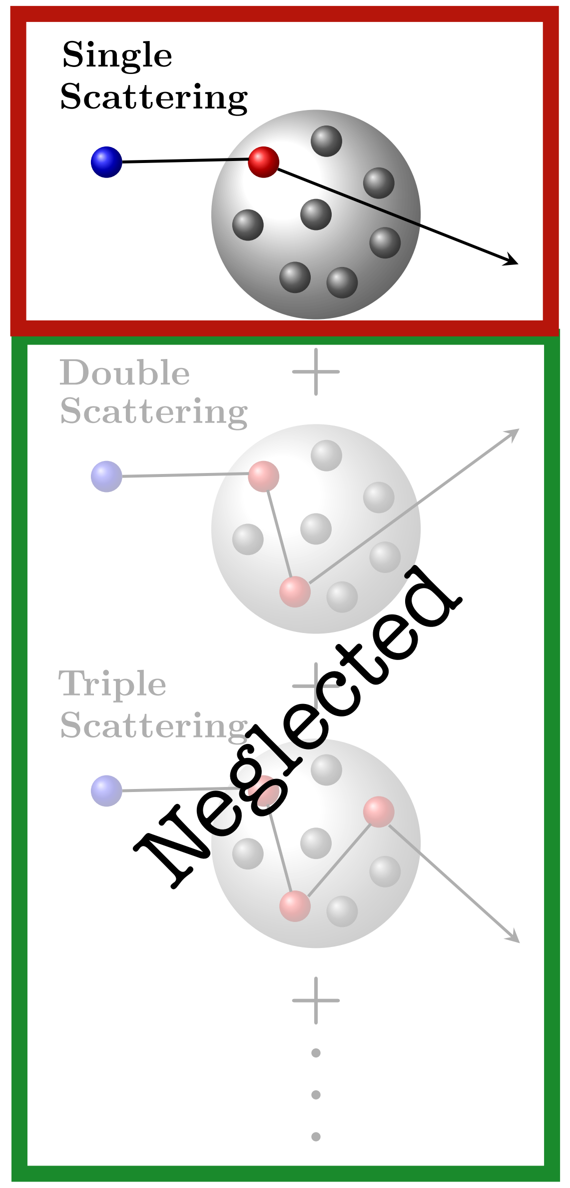
$$t_{0i} = v_{0i} + v_{0i}\, g_{0i}\, t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

$\mathbf{p} = (p, n, \bar{p})$



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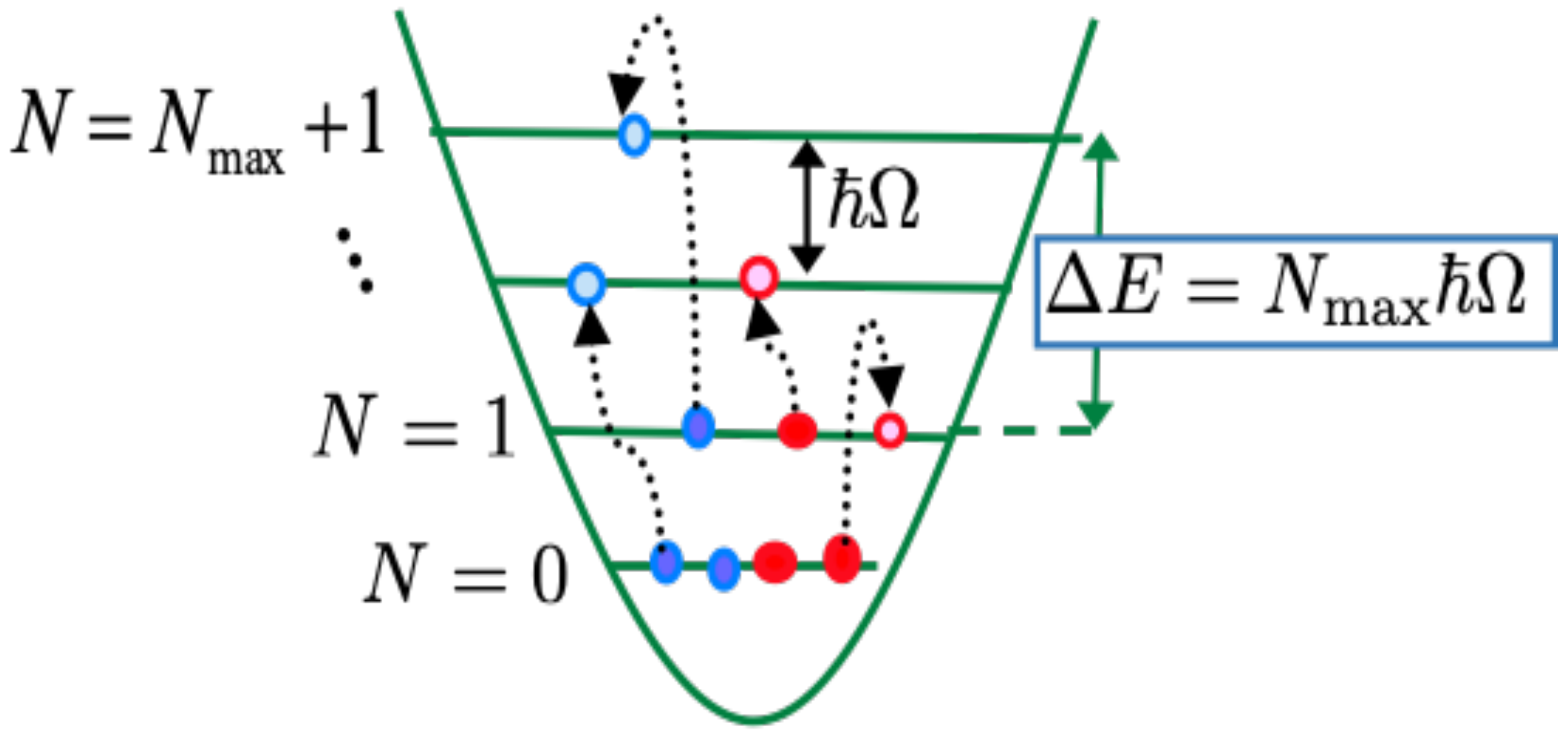
Nonlocal one-body density



- Computationally expensive
- Obtained from the No-Core Shell Model
- Calculation performed with **NN** and **3N** interaction

$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

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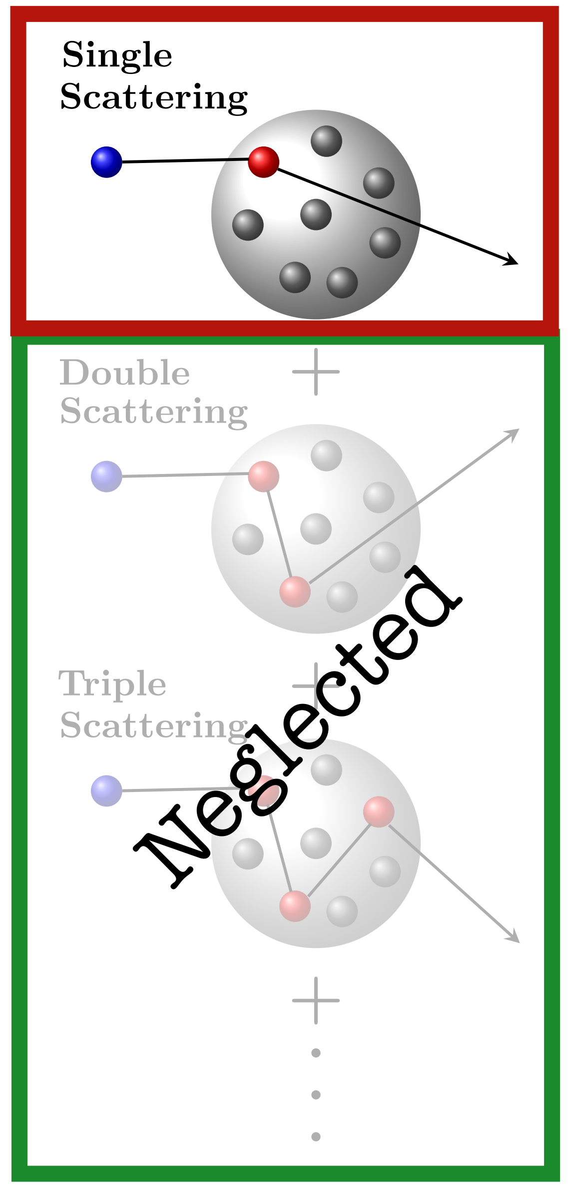

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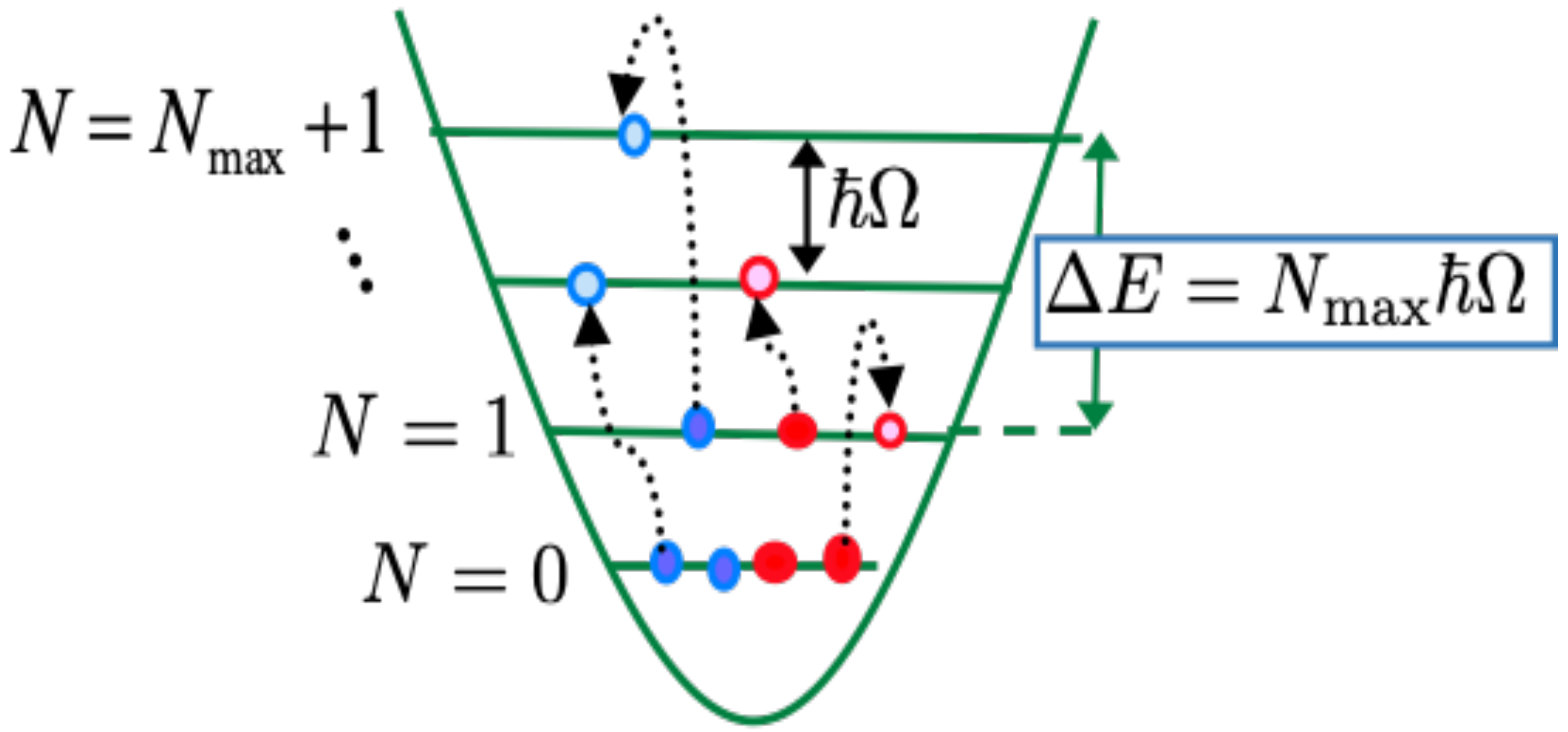

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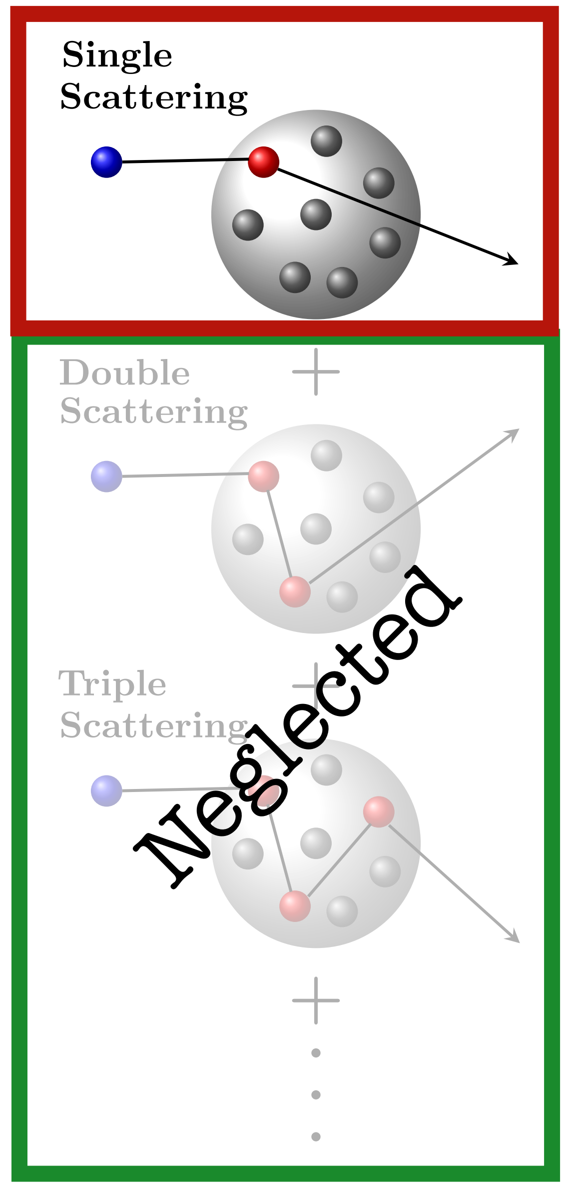
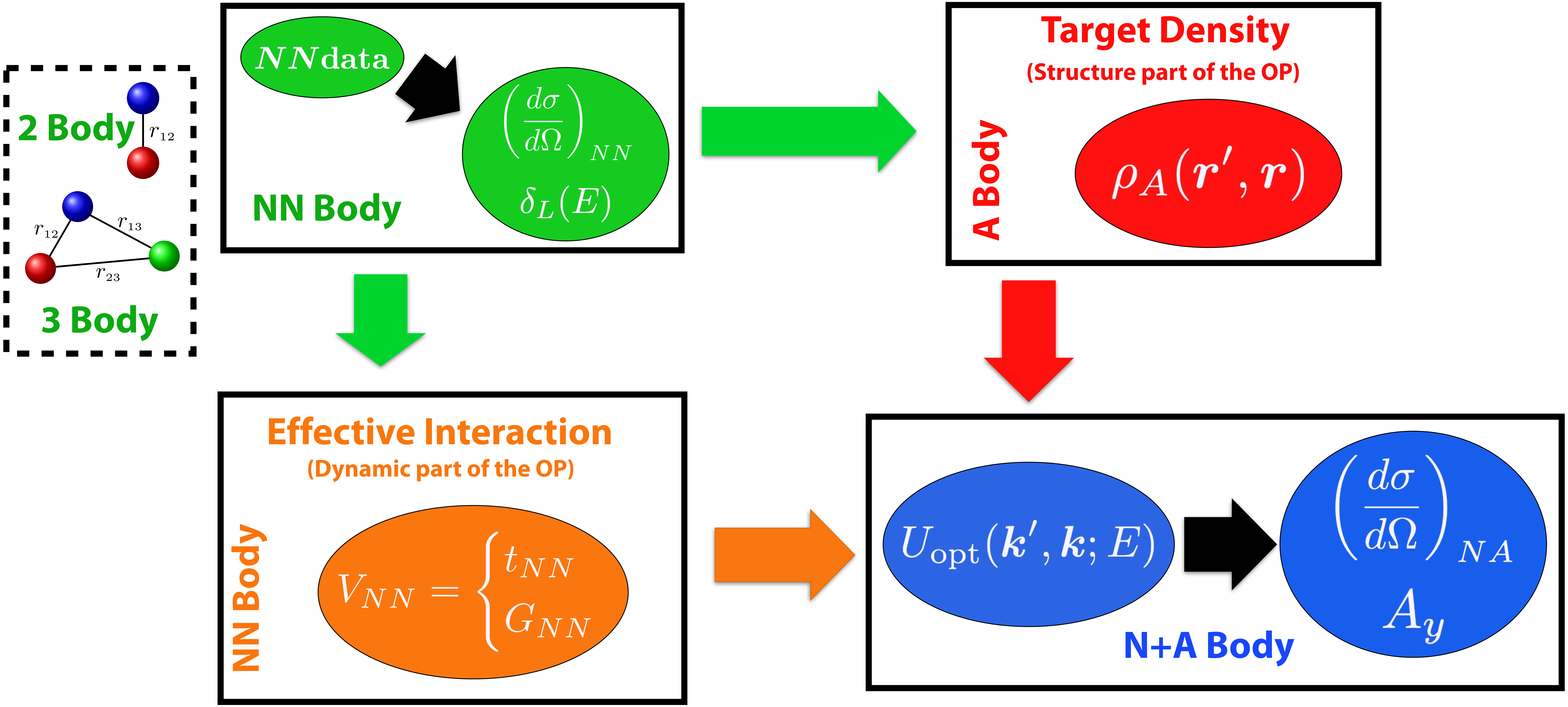


$$t^{(NA)}_{\mathbf{p}N} = \eta\, t^{(NN)}_{\mathbf{p}N}$$

It imposes the Lorentz invariance of flux when we pass from the NA to the NN frame where the t matrices are evaluated

Møller factor

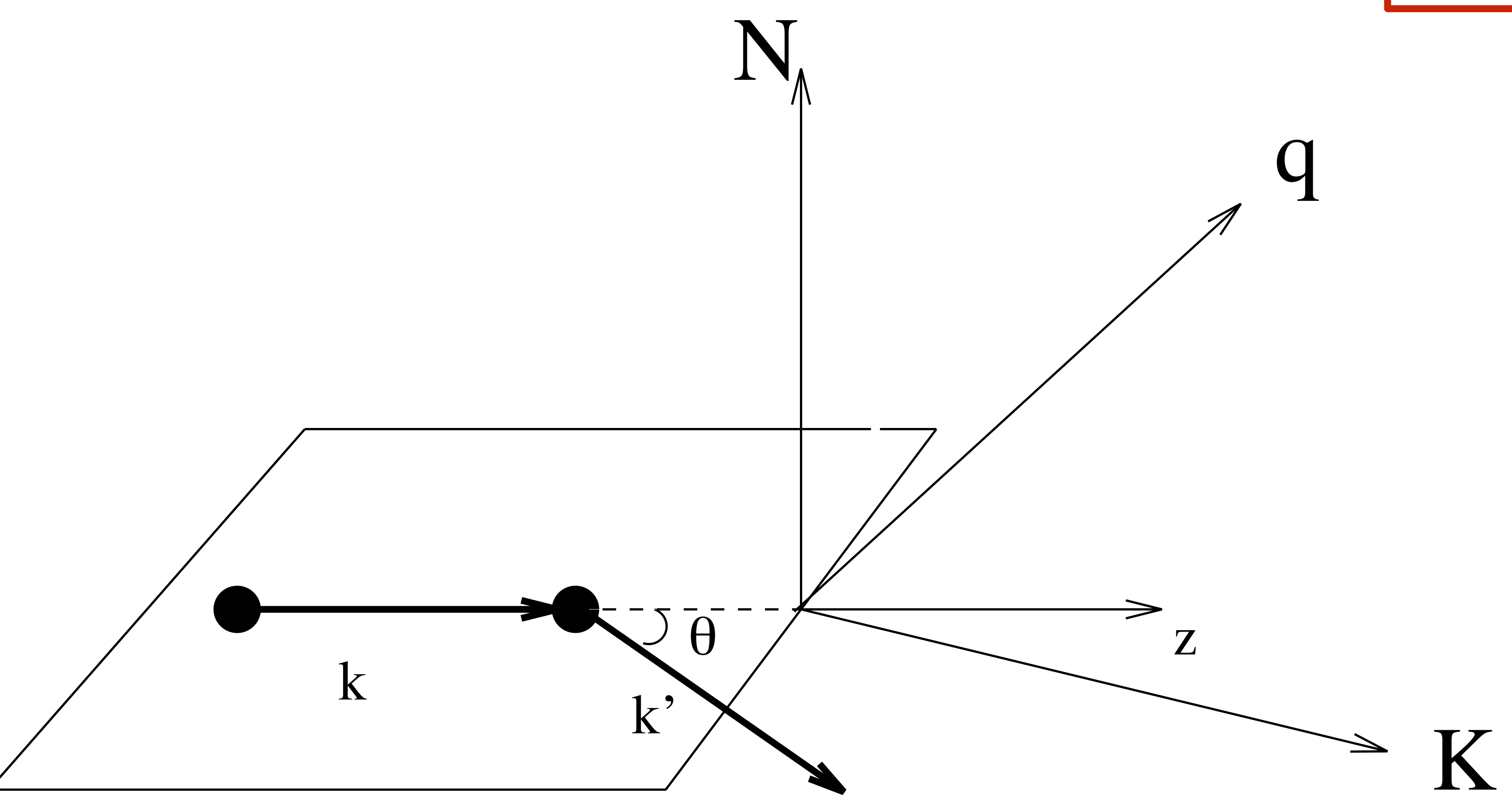
Road map to the microscopic optical potential



Theoretical framework - first order expansion

$$\boxed{\hat{U}(\boldsymbol{k}', \boldsymbol{k}; \omega) = (A-1)\,\langle \boldsymbol{k}', \Phi_A | t(\omega) | \boldsymbol{k}, \Phi_A \rangle}$$

$$\boldsymbol{q} \equiv \boldsymbol{k}' - \boldsymbol{k}\,, \qquad \boldsymbol{K} \equiv \frac{1}{2}(\boldsymbol{k}' + \boldsymbol{k})$$



$$\hat{U}(\boldsymbol{q}, \boldsymbol{K}; \omega) = \hat{U}^c(\boldsymbol{q}, \boldsymbol{K}; \omega) + \frac{i}{2}\boldsymbol{\sigma} \cdot \boldsymbol{q} \times \boldsymbol{K}\, \hat{U}^{ls}(\boldsymbol{q}, \boldsymbol{K}; \omega)$$

$$\hat{U}^c(\boldsymbol{q}, \boldsymbol{K}; \omega) = \frac{A-1}{A}\,\eta(\boldsymbol{q}, \boldsymbol{K}) \times \sum_{N=n,p} t^c_{pN}\left[\boldsymbol{q}, \frac{A+1}{A}\boldsymbol{K}; \omega\right] \rho_N(q)$$

Central component

$$\hat{U}(\boldsymbol{q}, \boldsymbol{K}; \omega) = \frac{A-1}{A}\,\eta(\boldsymbol{q}, \boldsymbol{K}) \times \sum_{N=n,p} t_{pN}\left[\boldsymbol{q}, \frac{A+1}{A}\boldsymbol{K}; \omega\right] \rho_N(q)$$

$\eta(\boldsymbol{q}, \boldsymbol{K}) =$ **Møller factor**

$$\left[\frac{E_{\text{proj}}(\boldsymbol{\kappa}')\, E_{\text{proj}}(-\boldsymbol{\kappa}')\, E_{\text{proj}}(\boldsymbol{\kappa})\, E_{\text{proj}}(-\boldsymbol{\kappa})}{E_{\text{proj}}(\boldsymbol{k}')\, E_{\text{proj}}\left(-\frac{\boldsymbol{q}}{2} - \frac{\boldsymbol{K}}{A}\right)\, E_{\text{proj}}(\boldsymbol{k})\, E_{\text{proj}}\left(\frac{\boldsymbol{q}}{2} - \frac{\boldsymbol{K}}{A}\right)}\right]^{\frac{1}{2}}$$

Kerman, McManus and Thaler, Ann. Phys. **8** (1959) 551 and many others

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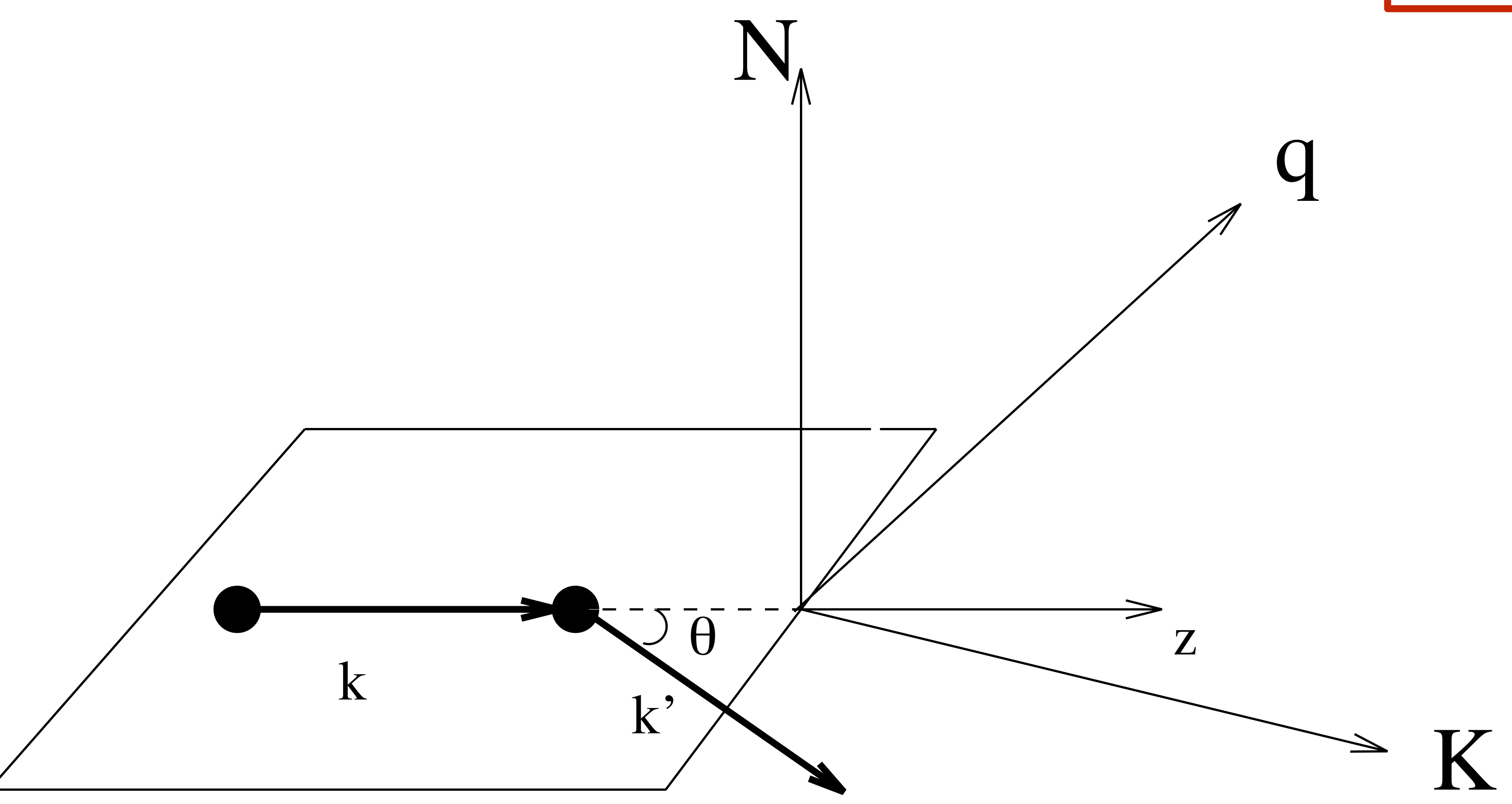

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$$\hat{U}(\boldsymbol{q}, \boldsymbol{K}; \omega) = \hat{U}^c(\boldsymbol{q}, \boldsymbol{K}; \omega) + \frac{i}{2}\boldsymbol{\sigma} \cdot \boldsymbol{q} \times \boldsymbol{K}\,\hat{U}^{ls}(\boldsymbol{q}, \boldsymbol{K}; \omega)$$

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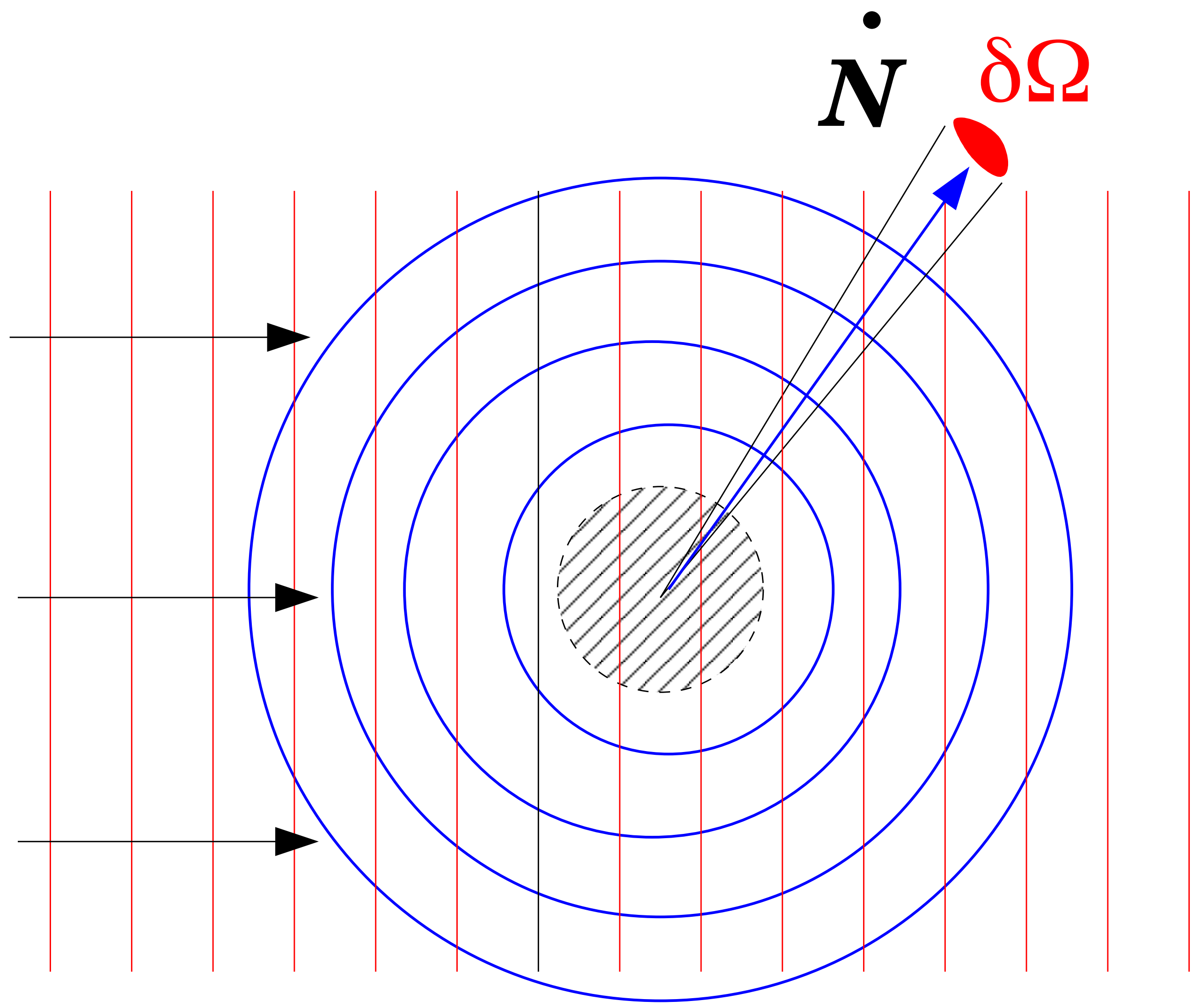
Central component

$$\hat{U}^{ls}(\boldsymbol{q}, \boldsymbol{K}; \omega) = \frac{A-1}{A}\,\eta(\boldsymbol{q}, \boldsymbol{K}) \left(\frac{A+1}{2A}\right) \times \sum_{N=n,p} t^{ls}_{pN}\left[\boldsymbol{q}, \frac{A+1}{A}\boldsymbol{K}; \omega\right] \rho_N(q)$$

Spin-orbit component

Kerman, McManus and Thaler, Ann. Phys. **8** (1959) 551 and many others

Theoretical framework - first order expansion



$$\dot{N} \sim \left\langle \frac{d\sigma}{d\Omega} \right\rangle \delta\Omega$$

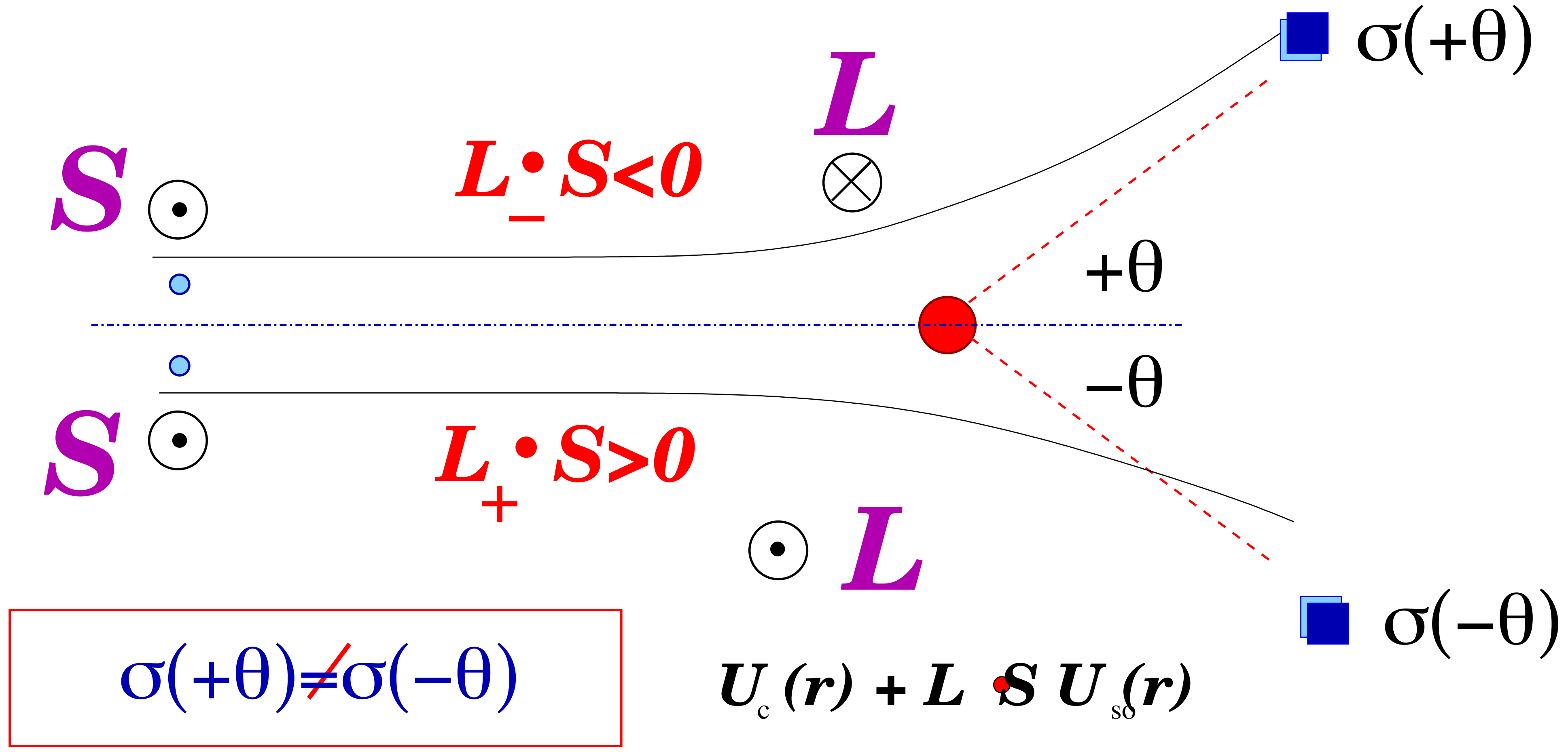
$$M(k_0,\theta) = A(k_0,\theta) + \boldsymbol{\sigma}\cdot\hat{\mathbf{N}}\,\underline{C(k_0,\theta)}$$

Spin-flip amplitude

$$A(\theta) = \frac{1}{2\pi^2}\sum_{L=0}^{\infty}\left[(L+1)F_L^+(k_0) + LF_L^-(k_0)\right]P_L(\cos\theta)$$

$$F_{LJ}(k_0) = -\frac{A}{A-1}4\pi^2\mu(k_0)\hat{T}_{LJ}(k_0,k_0;E)$$

$$C(\theta) = \frac{i}{2\pi^2}\sum_{L=1}^{\infty}\left[F_L^+(k_0) - F_L^-(k_0)\right]P_L^1(\cos\theta)$$



Differential cross section

$$\frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2$$

Analyzing power

$$A_y(\theta) = \frac{2\mathrm{Re}[A^*(\theta)\,C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

Spin rotation

$$Q(\theta) = \frac{2\mathrm{Im}[A(\theta)\,C^*(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

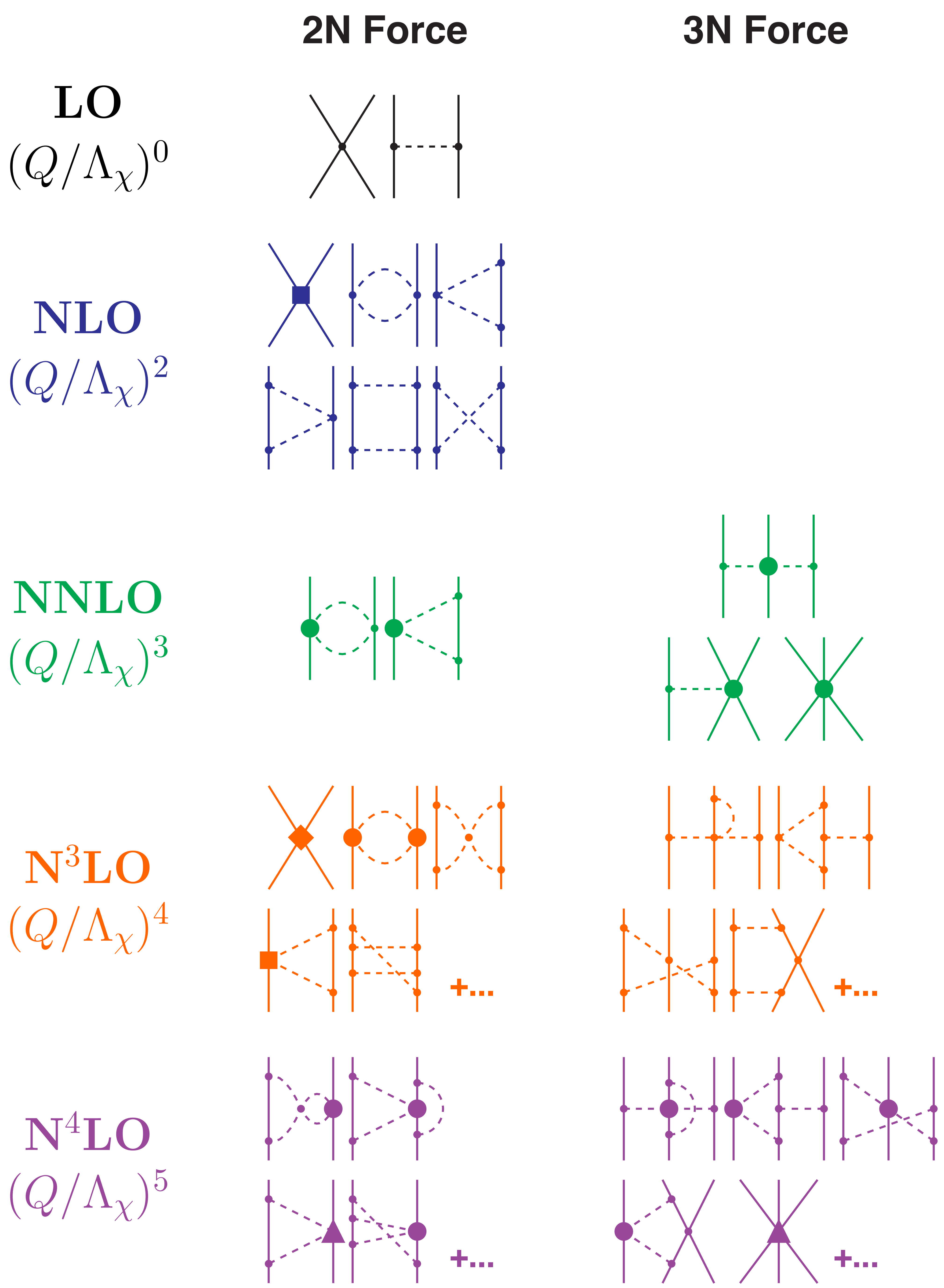
Rotation of the spin vector in the scattering plane, i.e. protons polarised along the **+*x*** axis have a finite probability of having the spin polarised along the **± *z*** axis after the collision

Chiral interactions

Advantages

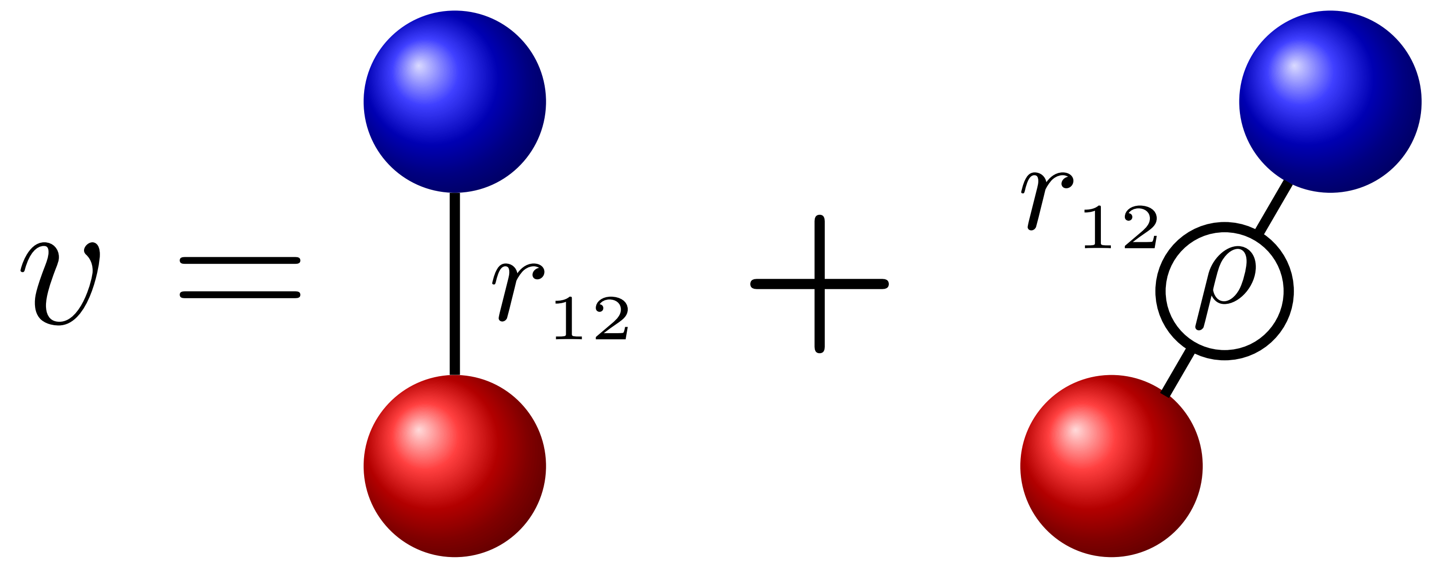
- QCD symmetries are consistently respected
- Systematic expansion (order by order we know exactly the terms to be included)
- Theoretical errors
- Two- and three-nucleon forces belong to the same framework

We use these interactions as the **only** input to calculate the **effective interaction** between projectile and target and the **target density**

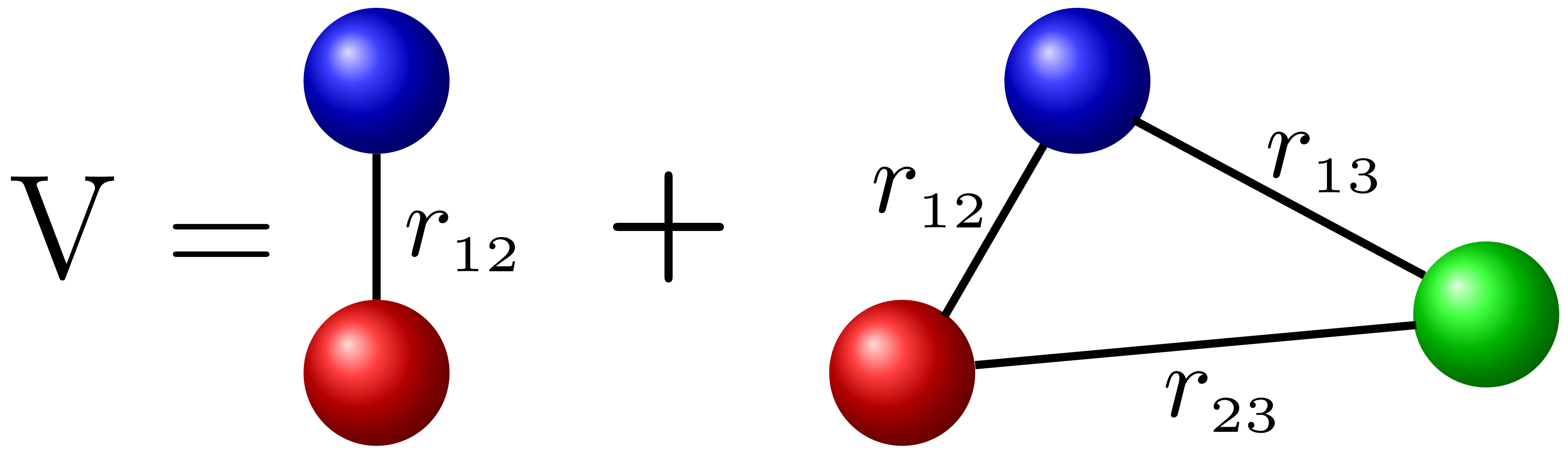


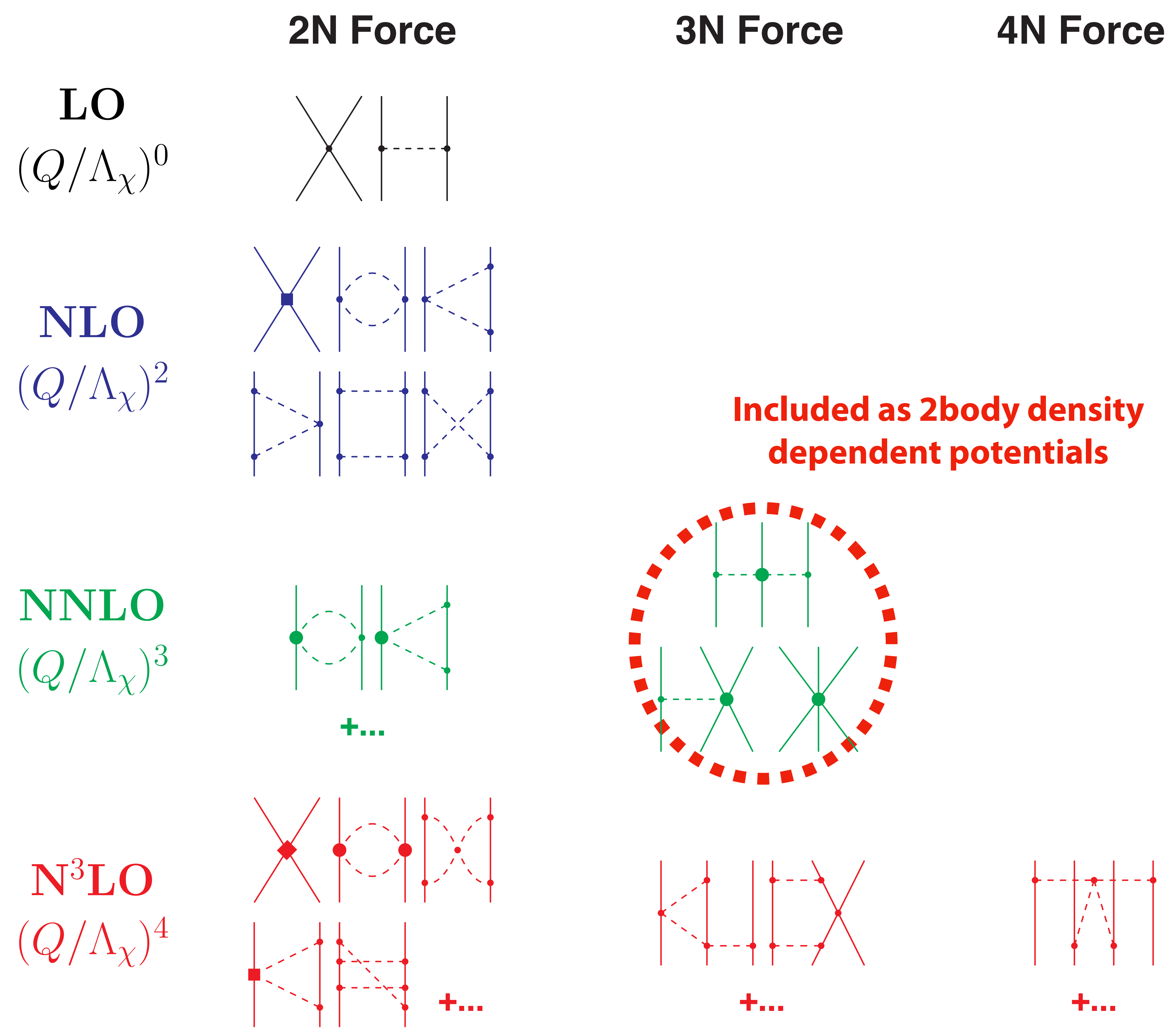
Chiral interactions

- **NN t matrix** computed with the addition of a density-dependent interaction



- **Nuclear density** computed with NN + 3N interaction





We employed both Machleidt and Epelbaum NN potentials at N3LO and N4LO order

Chiral effective field theory and nuclear forces, Phys. Rep. 503 (2011) 1-75
Modern Theory of Nuclear Forces, Rev. Mod. Phys. 81 (2009) 1773-1825

Spin-saturated nuclei

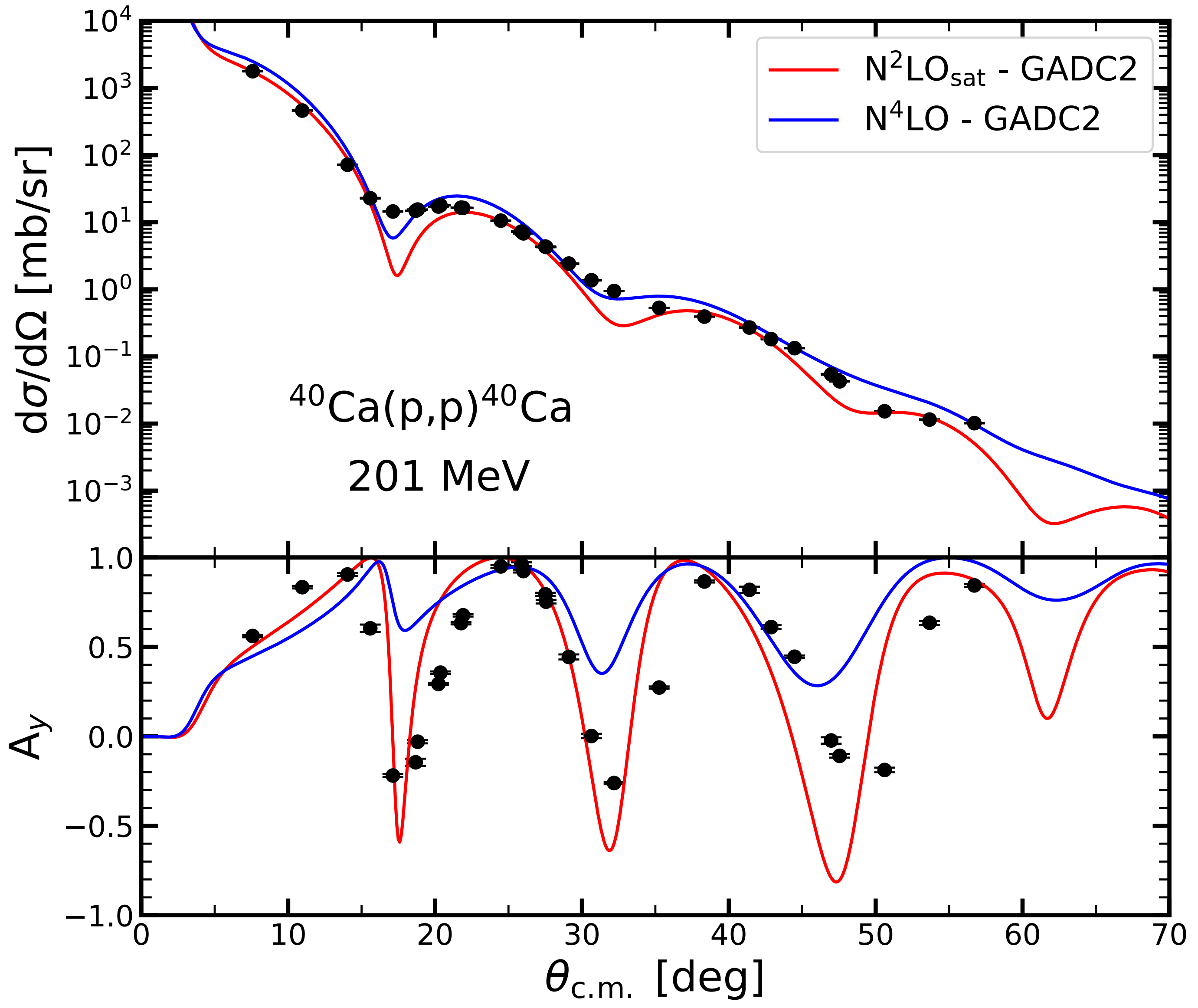
Differential cross section

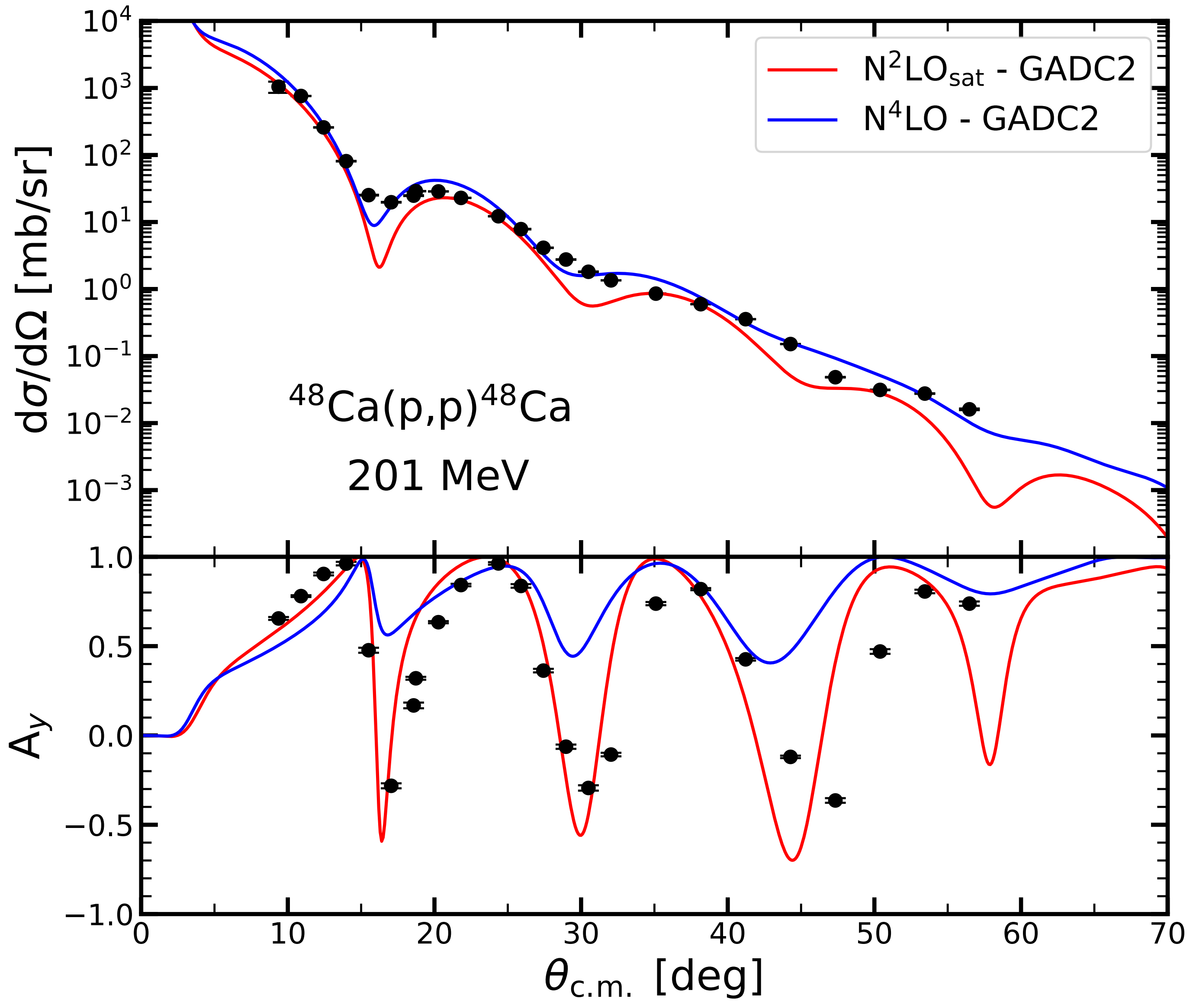
$$\frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2$$

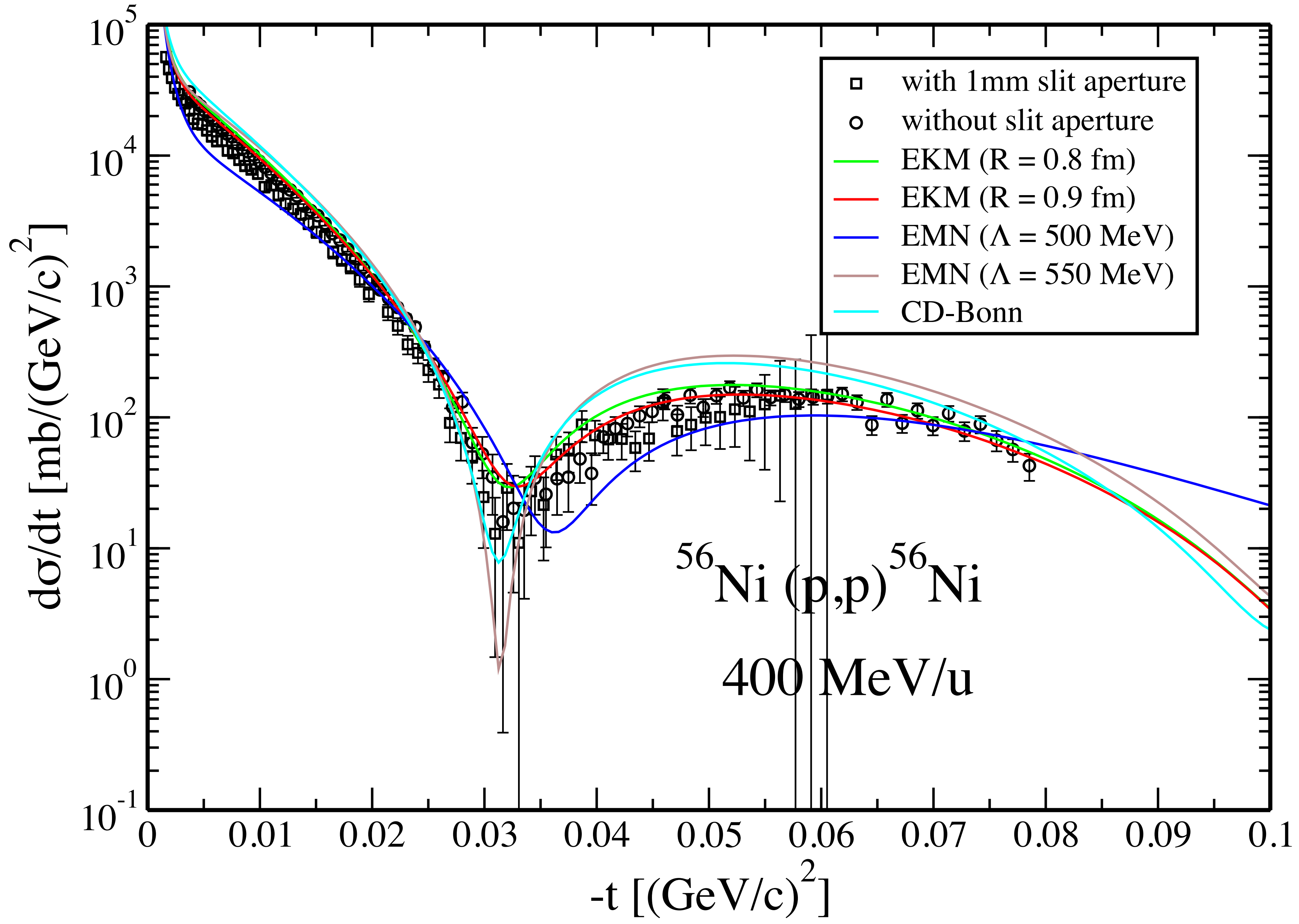
Analyzing power

$$A_y(\theta) = \frac{2\mathrm{Re}[A^*(\theta)\,C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

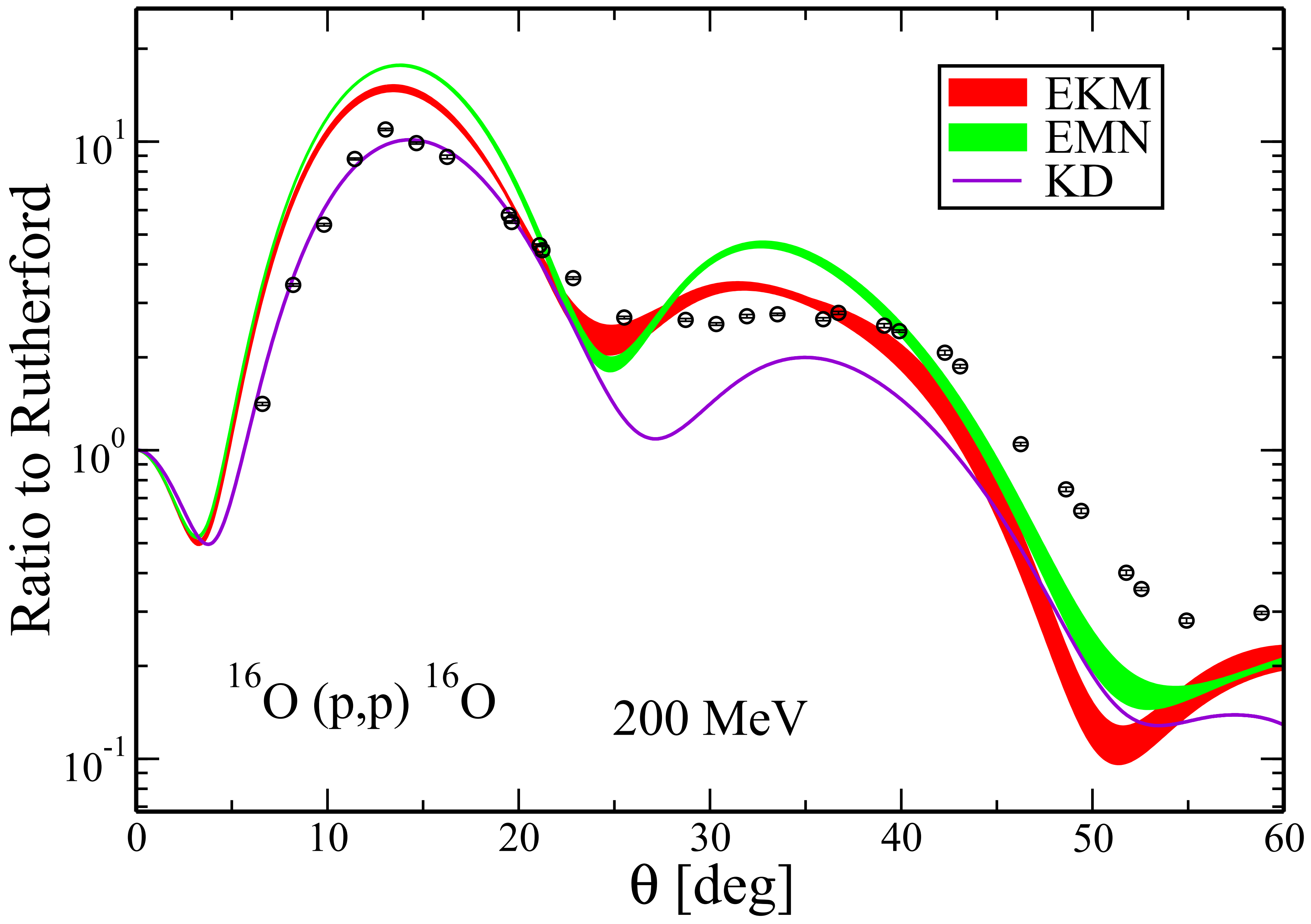
Vorabbi et al., in preparation



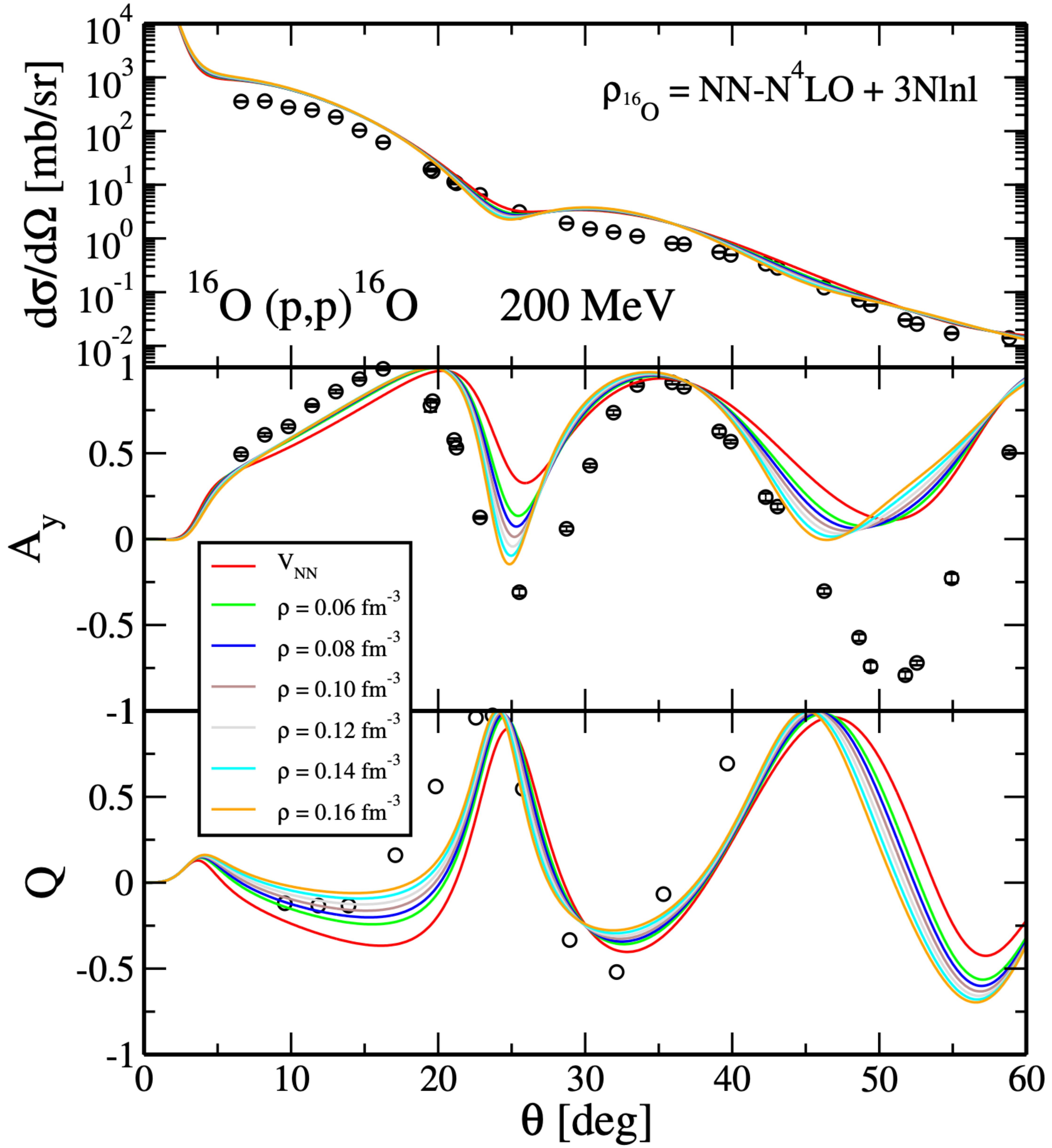




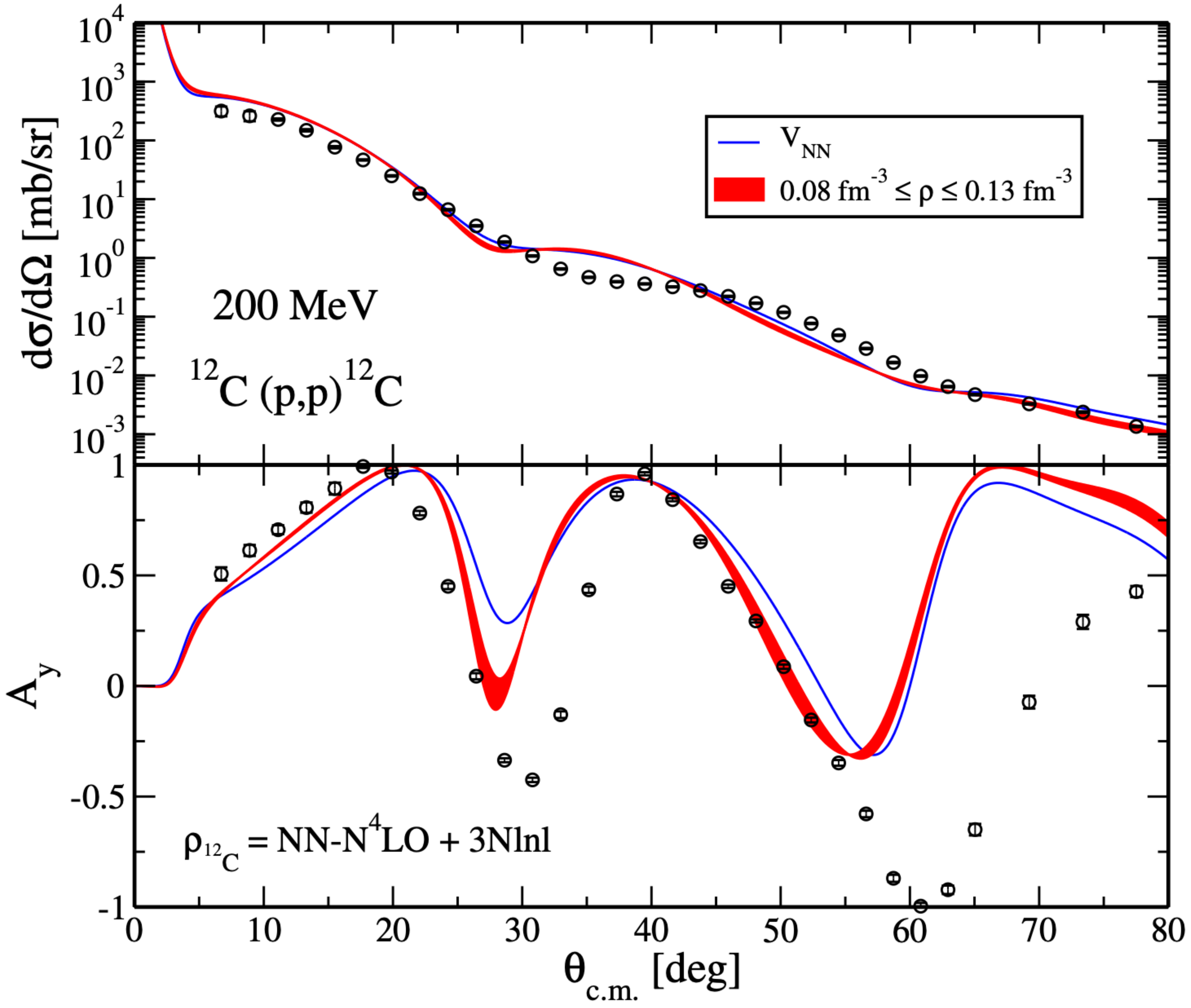
Vorabbi et al., (2016, 2018)



Assessing the impact of the 3N interaction



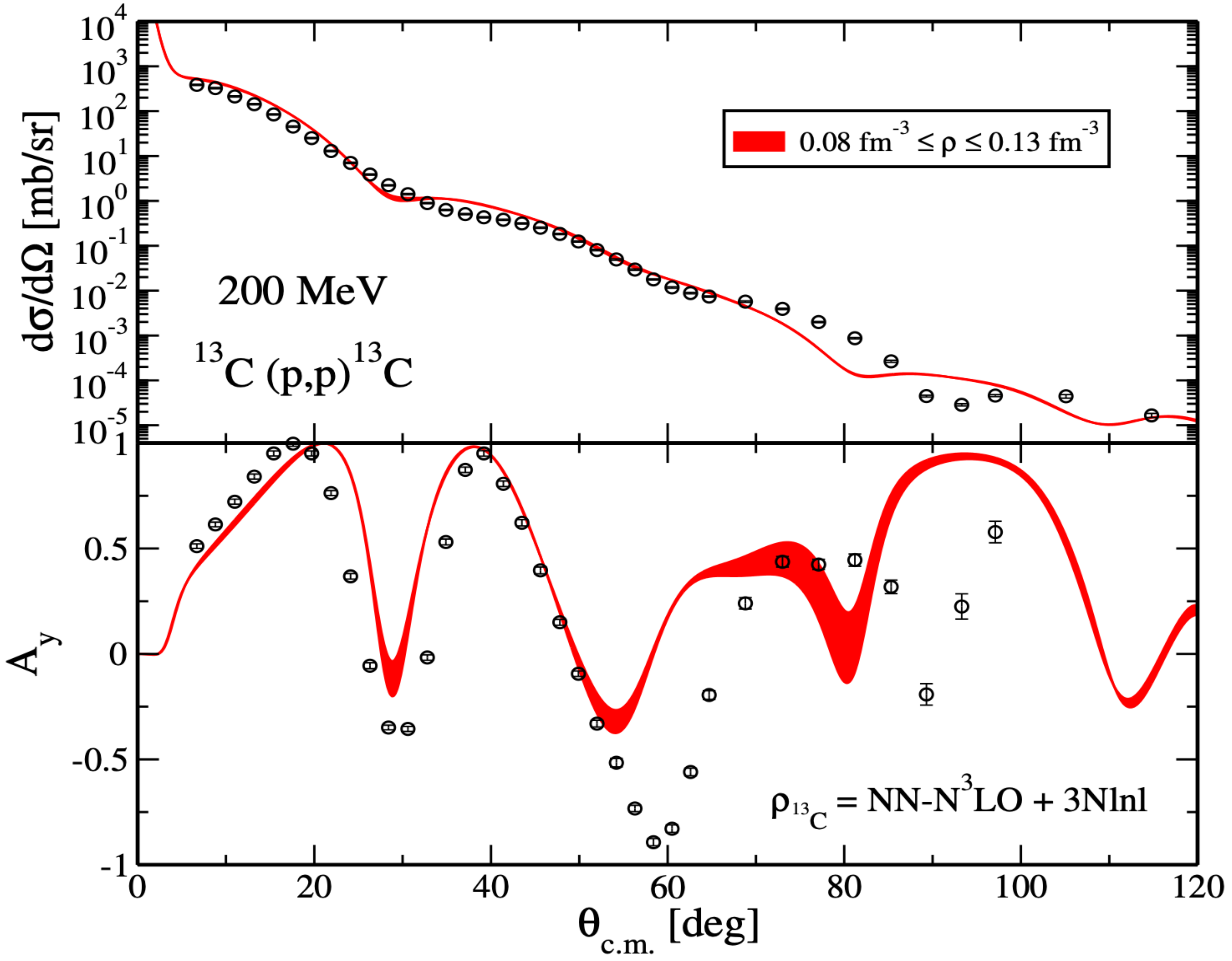
Vorabbi et al., PRC **103**, 024604 (2021)



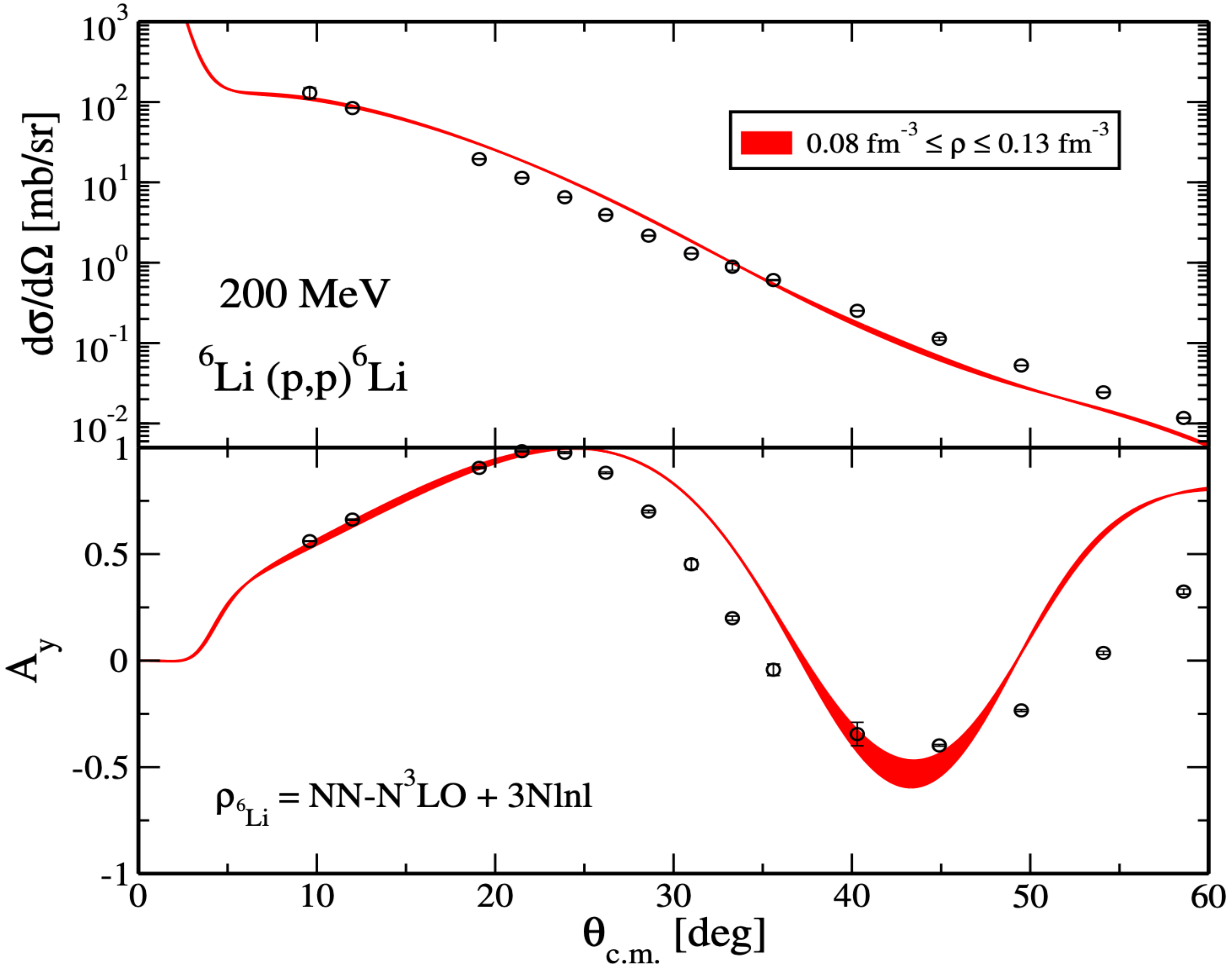
- For all nuclei we found very small contributions to the differential cross section
- The contributions to the spin observable are larger and they seem to improve the agreement with the data

Extension to non-zero spin targets

$J^\pi = 1/2^-$



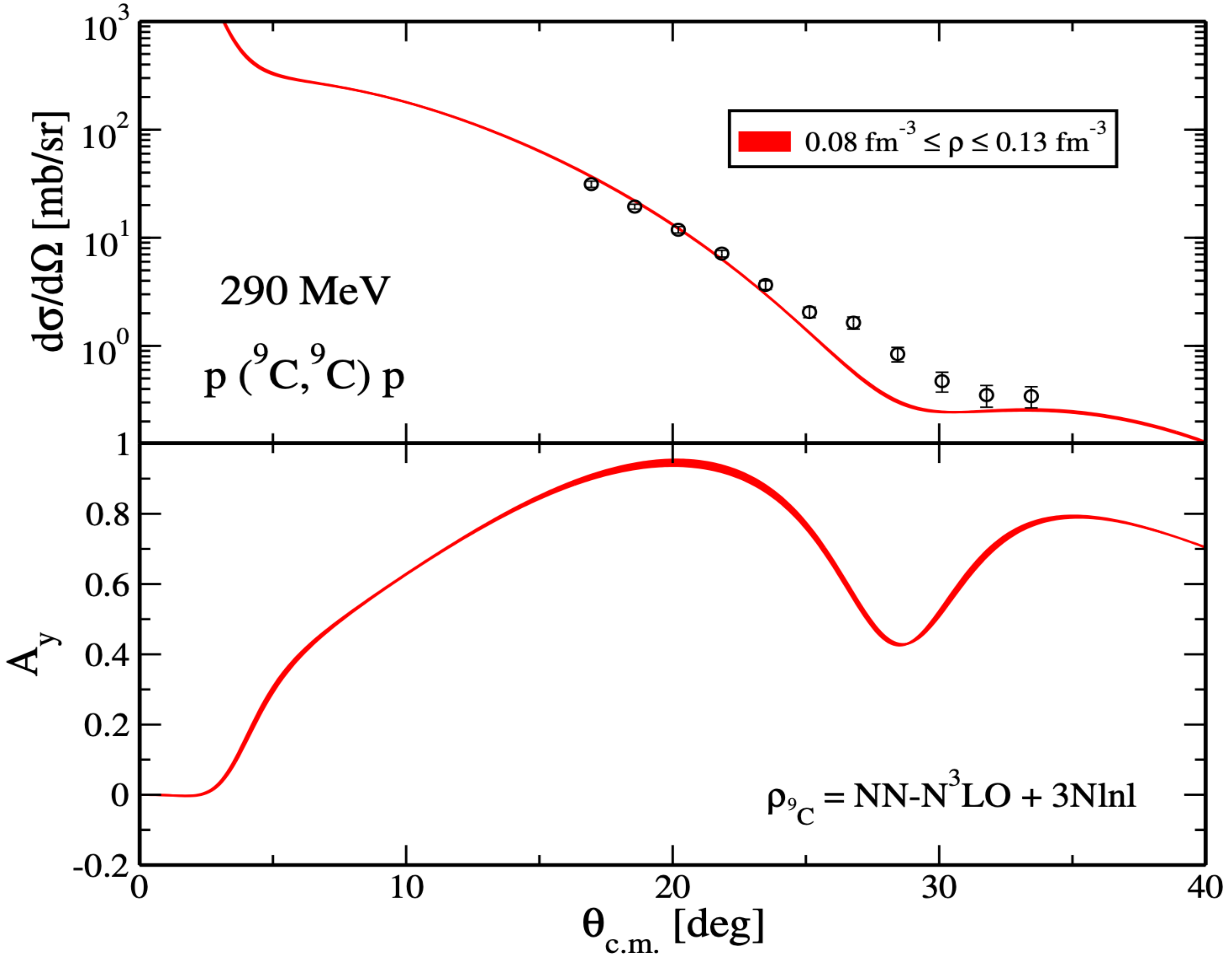
$J^\pi = 1^+$



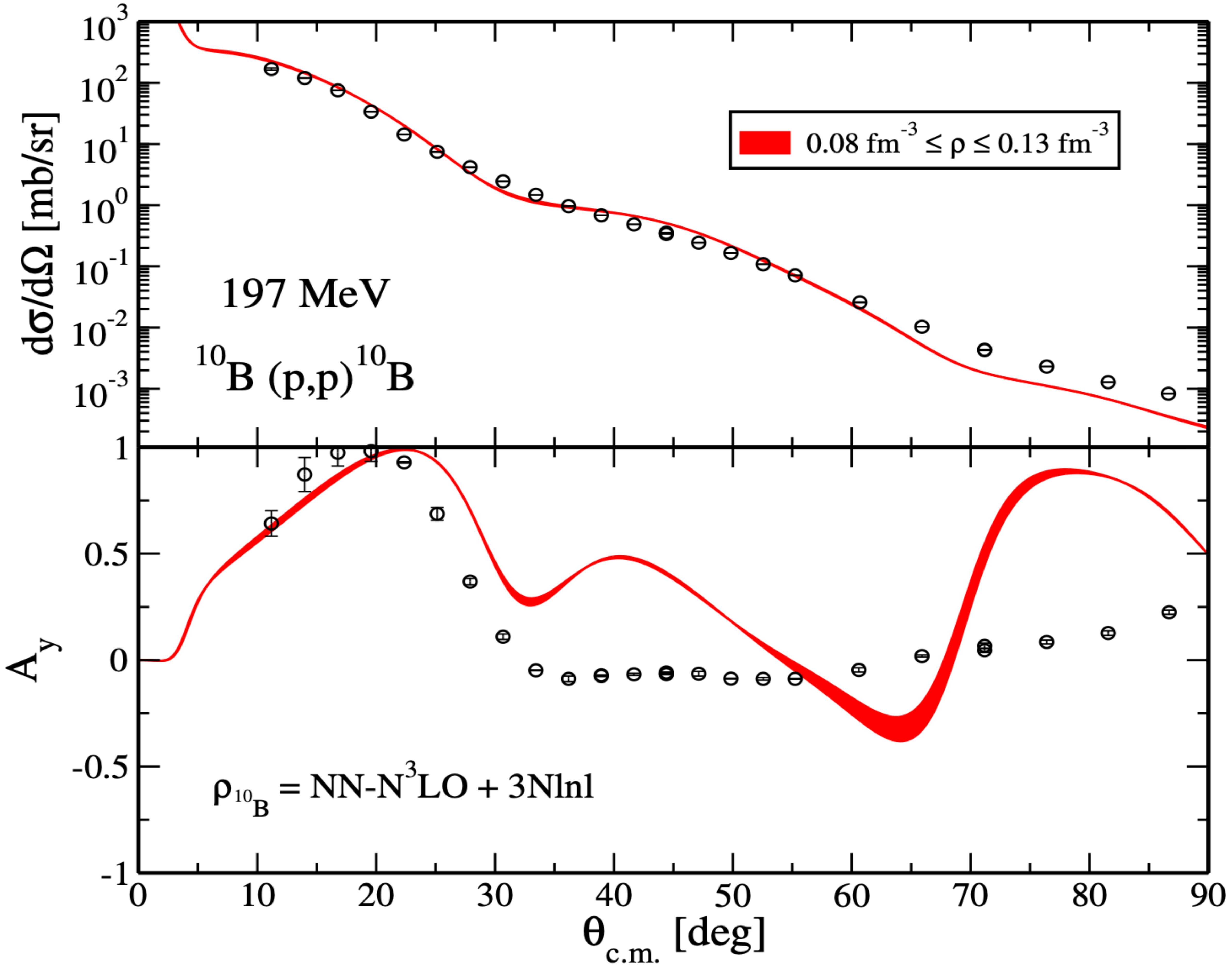
[Vorabbi et al., Phys. Rev. C **105**, 014621 (2022)]

Extension to non-zero spin targets

$J^\pi = 3/2^-$

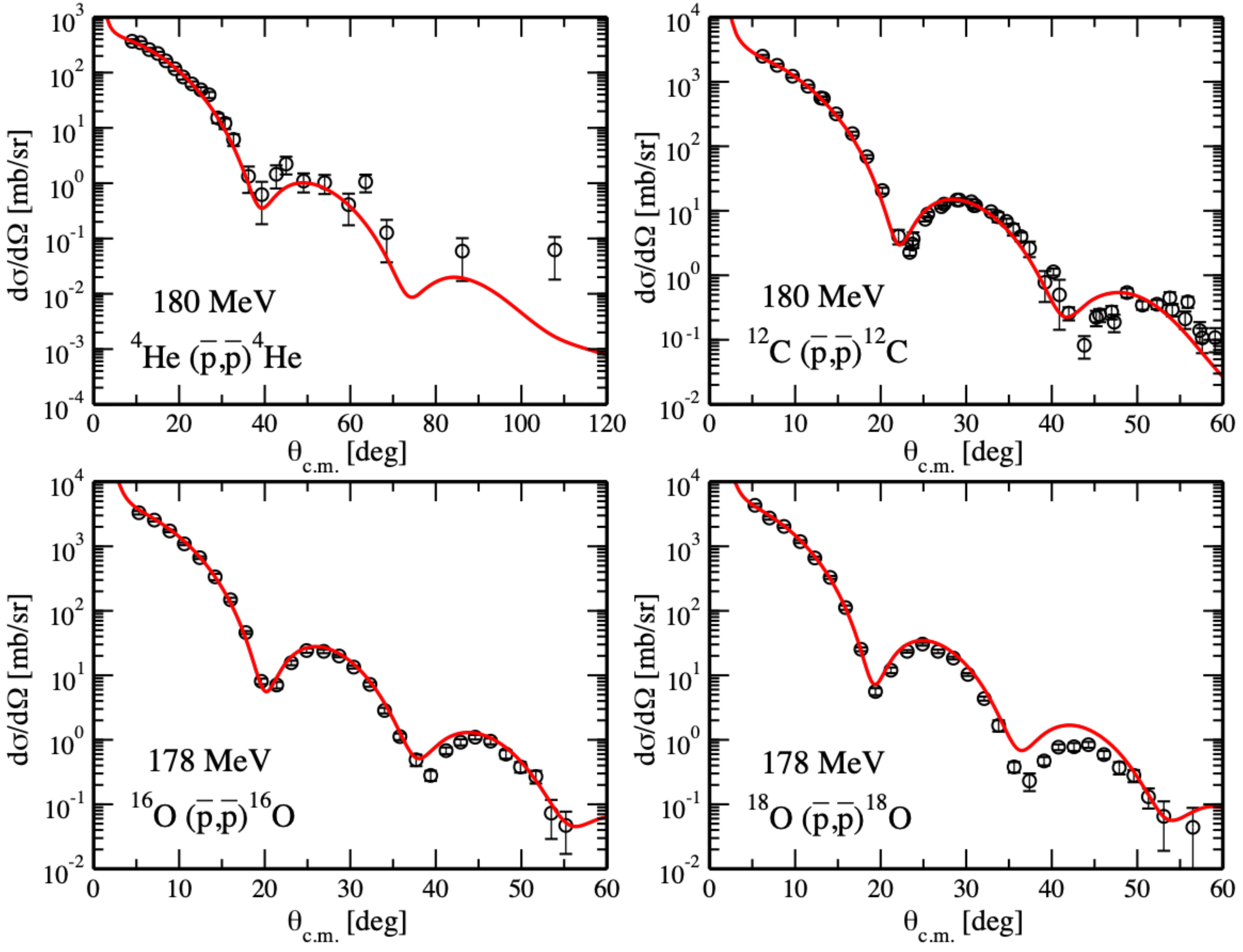


$J^\pi = 3^+$

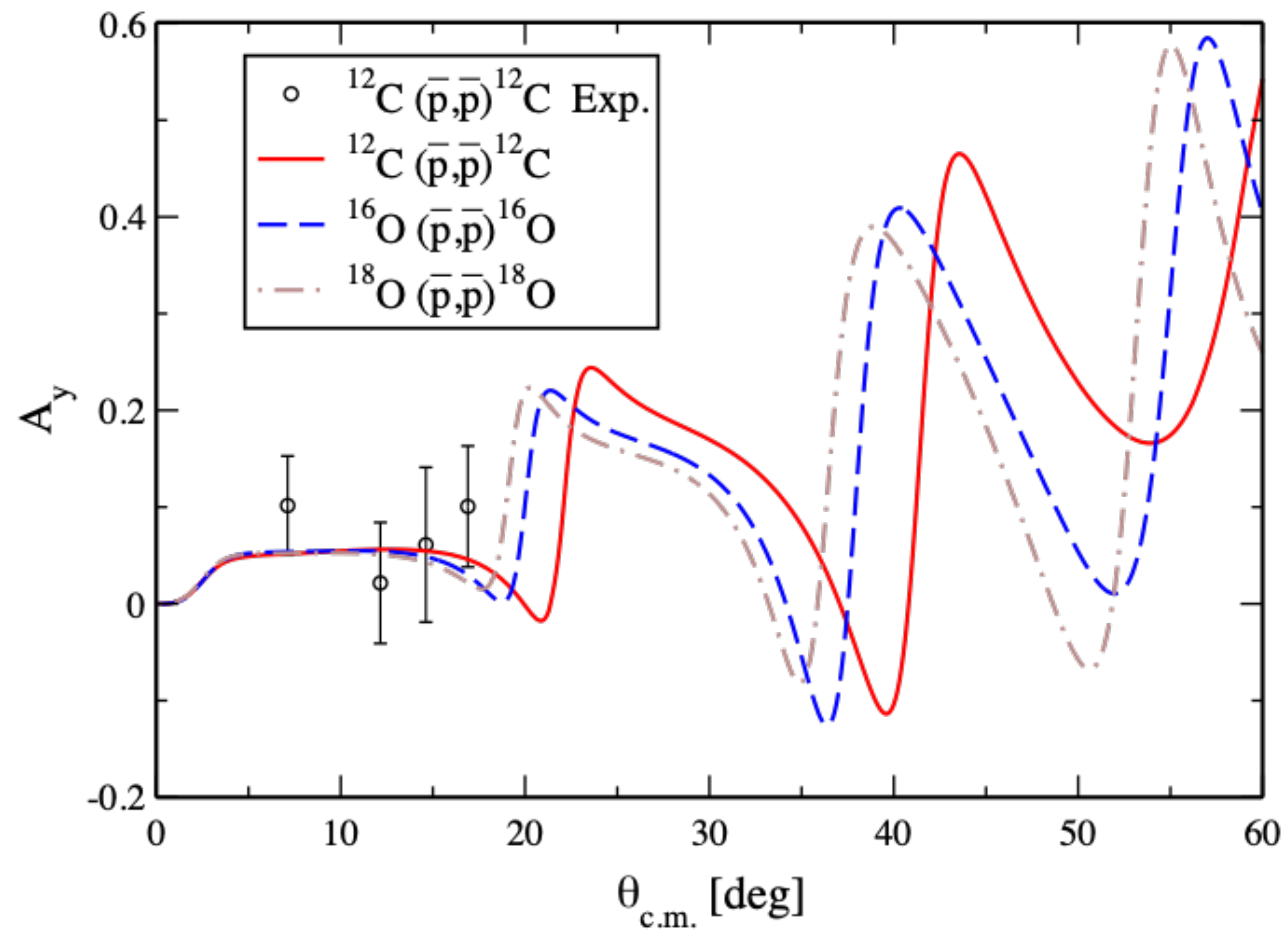


[Vorabbi et al., Phys. Rev. C **105**, 014621 (2022)]

Extension to antiproton-nucleus elastic scattering



$$t_{pN} \Rightarrow t_{\bar{p}N}$$



Elastic Antiproton-Nucleus Scattering from Chiral Forces

Matteo Vorabbi[1,2] Michael Gennari[2,3] Paolo Finelli[4] Carlotta Giusti[5] and Petr Navrátil[2]

PHYSICAL REVIEW LETTERS **124,** 162501 (2020)

Inclusion of double scattering

- Inclusion of the second-order term of the spectator expansion

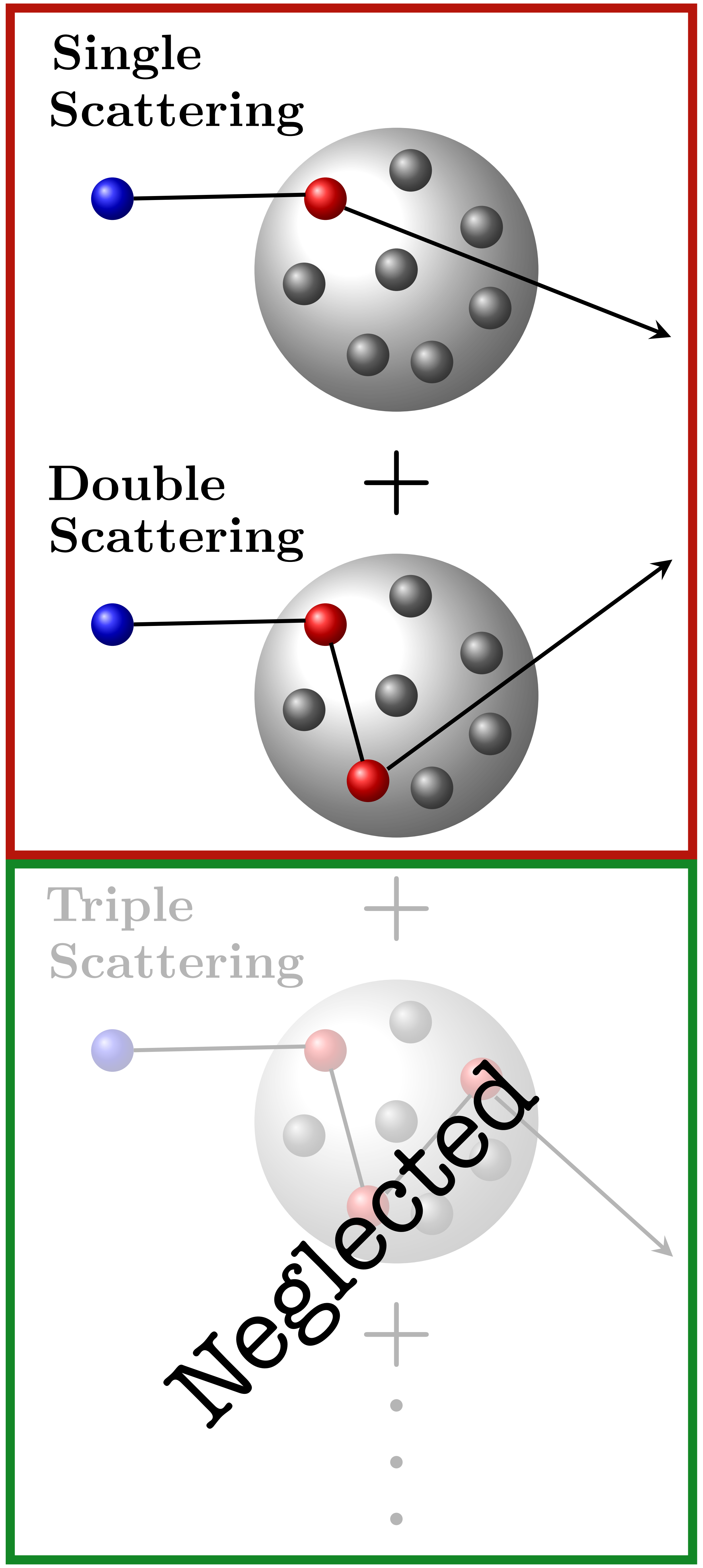
[Crespo *et al.*, PRC **46**, 279 (1992)]

$$U^{(2)} = \sum_{i=1}^{A} \tau_{0i} + \sum_{i,j\neq i}^{A} \tau_{0ij}$$

- Requires:
  1. Two-body density matrix (from NCSM)

$$\rho(\boldsymbol{r}'_1, \boldsymbol{r}'_2, \boldsymbol{r}_1, \boldsymbol{r}_2)$$

  2. Solution of the three-body scattering equation for τ_{0ij}



Inclusion of medium effects

First-order term of the spectator expansion

$$\tau_{0i} = v_{0i} + v_{0i} G_0(E) Q \tau_{0i}$$

(A+1)-body propagator

The problem is how to treat the *Q* operator!
The simplest approximation is

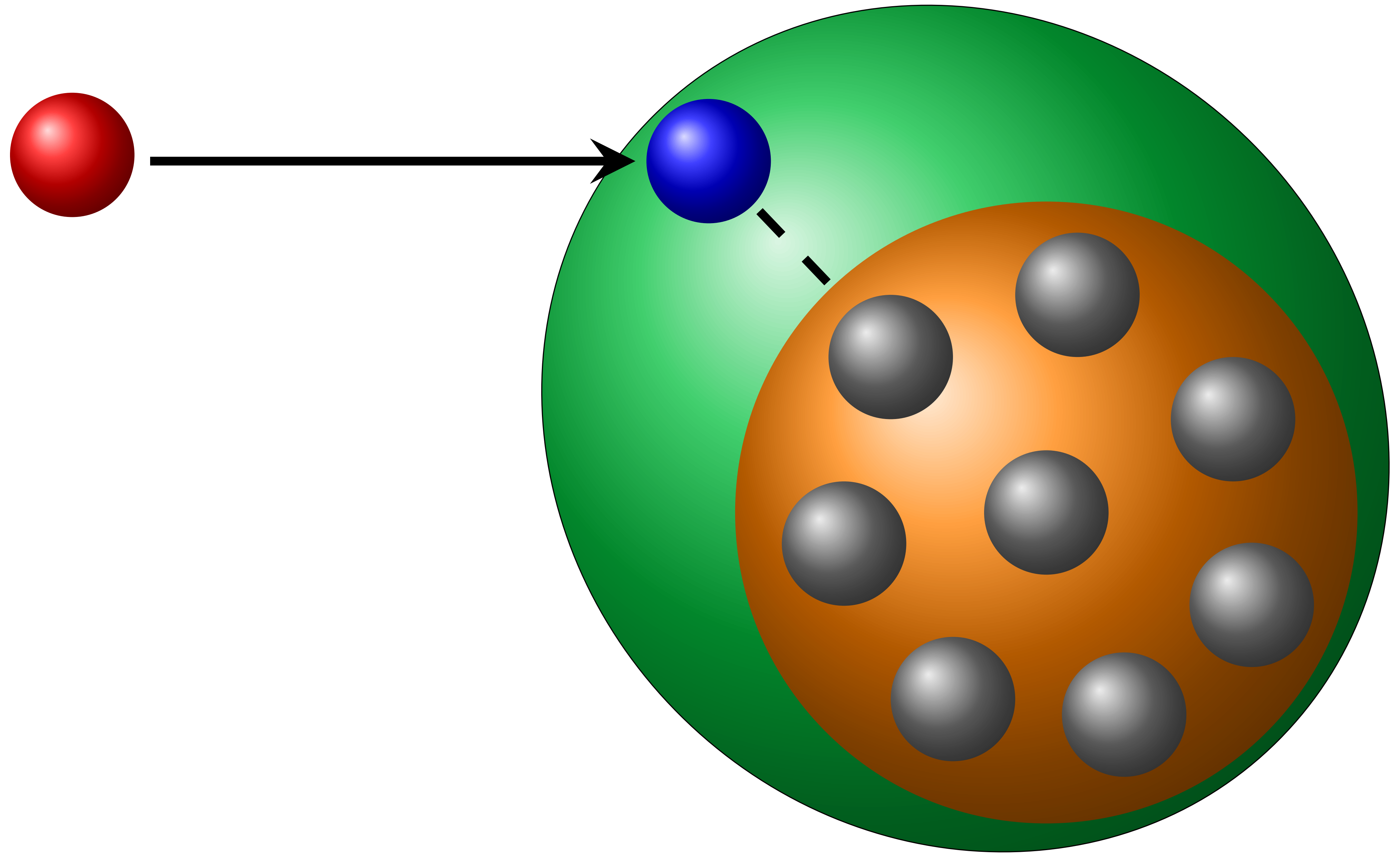
$$\tau_{0i} \approx t_{0i}$$

Inclusion of medium effects

- Work has been done to include these effects at a mean-field level
- We have to rethink the approach and do it with the NCSM and/or SCGF

[Chinn *et al*., PRC **52**, 1992 (1995)]

The first-order term is a 3-body problem



We have to include the interaction with the residual nucleus!

Distorted wave theory of inelastic scattering

The inelastic transition amplitude

[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1215 (1982)]
[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1233 (1982)]

$$T_{\mathrm{inel}}(\boldsymbol{k}_*, \boldsymbol{k}_0) = \int d\boldsymbol{r}' \int d\boldsymbol{r}\, \psi^\dagger(\boldsymbol{k}_*, \boldsymbol{r}')\, U_{\mathrm{tr}}(\boldsymbol{r}', \boldsymbol{r})\, \psi(\boldsymbol{k}_0, \boldsymbol{r})$$

Required potentials

$$U_{\mathrm{ex}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\mathrm{ex})}(\boldsymbol{q}, \boldsymbol{P})$$

$$U_{\mathrm{tr}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\mathrm{tr})}(\boldsymbol{q}, \boldsymbol{P})$$

$$U_{\mathrm{gs}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\mathrm{gs})}(\boldsymbol{q}, \boldsymbol{P})$$

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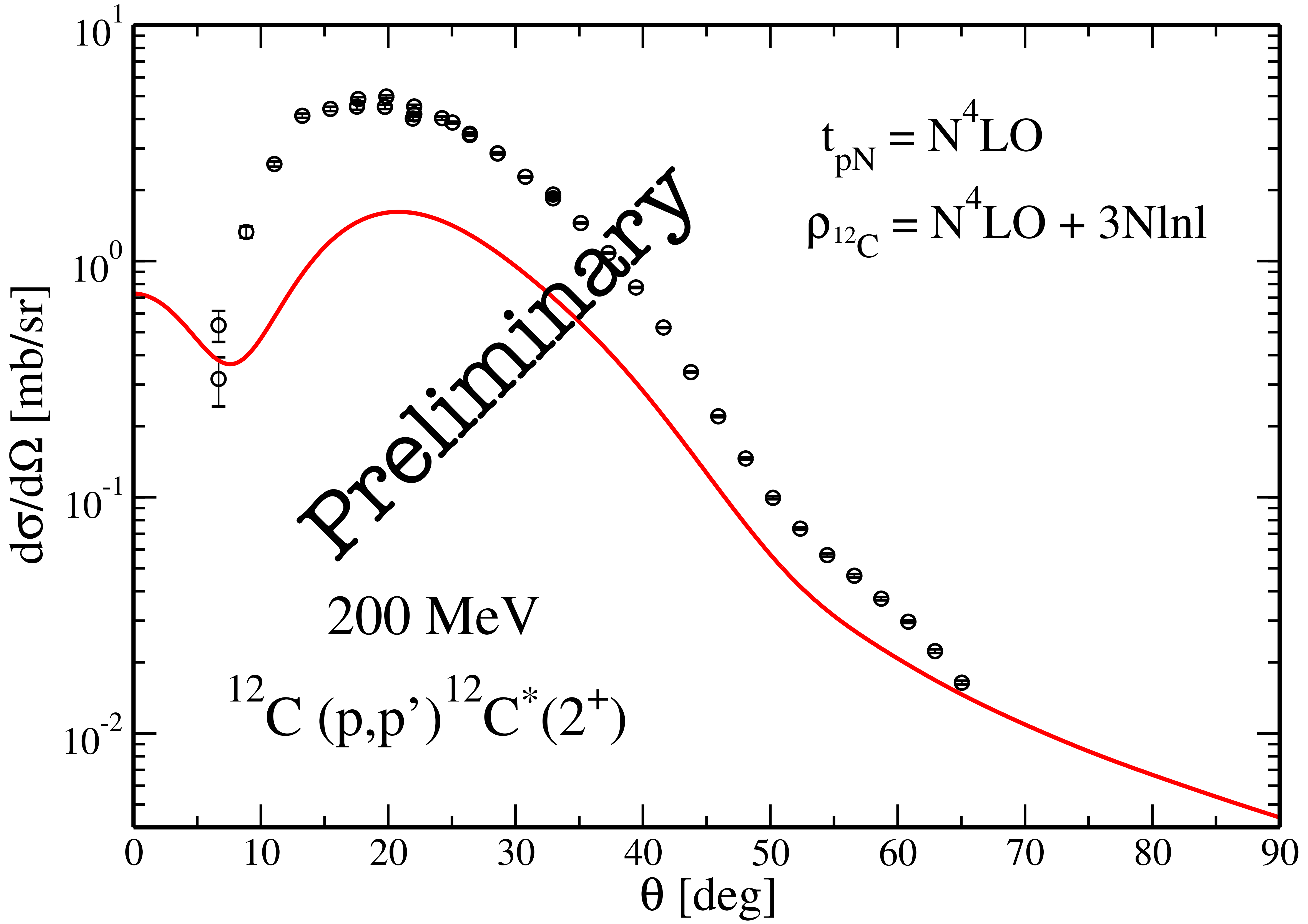
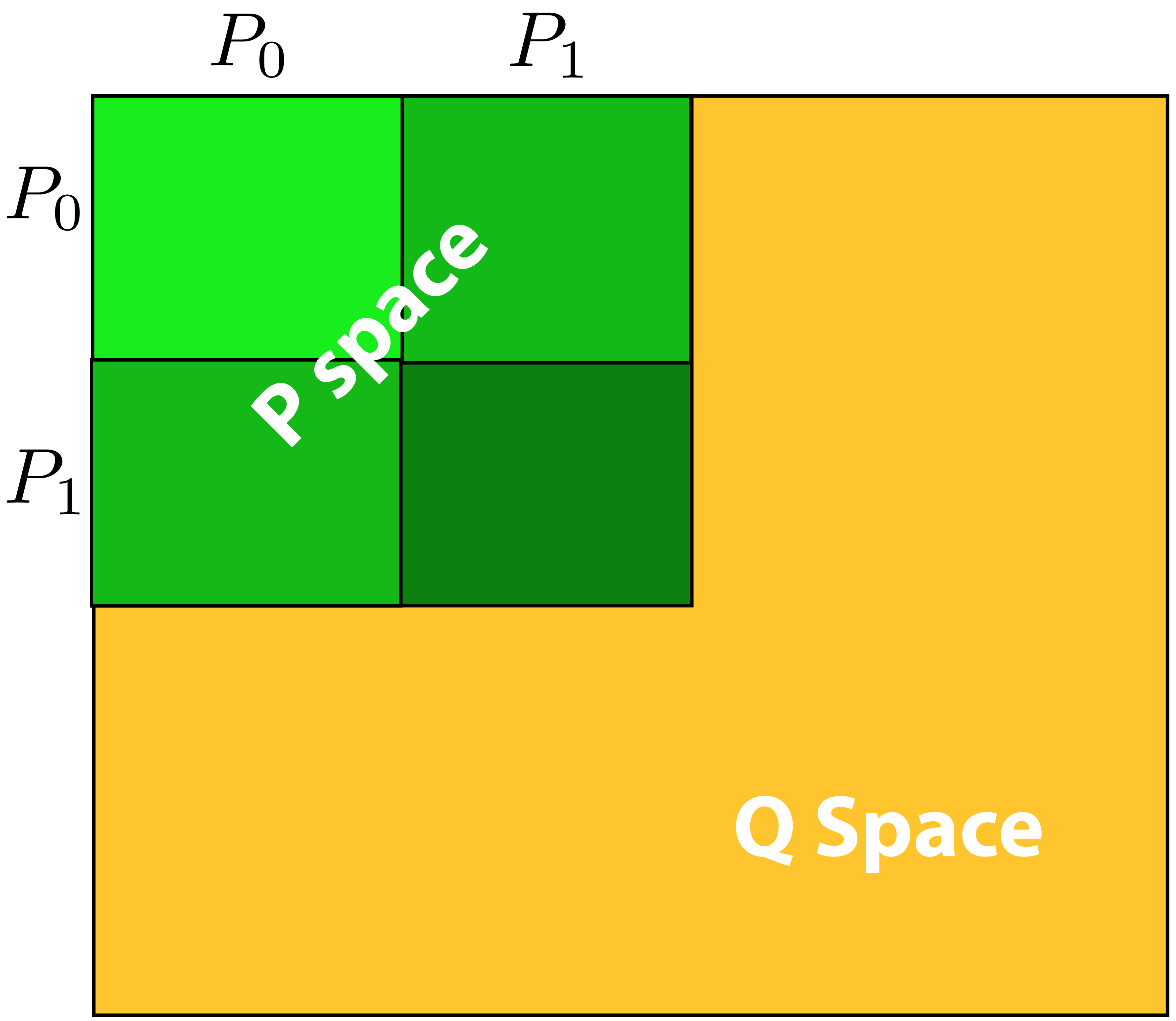


Toward a coupled channel approach?

New projection operators

$$P \equiv P_0 + P_1 + \ldots = |\Psi_0\rangle\langle\Psi_0| + |\Psi_1\rangle\langle\Psi_1| + \ldots$$

$$T_{00} \equiv P_0 T P_0\,, \quad T_{01} \equiv P_0 T P_1\ \ldots$$



Coupled-channel transition amplitude

$$T = U + UGT$$

It leads to a set of coupled Lippmann-Schwinger equations

Inputs

$$U_{00},\, U_{01},\, U_{10},\, U_{11},\, \ldots$$

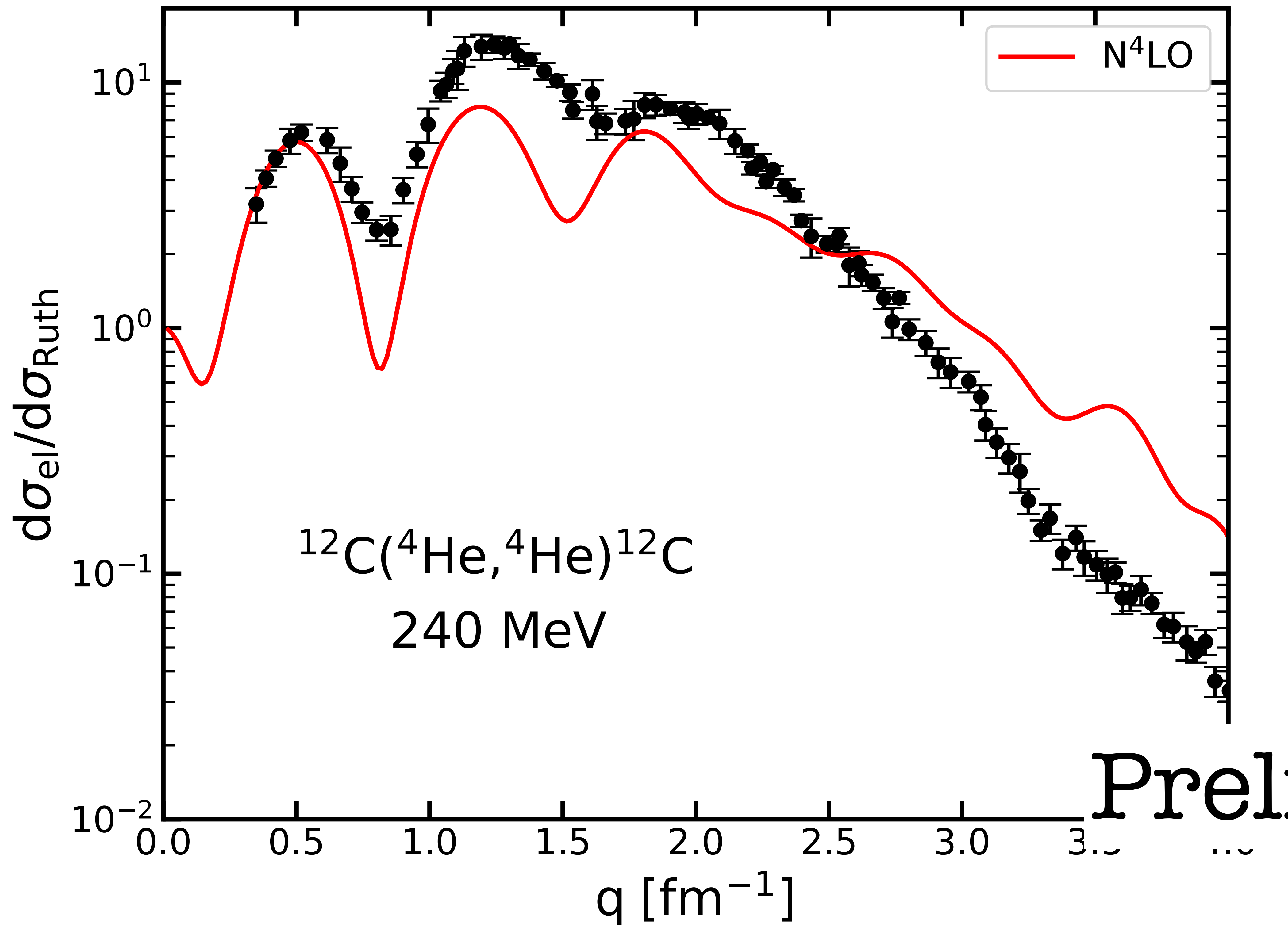
Optical potential

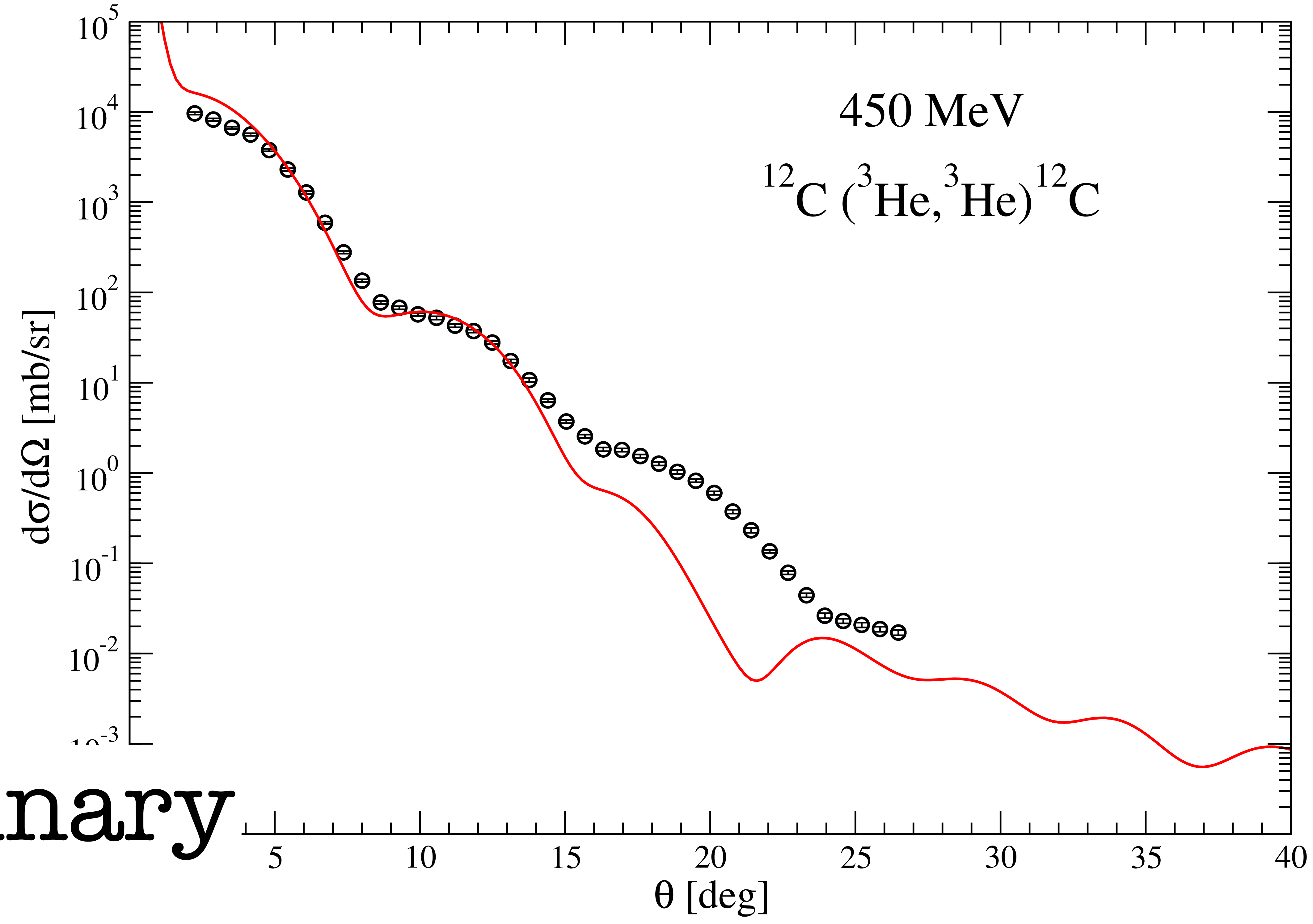
$$T \equiv \begin{pmatrix} T_{00} & T_{01} & \ldots \\ T_{10} & T_{11} & \ldots \\ \vdots & \vdots & \ddots \end{pmatrix} \qquad G \equiv \begin{pmatrix} G_{00} & 0 & \ldots \\ 0 & G_{11} & \ldots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Nucleus-nucleus collisions

$$T_{\mathrm{el}} = PUP + PUPG_0(E)T_{\mathrm{el}}$$

$$U = \sum_{i=1}^{A_{\mathbb{P}}} \sum_{j=1}^{A_{\mathbb{T}}} U_{ij} \qquad U_{ij} = v_{ij} + v_{ij} G_0(E) Q \sum_{k=1}^{A_{\mathbb{P}}} \sum_{l=1}^{A_{\mathbb{T}}} U_{kl}$$





Preliminary

Summary & outlook

✓ The choice of the NN interaction is crucial to define the energy limits of applicability of the optical potential

✓ The 3N interaction has a sizeable effect on polarisation observables

✓ The extension to nonzero spin targets provides a good description of the data for stable and unstable nuclei

✗ Extend the high- and low-energy limits of applicability of the optical potential

✗ Inclusion of the second-order term of the spectator expansion

✗ Consistent treatment of the full 3N interaction

✗ Development of a coupled-channel approach

✗ Evaluation of theoretical uncertainties

Long-term strategy: Ab-initio Nuclear Reactions

Extensions

- Non-zero spin targets
- Three-body
- Medium corrections
- 2nd order corrections

Elastic Nucleon-Nucleus scattering ✓

Inelastic Nucleon-Nucleus scattering ◁

(Work in progress)

Nucleus-Nucleus scattering ◁

(Work in progress)

Mass/Charge exchange scattering ✗

FRAGMENTATION

Numerical codes

- Factorized
- Full folding
- R-matrix
- Optimal approximation
- Coupled channel

Additional Project: Electroweak probes

PHYSICAL REVIEW LETTERS **125,** 182501 (2020)

Ab Initio **Computation of Charge Densities for Sn and Xe Isotopes**

P. Arthuis,[1,*] C. Barbieri,[1,2,3,†] M. Vorabbi,[4,‡] and P. Finelli[5,§]

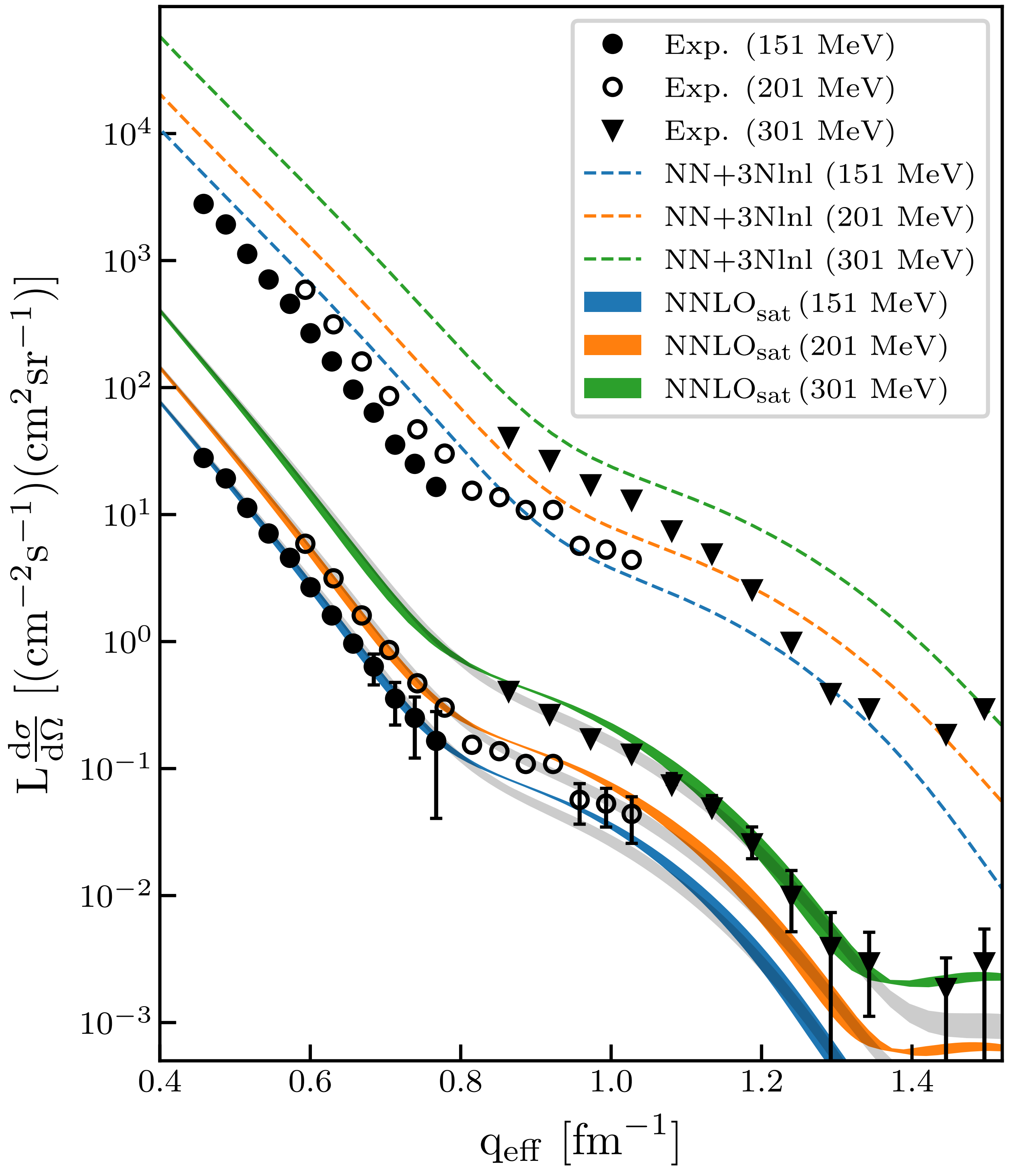


FIG. 3. Luminosity multiplied by the differential cross section for ^{132}Xe obtained from Gorkov SCGF calculations at ADC(2). The values for the $NN + 3N$(lnl) interaction have been scaled by 10^2 for clarity. The gray bands correspond to the two-point Fermi distribution with parameter and error bars extracted from Ref. [10]. Experimental values are taken from [10] and duplicated with a scaling of 10^2 for comparison with $NN + 3N$(lnl) values, where error bars have been removed for clarity.

New available code for elastic (PV) electron scattering

- Test with existing codes (ELSEPA,…)
- Beam-normal single spin asymmetry
- Extension to the inelastic case
- Standard Model test

PHYSICAL REVIEW C **105**, 055503 (2022)

Incorporating the weak mixing angle dependence to reconcile the neutron skin measurement on ^{208}Pb by PREX-II

M. Atzori Corona,[1,2] M. Cadeddu,[2,*] N. Cargioli,[1,2,†] P. Finelli,[3] and M. Vorabbi[4]

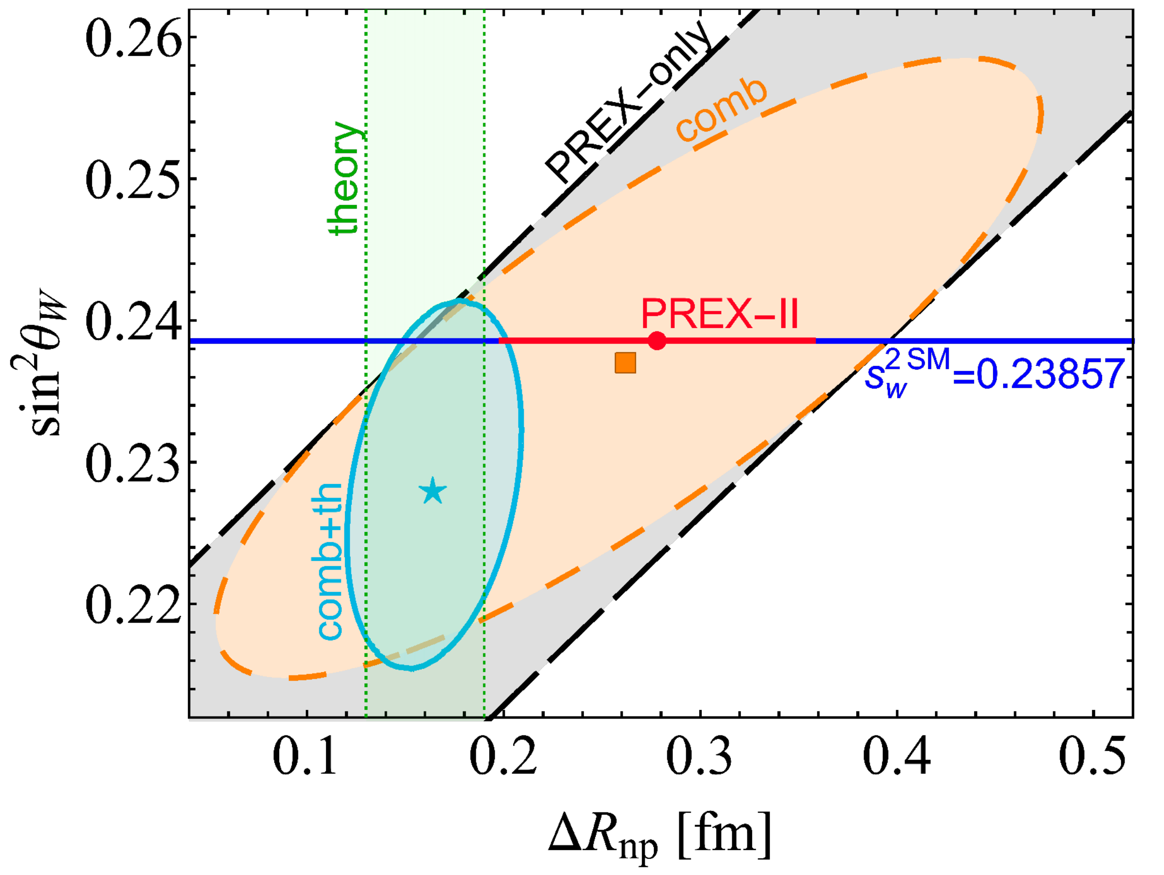


FIG. 3. Summary of the PREX-only (grey long dashed), combined (orange dashed) and combined+theory (cyan solid) 1σ confidence level contours in the $\sin^2\theta_W$ vs $\Delta R_{\rm np}(^{208}\rm Pb)$ plane. The orange square and the cyan star points are the best fits of combined and combined+theory, respectively. The green vertical band shows $\Delta R^{\rm th}_{\rm np}$, while the red dot the PREX-II result [1] for $s_W^{2\,\rm SM}$(blue line).

$$\langle A \rangle = \frac{\int d\theta \, \sin\theta \, A(\theta) \frac{d\sigma}{d\Omega} \epsilon(\theta)}{\int d\theta \, \sin\theta \frac{d\sigma}{d\Omega} \epsilon(\theta)}$$