Constraining the nuclear equation of state through nuclear experiments

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MoNStRe workshop

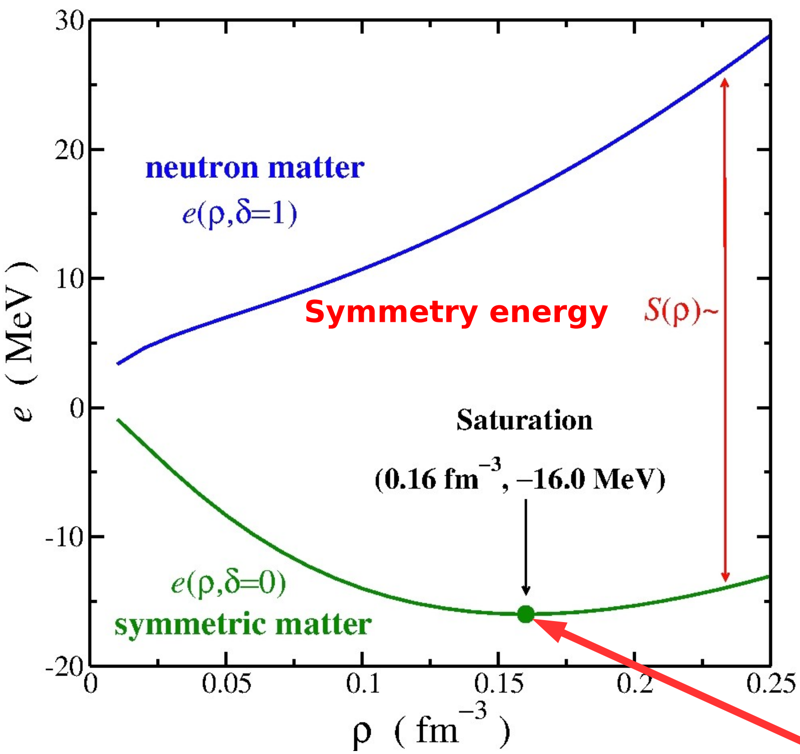
Milano, May 11th-12th 2023

Nuclear Equation of State (EoS)

Unpolarized, uniform nuclear matter at T=0 assuming isospin symmetry

Energy per nucleon (e) as a **function** of the **total density** [$\rho=\rho_n+\rho_p$] and their **relative difference** [$\delta=(\rho_n+\rho_p)/\rho$].

$$e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]$$



It is customary to also **expand** $e(\rho,0)$ and $S(\rho)$ around nuclear **saturation density** $\rho_0 \sim 0.16\ \mathrm{fm}^{-3}$

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

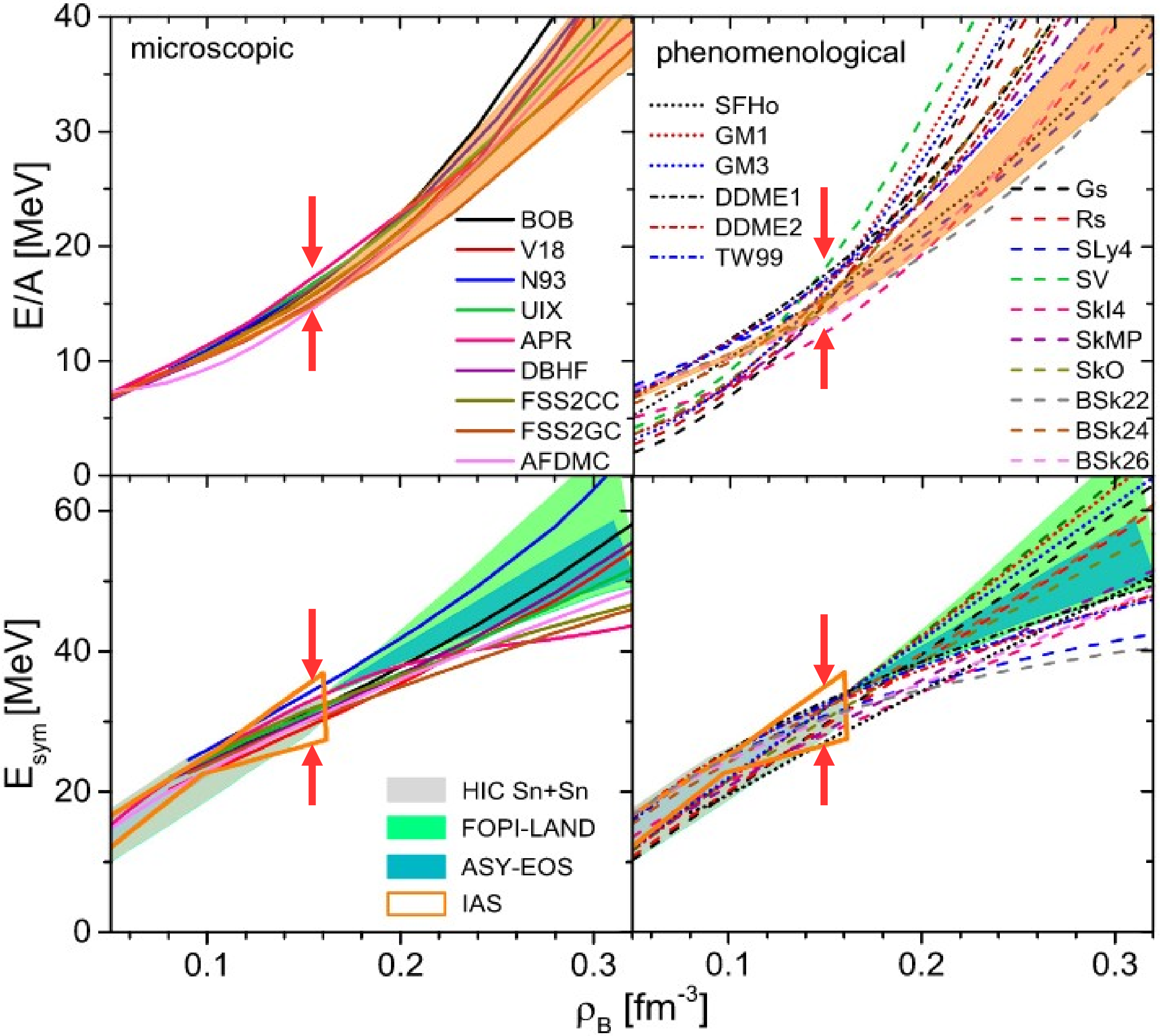
$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

- K_0 → how **compressible** is symmetric matter at ρ_0
- J (a_A) → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0
- L (a_s) → **neutron pressure** in neutron matter at ρ_0

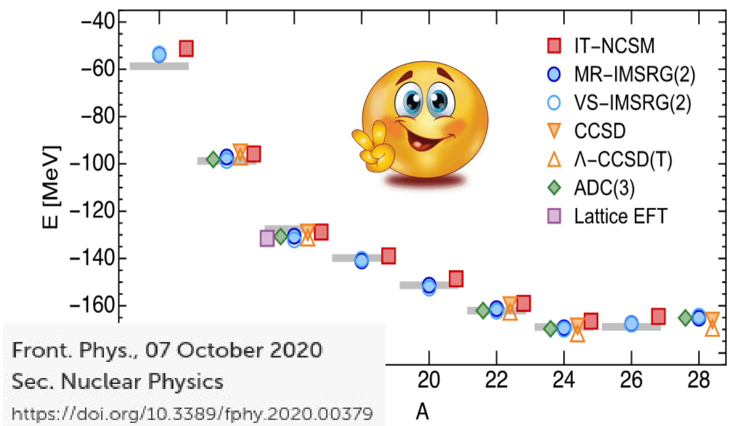
$$P(\rho=\rho_0,\delta=0) = 0\ \text{MeV fm}^{-3}$$

EoS from current nuclear models

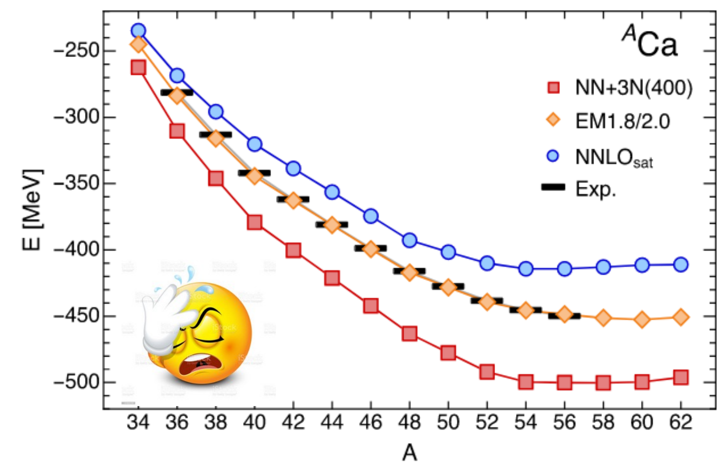
Micorscopic and phenomenological models constrainted by different data display similar discrepances on the EoS



Many-body methods have been shown to agree



Main source of uncertainty in the nuclear Hamiltonian



Progress in Particle and Nuclear Physics 120 (2021) 103879

Light orange bands from χEFT from Drischler C., et al. Phys. Rev. Lett., 125 202702 (2020).

Which information can be obtained on the EoS from nuclear observables?

(personal overview)

Exprimental binding energies and charge radii

are telling us the e_0 and ρ_0 with exquisite accuracy

→ A **small change** in the **saturation density** will **impact** the **size** of the **nucleus. Charge radii** are determined to an average accuracy of 0.016 fm (Angeli 2013).

For example, if one aims at determining the **r_{ch} = 5.5012±0.0013 fm** in **^{208}Pb** one must be **very precise** in the determination of **ρ_0**:

$$\frac{\delta\rho_0}{\rho_0} = -3\frac{\delta R}{R} \rightarrow \frac{\delta\rho_0}{\rho_0} \lesssim 0.1\%$$

<u>Note:</u> typical average theoretical deviation of accurate EDFs ~ 0.02 fm → **$\delta\rho_0/\rho_0$** is determined up to about a **1% accuracy** (That is, third digit in ρ_0).

→ In a similar way, a **small change** in the **saturation energy** (about $e_0 \approx$ -16 MeV) will **impact** on the **nuclear masses.**

For example, if one aims at determining the **B = 1636.4296±0.0012 MeV** in **^{208}Pb** one must be **very precise** in the determination of **e_0**:

$$\frac{\delta B}{B} = \frac{\delta e_0}{e_0} \rightarrow \frac{\delta e_0}{e_0} \lesssim 10^{-6}$$

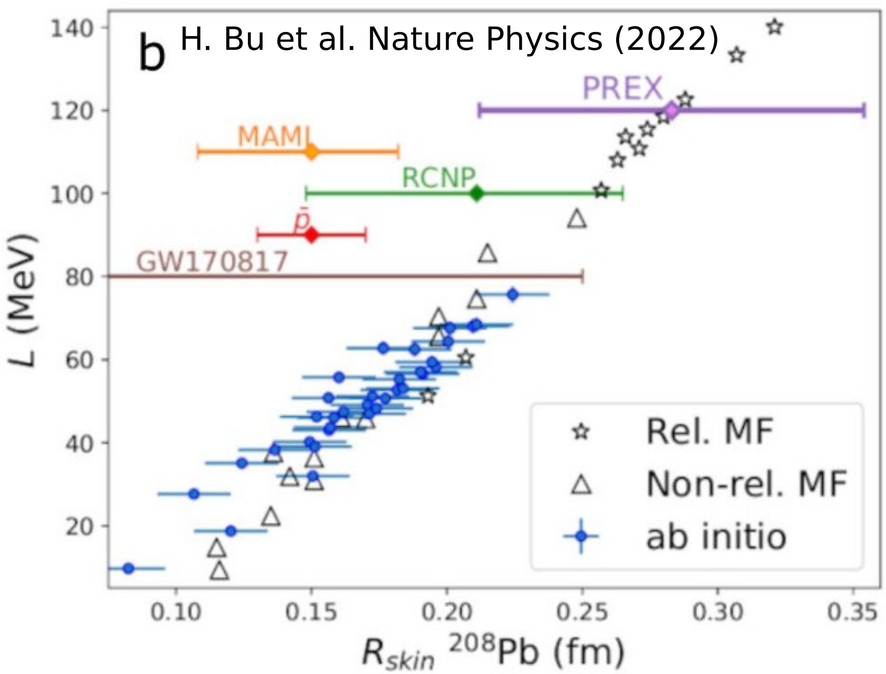
<u>Note:</u> typical average theoretical deviation of accurate EDFs ~ 1-2 MeV → **$\delta e_0/e_0$** is determined up to about a **0.1% accuracy** (That is, second decimal digit in e_0).

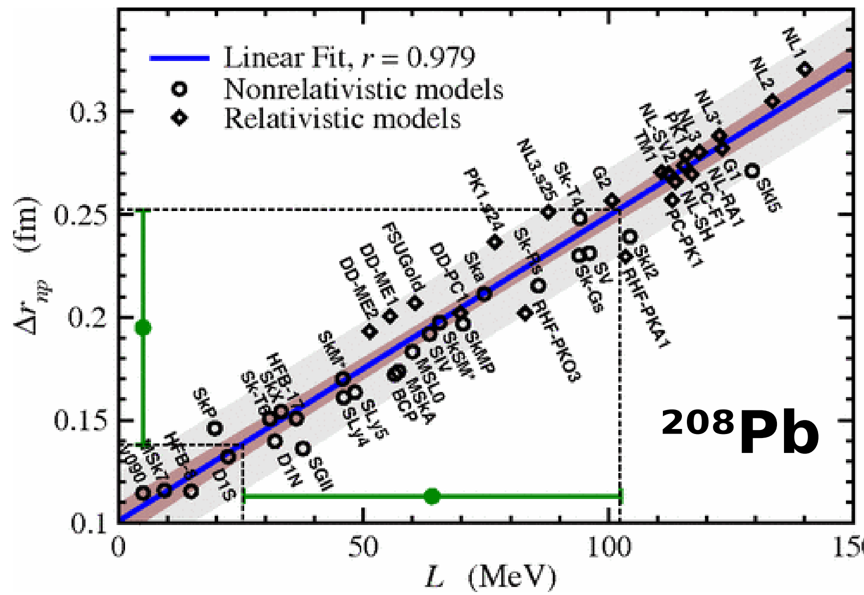
Neutron skin is a good proxy to L

B. Alex Brown Phys. Rev. Lett. 85, 5296 (2000)

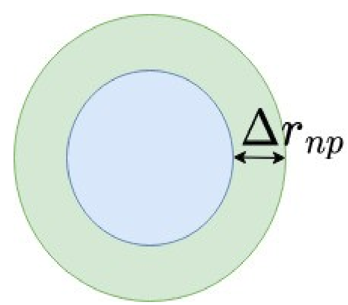
The **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in a heavy neutron rich nucleus is related to the **neutron pressure** ($\delta=1$) around ρ_0 (L).

$$P(\rho_0,\delta)=\rho_0^2\frac{\partial e(\rho,\delta)}{\partial\rho}\bigg|_{\rho=\rho_0}=\frac{1}{3}\rho_0\delta^2 L$$





Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



→ From the Droplet Model:

$$\Delta r_{np}\approx\frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

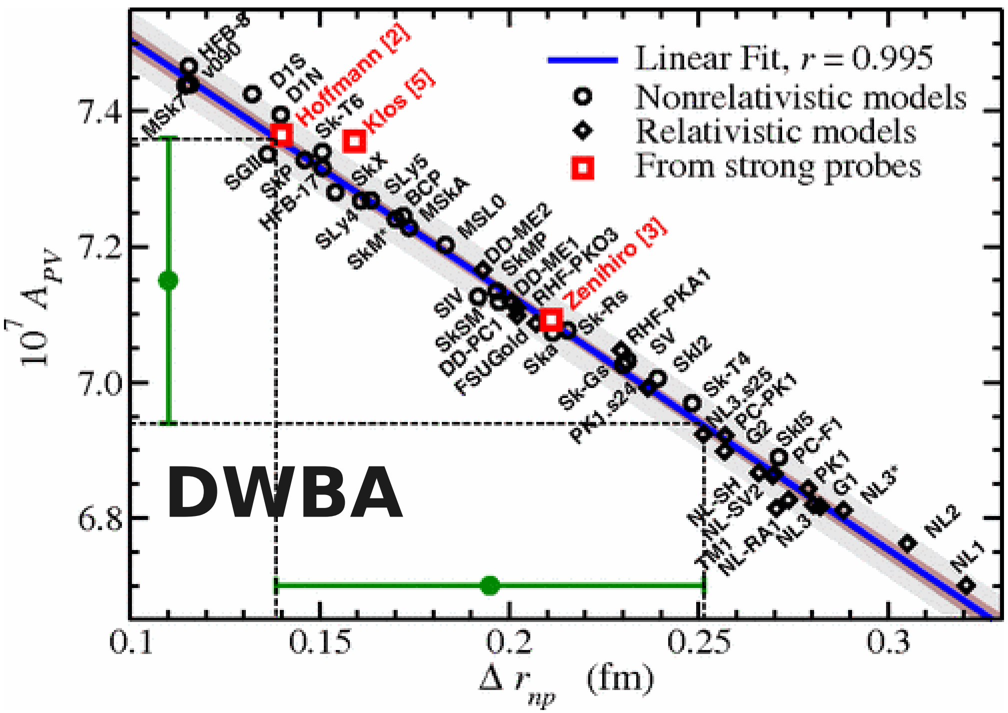
How to measure it? (in a direct way)

Parity violating and parity conserving elastic electron scattering

Polarized electron-Nucleus scattering:

→ In good approximation, the weak interaction probes the neutron distribution in nuclei while Coulomb interaction probes the proton distribution

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
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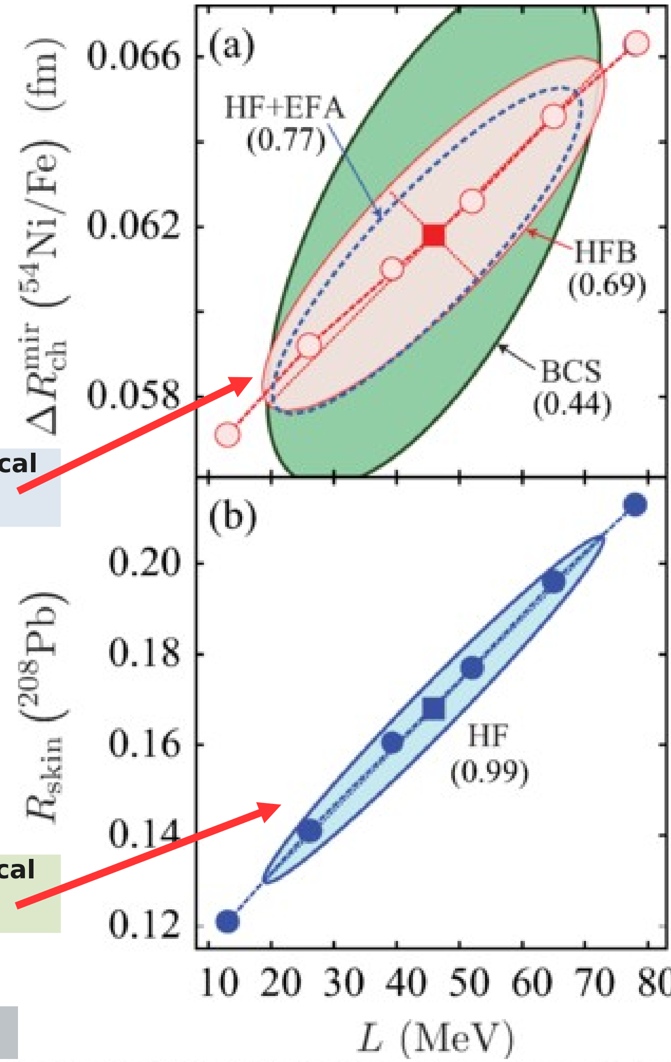
Isospin symmetry →
$\Delta r_{ch} = r_{ch}(^{54}Ni) - r_{ch}(^{54}Fe) = \Delta r_{np}(^{54}Fe)$

Large theoretical uncertainties

Small theoretical uncertainties



Paul-Gerhard Reinhard and Witold Nazarewicz
Phys. Rev. C **105**, L021301 – Published 3 February 2022

Skyy V. Pineda, Kristian König, Dominic M. Rossi, B. Alex Brown, Anthony Incorvati, Jeremy Lantis, Kei Minamisono, Wilfried Nörtershäuser, Jorge Piekarewicz, Robert Powel, and Felix Sommer
Phys. Rev. Lett. **127**, 182503 – Published 29 October 2021

Giant Monopole Resonance
do we understand it?

The compression-mode giant resonances and nuclear incompressibility

Umesh Garg [a], Gianluca Colò [b c]

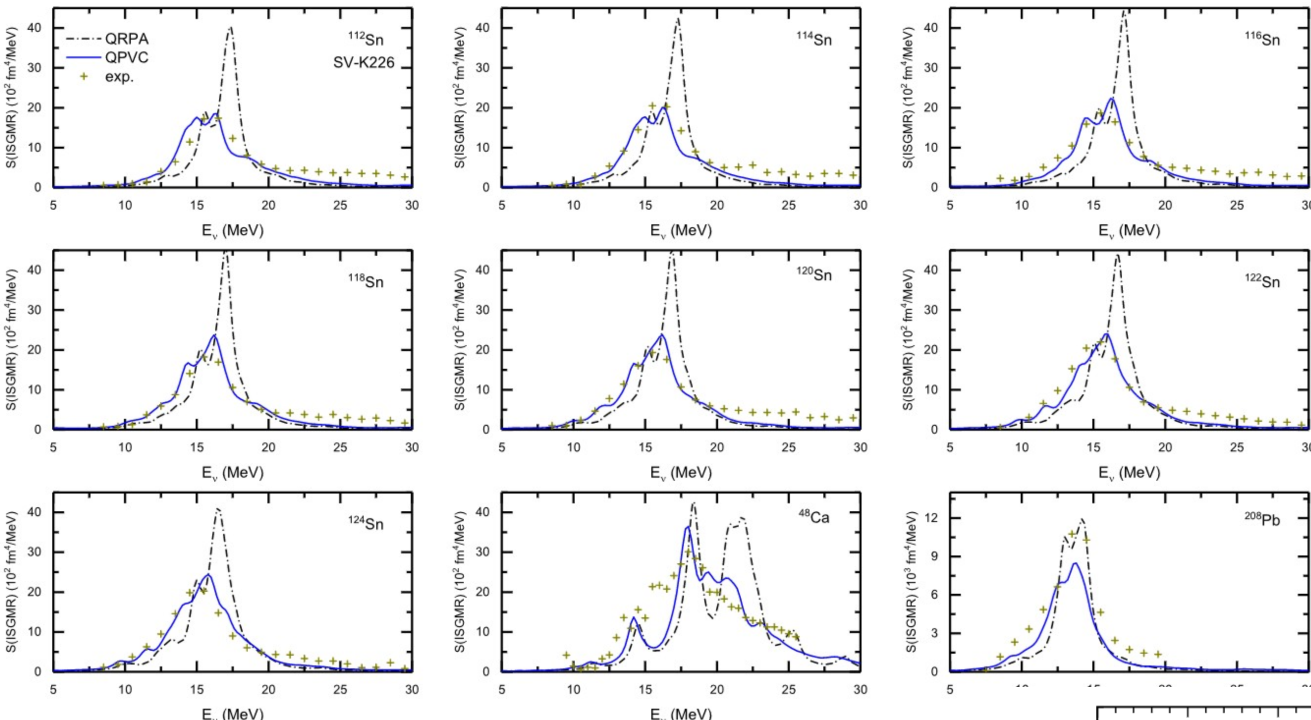
Progress in Particle and Nuclear Physics
Volume 101, July 2018, Pages 55-95

arXiv:2211.01264 [pdf, ps, other] nucl-th

Towards a Unified Description of Isoscalar Giant Monopole Resonances in a Self-Consistent Quasiparticle-Vibration Coupling Approach

Authors: Z. Z. Li, Y. F. Niu, G. Colò

SV-K226



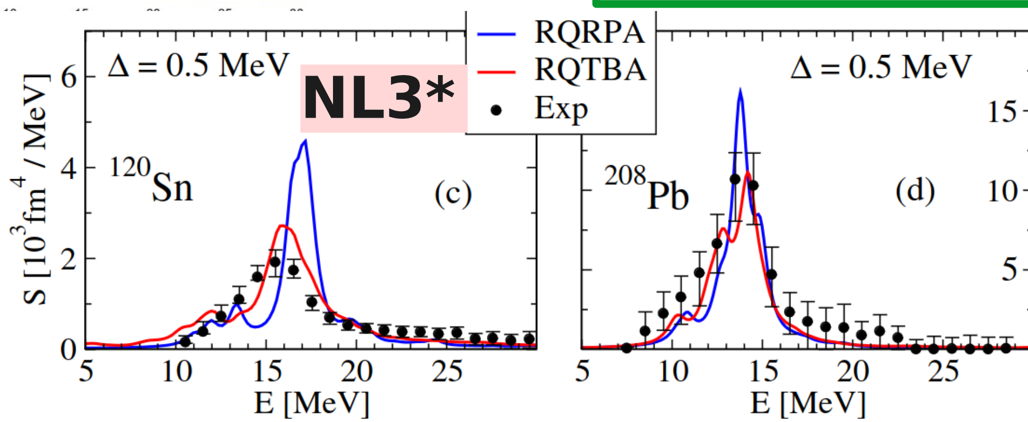
$K_\infty = 226$ MeV

These calculations points towards a **plausible estimate** on K = 242±16 MeV. **Is that the final word?** Further experiments are planned.

Very **recently** in arxiv two works explain **ISGMR** in different nuclei within the **PVC** approach

$$(E_x^{\mathrm{ISGMR}})^2 \equiv K_A \frac{\hbar^2}{m\langle r^2\rangle}$$

$$K_\infty = 258 \text{ MeV}$$

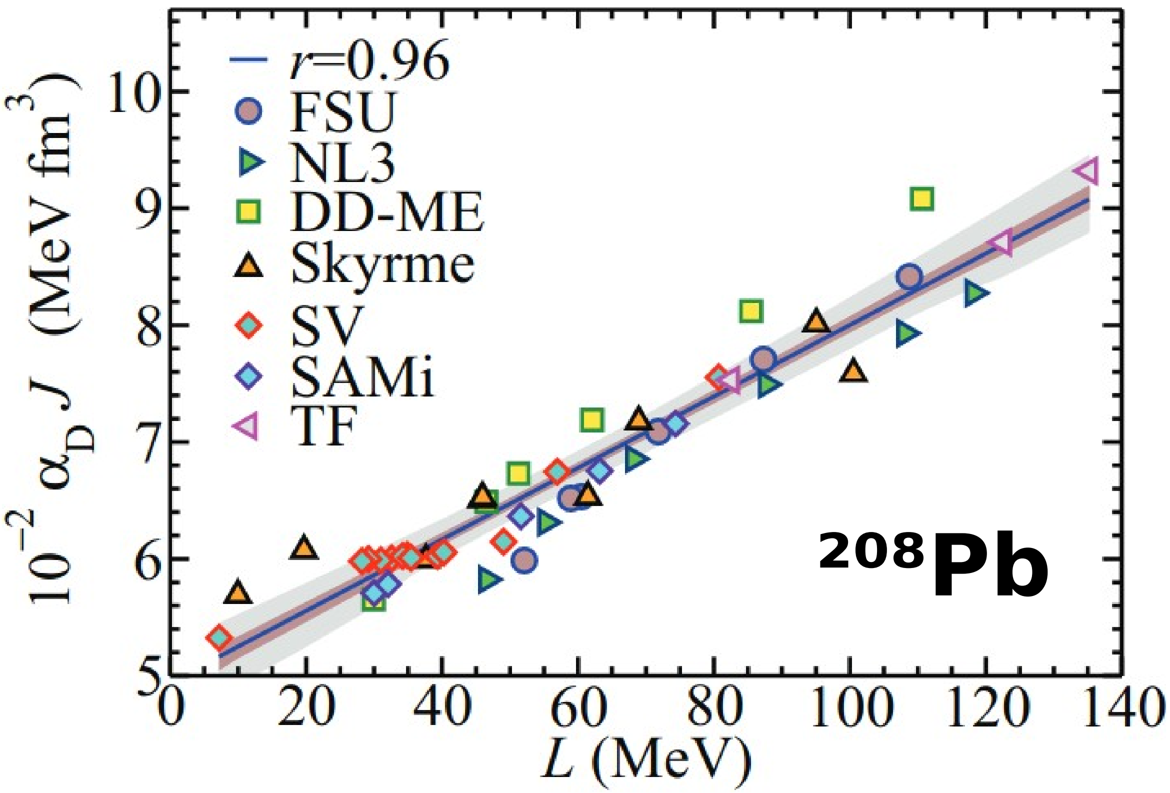


Elena Litvinova
Phys. Rev. C **107**, L041302 – Published 5 April 2023

Dipole polarizability, J and L

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1}(E1)$$

Determination of the J vs L relation from experimental data according to EDFs



$$\alpha_D \approx \frac{\pi e^2}{54} \frac{\langle r^2 \rangle}{J} A \left(1 + \frac{5}{2} \frac{\Delta r_{np} - \Delta r_{np}^{\text{surf}} - \Delta r_{np}^{\text{Coul}}}{\langle r^2 \rangle^{1/2} (I - I_{\text{Coul}})} \right)$$

$J = 25.0(2) + 0.19(2)L$ for ^{68}Ni,

$J = 25.4(1.1) + 0.17(1)L$ for ^{120}Sn,

$J = 24.5(8) + 0.17(1)L$ for ^{208}Pb .

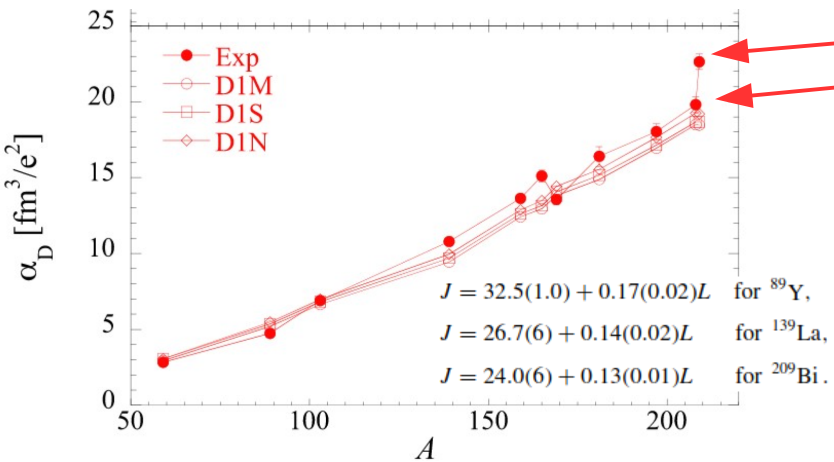
$$S(\langle \rho \rangle \approx 0.08 \text{ fm}^{-3}) \approx 25 \text{ MeV}$$

X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

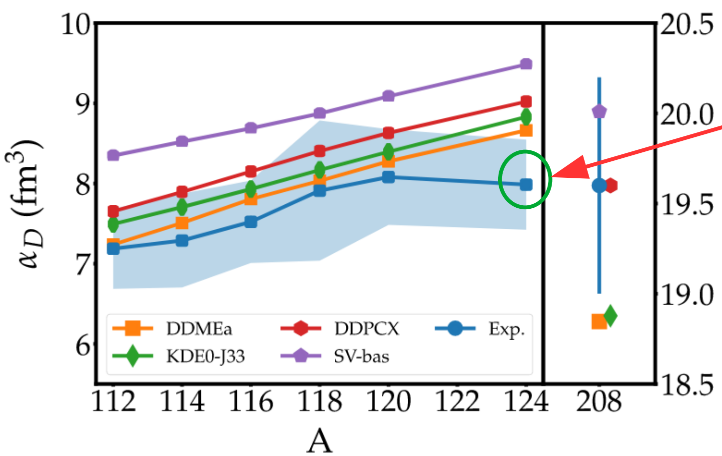
X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

Dipole polarizability:
do we understand it?

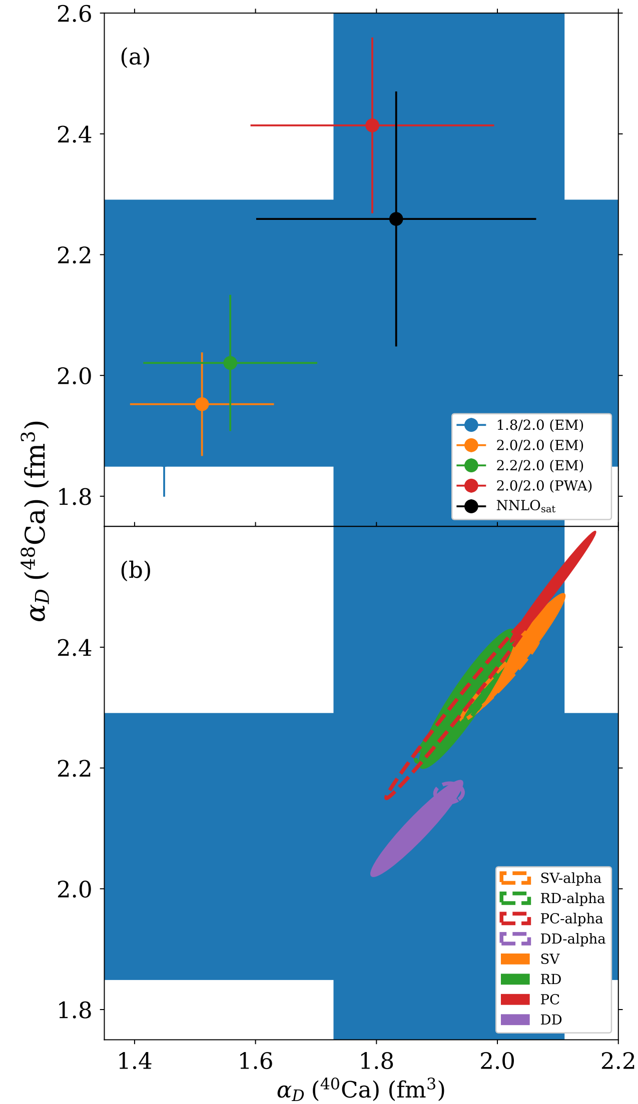
S. Goriely, S. Péru, G. Colò, X. Roca-Maza, I. Gheorghe, D. Filipescu, and H. Utsunomiya
Phys. Rev. C **102**, 064309 – Published 9 December 2020



^{209}Bi
^{208}Pb ?



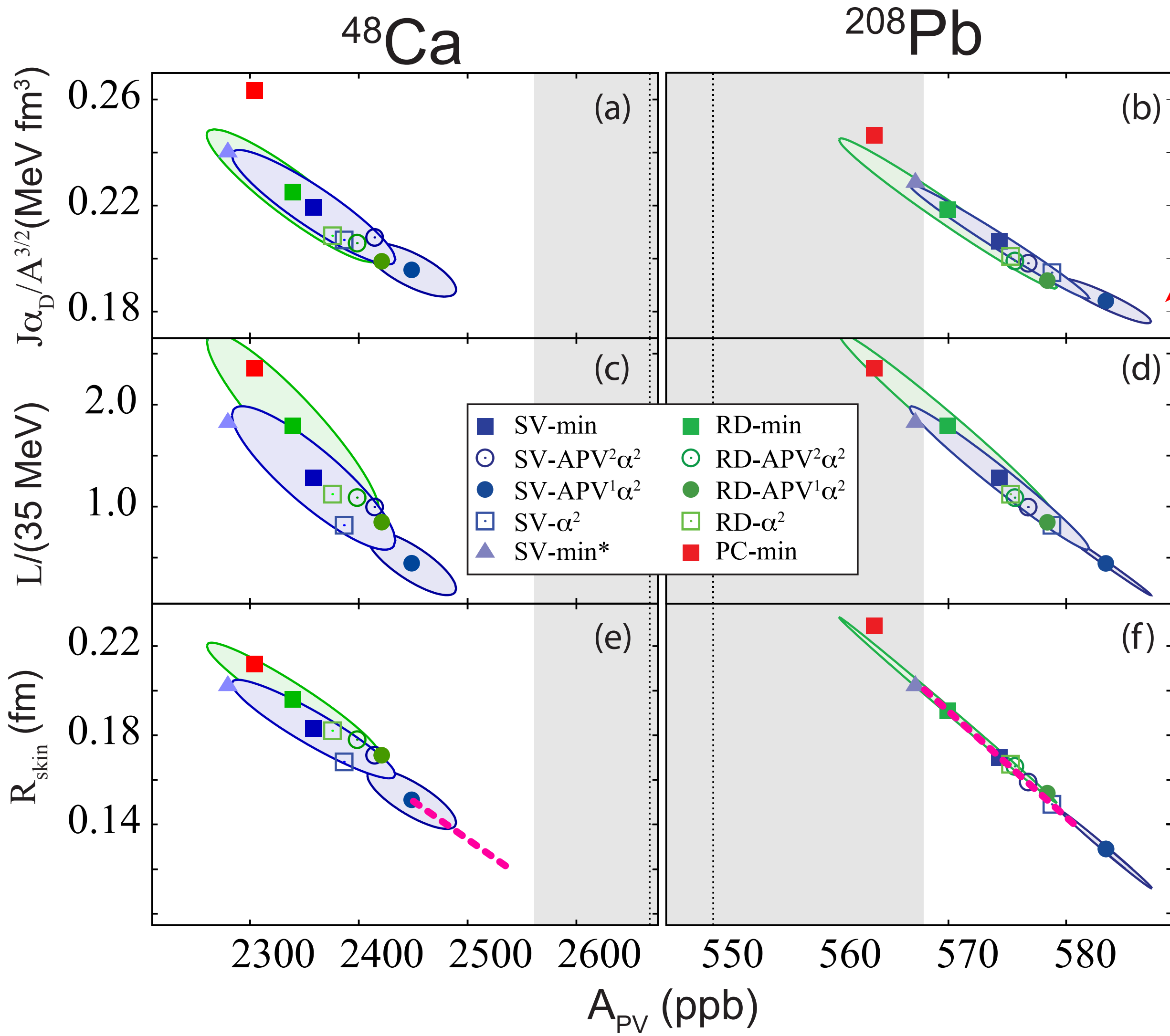
^{124}Sn?



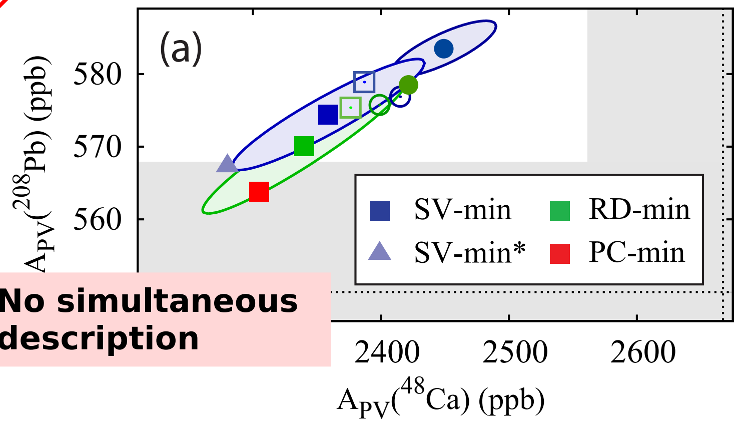
S. Bassauer, P. von Neumann-Cosel, P.-G. Reinhard et al.
Physics Letters B 810 (2020) 135804

Electric dipole polarizability of ^{40}Ca
R. W. Fearick, P. von Neumann-Cosel, S. Bacca, J. Birkhan, F. Bonaiti, I. Brandherm, G. Hagen, H. Matsubara, W. Nazarewicz, N. Pietralla, V. Yu. Ponomarev, P. -G. Reinhard, X. Roca-Maza, A. Richter, A. Schwenk, J. Simonis, and A. Tamii
Accepted 9 May 2023 Phys. Rev. Research

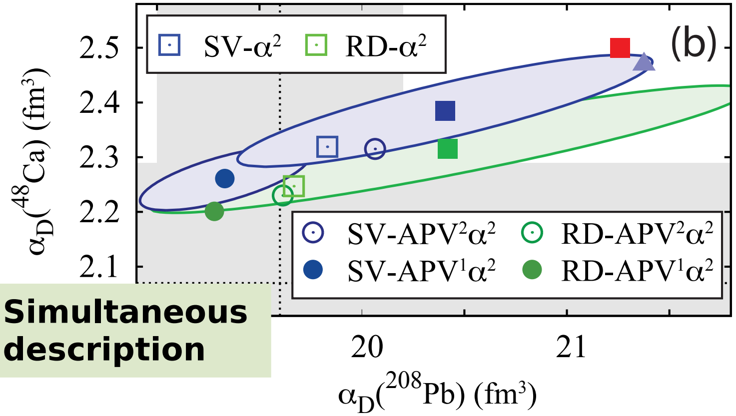
A_{PV} and α_D in ^{48}Ca and ^{208}Pb



Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca



No simultaneous description

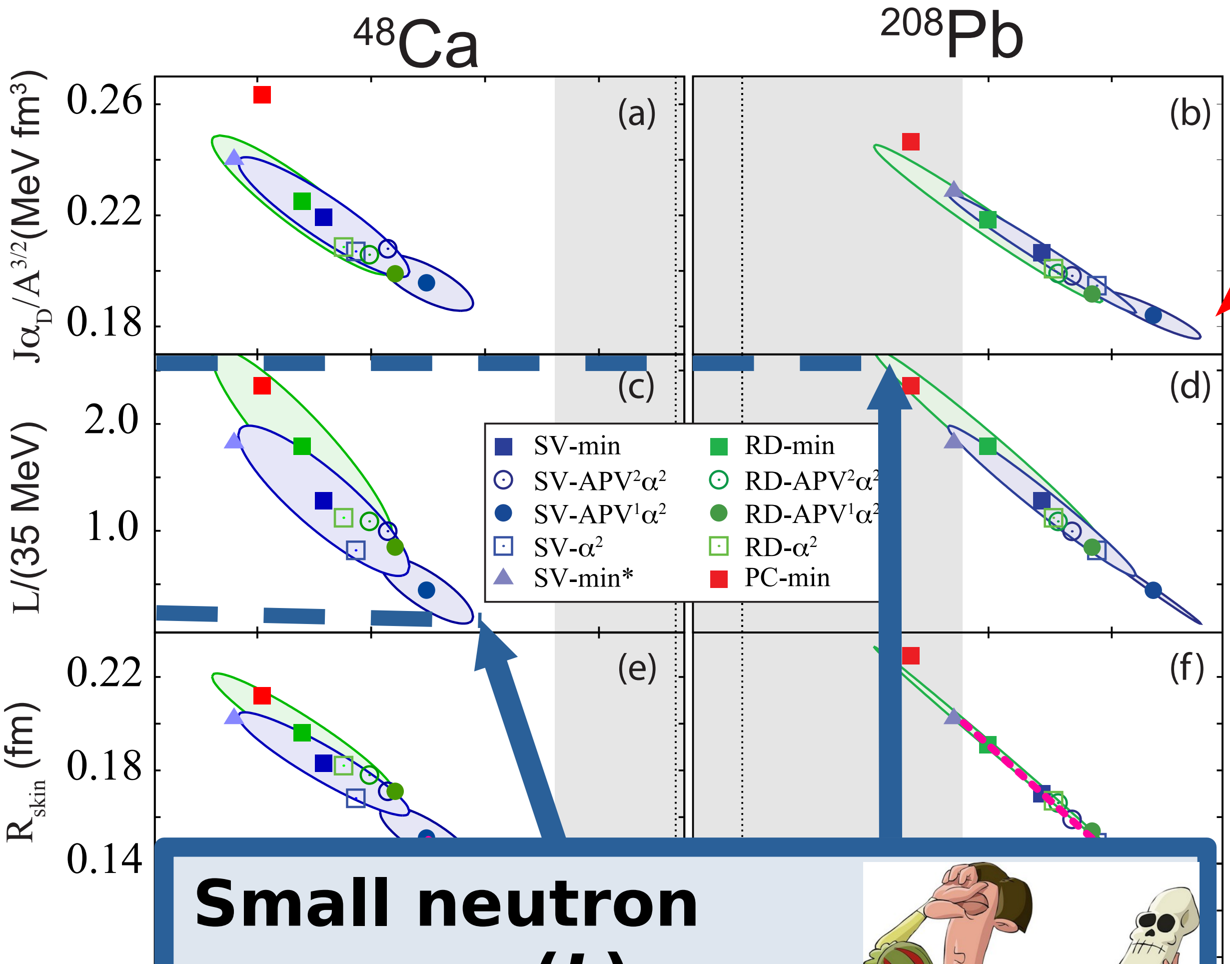


Simultaneous description

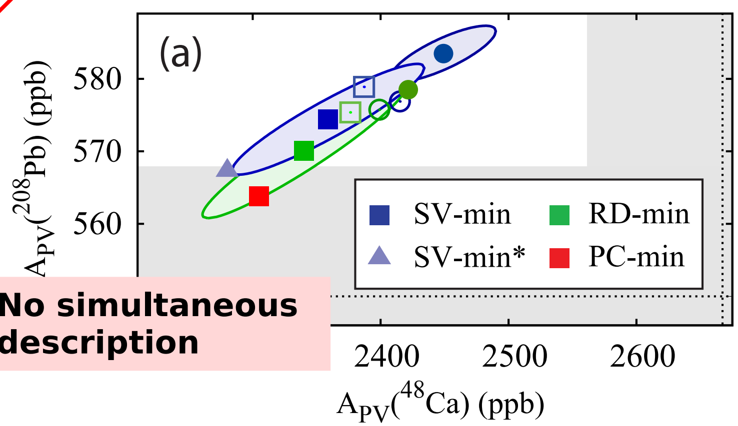
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz, PRL 127 232501 (2021) and PRL 129 232501 (2022)

Magenta dashed lines from Δr_{np} in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

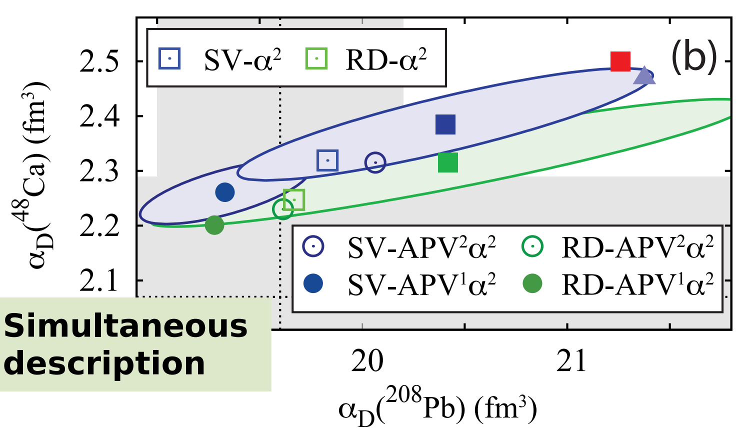
A_{PV} and α_D in ^{48}Ca and ^{208}Pb



Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca



No simultaneous description



Simultaneous description

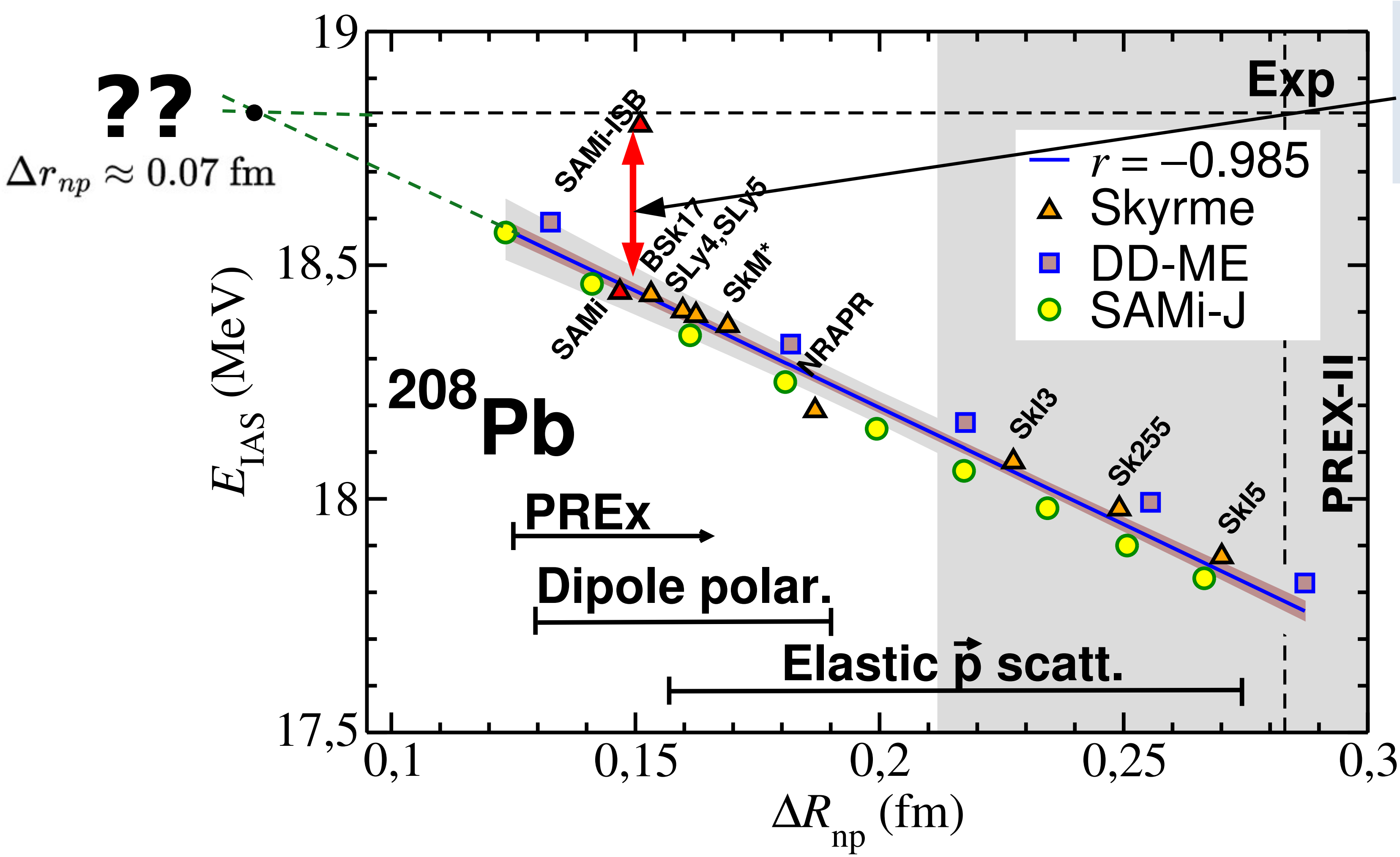
Small neutron pressure (*L*) or large neutron pressure?

...rom Δr_{np} in G.
...s 12, 186–190
...ure Physics (2022)

Isobaric Analog State, ISB and Δr_{np}

$$E_{\mathrm{IAS}} = \frac{\langle 0|T_+[\mathcal{H},T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle} \approx \frac{6}{5}\frac{Ze^2}{r_0A^{1/3}}\left(1-\sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta R_{\mathrm{np}}}{r_0A^{1/3}}\right)$$

$$F = T_\pm = \sum_i^A t_\pm(i)$$



Isospin symmetry breaking missing effects

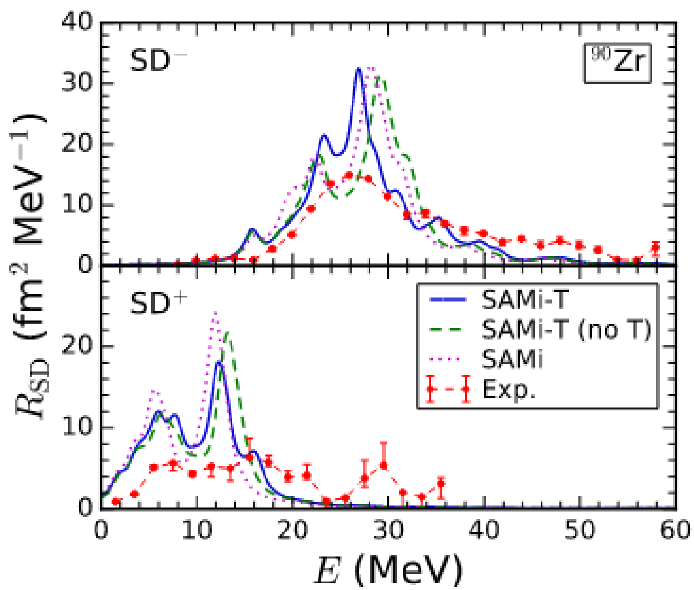
X. Roca-Maza, G. Colò, and H. Sagawa
Phys. Rev. Lett. **120**, 202501 – Published 18 May 2018

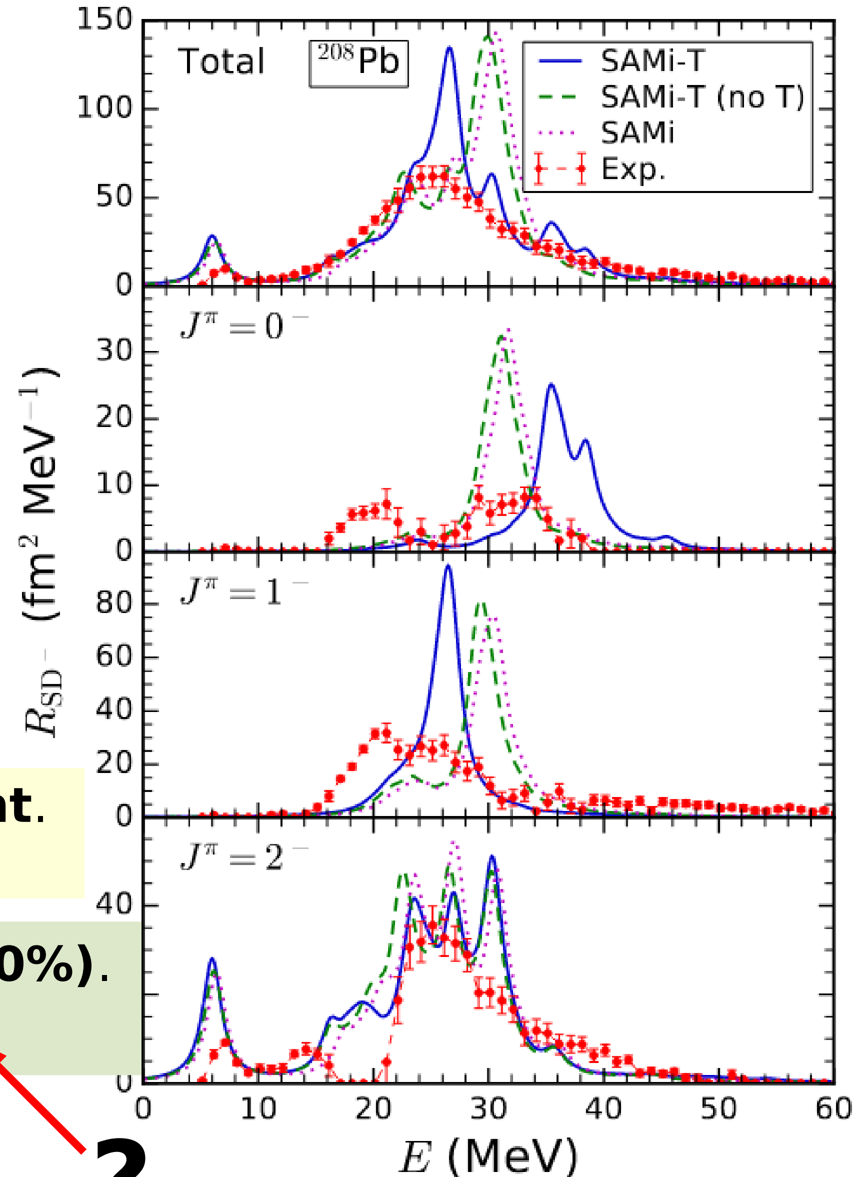
Spin Dipole Resonance and Δr_{np}

Difficult to measure and analyze?

$$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_i^L \left[Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i) \right]_{JM}$$

$$m_0(t_-) - m_0(t_+) = \frac{9}{4\pi}\left(N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right)$$





→ ^{90}Zr **exp and theo sum rules in agreement**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.04$ fm

→ ^{208}Pb **exp sum rule < theo sum rule (~20%)**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.03$ fm

?

Shihang Shen (申时行), Gianluca Colò, and Xavier Roca-Maza
Phys. Rev. C **99**, 034322 – Published 20 March 2019

Summary

with qualitative indication of accuracy needed to describe experiment (note that absolute values might be subject to systematics)

→ $\rho_0 \in$ [0.154,0.159] fm^{-3} → relative accuracy 2%

→ needed to describe experiment (R_{ch}) ≤0.1%

→ $e_0 \in$ [15.6,16.2] MeV→ relative accuracy 4%

→ needed to describe experiment (B) ≤0.0001%

→ $K_0 \in$ [200,260] MeV→ relative accuracy 25%

→ needed to describe experiment (E_x^{GMR}) ≤7%

→ $J \in$ [30,35] MeV → relative accuracy 15%

→ needed to describe experiment (α) ≤15%

→ $L \in$ [20,120] MeV → relative accuracy 150%

→ needed to describe experiment (α) ≤50%

→ ...

Nuclear equation of state from ground and collective excited state properties of nuclei

X. Roca-Maza[a], N. Paar[b]

Progress in Particle and Nuclear Physics
Volume 101, July 2018, Pages 96-176

Perspectives

→ Develop new EDFs:

→ **microscopic** (ab-initio based)

→ **phenomenologic** (R_{ch}, B, E_{GMR}, α_D and A_{PV}, ...)

→ Study isospin symmetry breaking in nuclei:

→ **microscopic** (QCD, ab-initio)

→ **phenomenologic** (mirror nuclei)

→ Test new EDFs on finite nuclei:

→ Are extrapolations to drip lines less affected by theoretical errors?

→ Does **GMR** be only explained by PVC? What about $\boldsymbol{\alpha_D}$?

→ Does **IAS** requires large/small ISB terms? Can theory support further experiments on the **SDR**?

→ Test new EDFs on Neutron Star observations

→ Does better EDFs put strong constraints on a neutron star EoS at low densities? Is that useful for the neutron star EoS at higher densities?

Collaborators

→ Gianluca **Colò** (University of Milan)

→ Hiroyuki **Sagawa** (University of Aizu & RIKEN)

→ Shihang **Shen** (Forschungszentrum Jülich)

→ Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)

→ Jorge **Piekarewicz** (Florida State University)

→ Nils **Paar** & Dario **Vretenar** (University of Zagreb)

→ Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)

→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)

→ Yifei **Niu** (Lanzhou University)

→ Witold **Nazarewicz** (FRIB and Michigan State University)

→ Stephane **Goriely** (Université Libre de Bruxelles)

→ Sophie **Péru** (Université Paris-Saclay, CEA)