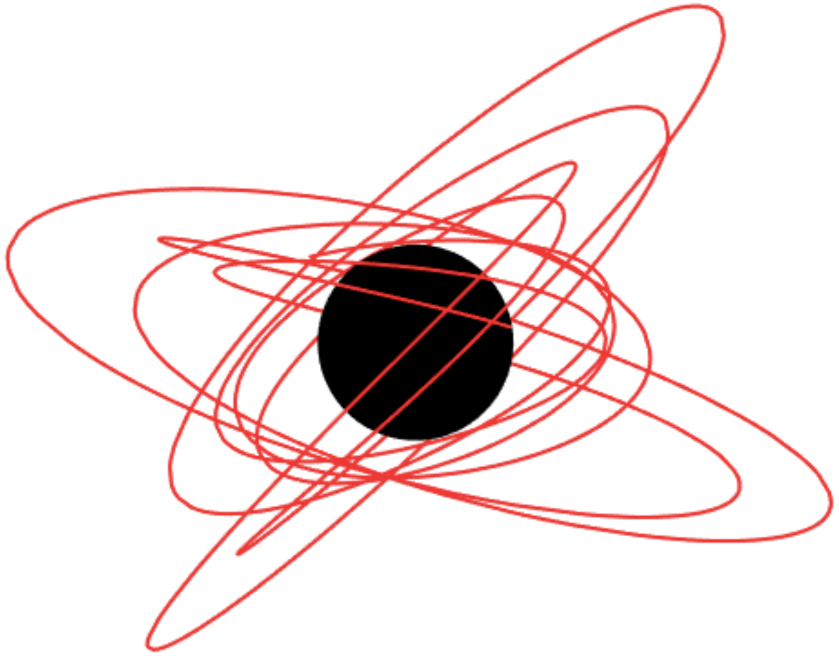
Detecting new fundamental fields with asymmetric binaries

PhD seminar - 31/05/2023

- *S.B+ : Phys.Rev.D 106 (2022) 4*
- *Phys. Rev. Lett 125, 141101 (2020)*
- *Nature Astron. 6 (2022) 4, 464-470*

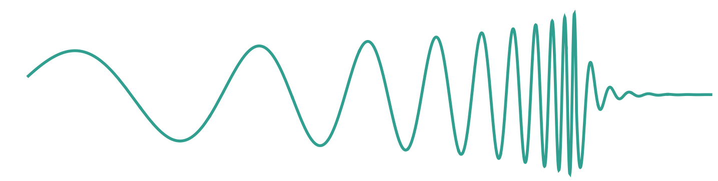
Collaboration with
A. Maselli, N. Franchini, L. Gualtieri, T. Sotiriou, P. Pani



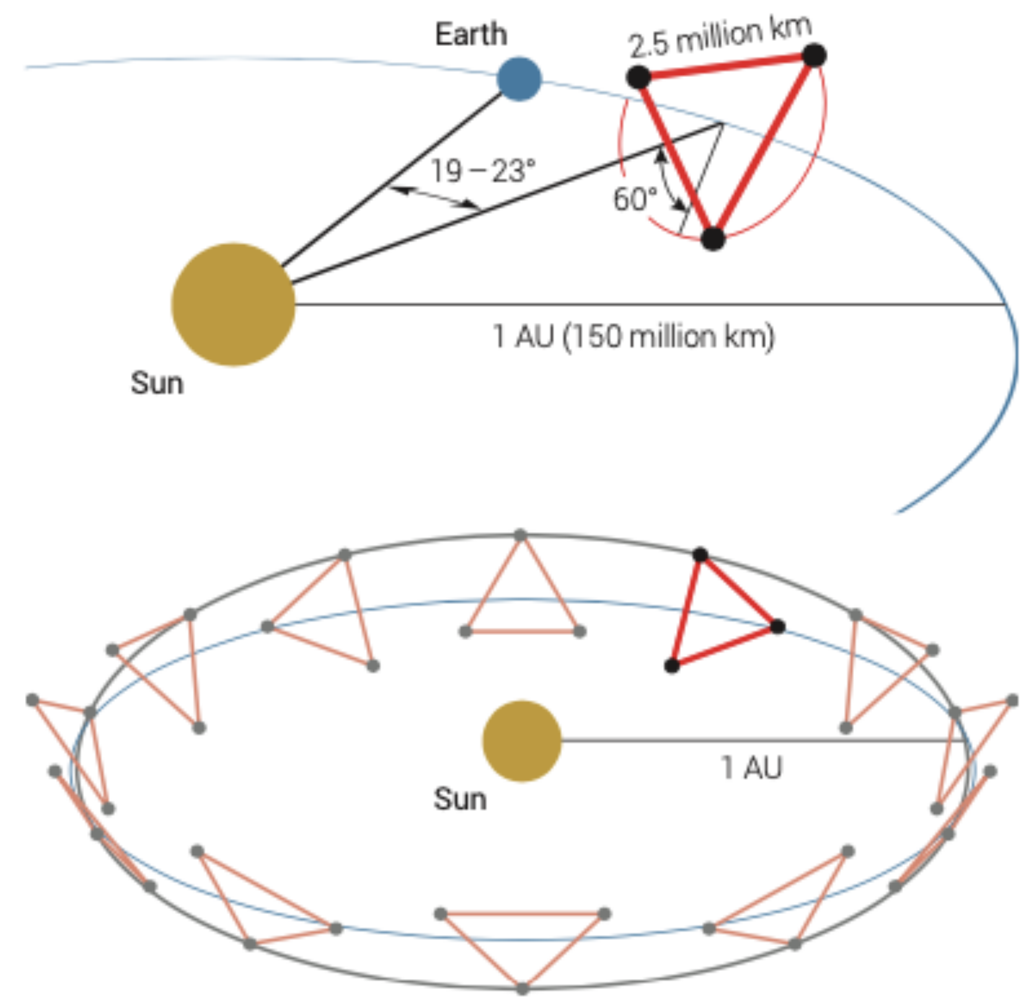
Susanna Barsanti
(She/her)
PhD student 36° cycle
Gravity theory group @Sapienza
University of Rome

Istituto Nazionale di Fisica Nucleare

Gravitational waves detection: LISA



- General Relativity (GR) to describe gravity
- Proved with extreme accuracy in the weak-field regime
- Detection of GWs to test GR in strong-field regime



- Laser Interferometer Space Antenna

  (LISA): future space based detector

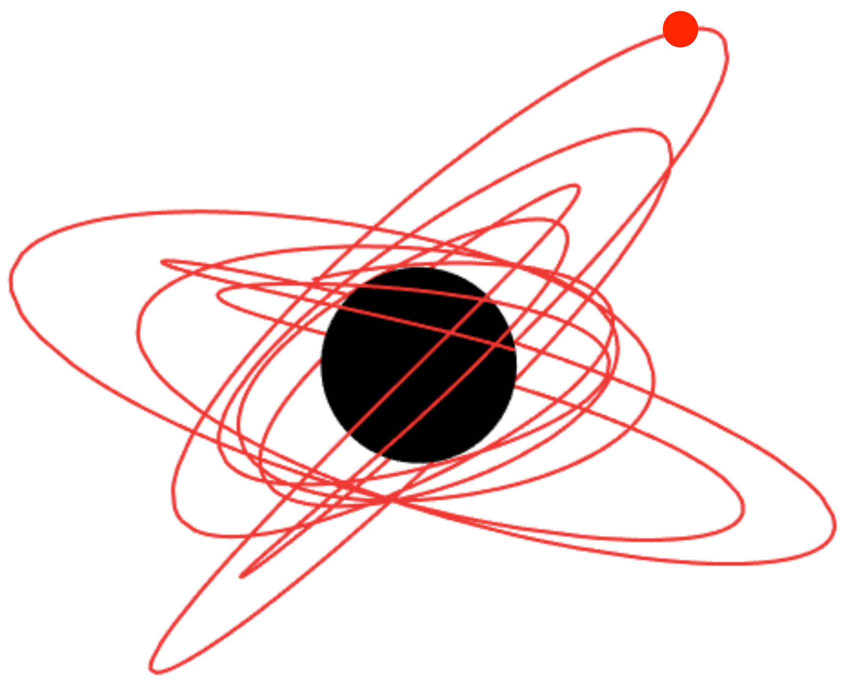
- Among the main targets:

  Extreme Mass Ratio Inspirals (EMRIs)

Using EMRIs to test GR

Binary systems with a stellar mass compact object inspiralling into a massive black hole

- Primary of $M \in (10^4, 10^7)\ M_\odot$
- Secondary of $\mu \ll M$, so that the mass ratio $q = \mu/M \sim (10^{-6} - 10^{-3})$
- Emit GWs in the mHz, main targets of LISA



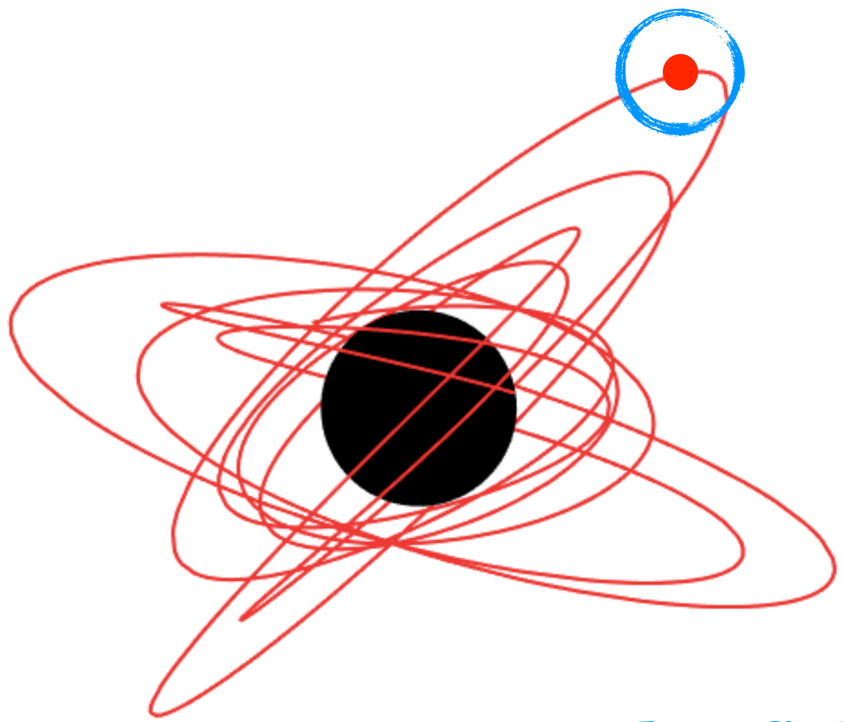
Complete $\sim 10^4 - 10^5$ orbits before the plunge

- Detailed map of the binary spacetime
- Pinpoint even small deviations from GR

Using EMRIs to test GR

Binary systems with a stellar mass compact object inspiralling into a massive black hole

- Primary of $M \in (10^4, 10^7)\ M_\odot$
- Secondary of $\mu \ll M$, so that the mass ratio $q = \mu/M \sim (10^{-6} - 10^{-3})$
- Emit GWs in the mHz, main targets of LISA



Complete $\sim 10^4 - 10^5$ orbits before the plunge

- Detailed map of the binary spacetime
- Pinpoint even small deviations from GR
- Beyond GR theories with additional scalar fields

* Use EMRIs to detect scalar fields with LISA

EMRIs in General Relativity

$G = c = 1$

$$S\left[\mathbf{g}, \Psi\right] = S_0\left[\mathbf{g}\right] + S_m\left[\mathbf{g}, \Psi\right]$$

matter fields Ψ

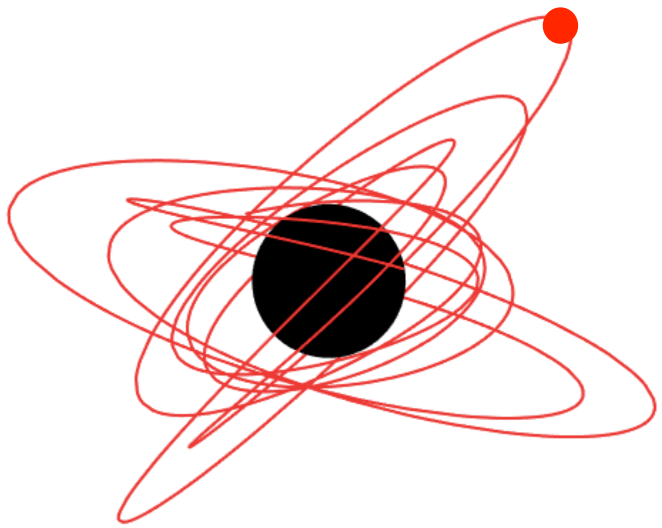
$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R$$

point particle

- R : function of $g_{\mu\nu}$, metric tensor which describes the spacetime

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$S_{pp} = -\int m \; ds = \int m \sqrt{g_{\mu\nu} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda}} d\lambda$$



EMRIs in General Relativity

$G = c = 1$

$$S\left[\mathbf{g}, \Psi\right] = S_0\left[\mathbf{g}\right] + S_m\left[\mathbf{g}, \Psi\right]$$

matter fields Ψ

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R$$

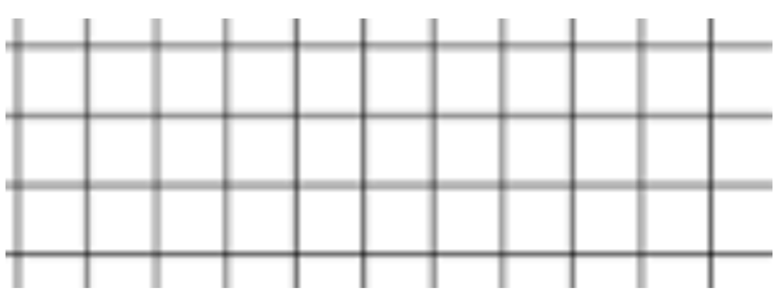
point particle

- R : function of $g_{\mu\nu}$, metric tensor which describes the spacetime

$$S_{pp} = -\int m\ ds = \int m \sqrt{g_{\mu\nu}\frac{dy_p^\mu}{d\lambda}\frac{dy_p^\nu}{d\lambda}} d\lambda$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

Flat: $ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$



EMRIs in General Relativity

$G = c = 1$

$$S\left[\mathbf{g}, \Psi\right] = S_0\left[\mathbf{g}\right] + S_m\left[\mathbf{g}, \Psi\right]$$

matter fields Ψ

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R$$

point particle

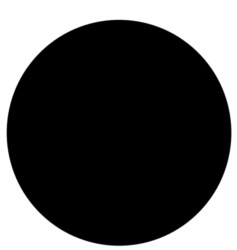
- R : function of $g_{\mu\nu}$, metric tensor which describes the spacetime

$$S_{pp} = -\int m \; ds = \int m \sqrt{g_{\mu\nu} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda}} d\lambda$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Flat: $ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$

BH: $ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$



EMRIs in General Relativity

$G = c = 1$

$$S\left[\mathbf{g}, \Psi\right] = S_0\left[\mathbf{g}\right] + S_m\left[\mathbf{g}, \Psi\right]$$

matter fields Ψ

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R$$

point particle

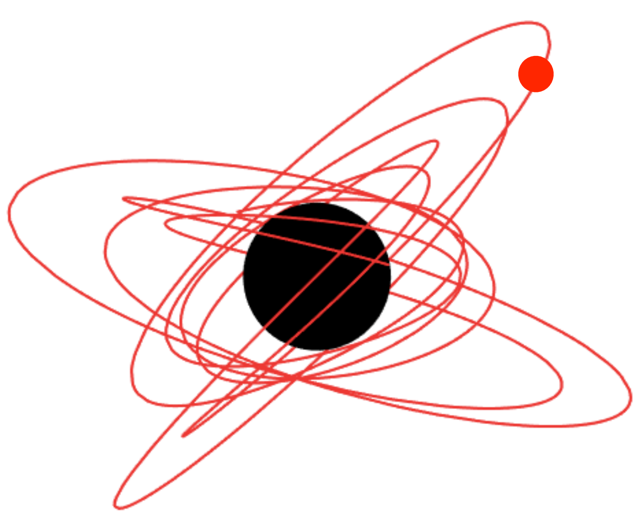
- R : function of $g_{\mu\nu}$, metric tensor which describes the spacetime

$$S_{pp} = -\int m\ ds = \int m \sqrt{g_{\mu\nu} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda}} d\lambda$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Flat: $ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$

BH: $ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$



$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

EMRIs in General Relativity

$G = c = 1$

$$S\left[\mathbf{g}, \Psi\right] = S_0\left[\mathbf{g}\right] + S_m\left[\mathbf{g}, \Psi\right]$$

matter fields Ψ

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R$$

point particle

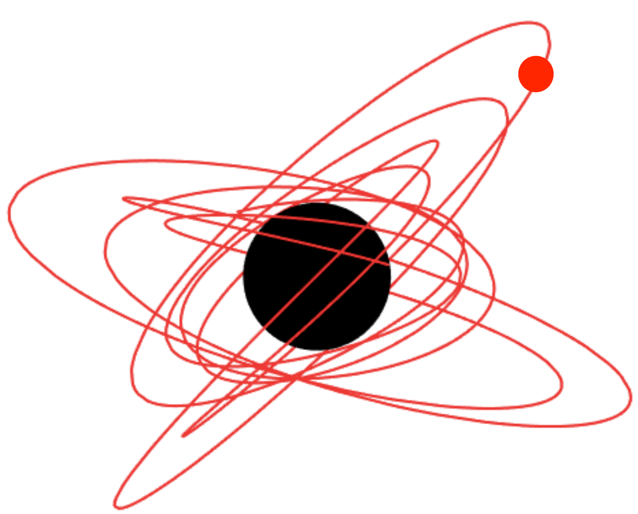
$$S_{pp} = -\int m\ ds = \int m\sqrt{g_{\mu\nu}\frac{dy_p^\mu}{d\lambda}\frac{dy_p^\nu}{d\lambda}}d\lambda$$

- R : function of $g_{\mu\nu}$, metric tensor which describes the spacetime

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

Flat: $ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$

BH: $ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$



$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

NO-HAIR THEOREM:

BHs are described only by three parameters: mass, spin, (negligible) electrical charge

EMRIs in General Relativity + scalar field

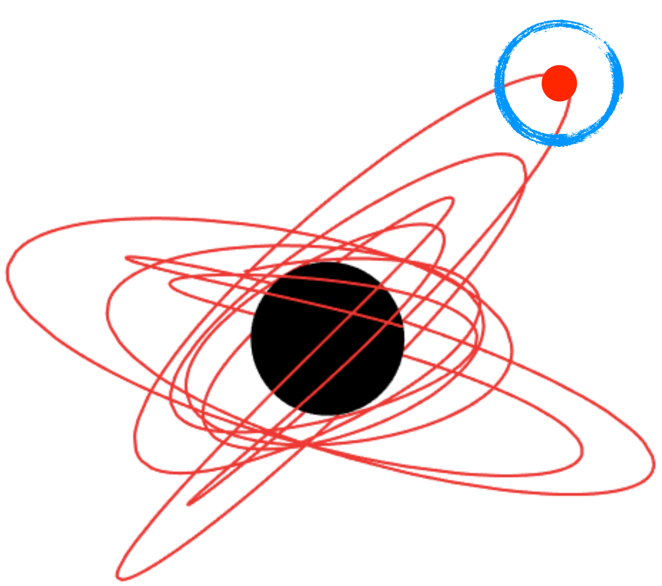
$G = c = 1$

$$S\left[\mathbf{g}, \varphi, \Psi\right] = S_0\left[\mathbf{g}, \varphi\right] + \alpha S_c\left[\mathbf{g}, \varphi\right] + S_m\left[\mathbf{g}, \varphi, \Psi\right]$$

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R - \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

Non minimal coupling

$$S_{pp} = -\int m(\varphi)\, ds = \int m(\varphi) \sqrt{g_{\mu\nu} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda}} d\lambda$$



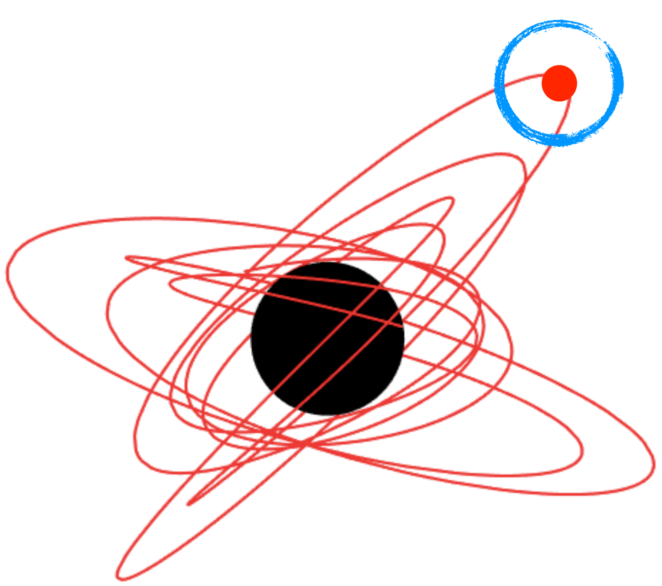
EMRIs in General Relativity + scalar field

$$S\left[\mathbf{g}, \varphi, \Psi\right] = S_0\left[\mathbf{g}, \varphi\right] + \alpha S_c\left[\mathbf{g}, \varphi\right] + S_m\left[\mathbf{g}, \varphi, \Psi\right]$$

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R - \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

Non minimal coupling

$$S_{pp} = -\int m(\varphi)\, ds = \int m(\varphi) \sqrt{g_{\mu\nu} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda}} d\lambda$$



Assumption:

- The theory tends to GR as $\alpha \to 0$

Case 1) α is dimensionless, and there is a no-hair theorem: i.e. black hole solutions as in GR

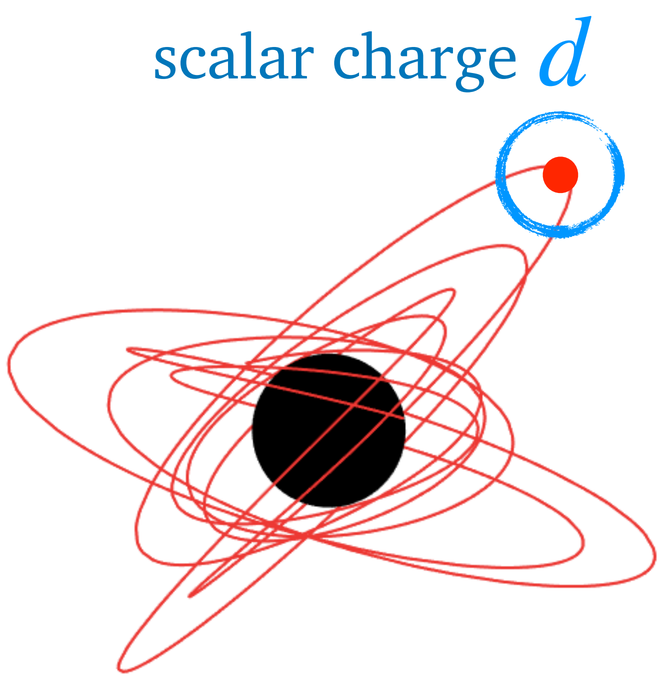
EMRIs in General Relativity + scalar field

$$S\left[\mathbf{g}, \varphi, \Psi\right] = S_0\left[\mathbf{g}, \varphi\right] + \alpha S_c\left[\mathbf{g}, \varphi\right] + S_m\left[\mathbf{g}, \varphi, \Psi\right]$$

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi} R - \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

Non minimal coupling

$$S_{pp} = -\int m(\varphi)\, ds = \int m(\varphi) \sqrt{g_{\mu\nu} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda}} d\lambda$$



Assumption:

- The theory tends to GR as $\alpha \to 0$

Case 2) $\alpha = [\text{mass}]^n$, and there is a scaling in the BH hair

- Any corrections to GR must depend on $\zeta \equiv \dfrac{\alpha}{(\text{mass})^n}$

For the **primary** $\zeta \equiv \dfrac{\alpha}{M^n} = q^n \dfrac{\alpha}{m_p^n} \ll 1$

Field equations

$$\frac{\delta S}{\delta g^{\mu\nu}} \xrightarrow{\zeta \ll 1} G_{\mu\nu} = T^p_{\mu\nu} = 8\pi m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy^p_\mu}{d\lambda} \frac{dy^p_\nu}{d\lambda} d\lambda$$

$$\frac{\delta S}{\delta \varphi} \xrightarrow{\zeta \ll 1} \Box \varphi = -4\pi\, d\, m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$

Field equations

$$G_{\mu\nu} = T^{p}_{\mu\nu} = 8\pi m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy^p_\mu}{d\lambda} \frac{dy^p_\nu}{d\lambda} d\lambda$$

→ same as in GR

$$\Box \varphi = -4\pi\, d\, m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$

Field equations

$$G_{\mu\nu} = T^p_{\mu\nu} = 8\pi m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy^p_\mu}{d\lambda} \frac{dy^p_\nu}{d\lambda} d\lambda$$

→ same as in GR

$$\Box \varphi = -4\pi \, d \, m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$

→ source term proportional to the scalar charge of the test body

→ universal: all the information of the theory are enclosed in d

→ for hairy BHs, if the little body is a BH, we find a relation $d(\alpha)$

Field equations

$$G_{\mu\nu} = T^{p}_{\mu\nu} = 8\pi m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy^p_\mu}{d\lambda} \frac{dy^p_\nu}{d\lambda} d\lambda$$

→ same as in GR

$$\Box \varphi = -4\pi \, d \, m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$

→ source term proportional to the scalar charge of the test body

→ universal: all the information of the theory are enclosed in d

→ for hairy BHs, if the little body is a BH, we find a relation $d(\alpha)$

Let's solve them in perturbation theory !

- Leading order in q : motion along geodesics

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu} \qquad \varphi = \varphi_0 + \varphi_1$$

Perturbations: Teukolsky formalism - s=0

$s = 0$

$$\psi^{(s=0)}(t, r, \theta, \phi) = \int d\omega \sum_{\ell m} R^{(s=0)}_{\ell m}(r, \omega) S^{(s)}_{\ell m}(\theta, \omega) e^{im\phi} e^{-i\omega t}$$

$$\psi(\omega, r) \equiv \sqrt{r^2 + a^2} R(\omega, r)$$

$$\frac{\partial^2 \psi_{\ell m}}{\partial r_\star^2} + \left(\omega^2 - V\right) \psi_{\ell m} = J$$

MASTER EQUATION

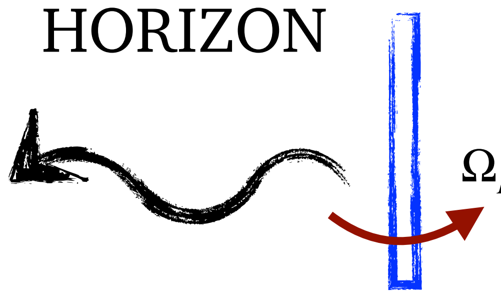
Green Method

- Homogenous solutions ψ_H and ψ_∞ with boundary conditions:

HORIZON

$\psi_H = e^{-ik_- r_\star}$

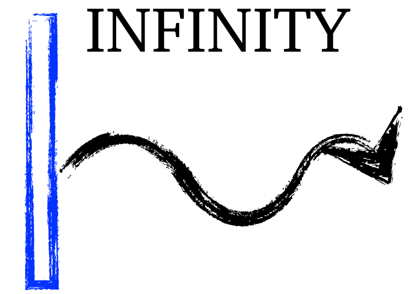
$k_- = \omega - \Omega_H$



$\Omega_H = \frac{a}{2Mr_+}$

Purely ingoing wave

INFINITY



$\psi_\infty = e^{ik_+ r_\star}$

$k_+ = \omega$

Purely outgoing wave

- General solution ψ obtained integrating over the source term:

$$\psi(\omega, r) = \psi_\infty \int_{-\infty}^{r_\star} \frac{\psi_H J dr'_\star}{W} + \psi_H \int_{r_\star}^{+\infty} \frac{\psi_\infty J dr'_\star}{W}$$

Wronskian
$W = \psi'_\infty \psi_H - \psi_\infty \psi'_-,$

$$\delta\varphi^{-,+}_{\ell m} = \int_{-\infty}^{+\infty} \frac{\psi_{\infty,H} J dr_\star}{W} \quad \propto \quad \delta(\omega - m\omega_p)$$

$$\omega_p = \frac{M^{1/2}}{r^{3/2} + aM^{3/2}}$$

Perturbations: GW emission

Spheroidal harmonics decomposition: $\psi^{(s)}(t,r,\theta,\phi) = \int d\omega \sum_{\ell m} R^{(s)}_{\ell m}(r,\omega) S^{(s)}_{\ell m}(\theta,\omega) e^{im\phi} e^{-i\omega t}$

Teukolsky formalism for the gravitational and scalar perturbations: $\frac{\partial^2 \psi_{\ell m}}{\partial r_\star^2} + \left(\omega^2 - V\right)\psi_{\ell m} = J$

$\psi(\omega,r) \equiv \sqrt{r^2+a^2} R(\omega,r)$

$$\dot{E}^{(\pm)}_{scal} = \frac{1}{16\pi} \sum_{\ell,m} \omega_m k^{\pm} |\delta\varphi^{\pm}_{\ell m}|^2 \qquad \omega_m = m\omega_p = m \frac{M^{1/2}}{r^{3/2} + aM^{3/2}}$$

TOTAL EMISSION:

$$\dot{E}_{GW} = \sum_{i=+,-} [\dot{E}^{(i)}_{\rm grav} + \dot{E}^{(i)}_{\rm scal}] = \dot{E}_{grav} + \dot{E}_{scal}$$

$\dot{E}_{scal} \propto d^2$

EXTRA emission *simply added* to the gravitational one!

only depends on the scalar charge d

Orbital Evolution

The emitted GW flux drives the adiabatic orbital evolution

- Balance law $\dot{E} = -\dot{E}_{GW}$

- From the rate of change of the integrals E, we obtain the time derivatives of r

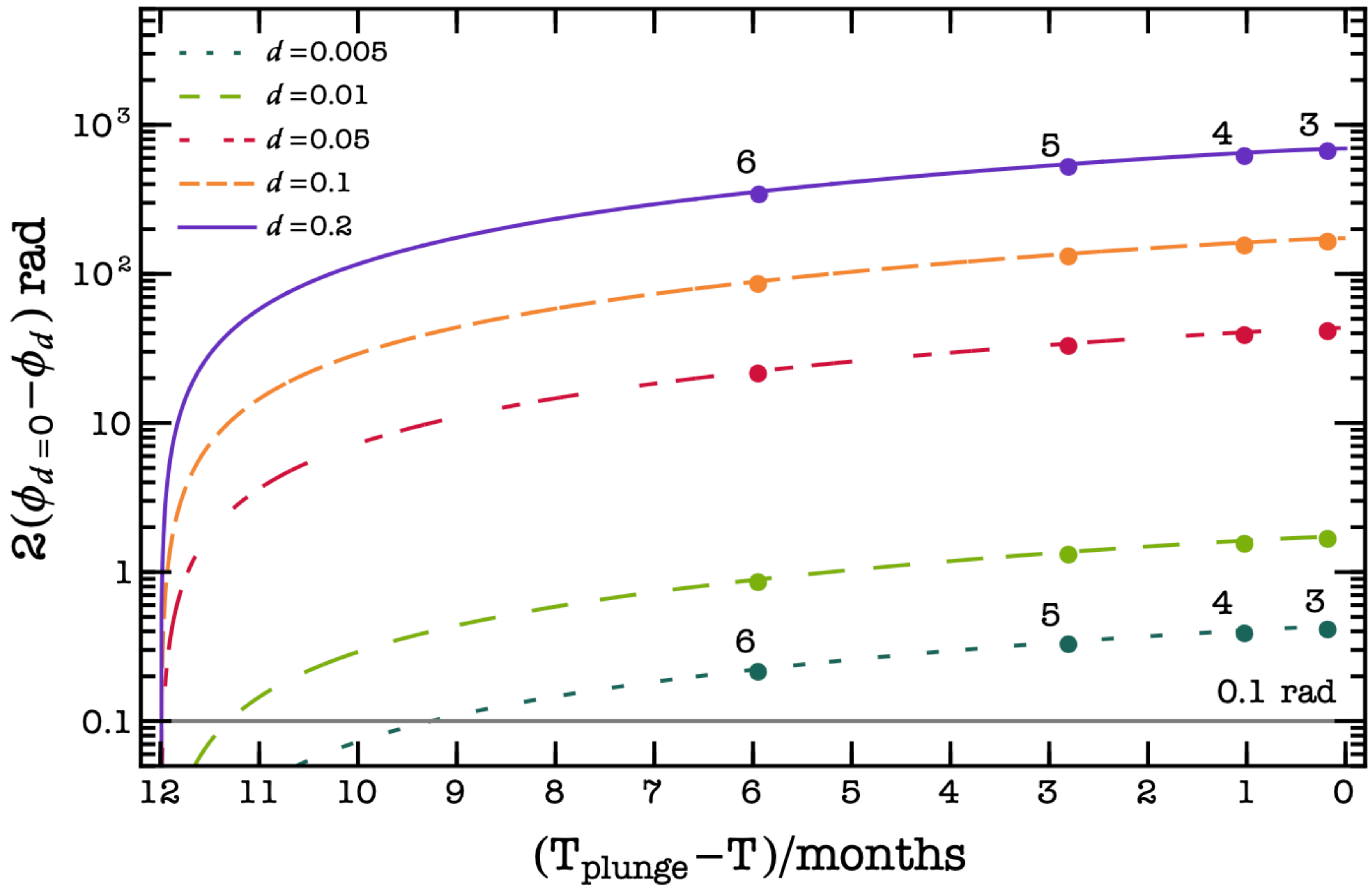
$$\frac{dr}{dt} = -\dot{E}\frac{dr}{dE_{orb}}$$

- And of the phases ϕ related to the frequencies $\omega_p = \frac{d\Phi}{dt} = \frac{M^{1/2}}{r^{3/2} + aM^{3/2}}$

- The extra emission accelerates the binary coalescence and affects the GW phase, causing a **dephasing** w.r.t the case $d = 0$

- Compute the dephasing $\Delta\phi = 2\int_0^{T_{obs}} \Delta\Omega_\phi dt$

Dephasing



- 1 year of inspiral before the plunge
- Horizontal line: threshold of phase resolution by LISA of $\Delta\psi_\phi = 0.1$ for $SNR = 30$

GW Signal

- Quadrupolar Approximation

$$h_{ij}^{TT} = \frac{2}{D}\left(P_{i\ell}P_{jm} - \frac{1}{2}P_{ij}P_{\ell m}\right)\ddot{I}_{\ell m}$$

$$I_{ij} = \int d^3x T^{tt}(t, x^i)x^i x^j = m_p x^i x^j$$

- Strain measured by the detector

$$h(t) = \frac{\sqrt{3}}{2}\left[h_+(t)F_+(t) + h_\times(t)F_\times(t)\right]$$

→ $$h_+ = -\left(\ddot{I}_{11} - \ddot{I}_{22}\right)\left(1 + \cos^2\iota\right)/2 = \mathscr{A}\cos[2\Phi(t) + 2\Phi_0]\left(1 + \cos^2\iota\right)$$

$$h_\times = 2\ddot{I}_{12}\cos\iota = -2\mathscr{A}\sin[2\Phi(t) + 2\Phi_0]\cos\iota$$

$$\mathscr{A} = \frac{2\mu}{D}\left[M\omega(t)\right]^{2/3}$$

→ $F_+, F_\times$ detector pattern functions, related to (together with ι)

(θ_s, ϕ_s) : source orientation angles

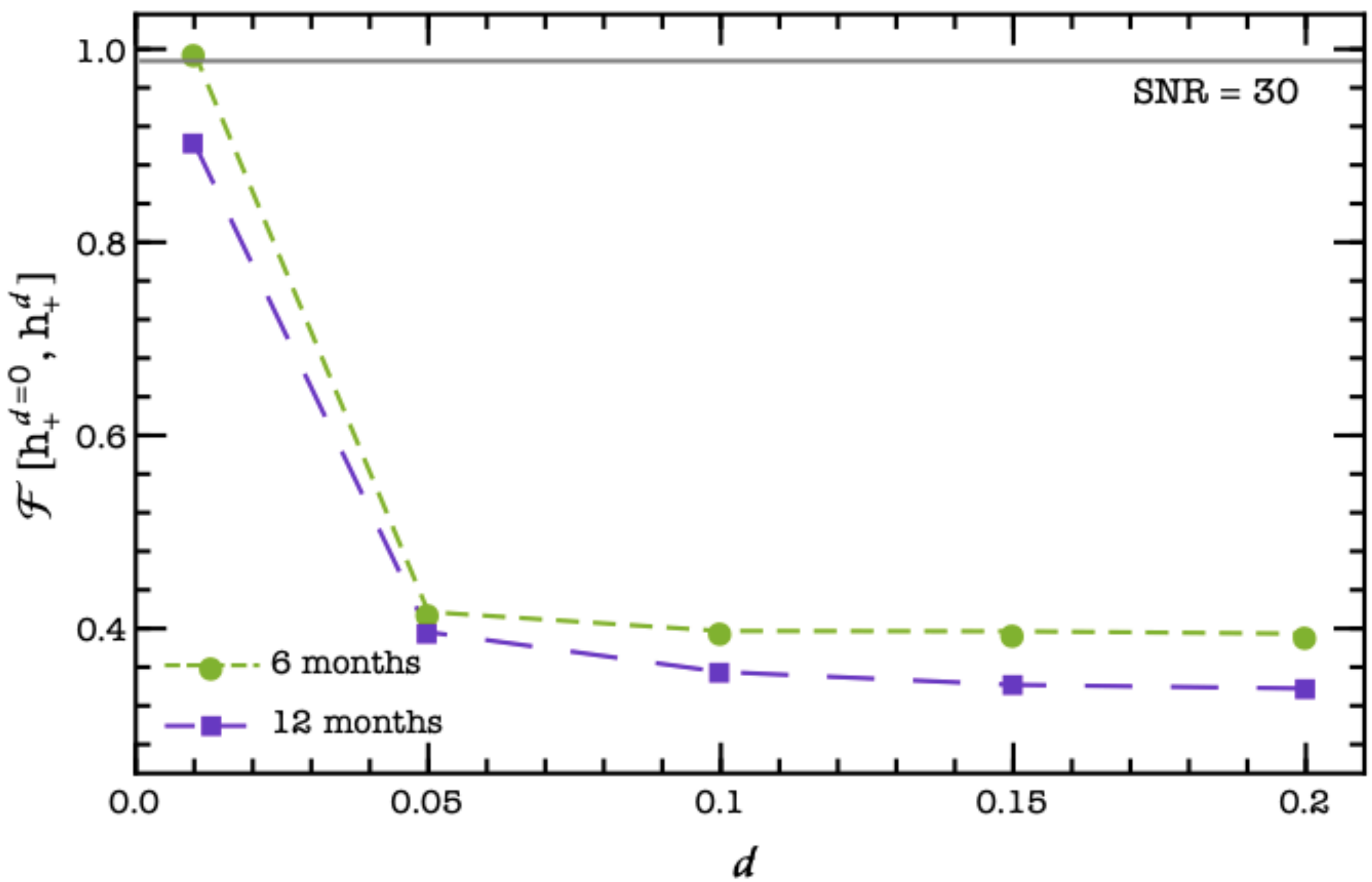
(θ_1, ϕ_1) : direction of the BH spin

in a Solar System reference frame

Faithfulness

Estimate of how much two signals differ: $\mathcal{F}[h_1, h_2] = \max_{\{t_c,\phi_c\}} \frac{\langle h_1|h_2\rangle}{\sqrt{\langle h_1|h_1\rangle\langle h_2|h_2\rangle}}$

Inner product $\langle h_1|h_2\rangle = 4\Re \int_{f_{min}}^{f_{max}} \frac{\tilde{h}_1(f)\tilde{h}_2^\star(f)}{S_n(f)} df$ LISA power spectral density



- Red line: threshold under which the signals are significantly different - $\mathcal{F} \lesssim 0.994$ for $SNR = 30$
- After 1year the faithfulness is always smaller than the threshold for scalar charges as small as $d = 0.01$

This first analysis suggest that LISA will be able to detect scalar charges as small as $d \sim 0.005 - 0.01$

What about the LISA ability to measure the scalar charge?

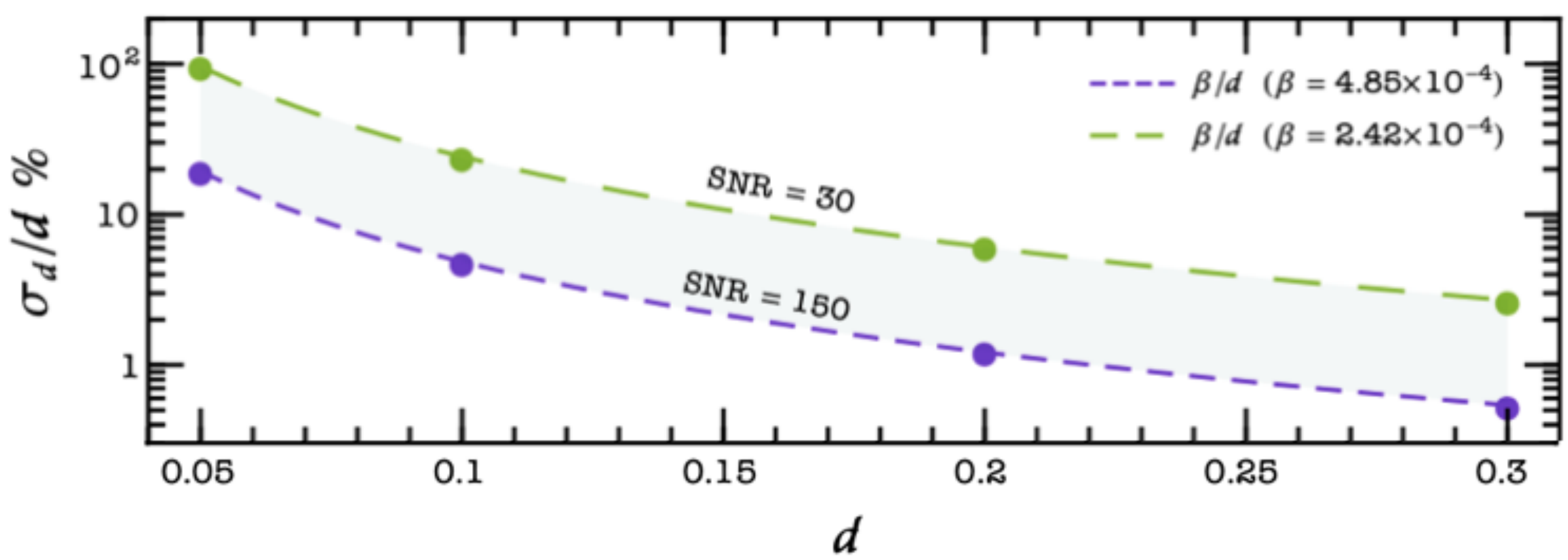
FIM: Relative error for the scalar charge

- Inject parameters to generate the waveform $\vec{\theta} = \left(\ln M, \ln m_p, \chi, \ln D, \theta_s, \phi_s, \theta_1, \phi_1, r_0, \Phi_0, d \right)$
- Fisher Information Matrix analysis

$$\Gamma_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle_{\theta=\hat{\theta}} \longrightarrow \mathbf{\Sigma} = \mathbf{\Gamma}^{-1} \longrightarrow \sigma_i = \Sigma_{ii}^{1/2} \quad , \quad c_{\theta_i\theta_j} = \Sigma_{ij}^{1/2}/(\sigma_{\theta_i}\sigma_{\theta_j})$$

- Results for $M = 10^6 M_\odot$, $\chi = 0.9$, $m_p = 10 M_\odot$

LISA potentially able to measure scalar charges with % error !



Conclusions

- EMRIs in a vast class of modified theories of gravity + scalar fields
- The extra energy loss accelerates the binary coalescence and leaves an imprint in the emitted GW
- The dephasing and the faithfulness show how scalar charges of $d \sim 0.01$ could be possibly detectable by LISA
- The Fisher analysis shows how LISA could be able to measure scalar charges with accuracy of the order of percent

To look forward ..

→ Easy extensions to multiple fields and couplings

→ Bayesian analysis

→ Self force corrections

Thank you for attention!

Back up slides

Field equations

$$S\left[\mathbf{g},\varphi,\Psi\right] = S_0\left[\mathbf{g},\varphi\right] + \alpha S_c\left[\mathbf{g},\varphi\right] + S_m\left[\mathbf{g},\varphi,\Psi\right]$$

$\zeta \ll 1$

$S_c \sim M^{-n} S_0$

$\dfrac{\delta S}{\delta g^{\mu\nu}}$

$$G_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi_1\partial_\nu\varphi_1 - \frac{1}{4}g_{\mu\nu}\left(\partial\varphi_1\right)^2 \quad -\frac{16\pi\alpha}{\sqrt{-g}}\frac{\delta S_c}{\delta g^{\mu\nu}} \sim \zeta G_{\mu\nu} \quad +8\pi\int m\left(\varphi\right)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}\frac{dy_p^\alpha}{d\lambda}\frac{dy_p^\beta}{d\lambda}d\lambda$$

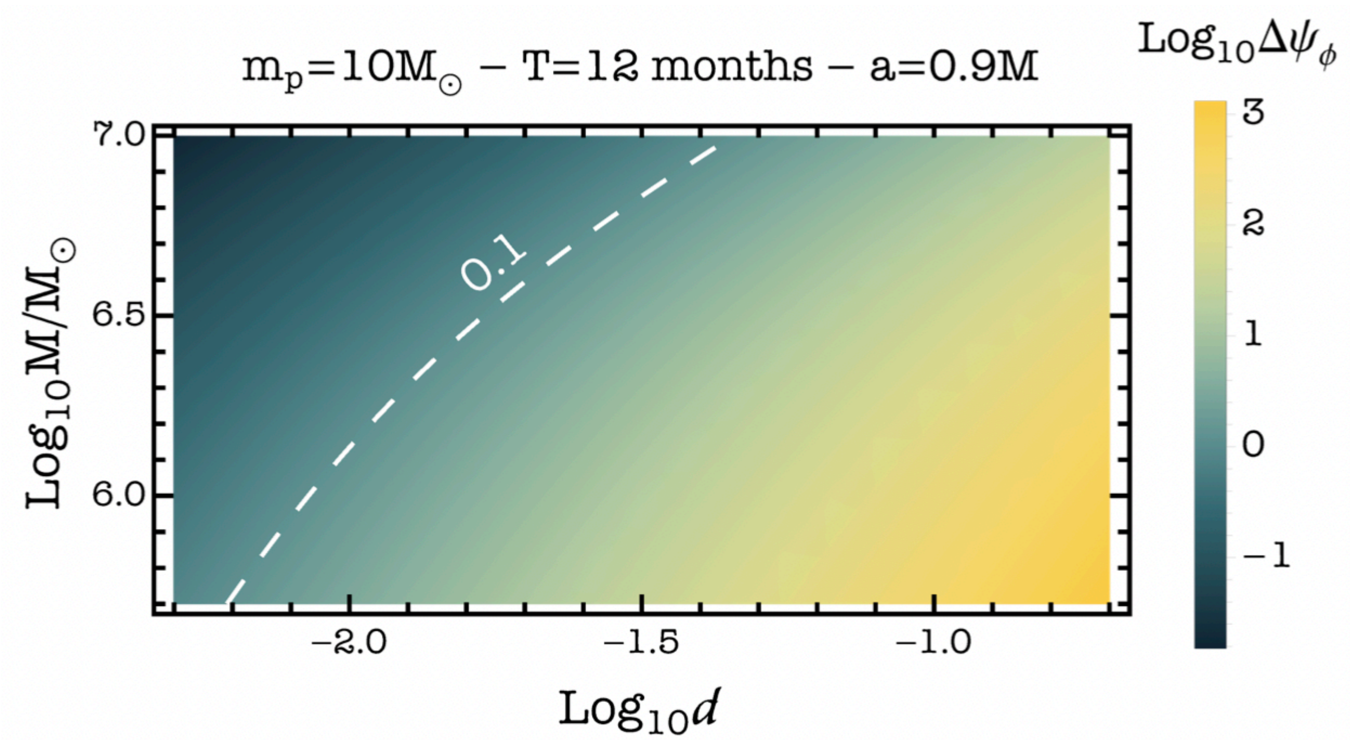
$\dfrac{\delta S}{\delta\varphi}$

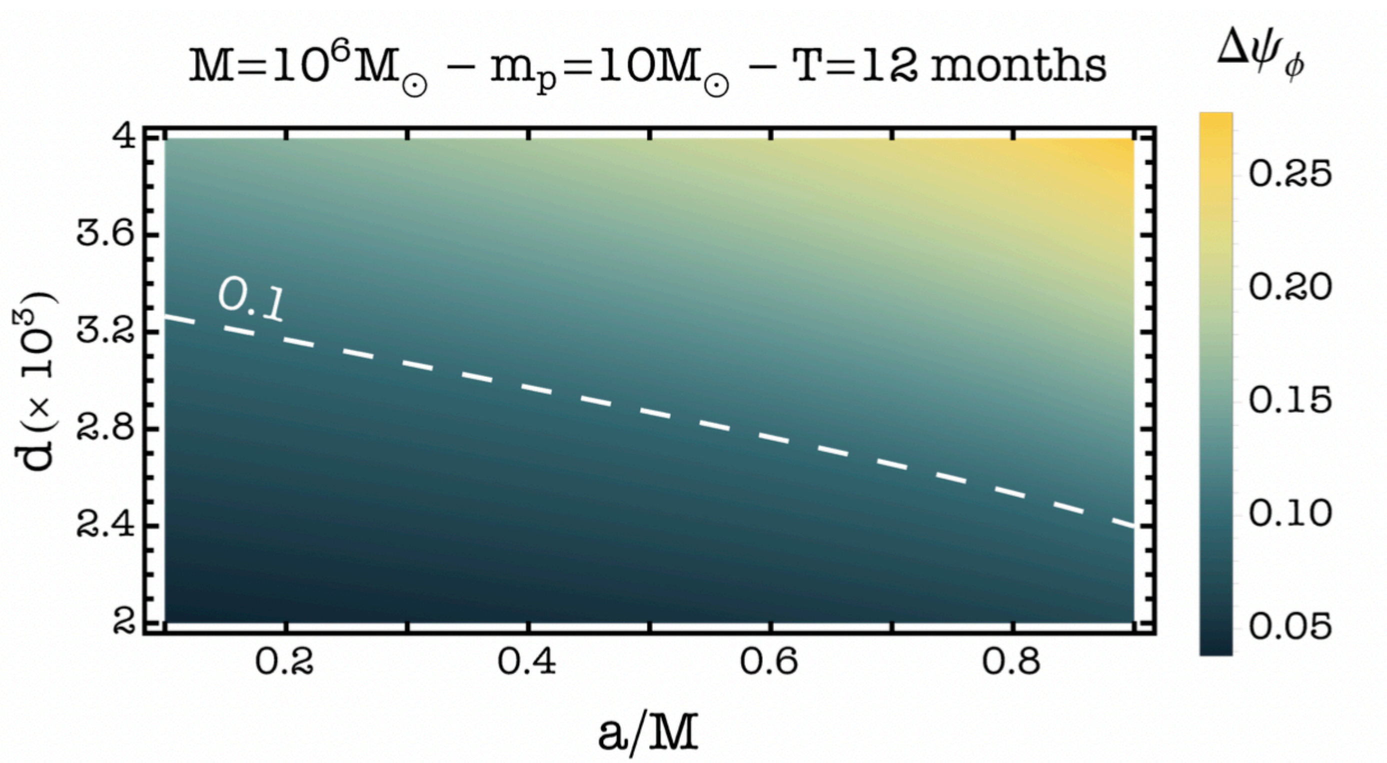
$$\Box\varphi = -\frac{16\pi\alpha}{\sqrt{-g}}\frac{\delta S_c}{\delta\varphi} \sim \zeta\Box\varphi \quad +16\pi\int m'\left(\varphi\right)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}d\lambda$$

- m, m' to be evaluated at φ_0
- In a reference frame centered on the particle : $\varphi = \varphi_0 + \dfrac{m_p d}{\tilde{r}} + O\left(\dfrac{m_p^2}{\tilde{r}^2}\right)$
- Matching with the scalar field eq. outside the world tube:
- (tt)-stress energy tensor in the weak field limit: matter density:

$$m'(\varphi_0) = -\frac{d}{4}m_p$$
$$m(\varphi_0) = m_p$$

Dephasing: circular orbits





- White dashed line: threshold of phase resolution by LISA of $\Delta\psi_\phi = 0.1$ for $SNR = 30$

- $\Delta\psi_\phi$ significant: for $M \lesssim 10^6 M_\odot$ it can be larger than 10^3 radians

- $\Delta\psi_\phi$ increases with the spin of the primary

GW Signal

[L. Barack and C. Cutler, Phys. Rev. D 69 (2004) 082005]

- Quadrupolar approximation $h_{ij}^{TT} = \frac{2}{D}\left(P_{i\ell}P_{jm} - \frac{1}{2}P_{ij}P_{\ell m}\right)\ddot{I}_{\ell m}$

$$I_{ij} = \int d^3x T^{tt}(t, x^i)x^i x^j = m_p x^i x^j$$

- Strain measured by the detector $h(t) = \sum_n h_n(t)$ $\quad h_n(t) = \frac{\sqrt{3}}{2}\left[F^+(t)A_n^+(t) + F^\times(t)A_n^\times(t)\right]$

$$F_+ = \frac{1+\cos^2\theta}{2}\cos 2\phi \cos 2\psi - \cos\theta \sin 2\phi \sin 2\psi$$

$$F_\times = \frac{1+\cos^2\theta}{2}\cos 2\phi \sin 2\psi + \cos\theta \sin 2\phi \cos 2\psi$$

LISA pattern functions

$$\phi(t) = \alpha_0 + \phi_t + \tan^{-1}\left[\frac{\sqrt{3}\cos\theta_s + \sin\theta_s\cos[\phi_t - \phi_s]}{2\sin\theta_s\sin[\phi_t - \phi_s]}\right]$$

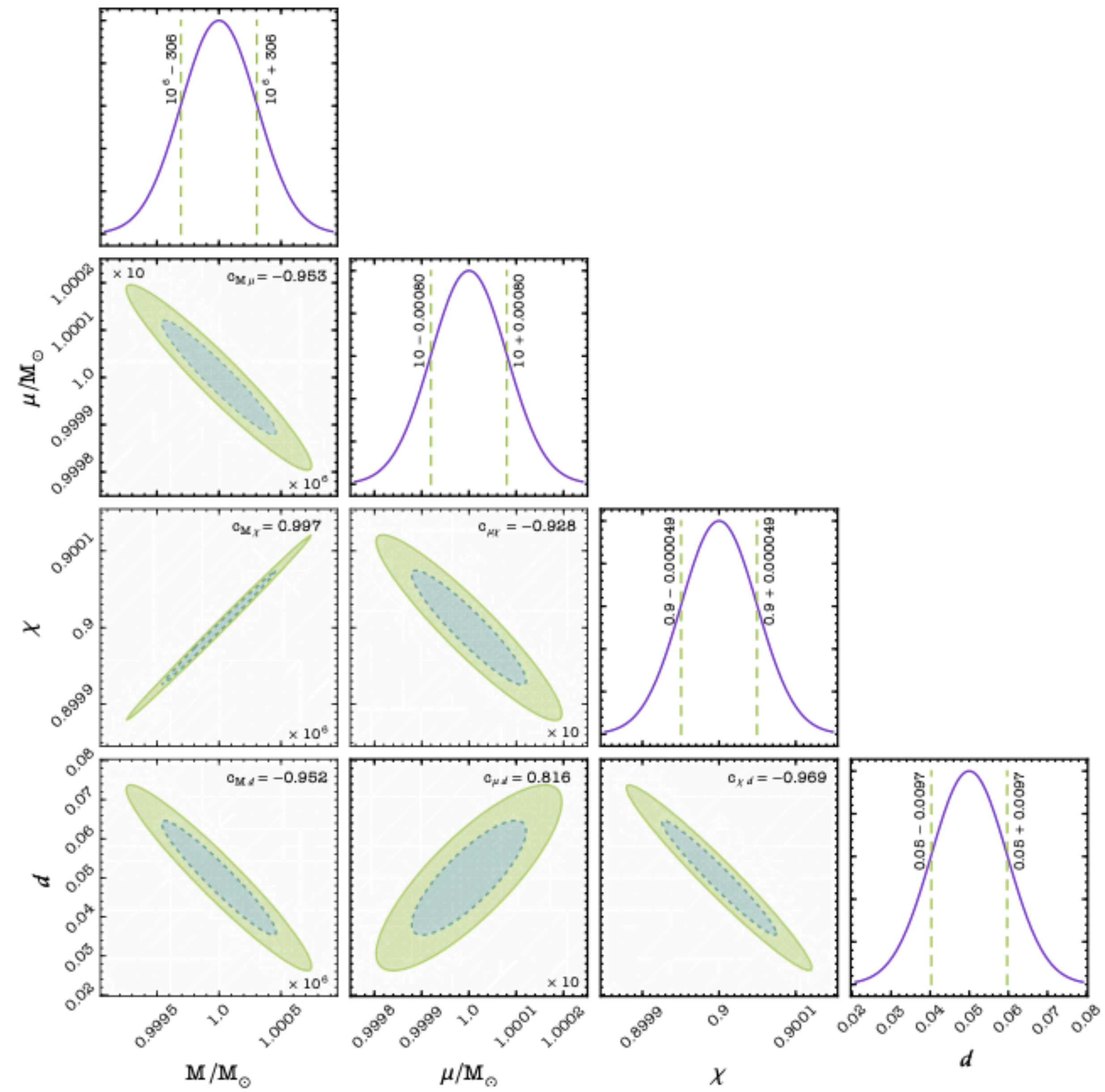
- In an ecliptic-based system:

$$\cos\theta(t) = \frac{1}{2}\cos\theta_s - \frac{\sqrt{3}}{2}\sin\theta_s\cos[\phi_t - \phi_s]$$

ψ polarization angle

- Doppler shifts: $\Phi(t) \rightarrow \Phi(t) + \Phi'(t)R_{AU}\sin\theta_s\cos(2\pi t/T - \phi_s)$

Probability distribution



- Corner plot of the probability distribution of (M, μ, χ, d), after 12 months of observation, with $d = 0.05$ and $SNR = 150$

- Vertical lines: 1-σ distribution for each waveform parameters

- Colored contours: 68 % and 95 % probability confidence intervals

- Measurement of the scalar charge with a relative error smaller than 10 % , with a probability distribution that does not have any support on $d = 0$ at more than 3-σ

- Scalar charge d highly correlated with μ and anti-correlated with M and χ

From the scalar charge to the coupling constant !

For theories with hairy BHs, it is possible to find a relation $\mathbf{d}(\alpha)$

Example of theories: scalar Gauss-Bonnet gravity (sGB)

$$\alpha S_c = \frac{\alpha}{4} \int d^4x \frac{\sqrt{-g}}{16\pi} f(\varphi)\, \mathcal{G}$$

$[\alpha] = (\mathrm{mass})^n$

- n=2
- Dimensionless coupling constant $\beta \equiv \alpha / m_p^2$
- Gauss-Bonnet invariant $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$

→ $f(\varphi) = e^{\varphi}$ → $d = 2\beta + \frac{73}{30}\beta^2 + O(\beta^3)$

→ $f(\varphi) = \varphi$ → $d = 2\beta + \frac{73}{60}\beta^3 + O(\beta^4)$

bounds on d can be translated to bounds on β

Coupling constant

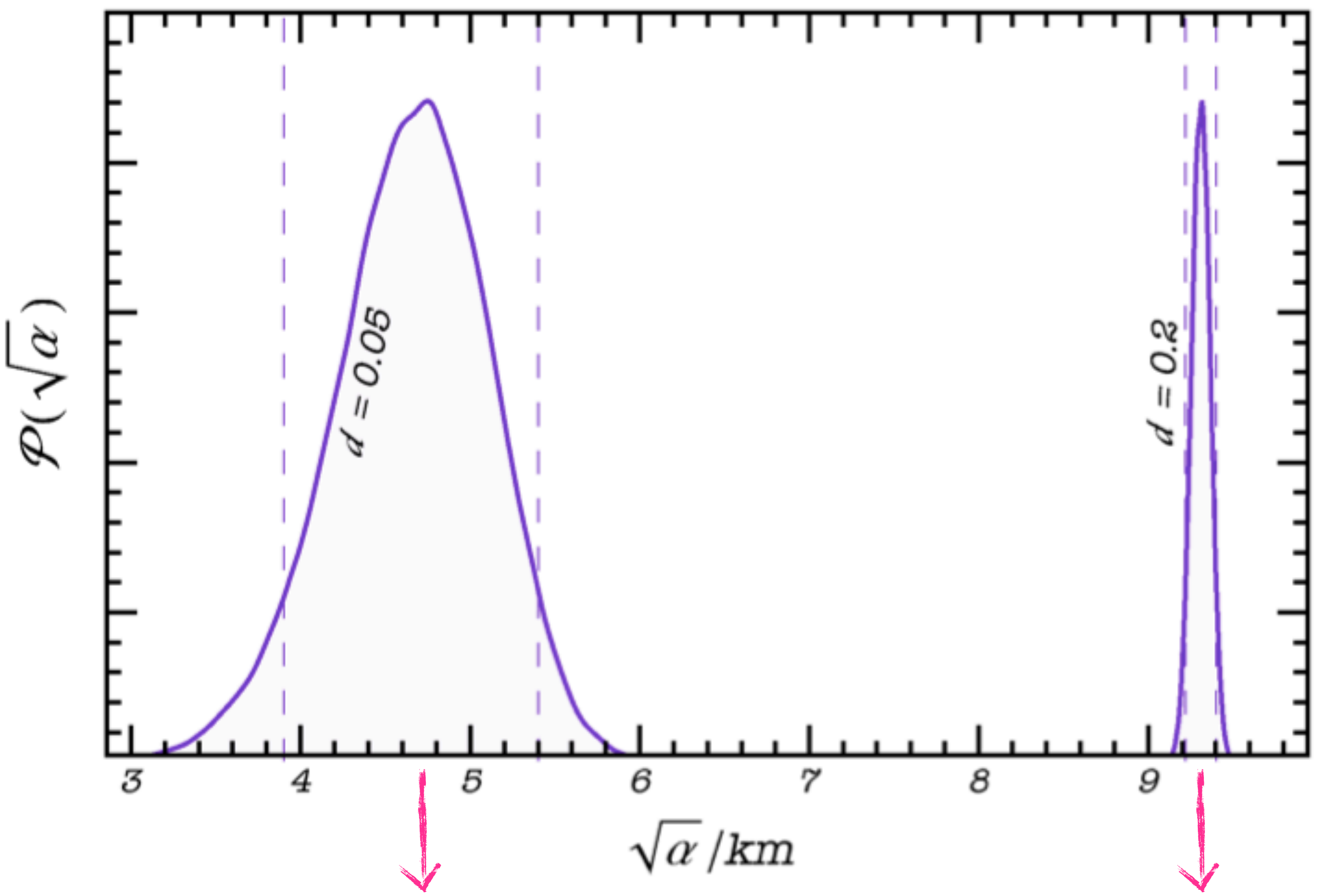
For hairy BHs, if the little body is a BH, we find a relation $d(\alpha)$

Shift-symmetric Gauss Bonnet gravity

$$S_c = \frac{\alpha}{4}\int d^4x \frac{\sqrt{-g}}{16\pi} f(\varphi)\mathcal{G}$$

$$f(\varphi) = \varphi$$

$$\alpha \simeq 2d\mu^2 - \frac{73}{240}d^3\mu^2$$



$\sqrt{\alpha}/km = 4.67^{+0.73}_{-0.77}$ $\sqrt{\alpha}/km = 9.312^{+0.092}_{-0.092}$

- Probability density function of $\sqrt{\alpha}$ obtained from the joint probability distribution of μ and d obtained from the Fisher analysis (SNR=150)
- Vertical lines: 90 % confidence interval
- Even for $d = 0.05$, the probability density functions do not have support with $\alpha = 0$