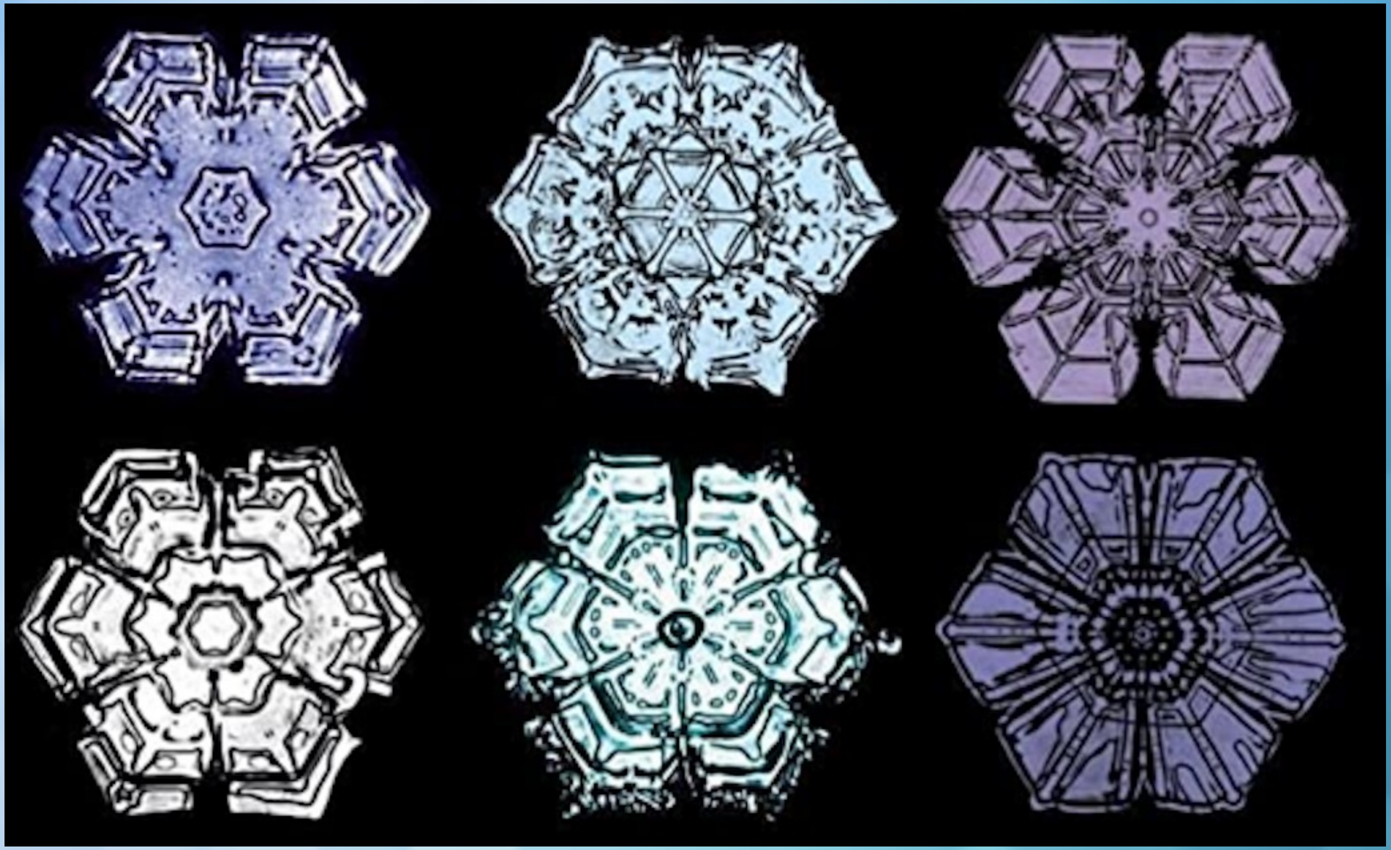
Fermion masses, universality, critical behavior and more …



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19^{th} May 2023
University of ROMA3, Roma

Plan

- critical behavior in Yukawa sector

  fermion masses, CP and modular invariance

  universality of modular-invariant predictions near $\tau = i$
  [FF. 2211.00659,2302.11580]

- **Modular invariance and the QCD angle**

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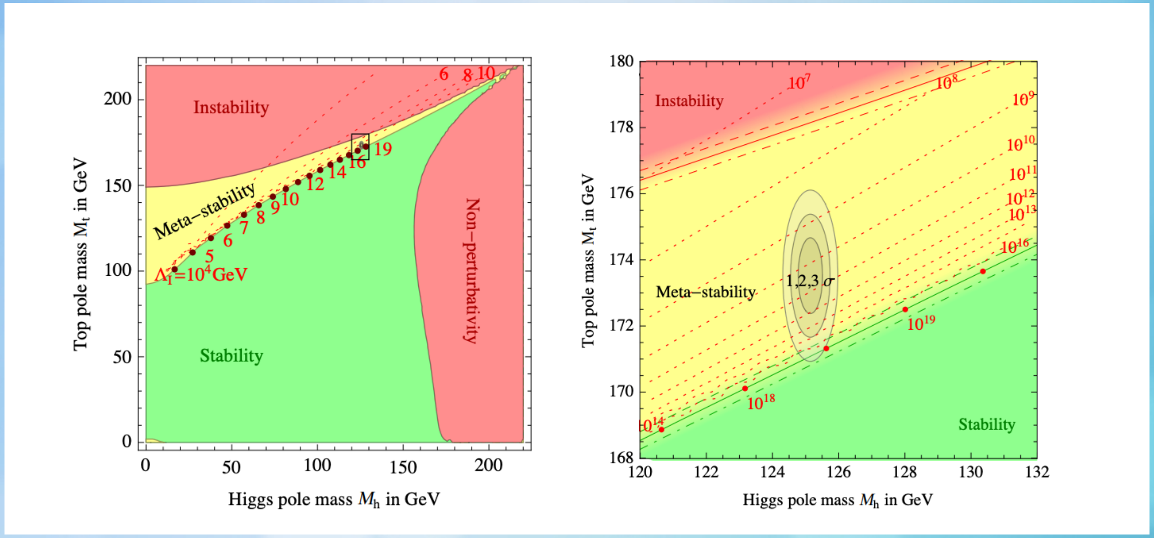
  [2305.08908]

hints of critical behavior in particle physics



SM electroweak vacuum close to metastability

Buttazzo, Degrassi, Giardinoa, Giudice Sala, Salvio Strumia 1307.3536

the Higgs quadratic coupling appears to be tuned to set the SM near the phase transition

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda = 1.1056 \times 10^{-52}\ \mathrm{m}^{-2}$$
$$= 2.888 \times 10^{-122}\ l_{\mathrm{P}}^{-2}$$

universe at the border between an expanding phase and a collapsing one

- hints from gravitational, gauge and Higgs sectors
  is the Yukawa sector close to a phase transition?

- near criticality = closeness to a symmetric phase

symmetric limit

from a symmetry group G
acting in generation space

$$m_{ij}(\tau) = m_{ij}^0$$

τ_α

symmetry breaking sector:
set of dimensionless, gauge invariant scalar fields, charged under G_{fl}

[τ_α stands for $\langle\tau_\alpha\rangle/\Lambda_F$ where the scale Λ_F has been set to 1]

symmetric limit

higher dimensional operators

SUSY breaking effects
RGE corrections
(Λ_{UV}, m_{SUSY}, $\tan\beta$)

$$m_{ij}(\tau) = m_{ij}^{0} + m_{ij}^{1\alpha}\,\tau_\alpha + m_{ij}^{1\bar{\alpha}}\,\bar{\tau}_{\bar{\alpha}} + m_{ij}^{2\alpha\beta}\,\tau_\alpha\tau_\beta + \ldots$$

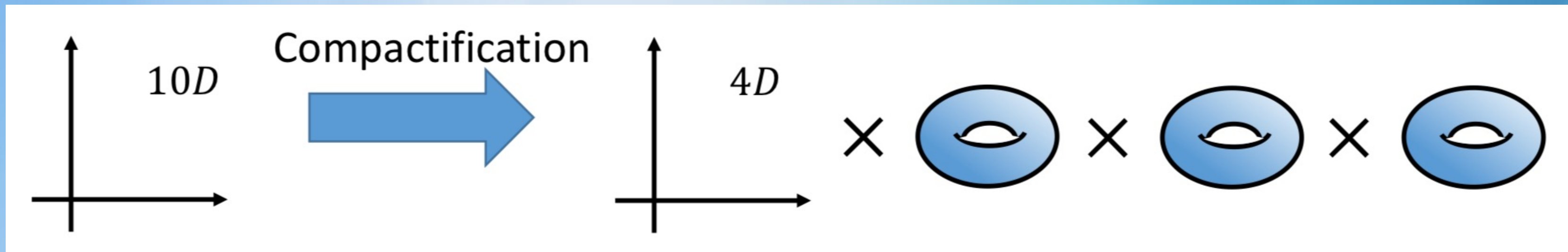
- near criticality = closeness to a symmetric phase
- problem: most models are conceived as a small deviation from a symmetric limit

huge number of models: G_{fl} continuous/discrete, global/local,.....
no baseline model in bottom-up approach

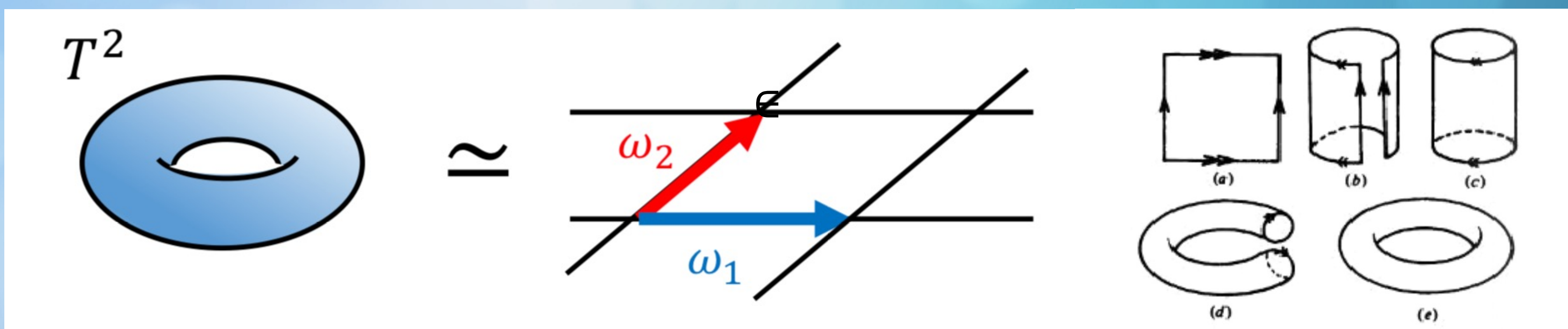
- we dismiss this class and explore a different type of symmetries

a motivated class of flavor symmetries

string theory in d=10 need 6 compact dimensions



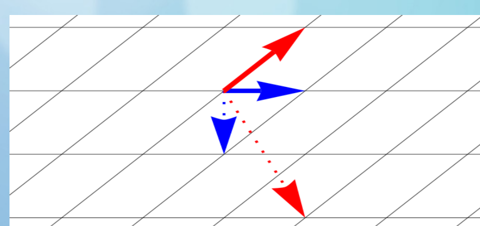
simplest compactification: 3 copies of a torus T^2



tori parametrized by

$$\mathcal{M} = \left\{ \tau = \frac{\omega_2}{\omega_1} \quad Im(\tau) > 0 \right\}$$

lattice left invariant by modular transformations:



$$\tau \to \gamma\tau \equiv \frac{a\tau + b}{c\tau + d} \in SL(2,Z)$$

a, b, c, d integers

$ad - bc = 1$

fixed point

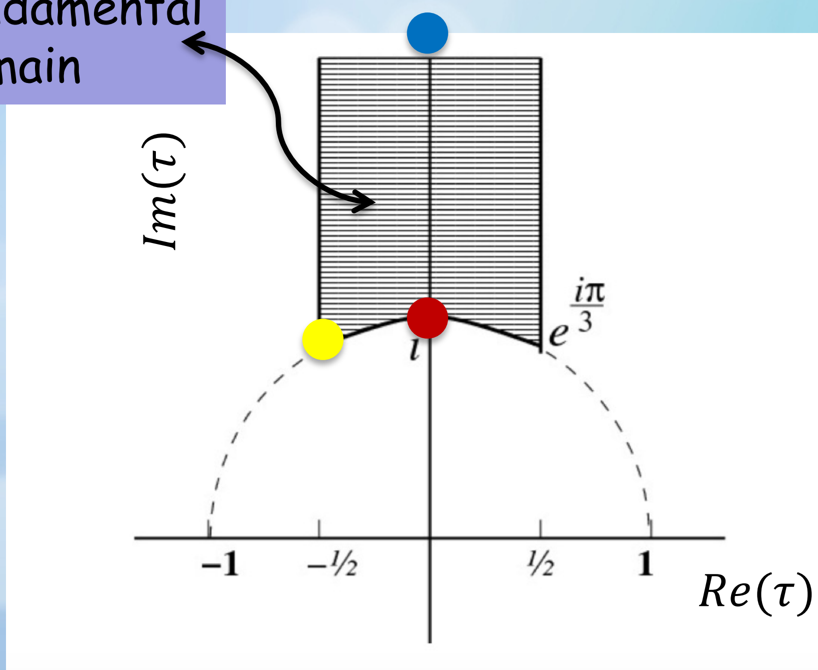
$\tau = i$ $\qquad \tau \xrightarrow{S} -\frac{1}{\tau}$ $\qquad \mathbb{Z}_4^S$

$\tau = e^{i\,2\pi/3}$ $\qquad \tau \xrightarrow{ST} -\frac{1}{\tau+1}$ $\qquad \mathbb{Z}_2^{ST} \times \mathbb{Z}_2^{S^2}$

$\tau = i\,\infty$ $\qquad \tau \xrightarrow{T} \tau + 1$ $\qquad \mathbb{Z}^T \times \mathbb{Z}_2^{S^2}$

residual symmetry

fundamental domain



modular invariance completely broken everywhere but at three fixed points

$SL(2,Z)$ generated by

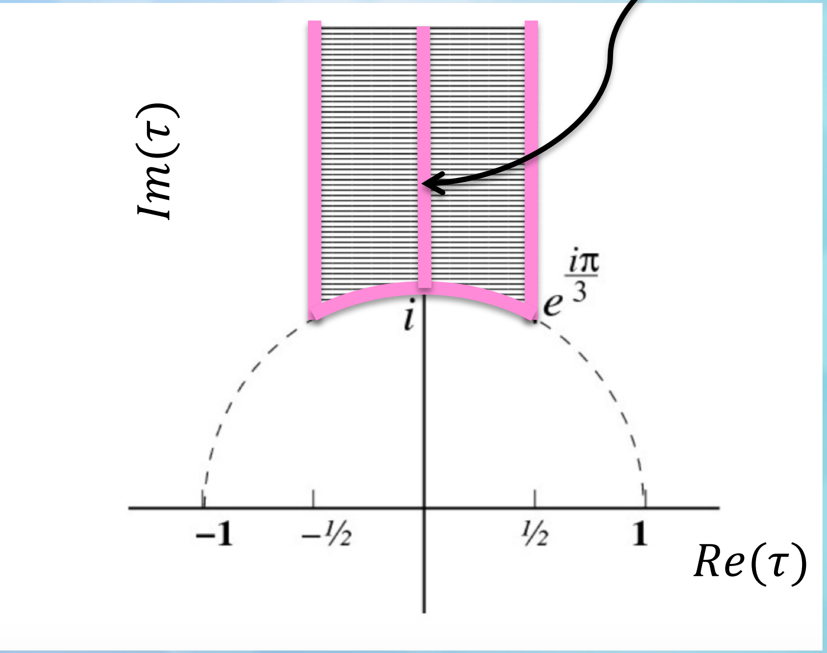
$$S: \tau \to -\frac{1}{\tau} \quad , \qquad T: \tau \to \tau + 1$$

strong indications that the four-dimensional CP symmetry is a gauge symmetry in string theory compactifications.

CP $\boxed{\tau \rightarrow -\tau^*}$ [up to modular transformations]

[Novichkov, Penedo, Petcov and Titov 1905.11970
Baur, Nilles, Trautner and Vaudrevange, 1901.03251]

unbroken CP



no point in the fundamental domain is a priori preferred by the data

$\mathcal{N}$=1 SUSY modular invariant theories

- $$\tau \rightarrow \gamma\tau \equiv \frac{a\tau + b}{c\tau + d}$$

  $$\varphi \rightarrow \rho_\varphi(\gamma)\,\varphi$$

unitary representation
of the finite modular group

level

$SL(2, Z_N)$

$\mathcal{N}$=1 SUSY modular invariant theories

- $$\tau \to \gamma\tau \equiv \frac{a\tau+b}{c\tau+d}$$

$$\varphi \to (c\,\tau+d)^{-k_\varphi}\,\rho_\varphi(\gamma)\,\varphi$$

the weight

unitary representation
of the finite modular group $SL(2,Z_N)$

level

- Yukawa interactions in $\mathcal{N}$=1 global SUSY [extension to $\mathcal{N}$=1 SUGRA straightforward]

$$S = \int d^4x d^2\theta\, w(\tau,\varphi) + h.c + \int d^4x d^2\theta d^2\bar\theta\, K(\tau,\varphi,\bar\tau,\bar\varphi)$$

superpotential =
Yukawa interactions

Kahler potential =
kinetic terms

$$w(\tau,\varphi) = \sum_n Y_{I_1 \dots I_n}(\tau)\varphi^{(I_1)} \dots \varphi^{(I_n)}$$

field-dependent
Yukawa couplings

invariance of $w(\Phi)$ guaranteed by $Y_{I_1 \dots I_n}(\tau)$ such that

$$Y_{I_1 \dots I_n}(\gamma\tau) = (c\tau + d)^{k_Y(n)} \rho(\gamma)\, Y_{I_1 \dots I_n}(\tau)$$

1. $-k_Y(n) + k_{I_1} + \cdots + k_{I_n} = 0$
2. $\rho \times \rho^{I_1} \times \ \dots \times \rho^{I_n} \supset 1$

assume no singularities ⇒ modular forms of level N and weight k_Y

form a linear space $\mathcal{M}_k(\Gamma_N)$ of finite dimension

$$m(\tau) = Z_A^{-\frac{1}{2}}(\tau, \bar{\tau})\ \mathcal{Y}(\tau)\, Z_B^{-\frac{1}{2}}(\tau, \bar{\tau})$$

non-holomorphic factors $Z_{A,B}^{-\frac{1}{2}}(\tau, \bar{\tau})$ from kinetic terms $K(\tau, \varphi, \bar{\tau}, \bar{\varphi})$
not constrained as strongly as $w(\Phi)$

Example

$SL(2,Z_3) \approx A_4{}'$

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \to (c\tau+d)^{-1}\rho(\gamma)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$k_\nu = +1$

~ 3 of $SL(2,Z_3)$

$$w(\tau,\nu) = m_0\, \nu\, \mathcal{Y}(\tau)\nu + h.c.$$

modular form of level 3
$k = +2$ and $\rho \subset 3+1+1'+1''$

$d(\mathcal{M}_2(\Gamma_3)) = 3$
$\rho = 3$

$$\mathcal{Y}(\tau) = \mathcal{Y}_0\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau)\\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau)\\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau)\end{pmatrix}$$

[F.F. 1706.08749]

Yukawas completely determined in terms of τ up to an overall constant

no corrections from higher order operators in the exact SUSY limit

models of lepton masses

- modular invariance successful in describing the lepton sector

reproduce neutrino masses (3), mixing angles (3) and CP phases (3) in terms of τ and 2 or 3 additional parameters.

- 1. Large freedom leads to > 100 viable models

$$\varphi \quad \rightarrow (c\,\tau + d)^{-k_\varphi}\, \rho_\varphi(\gamma)\, \varphi$$

$$SL(2, Z_N)$$

differ by the level N, the weights k_φ, the representations $\rho_\varphi(\gamma)$

- 2. non-holomorphic contribution assumed flavor universal

$$m(\tau) = Z_A^{-\frac{1}{2}}(\tau,\bar{\tau})\ \mathcal{Y}(\tau)\, Z_B^{-\frac{1}{2}}(\tau,\bar{\tau}) = z_0 \mathcal{Y}(\tau)$$

- what can we learn?

distribution of models in τ space

all CP invariant

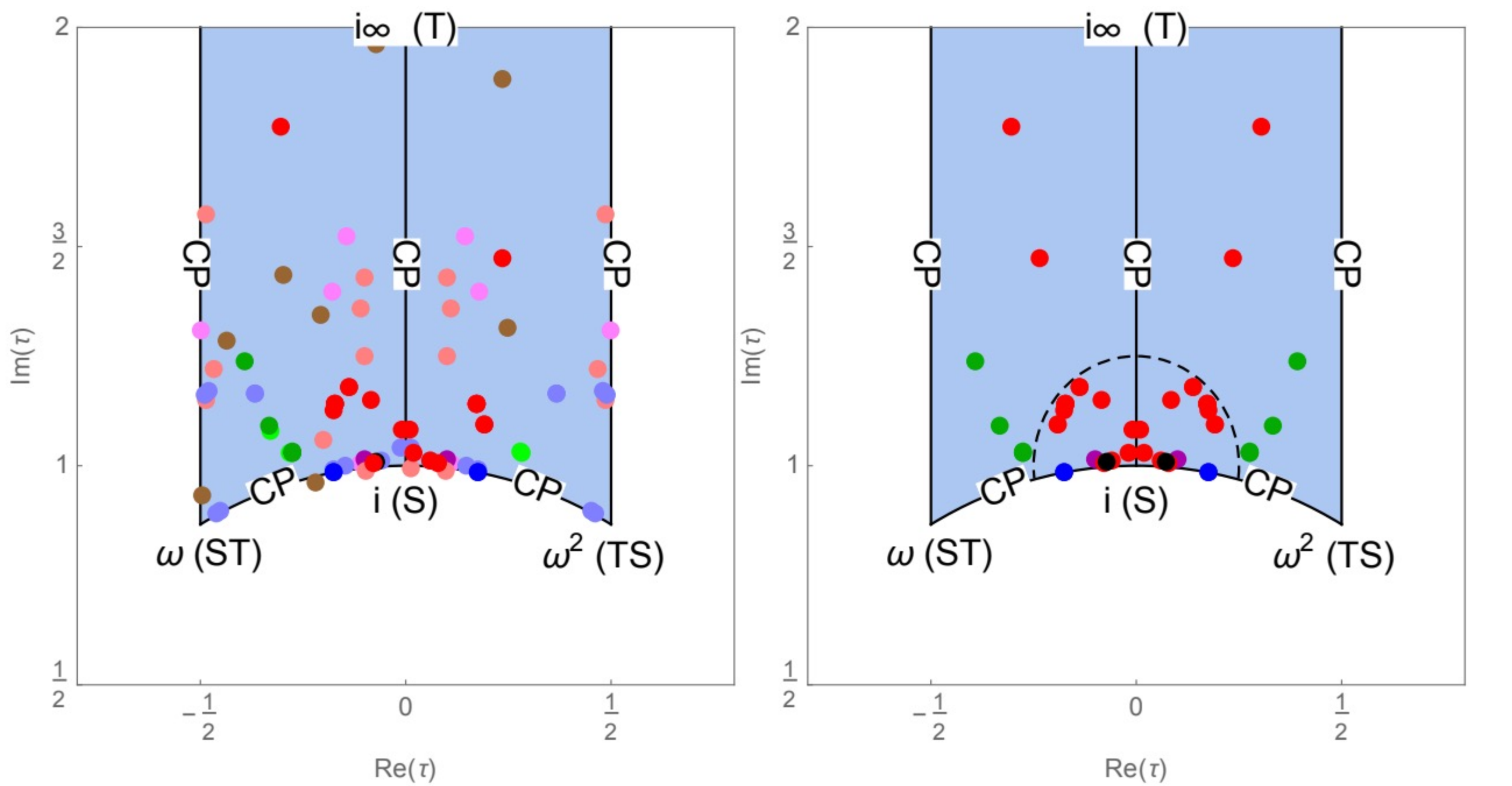


Figure 4: Fundamental domain $\mathcal{F}$ (light blue region) and fixed points (see text). Dots are the best-fit values of τ in models of ref. [66,68] (Γ_3 - light red), [69,70] ($\Gamma_3\&CP$ - red), [71] (Γ_4 - light magenta), [67] ($\Gamma_4\&CP$ - magenta), [72] (Γ_4' - light blue), [72] ($\Gamma_4'\&CP$ - blue), [73] ($\Gamma_5'\&CP$ - black), [74] (Γ_6' - light green), [74] ($\Gamma_6'\&CP$ - green), [75] (Γ_7 - brown). We use the notation $\Gamma_N' = SL(2,\mathbb{Z}_N)$ and $\Gamma_N = SL(2,\mathbb{Z}_N)/\{\pm\mathbb{1}\}$. In the left panel all models are displayed. The right panel includes only CP invariant models, for which the full pair of points τ and $-\bar{\tau}$ is shown. The dashed line represents the contour $|\tau - i| = 0.25$.

[F. 2211.00659,2302.11580]

assume $\tau \approx i$

make use of new coordinates: $u = \dfrac{\tau - i}{\tau + i}$

S

$u \to -u$

u is small

$$m_{ij}(u) = m_{ij}^{0} + m_{ij}^{1} u + m_{ij}^{\bar{1}} \bar{u} + m_{ij}^{2} u^2 + \ldots$$

near $\tau = i$, this looks like a $\mathbb{Z}_4^S$ theory spontaneously broken by the SB parameter u

independently on level N, the weights k_φ, representations $\rho_\varphi(\gamma)$ and on kinetic terms $K(\tau, \varphi, \bar{\tau}, \bar{\varphi})$

lepton doublets are in an irrep of $SL(2, Z_N)$

[F. 2211.00659,2302.11580]

for any choice of level N, weights k_φ and kinetic terms $Z_\varphi^{-\frac{1}{2}}(\tau, \bar{\tau})$

models fall into 3 classes, and the successful one predicts

$$m_\nu^{-1} = m_{0\nu}^{-1} \begin{pmatrix} a_{11}x & a_{12}^0 & a_{13}^0 \\ \cdot & a_{22}x & a_{23}x \\ \cdot & \cdot & a_{33}x \end{pmatrix} + \mathcal{O}(x^2)$$

$$x = |u| = \left|\frac{\tau - i}{\tau + i}\right|$$

a_{ij} real

$$sin^2\vartheta_{12} = \frac{1}{2}(1 + c_{12}\, x) + \cdots$$

$$sin^2\vartheta_{23} = c_{23} + \cdots$$

$$sin^2\vartheta_{13} = c_{13}\, x^2 + \cdots$$

$$m_3 = m_0 \frac{c_3}{x} + \cdots$$

$$m_{1,2} = m_0(1 \pm c_m x) + \cdots$$

$$\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} = c_{s/a}\, x^3 + \cdots$$

$$\delta_{CP} = c_\delta + \cdots$$

$$\alpha_{21} = \pi + \cdots$$

$$\alpha_{31} = c_\alpha + \cdots$$

data reproduced by

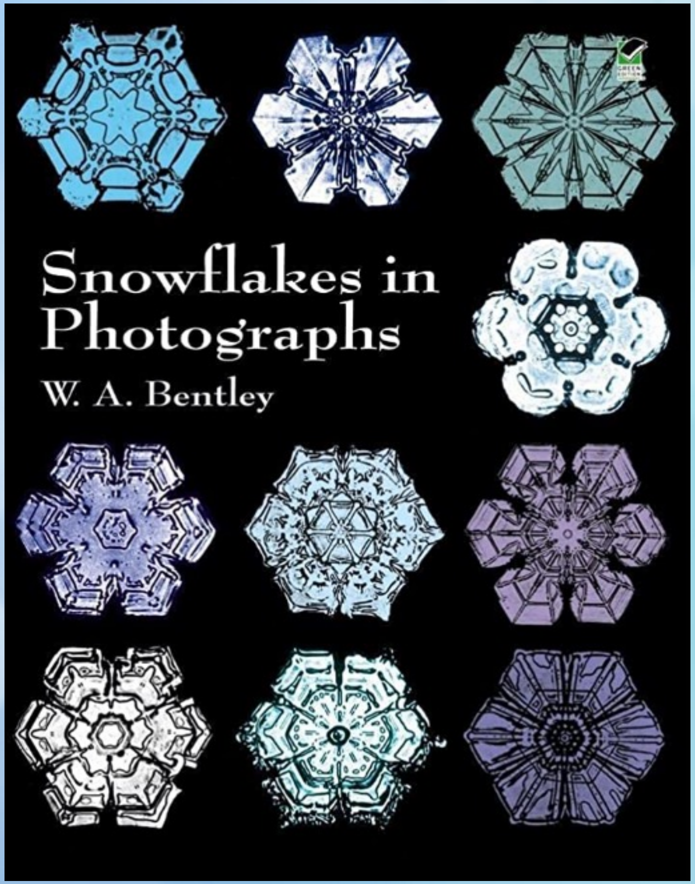
$m_0 \approx 10$ meV

$x = |u| \approx 0.1$

$c_i = O(1)$

Conclusion I

- models like $\tau \approx i$
- measurable quantities scale with powers of the order parameter, independently of the details of the theory
- as in systems belonging to the same universality class in 2nd-order phase transitions.



snowflakes share a six-fold symmetry but no two of them are identical

modular invariance and the strong CP problem

[FF., Strumia ,Titov 2305.08908]

$$\mathscr{L}_{\rm QCD} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}\,{\rm Tr}\, G^2 + \theta_{\rm QCD}\frac{g_3^2}{32\pi^2}\,{\rm Tr}\, G\tilde{G}.$$

$$\bar{\theta} = \theta_{\rm QCD} + \arg\det M_q,$$

$$d_n \simeq 1.2\times 10^{-16}\bar{\theta}\, e\cdot cm.$$

$\Rightarrow$ $|\bar{\theta}| \lesssim 10^{-10}$ & $\delta_{CKM} \approx \mathcal{O}(1)$

solutions

- $\bar{\theta}$ dynamically relaxed to zero by the axion, would-be GB of a global, anomalous $U(1)_{PQ}$ symmetry

- P or CP are symmetries of the UV theory, spontaneously broken in such a way to maintain $\bar{\theta}$ nearly vanishing while allowing for a large δ_{CKM}

A.E. Nelson, 'Naturally Weak CP Violation', Phys.Lett.B 136 (1984) 387.

S.M. Barr, 'Solving the Strong CP Problem Without the Peccei-Quinn Symmetry', Phys.Rev.Lett. 53 (1984) 329.

rigid supersymmetry

$$S = \int d^4x d^2\theta \left[w(\tau,\varphi) + \frac{f}{16} WW\right] + h.c + \int d^4x d^2\theta d^2\bar{\theta}\, K(\tau, e^{2V}\varphi, \bar{\tau}, \bar{\varphi})$$

$$f = \frac{1}{g_3^2} - i\frac{\theta}{8\pi^2}$$

unbroken supersymmetry ⇒ $\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y$

$\bar{\theta}$ holomorphic

G. Hiller, M. Schmaltz, 'Solving the Strong CP Problem with Supersymmetry', Phys.Lett.B 514 (2001) 263 [arXiv:hep-ph/0105254].

- modular invariance
- absence of singularities

⇒ strongly constrains holomorphic quantities

modular invariance at level $N = 1$

modular forms of level $N = 1$ are generated by $E_4(\tau)$ and $E_6(\tau)$

Modular weight k	0	2	4	6	8	10	12	14
Number of forms	1	0	1	1	1	1	2	1
Modular forms	1	–	E_4	E_6	$E_8 = E_4^2$	$E_{10} = E_4 E_6$	E_4^3, E_6^2	$E_{14} = E_4^2 E_6$

no modular forms of negative weight

CP-invariance, only source of (spontaneous) CP-breaking is τ

$$V \to V$$

$$\varphi \to (c\tau + d)^{-k_\varphi} \varphi$$

$$Y_{ij}^q(\tau) \to (c\tau + d)^{k_{ij}^q} \, Y_{ij}^q(\tau)$$

$\Rightarrow$

f = real constant

$$k_{ij}^u = k_{Q_j} + k_{U_i^c} + k_{H_u}$$

$$k_{ij}^d = k_{Q_j} + k_{D_i^c} + k_{H_d}$$

$$\det Y(\tau) \to (c\tau + d)^{k_{\det}} \quad \det Y(\tau)$$

$$k_{\det} \quad = \sum_{i=1}^3 \left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) + 3k_{H_u} + 3k_{H_d}$$

cancellation of modular anomalies

$$K(\tau,\varphi,\bar{\tau},\bar{\varphi}) = -h^2 \log(-i\tau + i\tau^+) + \sum_{\varphi} (-i\tau + i\tau^+)^{-k_\varphi} \varphi^+ e^{2V} \varphi$$

$$\psi_{can} \to \left(\frac{c\tau + d}{c\tau^+ + d}\right)^{-\frac{k_\varphi}{2}} \psi_{can}$$

conditions for gauge-modular anomaly cancellation

$$\sum_{i=1}^{3} \left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) = 0$$

$$\sum_{i=1}^{3} (3k_{Q_i} + k_{L_i}) + k_{H_u} + k_{H_d} = 0$$

$$\sum_{i=1}^{3} \left(k_{Q_i} + 8k_{U_i^c} + 2k_{D_i^c} + 3k_{L_i} + 6k_{E_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

simplest solution:

$$k_{H_u} + k_{H_d} = 0$$

$$k_{Q_i} = k_{U_i^c} = k_{D_i^c} = k_{L_i} = k_{E_i^c} = (-k, 0, k)$$

$$k_{\det} = 0$$

$\det Y(\tau)$ & f real constants

$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y = 0$

holds also after SUSY breaking, if no new phases in SUSY breaking sector

Example $k_{Q_i} = k_{U_i^c} = k_{D_i^c} = (-6,0,+6)$

$$Y_q(\tau) = \begin{pmatrix} 0 & 0 & c^q_{13} \\ 0 & c^q_{22} & c^q_{23}E_6 \\ c^q_{31} & c^q_{32}E_6 & c^q_{33}E_4^3 + c^{q\prime}_{33}E_6^2 \end{pmatrix}$$

$\tan\beta = 10 \quad \tau = 0.125 + i$

$$c^u_{ij} \approx 10^{-3}\begin{pmatrix} 0 & 0 & 1.56 \\ 0 & -1.86 & 0.87 \\ 1.29 & 4.14 & 3.51, 1.40 \end{pmatrix}, \qquad c^d_{ij} \approx 10^{-3}\begin{pmatrix} 0 & 0 & 1.55 \\ 0 & -2.59 & 4.59 \\ 0.378 & 0.710 & 0.734, 1.76 \end{pmatrix}$$

reproduce quark masses, mixing angles and CKM phase

$\delta_{CKM} \neq 0 \quad \Leftrightarrow \quad Im\,\det[Y_u^+ Y_u, Y_d^+ Y_d] \neq 0$ non-holomorphic

Leptons: $k_{L_i} = k_{E_i^c} = (-6,0,+6)$

$$c^e_{ij} = 10^{-3}\begin{pmatrix} 0 & 0 & 1.29 \\ 0 & 5.95 & 0.35 \\ -2.56 & 1.47 & 1.01, 1.32 \end{pmatrix}, \qquad c^\nu_{ij} = \frac{1}{10^{16}\,\mathrm{GeV}}\begin{pmatrix} 0 & 0 & 3.4 \\ 0 & 7.1 & 1.2 \\ 3.4 & 1.2 & 0.19, 0.95 \end{pmatrix}$$

heavy quarks and singularities

- heavy quarks not needed, but they can exist in the UV

  for example

  $$\underbrace{k_\varphi = (-6, -2, 0}_{\text{chiral}},\underbrace{+2, +6)}_{\text{heavy vector-like quark}}$$

  $$k_{H_u} + k_{H_d} = 0$$

- UV theory $\bar{\theta} = -8\pi^2 Im\, f_{UV} + \arg\det Y_{UV} = 0$

- IR theory has an anomalous field content, anomaly cancelled by:

  $$f_{IR} = f_{UV} - \frac{1}{8\pi^2}\log\det Y_{Heavy}$$

  $$\bar{\theta} = -8\pi^2 Im\, f_{IR} + \arg\det Y_{Light} =$$

  $$= +\arg\det Y_{Heavy} + \arg\det Y_{Light} = \arg\det Y_{UV} = 0$$

- $Y_{Light}(\tau)$ is singular at τ values such that $\det Y_{Heavy}(\tau) = 0$

$\mathcal{N} = 1$ supergravity

$$K = -h^2 \log(-i\tau + i\tau^+) + \cdots$$

$$k_W = \frac{h^2}{M_{Pl}^2} \to 0$$

corrections of $\mathcal{O}(k_W)$?

back to the rigid case

- K and w no more independent

$$\mathcal{G} = \frac{K}{M_{Pl}^2} + \log\left|\frac{w}{M_{Pl}^3}\right|^2$$

$$w(\tau) \to (c\tau + d)^{-k_W}\, w(\tau)$$

$$k_W > 0$$

no negative weight modular forms, $w(\tau)$ singular somewhere

V. Kaplunovsky, J. Louis, 'On Gauge couplings in string theory', Nucl.Phys.B 444 (1995) 191 [arXiv:hep-th/9502077].

- modular-QCD anomaly modified into

$$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) + 3k_W$$

can be rotated away if gluino is massless

quarks

gluino

J.P. Derendinger, S. Ferrara, C. Kounnas, F. Zwirner, 'On loop corrections to string effective field theories: Field dependent gauge couplings and sigma model anomalies', Nucl.Phys.B 372 (1992) 145.

L.J. Dixon, V. Kaplunovsky, J. Louis, 'Moduli dependence of string loop corrections to gauge coupling constants', Nucl.Phys.B 355 (1991) 649.

spontaneously broken supergravity

$$\bar{\theta} = -8\pi^2 Im\, f + \arg\det M_{quark} + 3\arg M_3 = 0$$

$$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) = k_{H_u} + k_{H_d} = 0 \quad \Rightarrow \quad \arg\det M_{quark} = 0$$

assume unique singularity at $\tau = i\infty$

$$w(\tau) = \cdots + c_0 M_{Pl}^3\, \eta(\tau)^{-2k_W}$$

$\eta(\tau)$ Dedekind eta function

H. Rademacher, H.S. Zuckeman, 'On the Fourier coefficients of certain modular forms of positive dimensions', Annals of Mathematics 39 (1938) 433.

$$f = \cdots + 3\frac{k_W}{4\pi^2}\log\eta(\tau)$$

cancels the gluino anomaly

To break SUSY, add a singlet, modular invariant S: $\langle S\rangle$ real

$$K = \cdots - M_{Pl}^2 \log(S + S^+)$$

$$M_3 = \frac{1}{2} e^{\frac{K}{2M_{Pl}^2}} K^{S\bar{S}} D_{\bar{S}} w^+ f_S$$

$$\arg M_3 = -\arg w = 2k_W \arg\eta(\tau)$$

$$\bar{\theta} = -8\pi^2 Im\, f + 3\arg M_3 = 0$$

deviations from $\bar{\theta} = 0$

SUSY unbroken

no corrections from K
no corrections from nonrenormalizable operators: $SL(2,\mathbb{Z})$
no corrections from additional moduli/singlets under reasonable assumptions

SUSY breaking corrections

potentially big if soft terms violate flavour in a generic way

minimized if $\Lambda_{CP} \gg \Lambda_{SUSY}$ (as e.g. in gauge mediation)

and soft breaking terms respect the flavour structure of the SM

$$\bar{\theta} \lesssim \frac{M_t^4 M_b^4 M_c^2 M_s^2}{v^{12}} J_{\rm CP} \tan^6\beta \sim 10^{-28} \tan^6\beta.$$

SM corrections

negligible: $\bar{\theta} \leq 10^{-18}$ at four loops

J.R. Ellis, M.K. Gaillard, 'Strong and Weak CP Violation', Nucl.Phys.B 150 (1979) 141.

I.B. Khriplovich, 'Quark Electric Dipole Moment and Induced θ Term in the Kobayashi-Maskawa Model', Phys.Lett.B 173 (1986) 193.

THANK YOU!

back-up slides

τ			mass ordering	$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
$\approx i$	k_S even	$m_\nu(0,0)$ regular	NO/IO	$\mathcal{O}(1)$	$\mathcal{O}(x^2)$	$\mathcal{O}(x^2)$	$\mathcal{O}(1)$
$\approx i$	k_S odd	$m_\nu(0,0)$ regular	IO	$\mathcal{O}(x)$	$\frac{1}{2}(1+\mathcal{O}(x))$	$\mathcal{O}(x^2)$	$\mathcal{O}(1)$
$\approx i$	k_S odd	$m_\nu(0,0)$ singular	NO	$\mathcal{O}(x^3)$	$\frac{1}{2}(1+\mathcal{O}(x))$	$\mathcal{O}(x^2)$	$\mathcal{O}(1)$
$\approx \omega$			NO/IO	$\mathcal{O}(x)$	$\frac{1}{2}(1+\mathcal{O}(x))$	$\mathcal{O}(x^2)$	$\mathcal{O}(x^2)$

Table 5. Synopsis of predictions in modular invariant flavor models of leptons, when the modulus τ falls in the vicinity of the fixed points $\tau = i$ or $\tau = \omega$ and ρ_l is an irreducible representation.

$$|\tau - i| = 0.20 \pm 0.04 \qquad |u| = \left|\frac{\tau - i}{\tau + i}\right| = 0.095 \pm 0.015$$

$$\sum_i m_i = 73.9 \pm 4.6 \text{ meV} \qquad m_{\beta\beta} = 5.5 \pm 4.2 \text{ meV}$$

$$\Delta m^2_{sol} = (74.04 \pm 0.27) \text{ meV}^2$$

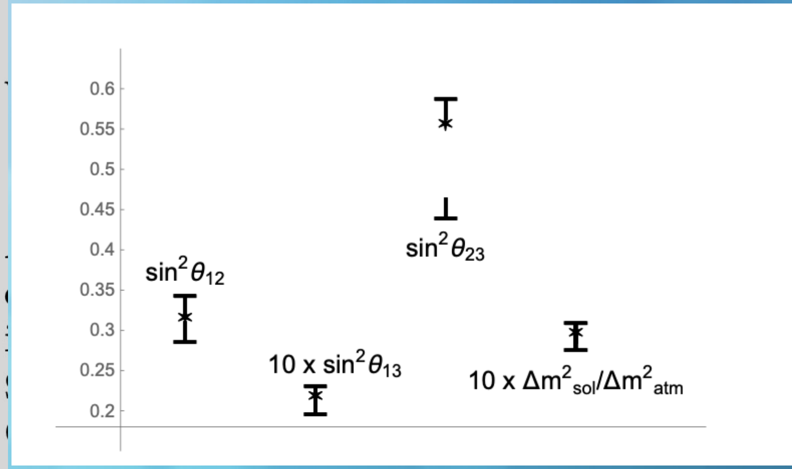
$$\Delta m^2_{atm} = (2456 \pm 32) \text{ meV}^2$$

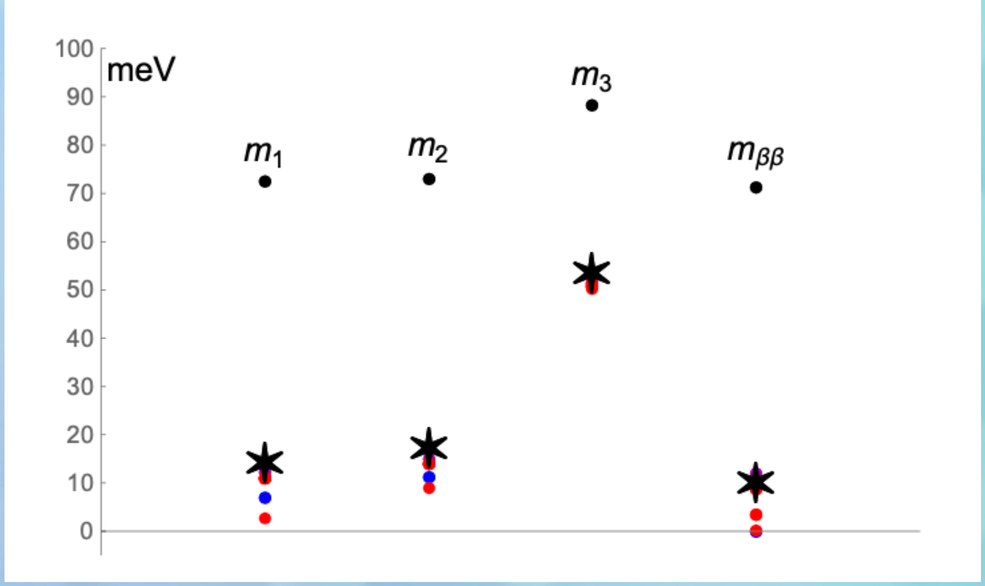
$$\frac{m_2 + m_1}{2} = 11.5 \pm 2.2 \text{ meV} \qquad \sqrt[3]{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} = 0.310 \pm 0.001$$

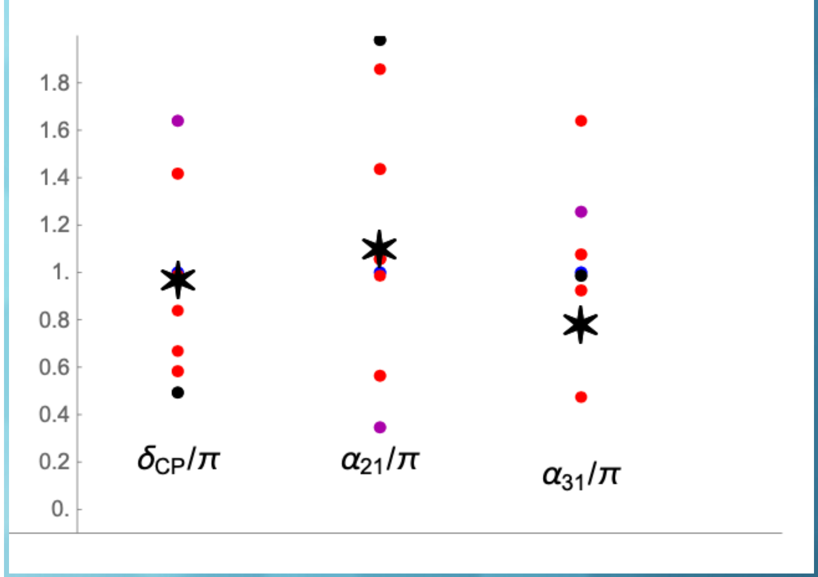
$$2\frac{m_2 - m_1}{m_2 + m_1} = 0.34 \pm 0.25 \qquad \frac{m_2 + m_1}{2m_3} = 0.23 \pm 0.04$$

$$\sin^2\theta_{13} = 0.02199 \pm 0.00036 \qquad \sin^2\theta_{23} = 0.565 \pm 0.027$$

$$1 - \sin^2\theta_{12} = 0.182 \pm 0.015 \; .$$







hints from gravitational, gauge and Higgs sectors
is the Yukawa sector close to a phase transition?

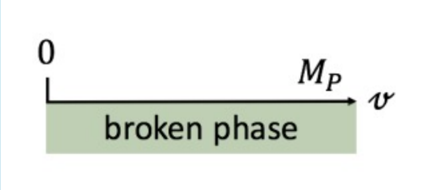
PHASE TRANSITION = a control parameter is tuned and produces a change in the organization of a system
(T, P, B, the Higgs quadratic coupling)

impossible to establish a PT in the Yukawa sector

no baseline theory of fermion masses and mixing angles,
only uncountable models

impossible to determine the set of control parameters

shift the focus from the control parameters the order parameter...



$v = 0$ disordered (symmetric) phase

$v \neq 0$ ordered (broken) phase

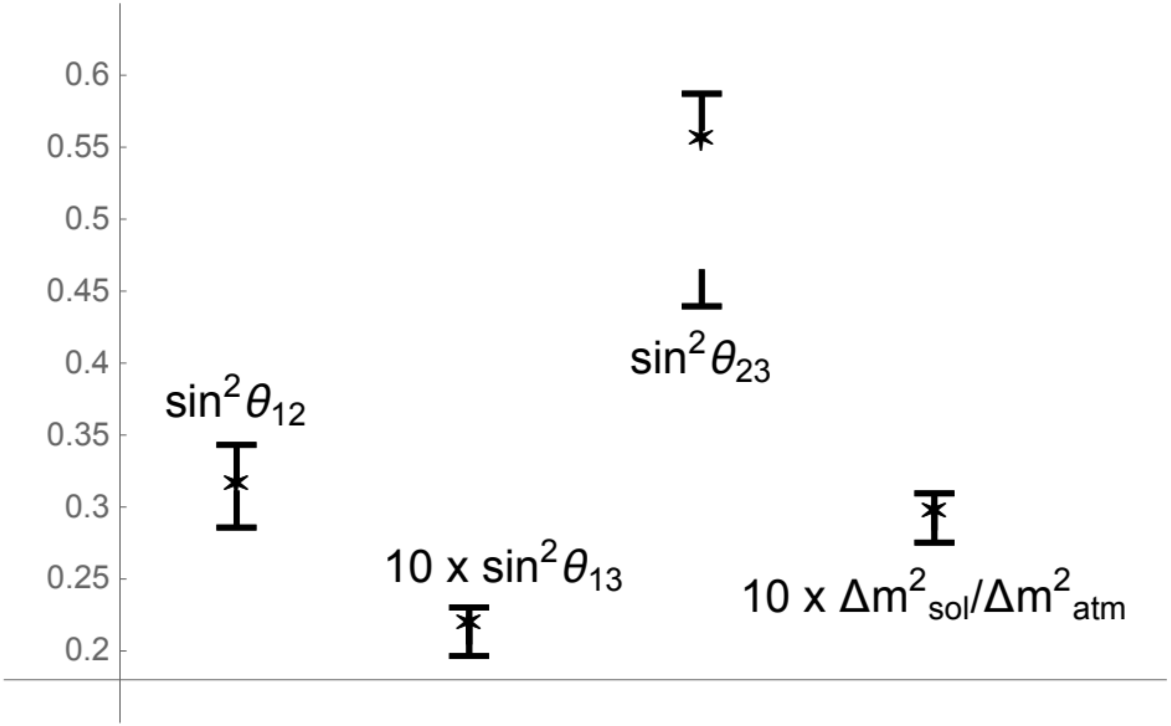


Figure 5: Mixing angles and $\Delta m^2_{sol}/\Delta m^2_{atm}$ of 14 pairs of CP and modular invariant models featuring normal ordering and $|\tau - i| < 0.25$ (see text). Shown in the plot are the intervals covered by the model predictions. A star indicates the average over the 14 models.

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

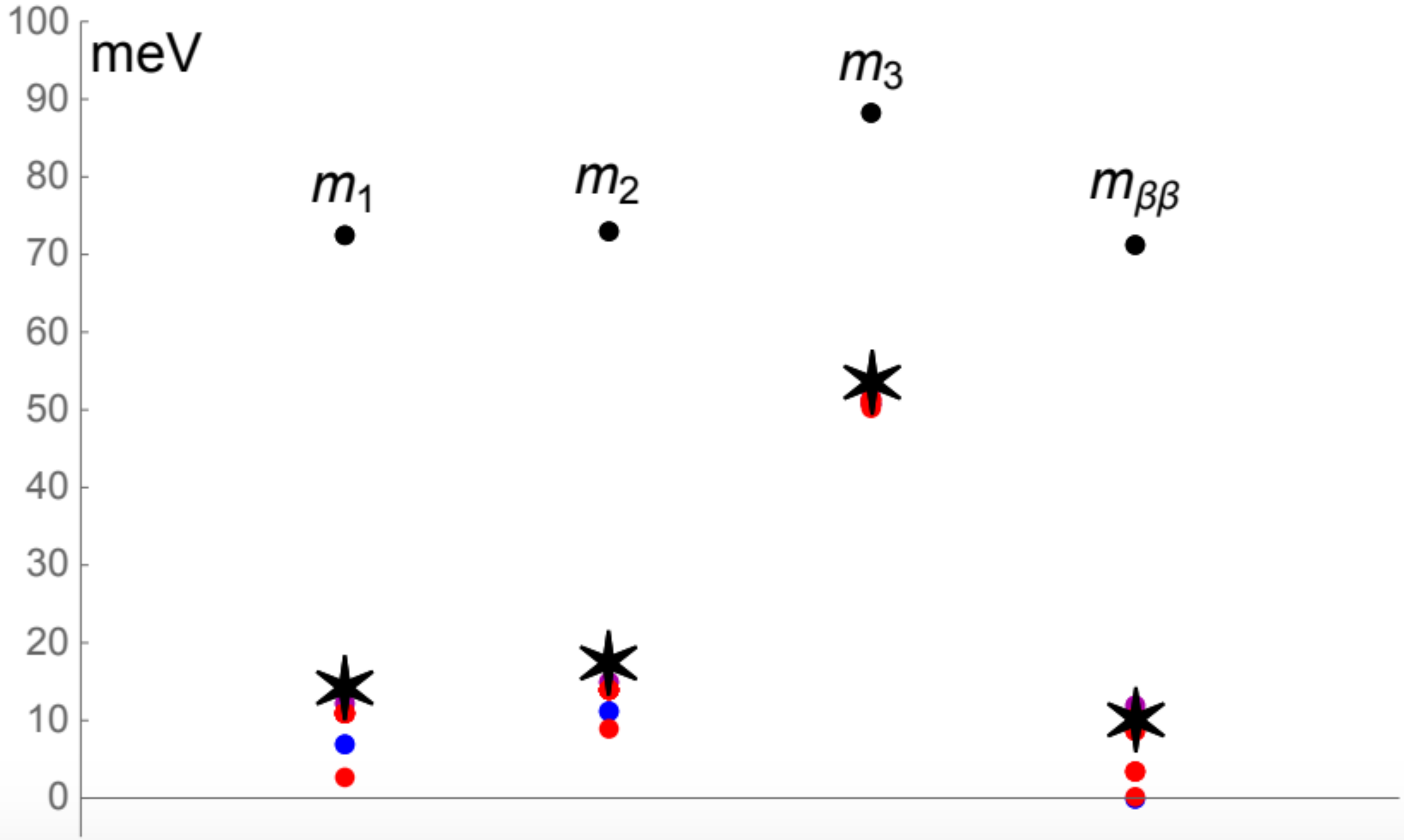


Figure 6: Mass parameters and phases of 14 pairs of CP and modular invariant models featuring normal ordering and $|\tau - i| < 0.25$ (see text). The full distributions of predictions are displayed. The color code is identical to the one in fig. 4. A star shows the average over the 14 models. CP violating phases refer to models where $\mathtt{Re}\tau > 0$.