From Extreme QCD to Conformal Window

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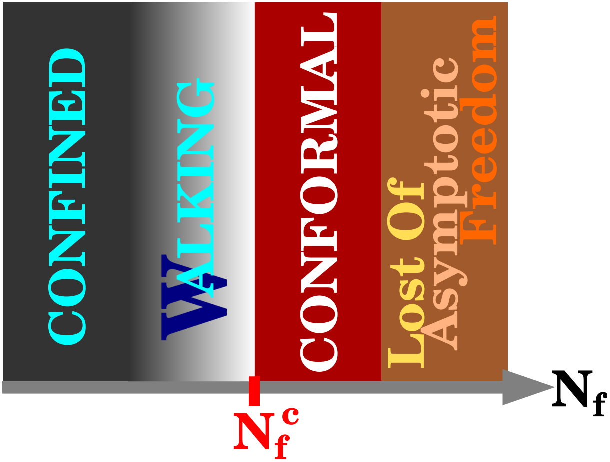
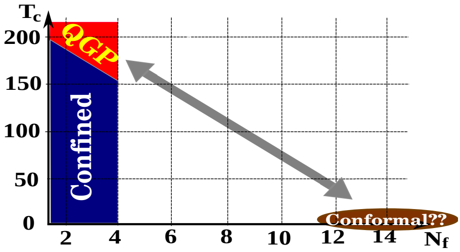
Rijksuniversiteit Groningen[B]

Talk at LNF-Spring-Workshop-2011, March 25

Large N_f QCD


CONFINED
WALKING
CONFORMAL
Lost Of
Asymptotic
Freedom
N_f
N_f^c

$T - N_f$ Phase Diagram: From Lattice Gauge Theory



- The physics of the chiral phase transition at finite T with $N_f = 2 - 4$ is well investigated by using the Lattice Monte-Carlo Simulations.
- Vanishing chiral phase transition would be a good indicator for the lower bound of the Conformal Window.
- Conection between QGP and Conformal Window?

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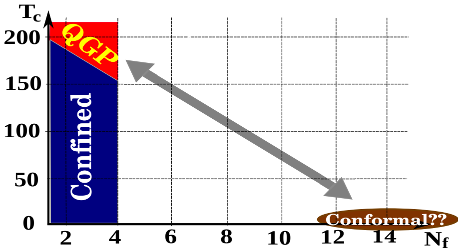

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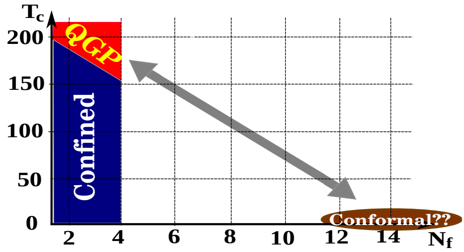

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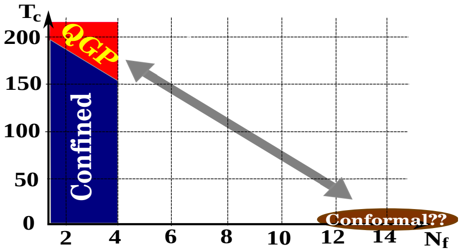

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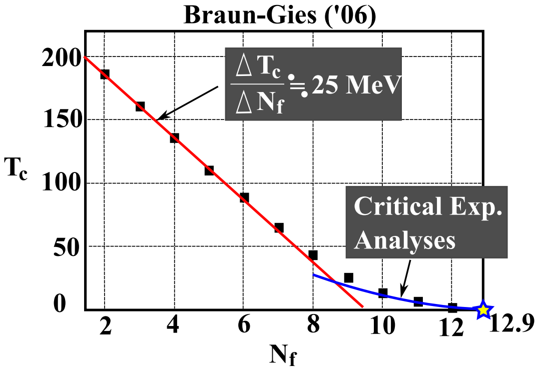
$T - N_f$ Phase Diagram: From Functional Renormalization Group



Status of Conformality In Strongly-Interacting Gauge Theory

On The Conformal Window...

- The dynamics of Strongly-Interacting Gauge Theory is the modern question in the theoretical physics. We know QED and QCD, why don't we expect CONFORMALITY?
- Electroweak Symmetry Breaking, Walking Technicolor, and AdS/CFT.

$$\frac{\langle\bar{\Psi}\Psi\rangle|_{\mathrm{ETC}}}{\langle\bar{\Psi}\Psi\rangle|_{\mathrm{TC}}} = \exp\left[\int_{\Lambda_{\mathrm{TC}}}^{\Lambda_{\mathrm{ETC}}} d(\log\mu)\ \gamma\left[g^2(\mu)\right]\right] \xrightarrow{\text{Conformal}} \left(\frac{\Lambda_{\mathrm{ETC}}}{\Lambda_{\mathrm{TC}}}\right)^{\gamma[g_*^2]} . \quad (1)$$

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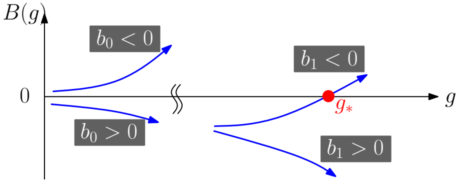
Banks-Zaks IR Fixed Point



$$B(g, N_c, N_f) = -g^3 \sum_n b_n(N_c, N_f) g^{2n} \ , \tag{2}$$

1-Loop: $$b_0 = \frac{1}{(4\pi)^2}\left[\frac{11}{3}C_2[G] - \frac{4}{3}N_f T[r]\right] , \tag{3}$$

2-Loop: $$b_1 = \frac{1}{(4\pi)^4}\left[\frac{34}{3}C_2^2[G] - \left[\frac{20}{3}C_2[G] + 4C_2[r]\right]N_f T[r]\right] . \tag{4}$$

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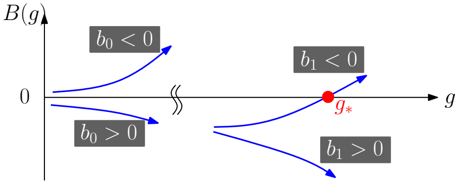

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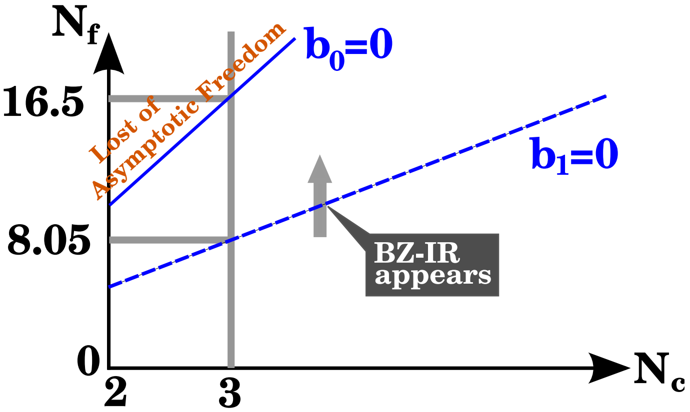
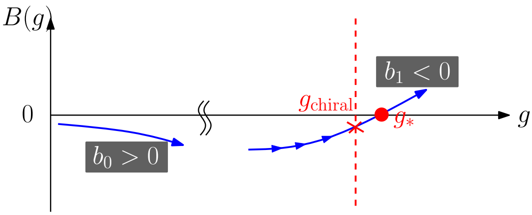
$N_f - N_c$ Phase Diagram: Fundamental Fermion



Banks-Zaks VS Chiral



$$g_* < g_{\rm chiral} \,, \quad (\text{For } \exists \text{Conformal Window}) \,. \tag{5}$$

$$g_*^2 = -\frac{b_0}{b_1} \,, \quad (\text{2-Loop}) \tag{6}$$

$$g_{\rm chiral}^2 = \frac{N_c}{6C_2[r]} \,, \quad (\text{Ladder Approx.}) \tag{7}$$

Appelquist-Lane-Mahanta ('88) , Dietrich-Sannino ('06)

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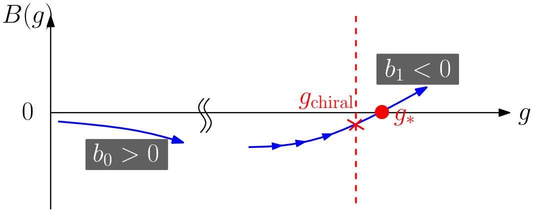

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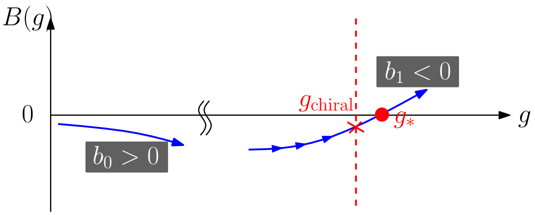

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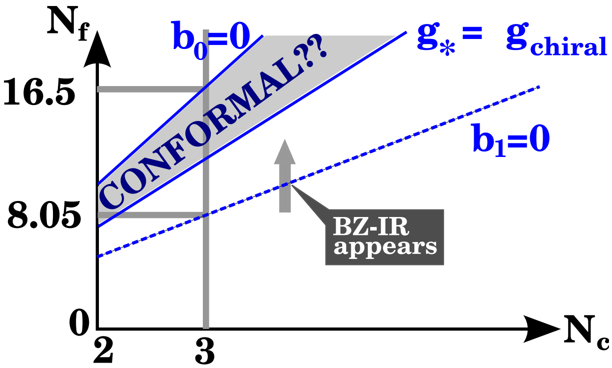

For $(N_c, N_f) = (3.0, 12.0)$, $g_*^2 = -b_0/b_1 \simeq 9.5$

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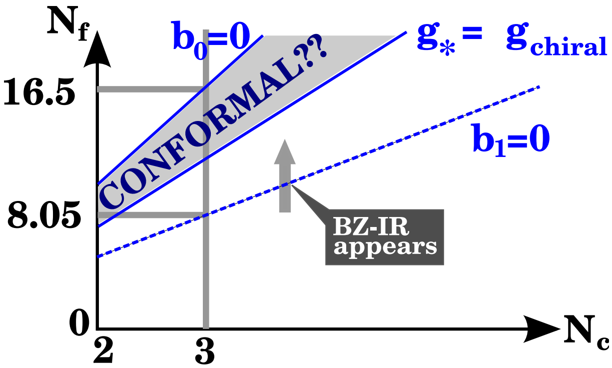

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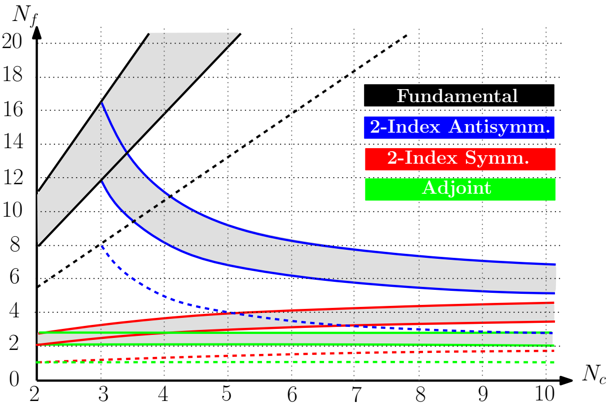
$N_f - N_c$ Phase Diagram III: With Two-Index Representations



Dietrich-Sannino ('07)

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Summary-Table I: Lattice Studies for Conformal Window

Fundamental Reps.

- $\mathbf{N_f = 16}$: Damgaard et al.('97, Meson Mass Analyses), A.Hasenfratz ('10, MCRG), Fodor et al.('09, Mass Scaling Low)
- $\mathbf{N_f = 12}$: Appelquist-Fleming-Neil('08, Step-Scaling, Schrodinger-Functional), Fodor et al.('09), A.Hasenfratz('10), Itou et al.(PoS2010, TPL), Deuzeman-Lombardo-Pallante('08, Bulk-Trans., Const. Phys.), Mawninney et al.('09), Kogut-Sinclair ('90).
- $\mathbf{N_f = 8}$: Appelquist-Fleming-neil('08), A.Hasenfratz('10), Deuzeman-Lombardo-Pallante ('08, Finite T, $a(\beta_c)/a(\beta_c') = N_\tau'/N_\tau$). Ohki et al.(PoS2010, TPL, $N_c = 2$).
- $\mathbf{N_f = 6}$: LSF Collaboration ('10, Enhancement of PBP/F_π^3) , Deuzeman-Lombardo-Miura-Pallante (Present, Finite T).

Summary-Table II: Lattice Studies for Conformal Window

Other Representations: $N_f = 2$

- Symm.: Degrand-Shamir-Svetitsky ('10, $N_c = 3$ Schrodinger-Functional), Fodor-Holland-Kuti-Nogradi-Schroeder ('09, $N_c = 3$ Random-Matrix-Therory)
- Adjoint: Hietanen-Rummukainen-Tuominen ('09, $N_c = 2$, Schrodinger-Functional), Catterall-Giedt-Sannino-Schneible ('08, $N_c = 2$, Meson Mass and String Tension)

Continuum Limit

$$B(g) = M\frac{dg}{dM} \ , \tag{8}$$

$$\int_{g_0}^{\infty}\frac{dg}{B(g)} = \int_{1/a}^{\Lambda}\frac{dM}{M} \ , \tag{9}$$

$$a\Lambda = (b_0 g_0)^{(b1/(2b_0^2))}\exp(-1/(2b_0 g_0^2)) \ . \tag{10}$$

- $g_0 \longrightarrow 0$ gives a continuum limit $a \longrightarrow 0$.
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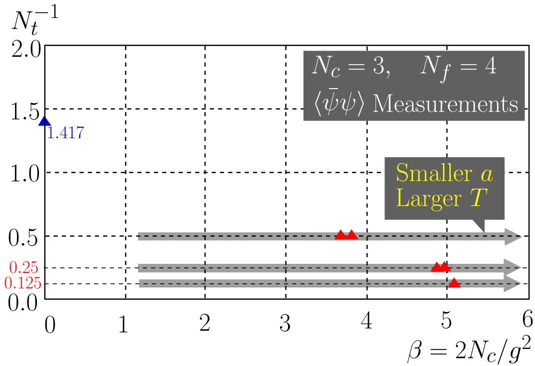
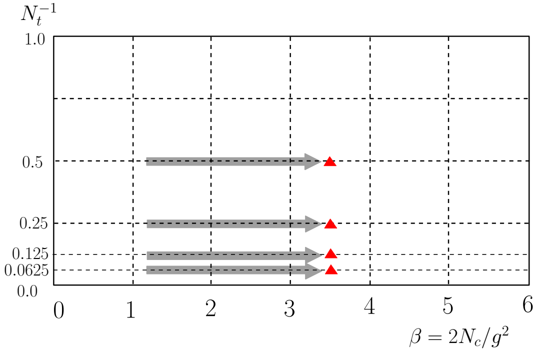
Thermal Transition

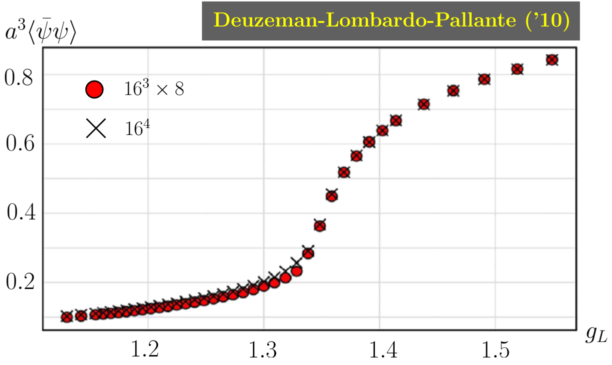


MC data: From the left, Forcrand,Fromm('09), Forcrand(private), Gottlieb et al.('87), D'Elia,M.Lombardo('03), Fodor,Katz('02), Gavai et al.('90)

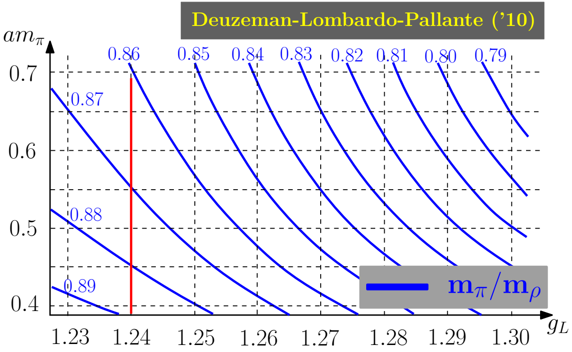
Bulk Transition



c.f. Bulk Transition at $N_f = 12$



Constant Physics on Meson Mass Spectrum, $N_f = 12$ Case



1. Input: (g_L, am_q) , Output: $(am_\pi, m_\pi/m_\rho)$
2. Constant Physics $\longleftrightarrow$ Relation Between a and $\bar{g}$
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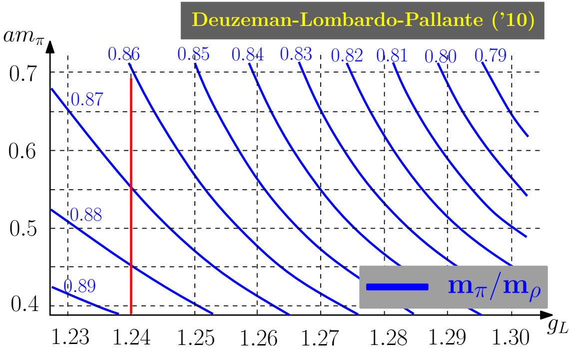

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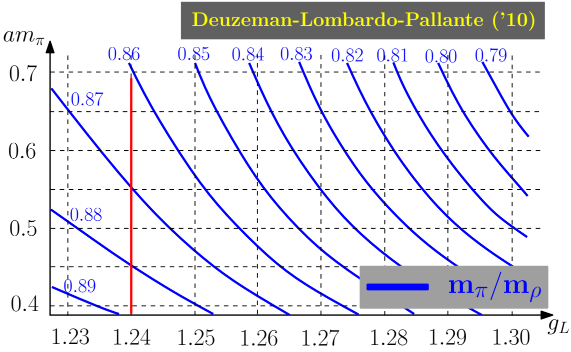

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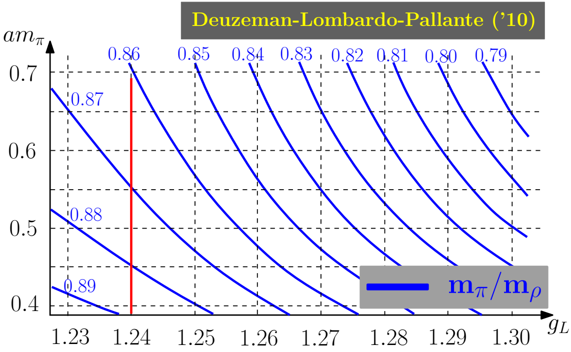

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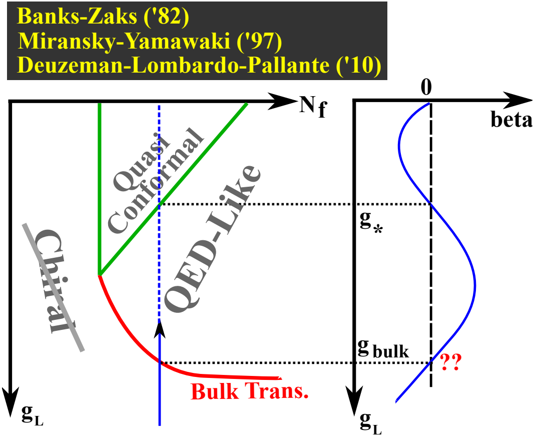
$g - N_f$ Phase Diagram



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Setups

GOAL

1. Observe the PBP and Polyakov loop, and find the transition point.
2. Determine whether the transition is bulk or thermal.
3. Investigate meson mass spectroscopy to get physical T_c (Thermal) or the B function (Bulk).

Conditions

- Staggered Fermions with 6-Flavors in Fundamental Representation
- On the Lattice $12^3 \times 4$, $16^3 \times 4$, and $16^3 \times 8$
- Rational Hybrid Molecular-Dynamics with Omelyan-Integrator

Code and Computers

- MILC-Code: http://www.physics.utah.edu/~detar/milc/milc_qcd.html
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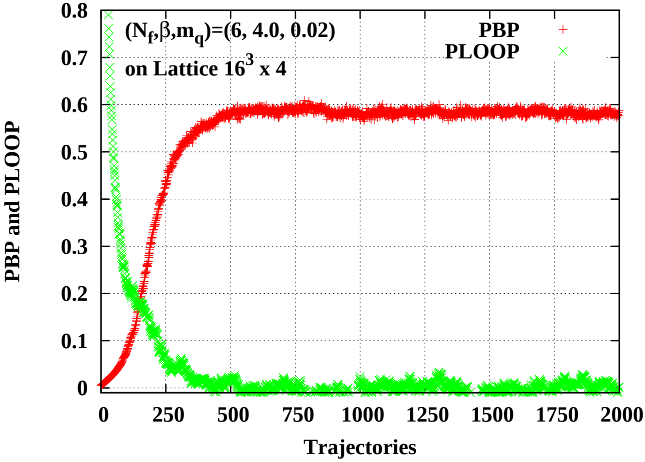
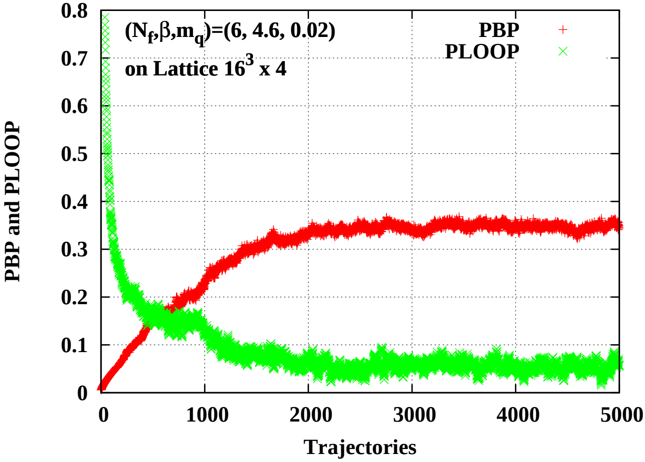
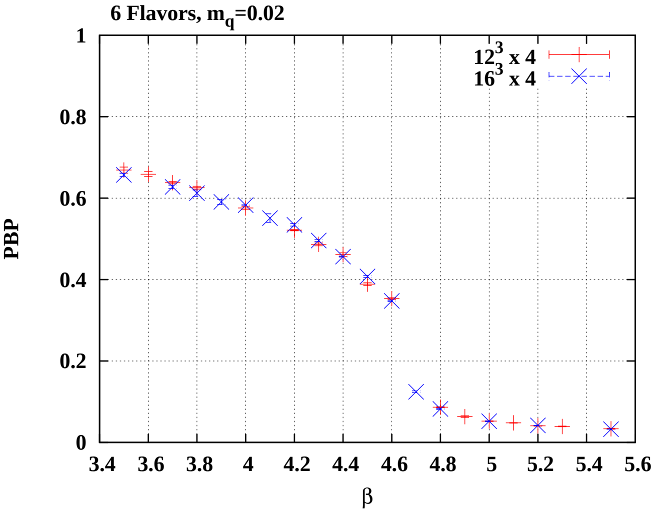
Trajectories at Strong Coupling: $\beta = 2N_c/g^2$



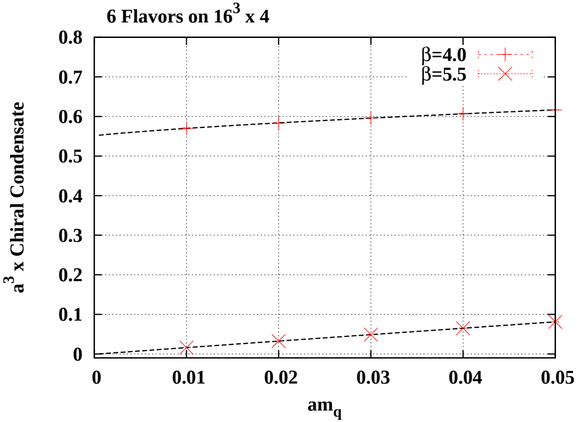
Trajectories in Critical Region



Finite-Size Effects

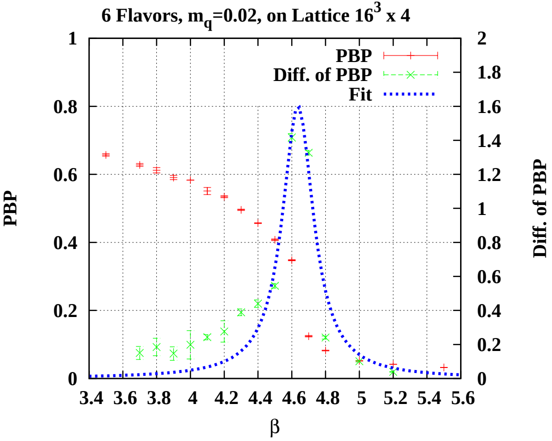


Mass Scaling Low of Chiral Condensate



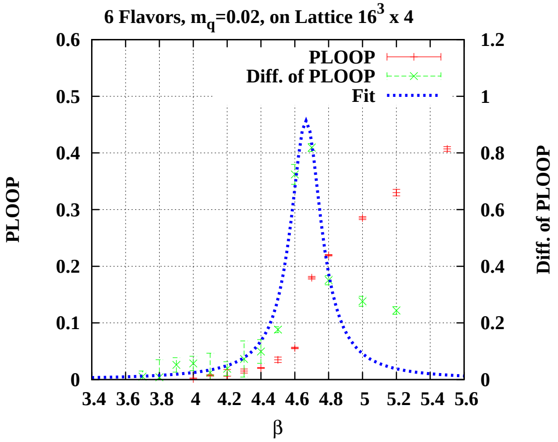
$$a^3\langle\bar{\psi}\psi\rangle = A(am_q) + B(am_q)\log[am_q] + a^3\langle\bar{\psi}\psi\rangle_0 \ . \tag{14}$$

β Dependence of PBP



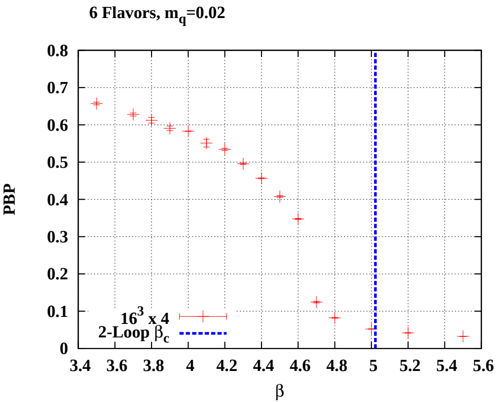
$$\beta_c = 4.6364 \pm 0.01204$$

β Dependence of PLOOP



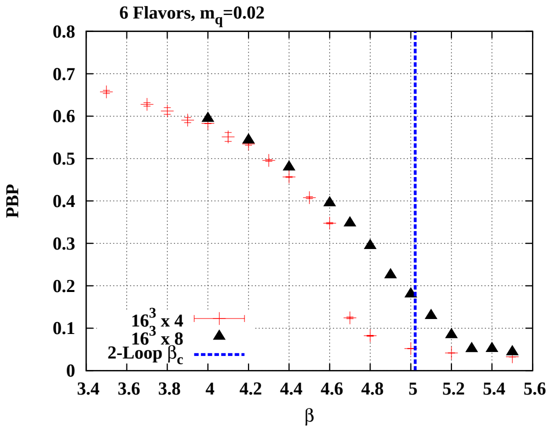
$$\beta_c = 4.66608 \pm 0.01221$$

PBP Scaling for N_τ I

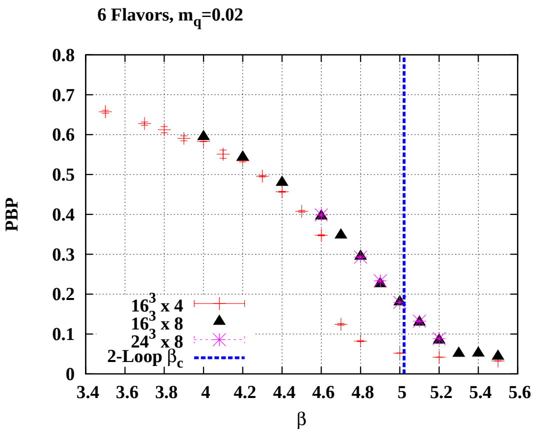


$$\frac{N_\tau^{-1}}{N_\tau'^{\,-1}} = \left(\frac{\beta_c}{\beta_c'}\right)^{b_1/(2b_0^2)} \exp\left[-\frac{\beta_c - \beta_c'}{4N_c b_0}\right] \tag{15}$$

PBP Scaling for N_τ II



PBP Scaling for N_τ III



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