Lattice Boltzmann Method applications: a characterization of thermal effects in plasma waves

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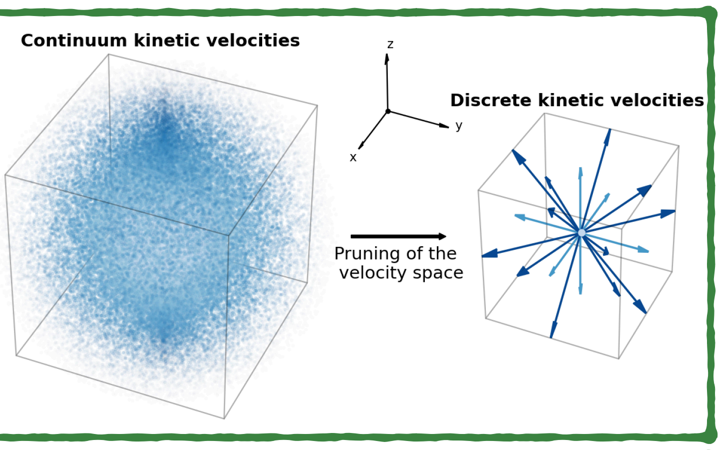
Introduction: the lattice Boltzmann (LB) method [1] is a well established approach applied for the first time in the community of plasma wakefield acceleration (PWFA) for the plasma evolution simulations [2]. In this work, we employ the LB method to investigate **the influence of temperature on plasma waves** [3][4]. Thermal effects can be relevant, for example, in PWFA processes with a high repetition rate, which holds significant importance for various PWFA applications. By utilizing the LB method, we explore and characterize well-known thermal features of plasma waves documented in the literature, including **acoustic motion**, **dispersion relations**, and **thermal anisotropies.**

Warm Fluid Model and closure schemes: including temperature effects in the plasma fluid equations leads to a *not-closed system* of equations. We make progress by considering two popular but different closure schemes:

- **Local Equilibrium Closure (LEC)** ⇨ the p.d.f. is set to the Maxwell-Jüttner distribution [5].
- **Warm Closure (WARMC)** ⇨ the third centered moment of the p.d.f. is negligible [4].

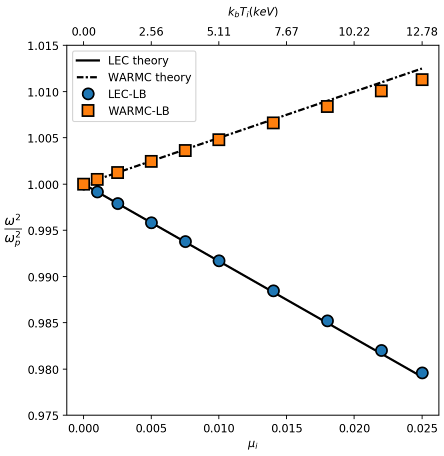
The LB method can reproduce these two closure schemes [6] by discretizing the phase space, in particular by *pruning the velocity space with a finite set* that reproduces the desired hydrodynamic.

$$f\left(\mathbf{x}, \boldsymbol{\xi}, t\right) \Rightarrow f\left(\mathbf{x}, \boldsymbol{\xi}_i, t\right) \text{ with } \Delta\mathbf{x} = \Delta\boldsymbol{\xi}_i t$$



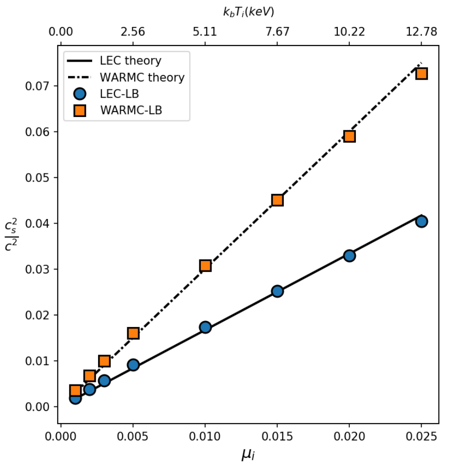
Dispersion relation (1D3V): theoretical predictions for the dependency of the normalized frequency ω/ω_p (ω_p being the plasma frequency) from the normalized temperature μ_i are well captured by the LB method.

$$\mu_i = \frac{k_b T_i}{m_e c^2}$$

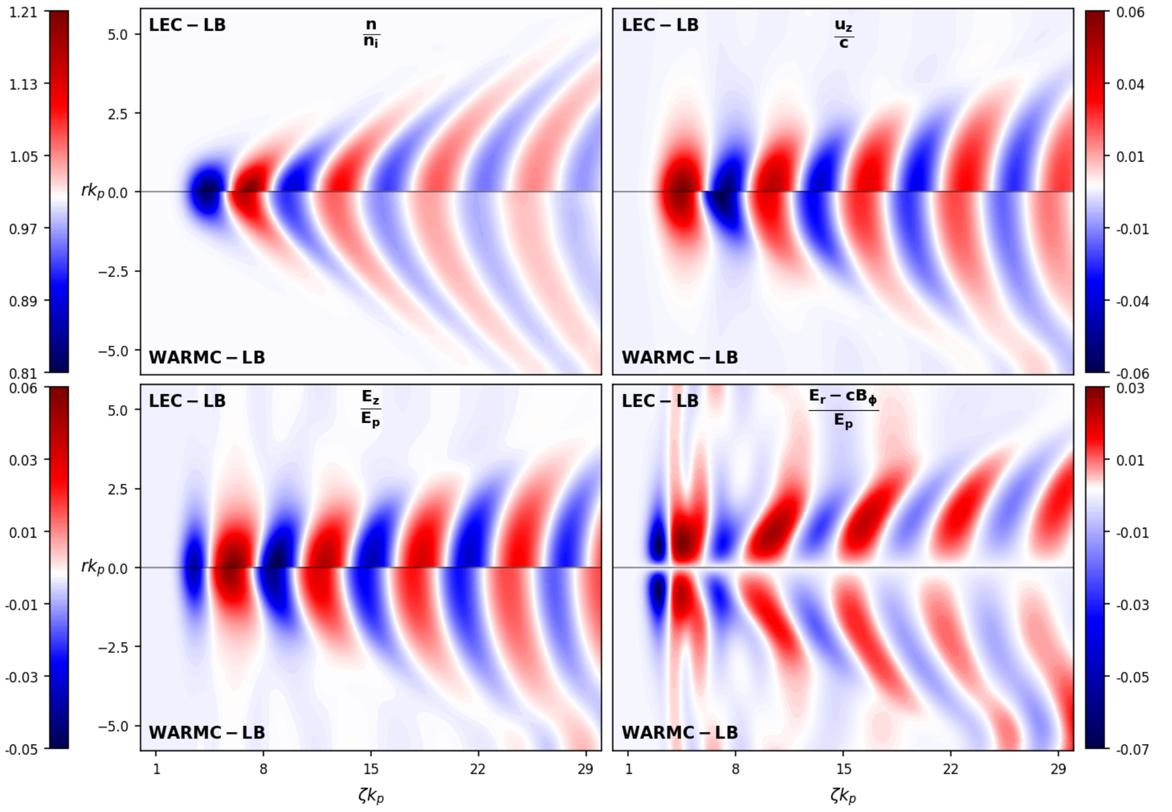


Sound velocity (3D3V): by tracking the angle of the Mach cone structure we are able to measure the sound velocity in the linear regime of the electron wave (c is the speed of light).

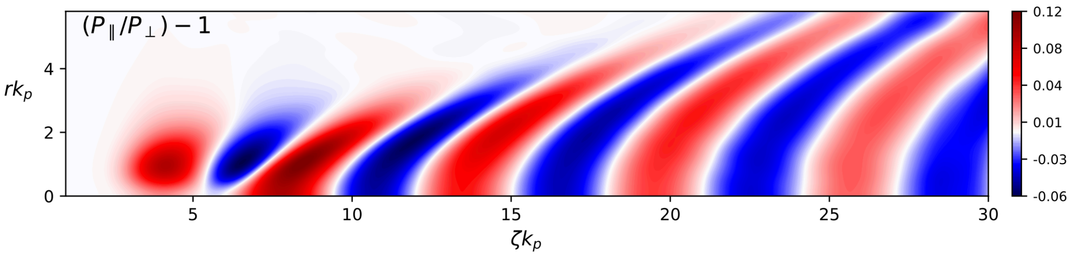
- **LEC** ⇨ $c_s^2/c^2 = 5/3\mu_i$
- **WARMC** ⇨ $c_s^2/c^2 = 3\mu_i$



Acoustic Motion (3D3V): LB method is able to simulate spatially resolved features of plasma quantities. A spatially resolved plot of the electron density n (normalized to ion density n_i) shows a typical *Mach cone structure*.



Thermal anisotropies (3D3V): the formulation of LEC leads to an isotropic pressure [5], while the WARMC implies pressure anisotropies [4]. Through the LB method we can monitor both the longitudinal ($P_{\|}$) and transverse ($P_{\perp}$) pressure (or thermal spreads) of the WARMC anisotropy and analyze them through the quantity $(P_{\|}/P_{\perp}) - 1 = 0$.



Conclusions: numerous physical processes are involved in PWFA. The **LB method has demonstrated remarkable** proficiency in reproducing PWFA results **as fluid solver in the cold limit [5]**. **Extensions to include thermal effects** face the non trivial problem of selecting an appropriate closure scheme for the warm plasma fluid equations. LB schemes can be adapted to comply with this task [6]. Here, we have shown some selected LB applications in presence of thermal effects. Ongoing efforts are dedicated to the incorporation of **ion motion**, detailed comparisons between LB schemes and **PIC simulations** and the inclusion of **laser-induced perturbations**.

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