ON THE BETATRON RADIATION IN CYLINDRICALLY SYMMETRIC PLASMA-ION CHANNELS

D. Francescone[1,2 †], F.Bosco[1,2], M. Carillo[1,2], G. J. Silvi[1,2], L. Giuliano[1,2], E.Chiadoni[1,4] , A.Curcio[4], A.Cianchi[5], M.Ferrario[4], M.Galletti [5], A.R.Rossi [3], V.Shpakov [4], A. Mostacci[1,2], L. Palumbo[1], and M. Migliorati[1]
[1]Sapienza University of Rome, 00161 Rome, Italy, [2]INFN-Sez. Roma1, Rome,00185, Italy,
[3]INFN-Sez Milano, 20133 Milano, Italy,
[4]Frascati National Laboratories INFN-LNF, 00044 Frascati (RM), 00044, Italy
[5]University of Rome Tor Vergata, Rome,Italy, 00133

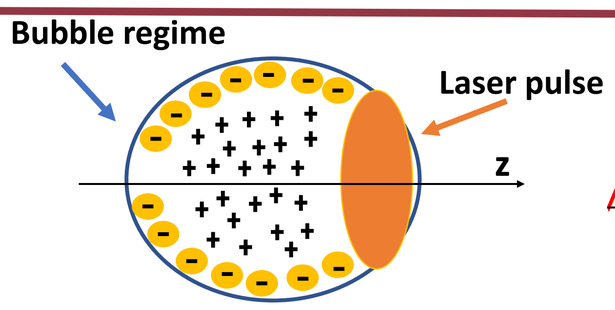
Abstract

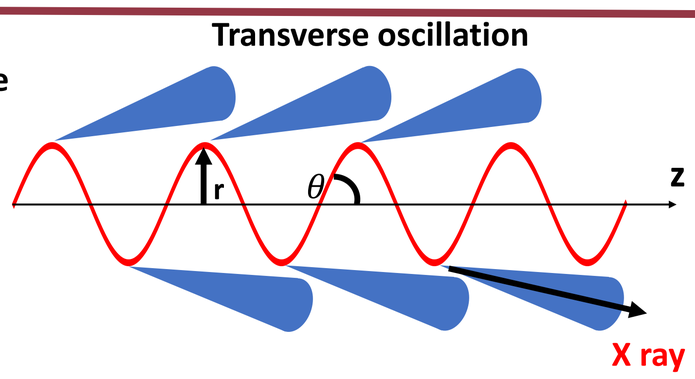
The relativistic interaction of short pulsed lasers or electrons with plasma has recently led to the birth of a new generation of femtosecond **X-ray sources**. Radiations with properties similar to those that can be observed from a wiggler or undulator, can be generated by the oscillations induced in the exited plasma by electrons (PWFA) or by lasers (**LWFA**), making plasma an interesting medium both for the acceleration as well as for the radiation source, with properties of being compact, providing collimated, incoherent, femtosecond radiation. Thus a lot of effort is being made to understand and improve this new source to make it really competitive, This paper summarizes and shows some preliminar theoretical results and numerical simulations of a simplified model called plasma ion column, using as a starting point the parameters expected for the **EuPRAXIA@SPARC_LAB facility,** highlighting strengths, limitations and scaling laws, which allow for a comparison with other types of more consolidated sources of light as Compton, Synchrotron and Free Electron Lasers (FEL).

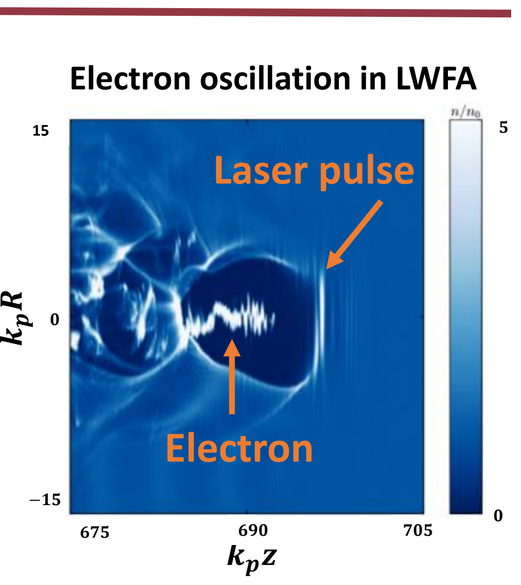
1. Electron dynamics

Betatron radiation is emitted for

- Oscillations of the beam barycenter (off-axis injection)
- Pure beam envelope oscillations (on-axis injection).
- Only Single particle dynamics is relevant.
- Incoherent radiation
- Restoring forces $\boldsymbol{F_{res}}$ created by the radial displacement.
- Range of emission **1 – 100 keV**







$$\omega_p = \sqrt{\frac{e^2 n_e}{m\,\epsilon_0}} = k_p c$$

Plasma frequency

$$\omega_\beta = \frac{\omega_p}{\sqrt{2\gamma}}$$

Betatron frequency

$$x = r\sin(k_\beta c t)$$

Transverse dynamics

2. Betatron strength parameter

- Analogy whit **Wiggler** and **Undulator**
- **Approximation**: we study the $\boldsymbol{K_\beta}$ of a single particle

$$K_\beta = \gamma r k_\beta$$

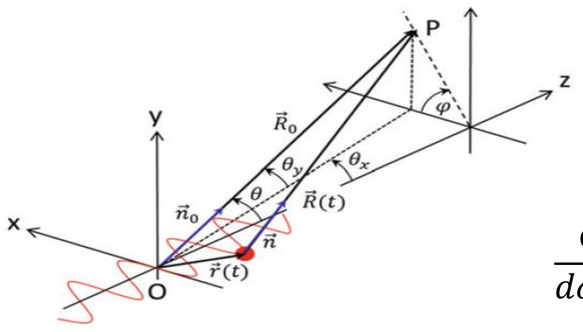
SINGLE PARTICLE APPROXIMATION

LIMIT OF THE APPROXIMATION

$$K_\beta(t) = \gamma(t) r(t) k_\beta(t)$$

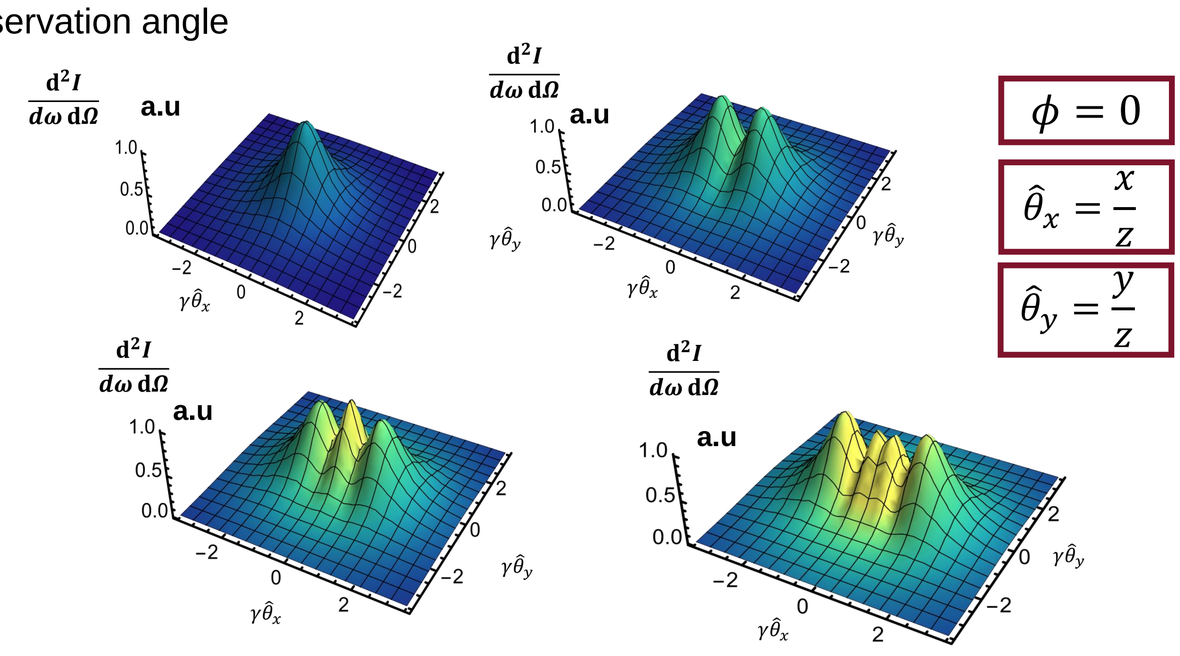
- Particle are accelerated
- Each particle has a different oscillation amplitude
- $k_\beta(t) = k_p/\sqrt{2\,\gamma(t)}$

3. Energy radiated



The $\mathbf{d^2I}\,/d\omega\,\mathbf{d\Omega}$ is the energy radiated per frequency $\boldsymbol{\omega}$ and per solid angle $\boldsymbol{\Omega}$ during the interaction time **T**, while **n** is a unit vector pointing the direction of observation, $\boldsymbol{\theta}$ is the observation angle with respect to the electron propagation axis and $\boldsymbol{\phi}$ is the azimuthal observation angle

$$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega} = \frac{\mathrm{e}^2\omega^2}{4\pi^2 c}\left|\int_{-T/2}^{T/2} \mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})\,\mathrm{e}^{\mathrm{i}\omega\left(\mathrm{t}-\frac{\mathbf{n}\cdot\mathbf{r}}{\mathrm{c}}\right)}\,\mathrm{d}t\right|^2$$



$$\phi = 0$$

$$\hat{\theta}_x = \frac{x}{z}$$

$$\hat{\theta}_y = \frac{y}{z}$$

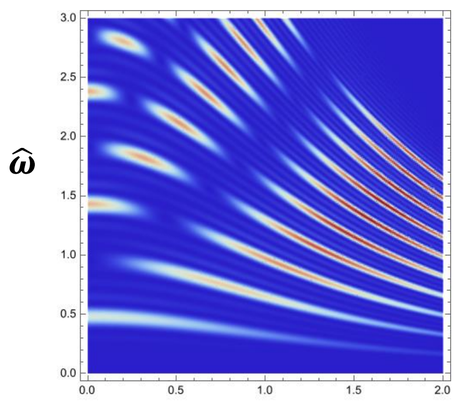
4. Spectrum

On axis θ=0

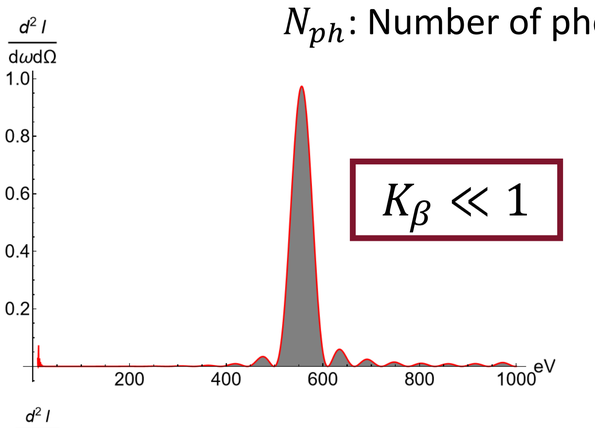
N_{ph}: Number of photons

Off axis $\theta \neq 0$

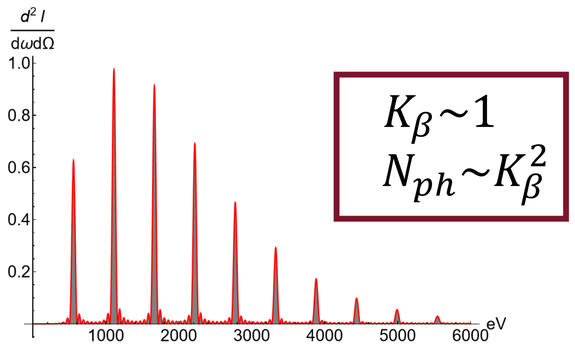
$\frac{\mathbf{d^2I}}{\boldsymbol{d\omega\,d\Omega}}$



Undulator-like

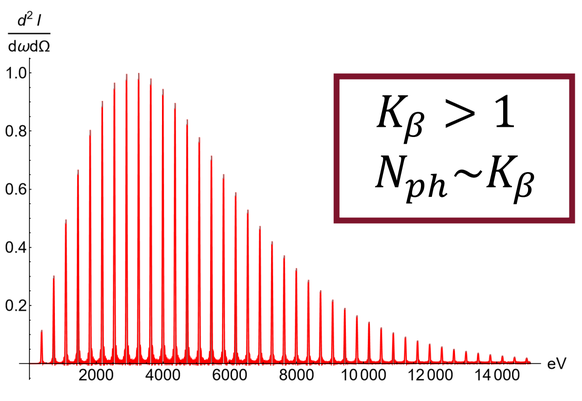


$K_\beta \ll 1$



$K_\beta \sim 1$

$N_{ph} \sim K_\beta^2$

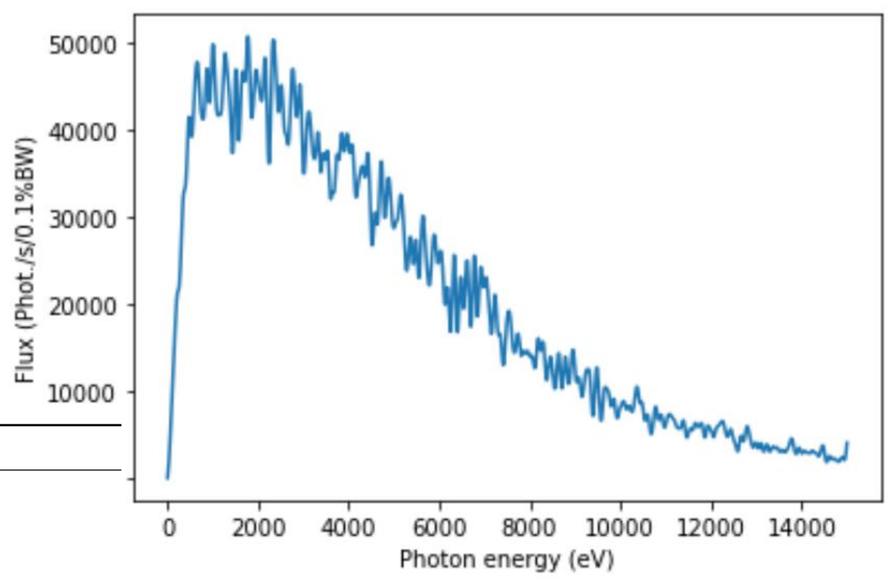


$K_\beta > 1$

$N_{ph} \sim K_\beta$

Wiggler-like

4. Photon flux



Possible EuPRAXIA@SPARC_LAB parameters

Parameter [Units]	Value
Beam energy [MeV]	600–950
Plasma density [cm^{-3}]	3.0×10^{16}
RMS transverse beam size [μm]	3.0
Charge [pC]	45
Capillary (plasma) length [m]	0.6
Pulse duration [fs]	10

References

- Jinchuan Ju. “Electron acceleration and betatron radiation driven by laser wakefield inside dielectric capillary tubes.” Theses. Université Paris Sud - Paris XI, June 2013.
- Esarey E, Shadwick BA, Catravas P, Leemans WP. Synchrotron radiation from electron beams in plasma-focusing channels. Phys Rev E Stat Nonlin Soft Matter Phys. 2002 May
- Francesco Stellato et al. “Plasma-Generated X-ray Pulses: Betatron Radiation Opportunities at EuPRAXIA@SPARCLAB.” In: Condensed Matter 7.1 (2022).