

## INTRODUCTION

X-rays production through betatron radiation emission from electron bunches is a valuable resource for several research fields. The EuAPS (EuPRAXIA Advanced Photon Sources) project, within the framework of the EuPRAXIA project, aims to provide 1-10 keV photons (soft X-rays) using a compact plasma based system designed to exploit self-injection processes that occur in highly nonlinear laser-plasma interaction (LWFA) to drive electron betatronic oscillations. While numerical analysis is being pursued, we also aim to gain insights into the emission process through an analytical approach. By generalizing well-known results (I. Kostyukov, S. Kiselev, and A. Pukhov), we derive a comprehensive analytical expression for the emission spectrum in solid angle for single particles subjected to constant longitudinal force and linear transverse force moving in a planar trajectory. Model's approximations are presented, along with intensity plots on the detector and trends in critical frequency for some plasma wiggler strength and longitudinal force values.

## ANALYTICAL MODEL

### EQUATIONS OF MOTION

$$\frac{dp_{x,s}}{dt} = m\dot{\gamma}v_{x,s} + m\gamma a_{x,s} = F_{x,s}$$

$$\begin{cases} \ddot{x} = -\frac{\dot{x}\dot{p} + \dot{k}x}{p_0 + \dot{p}t} \rightarrow x(t) = CJ_0(A\sqrt{1+Bt}) = CJ_0(\zeta) \\ \ddot{s} = \frac{\dot{p}(c-\dot{s})}{p_0 + \dot{p}t} \rightarrow s(t) = ct + \frac{k_1}{\dot{p}} \ln(p_0 + \dot{p}t) + k_2 \end{cases}$$

$$\dot{z} = \sqrt{\dot{s}^2 - \dot{x}^2} \approx \dot{s} - \frac{\dot{x}^2}{2\dot{s}} \approx c - \frac{cA^2}{\zeta^2}(1-\beta_0) - \frac{A^4B^2C^2J_1^2(\zeta)}{8c\zeta^2}$$

$$z(t) = \int \dot{z} dt = A\frac{\zeta^2}{2} + B\ln(\zeta) + C\frac{1}{2}(1 - J_0^2(\zeta) - J_1^2(\zeta))$$

### BESSEL APPROXIMATION

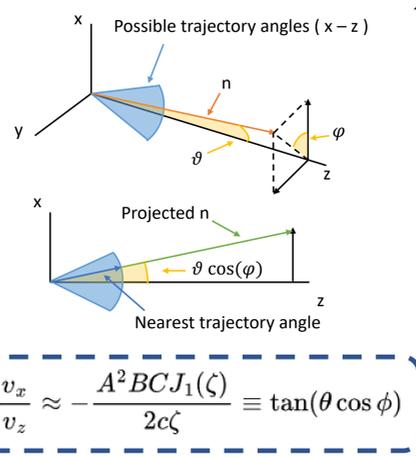
### BHĀSKARA APPROXIMATION

$$J_{1_n}(\zeta) \approx J_1(\zeta_{0_n}) \sin\left(\pi \frac{\zeta - \zeta_{1_n}}{\zeta_{1_{n+1}} - \zeta_{1_n}}\right)$$

$$\sin \zeta \approx \frac{16\zeta(\pi - \zeta)}{5\pi^2 - 4\zeta(\pi - \zeta)}$$

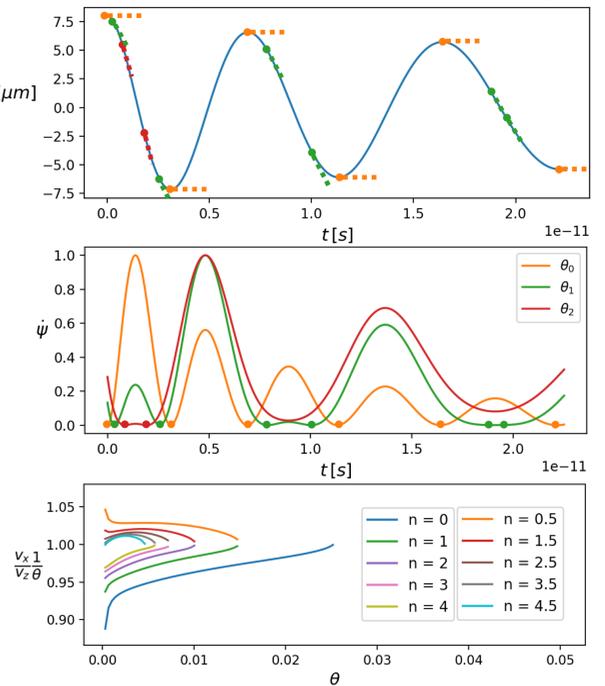
$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2c} \left| \int_{-\infty}^{\infty} \vec{n} \times (\vec{n} \times \vec{\beta}) e^{i\Psi} dt \right|^2$$

### NEARLY STATIONARY PHASE

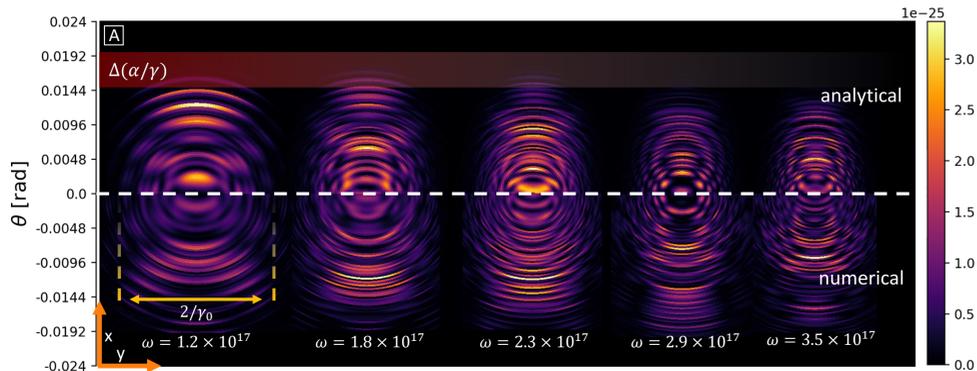


### 3° degree equation for stationary points

$$\Psi = \omega \left[ t - \frac{x(t) \sin \theta \cos \phi + z(t) \cos \theta}{c} \right]$$



## RESULTS



**A:** Analytical VS numerical radiation spots for several frequencies around instantaneous synchrotron critical frequency for accelerated case: expected divergencies along x and y axis, whose upper limit is given by particle initial energy, are observed. Slight discrepancies appear for growing frequency.

**B:** Intensity comparison between analytical model and numerical calculation along x axis (parallel to trajectory plane), with detailed spectrum profiles for some angles: spectra were calculated from an accelerated trajectory with initial plasma wiggler strength  $\alpha = 2$ , making the harmonics distinguishable.

**C:** Same as B but along y axis (perpendicular to trajectory plane).

**D:** Comparison between spectra emitted by non adiabatic (top) and adiabatic (bottom) acceleration in the same particle energy range: the full lines on the right plots show the instantaneous synchrotron critical frequency VS angle for several consecutive trajectory 'periods'. On the left plots, a neat linear trend in the spectrum profile is highlighted, extending across the first harmonic frequencies related to initial and final particle energy. For higher harmonics overlaps occur.

