Optimised density tailoring for dephasing mitigation in laser wakefield accelerators

in simulations…

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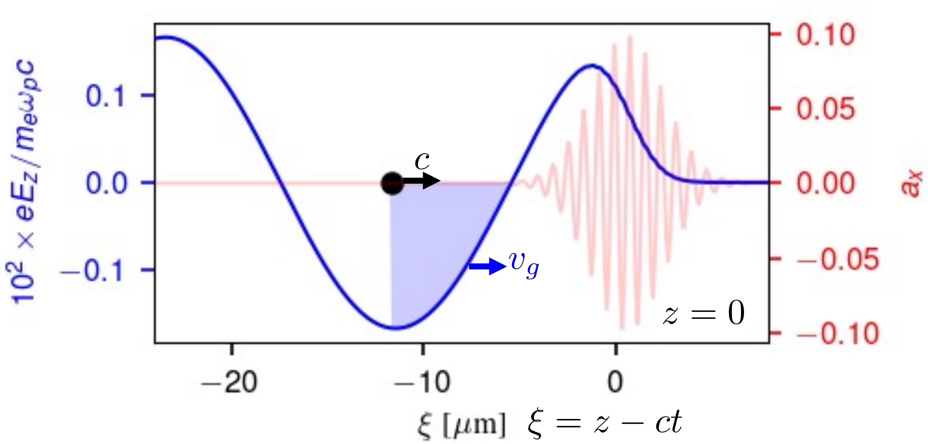
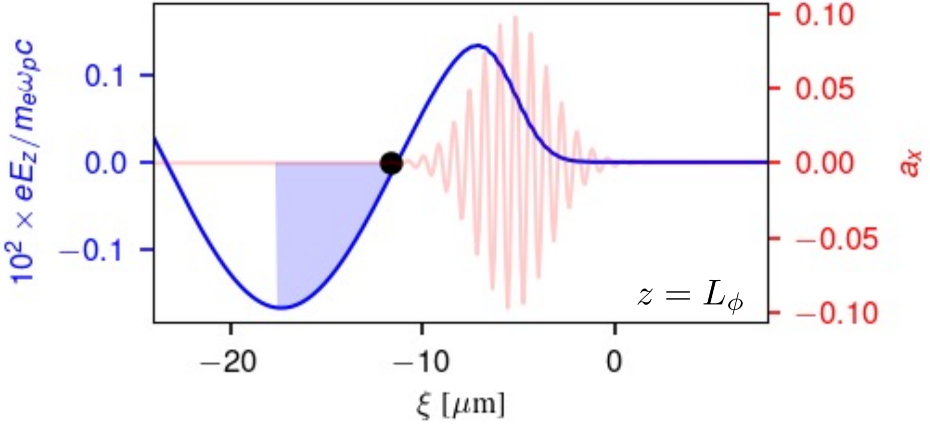
Dephasing in Laser Wakefield Accelerators





The usable phase region for an electron LWFA is accelerating and focusing.

For the linear regime

$$a_0 \ll 1 \qquad \omega_p \ll \omega_L$$

The group velocity of the laser is

$$v_g = c\sqrt{1 - \omega_p^2/\omega_L^2}$$

This sets the phase velocity of the wake and so the dephasing length is

$$L_\phi = \frac{\pi c}{2\omega_p}\frac{1}{\beta_g^{-1} - 1}$$

$$L_\phi \approx \pi c \frac{\omega_L^2}{\omega_p^3}$$

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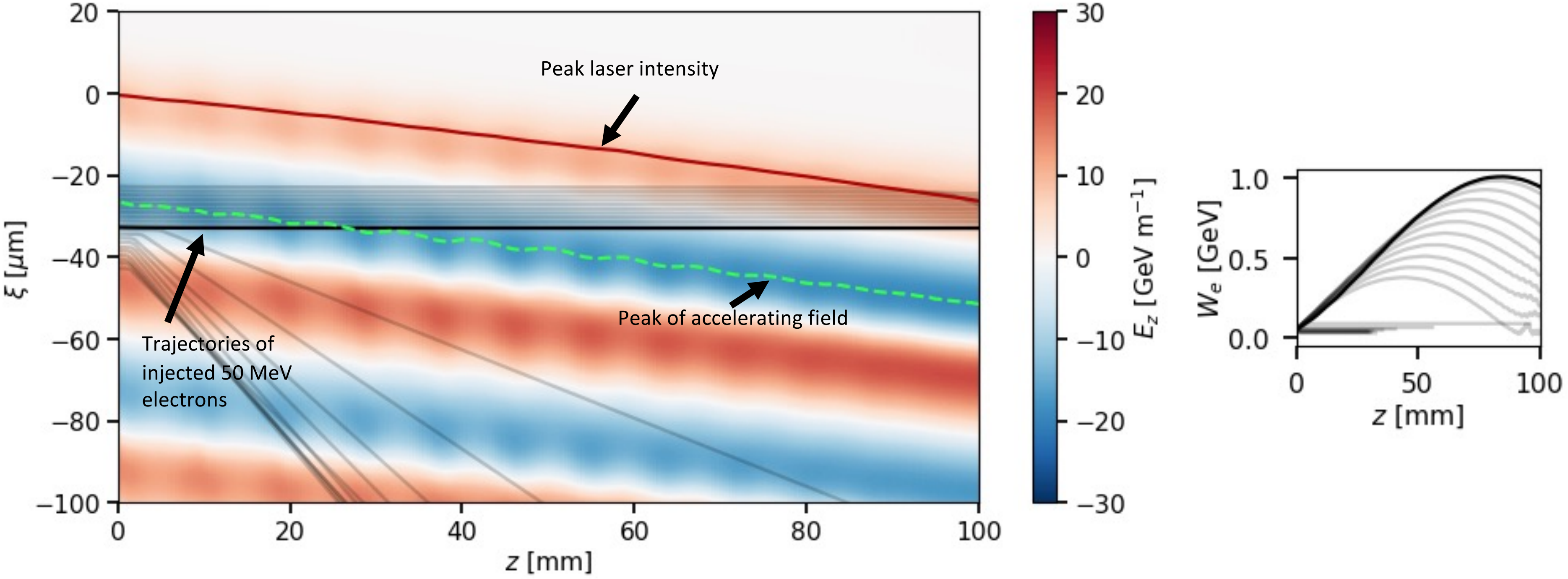


Peak laser intensity

Peak of accelerating field

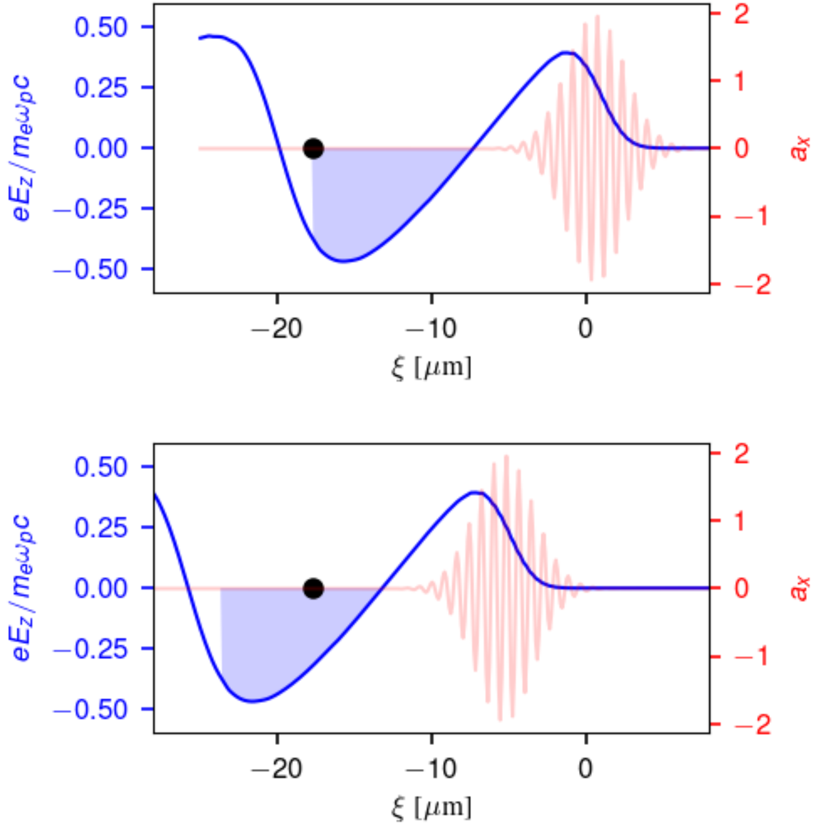
Trajectories of injected 50 MeV electrons

- Maximum electron energy reached for electrons injected behind peak field position

FBPIC simulation: a_0 = 0.95, guiding channel with n_e = 5 x 10^{17} cm^{-3}

FBPIC simulations: R. Lehe et al., CPC, 2016

Dephasing can be removed in a plasma density gradient



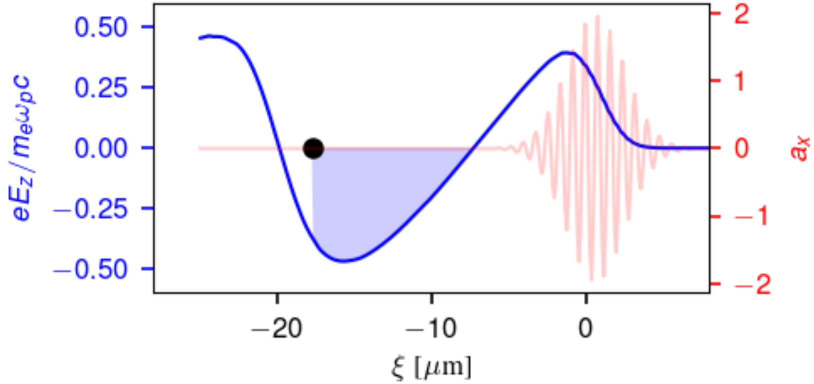
Non-Linear regime

$$a_0 \geq 1 \qquad \omega_p \ll \omega_L$$

$$v_g < c\sqrt{1-\omega_p^2/\omega_L^2}$$

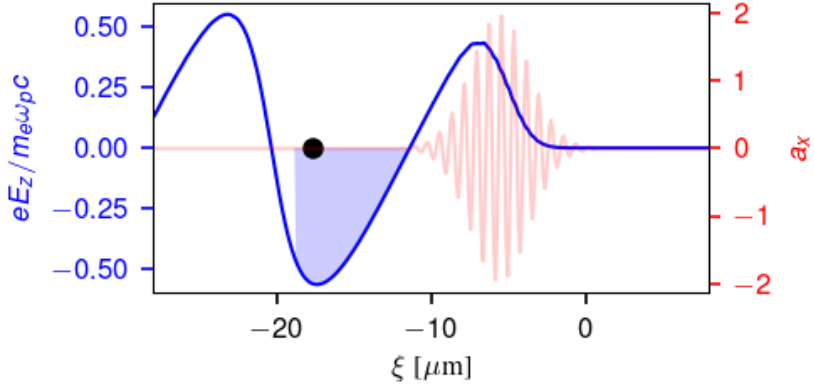
$$L_\phi \neq \frac{\pi c}{2\omega_p}\frac{1}{\beta_g^{-1}-1}$$

Dephasing can be removed in a plasma density gradient



$n_e = 2 \times 10^{18}\ \mathrm{cm}^{-3}$

Increasing plasma density reduces wavelength of plasma wave and re-phases electrons



$n_e = 4 \times 10^{18}\ \mathrm{cm}^{-3}$

Dephasing can be prevented in a plasma density gradient

For $n_e \ll n_c$, the requirement for preventing dephasing [1-4].

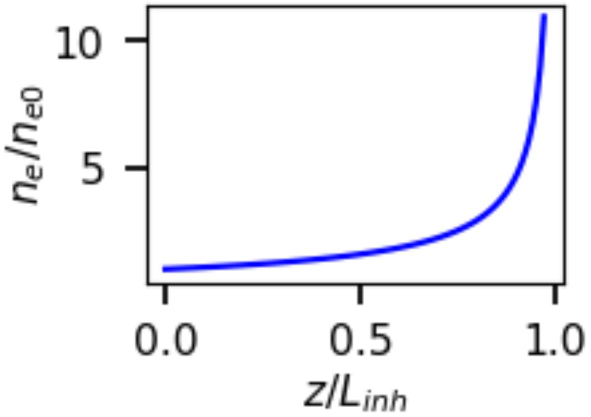
$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\phi}{\omega_p(z)}\right) = \frac{1}{c} - \frac{1}{v_g}$$

ϕ = electron phase in LWFA

For $a_0 \ll 1$, Pukhov [3] calculated the ideal density profile as,

$$n_e(z) = \frac{n_{e0}}{(1 - z/L_{\mathrm{inh}})^{2/3}}$$

$$L_{\mathrm{inh}} = \frac{c}{\omega}\left(\frac{n_c}{n_{e0}}\right)^{3/2}\frac{2\phi}{3}$$

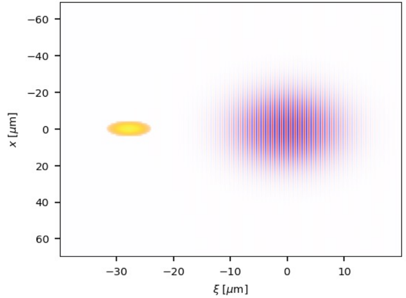


For highly non-linear LWFA, the above theory is not valid and the pulse evolution makes analytical treatment very difficult. Some experiments and simulations have shown approximate tailoring can still help [5-8].

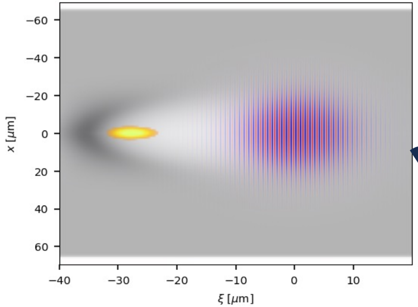
[1] T. Katsouleas, *Physical Review A* **33**, 2056–2064 (1986).
[2] P. Sprangle *et al.* Phys. Rev. E 63, 056405 (2001).
[3] A. Pukhov *Physical Review E* **77**, 025401 (2008).
[4] W. Rittershofer *et al. Phys. Plasmas* 17 (6) 063104 (2010)
[5] Z. Zhang *et al New J. Phys.* **17** 103011 (2015)
[6] E. Guillaume, *et al.* Phys. Rev. Lett. 115, 155002 (2015)
[7] A. Döpp *et al.* Phys. Plasmas 23 (5): 056702 (2016)
[8] C. Aniculaesei *et al.* Sci Rep 9, 11249 (2019).

Determining the ideal density profile “on-the-fly”

Initialise FBPIC simulation



Run for some initial time



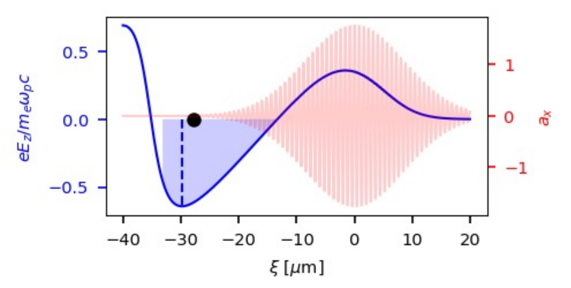
On-the-fly density tailoring

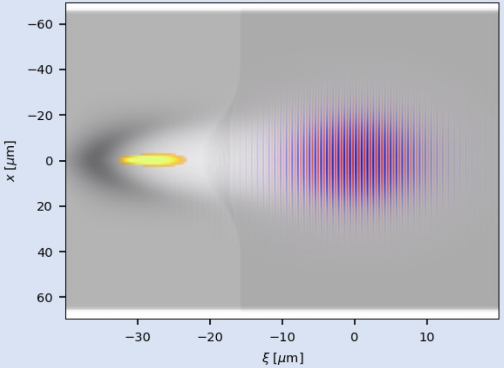
Get position of target particle

Calculate ideal particle position

Estimate optimal plasma density

Apply density correction to upcoming plasma

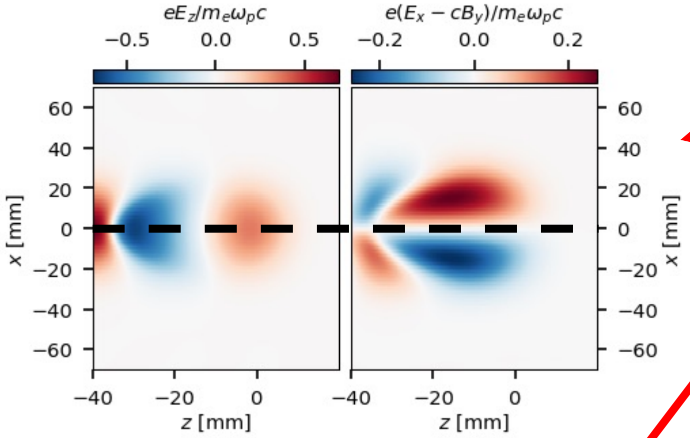




Check criteria for continuing simulation

Advance simulation to next measurement time

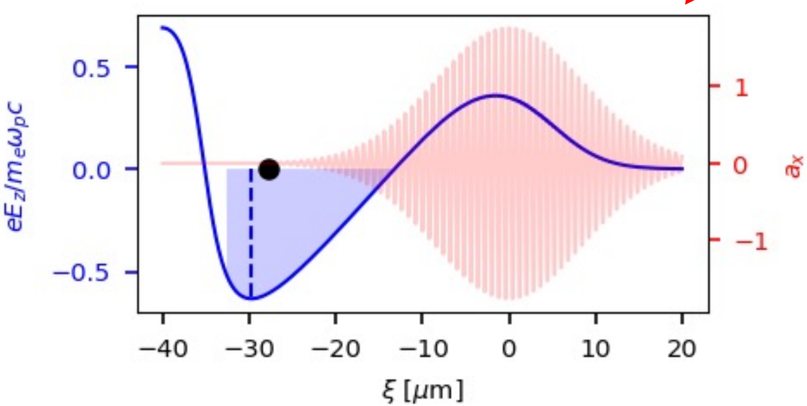
Calculating the next density step

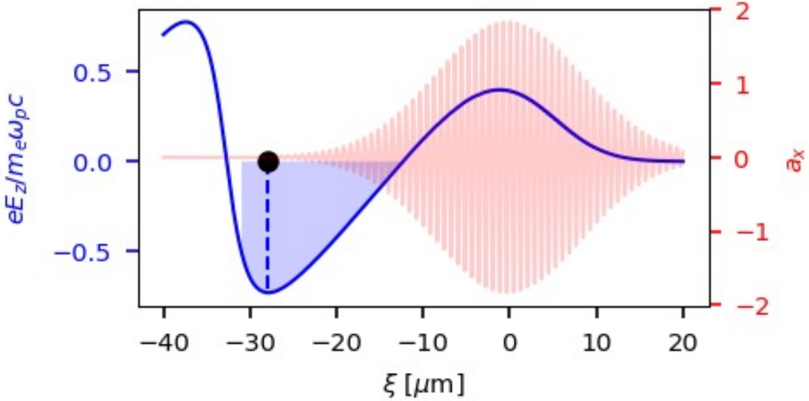


1. Measure accelerating and focusing forces
2. Determine optimum electron position
3. Compare to actual electron position and estimate required plasma density

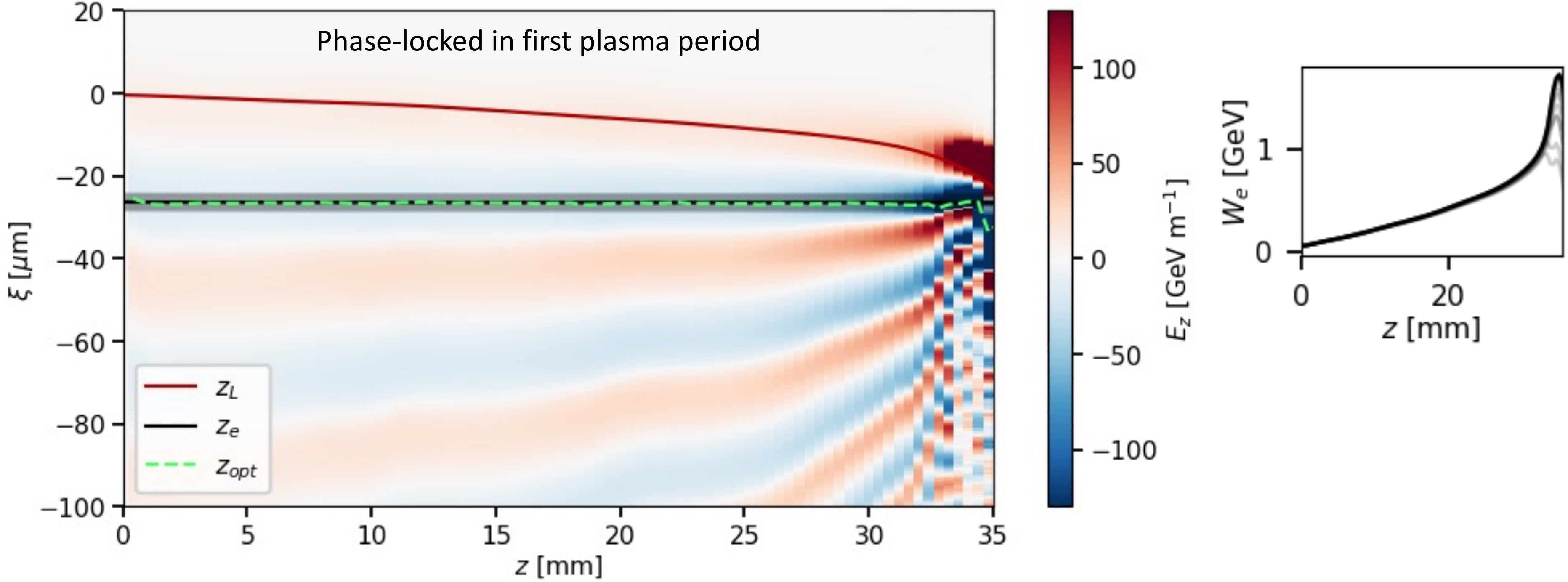
$$\lambda_{p,n+1} = \lambda_{p,n} + \frac{4}{3}\Delta z_n$$

4. Use PID controller to update next plasma density



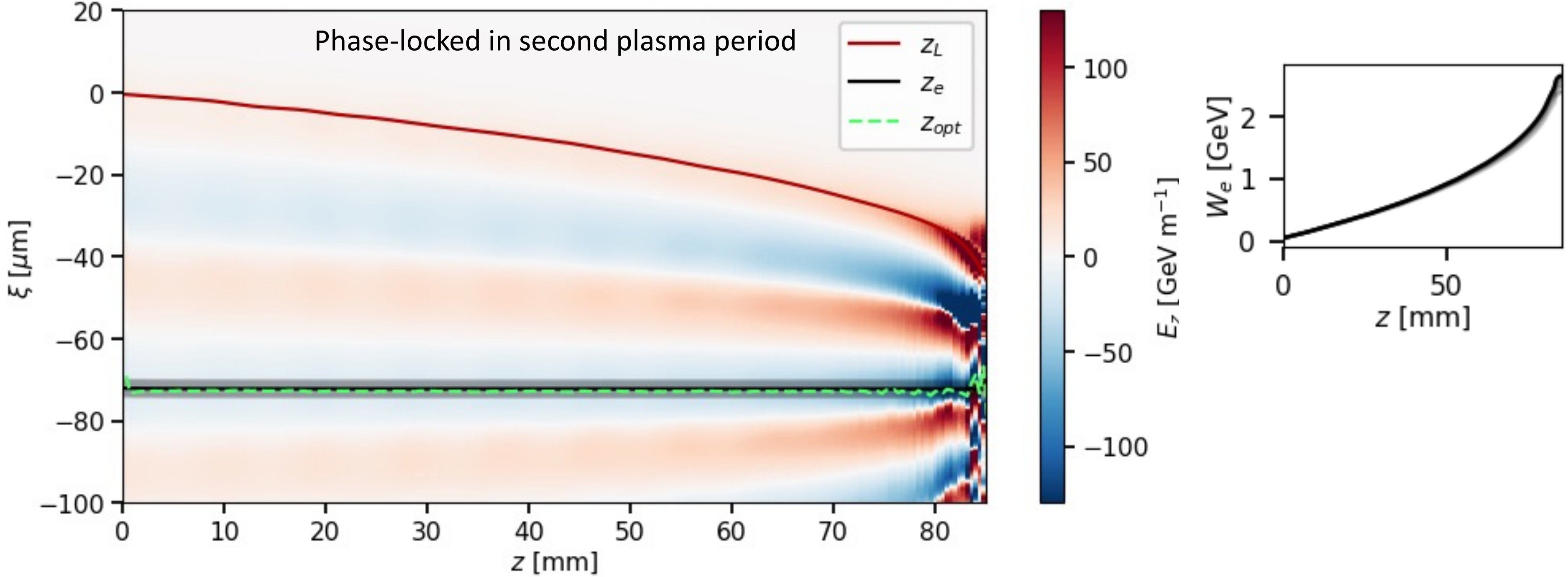


Case A: Quasi linear LWFA with external injection


Phase-locked in first plasma period

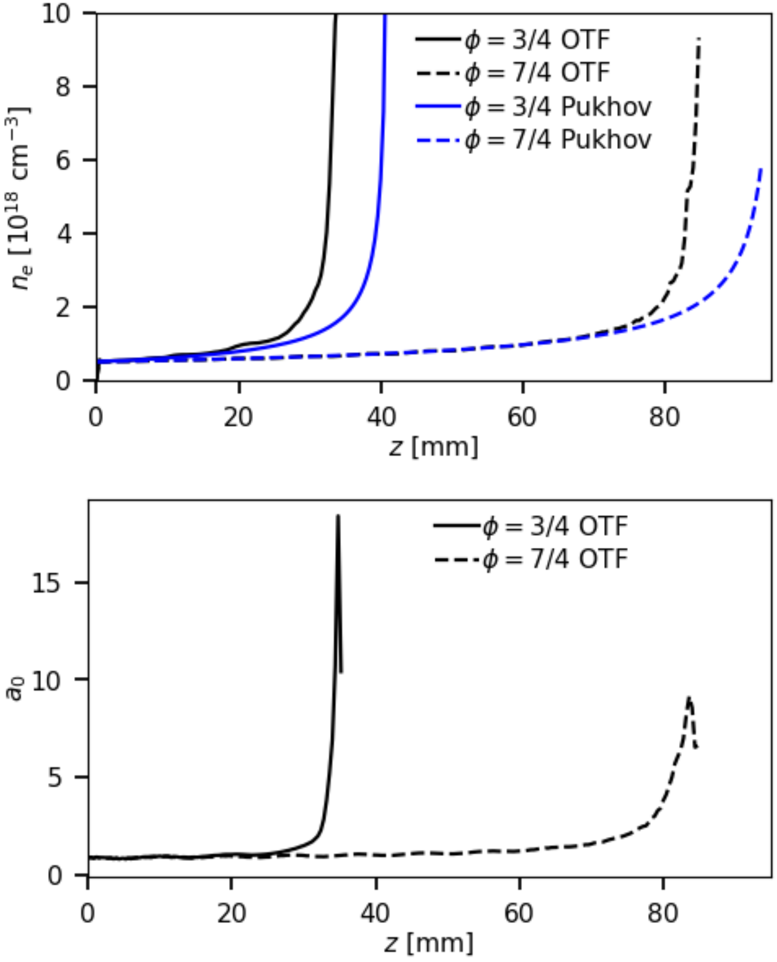
- a_0 = 0.95, guiding channel
- Initial on-axis plasma density n_e = 5 x 10^{17} cm^{-3}
- Initial laser energy = 1 J

Case A: Quasi linear LWFA with external injection

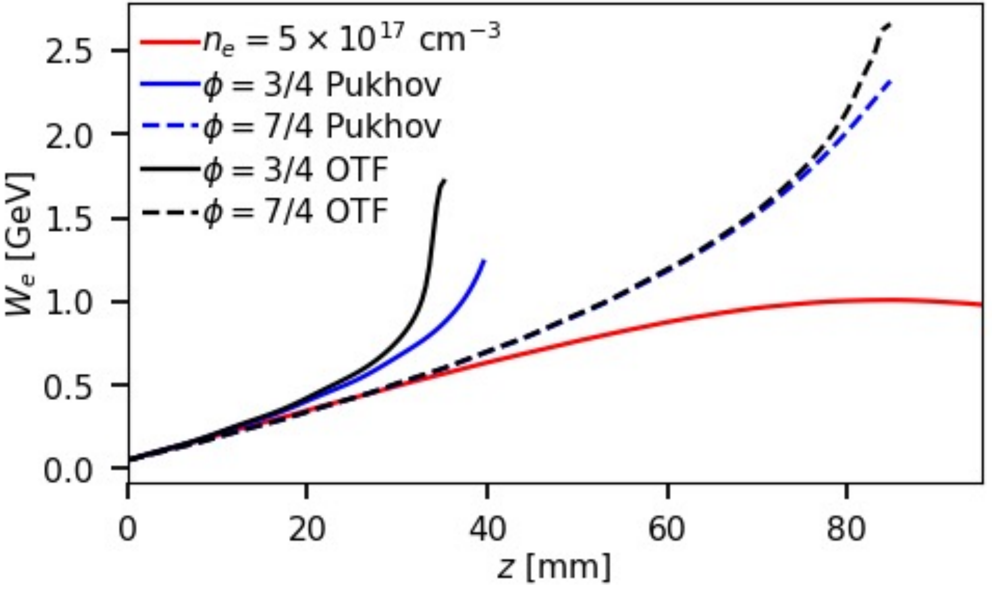


- a_0 = 0.95, guiding channel
- Initial on-axis plasma density n_e = 5 x 10^{17} cm^{-3}
- Initial laser energy = 1 J

Case A: Comparison of On-The-Fly simulation results to analytical solution

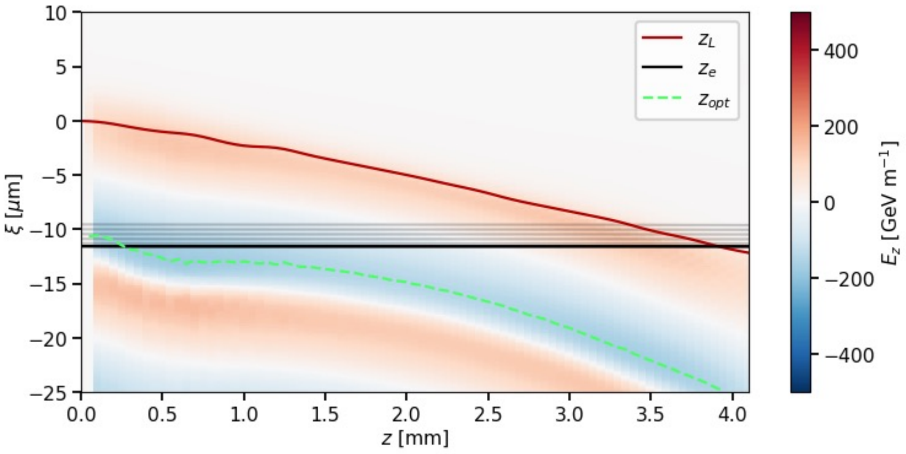


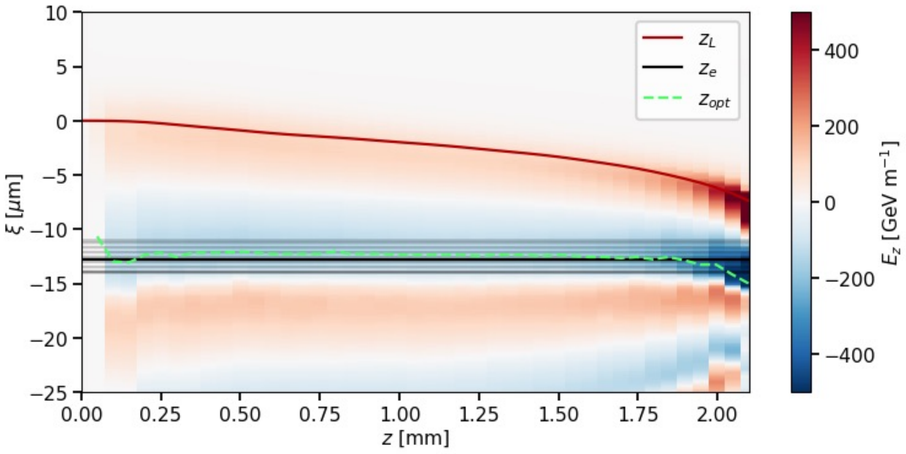
Ideal density matches theory until a_0 increases



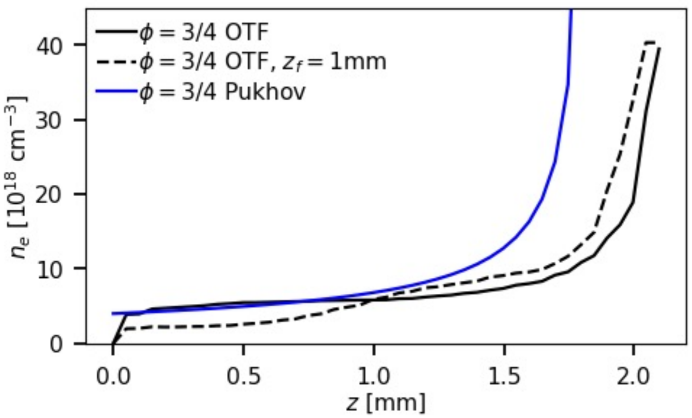
Large increase in energy gain – especially for acceleration in 2nd period

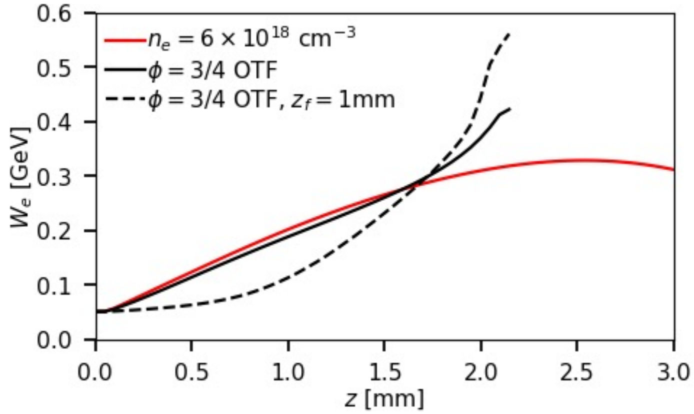
Case B: Non-linear LWFA with external injection



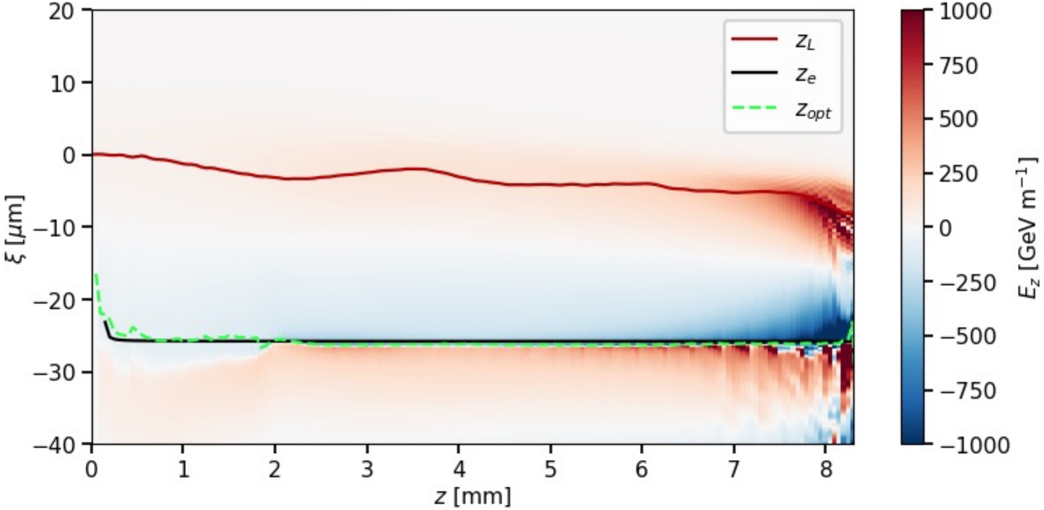


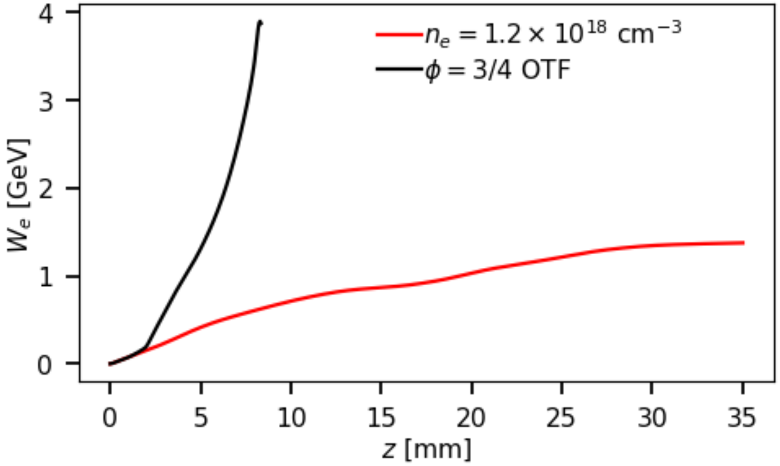
- a_0 =2 self-guiding only
- Plasma density
  n_e = 6 x 10^{18} cm^{-3}
- Initial laser energy
  = 200 mJ

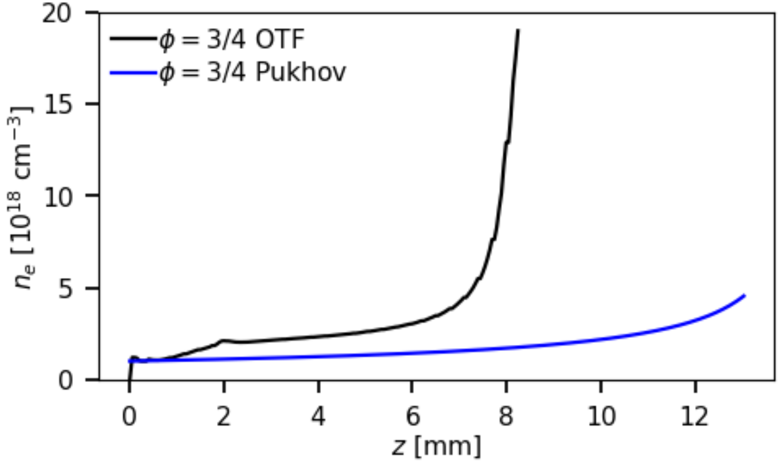




Case C: Highly Non-linear LWFA with ionisation injection







- a_0 = 1.8, self-guided
- Initial laser energy = 3.5 J
- Wakefield increases in non-linearity as laser focuses and compresses
- Produces 4 GeV, compared to 1.5 GeV from best constant density case

Summary and Outlook

- Simulations with on-the-fly density tuning can prevent dephasing even in highly non-linear LWFA
- Optimal tuning results in much higher electron energies
- This could allow LWFA to be purely depletion limited – implying high efficiency
- Further work required to look at beam loaded cases
- Could potentially find tuning for other beam quantities
  - E.g. emittance preservation
  - Optimise beam loading
- This approach can help optimise experimental design – practical realisation of density profiles is an important challenge