Accurate electron beam phase-space theory for ionization injection schemes

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Aims of the work

- Deep understanding of the ionization + post-extraction dynamics in the laser field

- Analytical (and accurate) description of the 3D phase-space, **also in the deep-saturation regime**

- Enabling the **accurate modeling** of the extraction/post-extraction in **fast envelope codes**

- Prediction of the synthetic phase space moments of the **whole electron bunch**

OUTLOOK

1. **Basics of tunnel ionization and post-ionization electron dynamics**
2. The starting point: theory of thermal emittance by C. Schroeder

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4. Accurate evaluation of rms values for particle extracted in a **single cycle**
5. **Saturation effects** in a single and double ionization process
6. **Whole beam rms parameters** in not saturated and saturated regimes

Basics of tunnel ionization and post-ionization electron dynamics

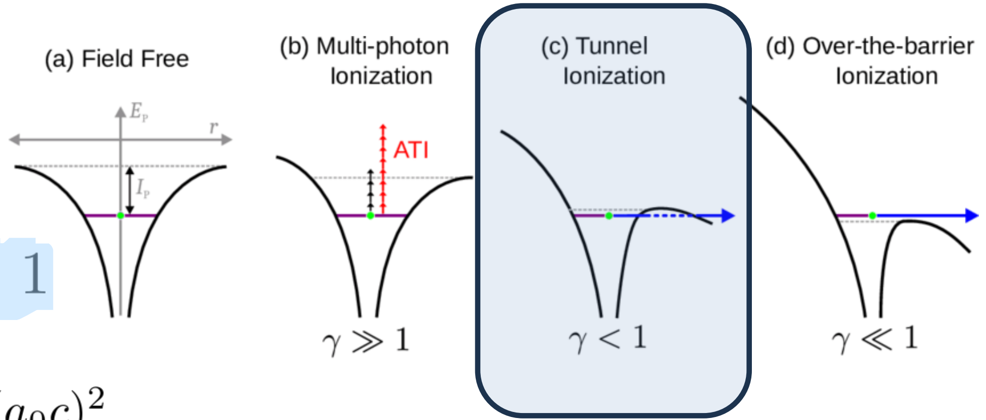
nuclear physics

Theory @ LDED

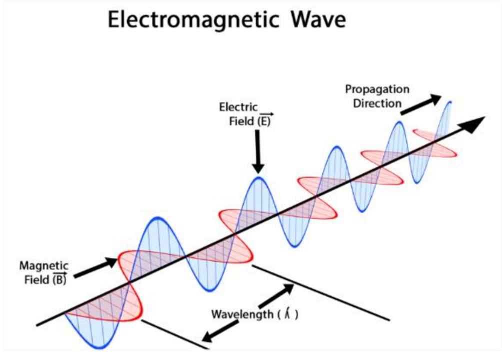
We will focus **on tunnel ionization,** which is the relevant regime for both standard and advanced ionization injection schemes.
Electrons are extracted with negligible initial momentum

$$\gamma_K = \sqrt{\frac{2U_I}{U_p}} \simeq \frac{t_{tunnel}}{t_{lasercycle}} < 1$$

$$U_p = mc^2(\sqrt{1 + a_0^2/2} - 1) \simeq \frac{1}{4}m(a_0 c)^2$$



After electrons leave the parent ion, they quiver in the laser field. **We neglect here the impact of ponderomotive and wakefield forces up to the time where electrons leave the laser pulse**.



$$\vec{u} \equiv \vec{p}/m_e c$$

Polarization along x

$$\dot{u}_x - \dot{a} \simeq 0 \rightarrow u_x - a \simeq constant$$
$$\dot{u}_y \simeq 0 \rightarrow u_y \simeq constant$$
$$d_t(\gamma + u_z) = 0 \rightarrow \gamma + u_z \equiv h_0 = constant$$

@extraction time

$$u_x \simeq a - a_e$$
$$u_y \simeq 0$$
$$\gamma \simeq \left[1 + \frac{1}{2}(a - a_e)^2\right]$$
$$u_z = \frac{1}{2}(a - a_e)^2$$

After pulse passage

$$u_x \simeq -a_e$$
$$u_y \simeq 0$$
$$\gamma \simeq \left[1 + \frac{1}{2}a_e^2\right]$$
$$u_z = \frac{1}{2}a_e^2$$

Basics of tunnel ionization and post-ionization electron dynamics

The instantaneous ionization probability in the adiabatic limit can be approximated as

$$W = C\left(2(2E_i)^{\frac{3}{2}}/E_{laser}\right)^{2n^*-|m|-3/2} e^{-(2(2E_i)^{\frac{3}{2}}/3E_{laser})}$$

Ammosov, M.V.; **D**elone, N.B.; **K**rainov, V.P.
SPIE, 1986, 294 Vol. 0664, pp. 138 – 1

where atomic units are used and $n^* = Z\sqrt{U_H/U_I}$

We can move to the usual LP units by using the notation
[P.Tomassini et al., PoP **24**, 103120 (2017)]

$$W = C \times \rho^{\mu} e^{-\frac{1}{\rho}}$$

$$\rho = \rho_0 |\cos(\xi)| \qquad \rho_0 \equiv \frac{3E_{max}}{2E_a}\left(\frac{U_H}{U_I}\right)^{3/2} = \frac{a_{max}}{a_c}$$

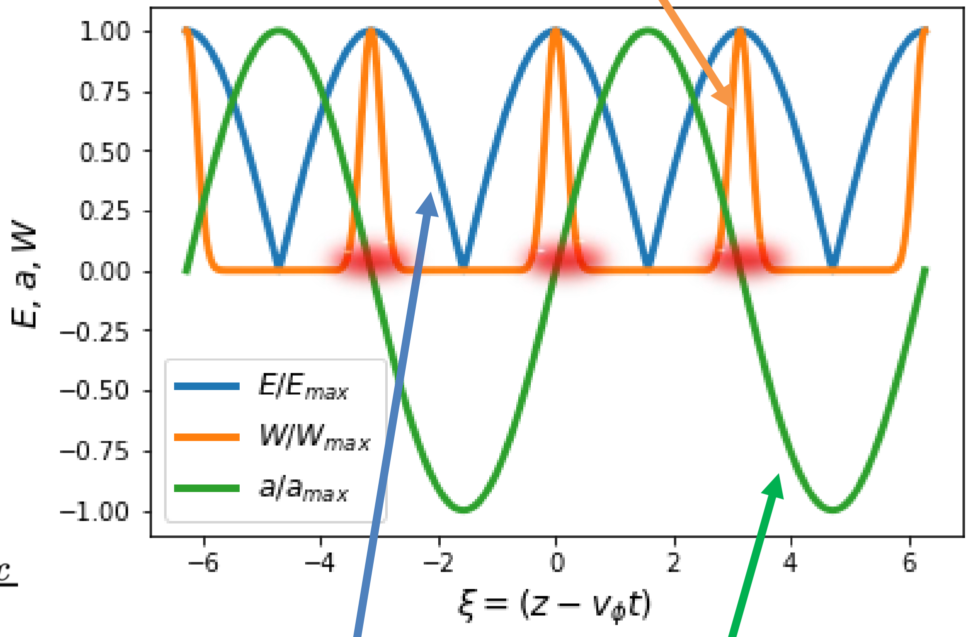
$$\xi = k_0(z - v_{\phi}t)$$

$$\mu = -2n^* + |m| + 1\,,$$

Critical amplitude

$$a_c \simeq 0.107\lambda_0 \left(\frac{U_I}{U_H}\right)^{3/2}$$

Ionization probability



Laser normalized electric field E

Laser normalized amplitude a

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The starting point: theory of thermal emittance by C. Schroeder

- **Opened the path for the comprehension and of the description** of the *rms* values of tranverse residual momentum, longitudinal and transverse size of the electron beam, with the discussion of the effects from the transverse ponderomotive force.

- **The theory of the rms residual momentum had been successfully implemented in laser-envelope** codes (e.g. **QFLUID** [P Tomassini and A R Rossi, PPCF **58** 034001 (2015)] and **SMILEI** [F. Massimo *et al.*, PRE 102, 3 03320 (2020)])

- In the following we will use their notation and the ones in [P.Tomassini et al., PoP **24**, 103120 (2017)]

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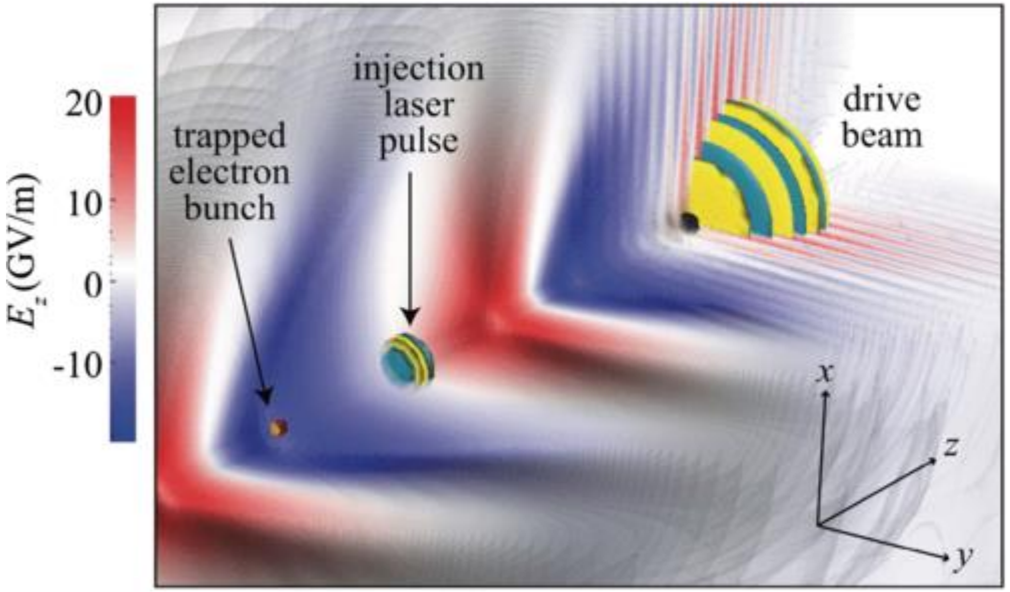
Thermal emittance from ionization-induced trapping in plasma accelerators

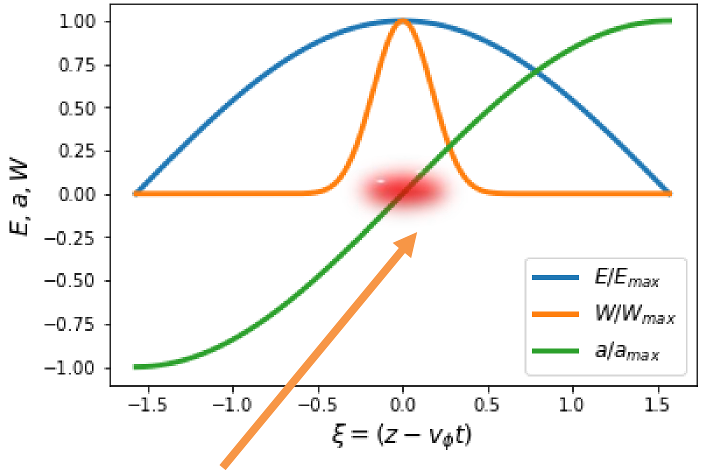
C. B. Schroeder, J.-L. Vay, E. Esarey, S. S. Bulanov, C. Benedetti, L.-L. Yu, M. Chen, C. G. R. Geddes, and W. P. Leemans

Phys. Rev. ST Accel. Beams **17**, 101301 – Published 3 October 2014



The starting point: theory of thermal emittance by C. Schroeder

Theory @ LDED



Electrons extracted in the E peak

They evaluated the rms extraction phase as $\Delta = \sqrt{\rho_0}$

$$\sigma(\xi) \equiv \sigma_\psi = \Delta[1 - (2n^* - |m| - 1)\Delta^2]^{-1/2},$$

$$\mu = -2n^* + |m| + 1\,,$$

C. B. SCHROEDER *et al.*

from which they were able to **evaluate the *rms* value of the residual transverse momentum** for electrons emitted in a single (half) cycle or for the entire beam

$$\langle u_x \rangle = 0 \quad \sigma(u_x) = \sigma(a_0 \sin \xi_e) \simeq a_0 \Delta[1 - \Delta^2(1+\mu)]$$

(*single peak case*)

UNSATURATED regime

SATURATED regime

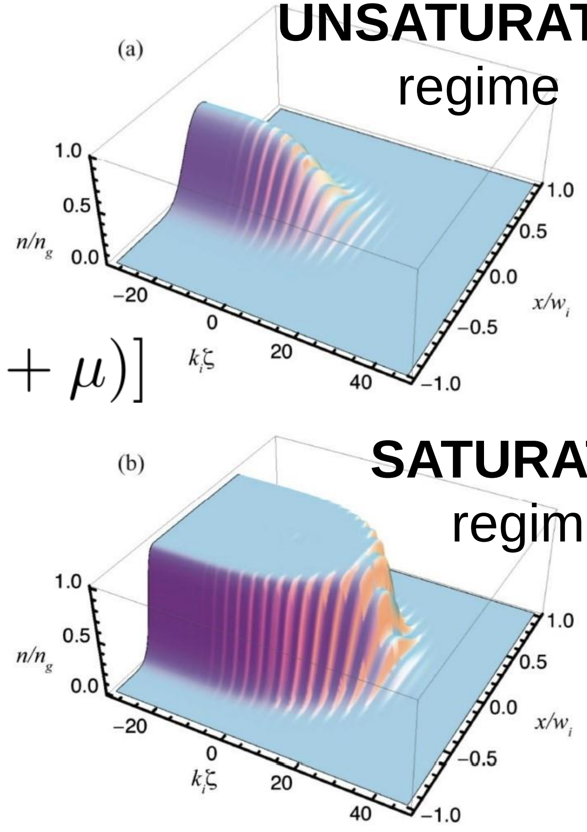


FIG. 5. Fractional ionization n/n_g for (a) $a_i = 0.14$ and (b) $a_i = 0.2$. The laser field has a wavelength of 0.4 μm and a Gaussian longitudinal envelope with length $k_i L_i = 32$.

In the case of saturation, evaluation of the asymptotic *rms* momentum $\sigma(u_x) \approx a_{sat}\Delta_{sat}$ where $a_{sat} < a_0$ is the laser amplitude at the saturation point and

$$\sigma(r) \approx w_0 \sqrt{\frac{2\ln(a_0/a_{sat})}{3}}$$

is the rms radius of the particles for a laser of waist w_0 .

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Accurate evaluation of rms values for particle extracted in a single cycle

eli nuclear physics

Theory @ LDED

High Power Laser Science and Engineering, (2022), Vol. 10, e15, 16 pages.
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HIGH POWER LASER SCIENCE AND ENGINEERING

RESEARCH ARTICLE

Accurate electron beam phase-space theory for ionization-injection schemes driven by laser pulses

Paolo Tomassini[1,2], Francesco Massimo[3], Luca Labate[1,4], and Leonida A. Gizzi[1,4]

$$\Delta^2 = \rho_0$$

The **correct** expansion of the ionization probability around their peaks up to $\mathcal{O}(\Delta^2)$ is

$$\begin{aligned} W(\xi) &= W_0 \cdot (\cos\xi)^\mu \exp\left[\frac{1}{\rho_0}\left(\frac{1}{\cos\xi} - 1\right)\right] \\ &\simeq W_0 \exp\left[-\frac{\xi^2}{2\rho_0}\right]\left(1 - \frac{\mu}{2}\xi^2 - \frac{5}{24\rho_0}\xi^4\right) \\ &\simeq W_0 \exp\left[-\frac{\xi^2}{2\sigma_\psi^2}\left(1 + \frac{5}{12}\xi^2\right)\right] \end{aligned}$$

$\mathcal{O}(\Delta^2)$

We keep all the $\mathcal{O}(\rho_0)$ orders, obtaining the following evaluation of the **cycle averaged ionization probability**

$$< W > \simeq \sqrt{\frac{2}{\pi}}\Delta_0 W_0 \left[1 - \frac{1}{2}(\mu + 5/4)\Delta_0^2\right]$$

$$< W > \simeq \sqrt{\frac{2}{\pi}} C\, \rho_0^{\mu+1/2} e^{-1/\rho_0} \times \left[1 - \frac{1}{2}(\mu + 5/4)\rho_0\right]$$

Accurate evaluation of rms values for particle extracted in a single cycle

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Case of **unsaturated ionization** $\frac{dn_e}{dt} = W(t)\,(n_{0,i} - n_e(t)) \simeq n_{0,i}W(t)$

The statistical distribution of the extraction time is the same of the ionization probability

- **Standard deviation of ξ (remember)**

$$\sigma^2_{\xi,0} \equiv \langle \xi^2 \rangle = \rho_0\,(1 - (\mu + 5/2)\rho_0)$$
$$\sigma^2_{\Psi} \equiv \langle \xi^2 \rangle = \rho_0\,(1 - \mu\rho_0)$$

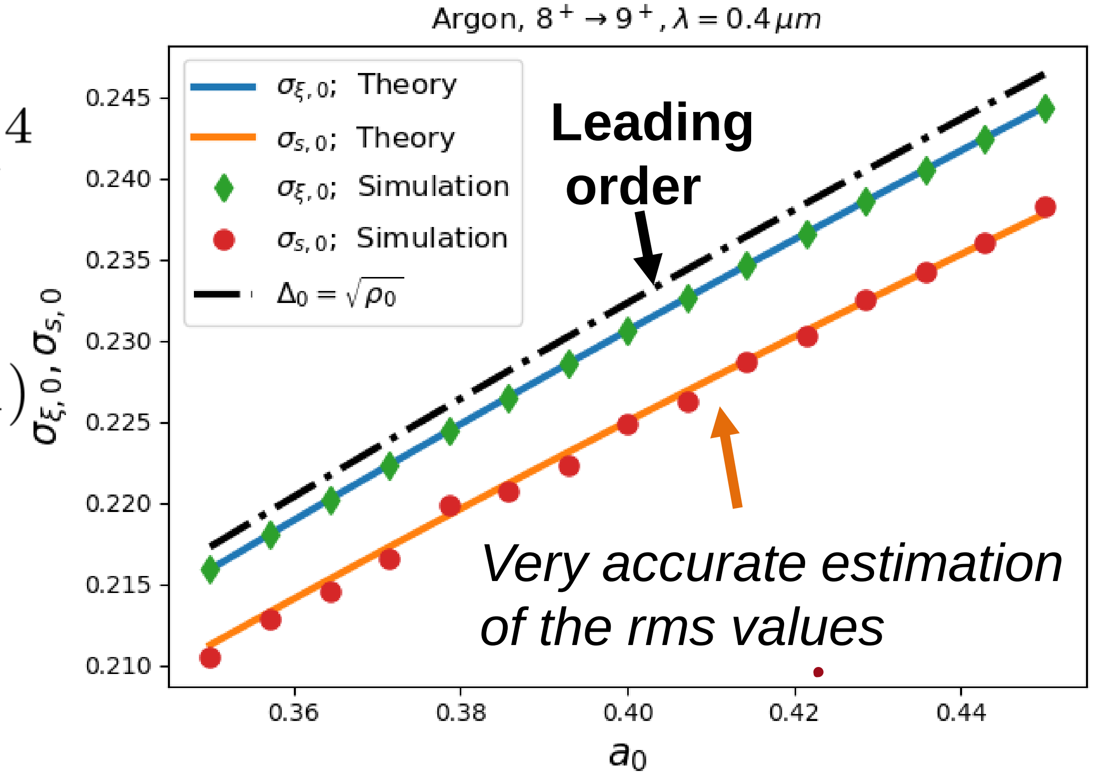
[from PRAB **17**(10) 101301 (2014)]

- **Variance of sin(ξ) with corrections up to $\rho_0^2 = \Delta^4$**

$$\sigma^2_{s,0} \equiv \langle \sin^2 \xi_e \rangle = \rho_0\left(1 + s_I \cdot \rho_0 + s_{II} \cdot \rho_0^2\right)$$
$$s_I = -(\mu + 5/2 + 1) \quad s_{II} = \frac{1}{8}(8\mu^2 + 68\mu + 131)$$

- **Variance of the residual transverse momentum**

$$u_x \simeq -a_0 \sin(\xi_e)$$
$$\sigma^2(u_x) \simeq a_0^2\sigma^2(\sin(\xi_e)) = a_0^2\sigma^2_{s,0}$$



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Saturation effects in a single ionization process

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Rate equation

$$\begin{cases} \frac{dn_e}{dt} = W \cdot (n_{0,i} - n_e), \\ W = C\left(\rho_0 |\cos\xi|\right)^{\mu} \exp\left(-\frac{1}{\rho_0 |\cos\xi|}\right) \end{cases}$$

$$\frac{1}{n_{0,i}} \frac{dn_e}{d\xi} = -\frac{\partial}{\partial_\xi} e^{-\Gamma(\xi)},$$

The cumulative ionization function Γ can be evaluated as

$$\Gamma(\xi) \equiv \frac{1}{k_{0,x}} \int_{-\pi/2}^{\xi} dx W(x)$$

$$= \frac{k_{ADK}}{k_0} \rho_0^{\mu} \int_{-\pi/2}^{\xi} dx (\cos x)^{\mu} e^{-\frac{1}{\rho_0 \cos x}}$$

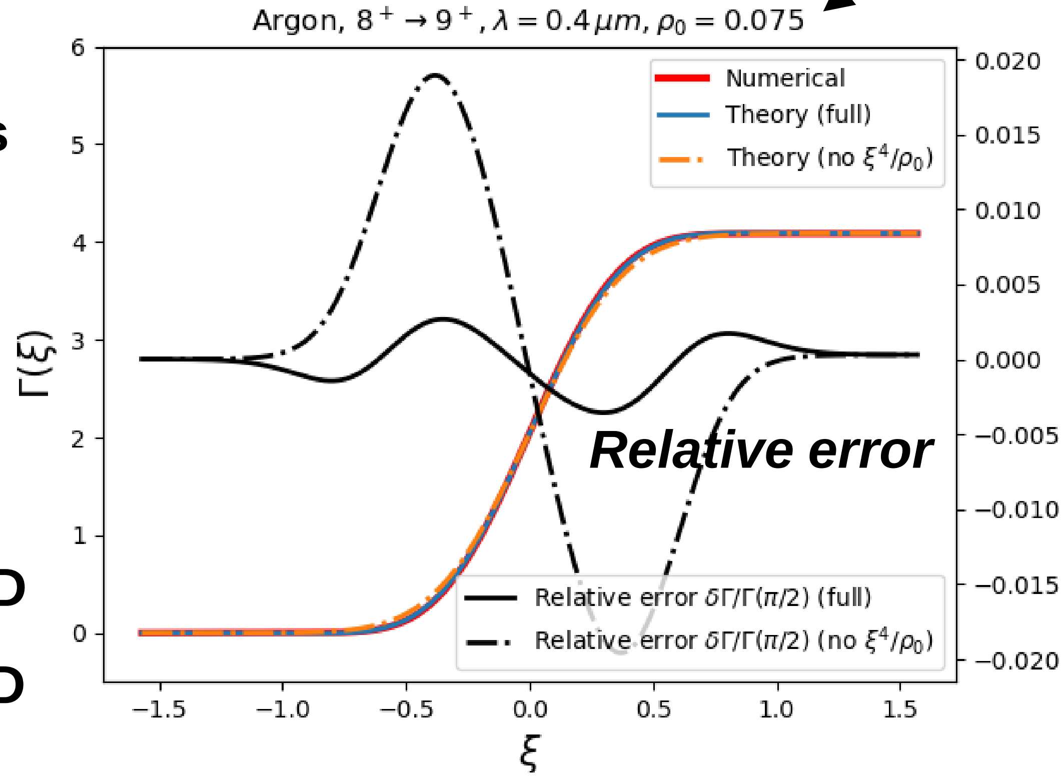
$$\simeq \nu_s(\rho_0) \mathcal{G}\left(\frac{\xi}{\sqrt{2\rho_0}}\right)$$

$\Gamma(\infty) \ll 1$ **NOT SATURATED**

$\Gamma(\infty) \geq 1$ **SATURATED**

(***see next slide***)

Deep saturation



Accurate evaluation of rms values for particle extracted in a single cycle

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The cumulative ionization function can be factorized as

$$\Gamma(\xi) \simeq \nu_s(\rho_0)\mathcal{G}\left(\frac{\xi}{\sqrt{2\rho_0}}\right)$$

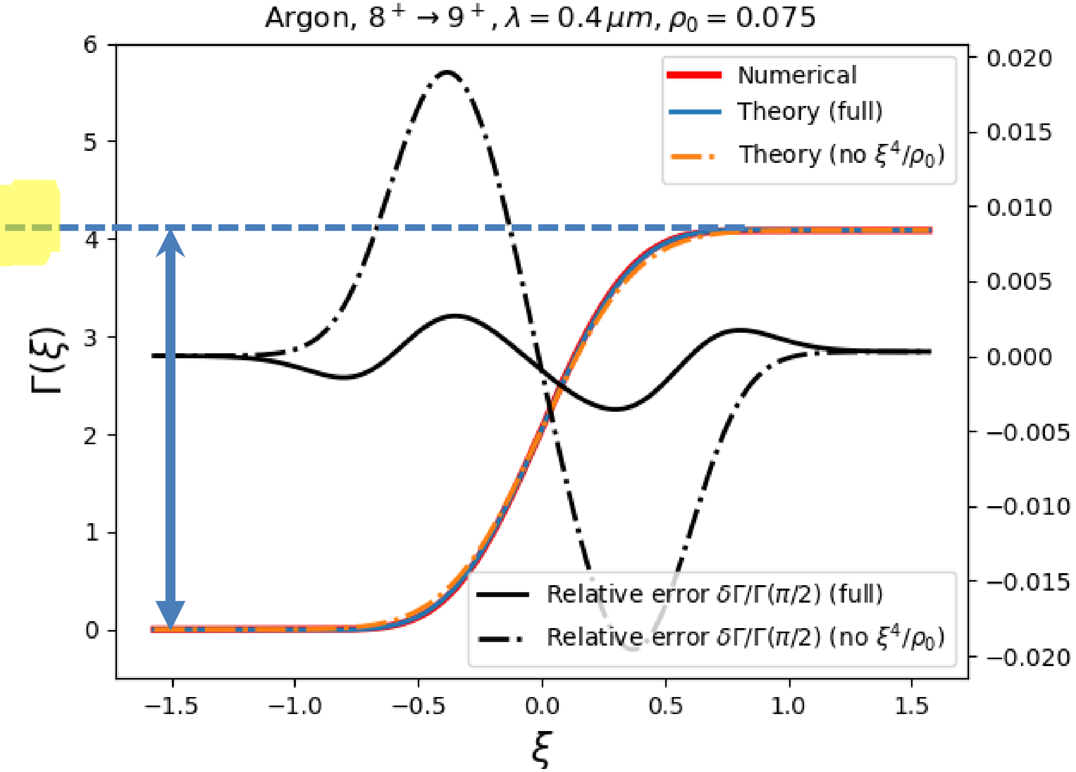
$\nu_s \simeq 4$

Saturation Amplitude

$$\nu_s \equiv \sqrt{2\pi}\frac{k_{ADK}}{k_{0,x}}\left[1 - \frac{(\mu+5/4)}{2}\rho_0\right]\rho_0^{\mu+1/2}e^{-\frac{1}{\rho_0}}$$

$$k_{ADK} = C(|m|)/c$$

Depends on the pulse amplitude through ρ_0



Universal Saturation Shape

$$\mathcal{G}(x) \equiv \frac{1}{2}(1 + E(x)) + \frac{\rho_0}{24\sqrt{\pi}}x(15 + 12\mu + 10x^2)e^{-x^2}$$

$$x \equiv \frac{\xi}{\sqrt{2\rho_0}}$$

Can be omitted for simplicity

Accurate evaluation of rms values for particle extracted in a single cycle

We finally got the ionisation rate for the model, valid also in the deep saturated regime:

$$\frac{1}{n_{0,i}}\frac{dn_e}{d\xi} = W_0 e^{-\frac{\xi^2}{2\rho_0}}\left(1-\frac{\mu}{2}\xi^2-\frac{5}{24\rho_0}\xi^4\right)\underbrace{e^{-\nu_s \mathcal{G}\left(\frac{\xi}{\sqrt{2\rho_0}}\right)}}_{\text{Asymmetric}}$$

Asymmetric

From that we get the extraction phase statistics FOR ANY PEAK

# of extracted electrons in the peak:

$$N_e = n_{0,i}\left(1-e^{-\nu_s}\right)$$

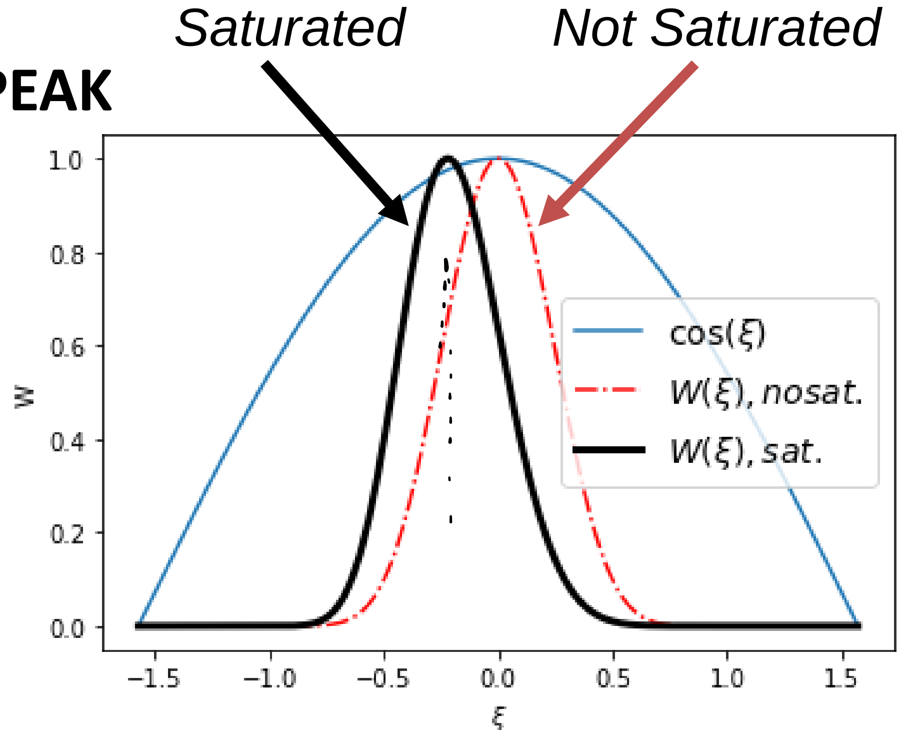
Asymmetric distribution:

$$\langle \xi_e \rangle \neq 0$$

$$\langle u_x \rangle \neq 0$$

$$\sigma(\xi_e)_{saturated} \neq \sigma(\xi_e)_{notsaturated} = \sigma_{\xi,0}$$

See next slides



Accurate evaluation of rms values for particle extracted in a single cycle

Theory @ LDED

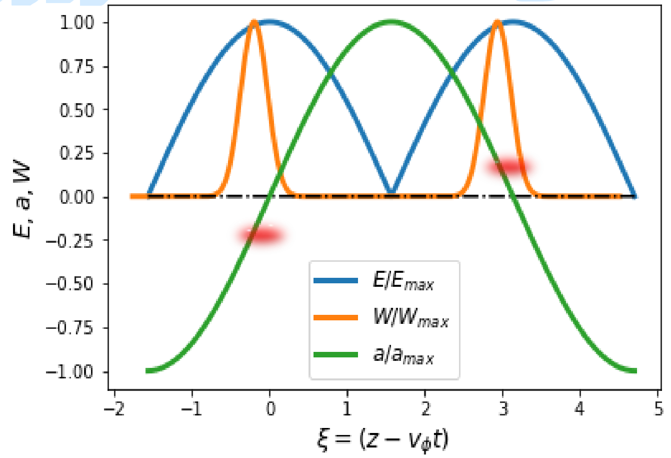
Given the number of available ions at that peak $n_{0,i}$ and the normalized laser amplitude a_0, do:

MODEL FOR ANY PEAK
particle extraction and residual momentum

1. Want to extract N_e electrons, with: $$N_e = n_{0,i}\left(1 - e^{-\nu_s}\right)$$
2. Generate the random variable x with : $$P(x) = \left[1 - \rho_0\left(\mu x^2 + \frac{5}{6}x^4\right)\right] e^{-x^2 - \nu_s \mathcal{G}(x)}$$
3. For any x evaluate the extraction phase: $$\xi_e = x \cdot \sqrt{2\rho_0}$$
4. **And finally get the residual transverse momentum** $$u_x = -a_0 \sin(\xi_e) \quad u_z = \frac{1}{2}u_x^2$$

5. **For a whole cycle consider two consecutive peaks.**

<u>**Note that the average momentum reverts for the two peaks**</u>



Accurate evaluation of rms values for particle extracted in a single cycle

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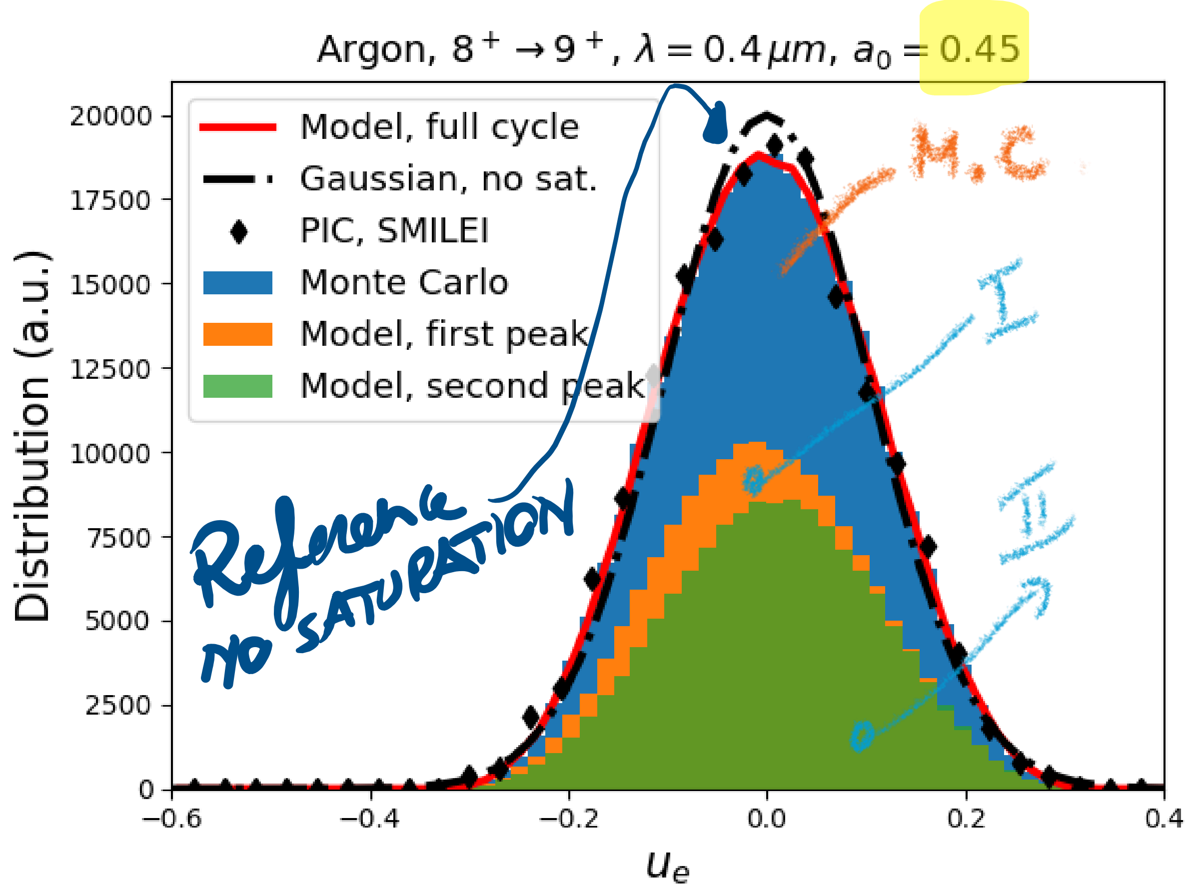
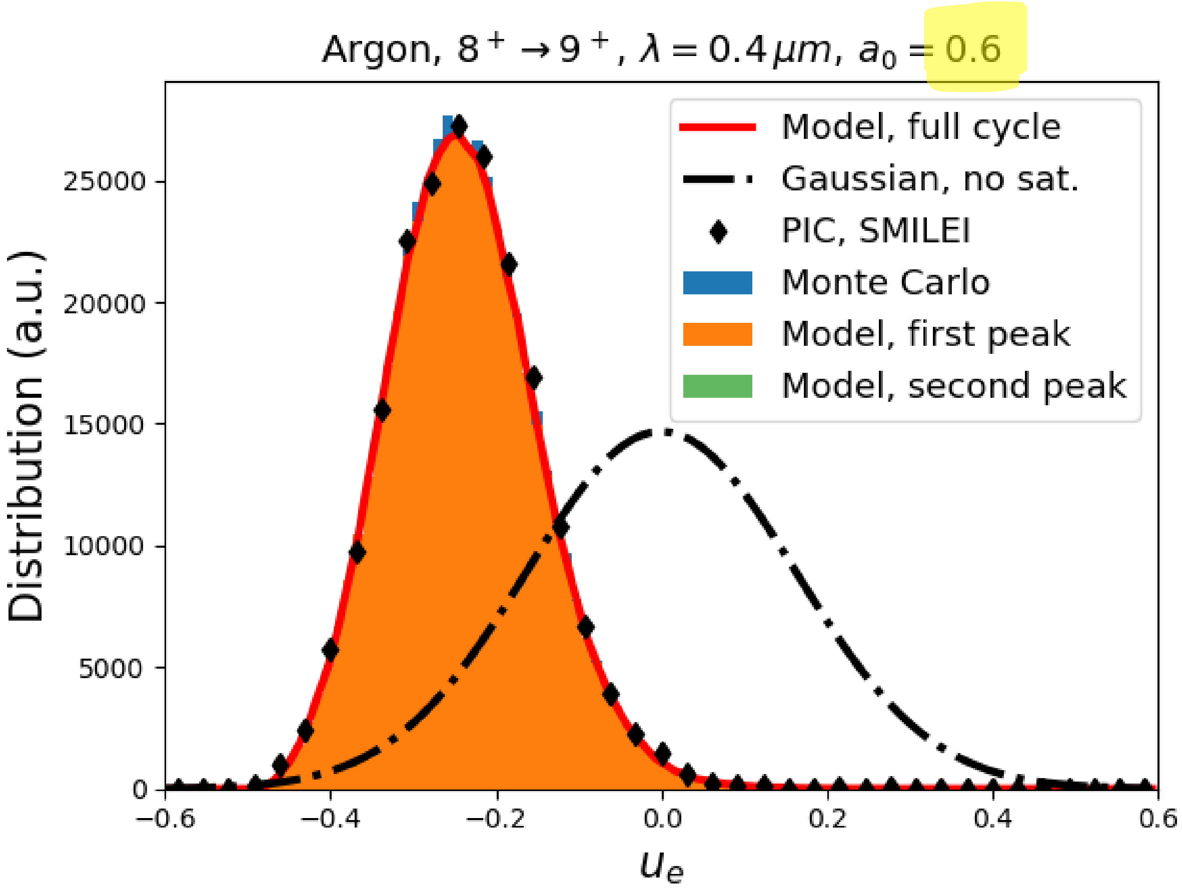
Results of the model, single cycle (two consecutive peaks)



Not Saturated



Saturated

Accurate evaluation of rms values for particle extracted in a single cycle

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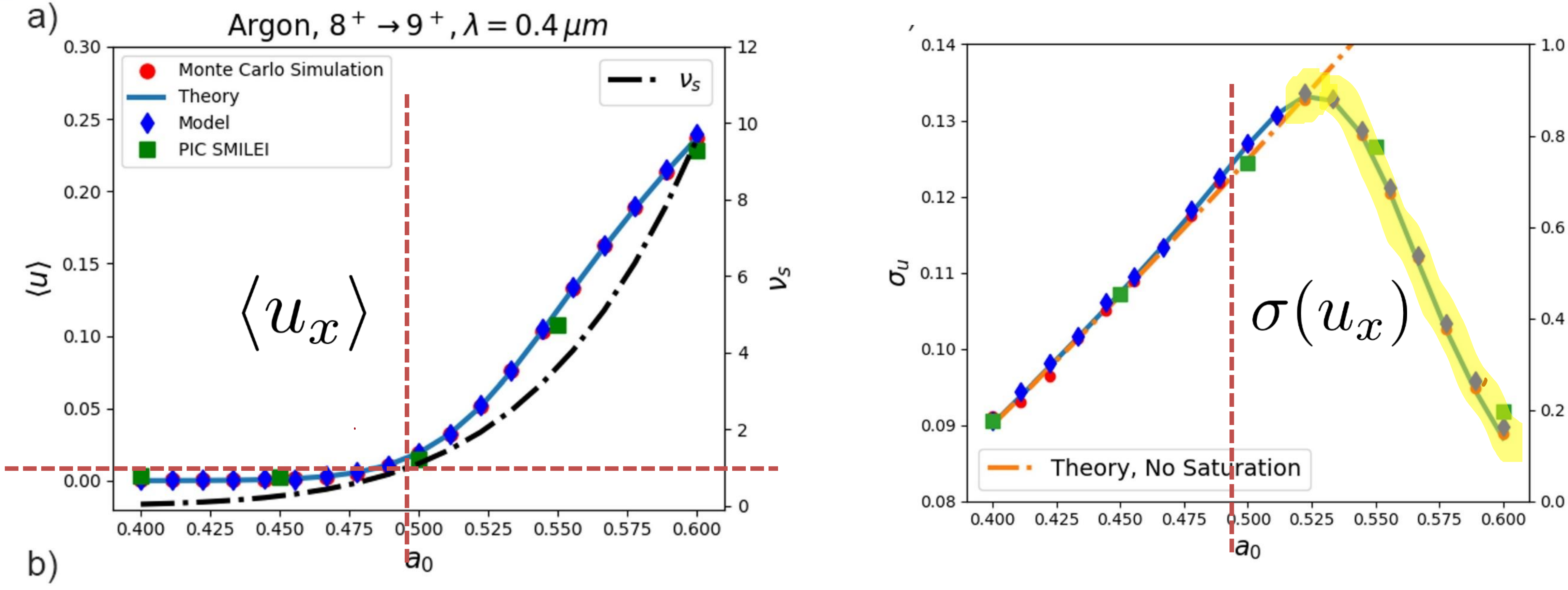
Results of the model, average and *rms* momentum, <u>single cycle</u>

We got analytical evaluation of $\langle u_x \rangle$ and $\sigma(u_x)$ (not shown here).

Full results in P.Tomassini et al., HPLSE 10, e15 (2022)



Accurate evaluation of rms values for particle extracted in a single cycle

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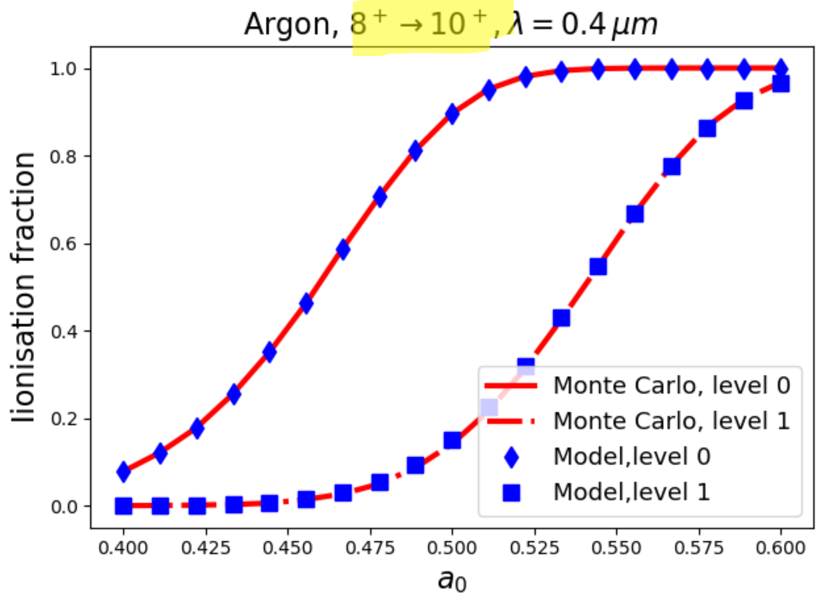
If the field strength is large enough to move into saturation within a single cycle, probably a <u>two-channels</u> (or more) <u>process is on</u>.

$$Kr^{8^+ \to 10^+} \quad Ar^{8^+ \to 10^+} \quad N^{5^+ \to 7^+}$$

- **The rate equation with a two-channels process reads**

(0): BASE process, *e.g.* $Ar^{8^+ \to 9^+}$
(1): SUBSEQUENT process, *e.g.* $Ar^{9^+ \to 10^+}$

$$\begin{cases} \frac{dn^{(0)}}{d\xi} = -n^{(0)}\nu_s^{(0)}\mathcal{G}^{(0)} \\ \frac{dn^{(1)}}{d\xi} = -n^{(1)}\nu_s^{(1)}\mathcal{G}^{(1)} + n^{(0)}\nu_s^{(0)}\mathcal{G}^{(0)} \end{cases}$$



- **and we solved as**

$$\begin{aligned} N_e^{(0)} &= n_i^{(0)}(1 - e^{-\nu_s^{(0)}}) \\ N_e^{(1)} &= n_i^{(1)}(1 - e^{-\nu_s^{(1)}}) + n_i^{(0)}\left(1 - e^{-\nu_s^{(0)}} - e^{-\nu_s^{(1)}}\mathcal{M}_{01}\right) \end{aligned}$$

Known function, not shown here

Accurate evaluation of rms values for particle extracted in a single cycle

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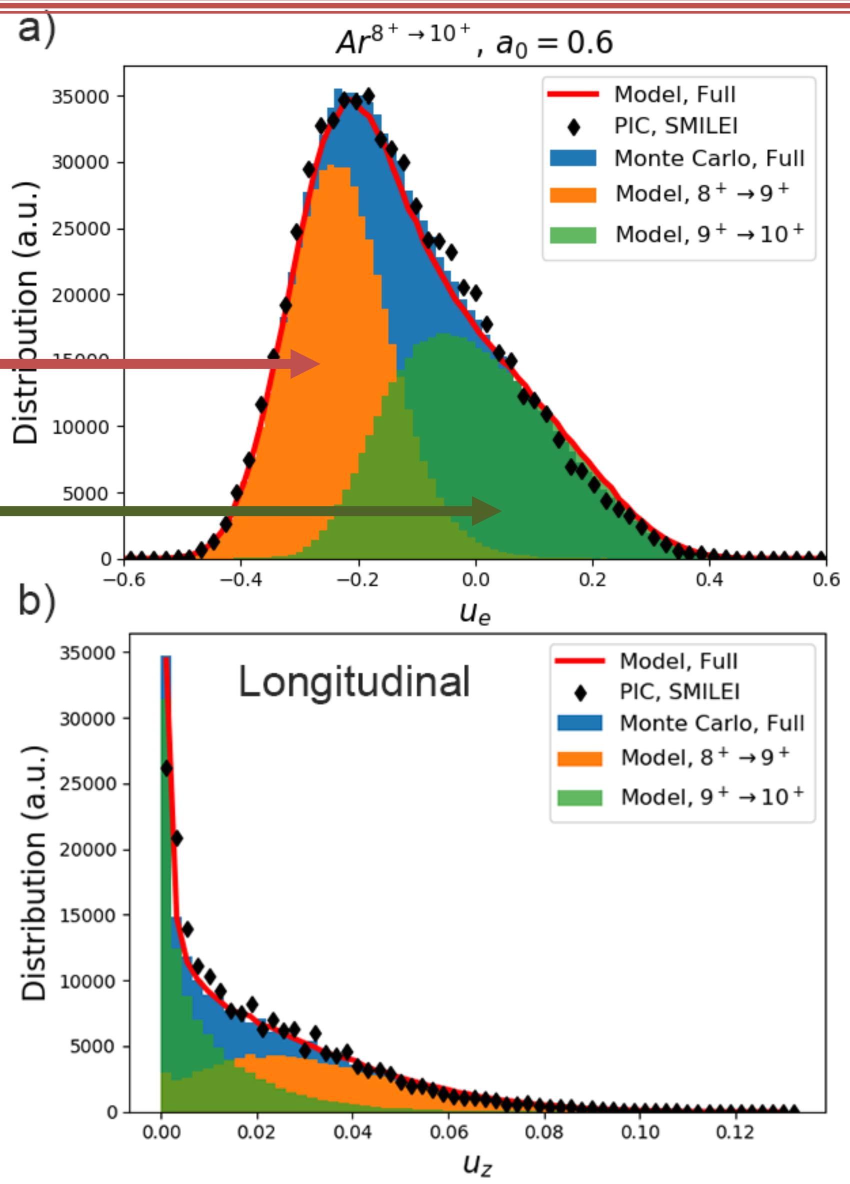
Theory outcome

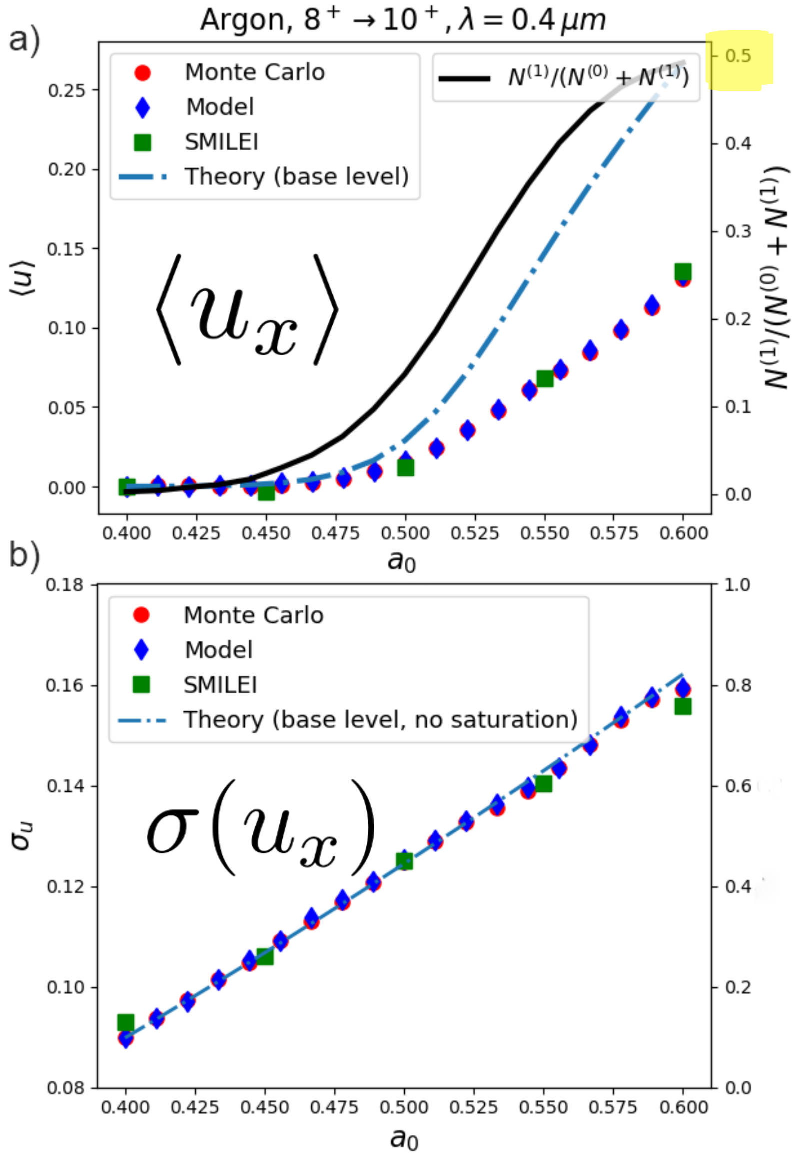
$Ar^{8+} \rightarrow 9^{+}$

$Ar^{9+} \rightarrow 10^{+}$

Longitudinal momentum

$$u_z \simeq \frac{1}{2} u_x^2$$

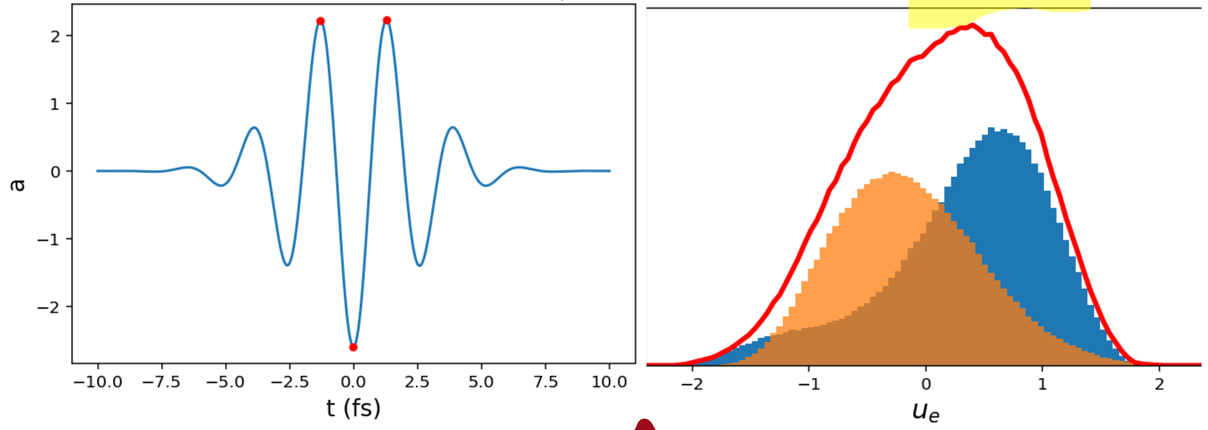




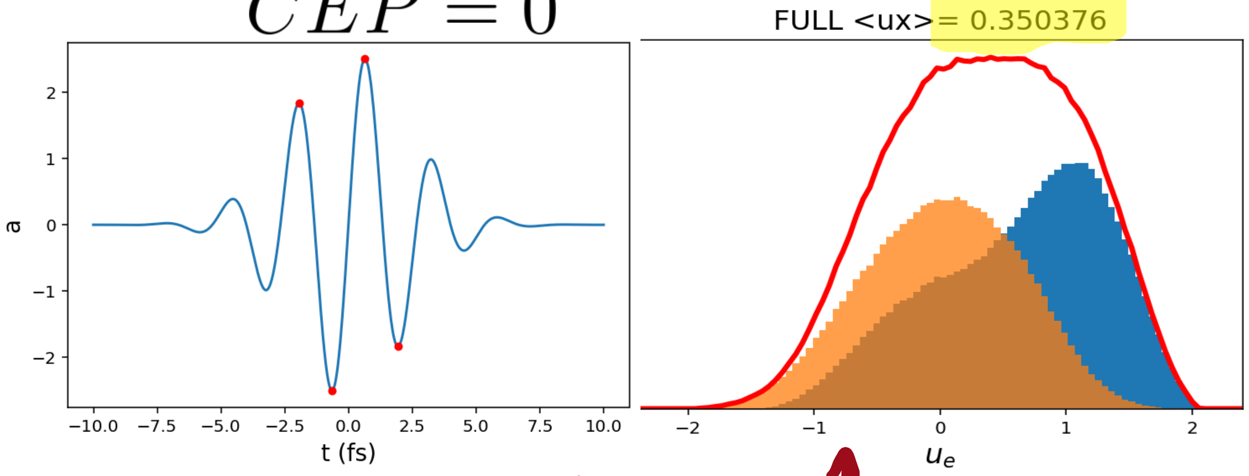
Ultrashort ionization pulse example

Theory @ LDED

$CEP = -\pi/2$



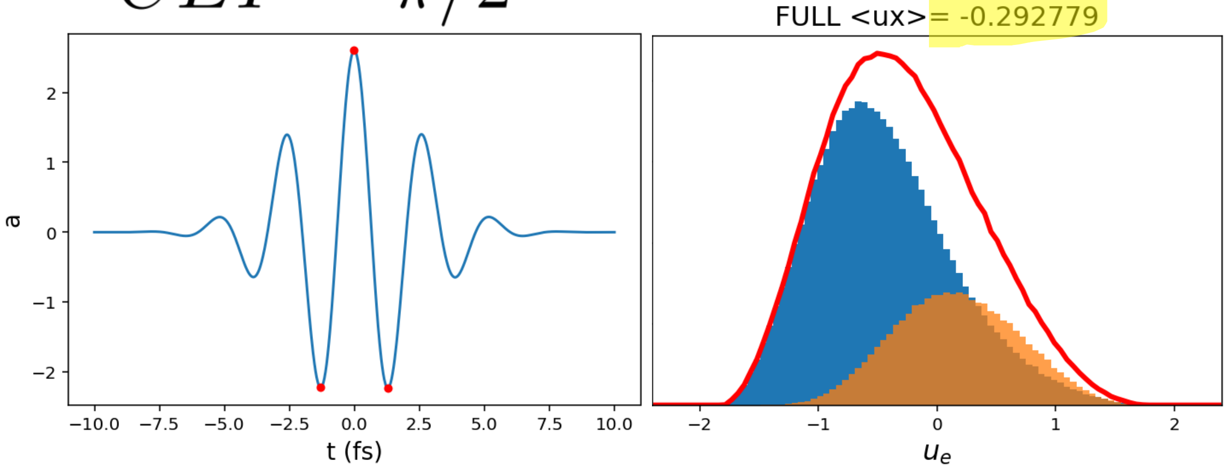
$CEP = 0$



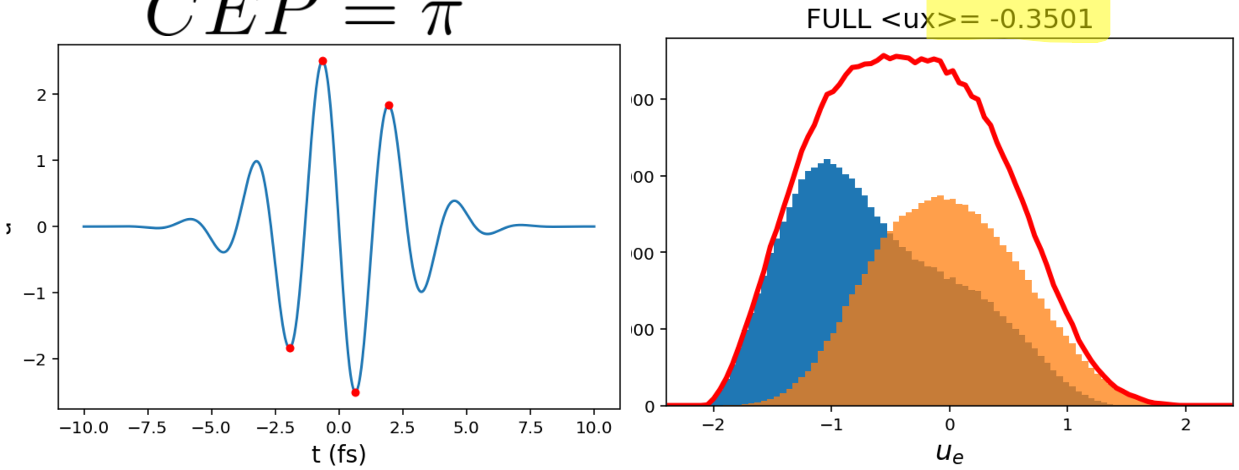
ASYMM

SYMM

$CEP = \pi/2$



$CEP = \pi$



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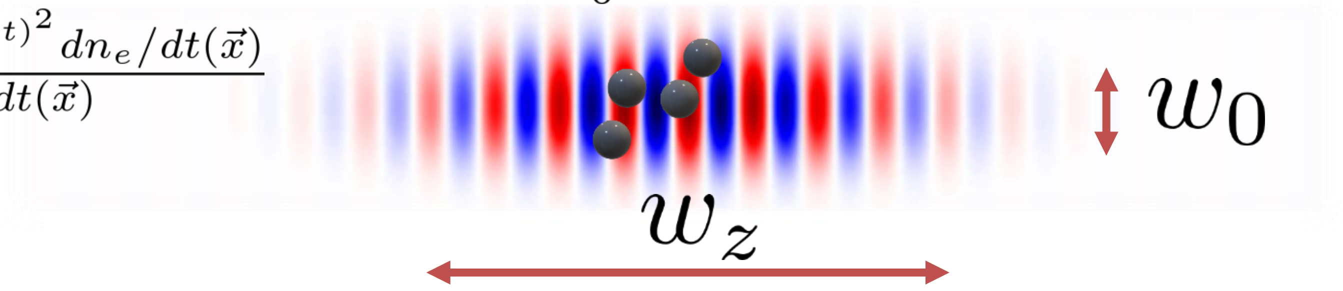
Whole beam rms parameters in not saturated regimes

Theory @ LDED

We can use the method of the *generating functionals*

$$E = E_0 e^{-\frac{r^2}{w_0^2} - \frac{(z - v_g t)^2}{w_z^2}}$$

$$\mathcal{G}(m,n) \equiv \langle e^{-mr^2 - n(z-ct)^2} \rangle = \frac{\int d^3x e^{-mr^2 - n(z-ct)^2} dn_e/dt(\vec{x})}{\int d^3x dn_e/dt(\vec{x})}$$



A (bi) Gaussian pulse shape is considered

Once we know $\mathcal{G}$

$$\langle r_e^2 \rangle = -\partial_m \mathcal{G}(m,0)_{m=0}$$

$$\langle z_e^2 \rangle = -\partial_n \mathcal{G}(0,n)_{n=0}$$

$$\langle \sigma_u^2 \rangle \equiv \frac{\int d^3x \sigma_{u_x}^2 \times dn_e/dt(\vec{x})}{\int d^3x dn_e/dt(\vec{x})} = a_0^2 \rho_0 \left[\mathcal{G}(3,3) + s_I \rho_0 \mathcal{G}(4,4) + s_{II} \rho_0^2 \mathcal{G}(5,5) \right]$$

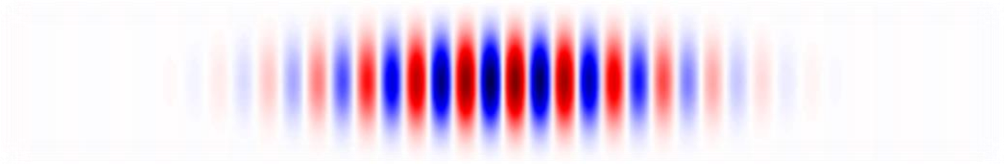
$$\sigma_{s,0}^2 \equiv \langle \sin^2 \xi \rangle = \rho_0 \left(1 + s_I \cdot \rho_0 + s_{II} \cdot \rho_0^2\right)$$

$$s_I = -(\mu + 5/2 + 1)$$

$$s_{II} = \frac{1}{8}(8\mu^2 + 68\mu + 131)$$

So once we know G we can get all we need

Whole beam rms parameters in not saturated regimes



To evaluate G ne need to perform the integrations:

$$I(k,\rho_0) \equiv \int_0^\infty dx^2 e^{\left[-(\mu+\frac{1}{2}+k)x-\frac{1}{\rho_0}(e^{x^2}-1)\right]} = e^{-(\mu+\frac{1}{2}+k)+\frac{1}{\rho_0}}\Gamma^{up}\left[-(\mu+\frac{1}{2}+k);\frac{1}{\rho_0}\right]$$

Upper incomplete Euler function

and to observe that, for each k,

$$\langle e^{-kx^2/w_0^2}\rangle = \langle e^{-ky^2/w_0^2}\rangle = \langle e^{-k(z-ct)^2/w_z^2}\rangle$$

thus obtaining G:

Spatial

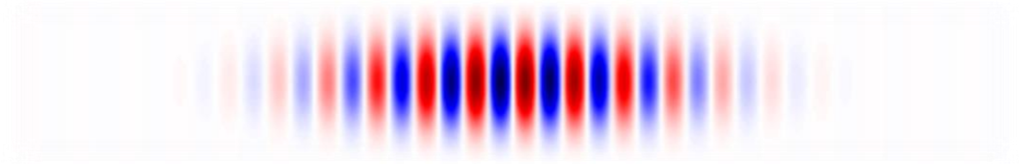
$$\begin{aligned}\mathcal{G}(k,0) &\equiv \langle e^{-kr^2/w_0^2}\rangle \\ &= \frac{I(k,\rho_0)-(\frac{\mu}{2}+\frac{5}{8})\rho_0 I(k+1,\rho_0)}{I(0,\rho_0)-(\frac{\mu}{2}+\frac{5}{8})\rho_0 I(1,\rho_0)} \\ &\simeq 1-k\rho_0+k(\mu+\frac{5}{2})\rho_0^2+\mathcal{O}(\rho_0)^3\end{aligned}$$

$$\mathcal{G}(0,k) = \sqrt{\mathcal{G}(k,0)}$$

Temporal

Whole beam rms parameters in not saturated regime

The final evaluation of the moments for the position and the momentum reads:



$$\sigma^2_{u_x,bunch,0} \equiv \langle \sigma_u^2 \rangle_{bunch} \simeq a_0^2 \rho_0 \times \left[1 - (\mu+8)\rho_0 + (\mu^2 + 19\mu + \tfrac{131}{2})\rho_0^2\right] .$$

$$\sigma^2_{x,bunch,0} \equiv \langle x^2 \rangle_{bunch} \simeq \tfrac{1}{2} w_0^2 \rho_0 \times \left[1 - (\mu+3)\rho_0 + \tfrac{1}{2}(3\mu + \tfrac{33}{4})\rho_0^2\right] .$$

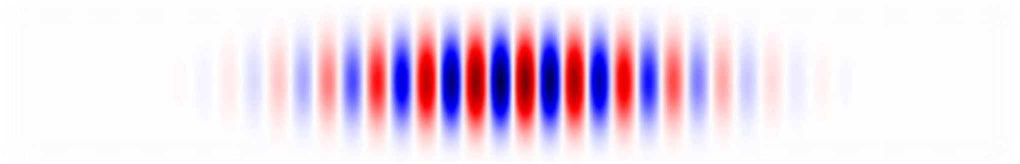
while the normalized emittance is:

$$\epsilon^2_{n,x} \equiv \langle x^2 \rangle_{beam} \langle u_x^2 \rangle_{beam} - \left(\langle x u_x \rangle_{beam}\right)^2 = \frac{1}{2} \left(a_0\, w_0\, \rho_0\right)^2 \mathcal{E}_n(\rho_0, \mu_0)$$

$$\mathcal{E}_n(\rho_0, \mu) \simeq 1 - (\mu + 11)\rho_0 + \left(2\mu^2 + \frac{63}{2}\mu + \frac{749}{8}\right)\rho_0^2$$

***Note**: we include corrections up to* $\rho_0^2 = \Delta^4$

Whole beam rms parameters in *saturated* regime



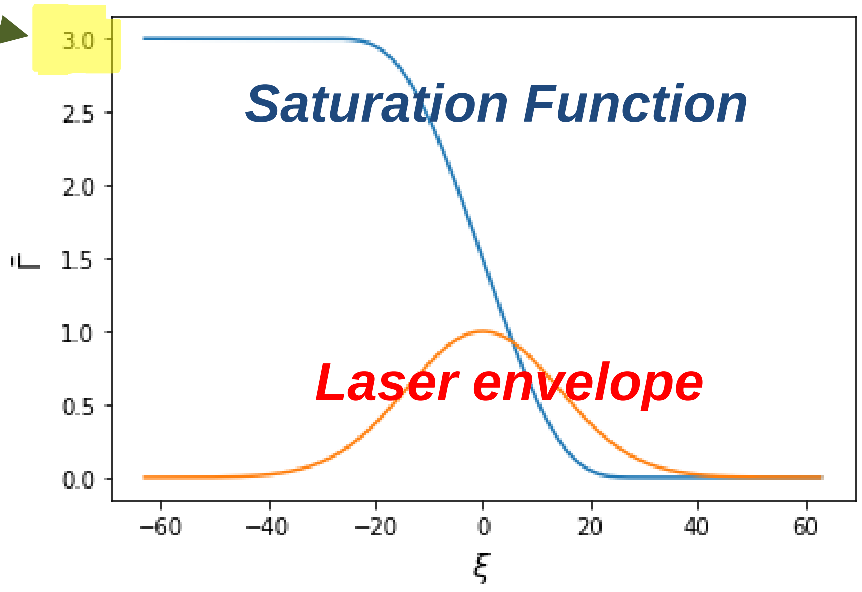
Saturation effects **along the pulse (**not single cycle)

can be evaluated by introducing the (cycle) **average saturation function along the pulse axis**:

$$\bar{\Gamma}(r, z-ct) = \int_{-\infty}^{z-ct} \langle W(r,\zeta)/c \rangle d\zeta \simeq \bar{\nu}_s e^{-\frac{r^2}{\rho_0 w_r^2}} \times \frac{1}{2}\left[1 + E\left(\frac{z-ct}{\sqrt{\rho_0} w_z}\right)\right]$$

where the (cycle averaged) saturation level is

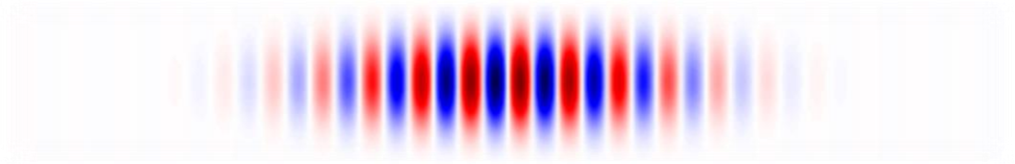
$$\bar{\nu}_s = \sqrt{2}(k_{ADK} w_z)\rho_0^{\mu+1} e^{-1/\rho_0} \, .$$



We got analytical estimation of the rms transverse position [not shown here] which shows an ***increase of the transverse size when saturation effects are on*** **[as anticipated by C. Schoeder et al**. PRAB **17**(10) 101301 (2014)],

Whole beam rms parameters in *saturated* regime

Theory@LDED

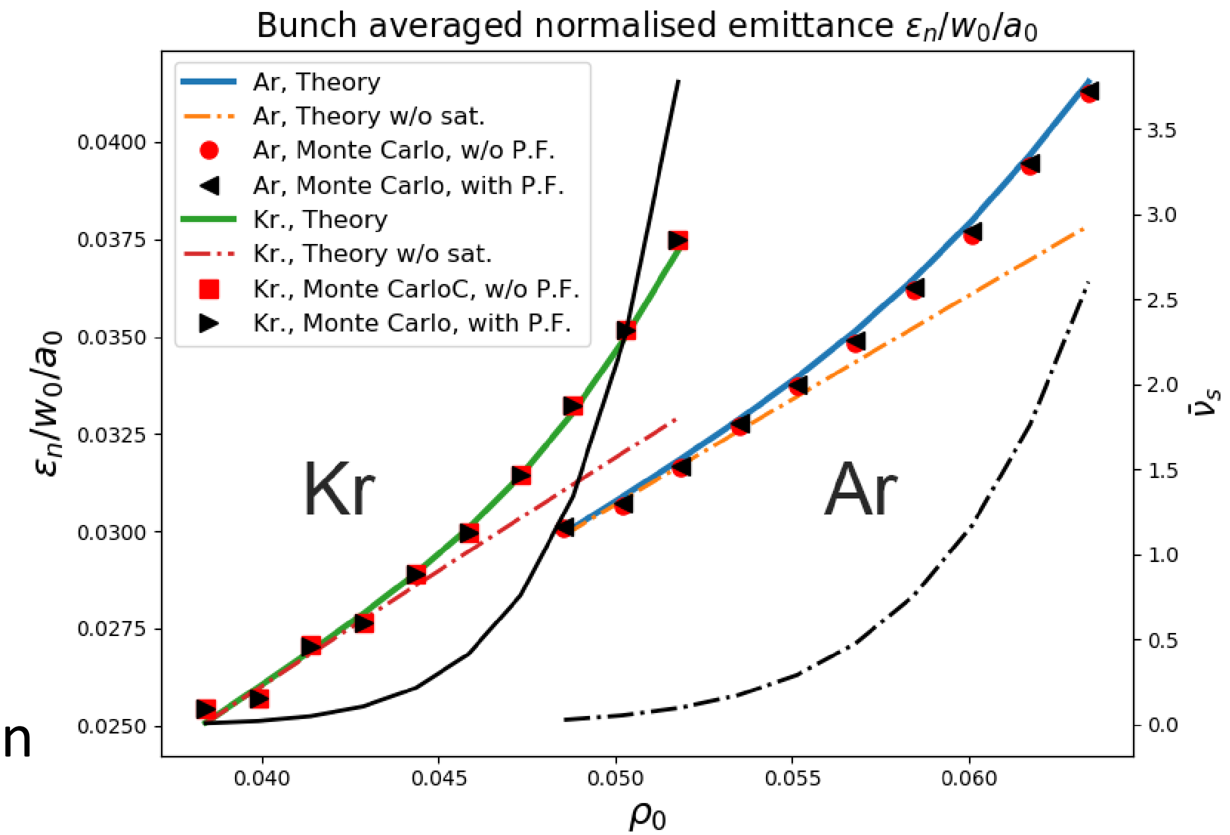


Beam radius increase brings to a **slight reduction of the overall transverse momentum**.

The final beam emittance reads

$$\epsilon_{n,x}^2 \simeq \frac{1}{2}\left(a_0\, w_0\, \rho_0\right)^2 \mathcal{E}_{n,sat}(\rho_0,\mu_0)\,,$$

$$\begin{aligned}\mathcal{E}_{n,sat} \;\simeq\; & \left(1+\frac{\bar{\nu}_s}{8}-\frac{5}{864}\bar{\nu}_s^2\right)\times \\ \times\; & \Big[1-(\mu+11+\frac{3}{8}\bar{\nu}_s)\rho_0 + \\ +\; & \left(2\mu^2+\frac{63}{2}\mu+\frac{749}{8}+\frac{3}{8}(\mu+11)\bar{\nu}_s\right)\rho_0^2\Big]\end{aligned}$$



and it **increases** when saturation effects are on

Conclusions

- We developed a **very accurate** and **detailed** model for the statistics of the electrons extracted by tunnel ionization in a single laser field peak.

This model **is valid also in the deep saturation regime** and can handle up to two consecutive ionization processes e.g. Kr(8 to 10).

The model can be employed in laser-envelope for an accurate, full 3D, description of the extracted electrons

- We also built **accurate analytical predictors** of the synthetic phase space moments of the **whole electron bunch, which are alos valid in the saturation regime.**

- **ELI-NP LDED is open on collaboration on this topic. Master/Ph.D students are welcome (also in co-tutoring)**