ToyMC Study for the Estimation of DCA

Shyam Kumar, Arianna Torres, Francesco Barile, Giuseppe Bruno, Gianfranco Tassielli, Mihail Bogdan Blidaru, Lukas Lautner, Alperen Yuncu, et al.

INFN Bari, Italy

Previous presentations

https://indico.cern.ch/event/1207616/contributions/5079140/attachments/2521089/4335005/Tracking_BeamTestData_Shyam.pdf

https://indico.cern.ch/event/1196877/contributions/5036272/attachments/2503083/4300248/WP3_meeting_06_09_2022.pdf

Hit Points in Beam test Data

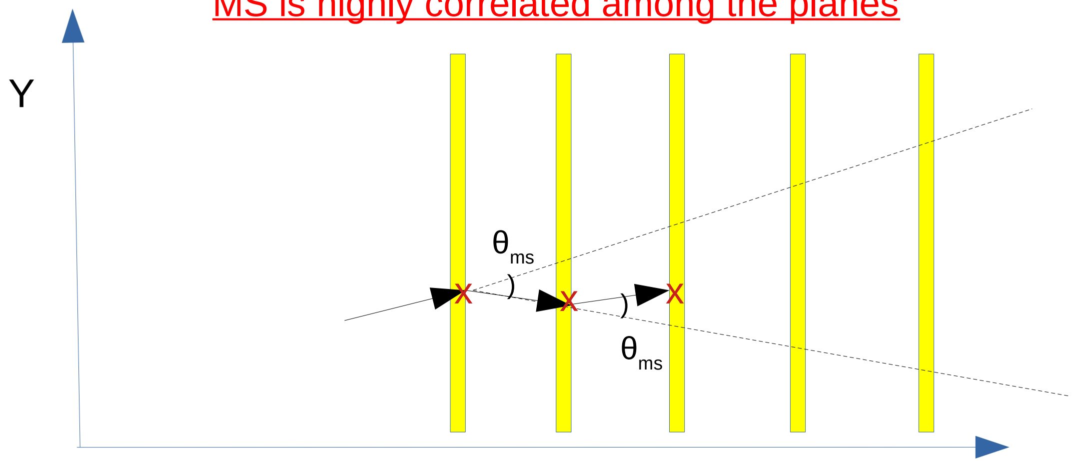
Simulation: Red Points we get during detector simulation after energy loss and multiple scattering

Digitization: Smear these red point by pixel resolution (spatial resolution)

Reconstruction: Fit the point after digitization

In the Beam test data, the point are already include both effects so we can directly fit the points

MS is highly correlated among the planes



$$C_{ij} = \langle (y_{hit} - y_{true})_i * (y_{hit} - y_{true})_j \rangle$$

Matrix by Werner Reigler (equal spacing)

arXiv:1805.12014

$$\mathbf{C}_y = \mathbf{M} = \frac{\sigma_\alpha^2 L^2}{N^2} \begin{pmatrix} \frac{N^2\sigma_0^2}{\sigma_\alpha^2 L^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \cdots \\ 0 & 2 & 5 & 8 & 11 & 14 & 17 & 20 & \cdots \\ 0 & 3 & 8 & 14 & 20 & 26 & 32 & 38 & \cdots \\ 0 & 4 & 11 & 20 & 30 & 40 & 50 & 60 & \cdots \\ 0 & 5 & 14 & 26 & 40 & 55 & 70 & 85 & \cdots \\ 0 & 6 & 17 & 32 & 50 & 70 & 91 & 112 & \cdots \\ 0 & 7 & 20 & 38 & 60 & 85 & 112 & 140 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Matrix by me statistically (equal spacing)

double radius[] = {1.8,2.4,3.0,3.6,4.2,4.8};

6x6 matrix is as follows

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2.003	3.016	4.024	5.046
2	0	2.003	5.001	8.019	11.01	14.05
3	0	3.016	8.019	14.07	20.07	26.15
4	0	4.024	11.01	20.07	30.06	40.15
5	0	5.046	14.05	26.15	40.15	55.31

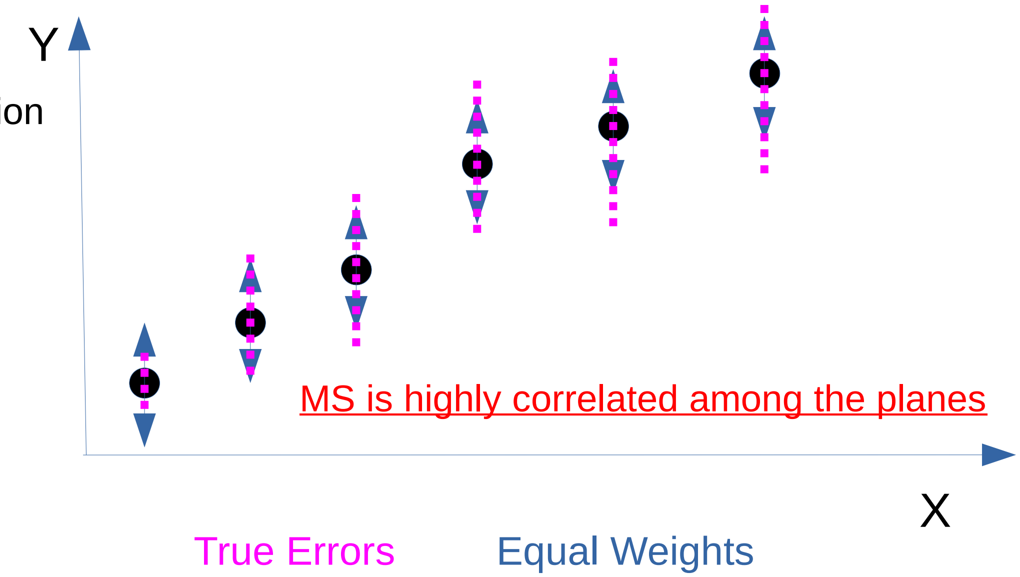
Global Chi2 fitting (Line Fit)

Global fit with Ignoring M.S.

In the case of global fit we are using only spatial resolution means we are giving the equal weights to each points

In reality points have different errors: Chi2 is overestimated and also fitting is biased

The points also has a correlation which is ignored if we ignore multiple scattering



$$\sigma_x = \sigma_y = 5\,\mu m \qquad \chi^2 = \sum_{i=1}^{n} \frac{dx_i^2}{\sigma_{x_i}^2} + \frac{dy_i^2}{\sigma_{y_i}^2}$$

Straight Line Fit (With Multiple Scattering)

<u>Global Chi2 fit with M.S. is complex</u>

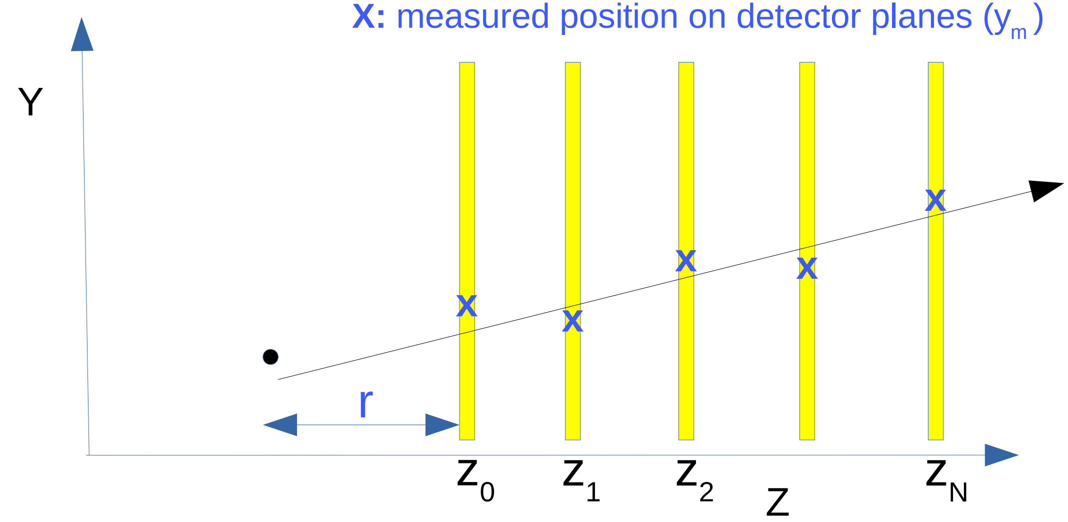
$$y = a + b\,z$$

If points on planes are uncorrelated

Minimize the quantity below (works for Spatial resolutions):

$$\chi^2 = \sum_{i=0}^{N} \frac{(y_m - y_i)^2}{\sigma_i^2} = \sum_{i=0}^{N} \frac{(y_m - a - b z_i)^2}{\sigma_i^2}$$

X: measured position on detector planes (y_m)



Multiple Scattering between planes are highly correlated, then quantity to be Minimized:

$$\chi^2 = (Y - Ap)^T (V_{SR} + V_{MS})^{-1} (Y - Ap)$$

$$\chi^2 = \sum_{i,j=0}^{N} \frac{(y_{m_i} - y_i)(y_{m_j} - y_j)}{\sigma_{ij}}$$

$$V_{SR} = \begin{pmatrix} \sigma_0^2 & \ldots\ldots \\ 0 & \sigma_1^2 \ldots\ldots \\ \ldots\ldots & \ldots\ldots \\ 0 & 0\ldots\ldots\ \sigma_N^2 \end{pmatrix} \qquad V_{MS} = \begin{pmatrix} \sigma_0^2 & \sigma_{01}\ldots\ldots \\ 0 & \sigma_1^2 \ldots\ldots \\ \ldots\ldots & \ldots\ldots \\ 0 & 0\ldots\ldots\ \sigma_N^2 \end{pmatrix}$$

For 100 points:
100x100 matrix difficult to Inverse (Chi2 fitting)
For Kalman filter 100 matrix of 5x5 dimensions

MS matrix is non-diagonal

Covariance Matrix

double radius[] = {1.8,2.4,3.0,5.0,7.5,10.};

p = 1 GeV/c V_{SR}

Theta MS: 0.000218366

Covariance Matrix for Spatial Resolution

6x6 matrix is as follows

	0	1	2	3	4	5
0	2.5e-07	0	0	0	0	0
1	0	2.5e-07	0	0	0	0
2	0	0	2.5e-07	0	0	0
3	0	0	0	2.5e-07	0	0
4	0	0	0	0	2.5e-07	0
5	0	0	0	0	0	2.5e-07

V_{MS} p = 1 GeV/c

Covariance Matrix for Multiple Scattering

6x6 matrix is as follows

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1.746e-08	3.497e-08	9.389e-08	1.672e-07	2.415e-07
2	0	3.497e-08	8.73e-08	2.629e-07	4.805e-07	7.015e-07
3	0	9.389e-08	2.629e-07	1.026e-06	1.974e-06	2.932e-06
4	0	1.672e-07	4.805e-07	1.974e-06	4.128e-06	6.303e-06
5	0	2.415e-07	7.015e-07	2.932e-06	6.303e-06	1.001e-05

scaled

Scaled Covariance Matrix for Multiple Scattering

6x6 matrix is as follows

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2.003	5.379	9.579	13.84
2	0	2.003	5.001	15.06	27.53	40.19
3	0	5.379	15.06	58.78	113.1	167.9
4	0	9.579	27.53	113.1	236.5	361.1
5	0	13.84	40.19	167.9	361.1	573.6

Diagonal Entries Scaled by Second Layer:

2^{nd} Layer: $(2.4-1.8)^2 = 0.36$ = **0.36/0.36 = 1**

3^{rd} Layer: $(3.0-2.4)^2+(3.0-1.8)^2 = 1.8$ = **1.8/0.36 = 5.0**

4^{th} Layer: $(5.0-3.0)^2+(5.0-2.4)^2+(5.0-1.8)^2 = 21.0$ = **21.0/0.36 = 58.33**

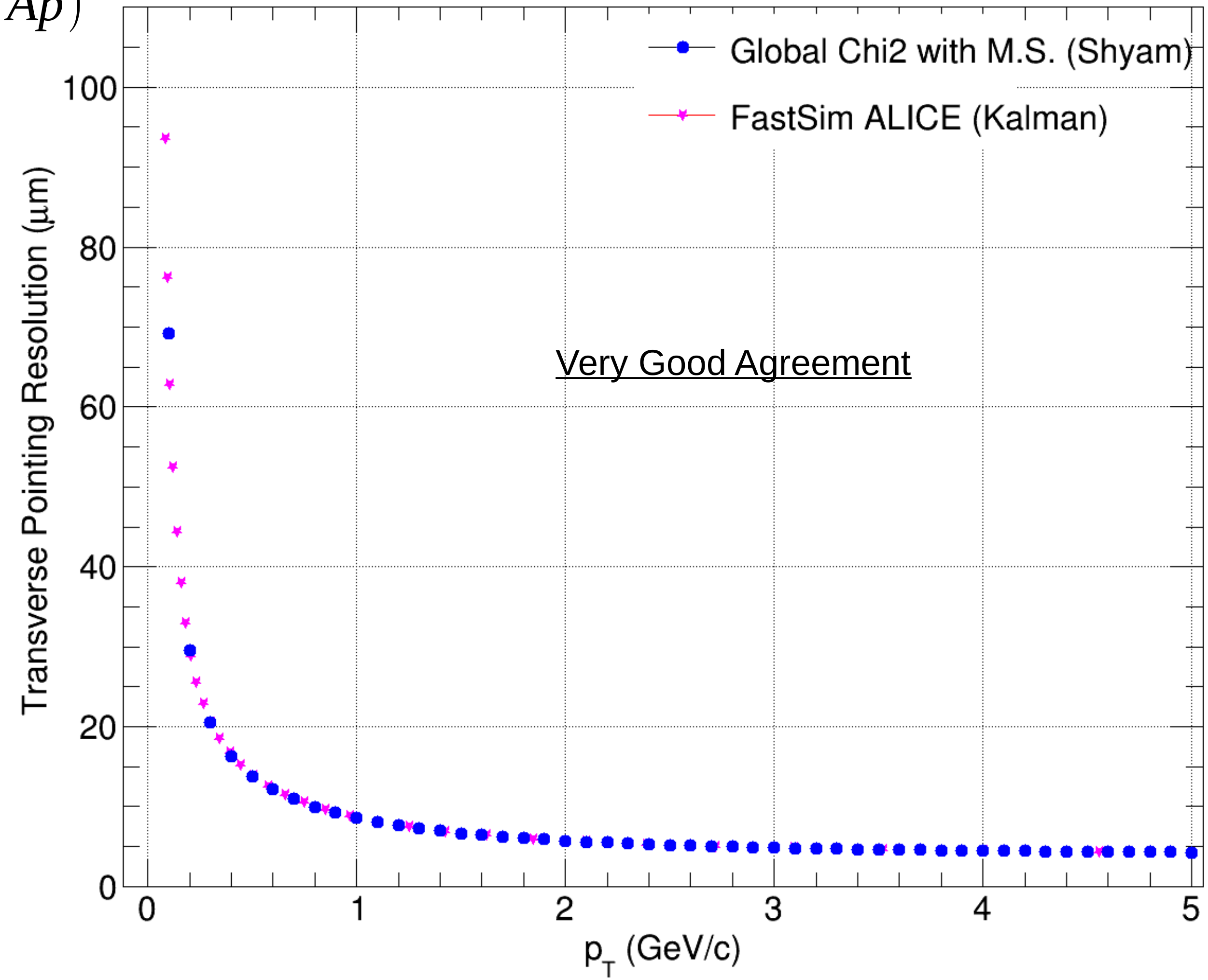
5^{th} Layer: $(7.5-5.0)^2+(7.5-3.0)^2+(7.5-2.4)^2+(7.5-1.8)^2 = 85.0$ = **85.0/0.36 = 236.11**

6^{th} Layer: $(10-7.5)^2+(10-5.0)^2+(10-3.0)^2+(10-2.4)^2+(10-1.8)^2 = 205.25$ = **205.25/0.36 = 570.14**

Global Chi2 Fitting:

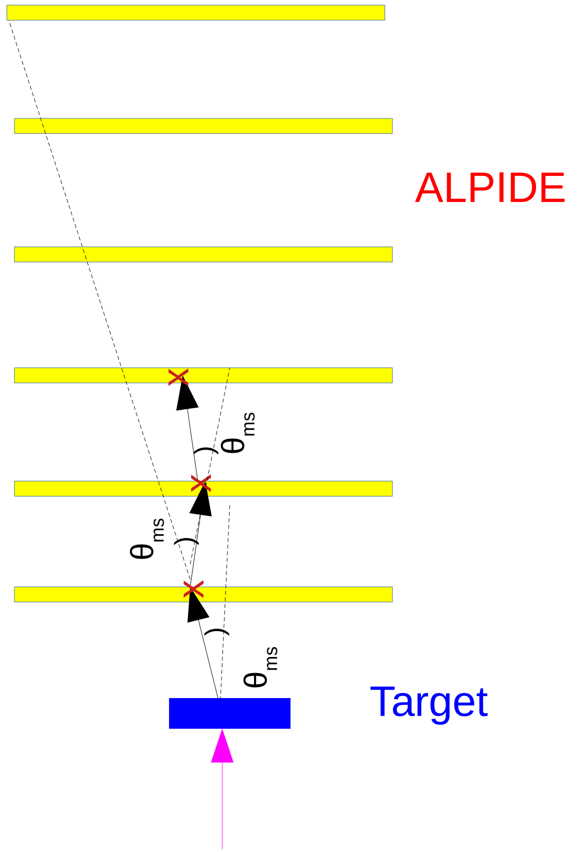
$$\chi^2 = (Y - Ap)^T (V_{SR} + V_{MS})^{-1} (Y - Ap)$$

double radius[] = {1.8,2.4,3.0,5.0,7.5,10.};



Global Chi2 Fitting:

$$\chi^2 = (Y - Ap)^T (V_{SR} + V_{MS})^{-1} (Y - Ap)$$

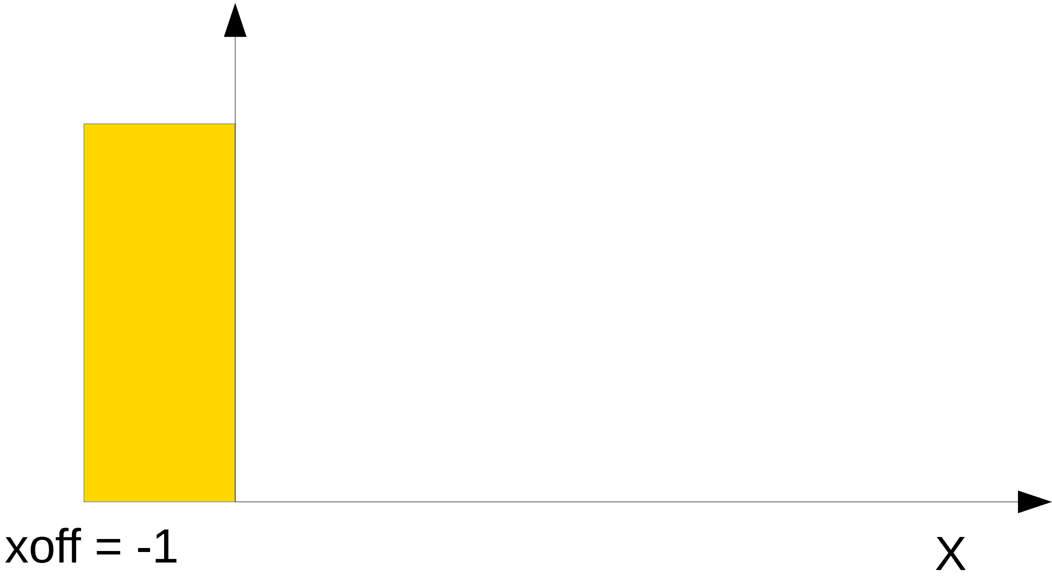


Multiple Scattering (V_{MS}) constructed dynamically for each track based on Momentum hypothesis from PYTHIA8

```
Double_t xhit =0., yhit = 0.;
TVector2 MCPos, HitPos;
 double prev_angle = TMath::ATan(slope); //+gRandom->Gaus(0.,theta_ms_target)
// Assuming target of thickness 1.16 mm
yhit = 1.16; xhit = yhit/TMath::Tan(prev_angle+gRandom->Gaus(0.,theta_ms_target));
HitPos.SetX(xhit); HitPos.SetY(yhit);

for (Int_t ihit=0;ihit<nlayers;++ihit){
yhit = radius[ihit];
TVector2 prevHit = HitPos;
double angle = prev_angle+gRandom->Gaus(0.,theta_ms); // theta_ms for ALPIDE
double ynew = yhit;
double xnew = prevHit.X()+(ynew-prevHit.Y())/TMath::Tan(angle);
HitPos.SetX(xnew);        HitPos.SetY(ynew);
prev_angle = angle;

hResX[ihit]->Fill((HitPos.X()-xhit));  hResY[ihit]->Fill((HitPos.Y()-yhit)); ResidualX[ihit] = (HitPos.X()-xhit);
} // Hits
```

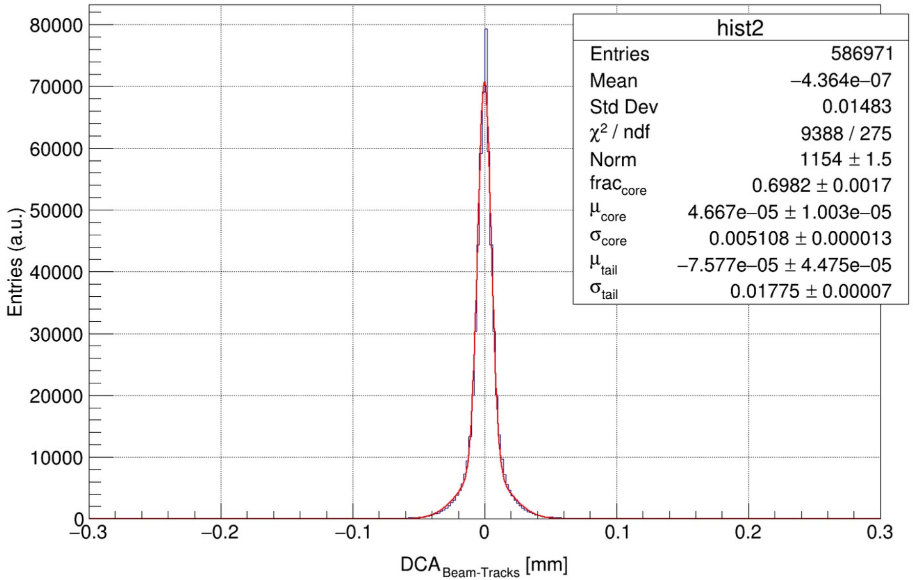
ToyMC (PYTHIA8)

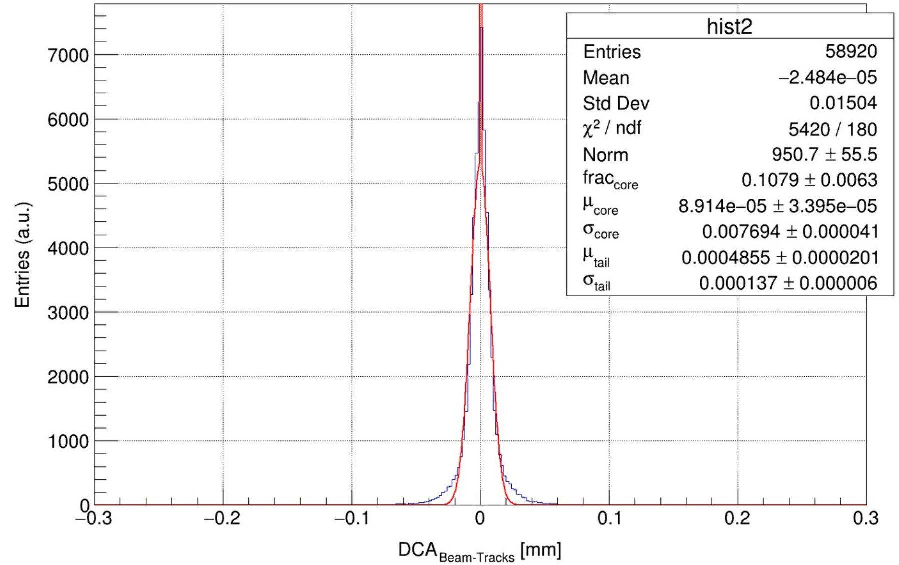
```
double thickness = 1.16; // mm
double offset = -1.0; // Expected target offset
double ztarget_end = thickness+offset;

// smear the vertex
double tmpz=gRandom->Uniform(0.,thickness);
bVTX.SetZ(tmpz+offset);
tmpz=thickness-tmpz;

// Produced Tracks
ParTrack[1] =Px[j]/Pz[j]; ParTrack[3] = Py[j]/Pz[j]; ParTrack[5] = 1.0;
ParTrack[0] = bVTX.X(); ParTrack[2] = bVTX.Y(); ParTrack[4] = bVTX.Z();

// Outgoing tracks from target
ParTrack[0] = bVTX.X()+ParTrack[1]*tmpz;
ParTrack[2] = bVTX.Y()+ParTrack[3]*tmpz;
ParTrack[4] = bVTX.Z()+tmpz;

TVector3 exPoint(ParTrack[0],ParTrack[2],ParTrack[4]);
double path_length = (exPoint-bVTX).Mag();
double effradlen = path_length/XX0_Cu;
double theta_MS = MultipleScattering(charge, part->Mass()*1000., p*1000., effradlen);

// Outgoing tracks from target with MS
TVector3 dir(ParTrack[1],ParTrack[3],ParTrack[5]);
dir.RotateX(gRandom->Gaus(0.,theta_MS));
dir.RotateY(gRandom->Gaus(0.,theta_MS));
ParTrack[1] = dir.X()/dir.Z(); ParTrack[3] = dir.Y()/dir.Z(); ParTrack[5] = 1.;
```

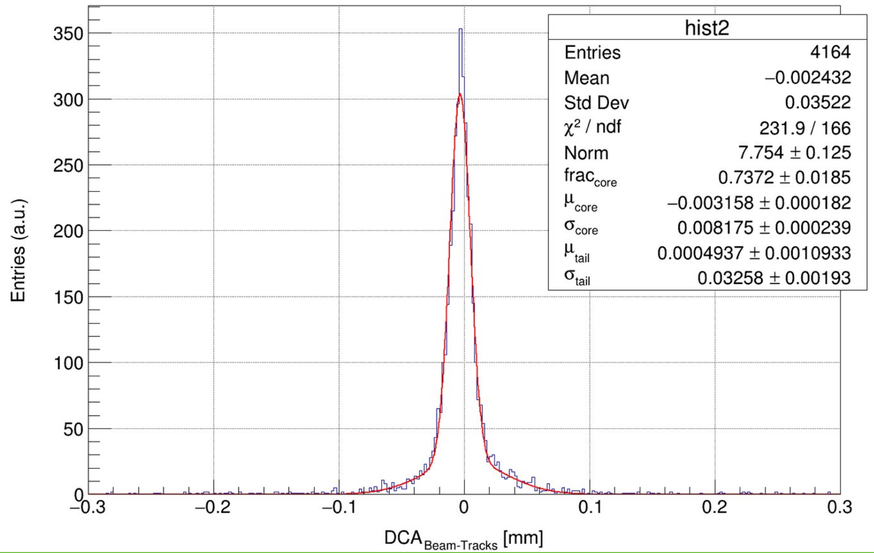


These final tracks are smeared with detector resolutions and fitted to estimate 3D DCA

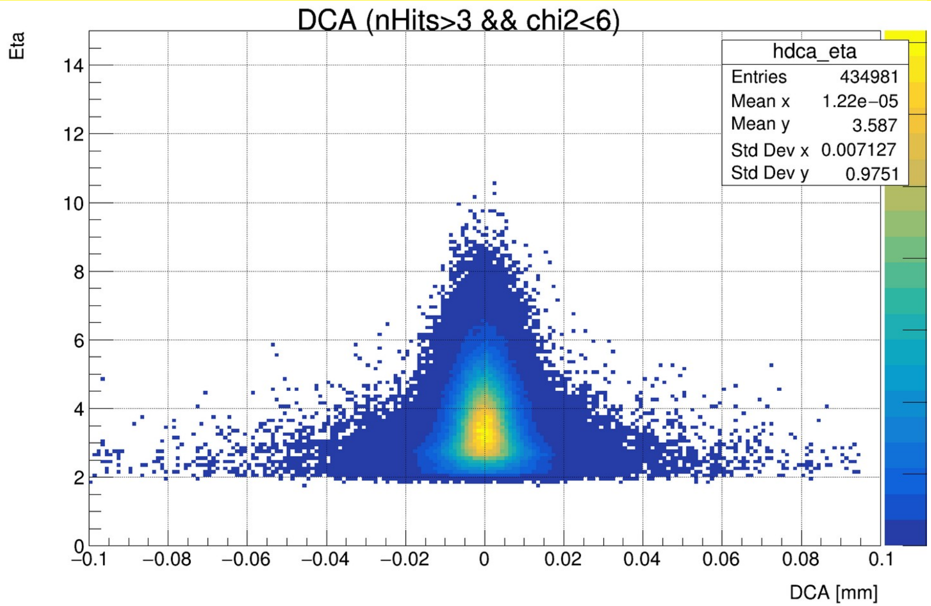
Comparison

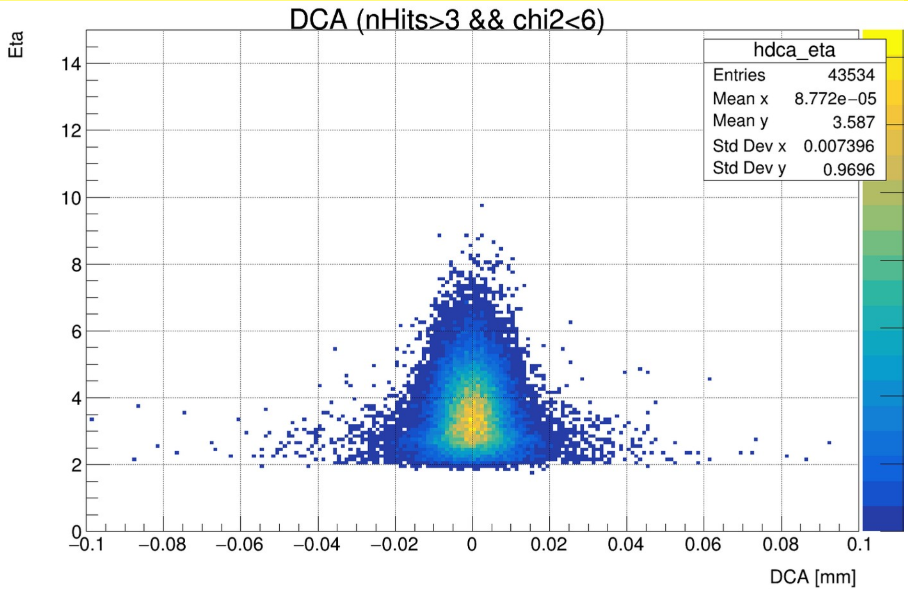

hist2
Entries 586971
Mean −4.364e−07
Std Dev 0.01483
χ² / ndf 9388 / 275
Norm 1154 ± 1.5
frac_core 0.6982 ± 0.0017
μ_core 4.667e−05 ± 1.003e−05
σ_core 0.005108 ± 0.000013
μ_tail −7.577e−05 ± 4.475e−05
σ_tail 0.01775 ± 0.00007
Entries (a.u.)
DCA Beam-Tracks [mm]

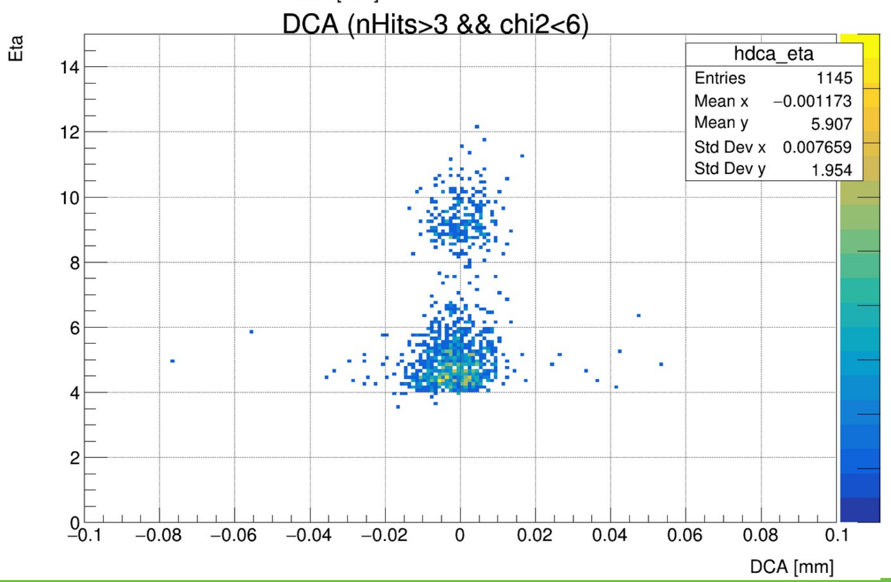

hist2
Entries 58920
Mean −2.484e−05
Std Dev 0.01504
χ² / ndf 5420 / 180
Norm 950.7 ± 55.5
frac_core 0.1079 ± 0.0063
μ_core 8.914e−05 ± 3.395e−05
σ_core 0.007694 ± 0.000041
μ_tail 0.0004855 ± 0.0000201
σ_tail 0.000137 ± 0.000006
Entries (a.u.)
DCA Beam-Tracks [mm]


hist2
Entries 4164
Mean −0.002432
Std Dev 0.03522
χ² / ndf 231.9 / 166
Norm 7.754 ± 0.125
frac_core 0.7372 ± 0.0185
μ_core −0.003158 ± 0.000182
σ_core 0.008175 ± 0.000239
μ_tail 0.0004937 ± 0.0010933
σ_tail 0.03258 ± 0.00193
Entries (a.u.)
DCA Beam-Tracks [mm]

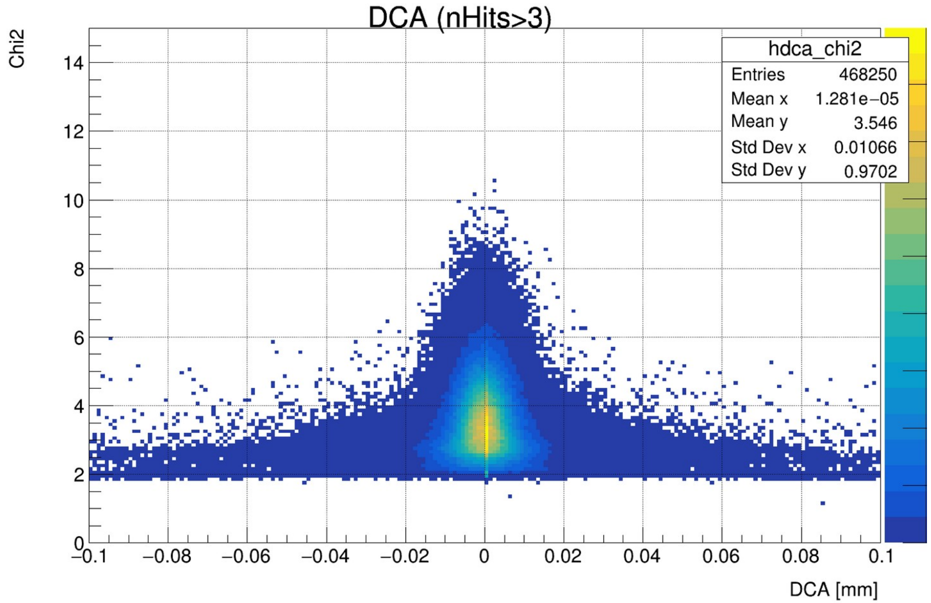
Comparison

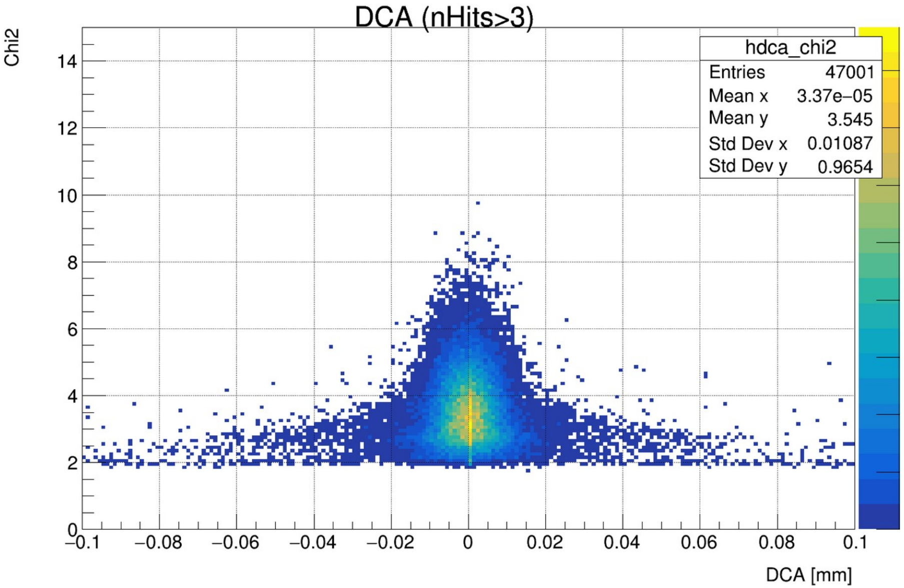

DCA (nHits>3 && chi2<6)
hdca_eta
Entries 434981
Mean x 1.22e−05
Mean y 3.587
Std Dev x 0.007127
Std Dev y 0.9751
Eta
DCA [mm]

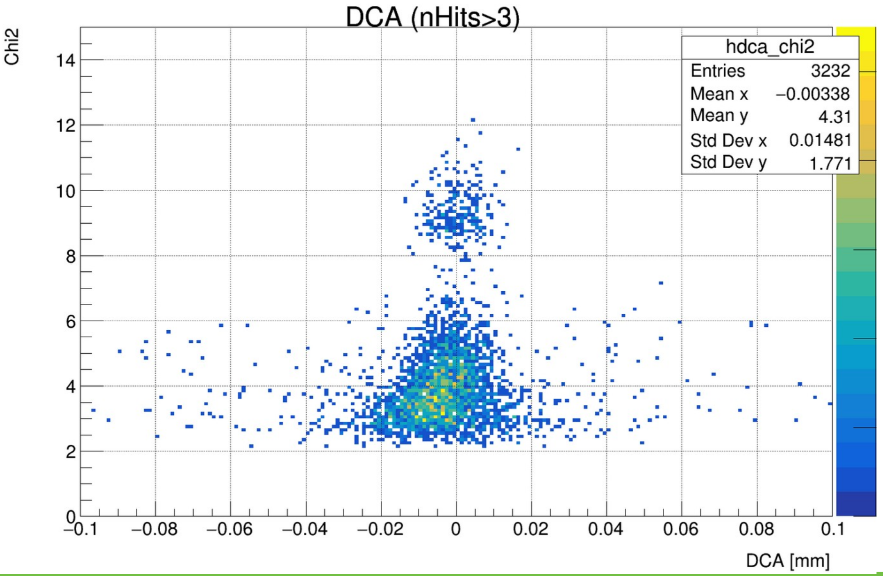

DCA (nHits>3 && chi2<6)
hdca_eta
Entries 43534
Mean x 8.772e−05
Mean y 3.587
Std Dev x 0.007396
Std Dev y 0.9696
Eta
DCA [mm]


DCA (nHits>3 && chi2<6)
hdca_eta
Entries 1145
Mean x −0.001173
Mean y 5.907
Std Dev x 0.007659
Std Dev y 1.954
Eta
DCA [mm]

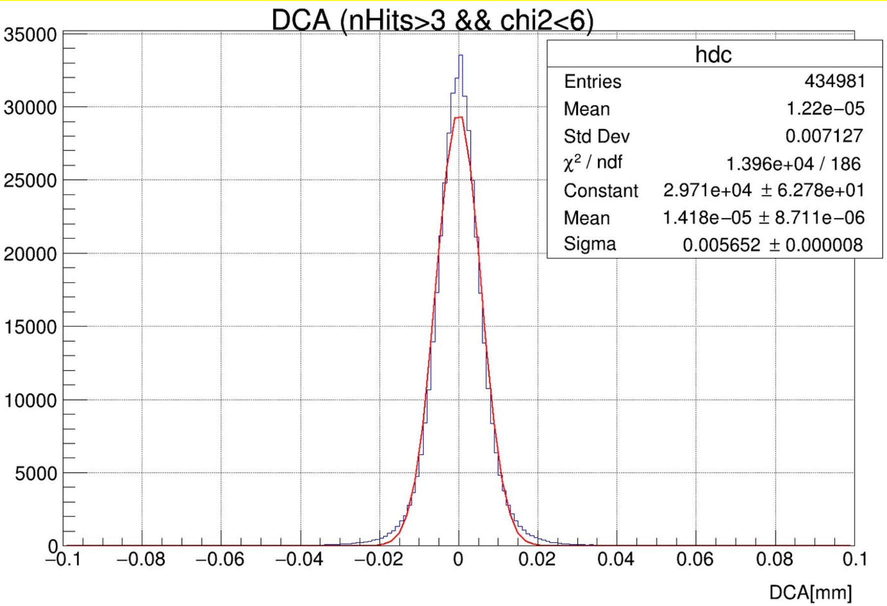
Comparison

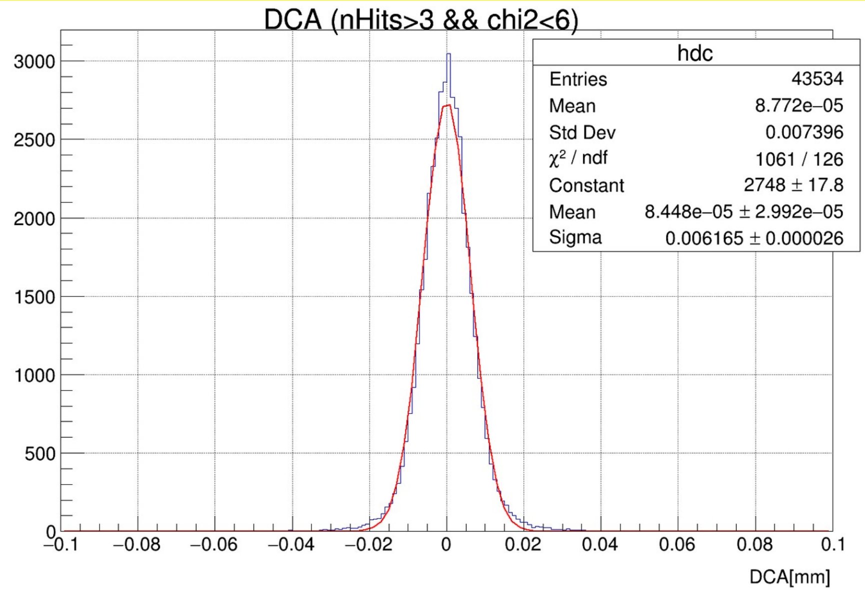

DCA (nHits>3)
hdca_chi2
Entries 468250
Mean x 1.281e−05
Mean y 3.546
Std Dev x 0.01066
Std Dev y 0.9702
Chi2
DCA [mm]

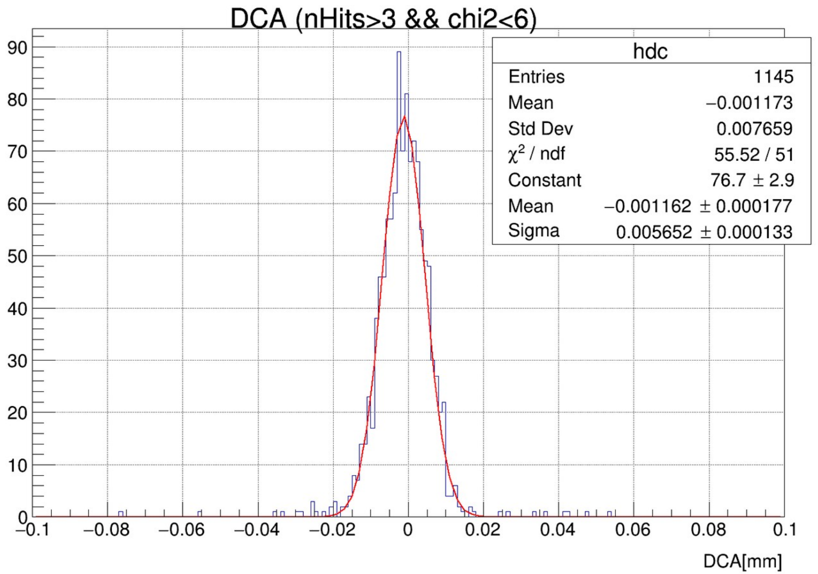

DCA (nHits>3)
hdca_chi2
Entries 47001
Mean x 3.37e−05
Mean y 3.545
Std Dev x 0.01087
Std Dev y 0.9654
Chi2
DCA [mm]


DCA (nHits>3)
hdca_chi2
Entries 3232
Mean x −0.00338
Mean y 4.31
Std Dev x 0.01481
Std Dev y 1.771
Chi2
DCA [mm]

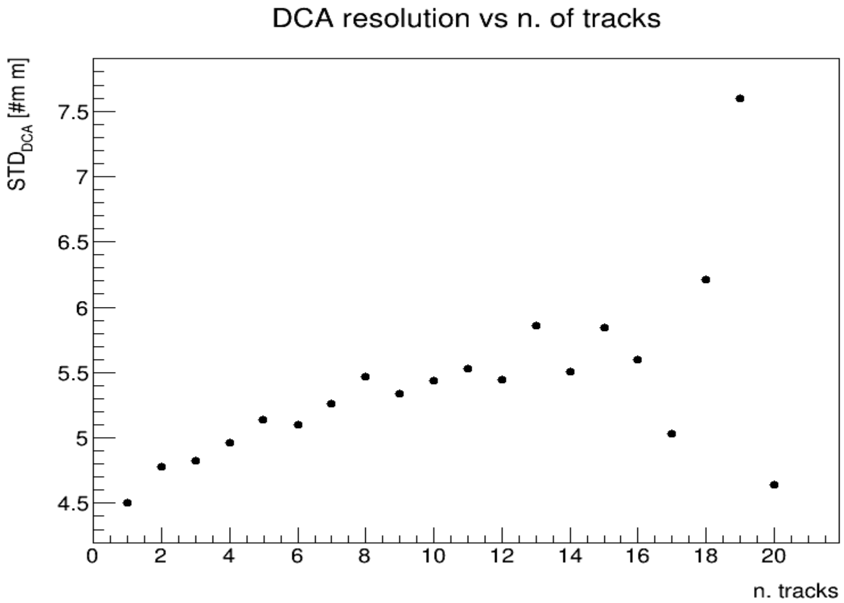
Comparison

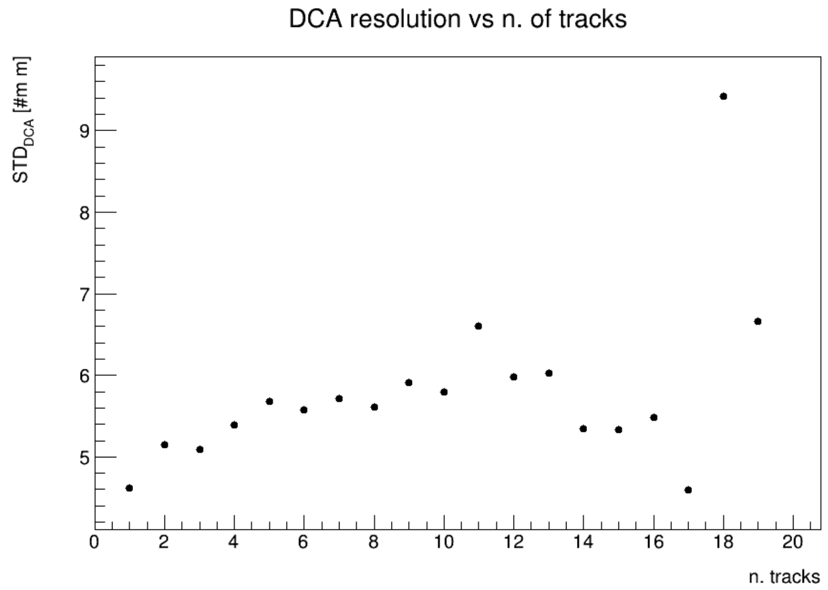

DCA (nHits>3 && chi2<6)
hdc
Entries 434981
Mean 1.22e−05
Std Dev 0.007127
χ2 / ndf 1.396e+04 / 186
Constant 2.971e+04 ± 6.278e+01
Mean 1.418e−05 ± 8.711e−06
Sigma 0.005652 ± 0.000008
DCA[mm]

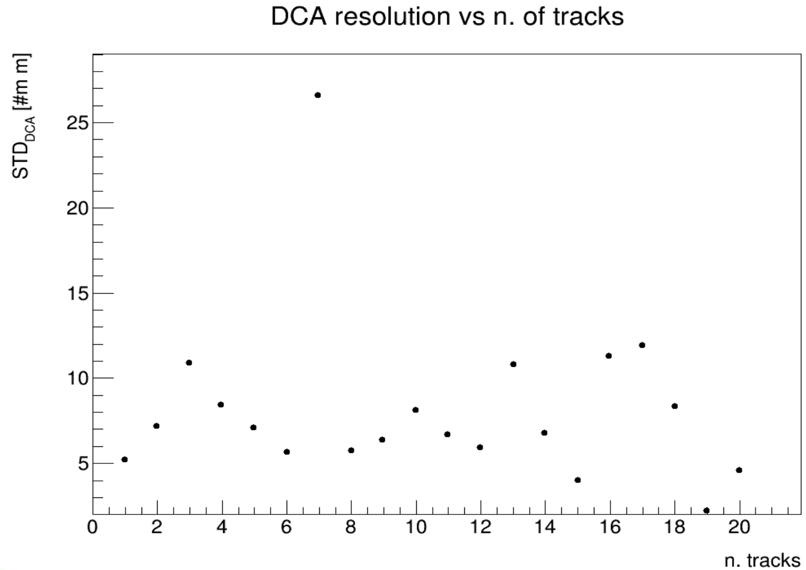

DCA (nHits>3 && chi2<6)
hdc
Entries 43534
Mean 8.772e−05
Std Dev 0.007396
χ2 / ndf 1061 / 126
Constant 2748 ± 17.8
Mean 8.448e−05 ± 2.992e−05
Sigma 0.006165 ± 0.000026
DCA[mm]


DCA (nHits>3 && chi2<6)
hdc
Entries 1145
Mean −0.001173
Std Dev 0.007659
χ2 / ndf 55.52 / 51
Constant 76.7 ± 2.9
Mean −0.001162 ± 0.000177
Sigma 0.005652 ± 0.000133
DCA[mm]

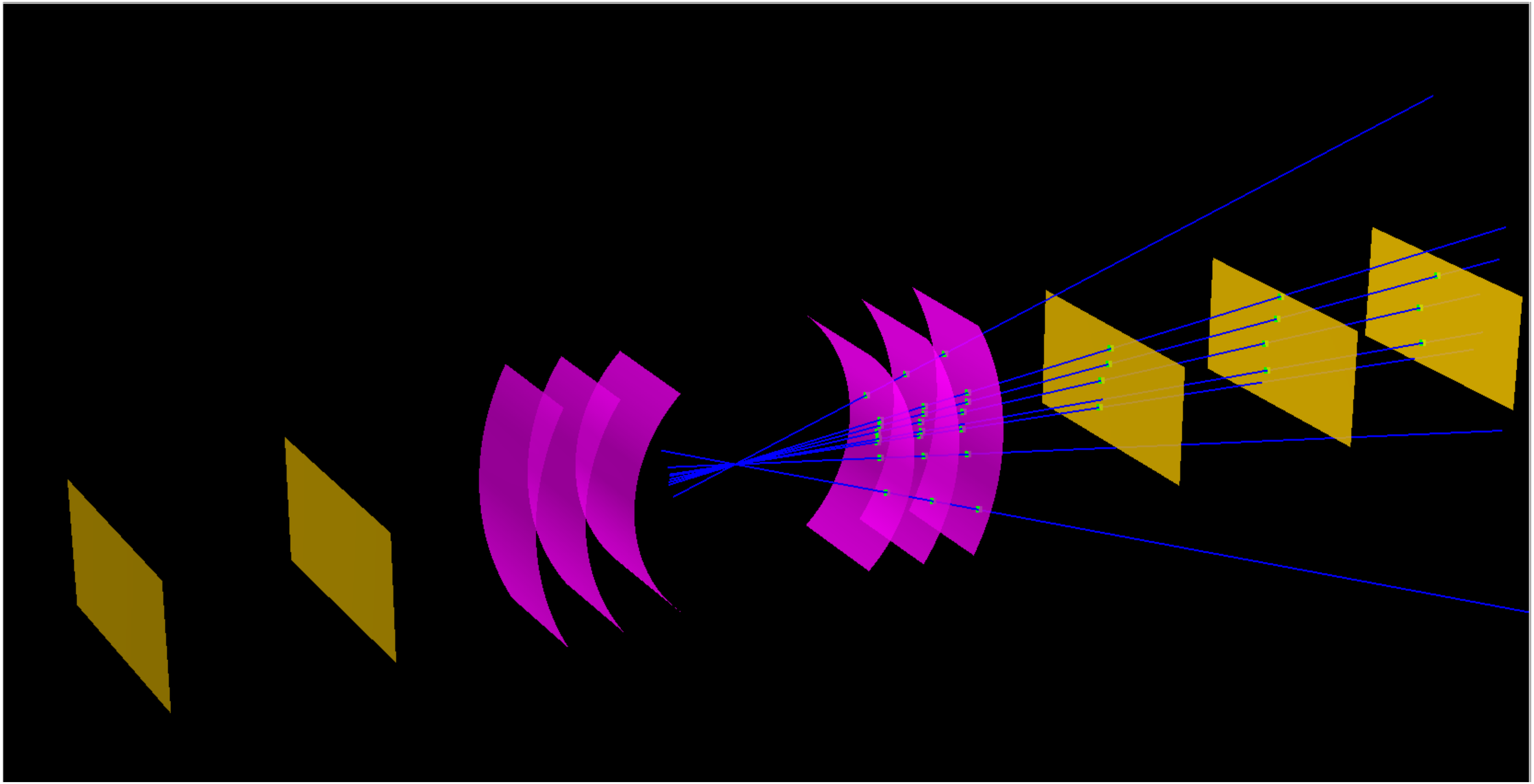
Comparison


DCA resolution vs n. of tracks
STD_{DCA} [#m m]
n. tracks


DCA resolution vs n. of tracks
STD_{DCA} [#m m]
n. tracks


DCA resolution vs n. of tracks
STD_{DCA} [#m m]
n. tracks

Event display



Summary & Future Plan

- Track fitting is done using Global Chi2 fitting (with and without M.S.) and Kalman filter method
- DCA between beam and tracks in space is evlauted using two methods
- Distance between beam and tracks is evaluated at z = 0
  - Full data sets provided by Arianna with final alignment (x 8 more events)