

# Il Flavor di Nando

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SAPIENZA  
UNIVERSITÀ DI ROMA



Roma June 9<sup>th</sup> 2023

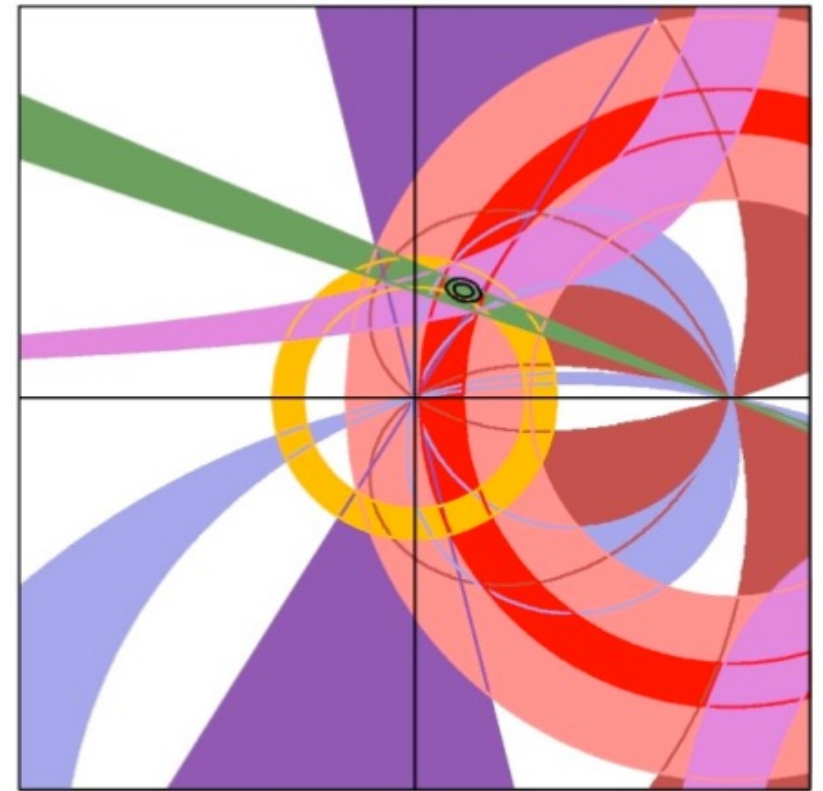


# ***PLAN OF THE TALK***

- *Synthetic (pre-)history of Nando & Guido*
- *Flavor & Babar*
- *The Unitary Triangle Fit*
- *SM Analysis*
- *Tensions and unknown*
- *BSM, Future directions, new/old ideas*
- *Conclusion*

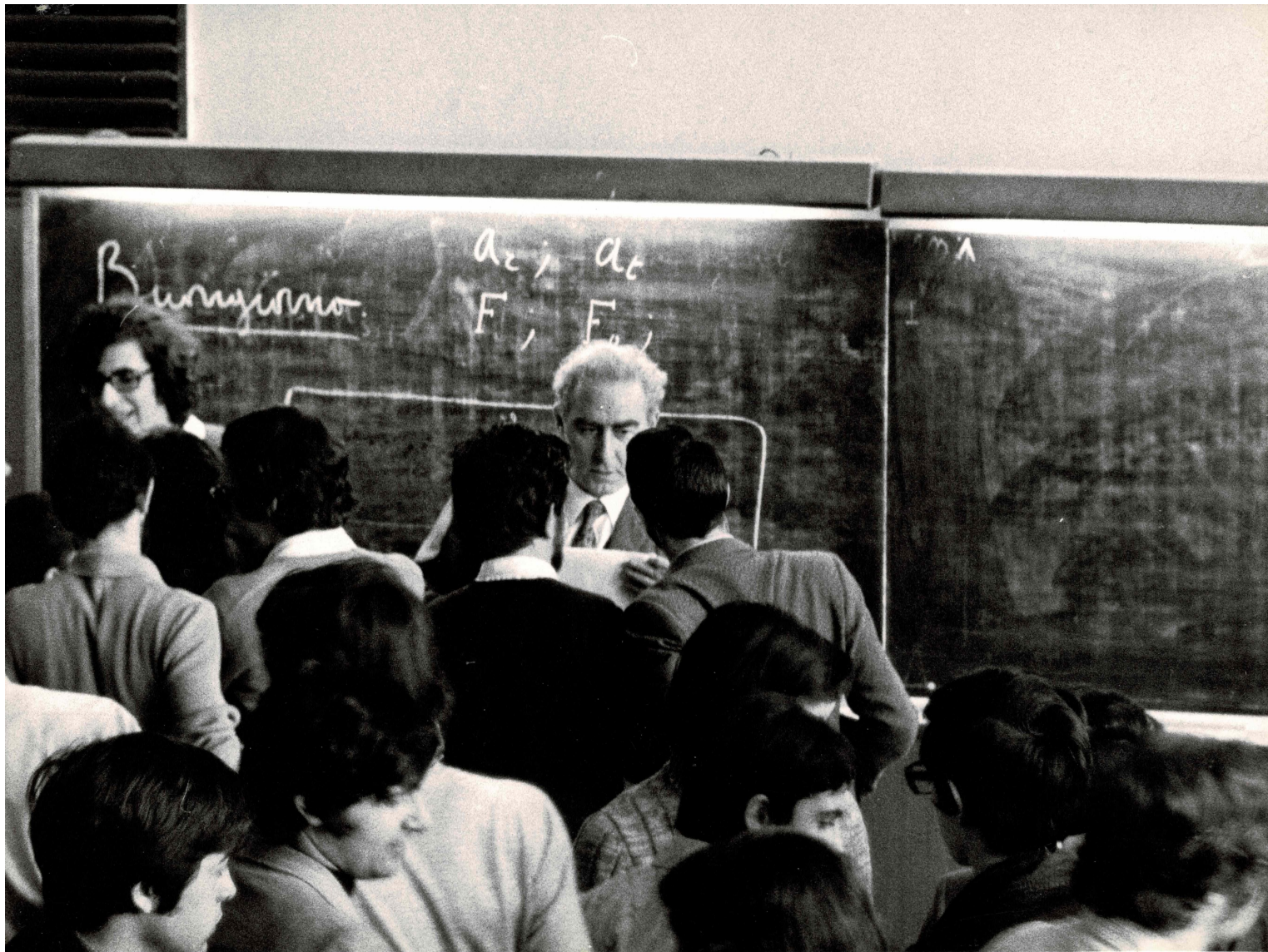
*New UFit Analysis of the Unitarity Triangle  
in the Cabibbo-Kobayashi-Maskawa scheme*

***Rend.Lincei Sci.Fis.Nat. 34 (2023) 37-57**  
arXiv:2212.03894*



Thanks to  
M. Bona, A. Di  
Domenico, C. Kelly, V.  
Lubicz, C. Sachrajda,  
L. Silvestrini, S. Simula,  
L. Vittorio,

*Non avendolo mai mozzicato – anche se a volte ne avrei avuto voglia - non conosco il sapore di Nando, ma ricordo quando ci siamo incontrati*



*Aula Amaldi, allora Aula di Fisica Generale, Maggio 1971  
Lezioni di Fisica 1 del prof. Salvini*

*A-L prof. Salvini*

*M-Z prof. Chiarotti*



*Aula Amaldi, allora Aula di Fisica Generale, Maggio 1971  
Lezioni di Fisica 1 del prof. Chiarotti  
Collettivo di Fisica & Massimo Pieri*

*Poi abbiamo preso strade diverse  
entrambi fisica delle particelle, uno sperimentale l'altro teorico  
Ci siamo ritrovati qualche anno più tardi con un lavoro sulle PdF  
al NLO*

**Parametrization of Proton Structure Functions** #5

CHARM Collaboration · J.V. Allaby (CERN) et al. (Aug, 1987)

Published in: *Phys.Lett.B* 197 (1987) 281-284

[DOI](#) [cite](#) [claim](#)

[reference search](#) [55 citations](#)

**Parton Densities from Deep Inelastic Scattering to Hadronic Processes at Super Collider Energies** #6

M. Diemoz (Rome U. and INFN, Rome), F. Ferroni (Rome U. and INFN, Rome), E. Longo (Rome U. and INFN, Rome), G. Martinelli (CERN) (Jun, 1987)

Published in: *Z.Phys.C* 39 (1988) 21

[DOI](#) [cite](#) [claim](#)

[reference search](#) [455 citations](#)

*con un breve ma importante seguito ....*

**NEUTRINO INTERACTIONS: STUDY OF NUCLEON STRUCTURE BY NEUTRINOS** #4

M. Diemoz (Rome U.), E. Longo (Rome U.), G. Martinelli (Rome U.), F. Ferroni (Ancona U.) (1989)

Published in: *Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol.* 1 (1991) 496-584

*al ristorante la Pergola dello Chef Heinz Beck*

*Evoluzione verso il Flavor (anche in senso culinario)*

*Indipendentemente Nando e collaboratori con Babar  
(vedi talk di Livio Lanceri) e*

*Enrico, Marco, Laura ed io (+ Luca+Ufit) ci siamo tuffati nella  
Fisica del Flavor: Nagoya 2006*



*Col sapore si  
mangia bene  
e Nando  
ama la  
buona cucina*

# *Evoluzione verso il Flavor non solo in senso culinario*

*Un periodo di intensa collaborazione con il gruppo Babar di Roma, tra i giovani Sharham, Gianluca, Federico (Mescia), ....*

3)



1)

2)

*Seminari con cadenza più o meno regolare per discutere insieme violazione di CP per mesoni B, decadimenti leptonici, decadimenti non-leptonici, fisica oltre il Modello Standard*

### 3) Arriva un mutante: lo studente in comune Nando-Guido, ovvero Maurizio Pierini



Università degli Studi di Roma "La Sapienza"  
Facoltà di Scienze Matematiche, Fisiche e Naturali  
Corso di Laurea in Fisica

**Decadimenti a due corpi del mesone B  
senza charm nello stato finale**

**Tesi di Laurea**  
di Maurizio Pierini  
matricola: 11108048

**Relatore:**

Prof. Fernando Ferroni

**Corelatore:**

Prof. Guido Martinelli

Anno Accademico 2000-2001



Università degli Studi di Roma "La Sapienza"  
Facoltà di Scienze Matematiche, Fisiche e Naturali  
Corso di Laurea in Fisica

**Time dependent Asymmetries in  $b \rightarrow s$   
decays in the Standard Model and  
beyond**

Tesi presentata da  
**Maurizio Pierini**  
per il conseguimento del titolo di Dottore di Ricerca

**Relatore:**

Prof. Fernando Ferroni

**Corelatore:**

Prof. Guido Martinelli

Anno Accademico 2000-2001



### 3) *Arriva un mutante: lo studente in comune Nando-Guido, ovvero Maurizio Pierini*

#### *Correlatore, Corelatore o Co-relattore*

*RR.1) tutti i vocabolari o dizionari della lingua italiana riportano "correlatore", ed anzi in molti il termine a singola R (e così pure co-relatore) non è neppure minimamente citato come possibile anche se più rara alternativa;*

*RR.2) si tratta di un caso di raddoppio sintattico (es.: correligionario, corresponsabile, ecc.) che viene spontaneo nel parlare, e quindi entrato definitivamente nell'uso comune della nostra lingua;*

*RR.3) la maggior parte di quelli che si ostinano a dire "corelatore" sono di Roma o di origine laziale; come risaputo, in quel vernacolo la riduzione di "RR" a "R" è molto comune nel linguaggio parlato MAURIZIO MANCO È DE ROMA*



*Oltre il flavor*



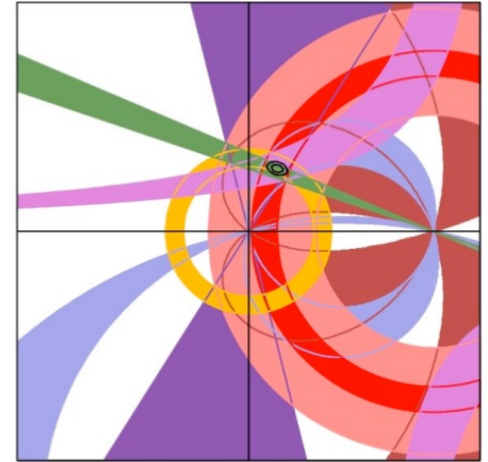
*Con Nando Presidente dell'INFN  
accordo con la SISSA per far nascere il GSSI  
(e non posso non menzionare Eugenio Coccia  
e i nostri surreali incontri col ministro Profumo)*

*non resta tanto tempo per parlare di Fisica,  
ovvero della fisica del sapore 20 anni dopo*

# *STANDARD MODEL*

## *UNITARITY TRIANGLE ANALYSIS*

*(Flavor Physics)*



- *Provides the best determination of the CKM parameters;*
- *Tests the consistency of the SM (“direct” vs “indirect” determinations) @ the quantum level;*
- *Provides predictions for SM observables (in the past for example  $\sin 2\beta$  and  $\Delta m_s$ )*
- *It could lead to new discoveries (CP violation, Charm, !?)*
- *The discovery potential of precision flavor physics should not be underestimated*

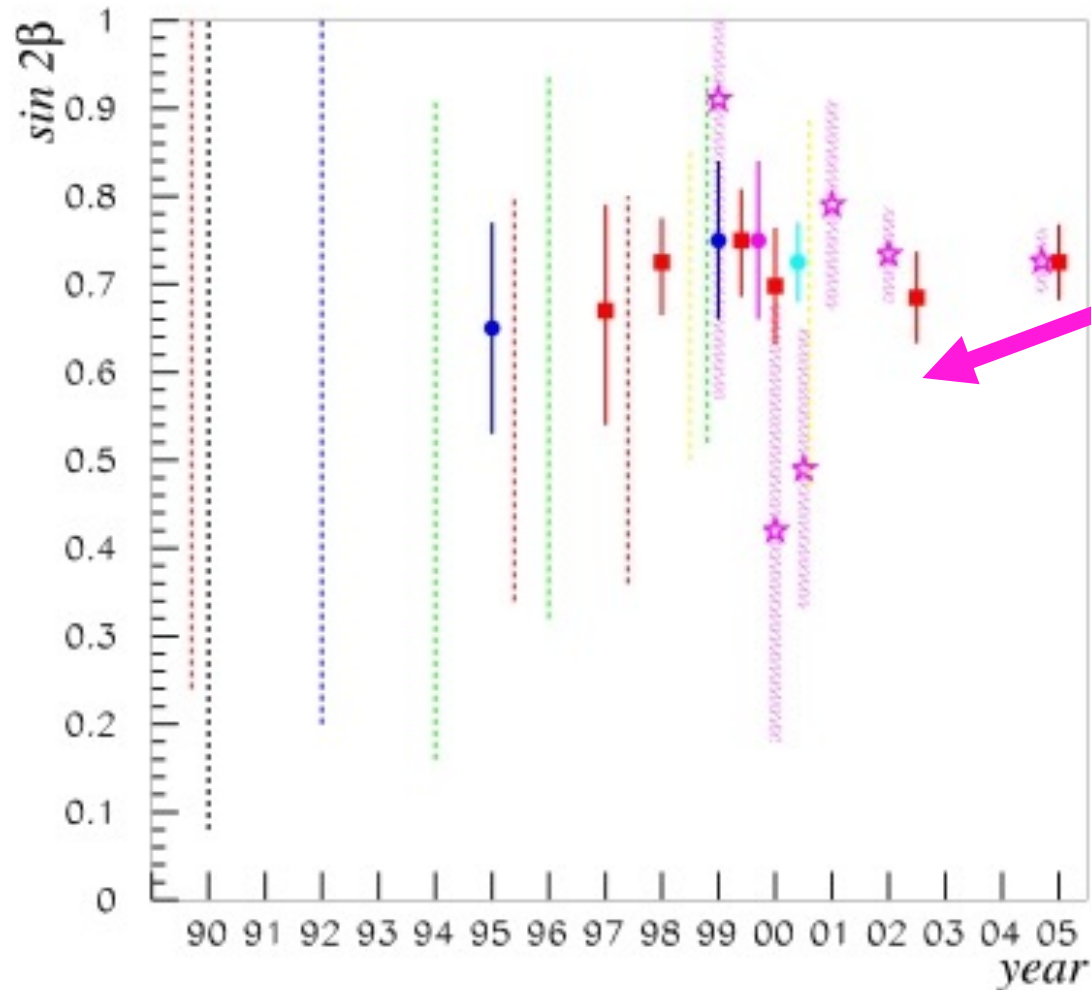
# Theoretical predictions of $\sin 2\beta$

2004

in the years

predictions  
exist since '95

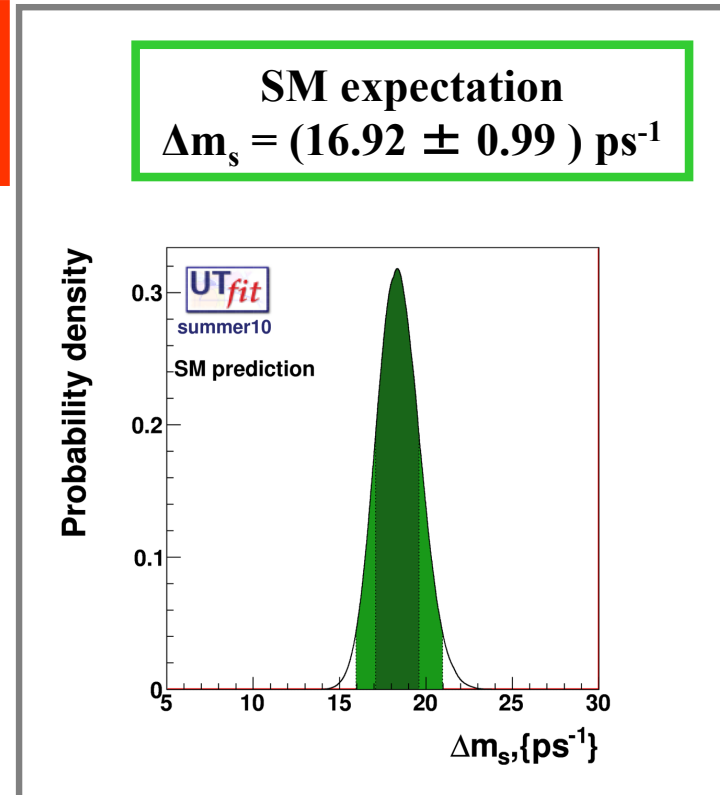
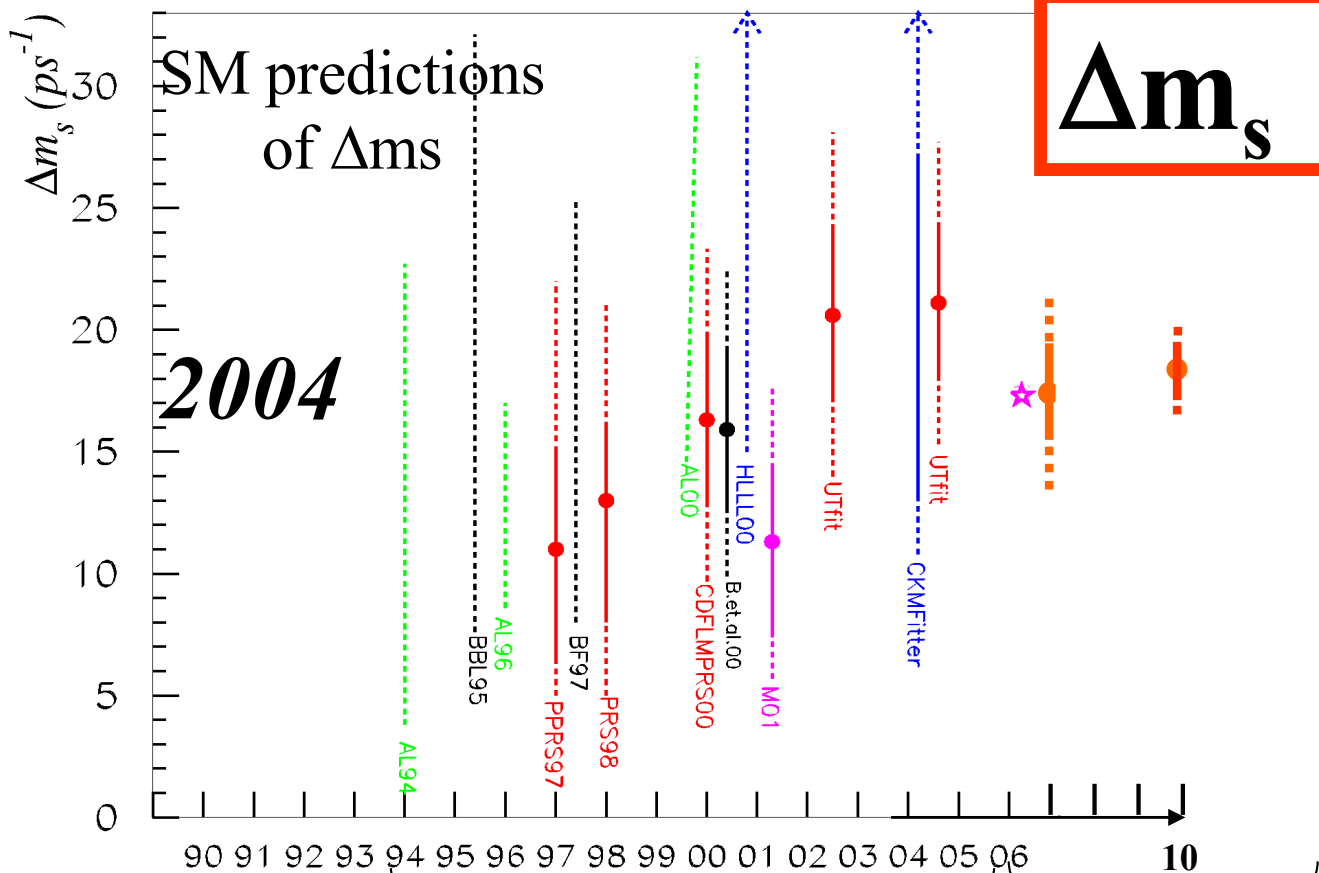
experiments



$\sin 2\beta_{\text{UTA}} = 0.65 \pm 0.12$   
Prediction 1995 from  
Ciuchini, Franco, G.M., Reina, Silvestrini

$\sin 2\beta_{\text{UTfit}} = 0.736 \pm 0.028$   
2023

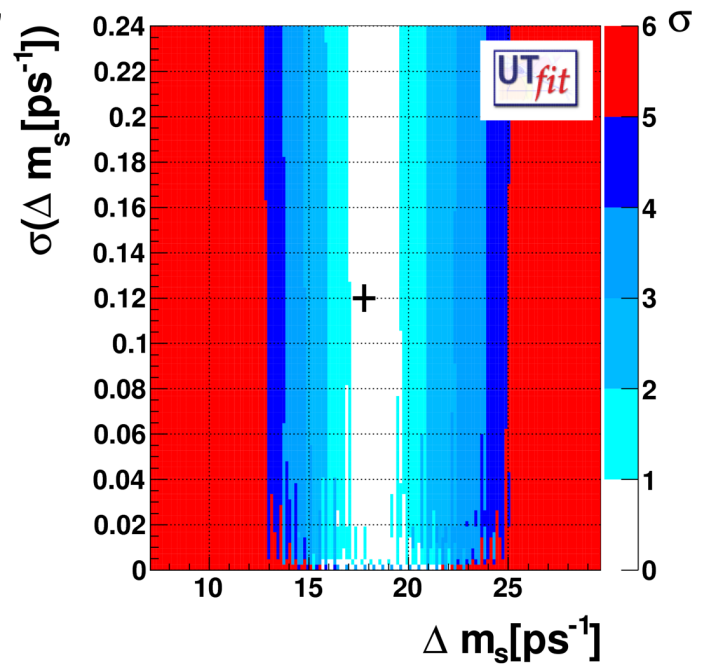
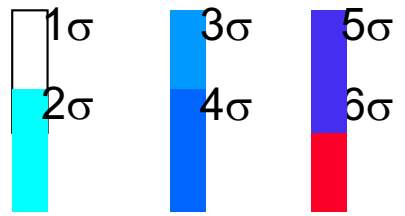
$\sin 2\beta_{\text{exp}} = 0.688 \pm 0.020$   
2023 pull=-1.4



**Exp**  
 $\Delta m_s = (17.72 \pm 0.04) \text{ ps}^{-1}$

**SM expectation 2023**  
 $\Delta m_s = (17.94 \pm 0.69) \text{ ps}^{-1}$   
**Exp**  
 $\Delta m_s = (17.741 \pm 0.020)_1 \text{ ps}^{-1}$

Prediction "era"      Monitoring "era"



# *Flavour Physics (on the Lattice)*

1963: Cabibbo Angle

1964: CP violation in K decays \*

1970 GIM Mechanism

1973: CP Violation needs at least three quark families (CKM) \*

1975: discovery of the tau lepton – 3<sup>rd</sup> lepton family \*

1977: discovery of the b quark - 3<sup>rd</sup> quark family \*

2003/4: CP violation in B meson decays

*\* Nobel Prize*



# Flavor physics in the Standard Model

In the SM, the quark mass matrix, from which the CKM matrix and  $\mathcal{CP}$  violation originate, is determined by the coupling of the Higgs boson to fermions.

$$\mathcal{L}_{\text{quarks}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Yukawa}}$$

$\mathcal{CP}$  invariant

$\mathcal{CP}$  and symmetry breaking are strictly correlated

$$\mathcal{L}(\Lambda_{\text{Fermi}}) = \mathcal{L}(\Lambda, H, H^\dagger) + \mathcal{L}^{\text{kin}} + \mathcal{L}_{\text{SM}}^{\text{gauge}} + \mathcal{L}_{\text{SM}}^{\text{Yukawa}} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \dots$$

EWSB

has many accidental symmetries

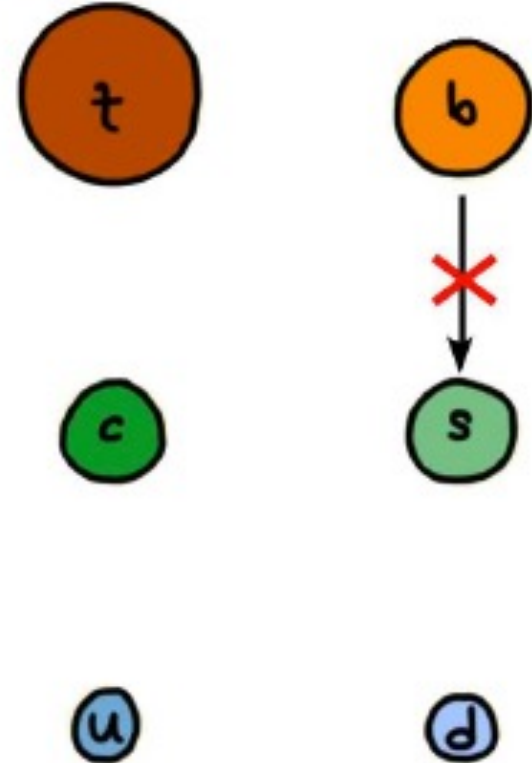
may violate accidental symmetries

*Absence of FCNC at tree level (& GIM suppression of FCNC @loop level)*

*Almost no CP violation at tree level*

*Flavour Physics is extremely sensitive to New Physics (NP)*

*In competition with Electroweak Precision Measurements*





# WHY RARE DECAYS ?

Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

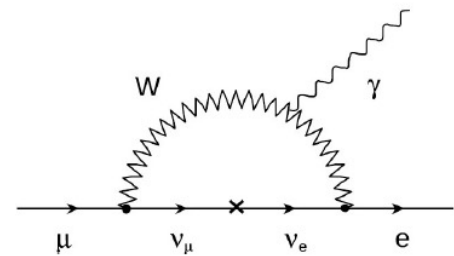
Proton decay

baryon and lepton number conservation

$\mu \rightarrow e + \gamma$

lepton flavor number

$\nu_i \rightarrow \nu_k$  **found !**



$$B(\mu \rightarrow e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

# RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

FCNC:

$$q_i \rightarrow q_k + \nu \bar{\nu}$$

$$q_i \rightarrow q_k + l^+ l^-$$

$$q_i \rightarrow q_k + \gamma$$

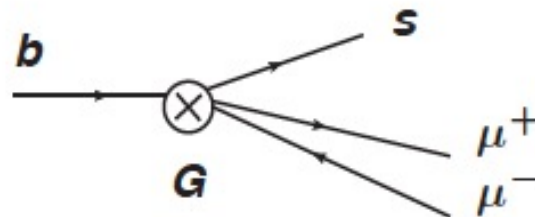
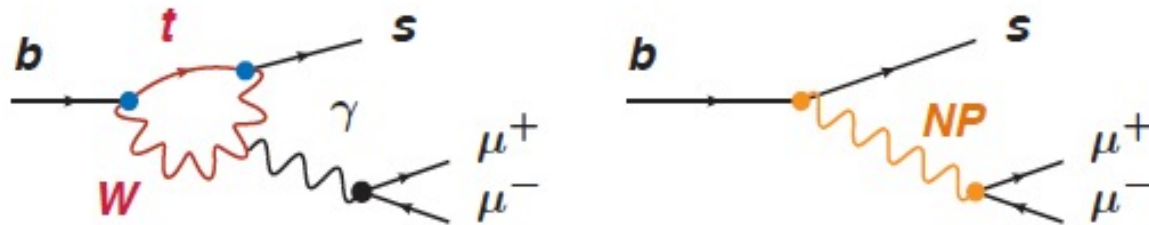
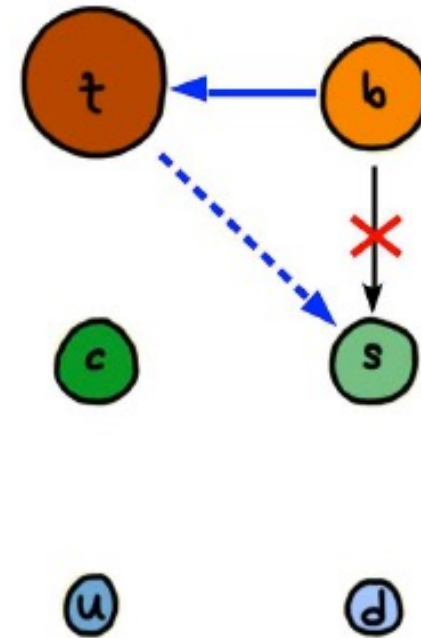
these decays occur only via loops because of GIM and are suppressed by CKM

**THUS THEY ARE SENSITIVE TO  
NEW PHYSICS**

# Flavor Changing Neutral Currents in the SM

In the SM, flavor changing neutral currents (FCNCs) are absent at the tree level

FCNCs can arise at the **loop level** they are suppressed by **loop factors** and small **CKM elements**



$$G \sim \frac{1}{16\pi^2} \frac{g^4}{m_W^2} \frac{m_t^2}{m_W^2} V_{tb} V_{ts}^* + \frac{C_{NP}}{\Lambda_{NP}^2}$$

→ measuring low energy flavor observables gives information on new physics flavor couplings and the new physics mass scale

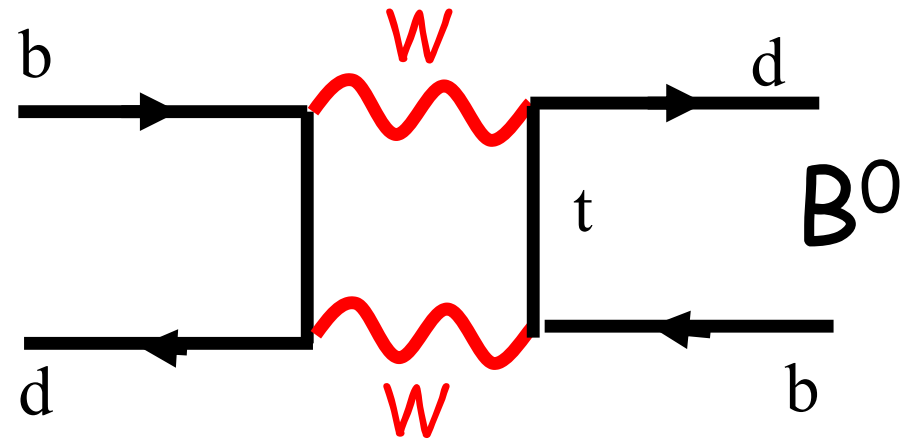
# B<sup>0</sup> - $\bar{B}^0$ mixing

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

## $\Delta B=2$ Transitions

$$\mathcal{H}_{eff}^{\Delta B=2} = \text{[Diagram: A circle with a blue 'O' inside, four black lines crossing at the center, representing an operator insertion.]}$$

$\bar{B}^0$



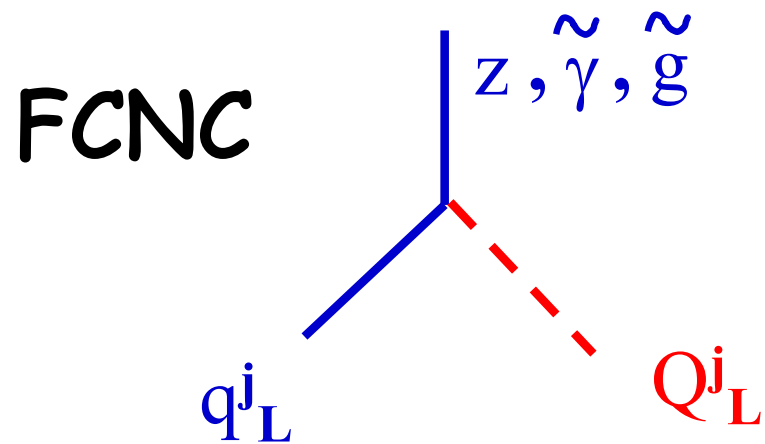
$$\propto \left( \bar{d} \gamma_\mu (1 - \gamma_5) b \right)^2$$

Hadronic matrix element

CKM

$$\Delta m_{d,s} = \frac{G_F^2 M_W^2}{16 \pi^2} A^2 \lambda^6 F_{tt} \left( \frac{m_t^2}{M_W^2} \right) \langle O \rangle$$

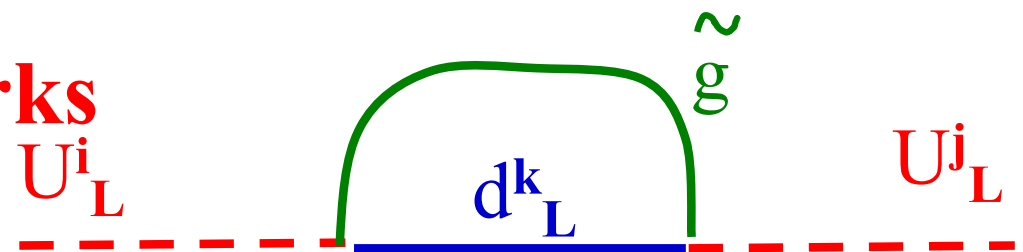
In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case **We may either**  
**Diagonalize the SMM**



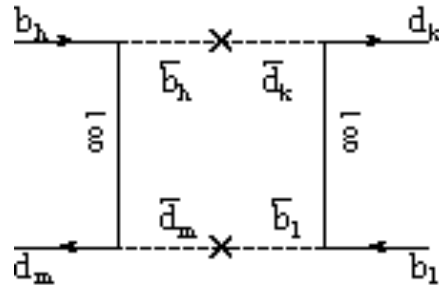
**or Rotate by the same matrices**

**the SUSY partners of the u- and d- like quarks**

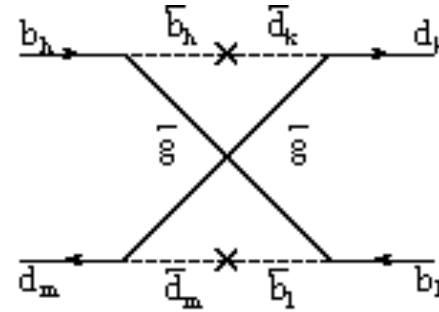
$$(Q_L^j)' = U_L^{ij} Q_L^j$$



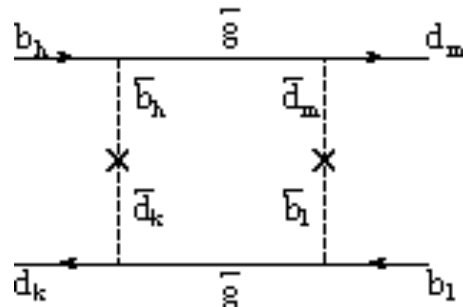
# In the latter case the Squark Mass Matrix is not diagonal



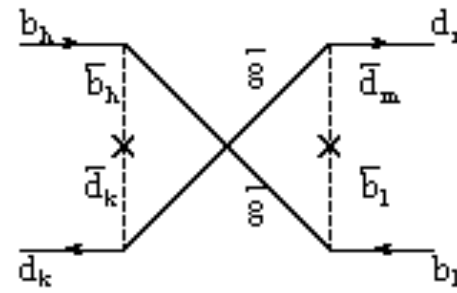
a)



c)



b)



d)

$$(m^2_Q)_{ij} = m^2_{average} \mathbf{1}_{ij} + \Delta m_{ij}^2 \quad \delta_{ij} = \Delta m_{ij}^2 / m^2_{average}$$

# CP Violation in the Standard Model

*After the diagonalisation of the quark mass matrix*

$$L_{CC}^{weak\ int} = \frac{g_W}{\sqrt{2}} (J_\mu^- W_\mu^+ + h.c.)$$
$$\rightarrow \frac{g_W}{\sqrt{2}} (\bar{u}_L \mathbf{V}^{CKM} \gamma_\mu d_L W_\mu^+ + \dots)$$

$N(N-1)/2$  angles and  $(N-1)(N-2)/2$  phases

$N=3$  3 angles + 1 phase KM  
 the phase generates complex couplings i.e. CP violation;

6 masses + 3 angles + 1 phase = 10 parameters

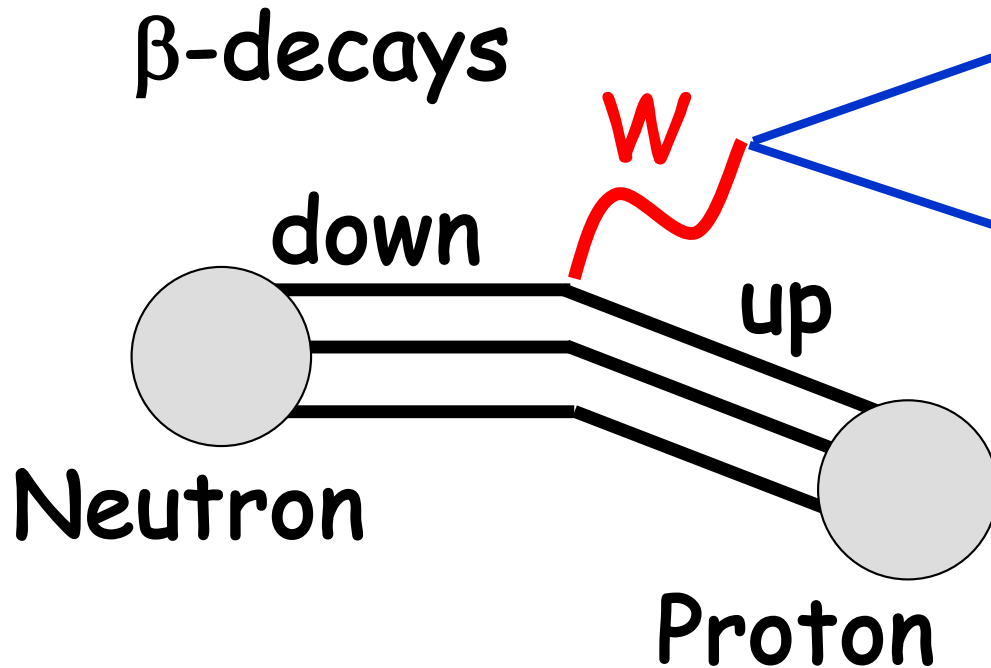
$V_{ud}$	$V_{us}$	$V_{ub}$
$V_{cd}$	$V_{cs}$	$V_{cb}$
$V_{td}$	$V_{ts}$	$V_{tb}$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$



# Quark masses & Generation Mixing

$V_{ud}$	$V_{us}$	$V_{ub}$
$V_{cd}$	$V_{cs}$	$V_{cb}$
$V_{td}$	$V_{ts}$	$V_{tb}$



$$|V_{ud}|$$

*updated values later* (0.999)

$ V_{ud}  = 0.9735(8)$
$ V_{us}  = 0.2196(23)$
$ V_{cd}  = 0.224(16)$
$ V_{cs}  = 0.970(9)(70)$
$ V_{cb}  = 0.0406(8)$
$ V_{ub}  = 0.00409(25)$
$ V_{tb}  = 0.99(29)$

# The Wolfenstein Parametrization

$1 - 1/2 \lambda^2$	$\lambda$	$A \lambda^3(\rho - i \eta)$
$-\lambda$	$1 - 1/2 \lambda^2$	$A \lambda^2$
$A \lambda^3 \times$ $(1 - \rho - i \eta)$	$-A \lambda^2$	$1$

$V_{ub}$

$+ O(\lambda^4)$

$V_{td}$

$$\lambda \sim 0.2 \quad A \sim 0.8$$

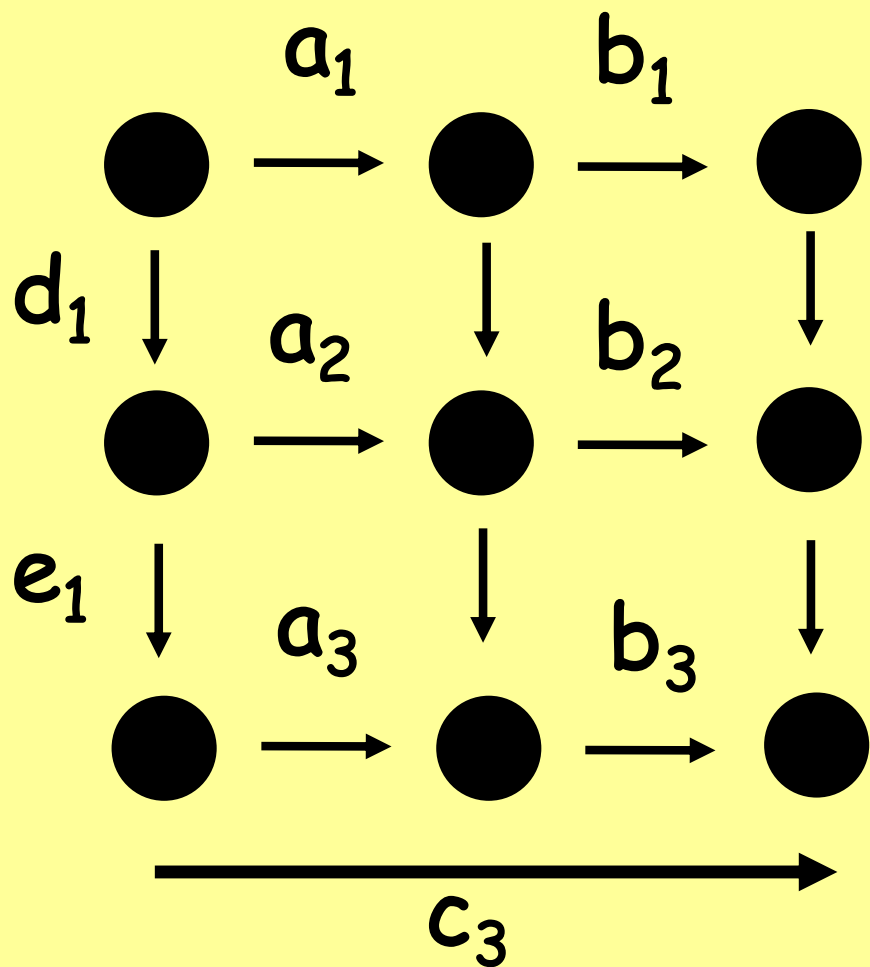
$$\eta \sim 0.2 \quad \rho \sim 0.3$$

$$\sin \theta_{12} = \lambda$$

$$\sin \theta_{23} = A \lambda^2$$

$$\sin \theta_{13} = A \lambda^3(\rho - i \eta)$$

# The Bjorken-Jarlskog Unitarity Triangle



$|V_{ij}|$  is invariant under phase rotations

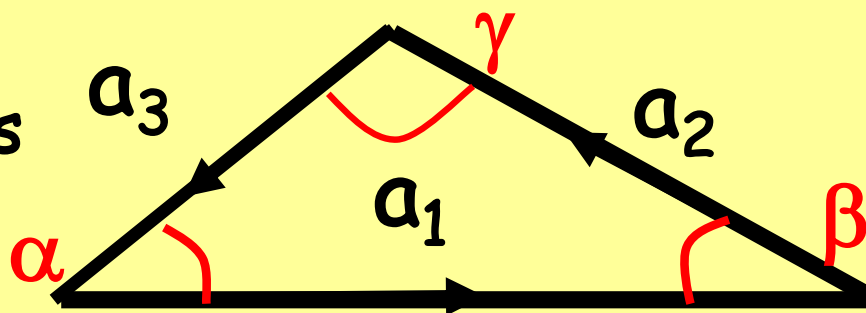
$$a_1 = V_{11} V_{12}^* = V_{ud} V_{us}^*$$

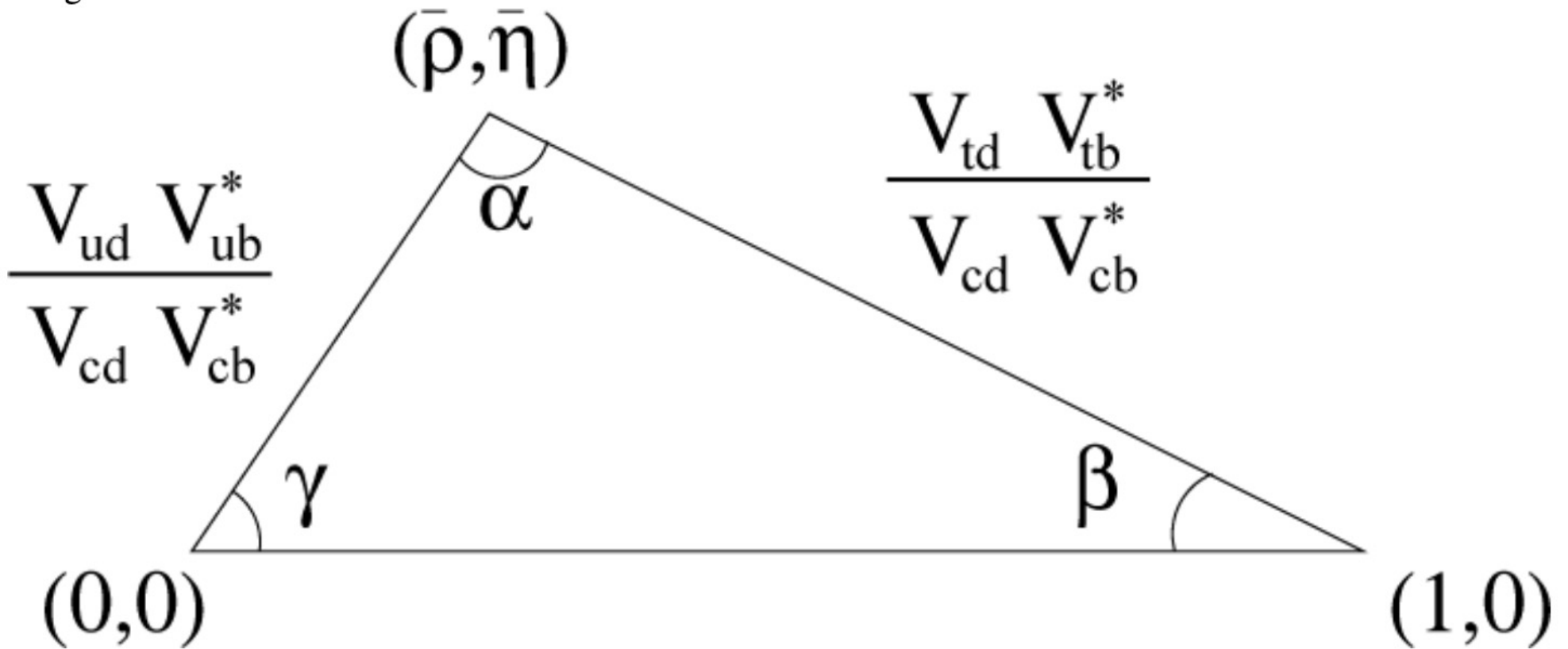
$$a_2 = V_{21} V_{22}^* \quad a_3 = V_{31} V_{32}^*$$

$$a_1 + a_2 + a_3 = 0$$

( $b_1 + b_2 + b_3 = 0$  etc.)

Only the orientation depends on the phase convention





***The Standard Triangle of the Standard Model***

# STRONG CP VIOLATION

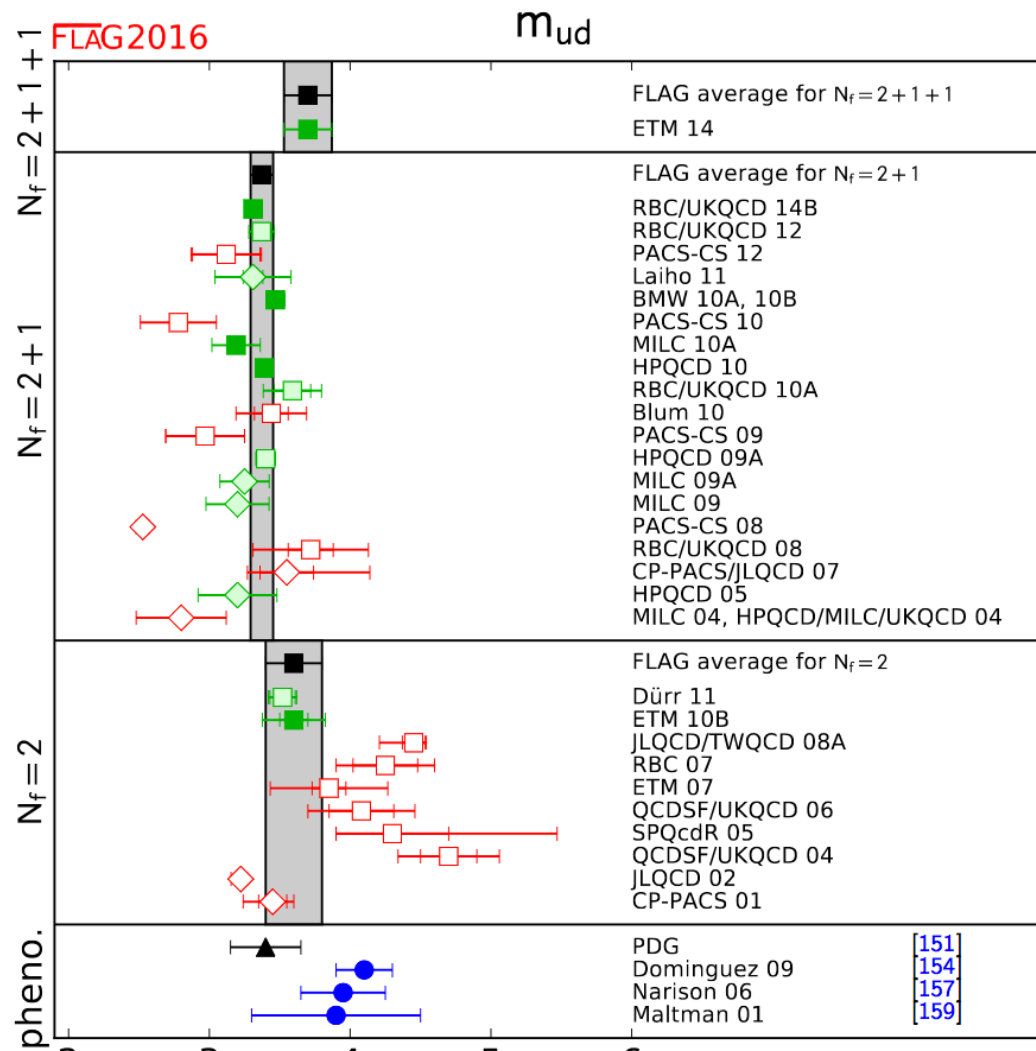
$$\mathcal{L}_\theta = \theta \tilde{G}^{\mu\nu a} G_{\mu\nu}^a \quad \tilde{G}_{\mu\nu}^a = \varepsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}^a$$

$$\mathcal{L}_\theta \sim \theta \vec{E}^a \cdot \vec{B}^a$$

This term violates  $CP$  and gives a contribution to the electric dipole moment of the neutron

$$e_n < 3 \cdot 10^{-26} \text{ e cm}$$

$$\theta < 10^{-10} \quad \text{which is quite unnatural !!}$$



$N_f$	$m_u$	$m_d$	$m_u/m_d$	$R$	$Q$
2+1+1	2.36(24)	5.03(26)	0.470(56)	35.6(5.1)	22.2 (1.6)
2+1	2.16(9)(7)	4.68(14)(7)	0.46(2)(2)	35.0(1.9)(1.8)	22.5(6)(6)
2	2.40(23)	4.80(23)	0.50(4)	40.7(3.7)(2.2)	24.3(1.4)(0.6)

*The extraordinary progress of the experimental measurements requires accurate theoretical predictions*

*Precision flavor physics requires the control of hadronic effects for which lattice QCD simulations are essential*

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

**SM**

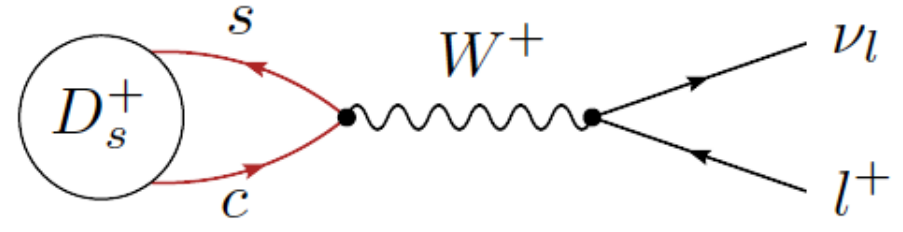
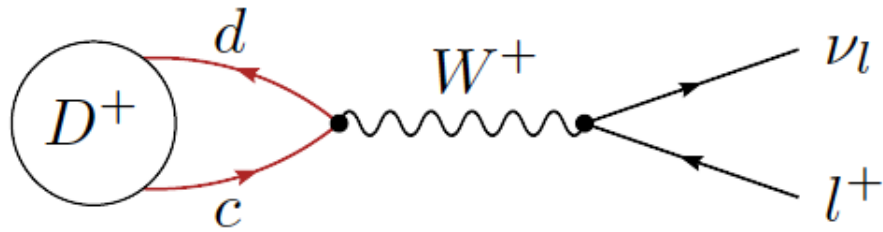
$$Q^{EXP} = \sum_i C_{SM}^i(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C_{Beyond}^{i'}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$

**BSM**

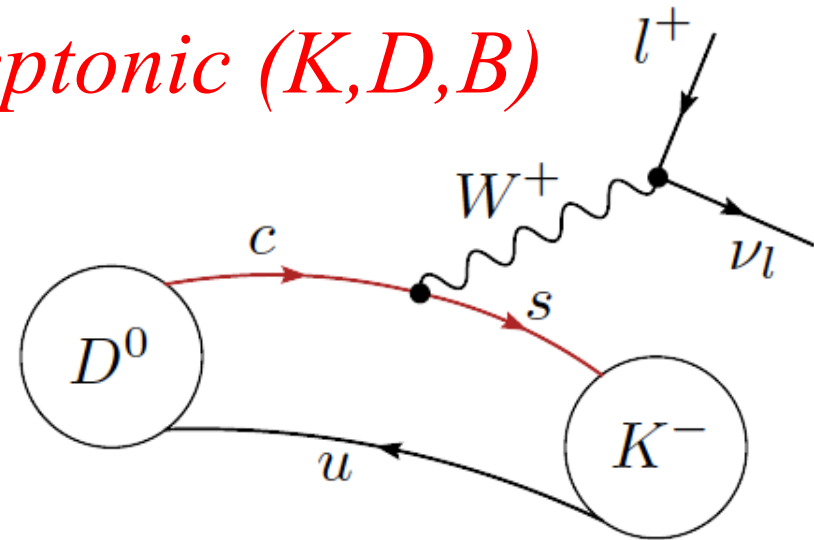
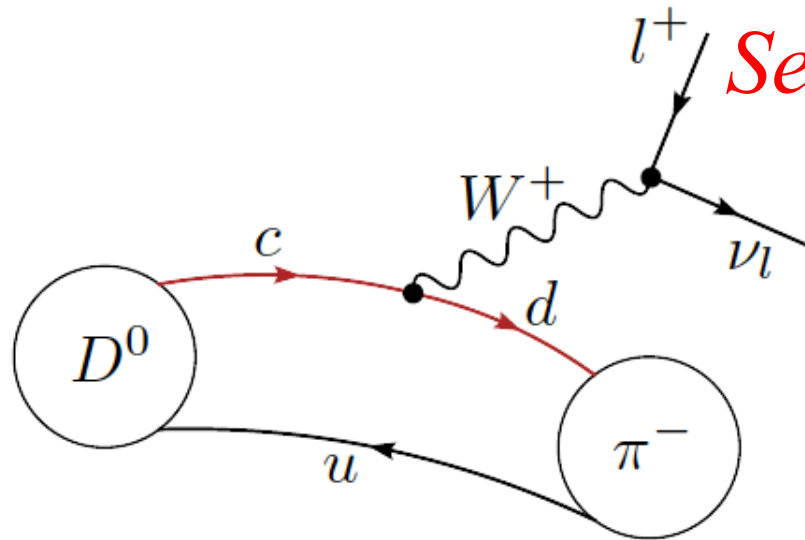
*What can be computed and what cannot be computed*



# Leptonic ( $\pi, K, D, B$ )

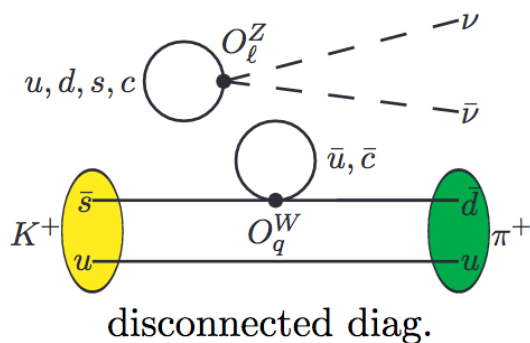
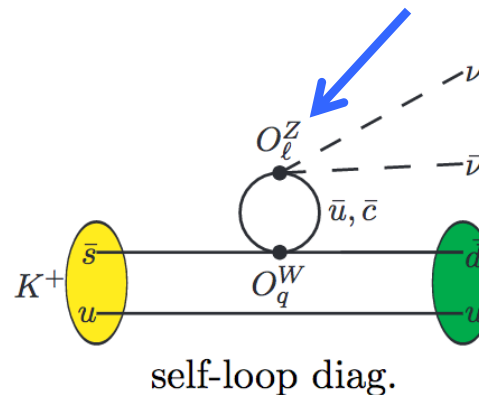
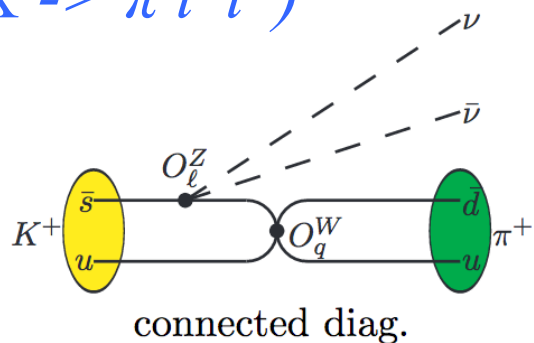


# Semileptonic ( $K, D, B$ )



(some) Radiative and Rare  
(also  $K \rightarrow \pi l^+ l^-$ )

long distance effects

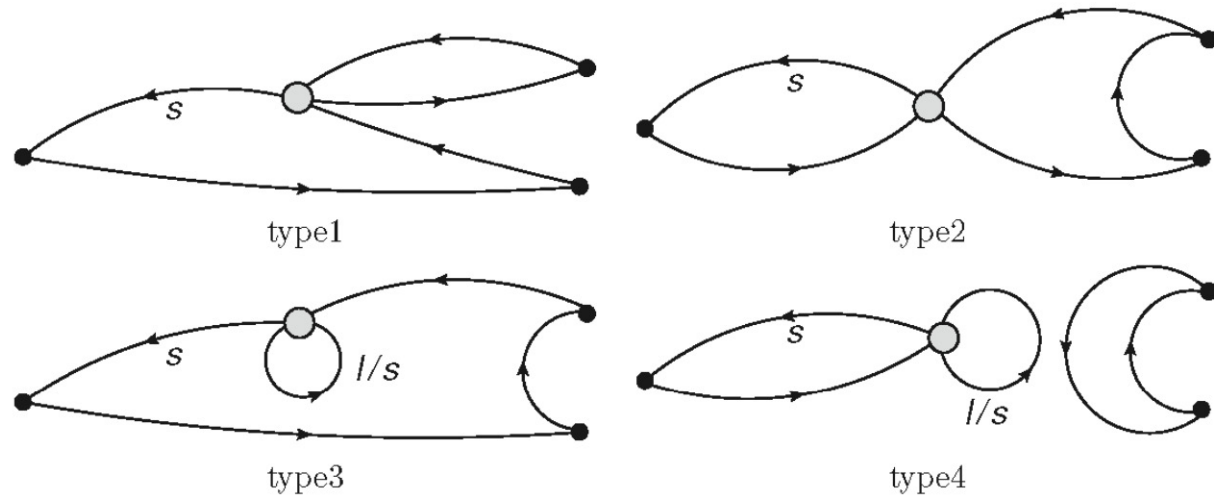




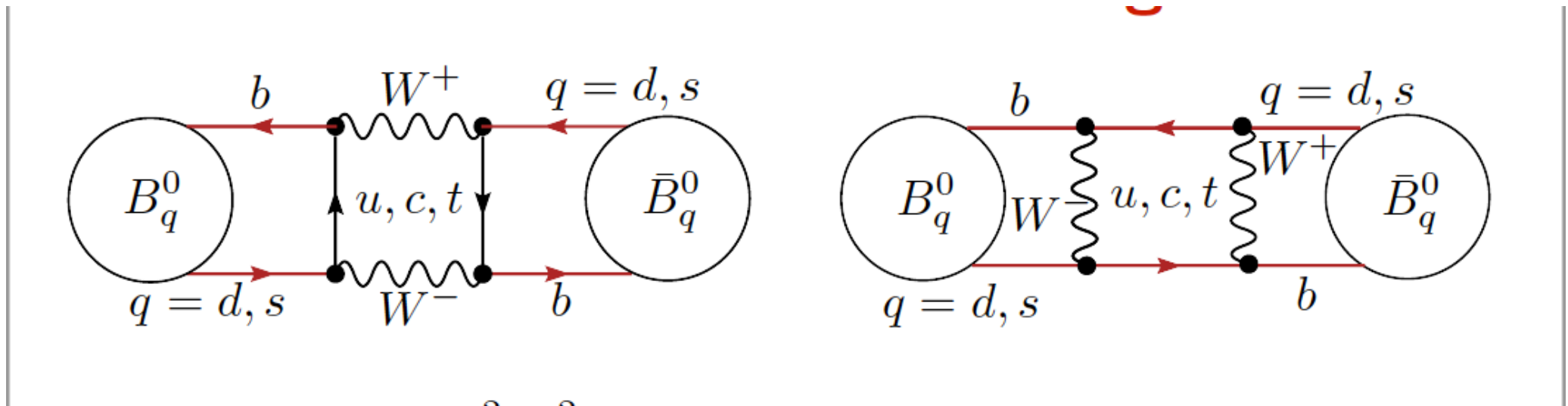
*Non-leptonic*

*but only below the inelastic threshold  
(may be also 3 body decays)*

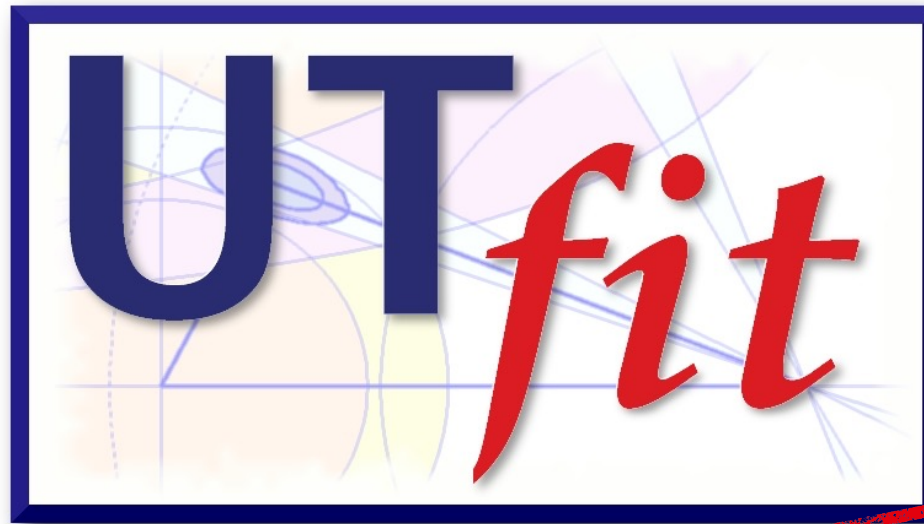
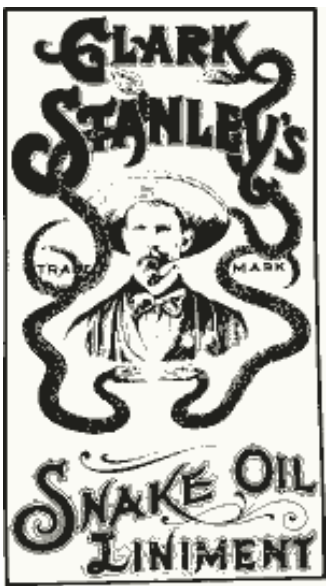
*$B \rightarrow \pi\pi, K\pi, \text{ etc. No !}$*



*Neutral meson mixing (local)*



*+ some long distance contributions to K and D neutral meson mixing + short distance contributions to  $B \rightarrow K^{(*)} l^+ l^-$*



[www.utfit.org](http://www.utfit.org)



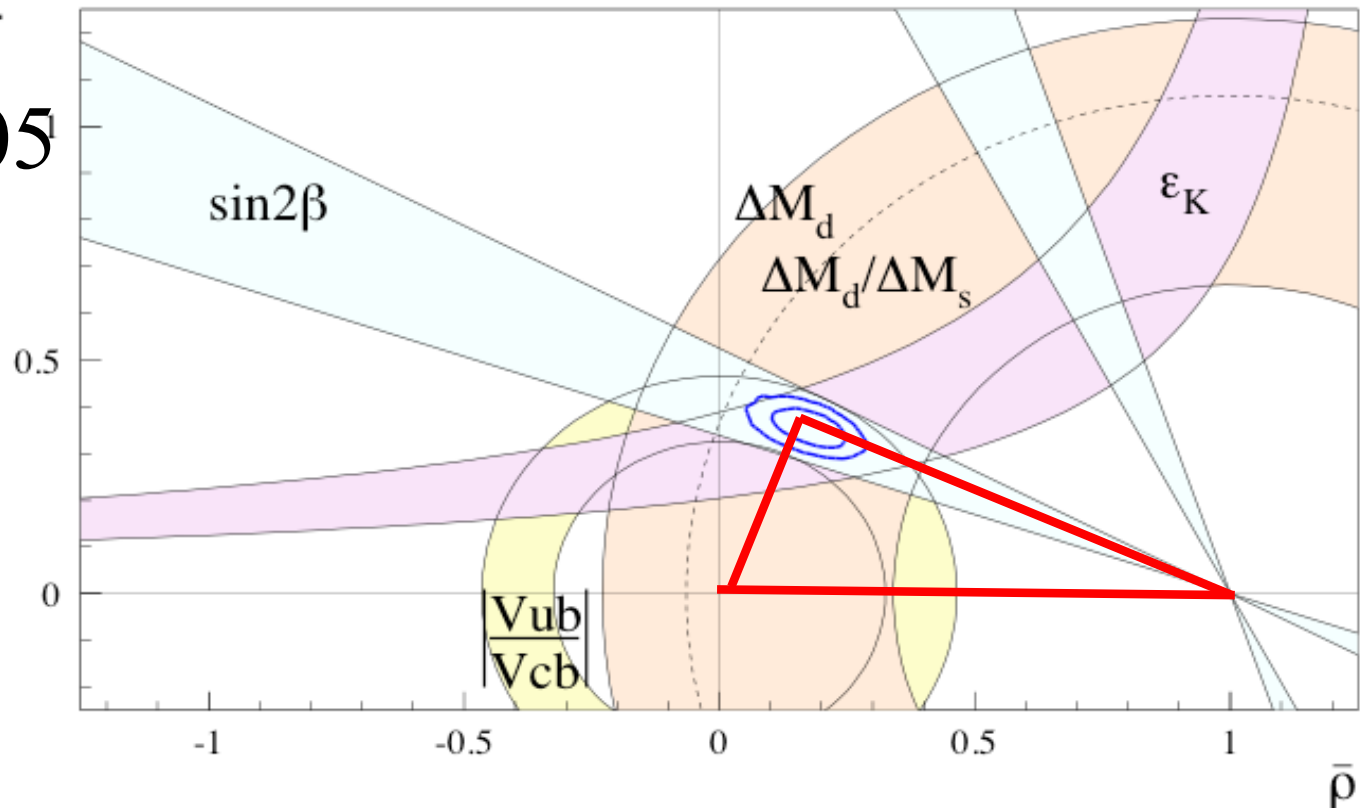
*M. Bona, M. Ciuchini, D. Derkach, F. Ferrari, E. Franco,  
V. Lubicz, G. Martinelli, D. Morgante, M. Pierini,  
L. Silvestrini, S. Simula, C. Tarantino, V. Vagnoni,  
M. Valli, and L. Vittorio*

*2023*

# Unitary Triangle SM

2005

semileptonic decays



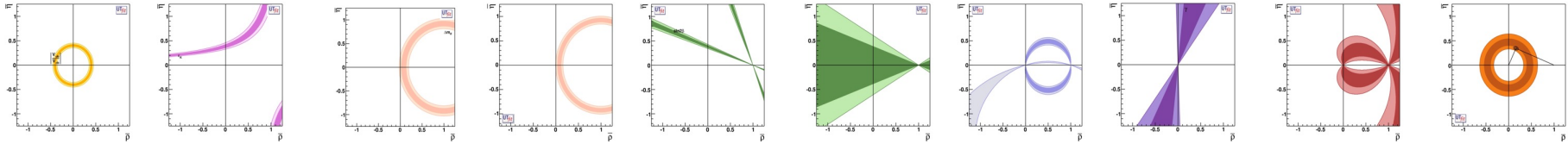
Experimental constraints

Meas.	$V_{CKM} \times \text{other}$	$(\bar{\rho}, \bar{\eta})$
$\frac{b \rightarrow u}{b \rightarrow c}$	$ V_{ub}/V_{cb} ^2$	$\bar{\rho}^2 + \bar{\eta}^2$
$\Delta m_d$	$ V_{td} ^2 f_{B_d}^2 B_{B_d}$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left  \frac{V_{td}}{V_{ts}} \right ^2 \xi^2$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
$\epsilon_K$	$f(A, \bar{\eta}, \bar{\rho}, B_K)$	$\propto \bar{\eta}(1 - \bar{\rho})$
$A(J/\psi K^0)$	$\sin 2\beta$	$\frac{2\bar{\eta}(1 - \bar{\rho})}{\sqrt{\bar{\eta}^2 + (1 - \bar{\rho})^2}}$

$B_{d,s}^0 - \bar{B}_{d,s}^0$  mixing

$K^0 - \bar{K}^0$  mixing

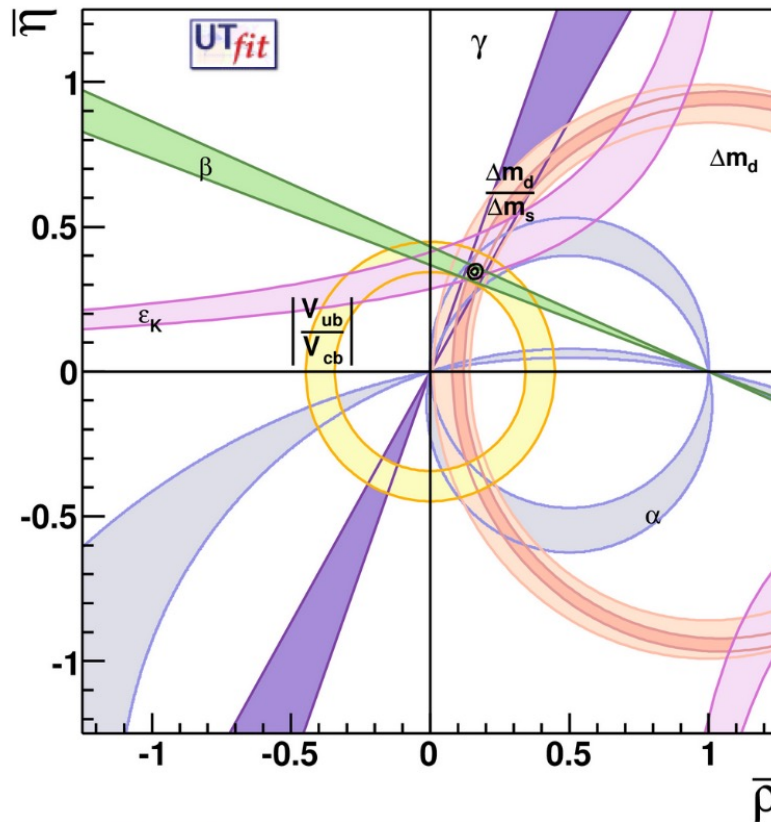
$B_d$



# 2023 results

$$\bar{\rho} = 0.1609 \pm 0.0095 \quad \bar{\eta} = 0.347 \pm 0.010$$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation



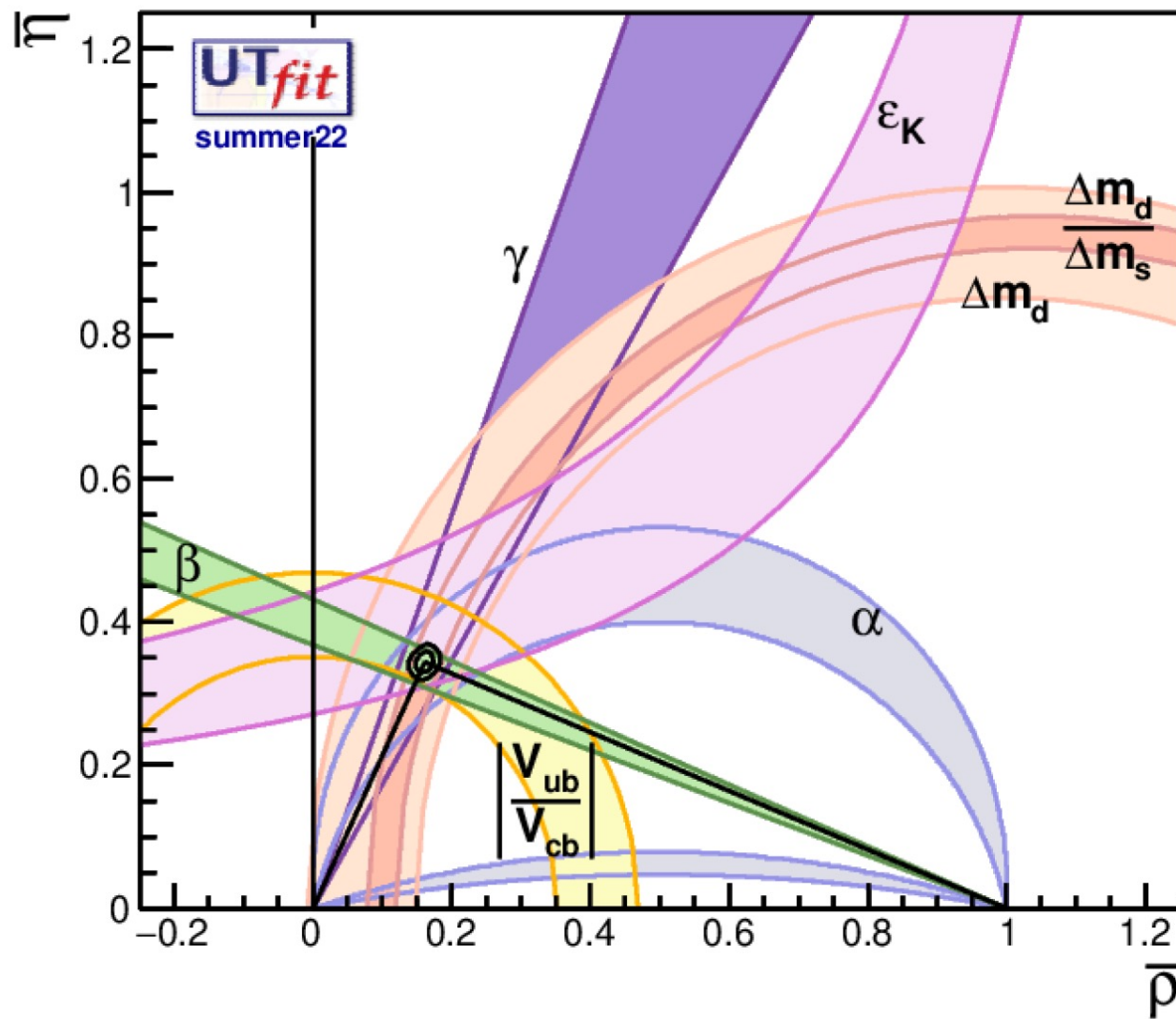
$$\begin{aligned} \alpha &= (94.9 \pm 4.7)^\circ \\ \sin 2\beta &= 0.688 \pm 0.0206 \\ \beta &= (22.46 \pm 0.68)^\circ \\ \gamma &= (66.1 \pm 3.5)^\circ \\ A &= 0.828 \pm 0.011 \\ \lambda &= 0.22519 \pm 0.00083 \end{aligned}$$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

# Unitarity Triangle analysis in the SM:

zoomed in..



levels @  
95% Prob

~6%

$$\rho = 0.1609 \pm 0.0095$$
$$\eta = 0.347 \pm 0.010$$

~3%

# Progressi dall'epoca della tesi di Pierini

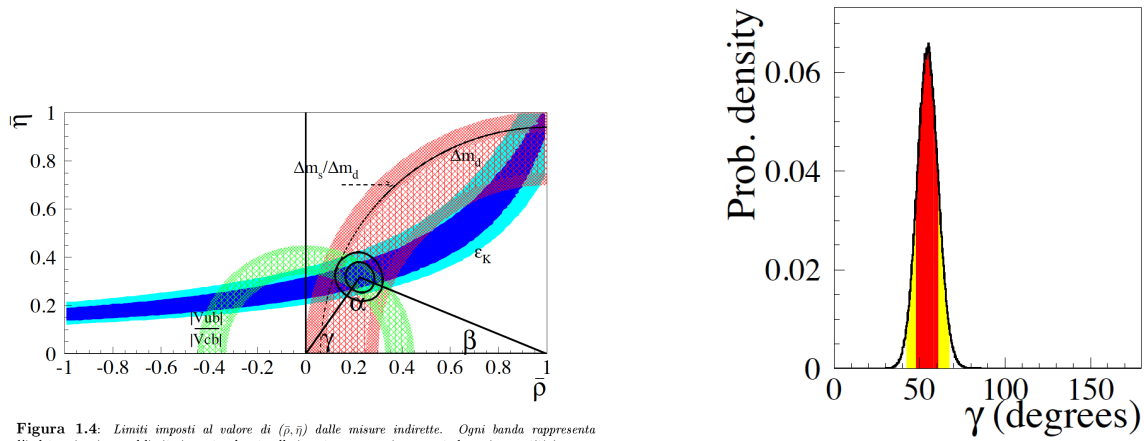
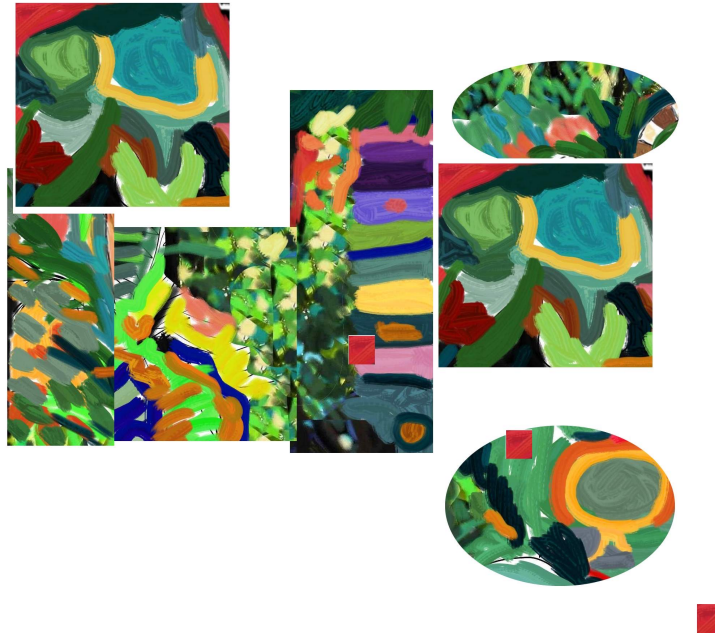
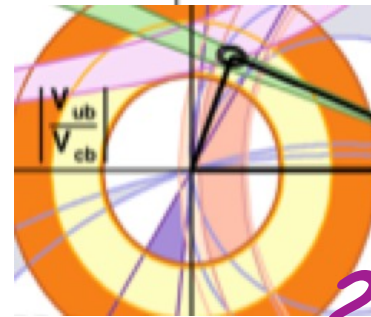
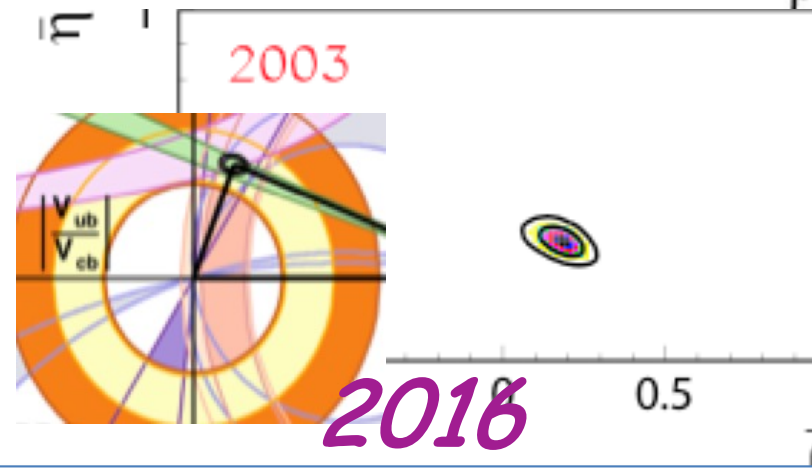
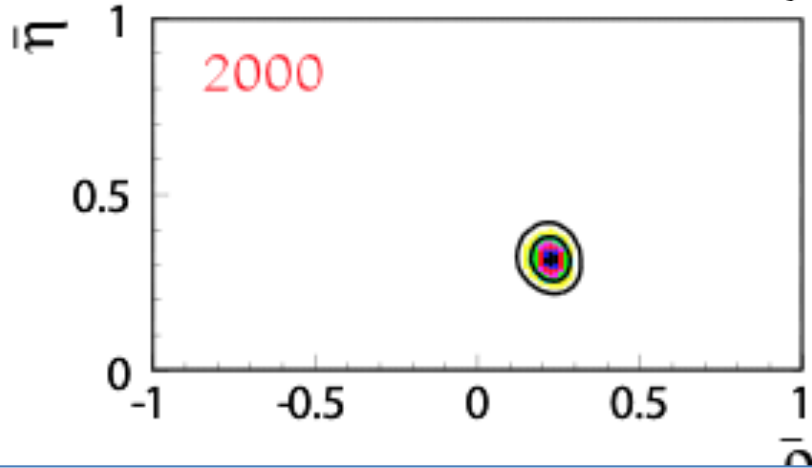
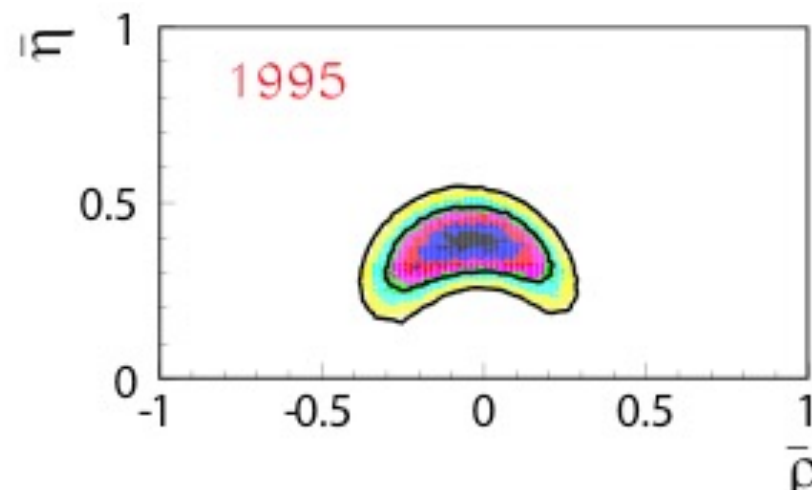
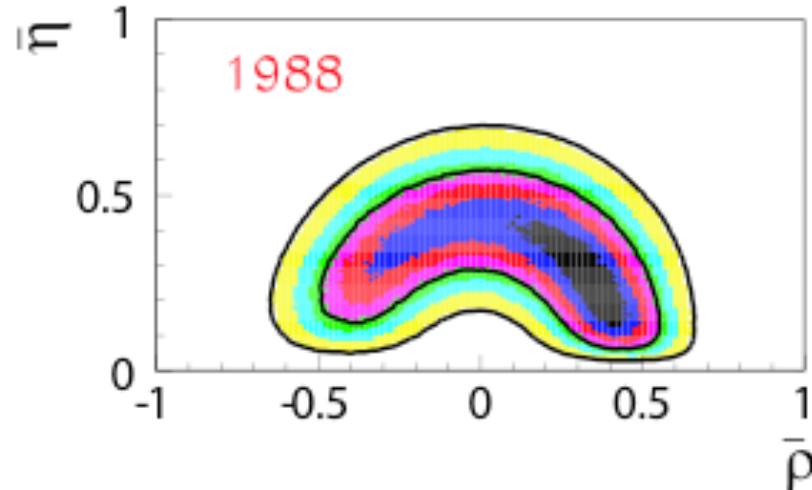


Figura 1.4: Limiti imposti al valore di  $(\rho, \eta)$  dalle misure indirette. Ogni banda rappresenta l'indeterminazione sul limite imposto, dovuto alle incertezze con cui sono note le varie quantità.



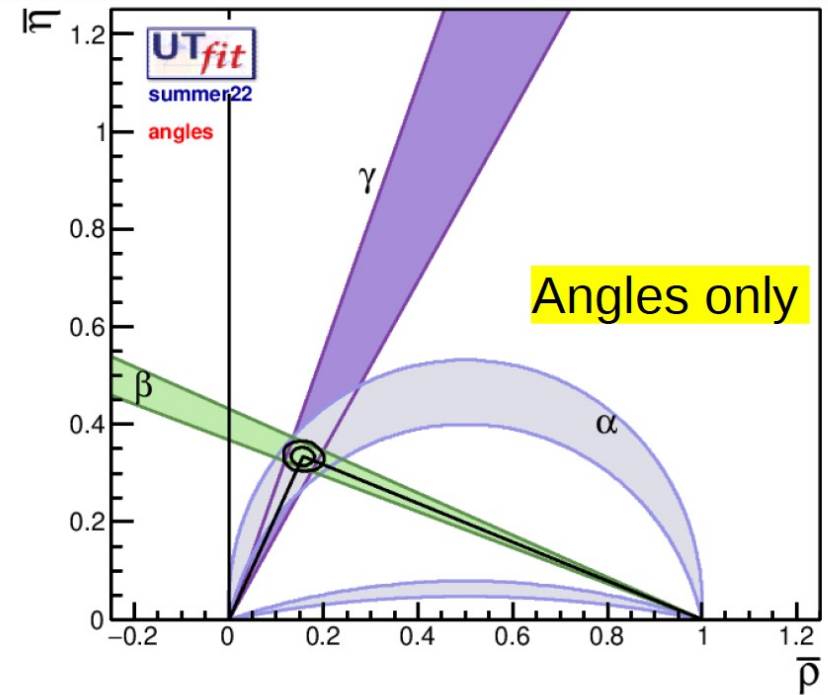
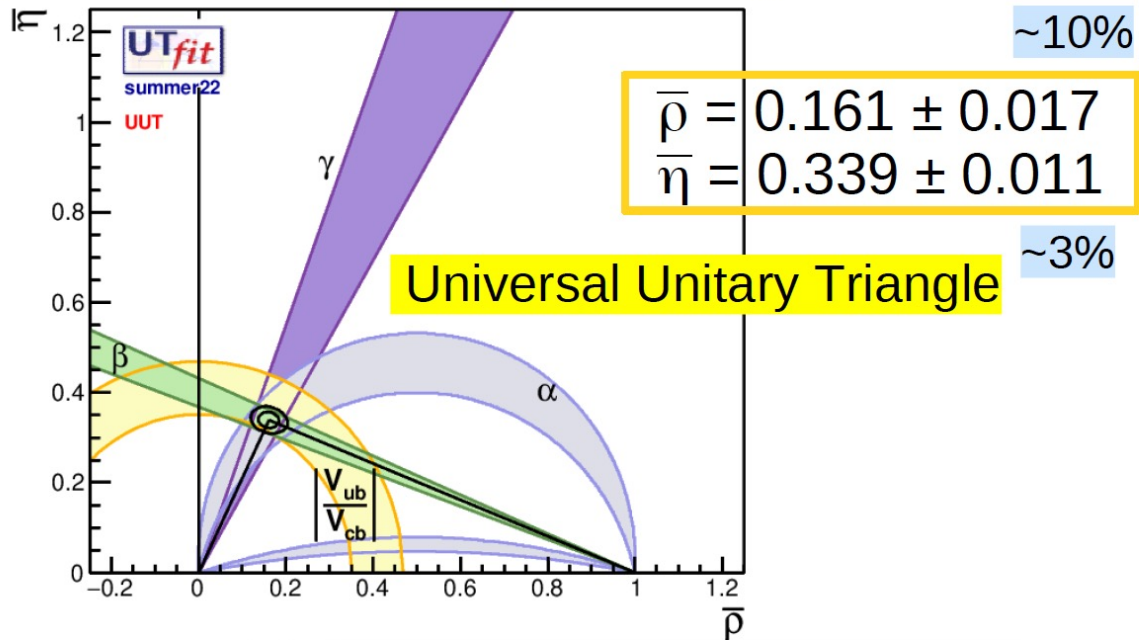
# PROGRESS SINCE 1988

*Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)*



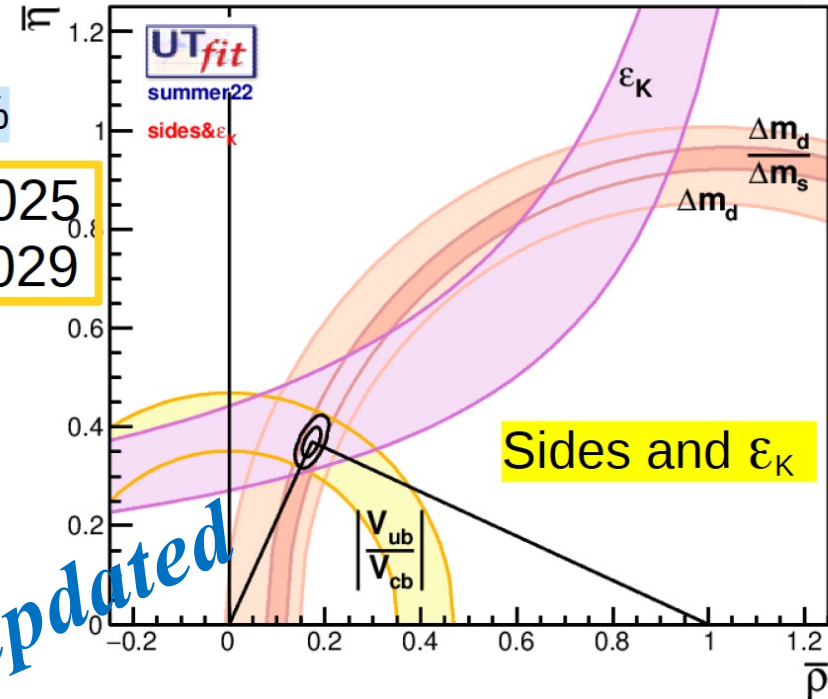
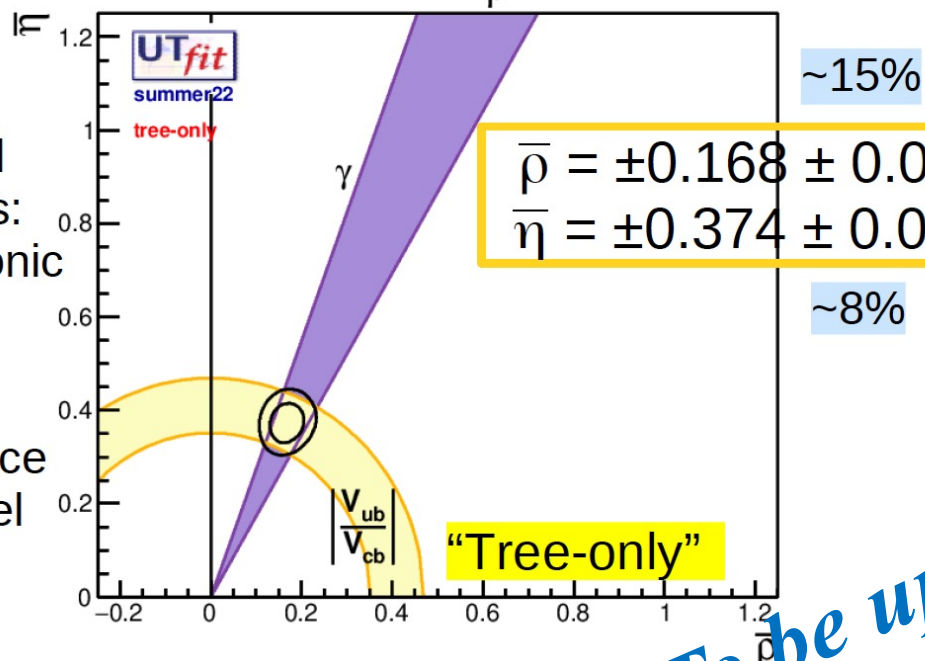
***BABAR played a fundamental role in this Progress***

# Some interesting configurations



Tree-level processes:  
Semileptonic  
and DK  
B decays

→ reference  
for model  
building



To be updated



# $V_{cb}$ and $V_{ub}$

from FLAG 2021

$$|V_{cb}| (excl) = (39.44 \pm 0.63) 10^{-3}$$

$$|V_{cb}| (incl) = (42.16 \pm 0.50) 10^{-3}$$

from Bordone et al.  $\sim 3.2\sigma$  discrepancy  
arXiv:2107.00604

$$|V_{ub}| (excl) = (3.74 \pm 0.17) 10^{-3}$$

$$|V_{ub}| (incl) = (4.32 \pm 0.29) 10^{-3}$$

from GGOU HFLAV 2021  $\sim 1.6\sigma$  discrepancy  
adding a flat uncertainty covering the spread of central values

$$|V_{ub} / V_{cb}| (LHCb) = (9.46 \pm 0.79) 10^{-2}$$

$$|V_{ub} / V_{cb}| (LHCb) = (7.9 \pm 0.6) 10^{-2}$$

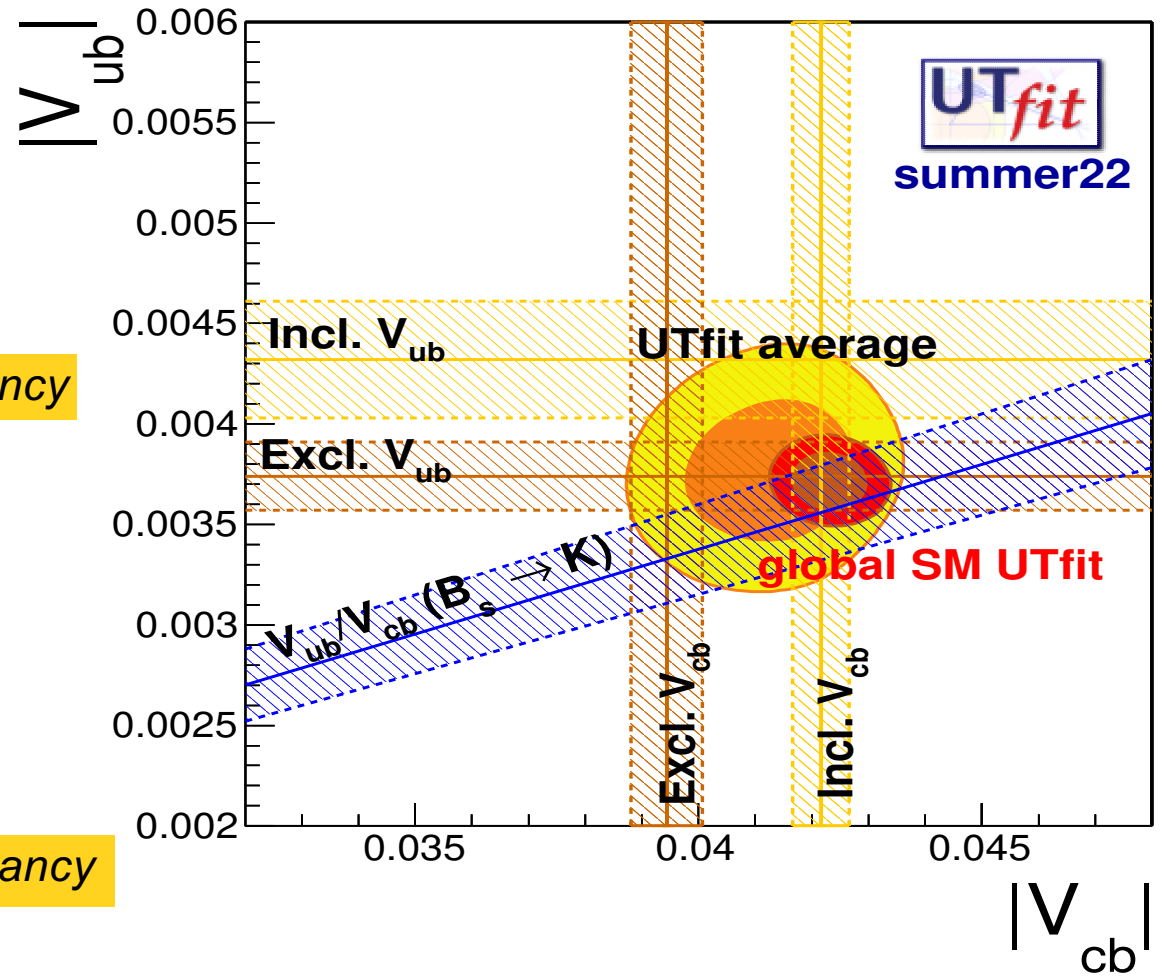
From global SM fit

$$|V_{cb}| = (42.00 \pm 0.47) 10^{-3} \quad |V_{ub}| = (3.715 \pm 0.093) 10^{-3}$$

$$Ufit Prediction V_{cb} = (42.22 \pm 0.51) 10^{-3}$$

$$V_{ub} = (3.70 \pm 0.11) 10^{-3}$$

# TENSIONS



From  $B_s$  to  $K$  at high  $q^2$

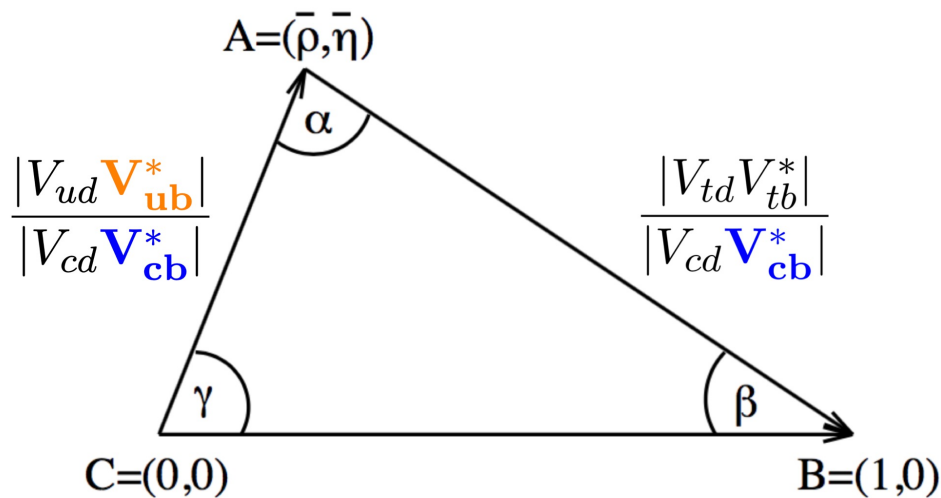
From  $\Lambda_b$ , excluded following FLAG guidelines

# *Exclusive semileptonic $B \rightarrow \{D(*), \pi\}$ decays through unitarity (and other developments)*

**Work in collaboration with M. Naviglio, S. Simula and L. Vittorio**

(PRD '21 (2105.02497), PRD '21 (2105.07851), PRD '22 (2105.08674), 2109.15248, 2204.05925, 2202.10285)

See talk by A. Vaquero



*Mr. Nosferatu  
from Transylvania*



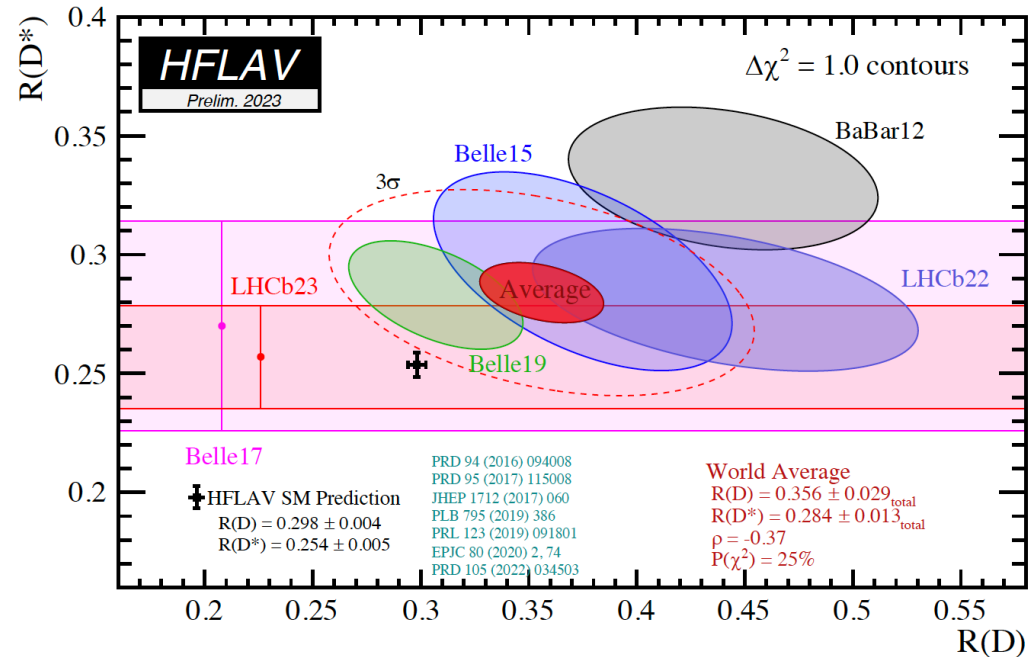
# State-of-the-art of the semileptonic $B \rightarrow \{D(^*), \pi\}$ decays

Two critical issues

- $V_{cb}$  - **exclusive/inclusive**  $|V_{cb}|$  **puzzle:**  
exclusive (FLAG '21):  $|V_{cb}|(BGL) \cdot 10^3 = 39.36$  (68)      inclusive (HFLAV '21):  $|V_{cb}| \cdot 10^3 = 42.19$  (78)  
 difference of  $\sim 2.7 \sigma$        $|V_{cb}| \cdot 10^3 = 42.16$  (50)  
 (Bordone et al. 2107.00604)
- $R_{D(^*)}$

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu_\ell)}$$

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \rightarrow D^*\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^*\ell\nu_\ell)}$$



HFLAV Collaboration, PRD '23 [arXiv:2206.07501] (updated plot)

3.2σ tension

# Main Results from the Dispersive Matrix Method

to show the relevant, attractive features of the **Dispersion Matrix (DM) approach** [arXiv:2105.02497], which is a rigorously model-independent tool for describing the hadronic form factors (FFs) in their whole kinematical range

- entirely based on first principles (i.e. lattice QCD simulations of 2- and 3-point Euclidean correlators)
- independent on any assumption about the momentum dependence of the FFs
- unitarization of the input theoretical data (including also kinematical constraints)
- **no mixing among theoretical calculations and experimental data to describe the shape of the FFs**

\* results for  $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu_\ell$  decays: extraction of  $|V_{cb}|$  and theoretical determination of  $R(D_{(s)}^{(*)})$  using LQCD results for the FFs (from FNAL/MILC and HPQCD) [2105.08674, 2109.15248, 2204.05925]

decay	$ V_{cb} ^{\text{DM}} \cdot 10^3$	inclusive	exclusive	observable	DM	experiment	difference
		[2107.00604]	[FLAG 21]	$R(D)$	0.296 (8)	0.340 (27) (13)	$\simeq 1.4 \sigma$
$B \rightarrow D$	$41.0 \pm 1.2$			$R(D^*)$	0.275 (8)	0.295 (11) (8)	$\simeq 1.3 \sigma$
$B \rightarrow D^*$	$41.3 \pm 1.7$			$R(D_s)$	0.298 (5)		
$B_s \rightarrow D_s$	$42.4 \pm 2.0$			$R(D_s^*)$	0.250 (6)		
$B_s \rightarrow D_s^*$	$41.4 \pm 2.6$						
average	$41.4 \pm 0.8$	$42.16 \pm 0.50$	$39.36 \pm 0.68$				
difference		$\simeq 0.8 \sigma$	$\simeq 1.9 \sigma$				

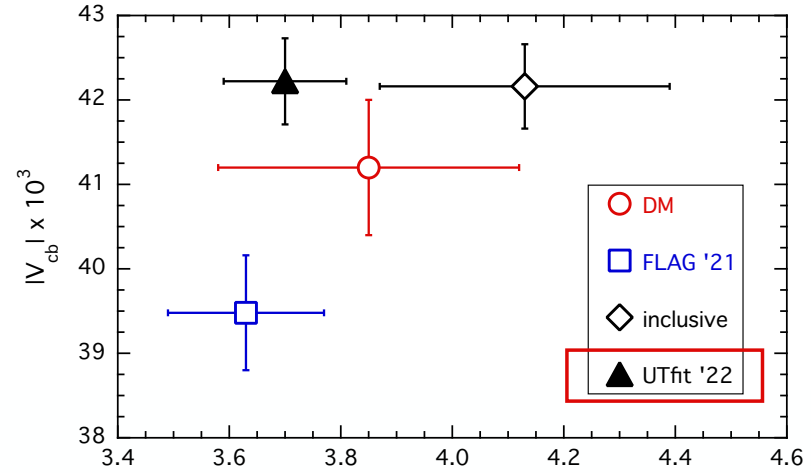
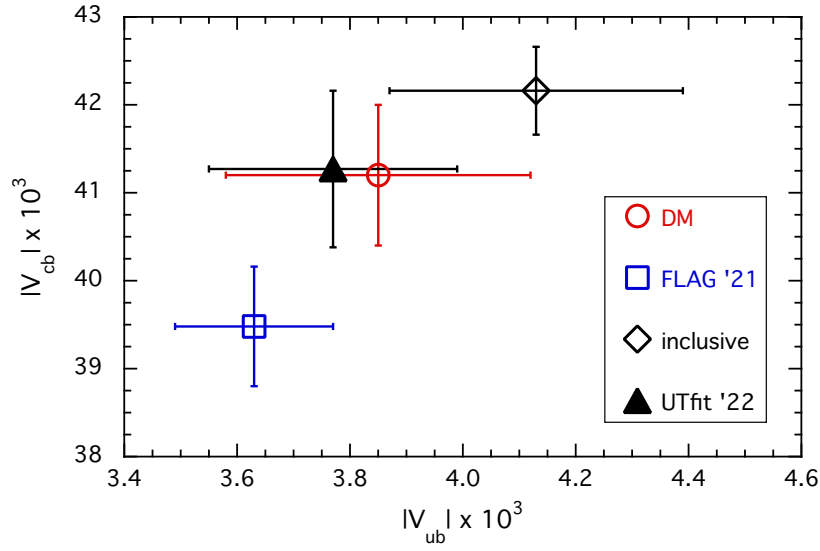
Ufit 42.22 (0.51)

\*\*\* reduced tensions in both  $|V_{cb}|$  and  $R(D^{(*)})$  \*\*\*

From S. Simula

- *universal: it can be applied to any exclusive semileptonic decays of mesons and baryons*

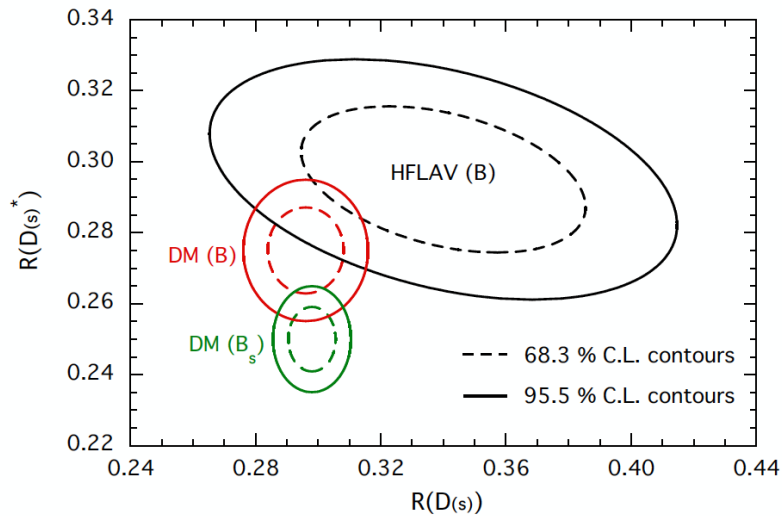
# Summary



	decays	$ V_{ub}  \times 10^3$	DM	FLAG '21	inclusive
$ V_{cb}  \cdot 10^3$	$B_{(s)} \rightarrow D_{(s)}^{(*)}$		41.4 (8)	39.48 (68)	42.16 (50)
$ V_{ub}  \cdot 10^3$	$B_{(s)} \rightarrow \pi, K$		3.85 (27)	3.63 (14)	4.13 (26)

*SU(3) breaking effects need further investigation*

	DM	HFLAV '19
R(D)	0.296 (8)	0.340 (27) (13)
R(D <sup>*</sup> )	0.275 (8)	0.295 (11) (8)
R(D <sub>s</sub> )	0.298 (5)	
R(D <sub>s</sub> <sup>*</sup> )	0.250 (6)	



reduced tensions in both  $|V_{cb}|$ ,  $|V_{ub}|$  and  $R(D^{(*)})$   
when theory and experiments are not fitted simultaneously

$$H_W^{\mu\nu}(k, \mathbf{p}) = H_{SD}^{\mu\nu}(k, \mathbf{p}) + H_{pt}^{\mu\nu}(k, \mathbf{p})$$

$$H_{SD}^{\mu\nu}(k, \mathbf{p}) = \frac{H_1(p \cdot k, k^2)}{M_{D_s}} [k^2 g^{\mu\nu} - k^\mu k^\nu] + \frac{H_2(p \cdot k, k^2)}{M_{D_s}} \frac{[(p \cdot k - k^2)k^\mu - k^2(p - k)^\mu]}{(p - k)^2 - M_{D_s}^2} (p - k)^\nu$$

$$-i \frac{F_V(p \cdot k, k^2)}{M_{D_s}} \varepsilon^{\mu\nu\gamma\beta} k_\gamma p_\beta + \frac{F_A(p \cdot k, k^2)}{M_{D_s}} [(p \cdot k - k^2)g^{\mu\nu} - (p - k)^\mu k^\nu]$$

$$H_{pt}^{\mu\nu}(k, \mathbf{p}) = f_{D_s} \left[ g^{\mu\nu} + \frac{(2p - k)^\mu (p - k)^\nu}{2p \cdot k - k^2} \right],$$

*real photon*

16

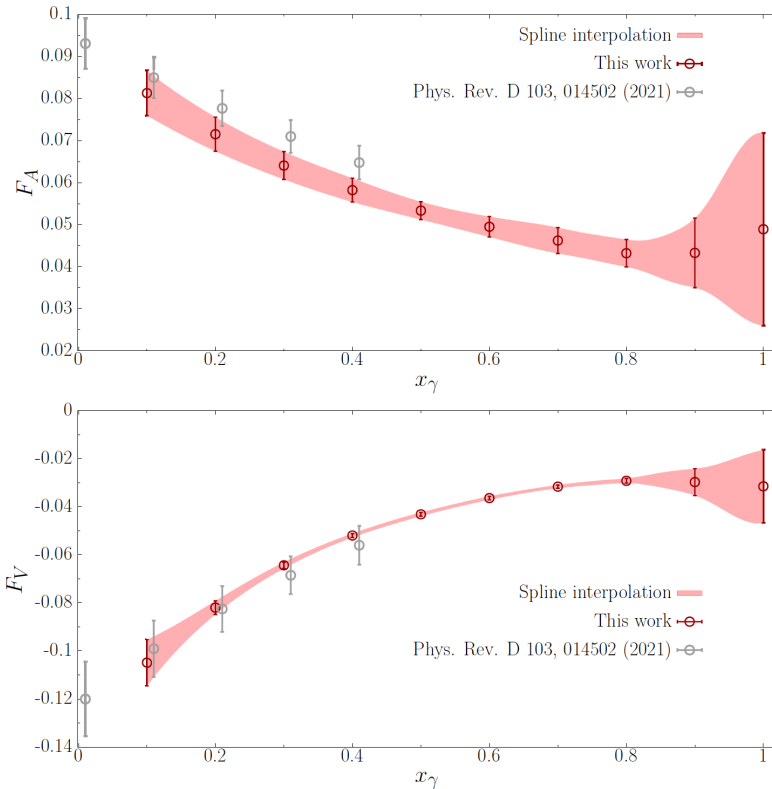
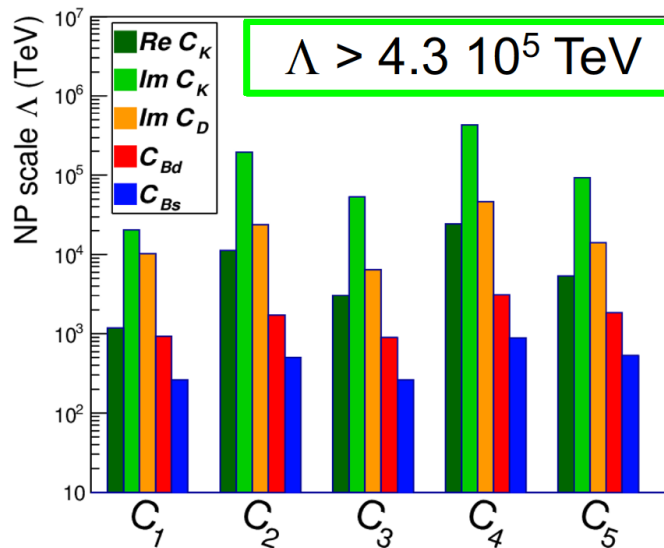


FIG. 8: The form factors  $F_A$  (top figure) and  $F_V$  (bottom figure), obtained after the extrapolation to the continuum limit, shown as a function of the dimensionless variable  $x_\gamma$ . In each of the two figures, the red band is the result of a smooth cubic spline interpolation to our data, while the gray data points correspond to the results from Ref. [15] which have been slightly shifted horizontally for visualization purposes.

# Beyond the SM

## Wilson Coefficients results

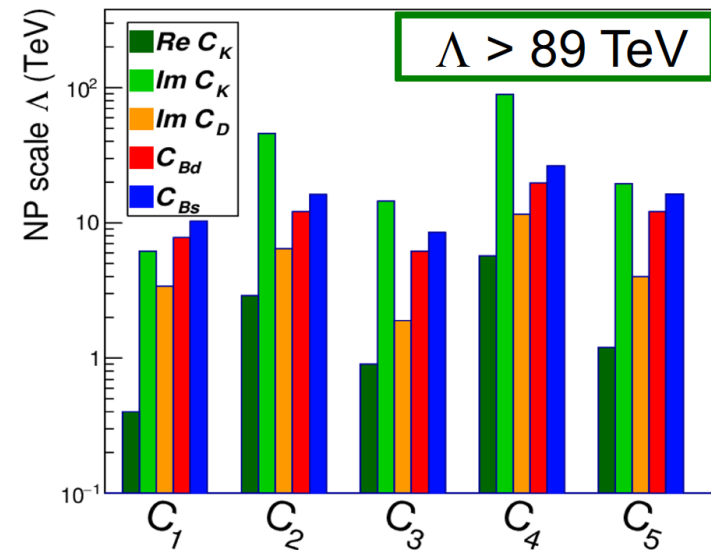
**Generic:**  $C(\Lambda) = \alpha/\Lambda^2$ ,  $F_i \sim 1$ , arbitrary phase,  $\alpha \sim 1$  for strongly coupled NP



- $\alpha \sim \alpha_w$  in case of loop coupling
  - through weak interactions\*
- $\Lambda > 1.3 \cdot 10^4 \text{ TeV}$

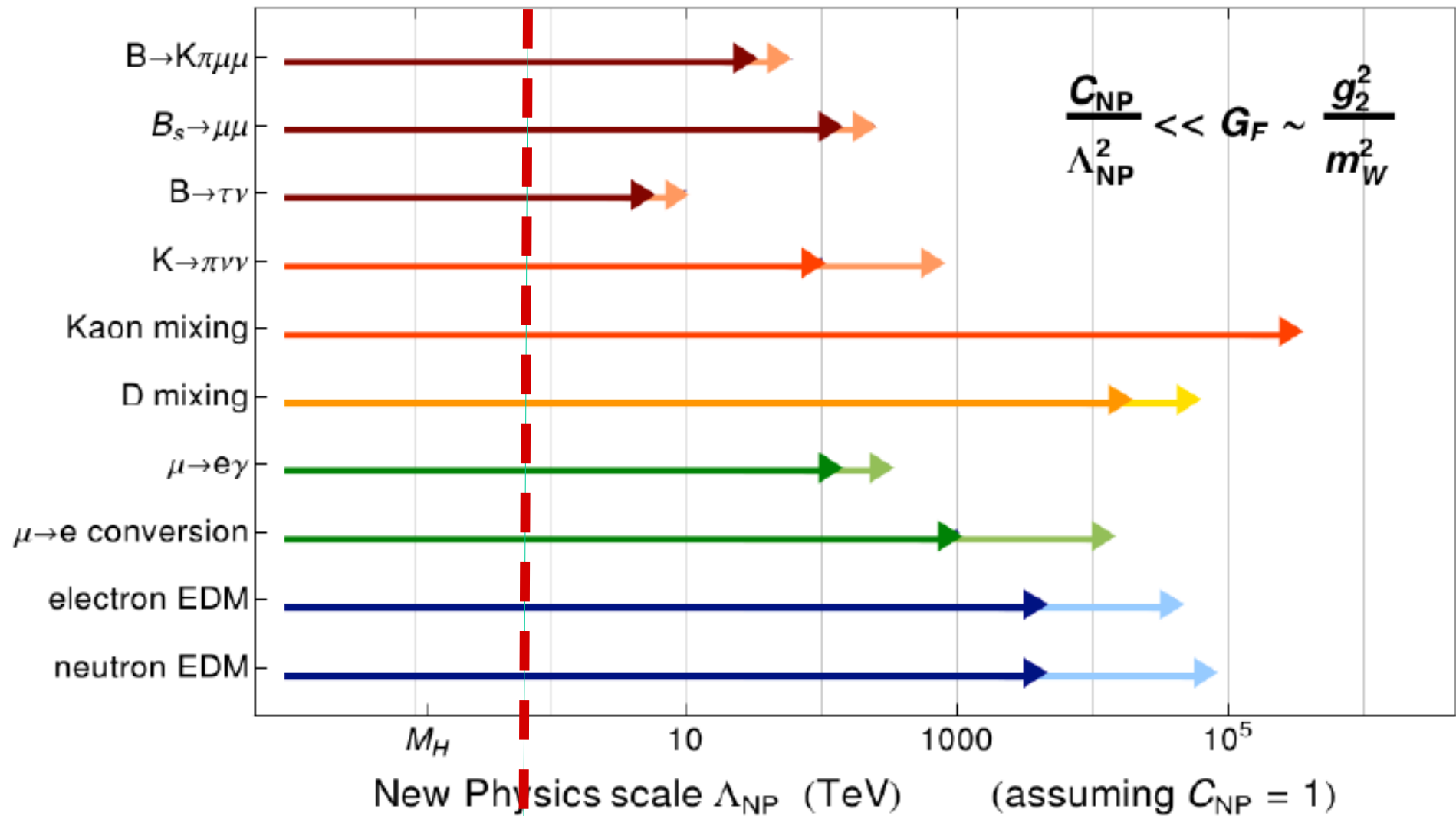
\*for lower bound for loop-mediated contributions, simply multiply by  $\alpha_s$  ( $\sim 0.1$ ) or by  $\alpha_w$  ( $\sim 0.03$ ).

**NMFV:**  $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$ ,  $F_i \sim |F_{SM}|$ , arbitrary phase



- $\alpha \sim \alpha_w$  in case of loop coupling
  - through weak interactions\*
- $\Lambda > 2.7 \text{ TeV}$

# *Sensitivity to New Physics from Flavor*



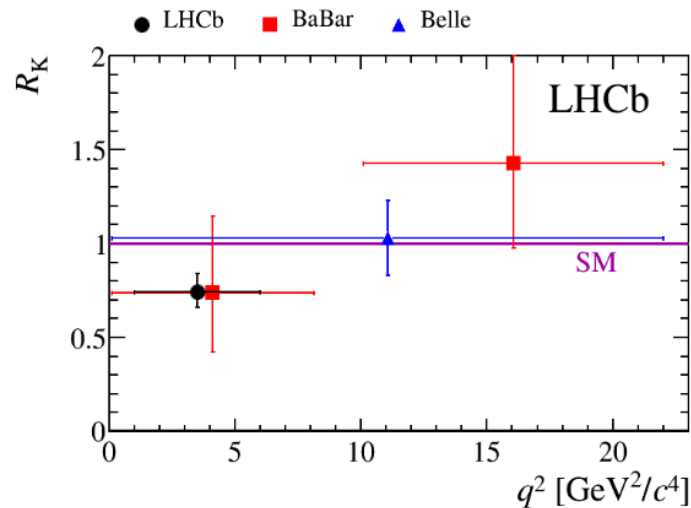
*Approximate LHC direct reach*



# Reminder:

$$R_K = B(B^+ \rightarrow K^+ \mu^+ \mu^-) / B(B^+ \rightarrow K^+ e^+ e^-)$$

- Test of lepton universality :  $R_K \sim 1$  in SM, with negligible theoretical uncertainties



LHCb, PRL 113 151601  
Belle, PRL 103 171801  
BaBar, PRD 86 032012

$$R_K(1 < q^2 < 6 \text{ GeV}^2) = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst})$$

- Compatible with SM at  $2.6\sigma$
- Experimentally challenging
  - lower trigger efficiency for electrons, resolution deteriorated by bremsstrahlung
- Other modes suitable for same test:  
 $B^0 \rightarrow K^{*0} l^+ l^-$ ,  $B_s \rightarrow \phi l^+ l^-$ ,  $\Lambda_B \rightarrow \Lambda l^+ l^-$

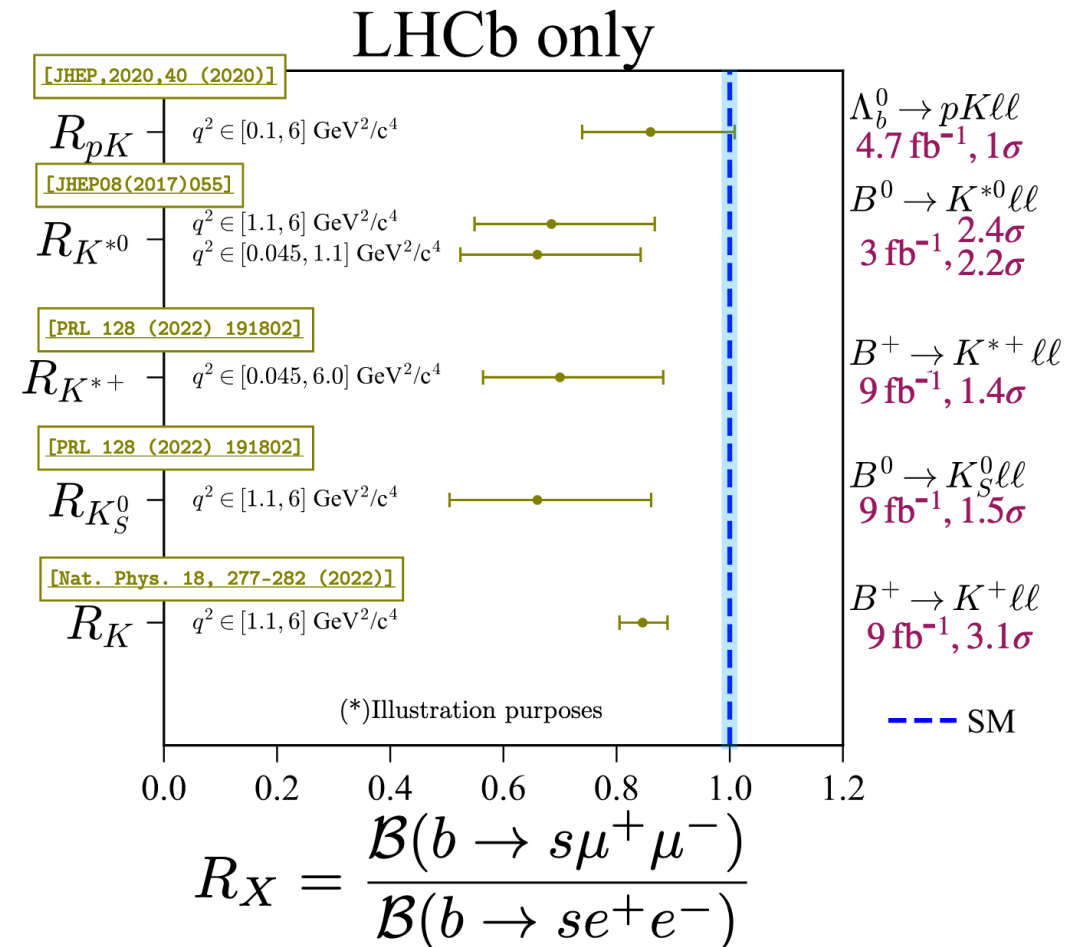
*Old slide*

# Excitement

Analysis

## Lepton Flavour Universality (LFU) tests in $b \rightarrow s\ell^+\ell^-$

- ◆ Coherent pattern of tension to SM in LFU test with  $b \rightarrow s\ell^+\ell^-$  transition:
- ◆  $R_X$  ratio extremely well predicted in SM
  - ▶ Cancellation of hadronic uncertainties at  $10^{-4}$
  - ▶  $\mathcal{O}(1\%)$  QED correction [Eur.Phys.J.C 76 (2016) 8]
  - ▶ Statistically limited
- ◆ Any departure from unity is a clear sign of New Physics



(\*) Measurements from Belle not shown (larger statistical uncertainties)

# Harakiri!



Analysis: results

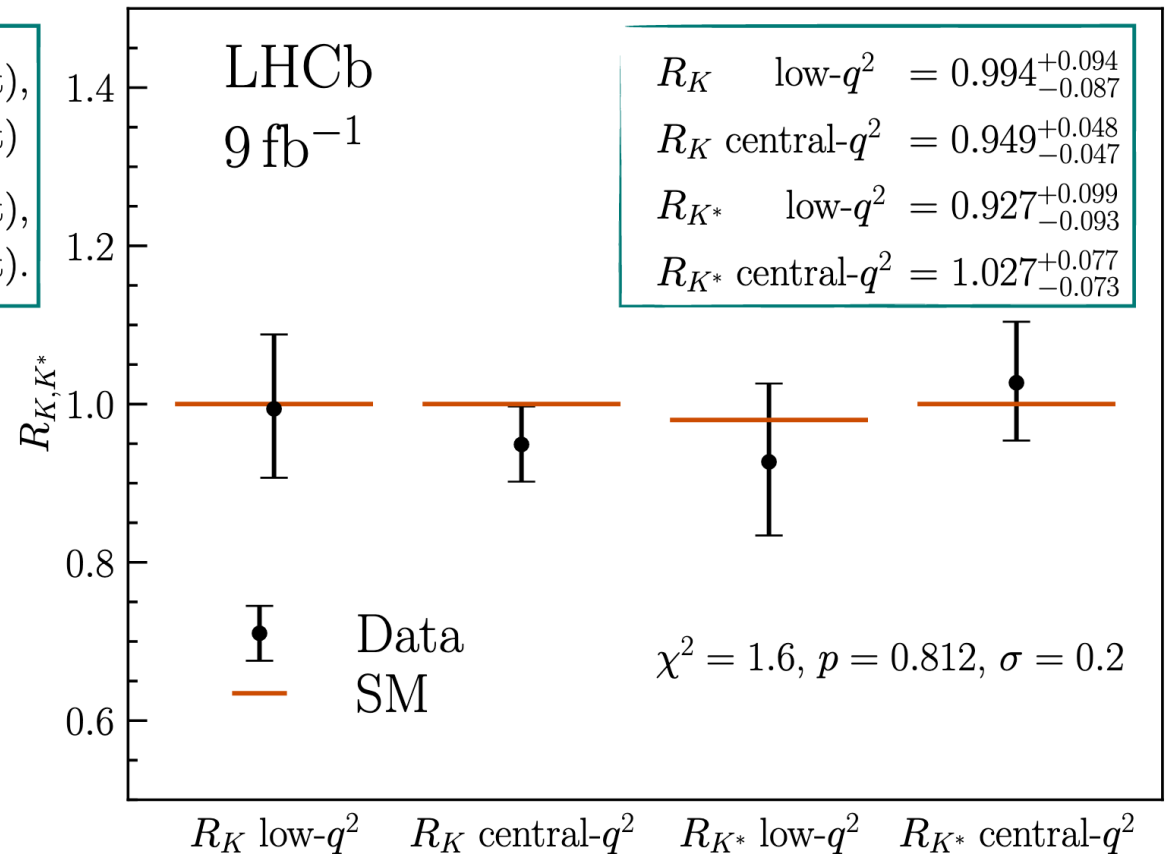
## Results

$$\text{low-}q^2 \begin{cases} R_K & = 0.994^{+0.090}_{-0.082} \text{ (stat)} \quad +0.027_{-0.029} \text{ (syst)}, \\ R_{K^*} & = 0.927^{+0.093}_{-0.087} \text{ (stat)} \quad +0.034_{-0.033} \text{ (syst)} \end{cases}$$

$$\text{central-}q^2 \begin{cases} R_K & = 0.949^{+0.042}_{-0.041} \text{ (stat)} \quad +0.023_{-0.023} \text{ (syst)}, \\ R_{K^*} & = 1.027^{+0.072}_{-0.068} \text{ (stat)} \quad +0.027_{-0.027} \text{ (syst)}. \end{cases}$$

◆ Most precise and accurate LFU test in  $b \rightarrow s\ell\ell$  transition

◆ Compatible with SM with a simple  $\chi^2$  test on 4 measurement at  $0.2 \sigma$



*Caro Nando,  
la nostra conoscenza e amicizia è > 50 anni,  
abbiamo fatto insieme cose di cui possiamo  
andare orgogliosi (e tu anche di più senza di  
me), spero che continueremo a profittare di  
questa nostra amicizia ancora per molto...*

THANKS FOR YOUR ATTENTION



# Back up Slides

# The Dispersive Matrix (DM) method $B \rightarrow D$

$$t \equiv q^2$$

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

$$\langle h_1 | h_2 \rangle = \int_{|z|=1} \frac{dz}{2\pi i z} \bar{h}_1(z) h_2(z)$$

$$g_t(z) \equiv \frac{1}{1 - \bar{z}(t)z}$$

The conformal variable  $z$  is related to the momentum transfer as:

$$z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1} \quad [0, t_{max}=t_-] \Leftrightarrow [z_{max}, 0]$$

$$t_{\pm} \equiv (m_B \pm m_D)^2$$

$$\det \mathbf{M} \geq 0$$

# The DM method

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

We also have to define the **kinematical functions**

$$\phi_0(z, Q^2) = \sqrt{\frac{2n_I}{3}} \sqrt{\frac{3t_+t_-}{4\pi}} \frac{1}{t_+ - t_-} \frac{1+z}{(1-z)^{5/2}} \left( \beta(0) + \frac{1+z}{1-z} \right)^{-2} \left( \beta(-Q^2) + \frac{1+z}{1-z} \right)^{-2},$$

$$\phi_+(z, Q^2) = \sqrt{\frac{2n_I}{3}} \sqrt{\frac{1}{\pi(t_+ - t_-)}} \frac{(1+z)^2}{(1-z)^{9/2}} \left( \beta(0) + \frac{1+z}{1-z} \right)^{-2} \left( \beta(-Q^2) + \frac{1+z}{1-z} \right)^{-3}, \quad \beta(t) \equiv \sqrt{\frac{t_+ - t}{t_+ - t_-}}$$

Thus, we need **these external inputs** to implement our method:

- estimates of the FFs, computed on the lattice, @  $\{t_1, \dots, t_n\}$ : from Cauchy's theorem (for generic  $m$ )

$$\langle g_{t_m} | \phi f \rangle = \phi(t_m, Q^2) f(t_m) \quad \langle g_{t_m} | g_{t_l} \rangle = \frac{1}{1 - \bar{z}(t_l)z(t_m) Q^2}$$

*LQCD data!*

- **non-perturbative** values of the **susceptibilities**, since from the dispersion relations (calling the Euclidean quadratic momentum)

$$\chi(Q^2) \geq \langle \phi f | \phi f \rangle$$

Since the susceptibilities are computed on the lattice, we can in principle use whatever value of  $Q^2 \equiv -q^2$

# The DM method

The positivity of the original inner products guarantees that  $\det \mathbf{M} \geq 0$  namely

$$\text{LOWER bound} \quad \boxed{\beta - \sqrt{\gamma}} \leq f(z) \leq \boxed{\beta + \sqrt{\gamma}} \quad \text{UPPER bound}$$

$$\beta = \frac{1}{d(z)\phi(z)} \sum_{j=1}^N f_j \phi_j d_j \frac{1 - z_j^2}{z - z_j} \quad \gamma = \frac{1}{d^2(z)\phi^2(z)} \frac{1}{1 - z^2} \left[ \chi - \sum_{i,j=1}^N f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

**UNITARITY FILTER:** unitarity is satisfied if  $\gamma$  is semipositive definite, namely if

$$\chi \geq \sum_{i,j=1}^N f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}$$

*This is a parametrization-independent unitarity test of the LQCD input data*

*A detailed discussion of the treatment of statistical errors and constraints was also presented (simplified with respect to L. Lellouch NPB, 479 (1996))*



# Non-perturbative computation of the susceptibilities

The possibility to compute the  $\chi$ s on the lattice allows us to choose *whatever value of  $Q^2$*  (i.e. near the region of production of the resonances)



**NOT POSSIBLE IN PERTURBATION THEORY!!!**

$$(m_b + m_c)\Lambda_{QCD} \ll (m_b + m_c)^2 - q^2$$

POSSIBLE IMPROVEMENT IN THE STUDY OF THE FFs through our method

Work in progress...

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\begin{aligned} \chi_{0+}(Q^2) &\equiv \frac{\partial}{\partial Q^2} [Q^2 \Pi_{0+}(Q^2)] = \int_0^\infty dt t^2 j_0(Qt) C_{0+}(t), \xrightarrow{W. I.} \frac{1}{4} \int_0^\infty dt' t'^4 \frac{j_1(Qt')}{Qt'} [(m_b - m_c)^2 C_S(t') + Q^2 C_{0+}(t')] \\ \chi_{1-}(Q^2) &\equiv -\frac{1}{2} \frac{\partial^2}{\partial^2 Q^2} [Q^2 \Pi_{1-}(Q^2)] = \frac{1}{4} \int_0^\infty dt t^4 \frac{j_1(Qt)}{Qt} C_{1-}(t) \\ \chi_{0-}(Q^2) &\equiv \frac{\partial}{\partial Q^2} [Q^2 \Pi_{0-}(Q^2)] = \int_0^\infty dt t^2 j_0(Qt) C_{0-}(t), \xrightarrow{W. I.} \frac{1}{4} \int_0^\infty dt' t'^4 \frac{j_1(Qt')}{Qt'} [(m_b + m_c)^2 C_P(t') + Q^2 C_{0-}(t')] \\ \chi_{1+}(Q^2) &\equiv -\frac{1}{2} \frac{\partial^2}{\partial^2 Q^2} [Q^2 \Pi_{1+}(Q^2)] = \frac{1}{4} \int_0^\infty dt t^4 \frac{j_1(Qt)}{Qt} C_{1+}(t) \end{aligned}$$

# Non-perturbative computation of the susceptibilities

Let us choose for the moment zero  $Q^2$ :

$$\chi_{0+}(Q^2 = 0) = \int_0^\infty dt t^2 C_{0+}(t) ,$$

$$\chi_{1-}(Q^2 = 0) = \frac{1}{12} \int_0^\infty dt t^4 C_{1-}(t) ,$$

$$\chi_{0-}(Q^2 = 0) = \int_0^\infty dt t^2 C_{0-}(t) ,$$

$$\chi_{1+}(Q^2 = 0) = \frac{1}{12} \int_0^\infty dt t^4 C_{1+}(t) .$$

$$\chi_{0+}(Q^2 = 0) = \frac{1}{12} (m_b - m_c)^2 \int_0^\infty dt t^4 C_S(t)$$

$$\chi_{0-}(Q^2 = 0) = \frac{1}{12} (m_b + m_c)^2 \int_0^\infty dt t^4 C_P(t)$$

$$C_{0+}(t) = \tilde{Z}_V^2 \int d^3x \langle 0 | T [\bar{q}_1(x) \gamma_0 q_2(x) \bar{q}_2(0) \gamma_0 q_1(0)] | 0 \rangle ,$$

$$C_{1-}(t) = \tilde{Z}_V^2 \frac{1}{3} \sum_{j=1}^3 \int d^3x \langle 0 | T [\bar{q}_1(x) \gamma_j q_2(x) \bar{q}_2(0) \gamma_j q_1(0)] | 0 \rangle ,$$

$$C_{0-}(t) = \tilde{Z}_A^2 \int d^3x \langle 0 | T [\bar{q}_1(x) \gamma_0 \gamma_5 q_2(x) \bar{q}_2(0) \gamma_0 \gamma_5 q_1(0)] | 0 \rangle ,$$

$$C_{1+}(t) = \tilde{Z}_A^2 \frac{1}{3} \sum_{j=1}^3 \int d^3x \langle 0 | T [\bar{q}_1(x) \gamma_j \gamma_5 q_2(x) \bar{q}_2(0) \gamma_j \gamma_5 q_1(0)] | 0 \rangle ,$$

$$C_S(t) = \tilde{Z}_S^2 \int d^3x \langle 0 | T [\bar{q}_1(x) q_2(x) \bar{q}_2(0) q_1(0)] | 0 \rangle ,$$

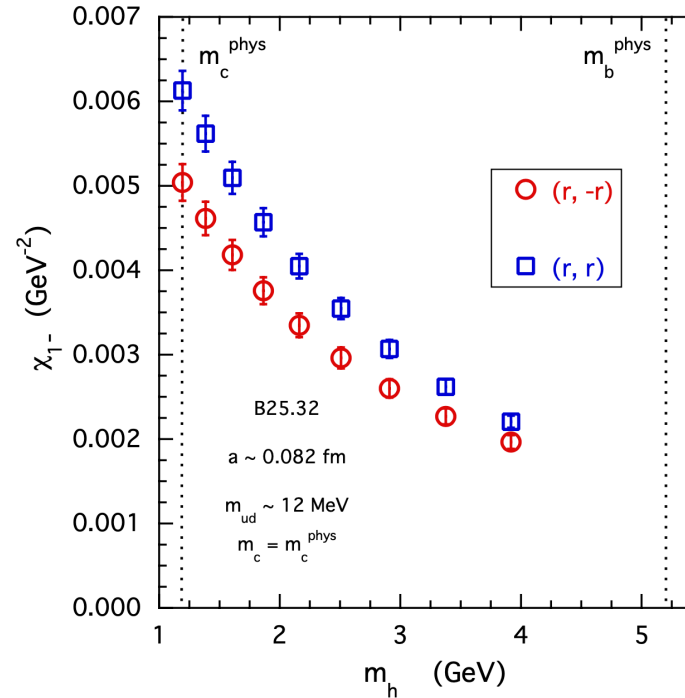
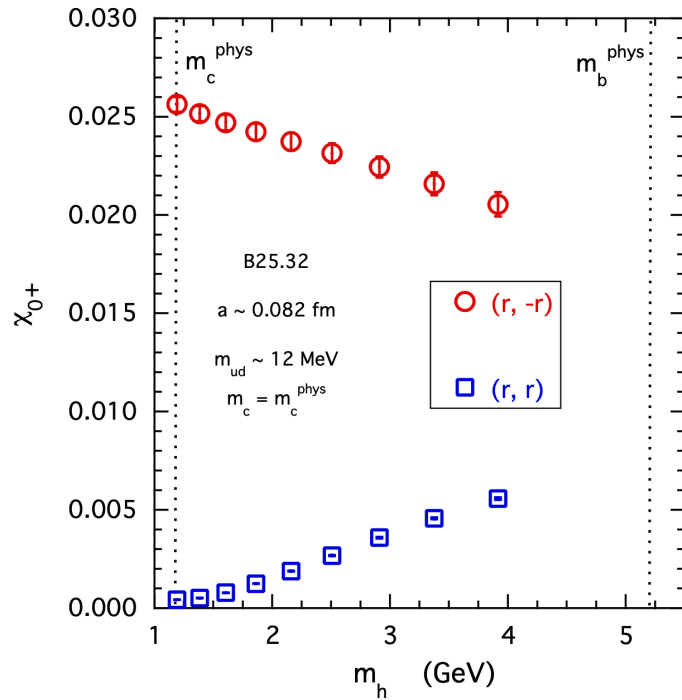
$$C_P(t) = \tilde{Z}_P^2 \int d^3x \langle 0 | T [\bar{q}_1(x) \gamma_5 q_2(x) \bar{q}_2(0) \gamma_5 q_1(0)] | 0 \rangle ,$$

**Z:** appropriate renormalization constants

N. Carrasco et al. [ETM Coll.], NPB 887 (2014) [arXiv:1403.4504]

# Non-perturbative computation of the susceptibilities

$r$ : (unphysical) Wilson parameter



Following set of nine quark masses:

$$m_h(n) = \lambda^{n-1} m_c^{phys} \quad \text{for } n = 1, 2, \dots$$

$$m_h(1) = m_c^{phys}$$

$$\lambda \equiv [m_b^{phys}/m_c^{phys}]^{1/10} = [5.198/1.176]^{1/10} \simeq 1.1602. \quad m_h(9) \simeq 3.9 \text{ GeV} \simeq 0.75 m_b^{phys}$$

$$m_h = a\mu_h/(Z_P a)$$

*Contact terms &  
Large discretisation effects*

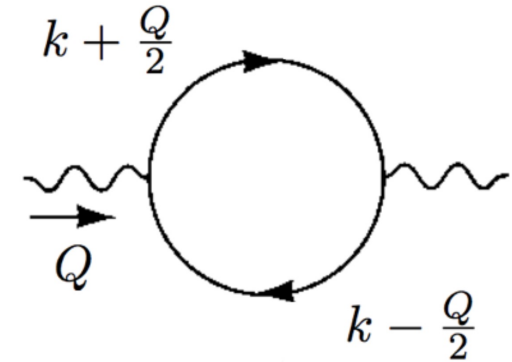
# Contact terms & perturbative subtraction

In **twisted mass LQCD** (tmLQCD):

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^\alpha G_1(k + \frac{Q}{2}) \gamma^\beta G_2(k - \frac{Q}{2}) \right],$$

$$G_i(p) = \frac{-i\gamma_\mu \hat{p}_\mu + \mathcal{M}_i(p) - i\mu_{q,i} \gamma_5 \tau^3}{\hat{p}^2 + \mathcal{M}_i^2(p) + \mu_{q,i}^2}, \quad i = 1, 2$$

$$\hat{p}_\mu \equiv \frac{1}{a} \sin(ap_\mu), \quad \mathcal{M}_i(p) \equiv m_i + \frac{r_i}{2} a \hat{p}_\mu^2, \quad \hat{p} \equiv \frac{2}{a} \sin\left(\frac{ap_\mu}{2}\right).$$



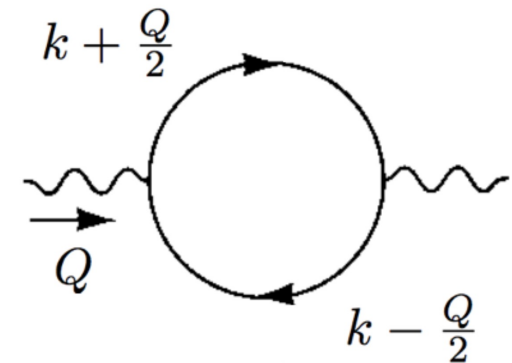
$$\begin{aligned} \Pi_V^{\alpha\beta} = & a^{-2} (Z_1^I + (r_1^2 - r_2^2) Z_2^I + (r_1^2 - r_2^2)(r_1^2 + r_2^2) Z_3^I) g^{\alpha\beta} \\ & + (\mu_1^2 Z^{\mu_1^2} + \mu_2^2 Z^{\mu_2^2} + \mu_1 \mu_2 Z^{\mu_1 \mu_2}) g^{\alpha\beta} + (Z_1^{Q^2} + (r_1^2 - r_2^2) Z_2^{Q^2}) Q \cdot Q g^{\alpha\beta} \\ & + (Z_1^{Q^\alpha Q^\beta} + (r_1^2 - r_2^2) Z_2^{Q^\alpha Q^\beta}) Q^\alpha Q^\beta + \underline{r_1 r_2 (a^{-2} Z_1^{r_1 r_2} g^{\alpha\beta} + (Z_2^{r_1 r_2} + (r_1^2 + r_2^2) Z_3^{r_1 r_2} \\ & + (r_1^4 + r_2^4) Z_4^{r_1 r_2}) Q \cdot Q g^{\alpha\beta} + (\mu_1^2 Z_5^{r_1 r_2} + \mu_2^2 Z_6^{r_1 r_2}) g^{\alpha\beta})} + O(a^2), \end{aligned} \quad \text{CONTACT TERMS!!!}$$

# Contact terms & perturbative subtraction

In twisted mass LQCD (tmLQCD):

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^\alpha G_1(k + \frac{Q}{2}) \gamma^\beta G_2(k - \frac{Q}{2}) \right],$$

Thus, by separating the *longitudinal* and the *transverse* contributions, we can compute the susceptibilities for all the spin-parity quantum numbers in the free theory on the lattice, i.e. at order  $\mathcal{O}(\alpha_s^0)$  using twisted-mass fermions!



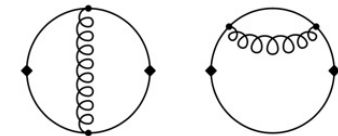
$$\chi_j^{free} = \chi_j^{LO} + \chi_j^{discr}$$

LO term of PT @  $\mathcal{O}(\alpha_s^0)$       contact terms and discretization effects @  $\mathcal{O}(\alpha_s^0 a^m)$  with  $m \geq 0$

**Perturbative subtraction:**

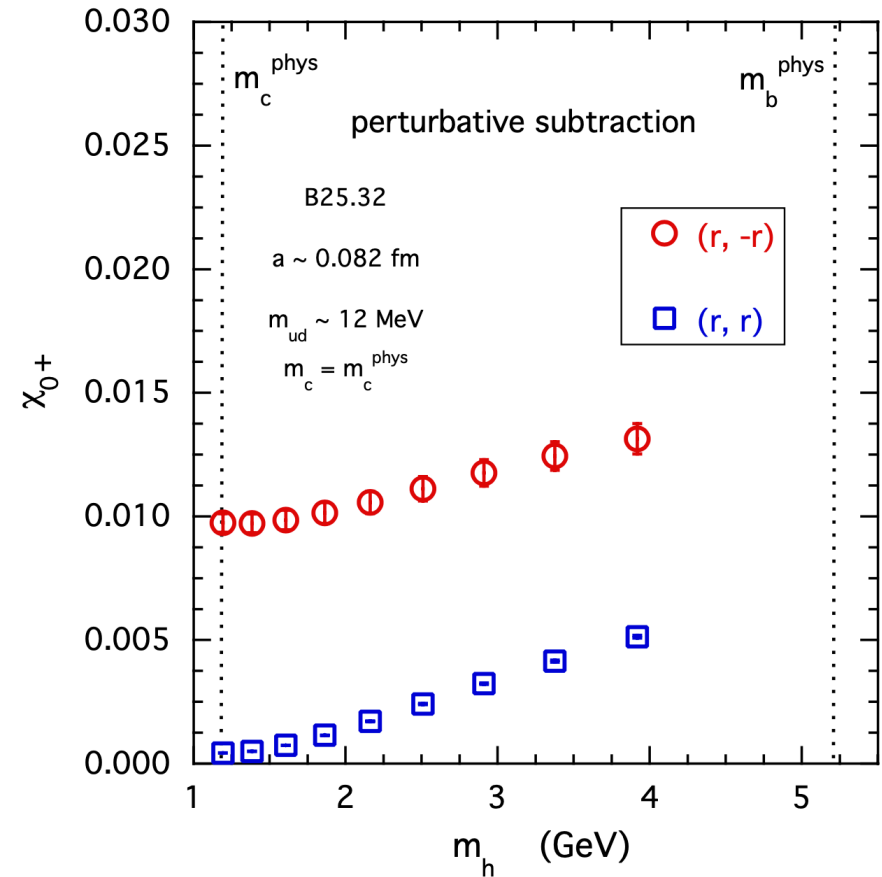
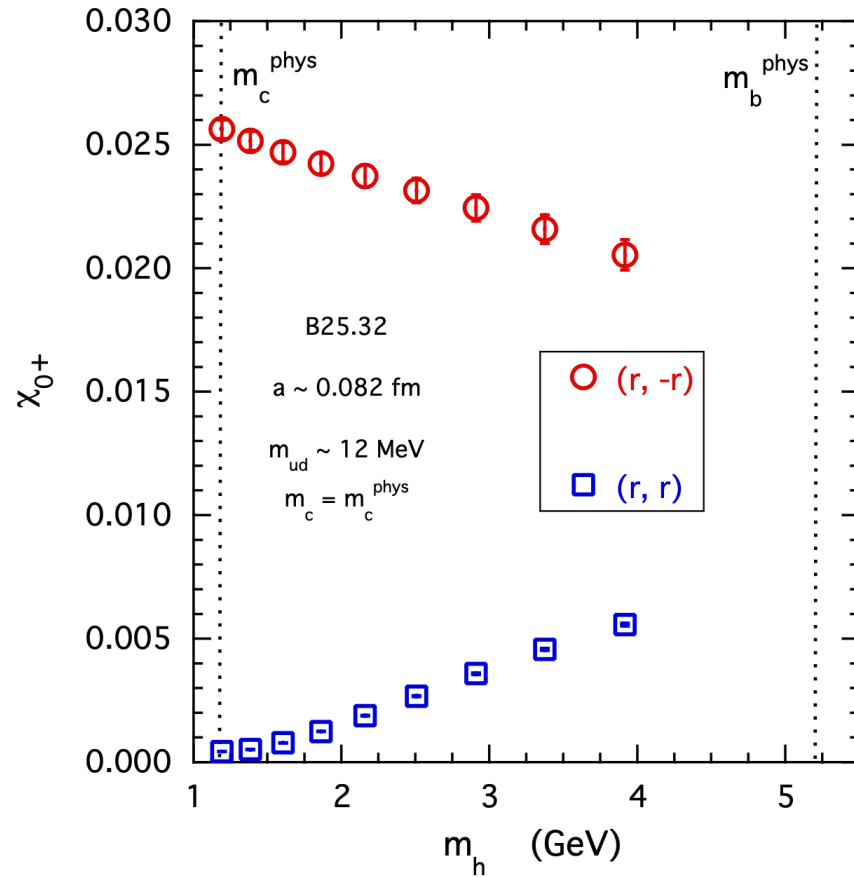
$$\chi_j \rightarrow \chi_j - \left[ \chi_j^{free} - \chi_j^{LO} \right]$$

**Higher order corrections?**



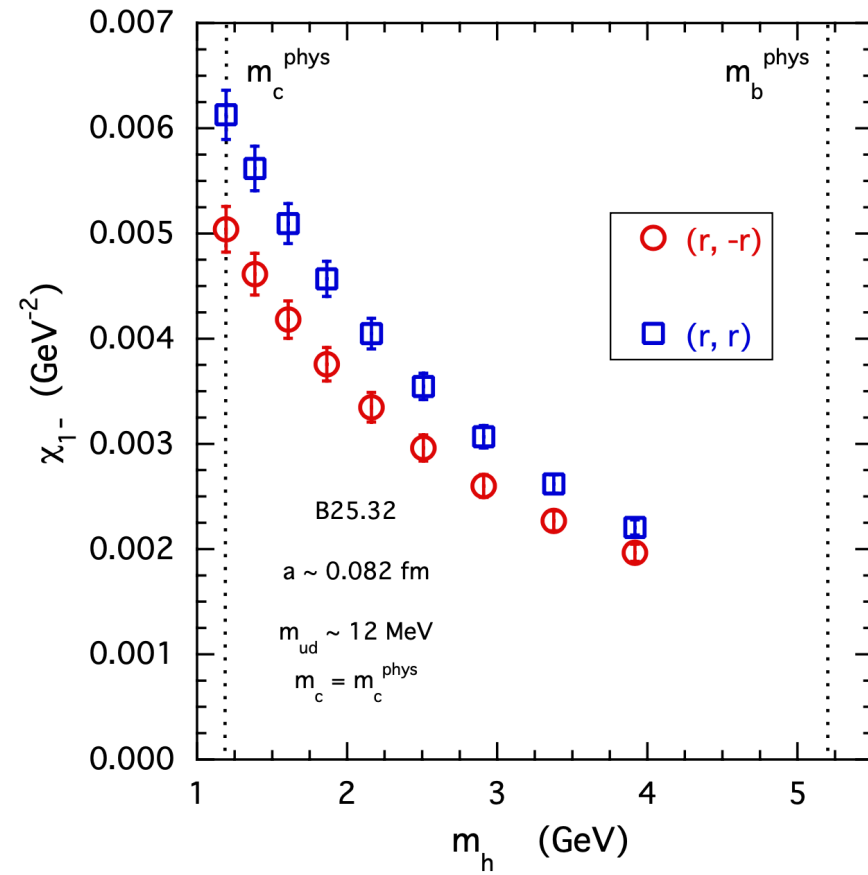
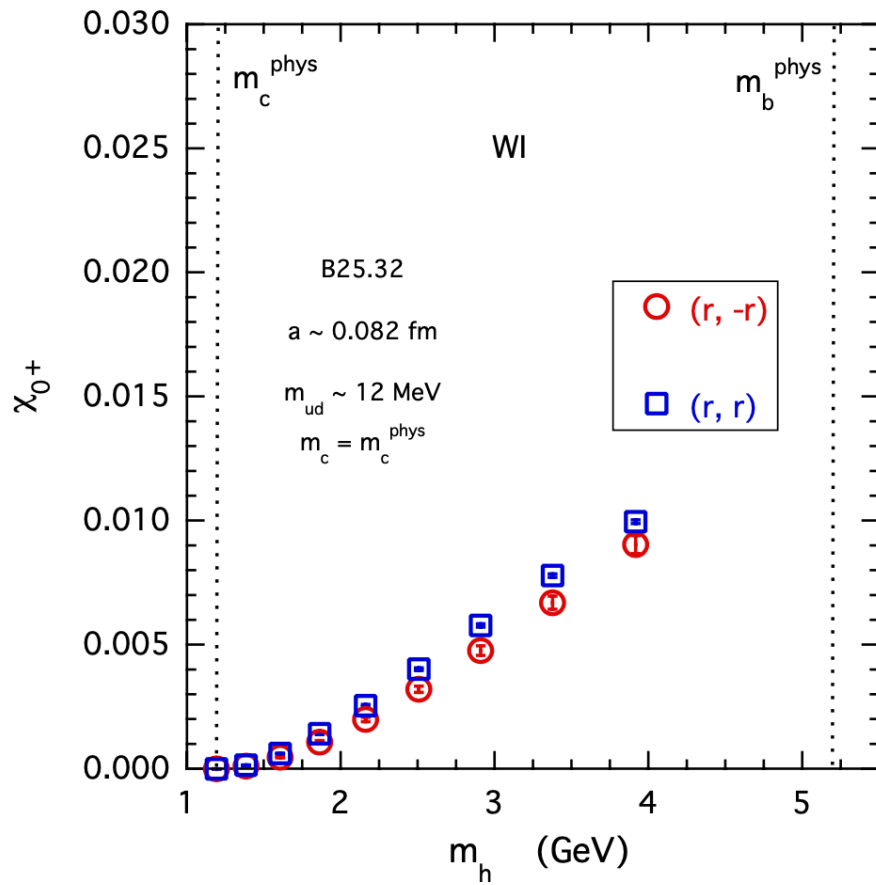
**Work in progress...**

# Contact terms & perturbative subtraction



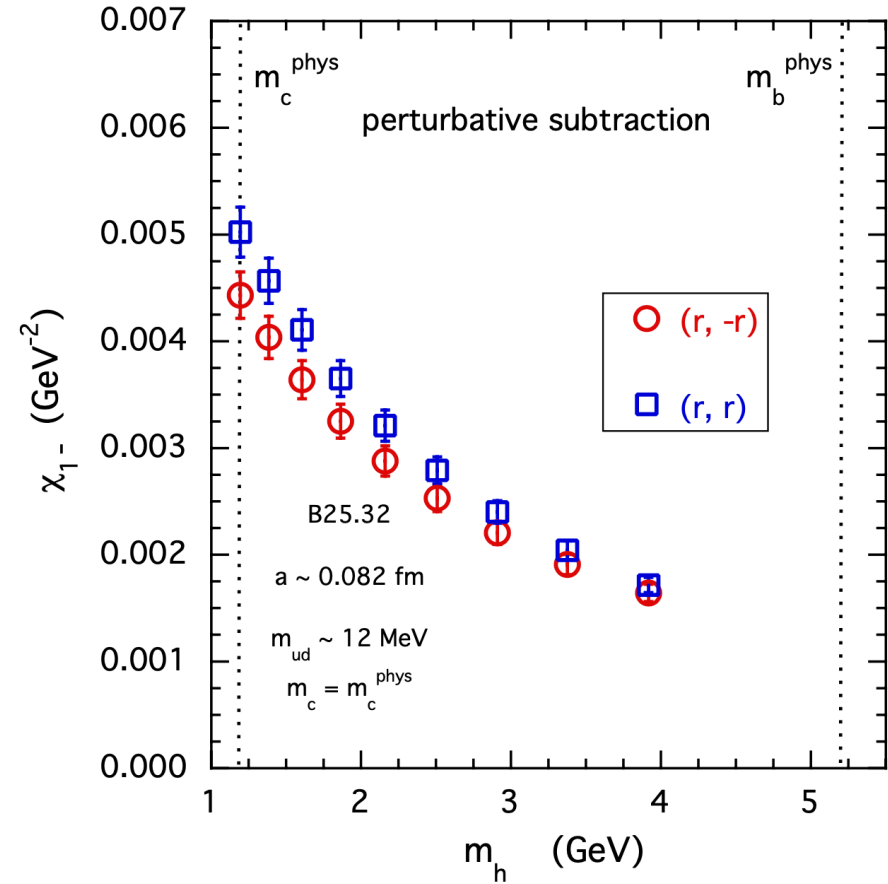
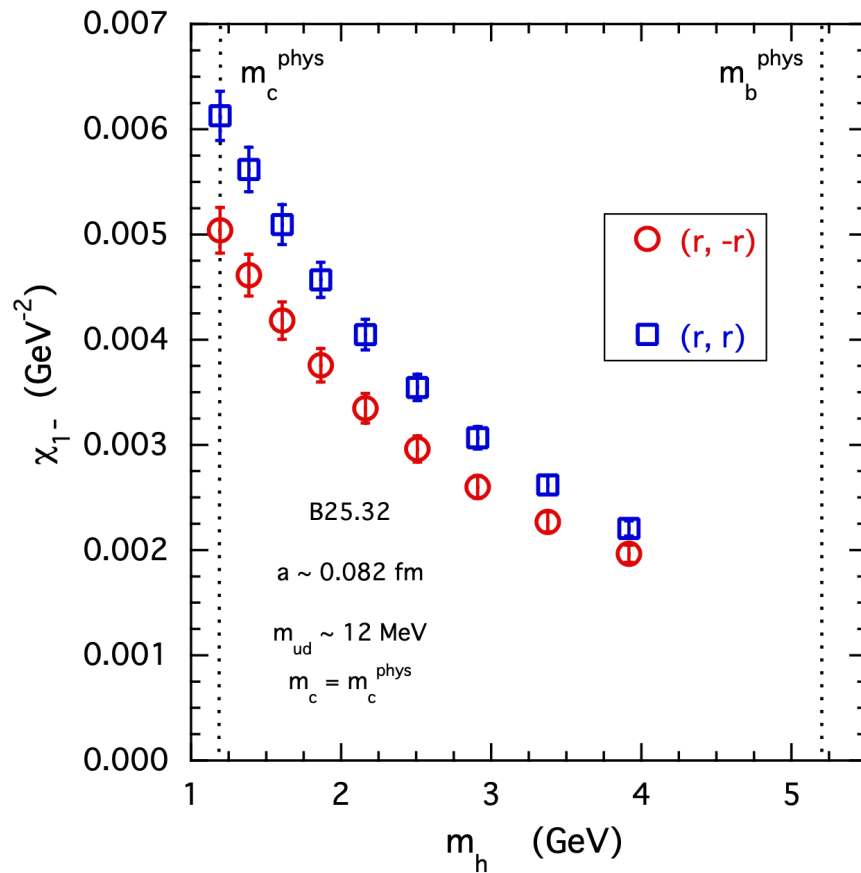
**NOT ENOUGH...**

# Non-perturbative computation of the susceptibilities



*Much better using the Ward Identity*

## Contact terms & perturbative subtraction



OK

*An extrapolation to the continuum limit was implemented*



# ETMC ratio method & final results

For the extrapolation to the physical  $b$ -quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]} \quad \text{to ensure that} \quad \lim_{n \rightarrow \infty} R_j(n) = 1$$



$$\begin{aligned} \rho_{0+}(m_h) &= \rho_{0-}(m_h) = 1, \\ \rho_{1-}(m_h) &= \rho_{1+}(m_h) = (m_h^{pole})^2 \end{aligned}$$

All the details are deeply discussed in [arXiv:2105.07851](https://arxiv.org/abs/2105.07851). In this way, we have obtained **the first lattice QCD determination of susceptibilities of heavy-to-heavy (and heavy-to-light, in prep.) transition current densities:**

$b \rightarrow c$

$b \rightarrow u$

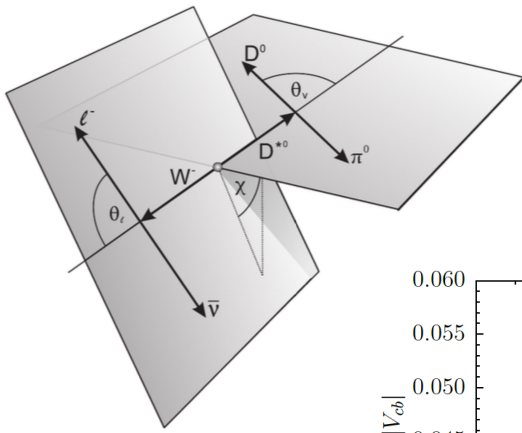
	Perturbative	With subtraction	Non-perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L} [10^{-3}]$	6.204(81)	—	7.58(59)	—	2.04(20)	—
$\chi_{A_L} [10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)	2.34(13)	—
$\chi_{V_T} [10^{-4} \text{ GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)	4.88(1.16)	4.45(1.16)
$\chi_{A_T} [10^{-4} \text{ GeV}^{-2}]$	3.894	—	4.69(30)	—	4.65(1.02)	—

Differences with PT? ~4% for 1<sup>-</sup>, ~7% for 0<sup>-</sup>, ~20 % for 0<sup>+</sup> and 1<sup>+</sup>

Bigi, Gambino PRD '16

Bigi, Gambino, Schacht PLB '17

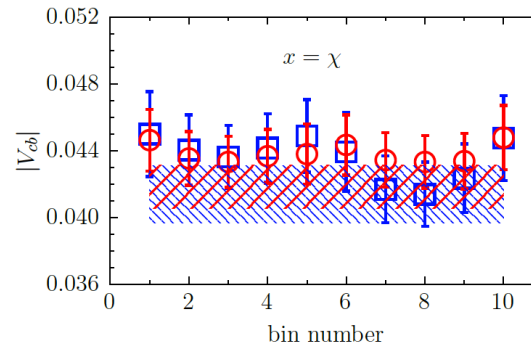
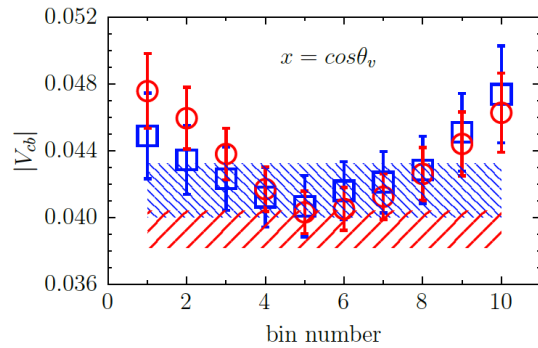
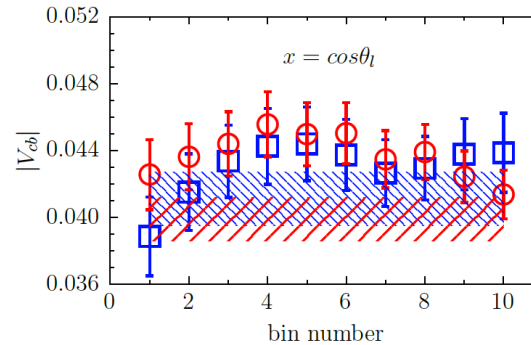
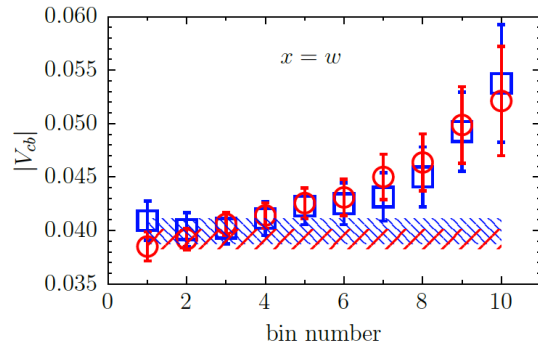
Bigi, Gambino, Schacht JHEP '17



# Exclusive Vcb determination from $B \rightarrow D^*$

$$d\Gamma/dx,$$

$$x = w, \cos \theta_l, \cos \theta_v, \chi$$



**Blue squares:**  
arXiv:1702.01521

**Red points:**  
arXiv:1809.03290

*In the  $w$  differential decay rate data systematically above the result of the fit  
This problem is known and has been studied, for example, in Nucl. Instrum. Meth.  
A346 (1994) 306-311*

*Our interpretation is that there is a problem related to the  
experimental calibration and to the covariance matrix*

# experimental data for $B \rightarrow D^* \ell \nu_\ell$ decays

- two sets of data from Belle collaboration arXiv:1702.01521 and arXiv:1809.03290 total of 80 data points
- four different differential decay rates  $d\Gamma/dx$  where  $x = \{w, \cos\theta_v, \cos\theta_\ell, \chi\}$ : 10 bins for each variable

\*\*\* we do not mix theoretical calculations with experimental data to describe the shape of the FFs \*\*\*

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}} \quad i = 1, \dots, N_{bins}$$

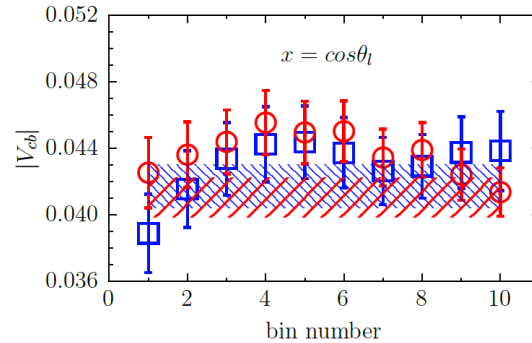
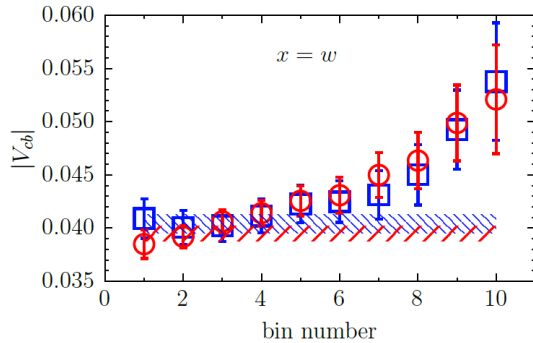
\* issue with the covariance matrix  $C_{ij}^{exp.}$  of the Belle data:  $\Gamma^{exp.} \equiv \sum_{i=1}^{10} \left( \frac{d\Gamma}{dx} \right)_i^{exp.}$  should be the same for all the variables  $x$   
 (see D'Agostini, arXiv: 2001.07562)

- we recover the above property by evaluating the correlation matrix of the experimental ratios

$$\frac{1}{\Gamma^{exp.}} \left( \frac{d\Gamma}{dx} \right)_i$$

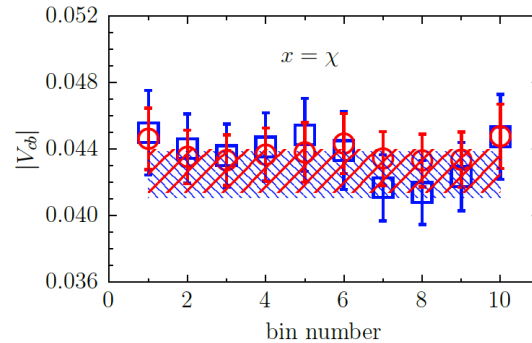
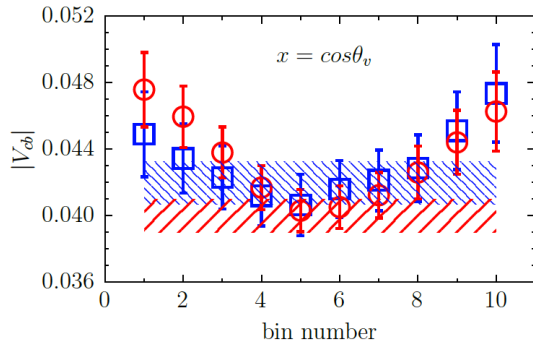
and by considering the new covariance matrix of the experimental data given by (see arXiv:2105.08674)

$$\widetilde{C}_{ij}^{exp.} \rightarrow \rho_{ij}^{ratios} \sqrt{C_{ii}^{exp.} C_{jj}^{exp.}}$$



blue data: Belle 1702.01521

red data: Belle 1809.03290



significant improvement of the  $\chi^2/(d.o.f.)$

bands are (correlated) weighted averages

$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}},$$

$$\sigma_{|V_{cb}|}^2 = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}},$$

# Exclusive $V_{cb}$ determination from $B \rightarrow D^*$

Belle 1702.01521

Belle 1809.03290

experiment	$ V_{cb} (x = w)$	$ V_{cb} (x = \cos\theta_l)$	$ V_{cb} (x = \cos\theta_v)$	$ V_{cb} (x = \chi)$
Ref. [11]	0.0404 (9)	0.0417 (13)	0.0420 (13)	0.0425 (14)
$\chi^2/(d.o.f.)$	1.05	0.89	0.69	0.73
Ref. [12]	0.0395 (7)	0.0410 (12)	0.0400 (10)	0.0427 (13)
$\chi^2/(d.o.f.)$	1.22	1.36	2.02	0.41

averaging procedure

$$\mu_x = \frac{1}{N} \sum_{k=1}^N x_k,$$

$$\sigma_x^2 = \frac{1}{N} \sum_{k=1}^N \sigma_k^2 + \frac{1}{N} \sum_{k=1}^N (x_k - \mu_x)^2,$$



$$|V_{cb}| = (41.2 \pm 1.6) \cdot 10^{-3}$$

with the original covariance of Belle data

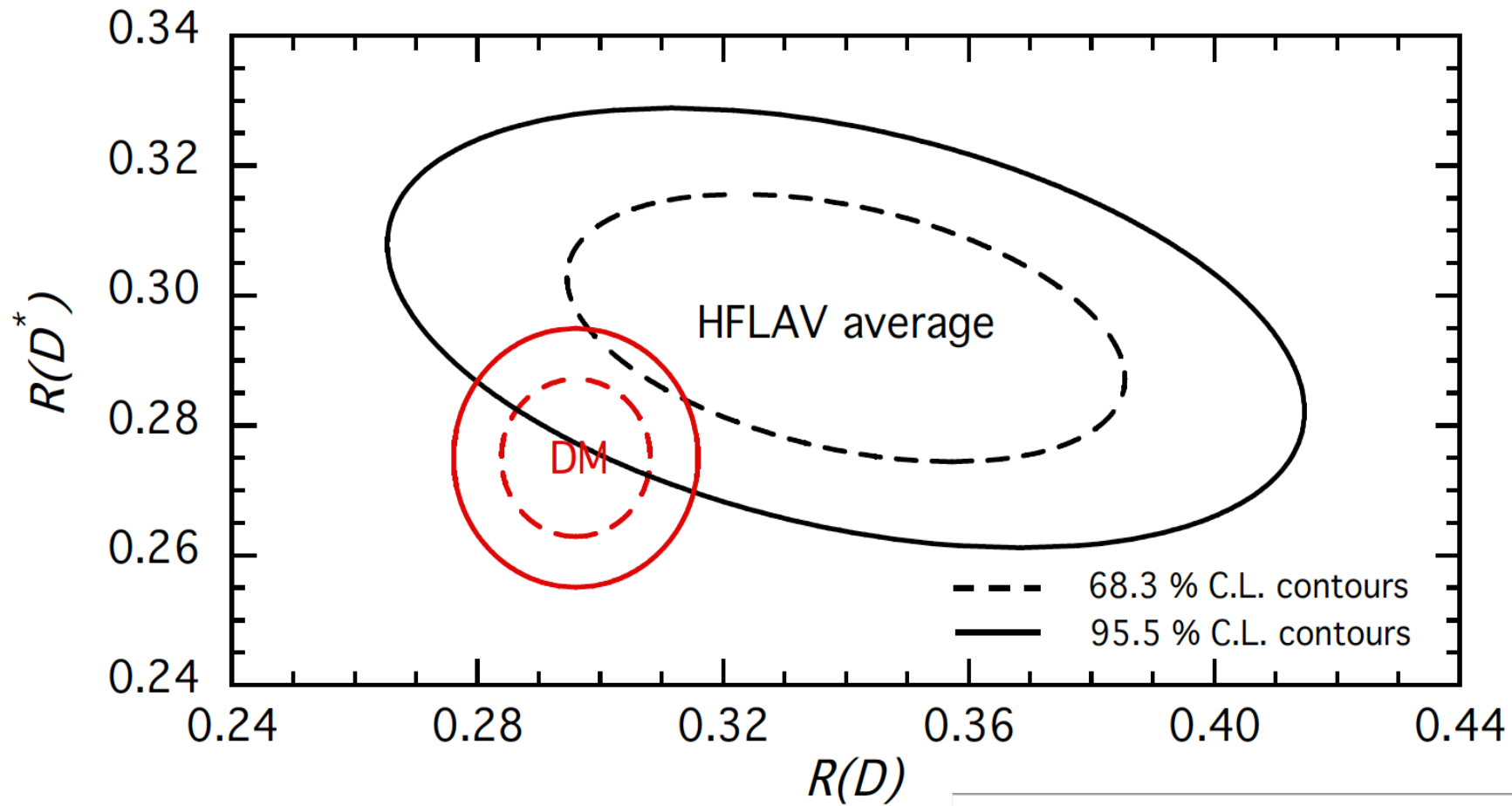
$$|V_{cb}| = (40.5 \pm 1.7) \cdot 10^{-3}$$



$ V_{cb} _{excl.} \cdot 10^3$	=	$39.6_{-1.0}^{+1.1}$	Gambino et al., arXiv:1905.08209
$ V_{cb} _{excl.} \cdot 10^3$	=	$39.56_{-1.06}^{+1.04}$	Jaiswal et al., arXiv:2002.05726
$ V_{cb} _{excl.} \cdot 10^3$	=	$38.86 \pm 0.88$	FLAG '21, arXiv:2111.09849

the use of exp. data to describe the shape of the FFs leads to smaller errors, but the use of truncated BGL fits does not guarantee that the final error is not underestimated

$$|V_{cb}|_{incl.} \cdot 10^3 = 42.16 \pm 0.50 \quad (\text{Bordone et al: arXiv:2107.00604})$$

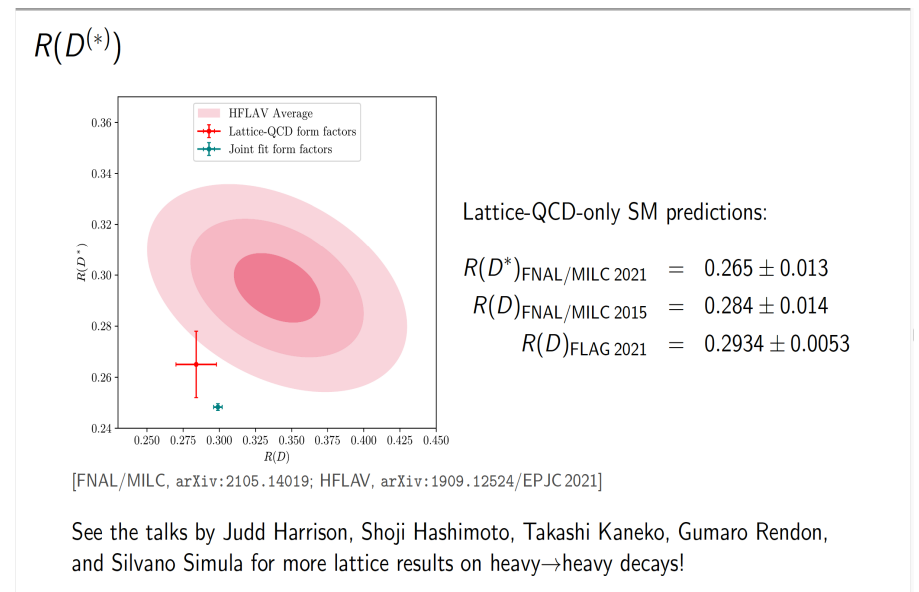


*Is there really a problem with  
Lepton Flavor Universality in  
 $B \rightarrow D^{(*)}$  decays ?*

*or*

*Much ado about nothing*

*S. Meinel CKM21*



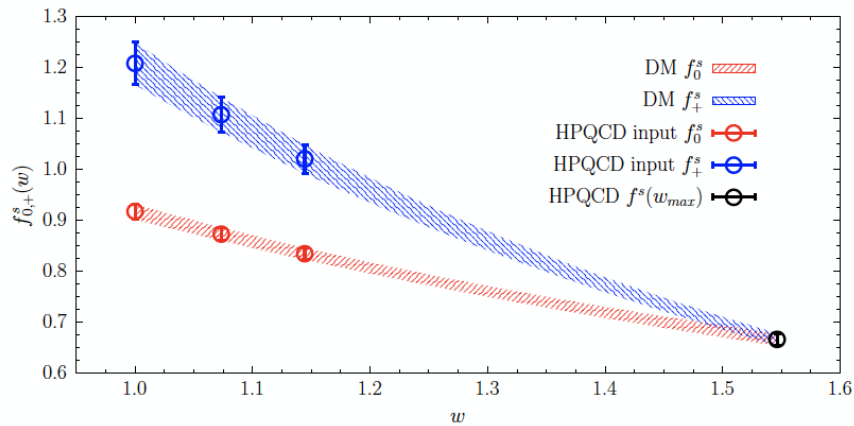
# form factors for $B_s \rightarrow D_s^{(*)} \ell \nu_\ell$ decays

[arXiv:2204.05925]

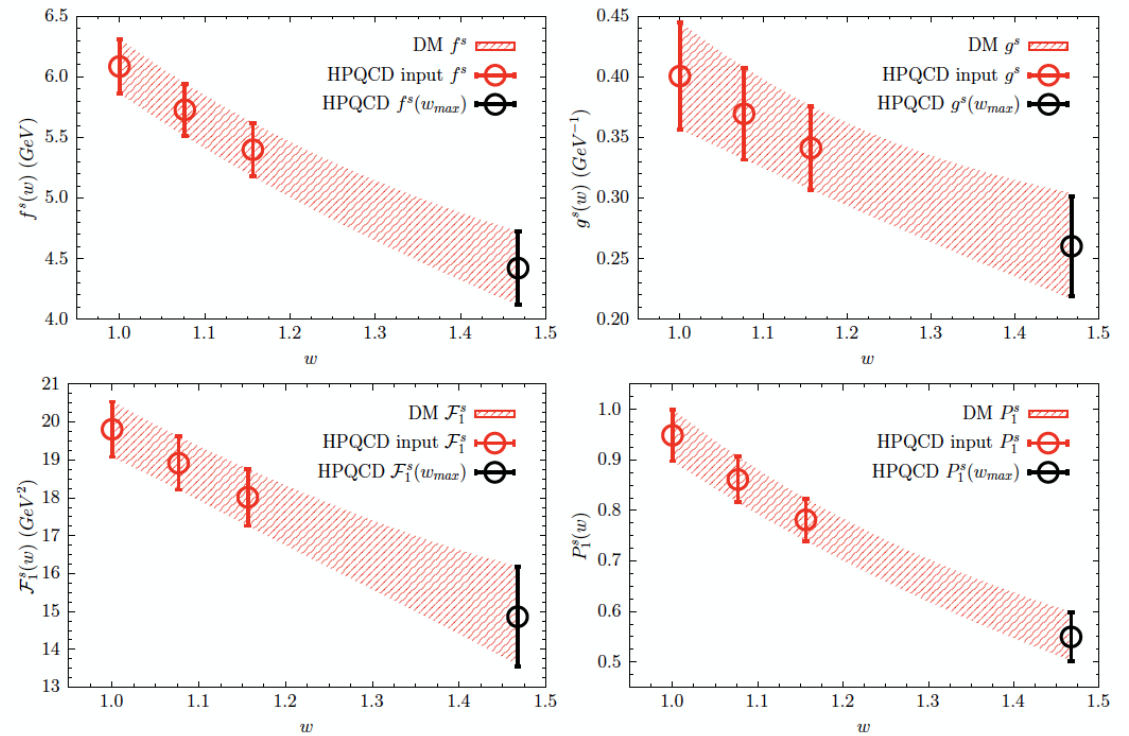
- \* lattice QCD form factors from **HPQCD arXiv:1906.00701** ( $B_s \rightarrow D_s$ ) and **arXiv:2105.11433** ( $B_s \rightarrow D_s^*$ ) in the form of BCL fits in the whole kinematical range
- \* we extract 3 data points for the FFs at small values of the recoil and we apply the DM approach

$$B_s \rightarrow D_s^* \ell \nu_\ell$$

$$B_s \rightarrow D_s \ell \nu_\ell$$



\* nice agreement in the whole kinematical range



# DM confronts BGL

two important differences in the DM method with respect to BGL parametrization

- No series expansion to describe the FFs  **NO TRUNCATION ERRORS**

particularly relevant for semileptonic decays characterized by a very large  $q^2$  range

$$B \rightarrow \pi \ell \nu$$

$$\text{Maximum } q^2 = 26.46 \text{ GeV}^2$$

$$\Lambda_b \rightarrow p \ell \nu$$

$$\text{Maximum } q^2 = 21.9 \text{ GeV}^2$$

- Unitarity check of FFs data completely independent of the parameterization

## The DM approach

- i) reproduces exactly the known data
- ii) allows to extrapolate the form factor in the whole kinematical range
- iii) in a parameterization-independent way
- iv) providing a band of values representing the results of all possible BGL fits satisfying unitarity and passing through the known points (important for estimating uncertainties)

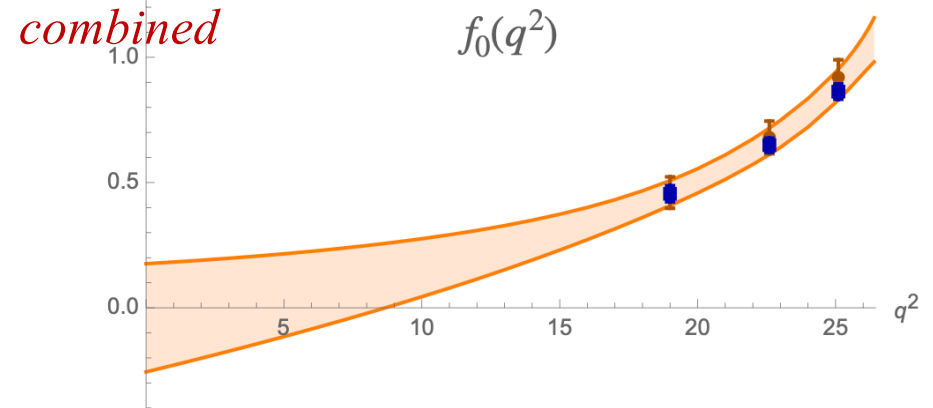
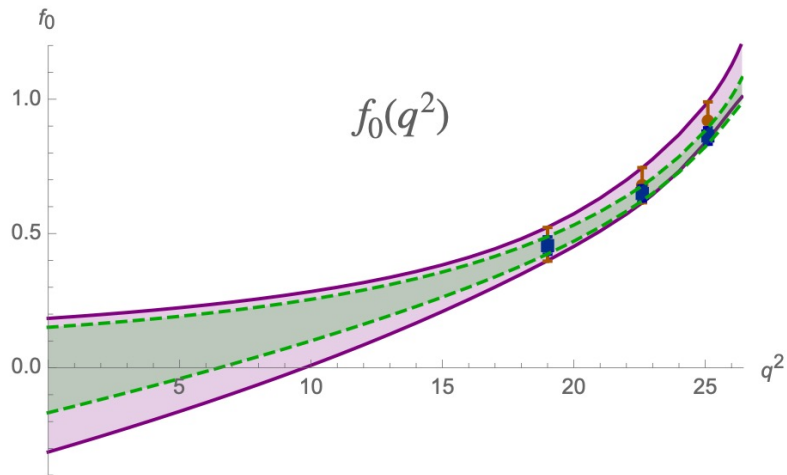
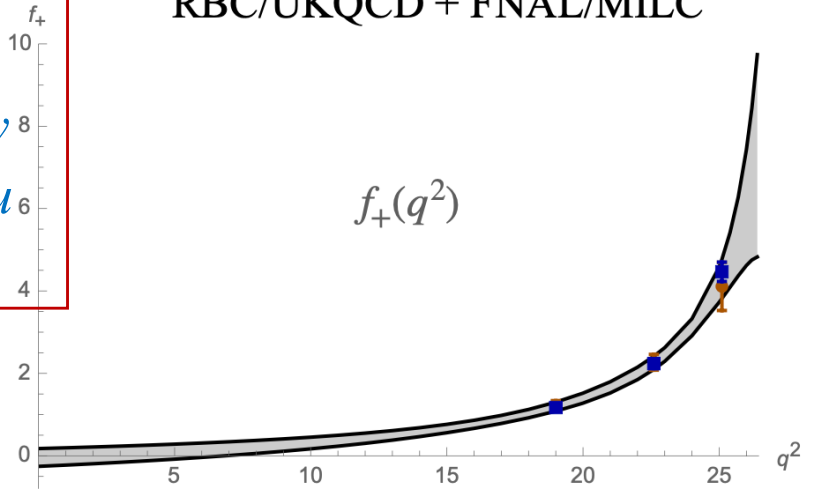
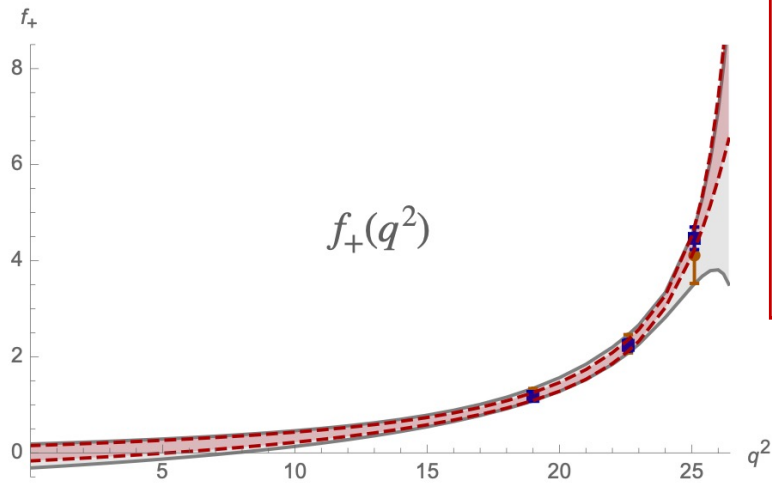
# Semileptonic $B \rightarrow \pi$ decays (in prep.)

Solid: RBC/UKQCD

Dashed: FNAL/MILC

RBC/UKQCD + FNAL/MILC

*Non-perturbative susceptibility for the  $b \rightarrow u$  current*

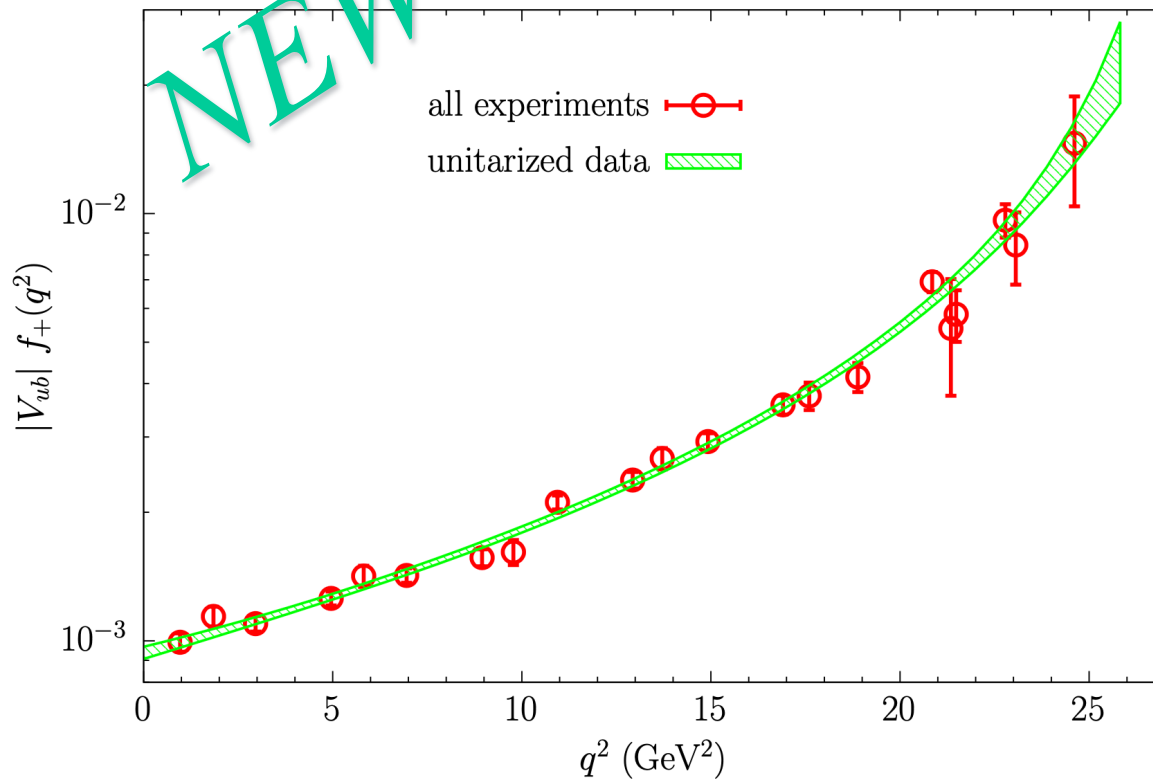


	$f_+(0) = f_0(0)$
RBC/UKQCD	$-0.06 \pm 0.25$
FNAL/MILC	$-0.01 \pm 0.16$
Combined	$-0.04 \pm 0.22$
LCSR	$0.28 \pm 0.03$

- 3 RBC/UKQCD data (points) for each FF [arXiv:1501.05363]
- 3 FNAL/MILC data (squares) for each FF [arXiv:1503.07839]



# Unitarization of the experimental data



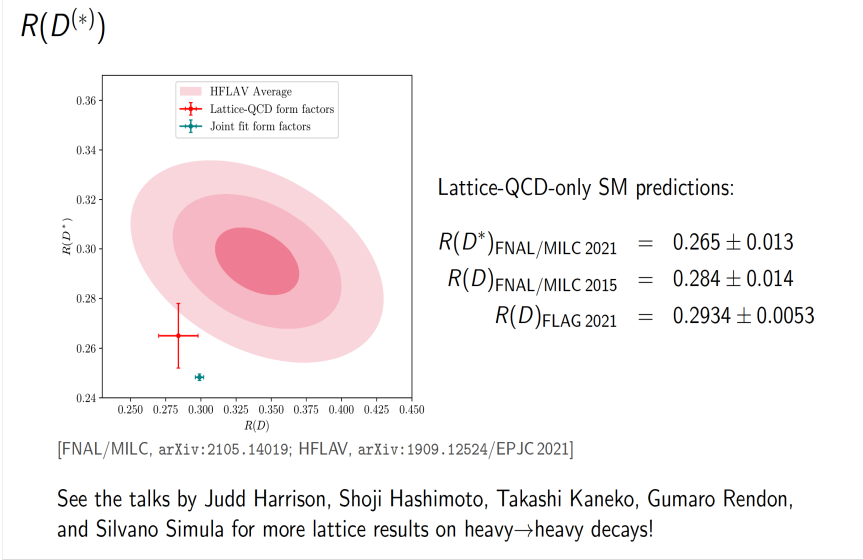
$$|V_{ub}|_{\text{DM}} \times 10^3 = 3.88 \pm 0.32$$

Reference	$ V_{ub}  \times 10^3$
FLAG '21	$3.74 \pm 0.17$
HFLAV '18 & PDG '20	$4.32 \pm 0.29$
Belle Coll. '21	$4.10 \pm 0.28$

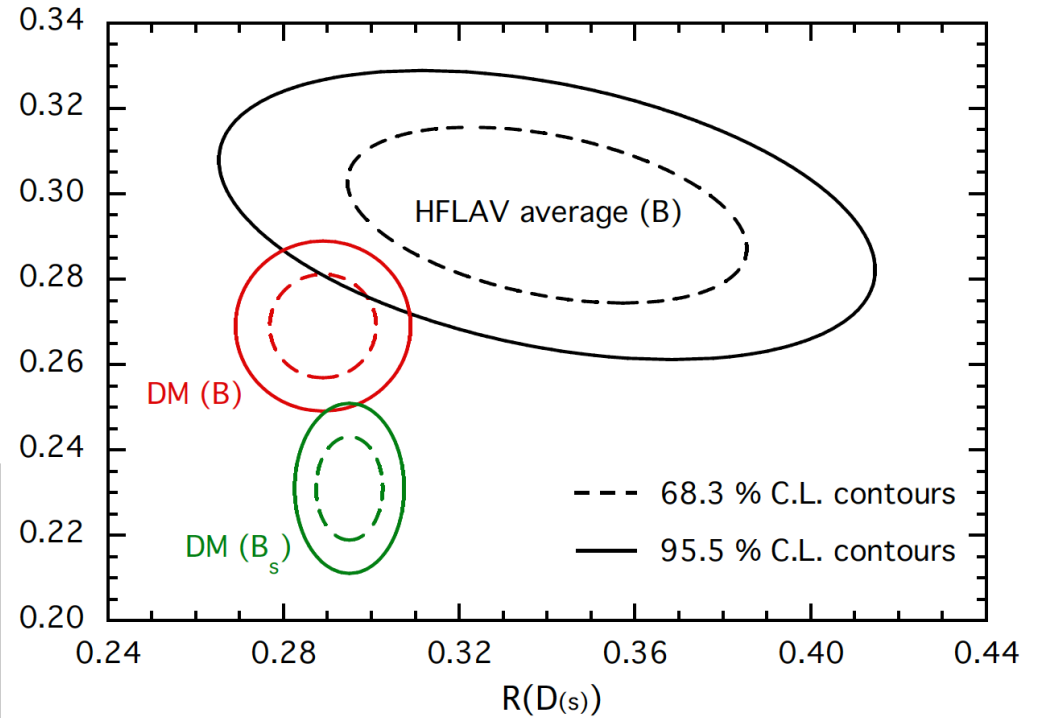
Exclusive and the inclusive values are compatible at the  $1\sigma$  level

- \* construct the experimental values of  $|V_{ub}f_+(q_i^2)| = \sqrt{\Delta\Gamma_i/z_i}$  ( $z_i$  = kinematical coefficient in the i-th bin)
- \* apply the DM method on the data points  $|V_{ub}f_+(q_i^2)|$  using the unitarity bound  $|V_{ub}|^2 \chi_{1-}(0)$  with an initial guess for  $|V_{ub}|$
- \* determine  $|V_{ub}|$  using the theoretical DM bands and iterate the procedure until consistency for  $|V_{ub}|$  is reached

# S. Meinel CKM21



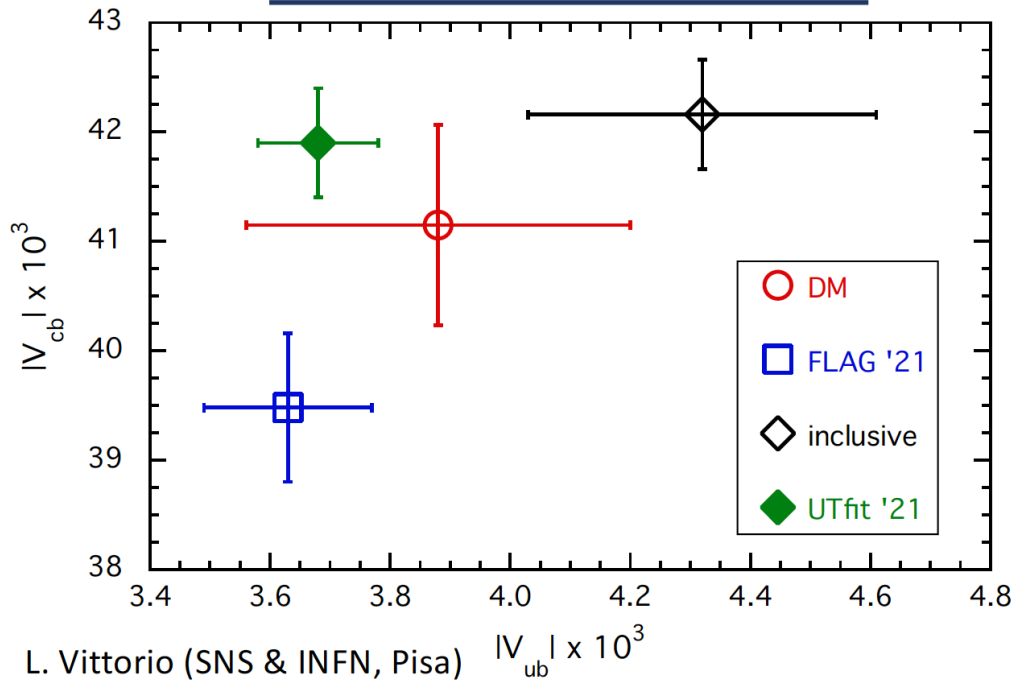
$R(D_{(s)})$



## LFU observables

**IMPORTANT:** the difference between the red and the green area comes from the difference in the LQCD computations by FNAL/MILC and HPQCD Collaborations

## CKM matrix elements



# Conclusions

*The Dispersion Matrix approach is a powerful tool to implement unitarity in the analysis of exclusive semileptonic decays of mesons and baryons*

- it does not rely on any assumption about the momentum dependence of the hadronic form factors
- it can be based entirely on first principles (*i.e.* unitarity and analyticity) using non-perturbative lattice determinations of both the relevant form factors and the dispersive bounds (the susceptibilities) from appropriate 2-point and 3-point (Euclidean) correlation functions
- it predicts band of values that are equivalent to all possible BGL fits satisfying unitarity and reproducing exactly a given set of data points. Larger but more reliable uncertainties
- It is not biased by the fit of the experimental data
- it is universal, namely it can be applied to any exclusive semileptonic decay e.g. baryon decays

# Conclusions 2

*New insight on both:*

- *the  $|V_{cb}|$ ,  $|V_{ub}|$  puzzles (exclusive and inclusive determinations compatible @ the  $1\sigma$  level)*

*We found problems with the Belle covariance matrix*

- *the  $R(D^*)$  anomalies (theoretical values and measurements compatible @ the  $1.6\sigma$  level)*

*• No apparent deviation in the down sector, what about the up one?*