

Tensor network simulations of quantum circuits with finite fidelity

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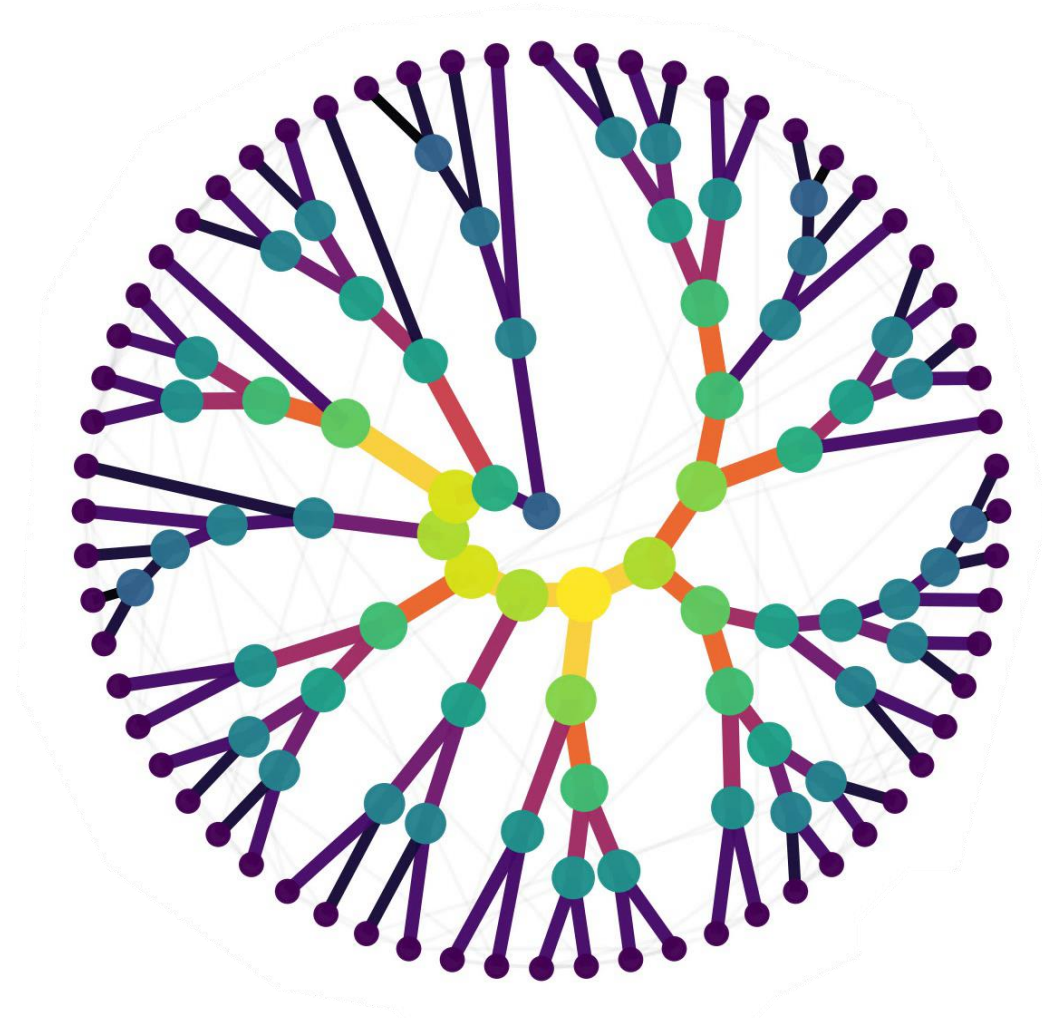
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In collaboration with:

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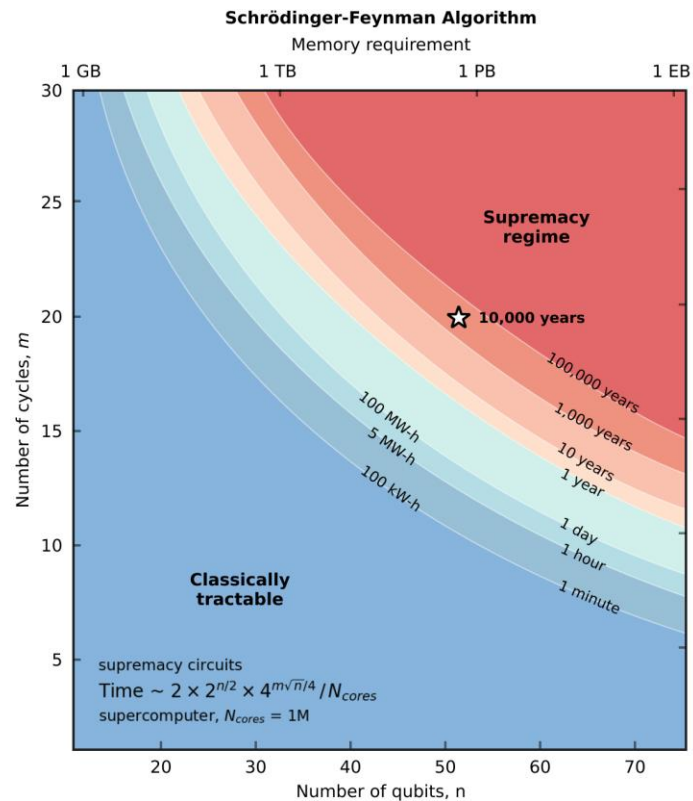
Giuseppe Magnifico (UniBa)

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Quantum advantage

“...classical systems cannot simulate highly entangled quantum systems efficiently, and we hope to hasten the day when well controlled quantum systems can perform tasks surpassing what can be done in the classical world.” John Preskill



Google’s original division of classically tractable vs. supremacy regimes

Quantum advantage and tensor networks

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Quantum supremacy using a programmable superconducting processor



Hyper-optimized tensor network contraction

Johnnie Gray^{1,2} and Stefanos Kourtis^{1,3,4}

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[Ivan Bacon](#), [Joseph C. Bardin](#), [Rami Barends](#), [Rupak Biswas](#), [David A. Buell](#), [Brian Burkett](#), [Yu Chen](#), [Zijun Chen](#), [Ben Chiaro](#), [Dunsworth](#), [Edward Farhi](#), [Brooks Foxen](#), [Austin Fowler](#), [Craig](#)

[arXiv](#) > [quant-ph](#) > [arXiv:2005.06787](#)

Quantum Physics

[Submitted on 14 May 2020]

Classical Simulation of Quantum Supremacy Circuits

[Cupjin Huang](#), [Fang Zhang](#), [Michael Newman](#), [Junjie Cai](#), [Xun Gao](#), [Zhengxiong Tian](#), [Junyin Wu](#), [Haihong Xu](#), [Huanjun Yu](#), [Bo Yuan](#), [Mario Szegedy](#), [Yaoyun Shi](#), [Jianxin Chen](#)

A performance of 1.2 Eflops (single-precision), or 4.4 Eflops (mixed-precision) for simulating a $10 \times 10 \times (1+40+1)$ circuit (a new milestone for classical simulation of RQC), using about 42 million Sunway cores. The time to sample Goolge Sycamore in a simulation way is reduced from years to 304 seconds.

QuanTeN.jl

Hyper-optimized contractions + simulations with finite fidelity

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QuanTeN.jl

A Julia package for optimised quantum simulations with tensor networks

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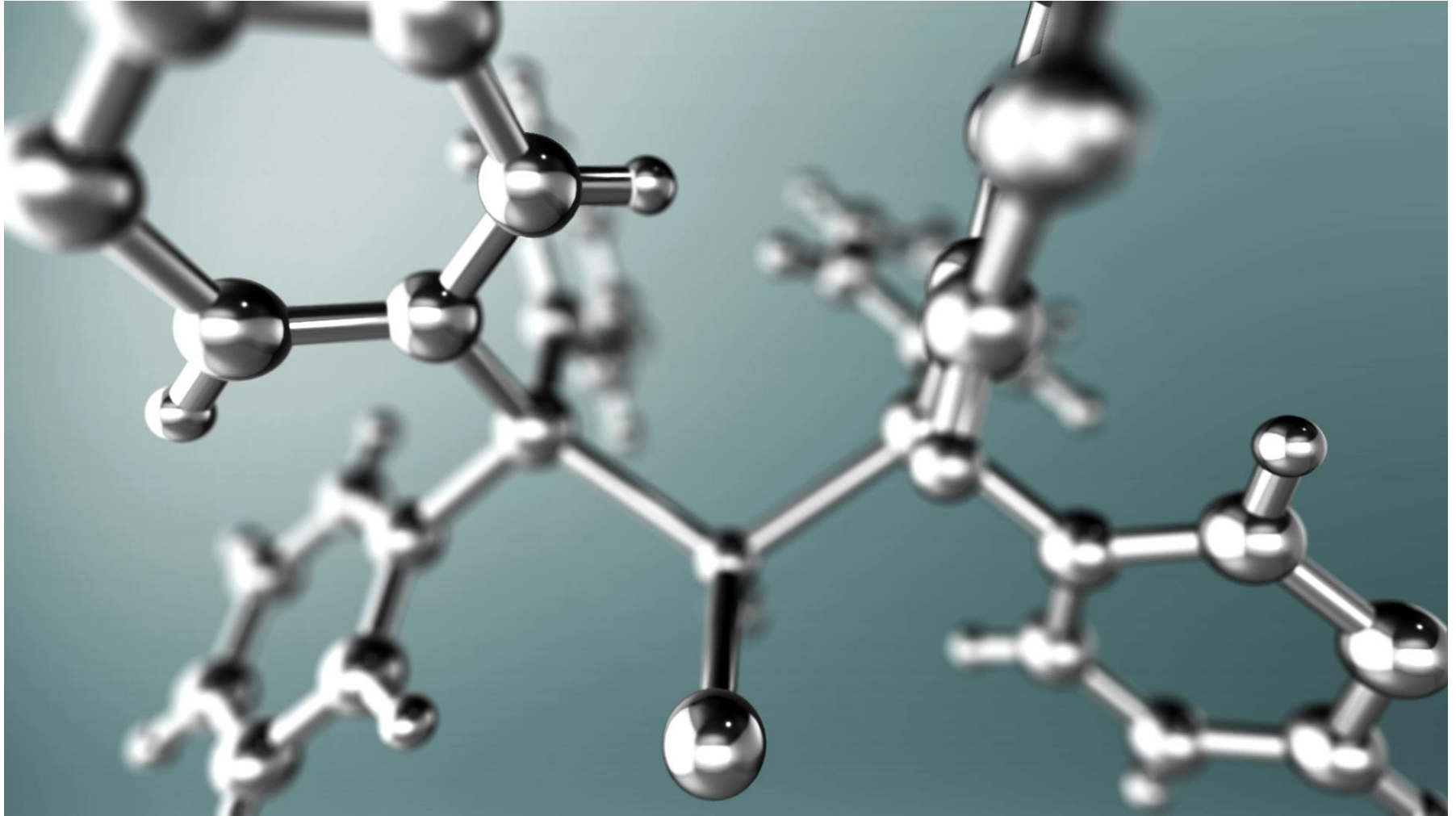
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Languages

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2. Exact contractions



Tensor notation

vector

$$v_j$$



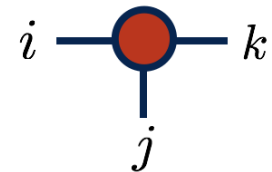
matrix

$$M_{ij}$$



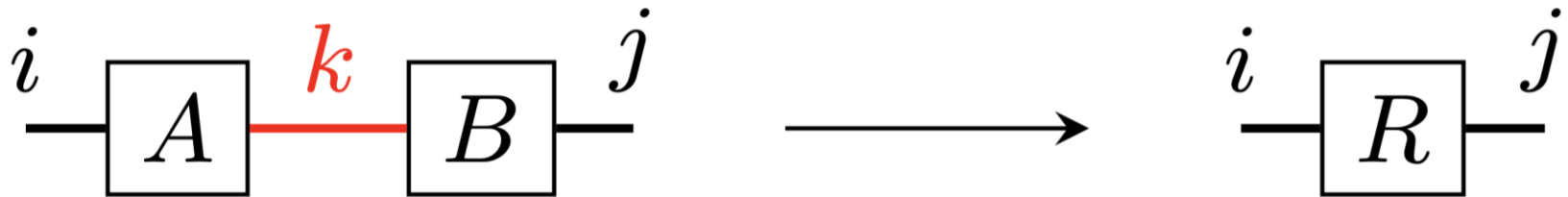
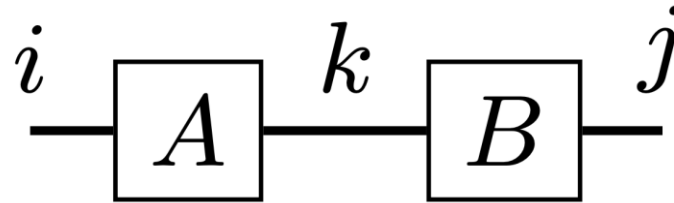
3-index
tensor

$$T_{ijk}$$



Source: <https://tensornetwork.org/>

Contractions

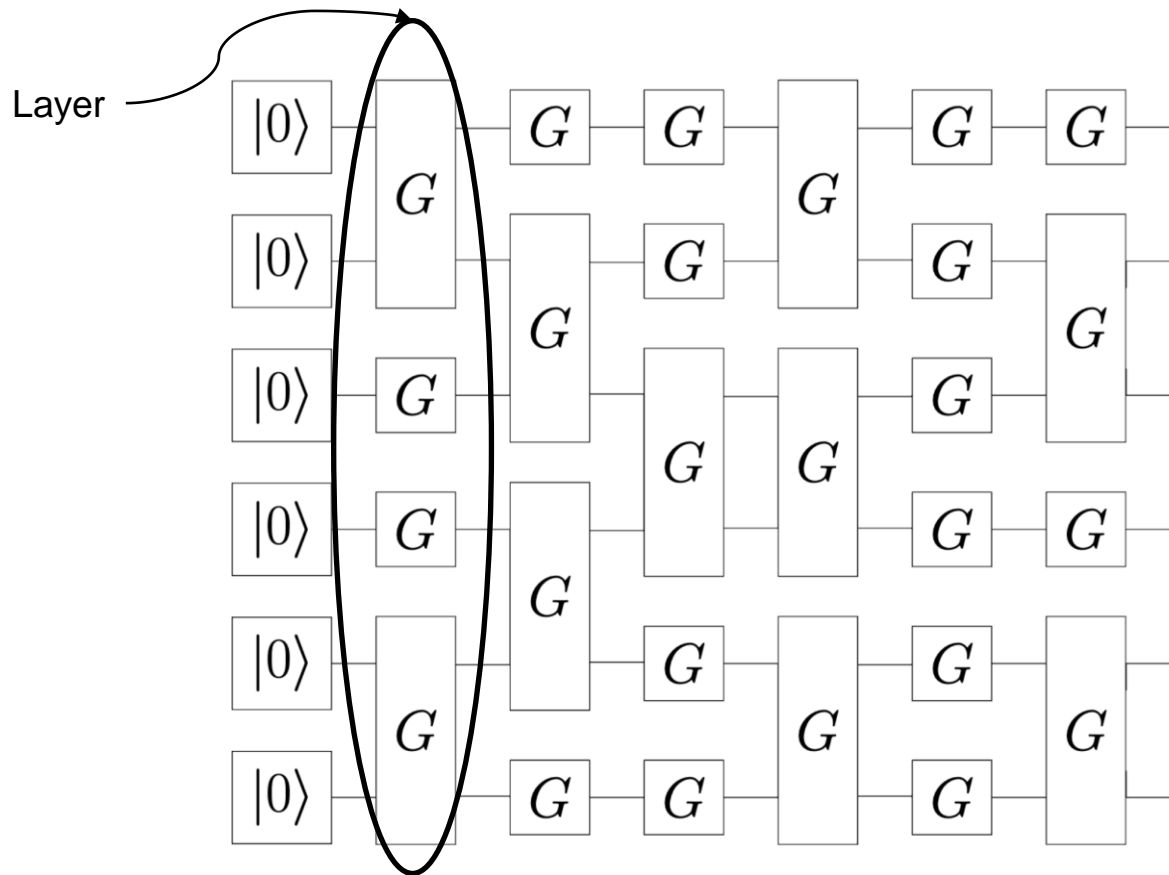


$$R_{ij} = \sum_k A_{ik} B_{kj}$$

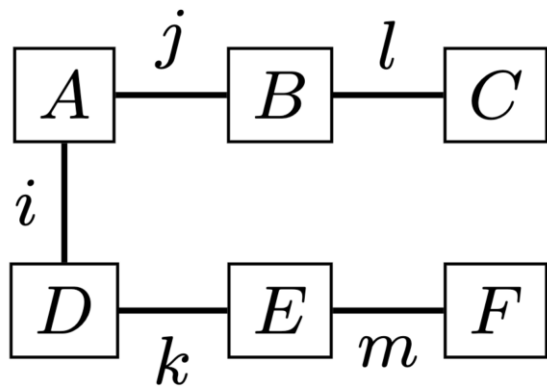
Tensor network approach for quantum circuits

Initial state: MPS

Evolution: n -qubit gate \rightarrow rank- $2n$ tensor

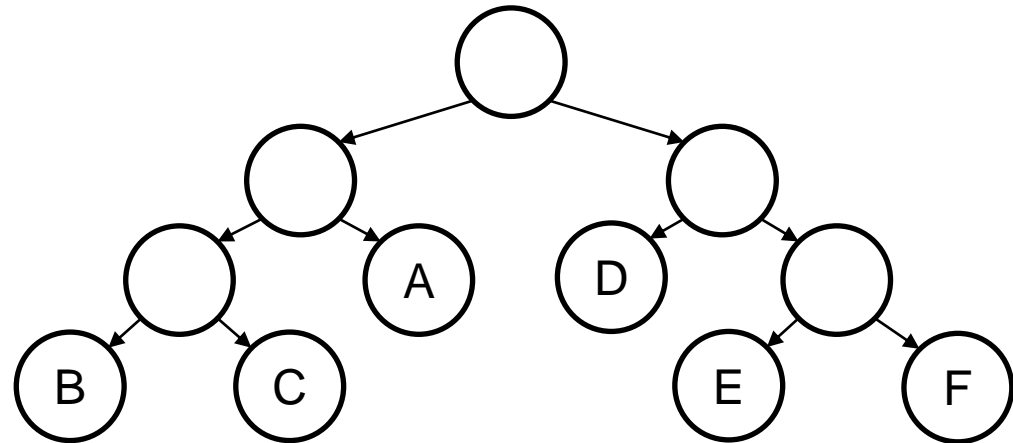


Schrödinger method

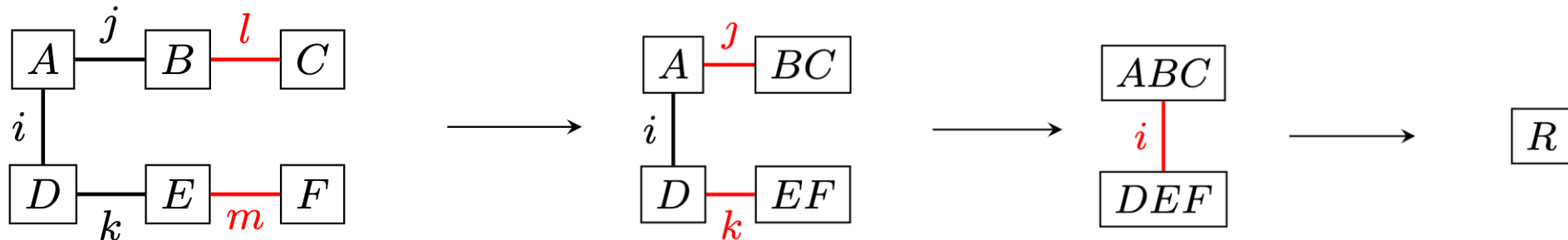


Heuristics

Contraction sequence:
[[A, [B, C]], [D, [E, F]]]



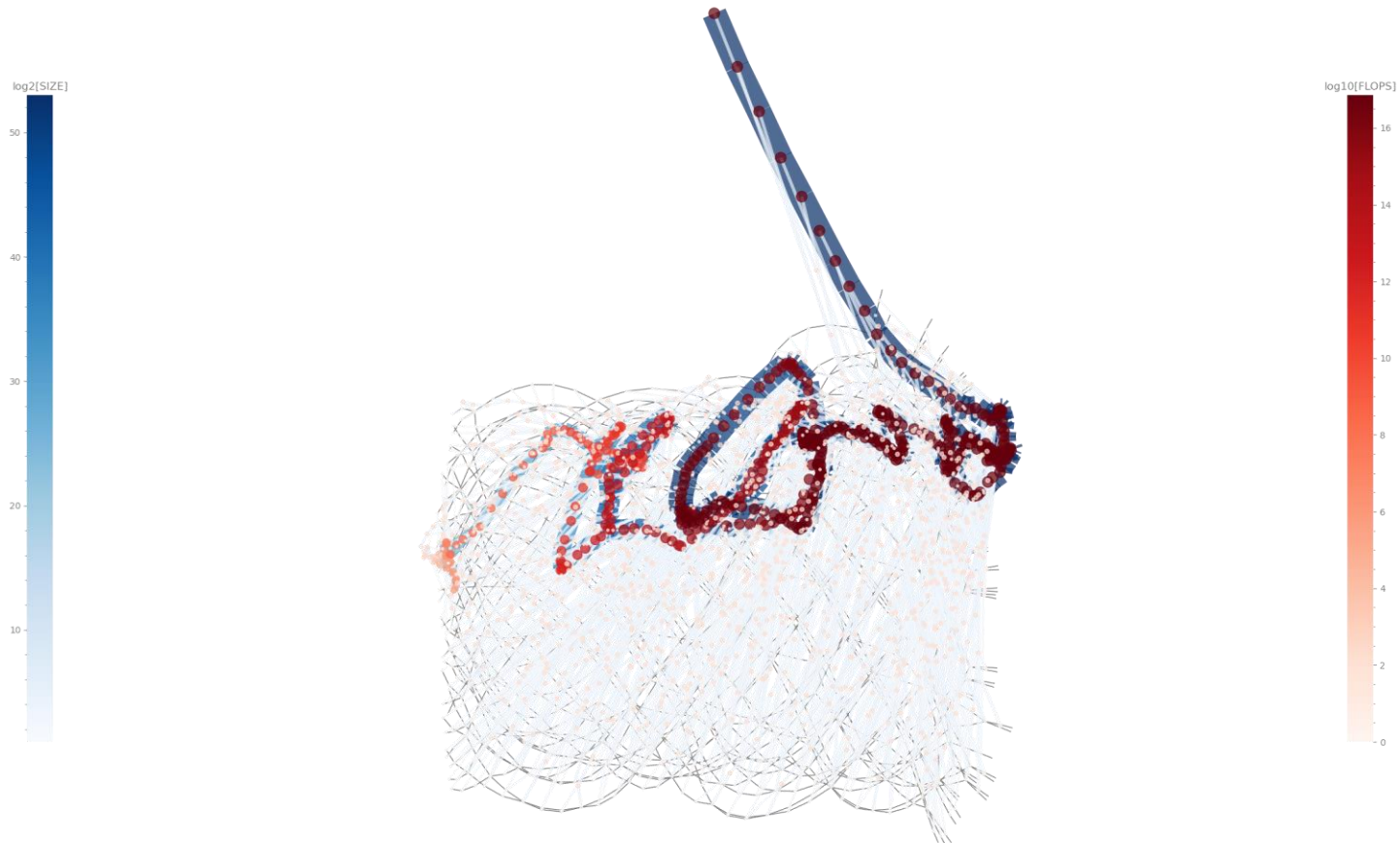
Contractor



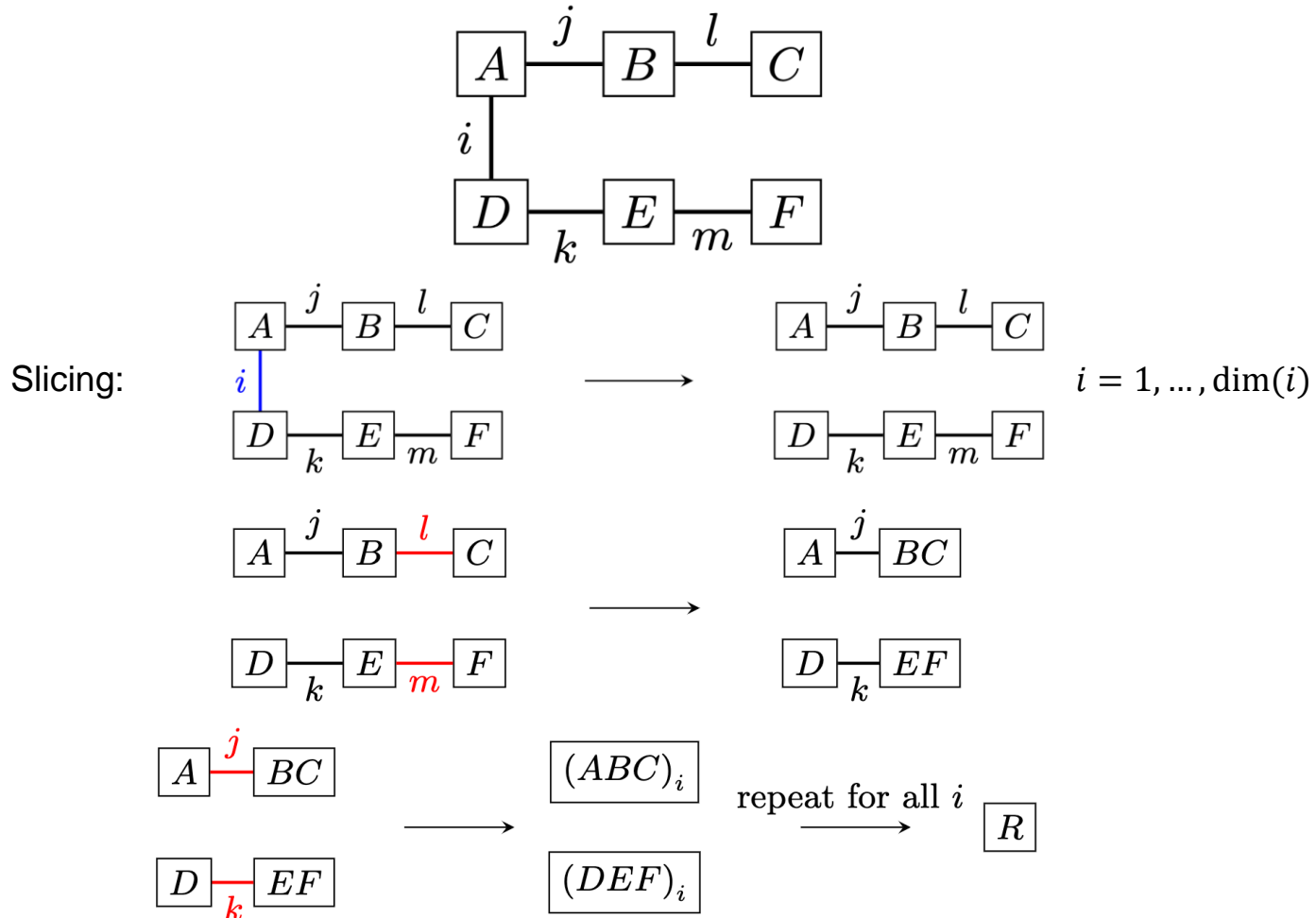
Heuristics – `opt_einsum` and `cotengra`

`opt_einsum`: random-greedy approach to minimize size of contractions

`cotengra`: collection of many heuristics to optimize the contraction tree



Schrödinger-Feynman method



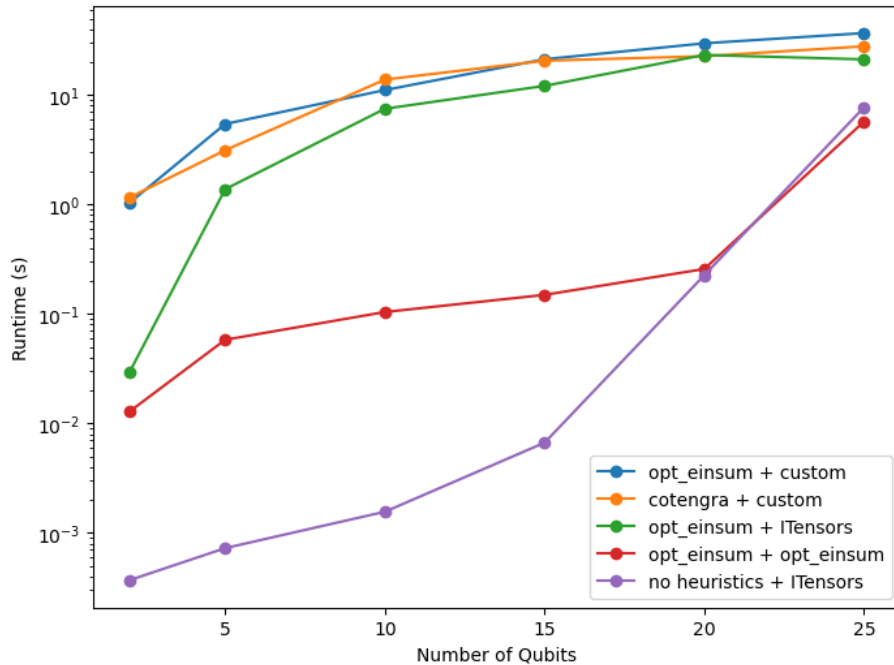
Results – Benchmarking exact contractions

Heuristics	Contractors
opt_einsum	custom
cotengra	custom
opt_einsum	ITensors.jl
opt_einsum	opt_einsum
Left to right	ITensors.jl

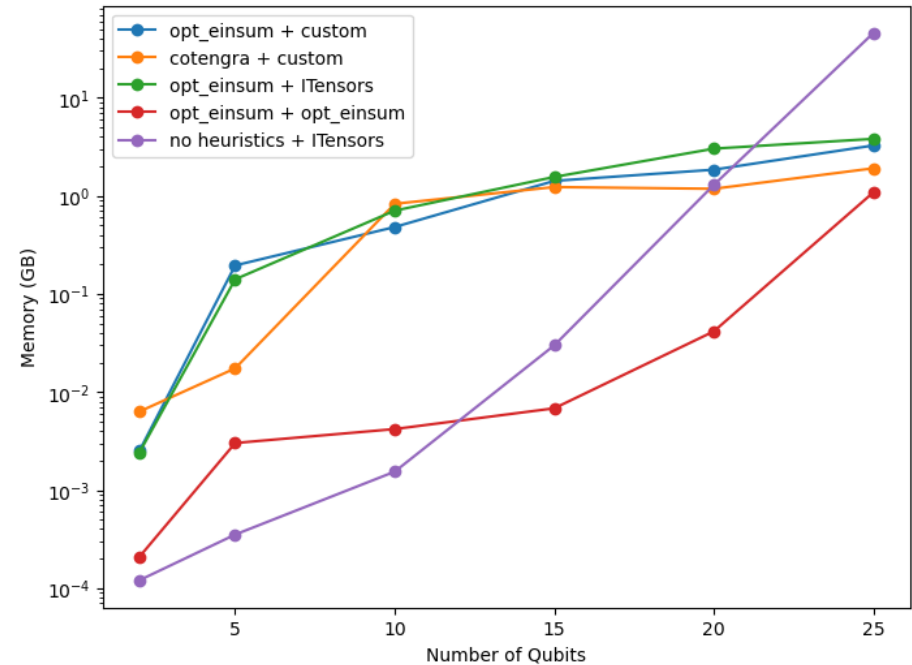
Results – Benchmarking exact contractions

Low number of layers:

Runtime of Random Quantum Circuit for 5 Layers



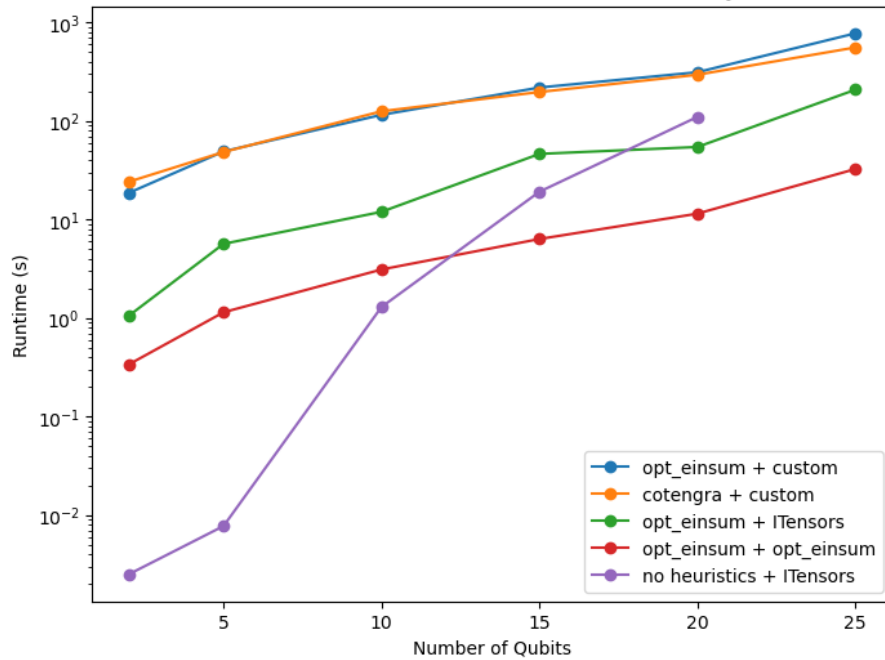
Memory Usage of Random Quantum Circuit for 5 Layers



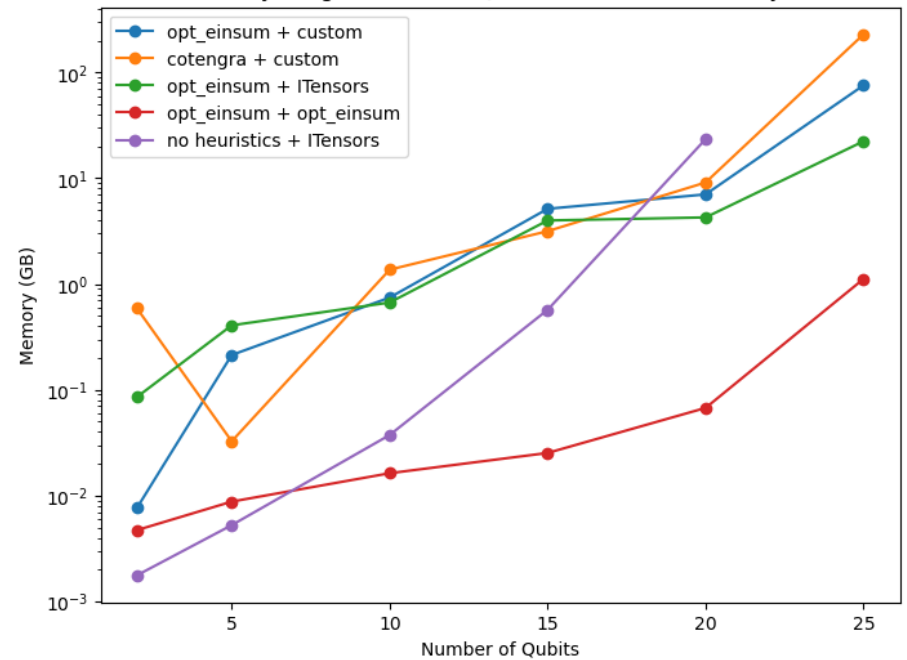
Results – Benchmarking exact contractions

High number of layers:

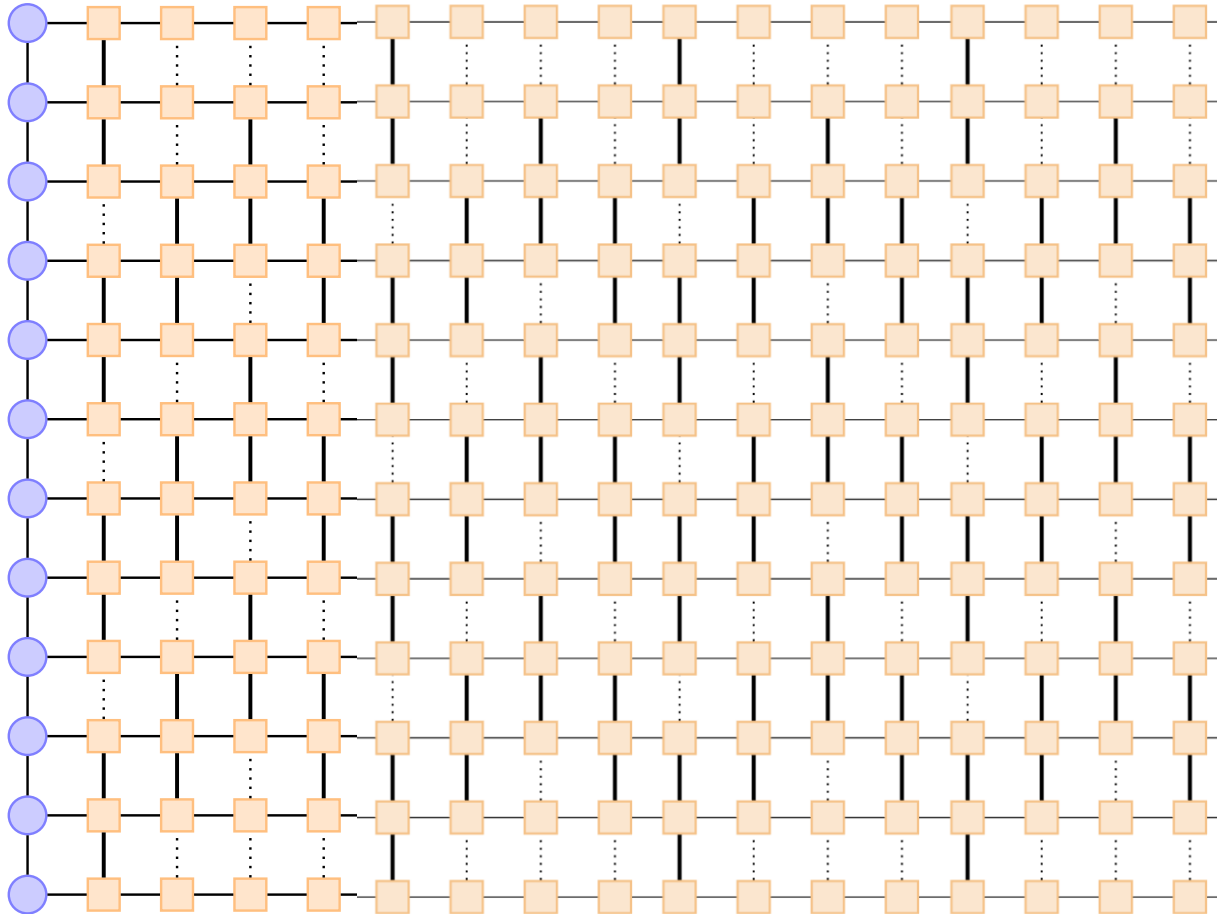
Runtime of Random Quantum Circuit for 100 Layers



Memory Usage of Random Quantum Circuit for 100 Layers



3. Simulations with finite fidelity



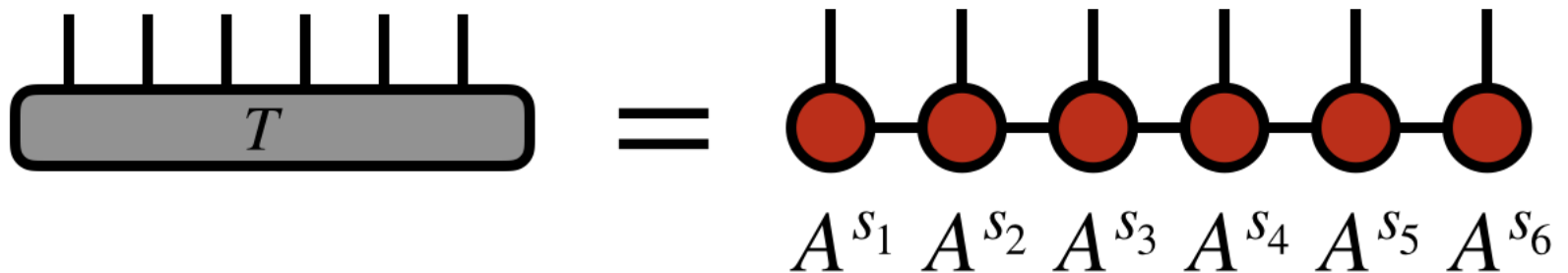
SVD and MPS

Singular Value Decomposition (SVD):



Successive SVDs and truncation of singular values produce a matrix-product state (MPS) approximation of a tensor:

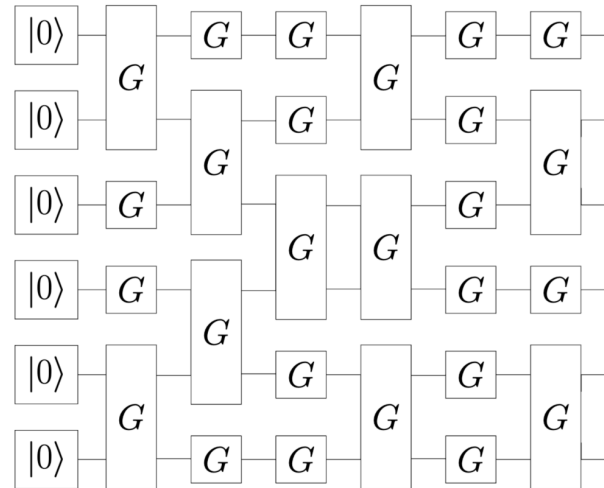
$$T^{s_1 s_2 s_3 s_4 s_5 s_6} = \sum_{\{\alpha\}} A_{\alpha_1}^{s_1} A_{\alpha_1 \alpha_2}^{s_2} A_{\alpha_2 \alpha_3}^{s_3} A_{\alpha_3 \alpha_4}^{s_4} A_{\alpha_4 \alpha_5}^{s_5} A_{\alpha_5}^{s_6}$$



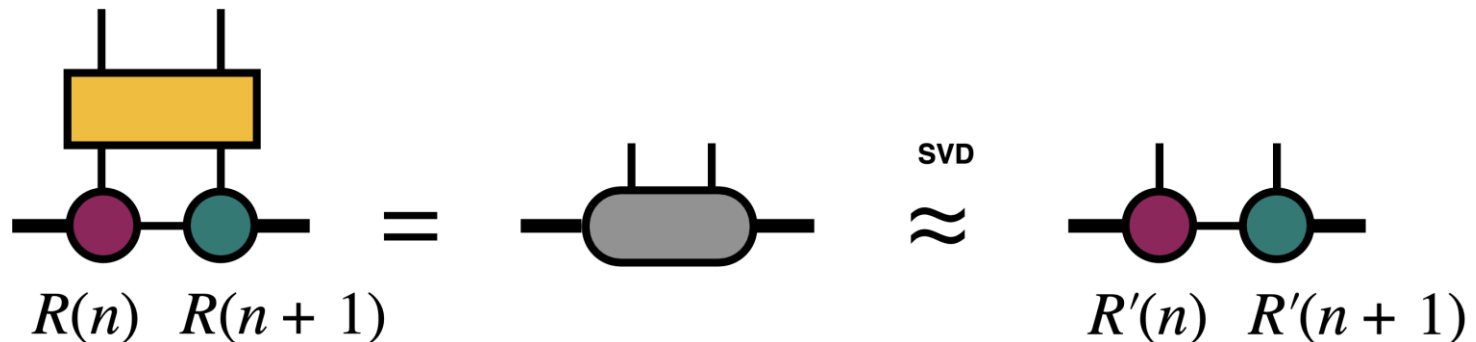
Bond dimension grows exponentially with the number of entangling gates applied

Time-Evolving Block Decimation (TEBD)

Remember random quantum circuit:

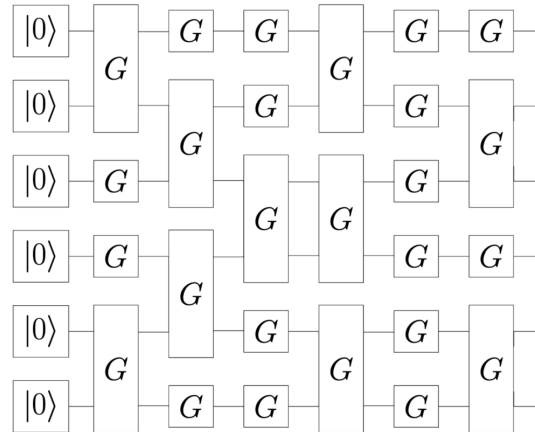


For each layer, apply gates and, for entangling gates, perform truncated SVD [6]

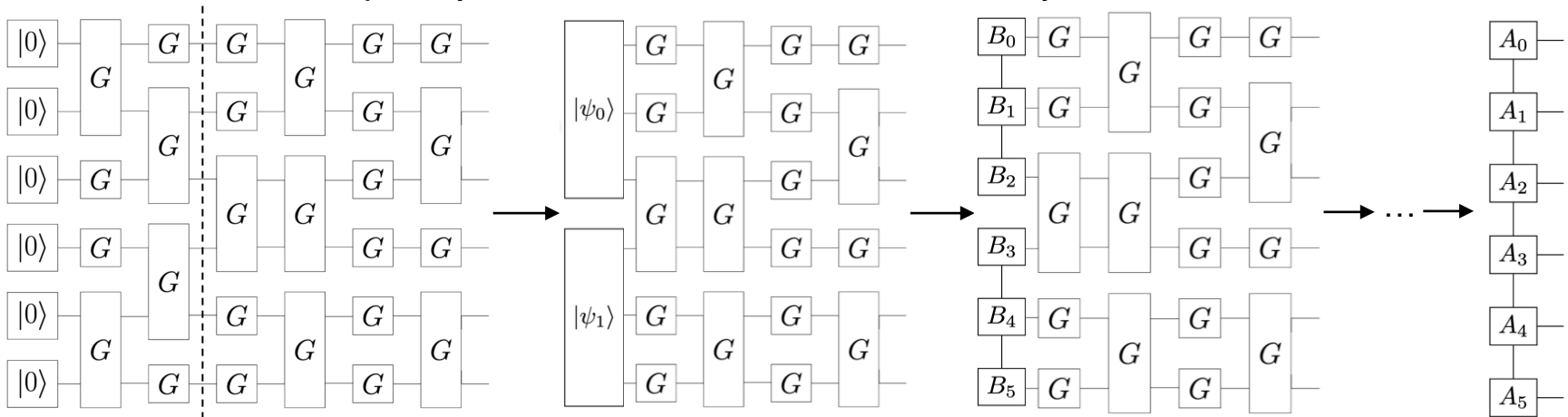


Cluster-TEBD

Remember random quantum circuit:

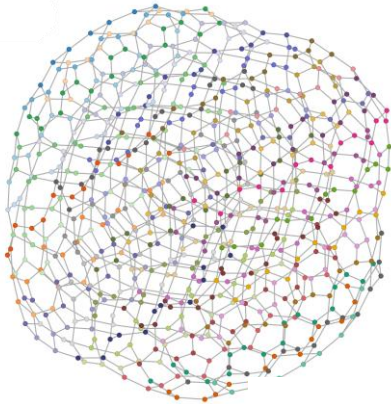


Instead, contract multiple layers until exact state fits in memory

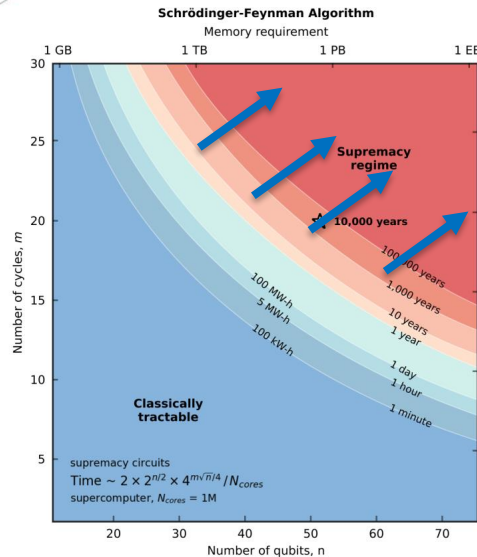
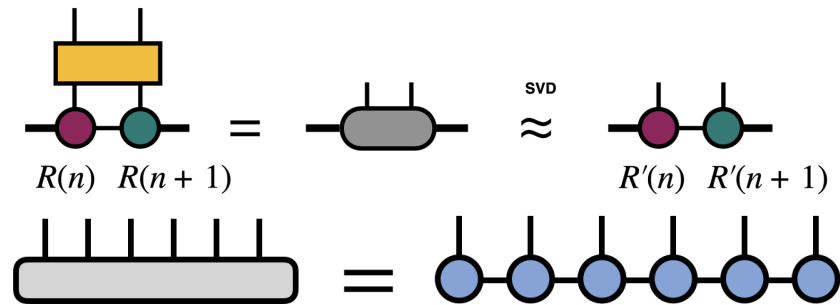


Summary

Heuristics for optimized exact contractions of deep quantum circuits



SVD and MPS to store many-qubit quantum circuits



Push boundaries to find actual quantum advantage threshold

References

- [1] John Preskill, “Quantum computing and the entanglement frontier”. arXiv:1203.5813 (2012)
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- [3] Daniel G. A. Smith and Johnnie Gray, “opt_einsum - A Python package for optimizing contraction order for einsum-like expressions”. *Journal of Open Source Software*, 3(26), 753 (2018)
- [4] Johnnie Gray and Stefanos Kourtis, “Hyper-optimized tensor network contraction”. *Quantum* 5, 410 (2021)
- [5] Cupjin Huang *et al.*, “Classical Simulation of Quantum Supremacy Circuits”. arXiv:2005.06787 (2020)
- [6] Yiqing Zhou, E. Miles Stoudenmire and Xavier Waintal, “What limits the simulation of quantum computers?” *Phys. Rev. X* **10**, 041038 (2020)