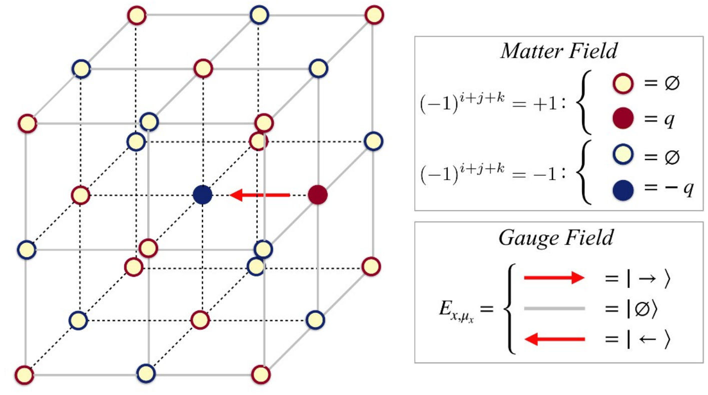
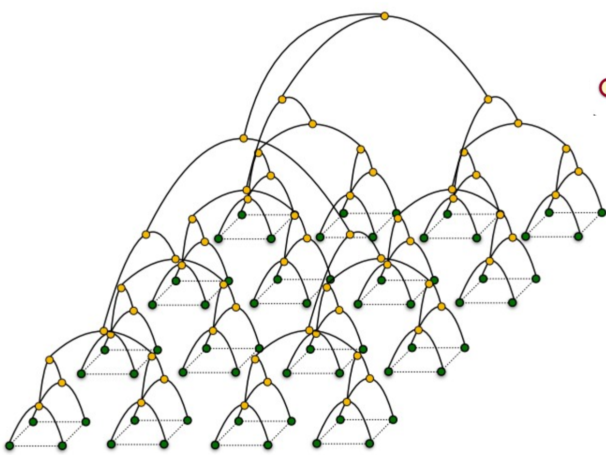
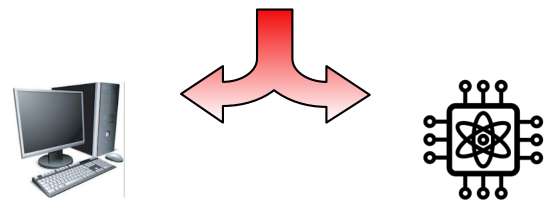
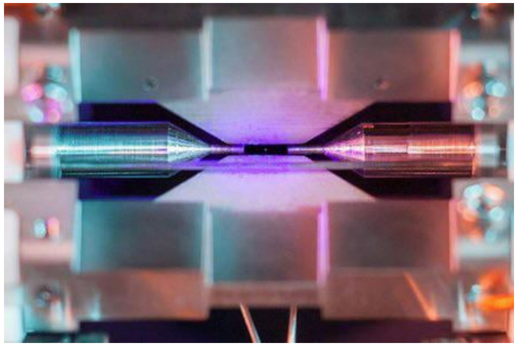
Advances in the Simulation of Hamiltonian Lattice Gauge Theories









Speaker: Pietro Silvi

Department of Physics and Astronomy – University of Padua & INFN Padova
Padua Quantum Technologies Research Center (PaduaQTech)
Quantum Computing and Simulation Center (QCSC)

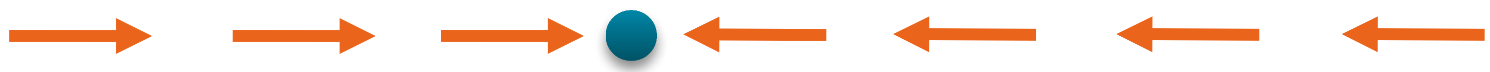
Lattice Gauge Theories (LGT)

1) Quantum Matter and Quantum Fields

2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$

$$-e$$



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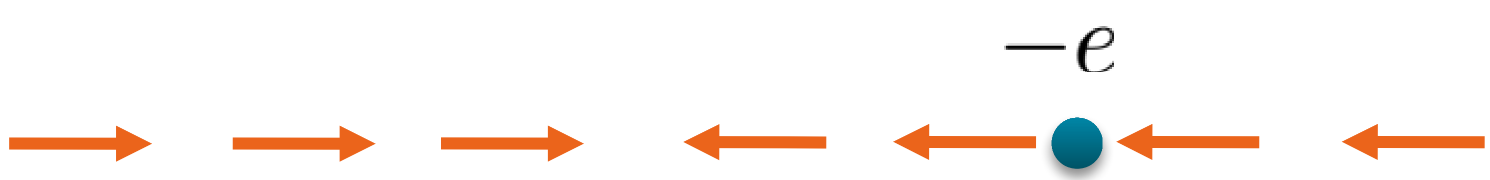

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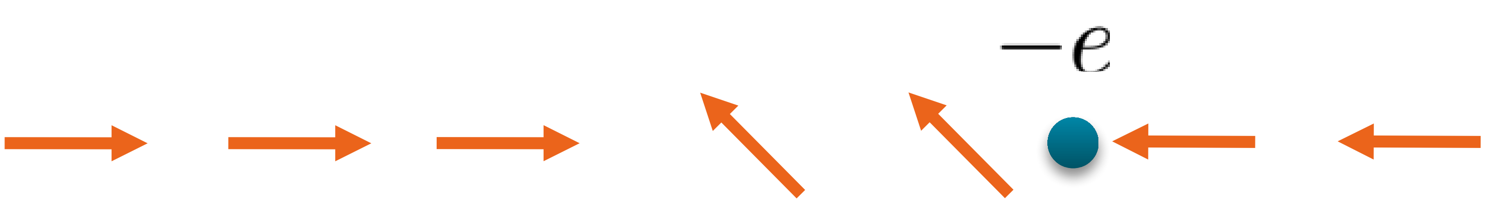

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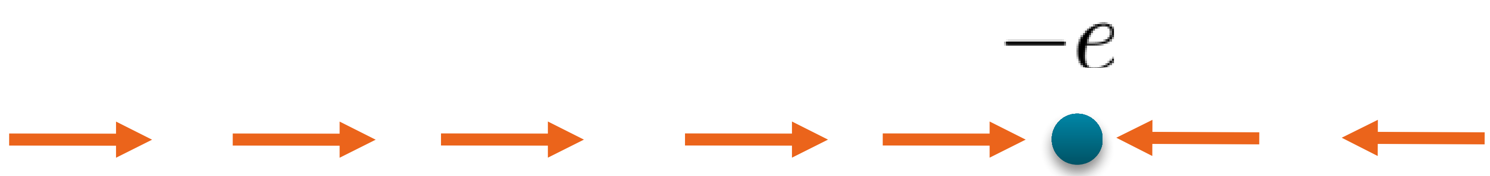
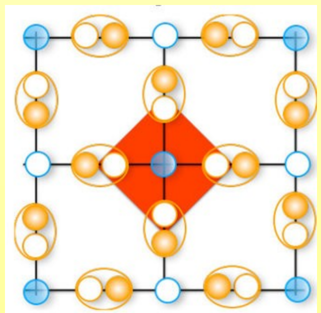
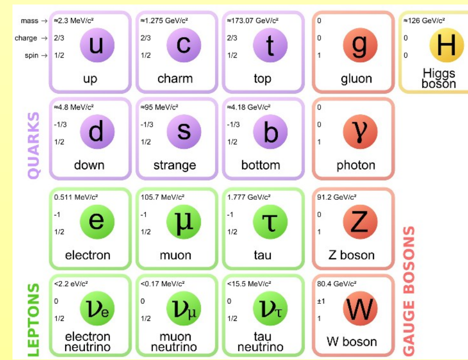


LGT are almost everywhere in theoretical physics!

As emergent theories in condensed matter: **high-Tc superconductors, frustrated systems, spin liquids.**



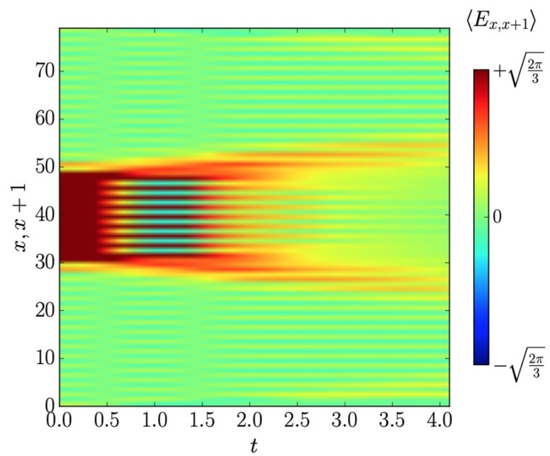
As fundamental description in particle physics: **Standard Model**



- They are <u>**extremely demanding**</u> from a numerical point of view.
- Powerful numerical methods, such as Monte Carlo, fail in several regimes of finite-density or for non-equilibrium phenomena (sign-problem).
- Ideal goal for quantum-inspired efficient algorithms and quantum simulation/computation!

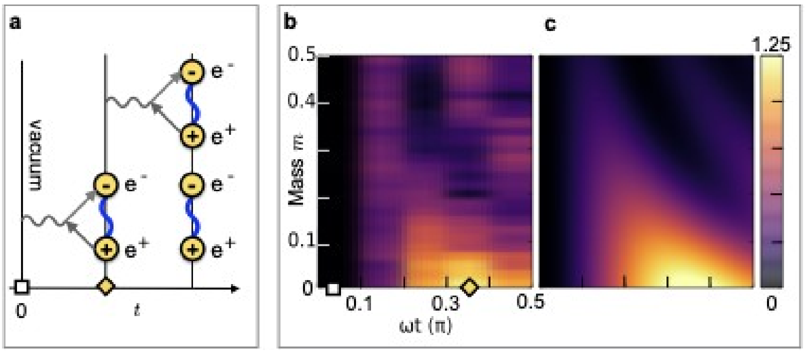
Quantum Technologies for LGT

Efficient quantum-inspired algorithms: Tensor Networks (no sign-problem)



Quantum **4**, 281 (2020)

First implementation of U(1) LGT on digital quantum computer



Nature **534,** 516–519 (2016).

Simulating Lattice Gauge Theories within Quantum Technologies

M.C. Bañuls[1,2], R. Blatt[3,4], J. Catani[5,6,7], A. Celi[3,8], J.I. Cirac[1,2], M. Dalmonte[9,10], L. Fallani[5,6,7], K. Jansen[11], M. Lewenstein[8,12,13], S. Montangero[7,14 a], C.A. Muschik[3], B. Reznik[15], E. Rico[16,17 b], L. Tagliacozzo[18], K. Van Acoleyen[19], F. Verstraete[19,20], U.-J. Wiese[21], M. Wingate[22], J. Zakrzewski[23,24], and P. Zoller[3]

Eur. Phys. J. D **74,** 165 (2020)

Dipartimento di Fisica e Astronomia Galileo Galilei

INFN

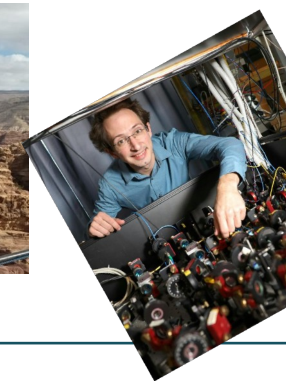
PASQuanS2

QuantHEP

T-NiSQ

Tensor Networks in Simulation of Quantum Matter


Non-Abelian

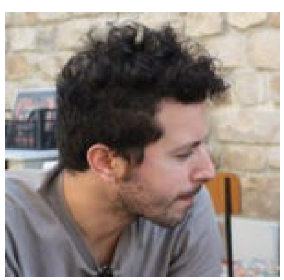
Classical Simulation

(Tensor Networks)



Quantum Simulation

(Trapped ion qudits)

Abelian

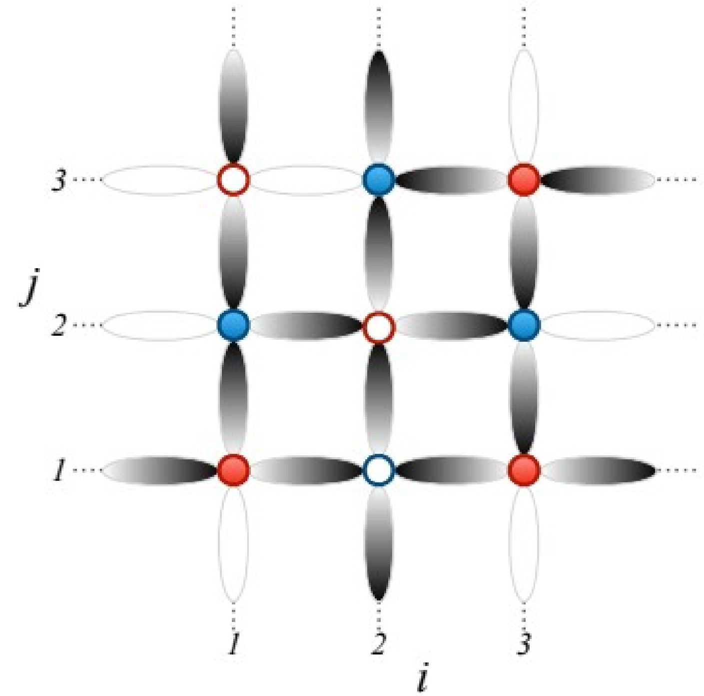
New J. Phys. 16 103015 (2014)
Quantum 1, 9 (2017)
Phys. Rev. D 100, 074512 (2019)
Phys. Rev. X 10, 041040 (2020)
Nature communications 12.1, 3600 (2021)
Philosophical Transactions of the Royal Society A 380.2216 (2022)
ArXiv:2307.09396 (2023)
ArXiv:2308.04488 (2023)

Framework choices

Hamiltonian Lattice Gauge Formulation[1]

- Space discretized
- Time continuous
- Matter quantum fields on sites
- Gauge quantum fields on bonds

[1] Phys. Rev. D 11, 395 (1975)



Suitable for:
Real time dynamics

Requirements:
Sign-problem free methods

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Tensor Network Methods

- Find ground states
- Track real-time evolution

...on equal footing[2]

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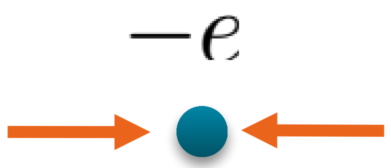
Quantum Simulators

- Optical lattices
- Rydberg atoms
- Trapped ions[3]

[3] Nature Physics 18, 1053 (2022)

Framework choices



Leptons and quarks are fermions

Tensor Networks and Quantum Simulators *could* take care of fermionic statistics[4,5]

...or we simply **eliminate** the fermionic statistics!!!

Tensor Network Methods

- Find ground states
- Track real-time evolution

...on equal footing[2]

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Quantum Simulators

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- Trapped ions[3]

[3] Nature Physics 18, 1053 (2022)

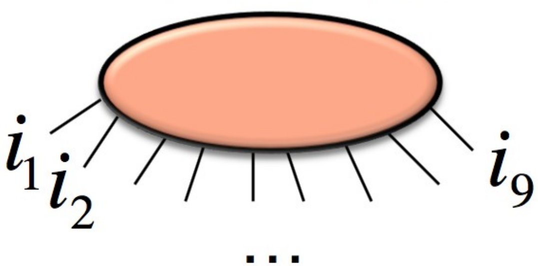
[4] Phys. Rev. B 80, 165129 (2009)
[5] arXiv:2303.08683 (2023)

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d-level systems

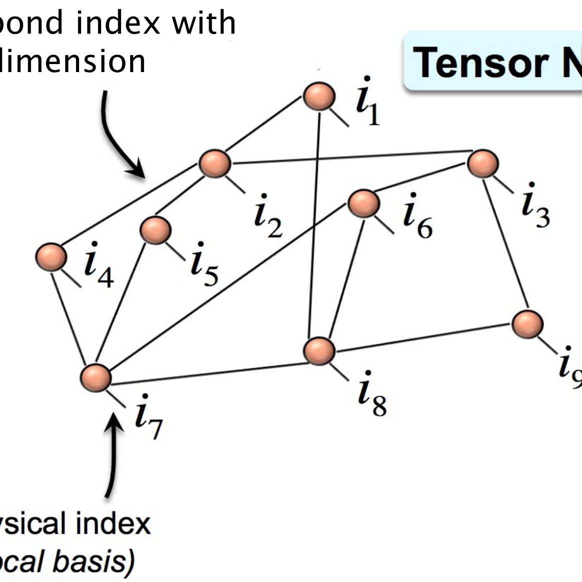
Tensor (multidimensional array of complex numbers)

$C_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$



replacement

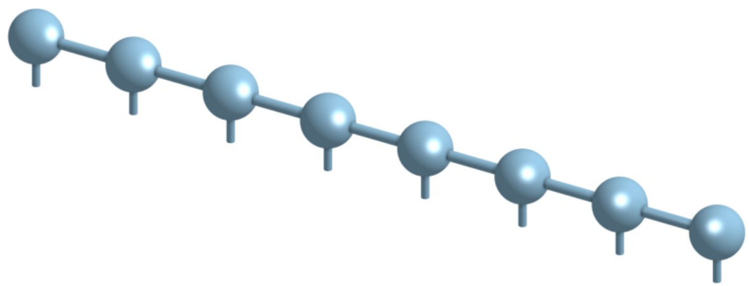
multiple SVD



representation, exponentially large in the system size. Inefficient.

Physical indices with dimension .
Bond indices with dimension .
The number of parameters is

Examples



Matrix Product States (MPS)

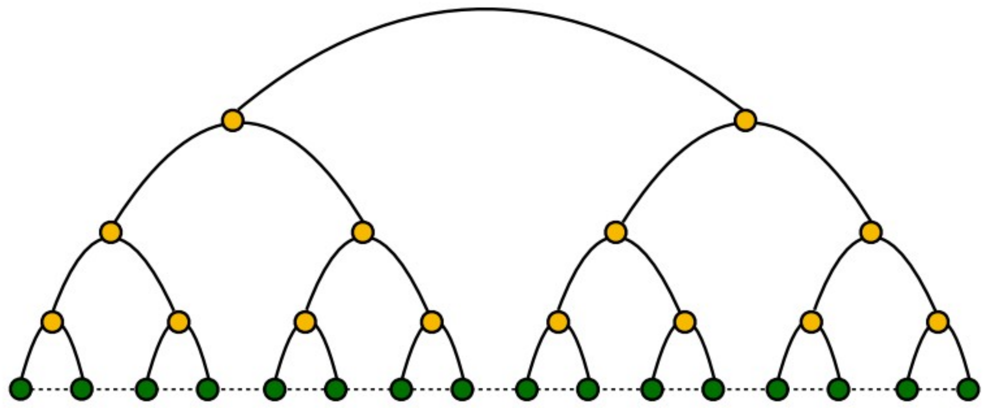
minimize $E = \langle \psi | H | \psi \rangle$

$O(\chi^3)$

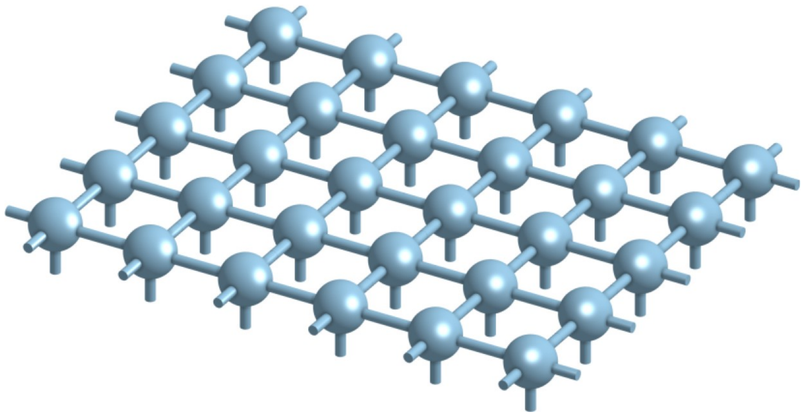
Tree Tensor Networks (TTN)

$O(\chi^4)$

- strong connectivity
- distance between two lattice sites scales logarithmically within the network



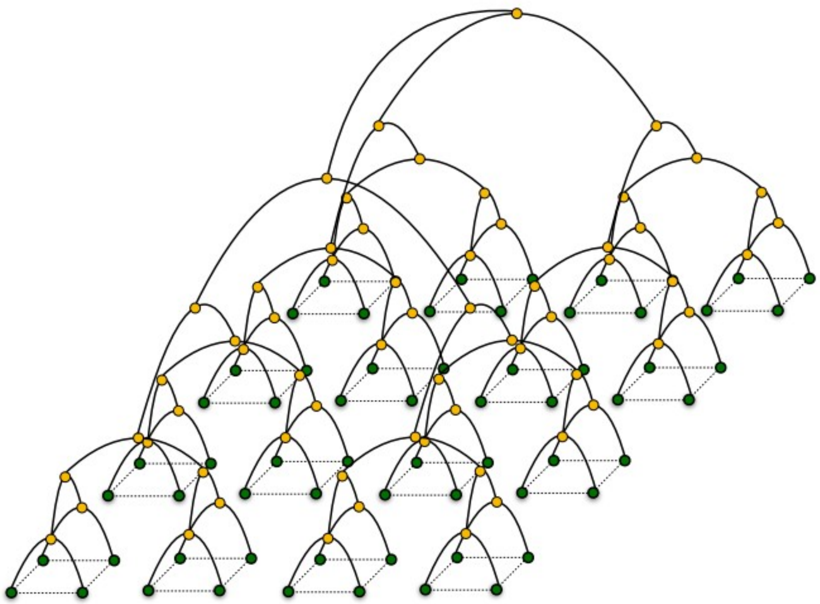
Projected Entangled Pair States (PEPS)



- they automatically reproduce the area-law of entanglement
- the optimization has a complexity

Tree Tensor Networks (TTN)

- they do not automatically reproduce the area-law of entanglement
- the optimization has a complexity



Model 1 (Abelian): Hamiltonian Lattice QED

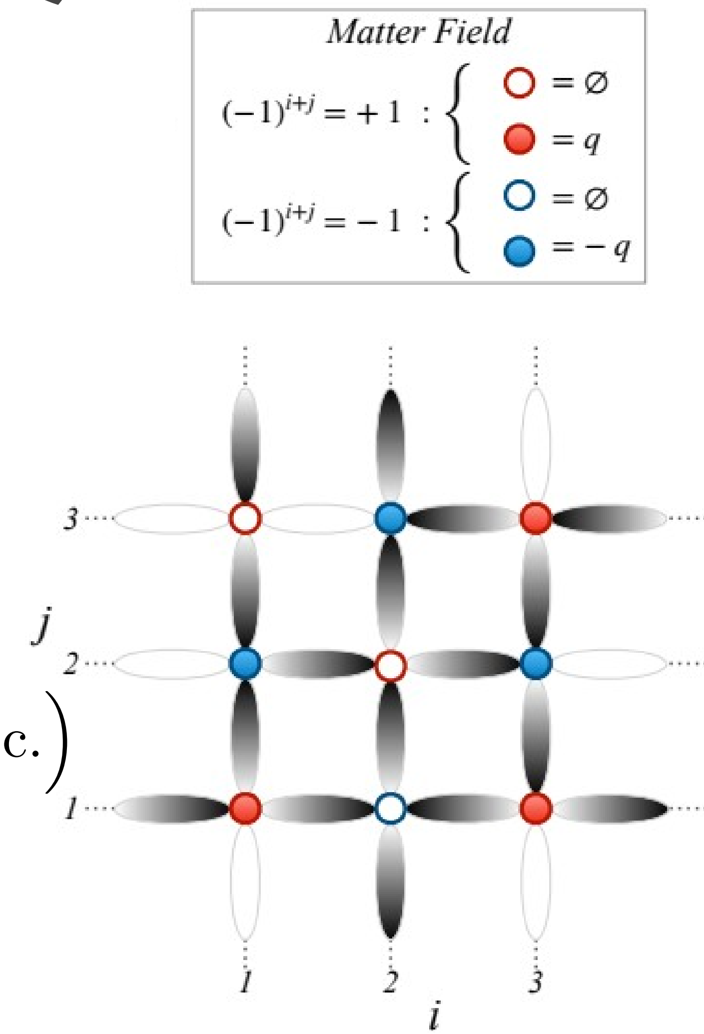
For example in 2+1D:

$$H = \frac{c\hbar}{2a}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{U}_{\mathbf{j},\mu}\hat{\psi}_{\mathbf{j}+\mu} + \mathrm{H.c.}\right)$$

$$+m_e c^2 \sum_{\mathbf{j},\mu}(-1)^{j_x+j_y}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{\psi}_{\mathbf{j}}$$

$$+g^2\frac{c\hbar}{2a}\sum_{\mathbf{j},\mu}\hat{E}^2_{\mathbf{j},\mu}$$

$$-\frac{1}{g^2}\frac{c\hbar}{2a}\sum_{\mathbf{j}}\left(\hat{U}_{\mathbf{j},\mu_x}\hat{U}_{\mathbf{j}+\mu_x,\mu_y}\hat{U}^{\dagger}_{\mathbf{j}+\mu_y,\mu_x}\hat{U}^{\dagger}_{\mathbf{j},\mu_y} + \mathrm{H.c.}\right)$$



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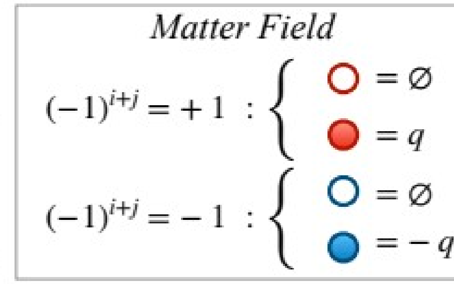

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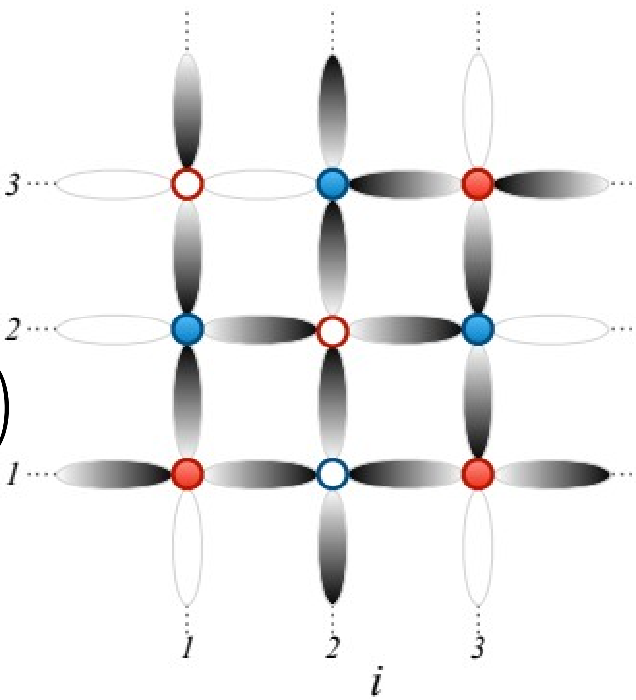

Tricky. Let's delve deeper

Matter field: Staggered (Dirac) fermions

$$H = \frac{c\hbar}{2a}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{\psi}^{\dagger}_{\mathbf{j}}\cancel{\hat{U}_{\mathbf{j},\mu}}\hat{\psi}_{\mathbf{j}+\mu} + \mathrm{H.c.}\right)$$
$$+m_e c^2 \sum_{\mathbf{j},\mu}(-1)^{j_x+j_y}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{\psi}_{\mathbf{j}}$$

Lattice Dirac Hamiltonian (2-spinor field)

But what is a fermion, **really**?

$$\left\{\hat{\psi}_{\mathbf{j}}, \hat{\psi}^{(\dagger)}_{\mathbf{j}'\neq\mathbf{j}}\right\} = 0$$

Mutual anticommutation

Local algebra rules determine the “fermion type”

$$\left\{\hat{\psi}_{\mathbf{j}}, \hat{\psi}^{\dagger}_{\mathbf{j}}\right\} = 1$$

$$\left\{\hat{\psi}_{\mathbf{j}}, \hat{\psi}_{\mathbf{j}}\right\} = 0$$

Dirac Fermion

Matter field: Staggered (Dirac) fermions

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But what is a fermion, **really**?

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Mutual anticommutation

Local algebra rules determine the “fermion type”

$$\hat{\psi}_{\mathbf{j}} = \hat{\psi}_{\mathbf{j}}^{\dagger}$$
$$\left\{\hat{\psi}_{\mathbf{j}},\hat{\psi}_{\mathbf{j}}\right\} = 2$$

Majorana Fermion

Matter field: Staggered (Dirac) fermions

$$H = \frac{c\hbar}{2a} \sum_{\mathbf{j},\mu} \left(e^{i\phi_{\mathbf{j}\mu}} \hat{\psi}^\dagger_{\mathbf{j}} \hat{U}_{\mathbf{j},\mu} \hat{\psi}_{\mathbf{j}+\mu} + \text{H.c.} \right) + m_e c^2 \sum_{\mathbf{j},\mu} (-1)^{j_x + j_y} \hat{\psi}^\dagger_{\mathbf{j}} \hat{\psi}_{\mathbf{j}}$$

Lattice Dirac Hamiltonian (2-spinor field)

But what is a fermion, **really**?

$$\left\{ \hat{\psi}_{\mathbf{j}}, \hat{\psi}^{(\dagger)}_{\mathbf{j}' \neq \mathbf{j}} \right\} = 0$$

Mutual anticommutation

Local algebra rules determine the "fermion type"

$$\left\{ \hat{\psi}_{\mathbf{j}}, \hat{\psi}^{(\dagger)}_{\mathbf{j}} \right\} = ??$$

"Whatever" Fermion

Build Fermions from local operators (matrices)

$$\hat{b} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A ‘standard’ quantum operator.
You can call it:

- Spin-like
- Boson-like
- Genuinely local

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You can call it:

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- Genuinely local

Meaning that *globally*, it acts this way

$$\hat{b}_j = \cdots \otimes 1_{j-2} \otimes 1_{j-1} \otimes \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_j \otimes 1_{j+1} \otimes 1_{j+2} \otimes \cdots$$

Build Fermions from local operators (matrices)

$$\hat{\xi} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_F$$

A ‘fermionic’ quantum operator
Defined via matrix (Fermatrix)

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$$\hat{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

I also need to define the local fermion *parity* $\hat{P} = \hat{P}^{-1} = \hat{P}^\dagger$

$\hat{\xi}$ Must invert the parity $\{\hat{\xi}, \hat{P}\} = 0$

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I say that, *globally*, it acts this way

$$\hat{\xi}_j = \cdots \otimes P_{j-2} \otimes P_{j-1} \otimes \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_j \otimes 1_{j+1} \otimes 1_{j+2} \otimes \cdots$$

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With this definition two fermatrices will mutually anticommute (also mismatched types)

$$\{\hat{\xi}^{A}_{\mathbf{j}}, \hat{\xi}^{B}_{\mathbf{j}' \neq \mathbf{j}}\} = 0$$

ALWAYS (ordering is irrelevant)

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ALWAYS (ordering is irrelevant)

Notable examples:

Dirac Fermion

$$\hat{\psi} = \left(\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array}\right)_F$$

$$\hat{P} = \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right)$$

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ALWAYS (ordering is irrelevant)

Notable examples:

Dirac Fermion $\hat{\psi} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_F$

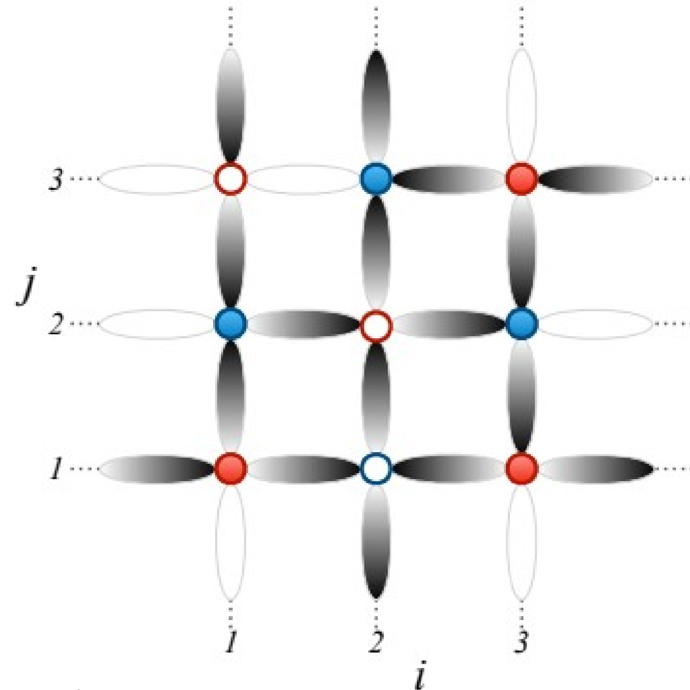
Majo. Fermion $\hat{\psi}_M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_F$

In both cases

$$\hat{P} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Practical formalism to define exotic fermions

Where were we? Oh right...



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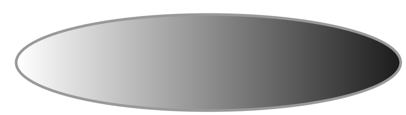
Gauge Field Operators

$$\hat{U}_{\mathbf{j},\mu}, \hat{E}_{\mathbf{j},\mu}$$

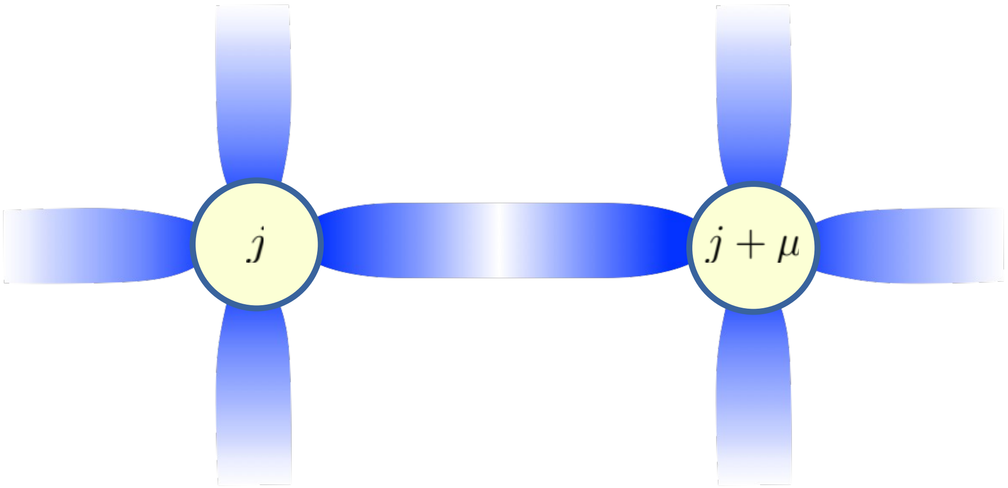
} act on gauge fields, sitting on the lattice bonds



$$\hat{E}^{\dagger}_{\mathbf{j},\mu} = \hat{E}_{\mathbf{j},\mu} \qquad \hat{U}^{\dagger}_{\mathbf{j},\mu} = \hat{U}^{-1}_{\mathbf{j},\mu}$$

$$[\hat{U}_{\mathbf{j},\mu}, \hat{E}_{\mathbf{j}',\mu'}] = \hat{U}_{\mathbf{j},\mu}\delta_{\mathbf{j},\mathbf{j}'}\delta_{\mu,\mu'}$$

Their algebra tells us...



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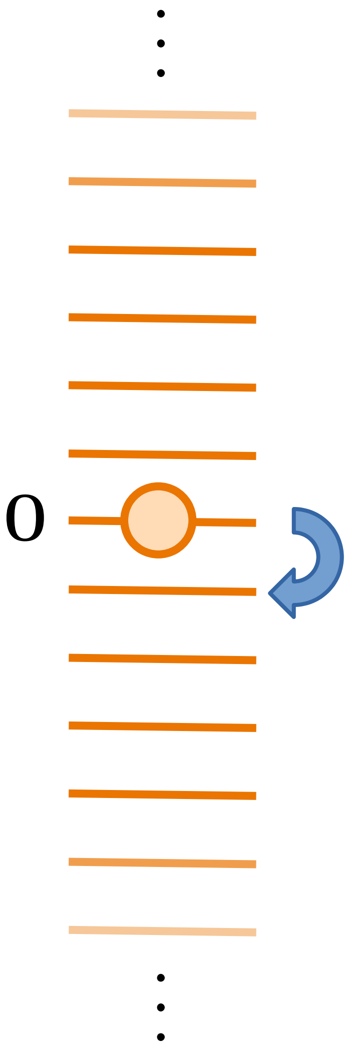
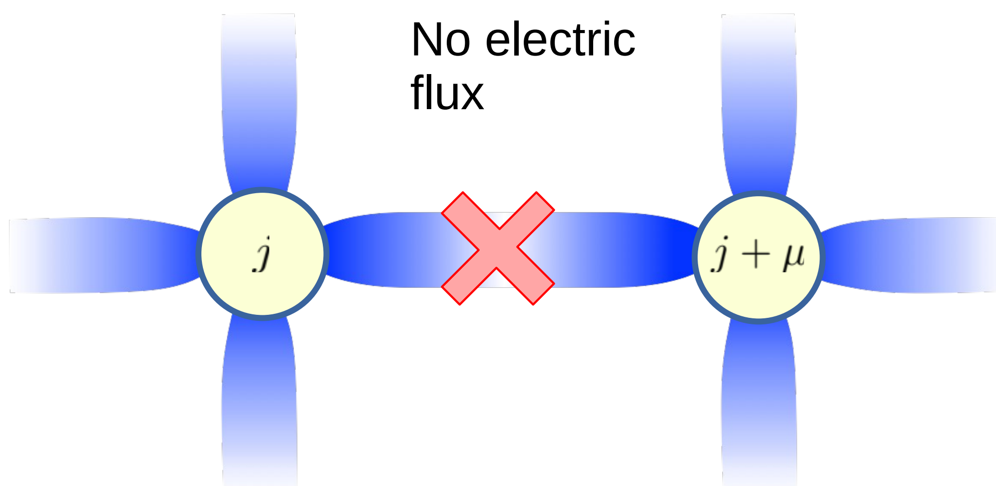


Infinite ladder of quantum levels:
Eigenstates of $\hat{E}_{\mathbf{j},\mu}$
With a defined electric flux

$\hat{U}_{\mathbf{j},\mu}$ acts as a lowering operator



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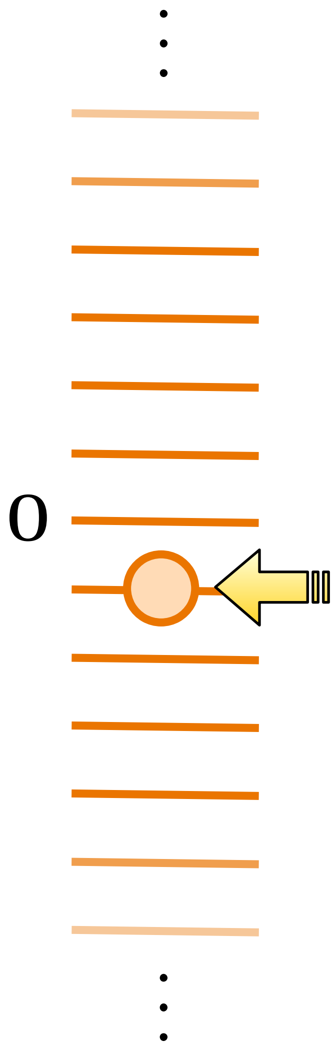
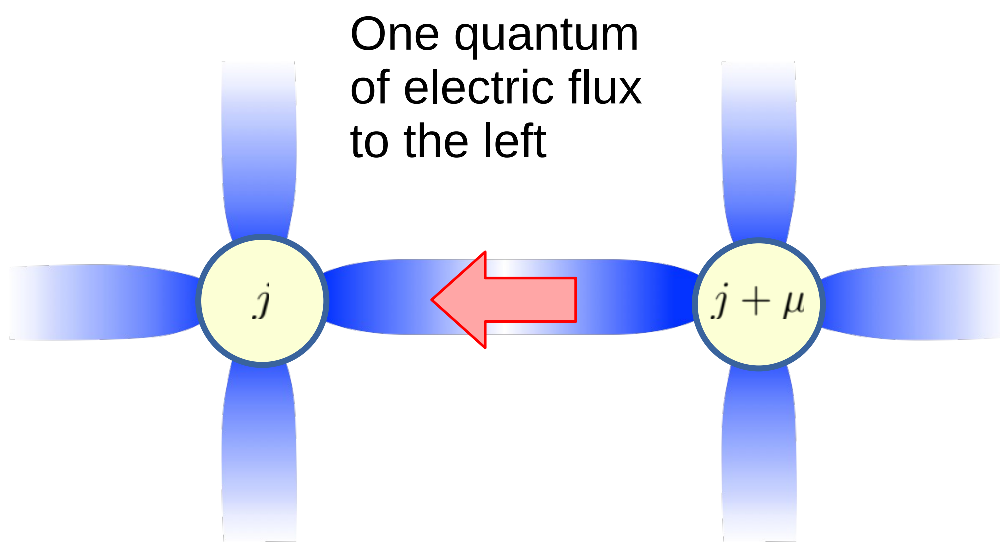
$$[\hat{U}_{\mathbf{j},\mu}, \hat{E}_{\mathbf{j}',\mu'}] = \hat{U}_{\mathbf{j},\mu}\delta_{\mathbf{j},\mathbf{j}'}\delta_{\mu,\mu'}$$

Their algebra tells us...



Infinite ladder of quantum levels:
Eigenstates of $\hat{E}_{\mathbf{j},\mu}$
With a defined electric flux

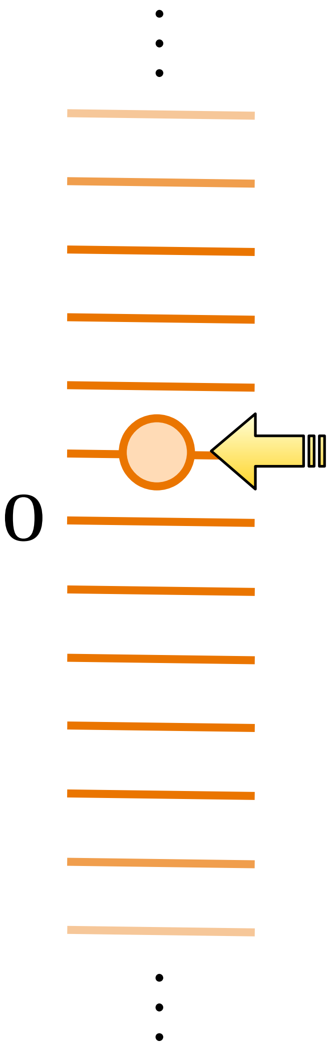
$\hat{U}_{\mathbf{j},\mu}$ acts as a lowering operator



$$\hat{E}^{\dagger}_{\mathbf{j},\mu} = \hat{E}_{\mathbf{j},\mu} \qquad \hat{U}^{\dagger}_{\mathbf{j},\mu} = \hat{U}^{-1}_{\mathbf{j},\mu}$$

$$[\hat{U}_{\mathbf{j},\mu}, \hat{E}_{\mathbf{j}',\mu'}] = \hat{U}_{\mathbf{j},\mu}\delta_{\mathbf{j},\mathbf{j}'}\delta_{\mu,\mu'}$$

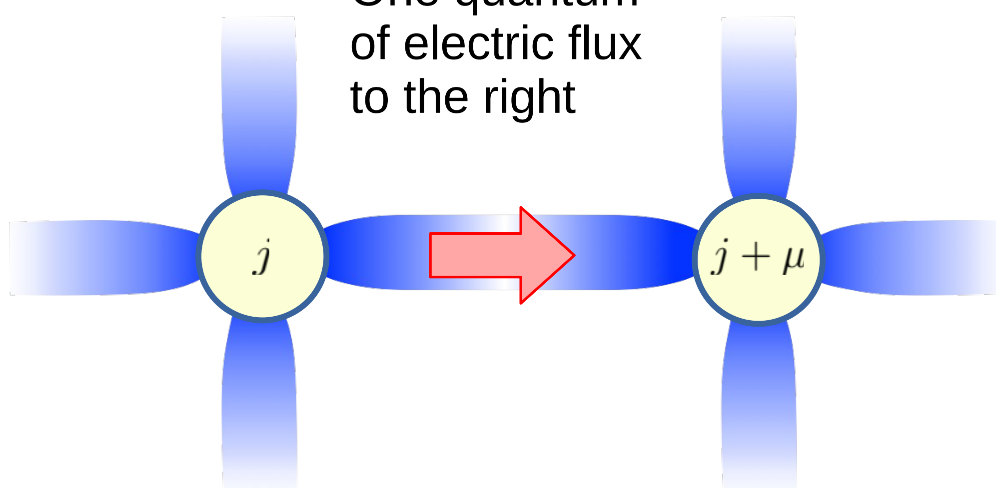
Their algebra tells us...



Infinite ladder of quantum levels:
Eigenstates of $\hat{E}_{\mathbf{j},\mu}$
With a defined electric flux

$\hat{U}_{\mathbf{j},\mu}$ acts as a lowering operator

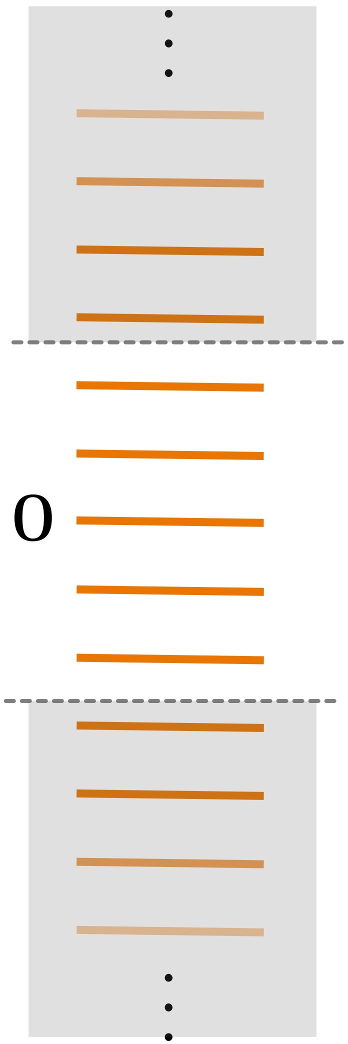
One quantum
of electric flux
to the right



$$\hat{E}^{\dagger}_{\mathbf{j},\mu} = \hat{E}_{\mathbf{j},\mu} \qquad \hat{U}^{\dagger}_{\mathbf{j},\mu} = \hat{U}^{-1}_{\mathbf{j},\mu}$$

$$[\hat{U}_{\mathbf{j},\mu}, \hat{E}_{\mathbf{j}',\mu'}] = \hat{U}_{\mathbf{j},\mu}\delta_{\mathbf{j},\mathbf{j}'}\delta_{\mu,\mu'}$$

Quantum Link Model



Energy cutoff in $\hat{E}^2_{\mathbf{j},\mu}$

$$\hat{E}^{\dagger}_{\mathbf{j},\mu} = \hat{E}_{\mathbf{j},\mu} \qquad \hat{U}^{\dagger}_{\mathbf{j},\mu} = \hat{U}^{-1}_{\mathbf{j},\mu}$$

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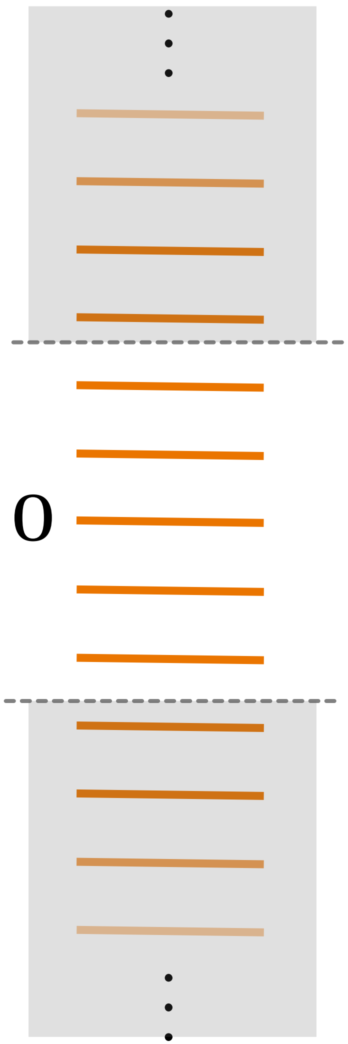
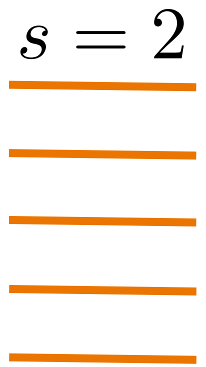
Quantum Link Model



Energy cutoff in $\hat{E}^2_{\mathbf{j},\mu}$

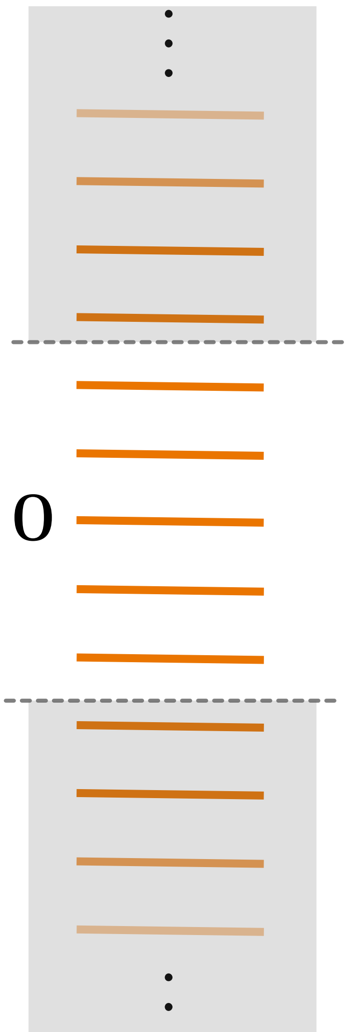


Finite spin-shell

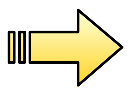
$$\hat{E}^{\dagger}_{\mathbf{j},\mu} = \hat{E}_{\mathbf{j},\mu} \qquad \hat{U}^{\dagger}_{\mathbf{j},\mu} = \hat{U}^{-1}_{\mathbf{j},\mu}$$

$$[\hat{U}_{\mathbf{j},\mu}, \hat{E}_{\mathbf{j}',\mu'}] = \hat{U}_{\mathbf{j},\mu}\delta_{\mathbf{j},\mathbf{j}'}\delta_{\mu,\mu'}$$

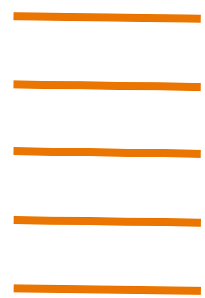
Quantum Link Model



Energy cutoff in $\hat{E}^2_{\mathbf{j},\mu}$



$s = 2$



Finite spin-shell

Replace e.g.

$$\hat{E} \to \hat{S}^z$$

$$\hat{U} \to \frac{1}{s}\hat{S}^-$$

Unitarity is sacrificed, the rest is fine

Other strategies are known: e.g.
[1] Phys. Rev. D 106, 114511 (2002)
[2] arXiv:2304.02527 (2023)

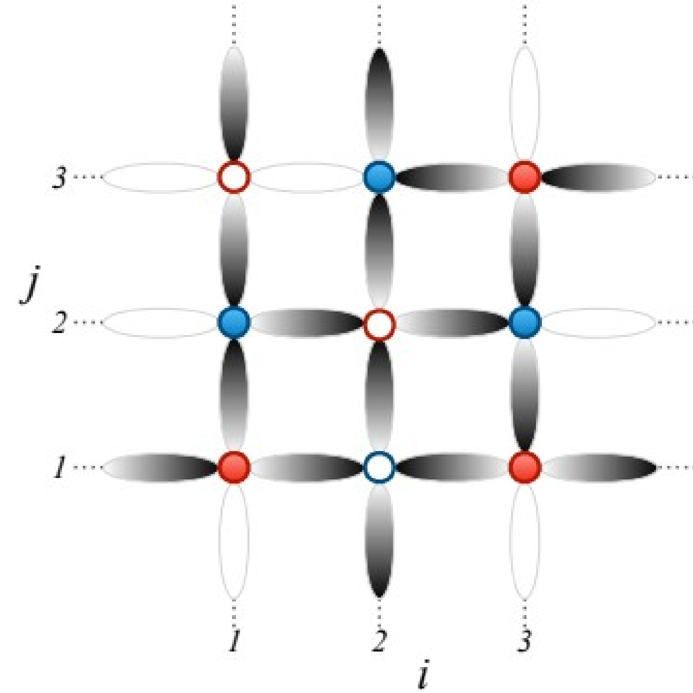
Back again

$$H_{\text{QED}} = \frac{c\hbar}{2as}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{S}^{-}_{\mathbf{j},\mu}\hat{\psi}_{\mathbf{j}+\mu} + \text{H.c.}\right)$$

$$+m_e c^2\sum_{\mathbf{j},\mu}(-1)^{j_x+j_y}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{\psi}_{\mathbf{j}}$$

$$+g^2\frac{c\hbar}{2a}\sum_{\mathbf{j},\mu}\left(\hat{S}^{z}_{\mathbf{j},\mu}\right)^2$$

$$-\frac{1}{g^2}\frac{c\hbar}{2as^4}\sum_{\mathbf{j}}\left(\hat{S}^{-}_{\mathbf{j},\mu_x}\hat{S}^{-}_{\mathbf{j}+\mu_x,\mu_y}\hat{S}^{+}_{\mathbf{j}+\mu_y,\mu_x}\hat{S}^{+}_{\mathbf{j},\mu_y} + \text{H.c.}\right)$$



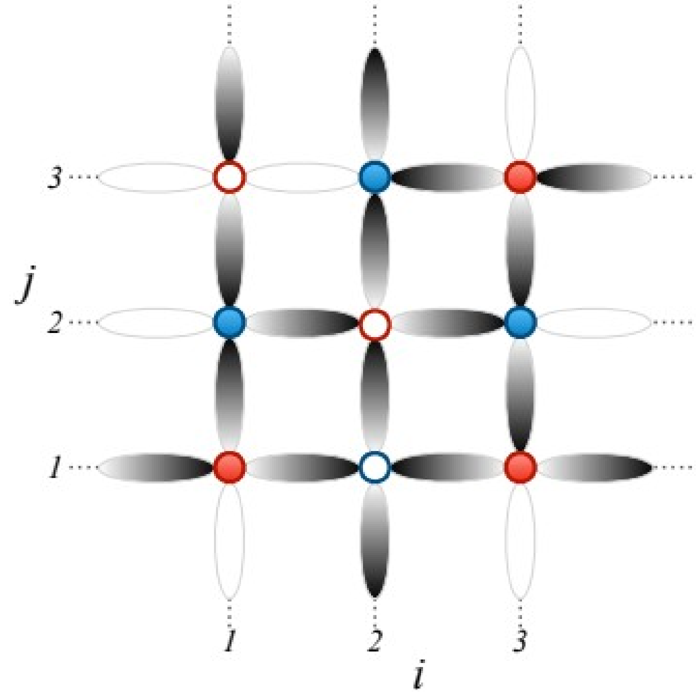
Back again

$$H_{\rm QED} = \frac{c\hbar}{2as} \sum_{\mathbf{j},\mu} \left(e^{i\phi_{\mathbf{j}\mu}} \hat{\psi}^\dagger_{\mathbf{j}} \hat{S}^-_{\mathbf{j},\mu} \hat{\psi}_{\mathbf{j}+\mu} + \text{H.c.} \right)$$

$$+ m_e c^2 \sum_{\mathbf{j},\mu} (-1)^{j_x + j_y} \hat{\psi}^\dagger_{\mathbf{j}} \hat{\psi}_{\mathbf{j}}$$

$$+ g^2 \frac{c\hbar}{2a} \sum_{\mathbf{j},\mu} \left(\hat{S}^z_{\mathbf{j},\mu} \right)^2$$

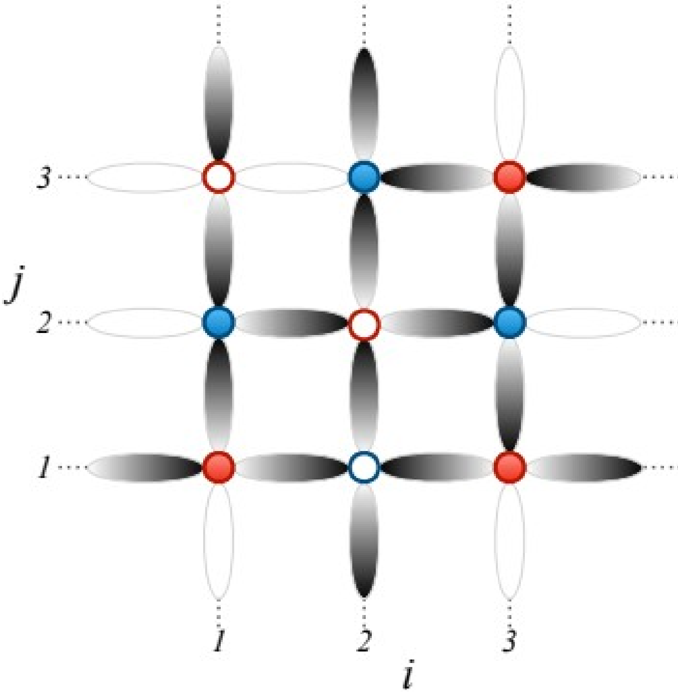
$$- \frac{1}{g^2} \frac{c\hbar}{2as^4} \sum_{\mathbf{j}} \left(\hat{S}^-_{\mathbf{j},\mu_x} \hat{S}^-_{\mathbf{j}+\mu_x,\mu_y} \hat{S}^+_{\mathbf{j}+\mu_y,\mu_x} \hat{S}^+_{\mathbf{j},\mu_y} + \text{H.c.} \right)$$



Gauss' Law (gauge symmetry) $\forall \mathbf{j}$

$$\hat{S}^z_{\mathbf{j},\mu_x} + \hat{S}^z_{\mathbf{j},\mu_y} - \hat{S}^z_{\mathbf{j}-\mu_x,\mu_x} - \hat{S}^z_{\mathbf{j}-\mu_y,\mu_y} = \hat{q}_{\mathbf{j}} = \frac{1}{2} + (-1)^{\mathbf{j}} \left(\frac{1}{2} - \hat{\psi}^\dagger_{\mathbf{j}} \hat{\psi}_{\mathbf{j}} \right) + q^{(0)}_{\mathbf{j}}$$

Back again



$$H_{\mathrm{QED}} = \frac{c\hbar}{2as} \sum_{\mathbf{j},\mu} \left(e^{i\phi_{\mathbf{j}\mu}} \hat{\psi}^{\dagger}_{\mathbf{j}} \hat{S}^{-}_{\mathbf{j},\mu} \hat{\psi}_{\mathbf{j}+\mu} + \mathrm{H.c.} \right)$$

$$+ m_e c^2 \sum_{\mathbf{j},\mu} (-1)^{j_x + j_y} \hat{\psi}^{\dagger}_{\mathbf{j}} \hat{\psi}_{\mathbf{j}}$$

$$+ g^2 \frac{c\hbar}{2a} \sum_{\mathbf{j},\mu} \left(\hat{S}^{z}_{\mathbf{j},\mu} \right)^2$$

$$- \frac{1}{g^2} \frac{c\hbar}{2as^4} \sum_{\mathbf{j}} \left(\hat{S}^{-}_{\mathbf{j},\mu_x} \hat{S}^{-}_{\mathbf{j}+\mu_x,\mu_y} \hat{S}^{+}_{\mathbf{j}+\mu_y,\mu_x} \hat{S}^{+}_{\mathbf{j},\mu_y} + \mathrm{H.c.} \right)$$

Gauss' Law (gauge symmetry) $\forall \mathbf{j}$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

Back again

$$H_{\mathrm{QED}} = \frac{c\hbar}{2as}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{S}^{-}_{\mathbf{j},\mu}\hat{\psi}_{\mathbf{j}+\mu} + \mathrm{H.c.}\right)$$
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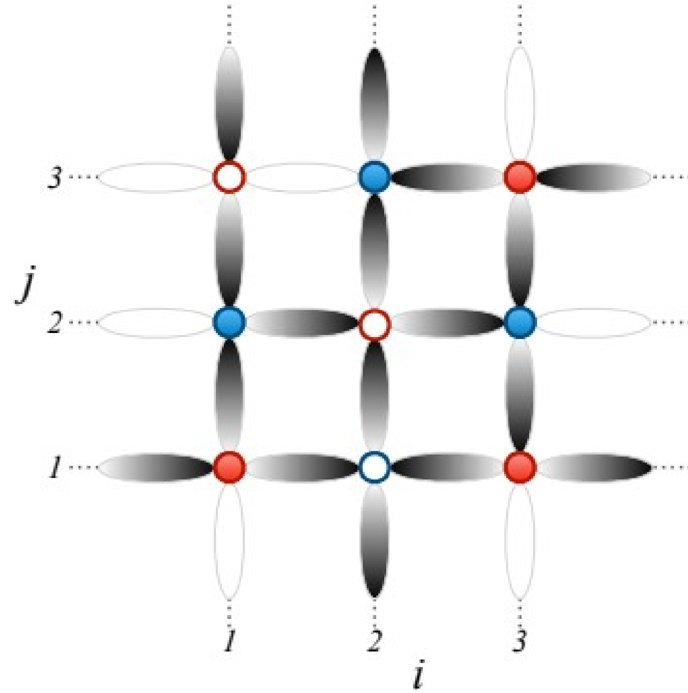
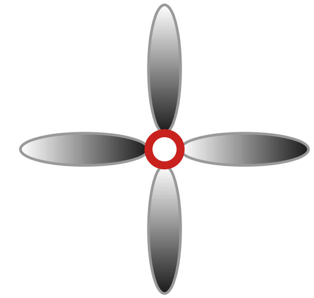
$$-\frac{1}{g^2}\frac{c\hbar}{2as^4}\sum_{\mathbf{j}}\left(\hat{S}^{-}_{\mathbf{j},\mu_x}\hat{S}^{-}_{\mathbf{j}+\mu_x,\mu_y}\hat{S}^{+}_{\mathbf{j}+\mu_y,\mu_x}\hat{S}^{+}_{\mathbf{j},\mu_y} + \mathrm{H.c.}\right)$$



Gauss' Law (gauge symmetry) $\forall \mathbf{j}$

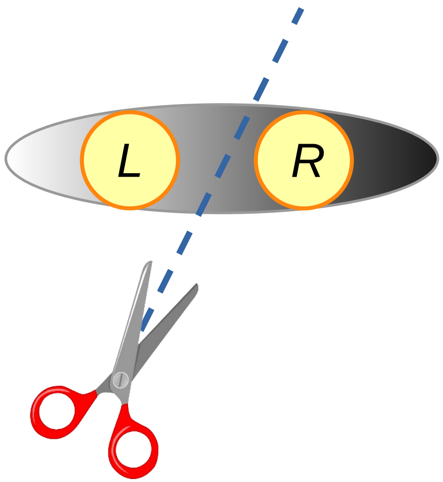
on vertices

$$\vec{\nabla}\cdot\vec{E} = \rho$$



Fermionic Rishons

I split the gauge field into two "copies"



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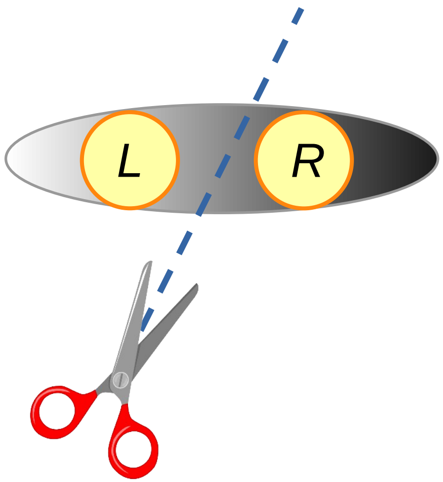

$$|m\rangle \to |m\rangle_L \otimes |-m\rangle_R$$

Extra selection rule needed

$$\hat{S}^z_L + \hat{S}^z_R = 0$$

Fermionic Rishons

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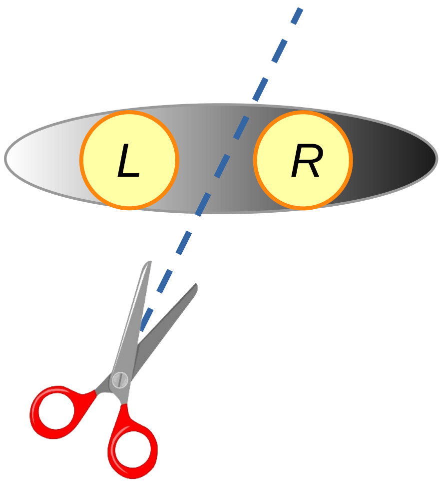


$$|m\rangle \rightarrow |m\rangle_L \otimes |-m\rangle_R$$

Extra selection rule needed

$$\hat{S}^z_L + \hat{S}^z_R = 0$$

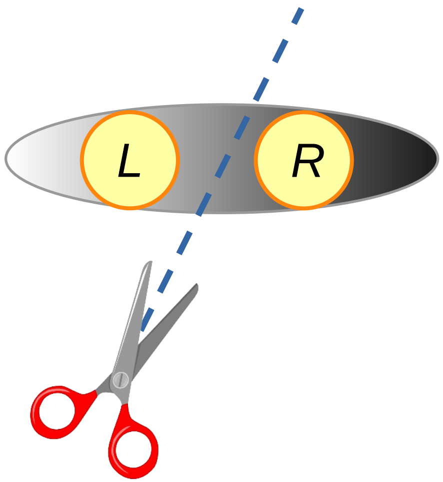
I can decompose:

$$\hat{S}^-_{\mathbf{j},\mu} = \hat{\xi}_{\mathbf{j},\mu,L}\hat{\xi}^\dagger_{\mathbf{j},\mu,R}$$

Exotic fermion operators

Fermionic Rishons

I split the gauge field into two "copies"



$$|m\rangle \rightarrow |m\rangle_L \otimes |-m\rangle_R$$

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Exotic rishon operator as a fermatrix

I can decompose:

$$\hat{S}^-_{\mathbf{j},\mu} = \hat{\xi}_{\mathbf{j},\mu,L}\hat{\xi}^\dagger_{\mathbf{j},\mu,R}$$

Exotic fermion operators

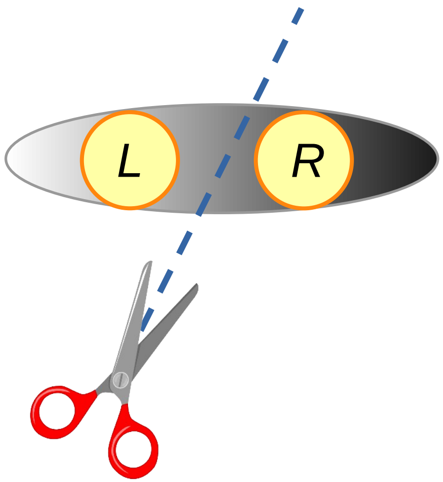
$$\hat{\xi} = \sqrt[4]{2}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}_F$$

s = 1

$$\hat{P} = (-1)^{\hat{S}^z - s}$$

Fermionic Rishons

I split the gauge field into two "copies"



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Exotic fermion operators

Exotic rishon operator as a fermatrix

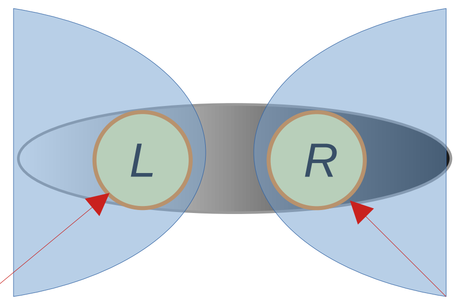
$$\hat{\xi} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt[4]{3} & 0 & 0 & 0 \\ 0 & \sqrt[4]{4} & 0 & 0 \\ 0 & 0 & \sqrt[4]{3} & 0 \end{pmatrix}_F$$

s = 3/2

$$\hat{P} = (-1)^{\hat{S}^z - s}$$

Fermionic Rishons

I split the gauge field into two "copies"



belongs here

belongs here

$$|m\rangle \rightarrow |m\rangle_L \otimes |-m\rangle_R$$

Extra selection rule needed

$$\hat{S}^z_L + \hat{S}^z_R = 0$$

I can decompose:

$$\hat{S}^-_{\mathbf{j},\mu} = \hat{\xi}_{\mathbf{j},\mu,L}\hat{\xi}^\dagger_{\mathbf{j},\mu,R}$$

Exotic fermion operators

Exotic rishon operator as a fermatrix

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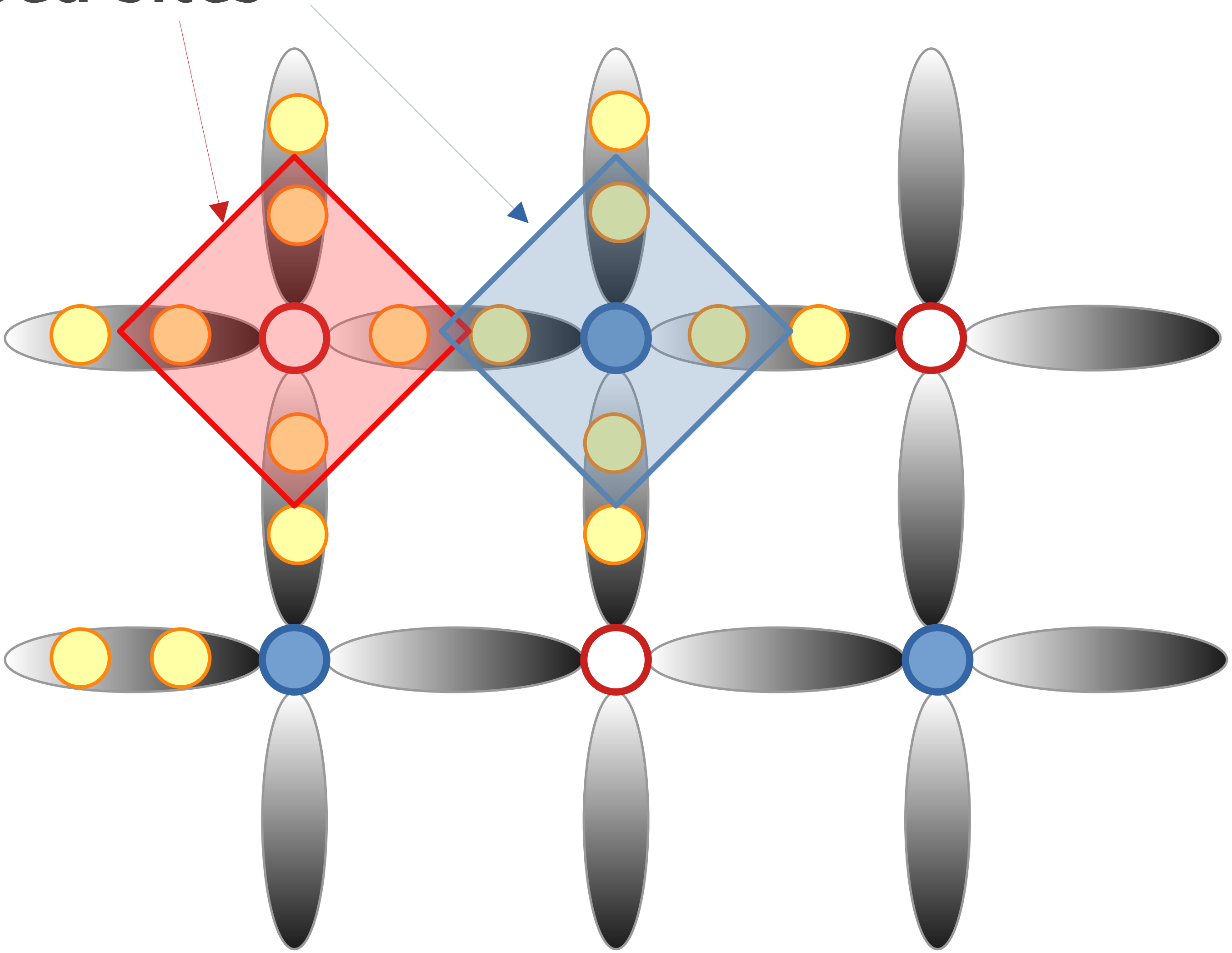
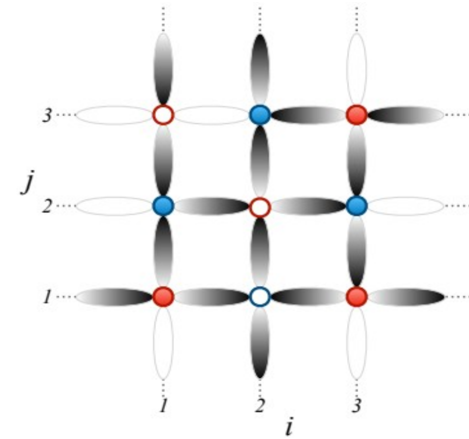
s = 3/2

$$\hat{P} = (-1)^{\hat{S}^z - s}$$

Dressed sites



Why all the fuss?



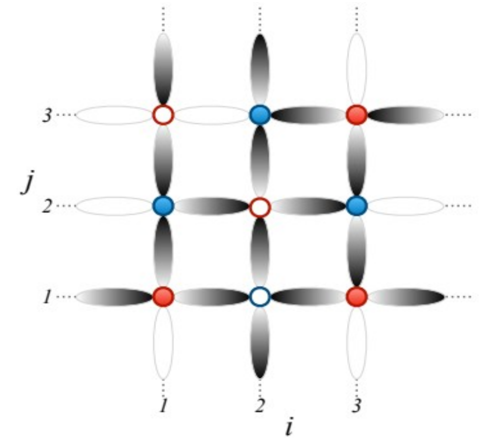
$$H_{\text{QED}} = \frac{c\hbar}{2as} \sum_{\mathbf{j},\mu} \left(e^{i\phi_{\mathbf{j}\mu}} \hat{\psi}^{\dagger}_{\mathbf{j}} \hat{\xi}_{\mathbf{j},+\mu} \hat{\xi}^{\dagger}_{\mathbf{j}+\mu,-\mu} \hat{\psi}_{\mathbf{j}+\mu} + \text{H.c.} \right)$$

$$+ m_e c^2 \sum_{\mathbf{j},\mu} (-1)^{j_x+j_y} \hat{\psi}^{\dagger}_{\mathbf{j}} \hat{\psi}_{\mathbf{j}}$$

$$+ g^2 \frac{c\hbar}{4a} \sum_{\mathbf{j},\mu} \left(\hat{S}^z_{\mathbf{j},+\mu} \right)^2 + \left(\hat{S}^z_{\mathbf{j},-\mu} \right)^2$$

$$- \frac{1}{g^2} \frac{c\hbar}{2as^4} \sum_{\mathbf{j}} \left(\hat{\xi}_{\mathbf{j},+\mu_x} \hat{\xi}^{\dagger}_{\mathbf{j},+\mu_y} \cdots \hat{\xi}_{\mathbf{j}+\mu_y,-\mu_y} \hat{\xi}^{\dagger}_{\mathbf{j}+\mu_y,+\mu_y} + \text{H.c.} \right)$$

Why all the fuss?



$$H_{\mathrm{QED}} = \frac{c\hbar}{2as}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{\psi}_{\mathbf{j}}^{\dagger}\hat{\xi}_{\mathbf{j},+\mu}\hat{\xi}_{\mathbf{j}+\mu,-\mu}^{\dagger}\hat{\psi}_{\mathbf{j}+\mu} + \mathrm{H.c.}\right)$$

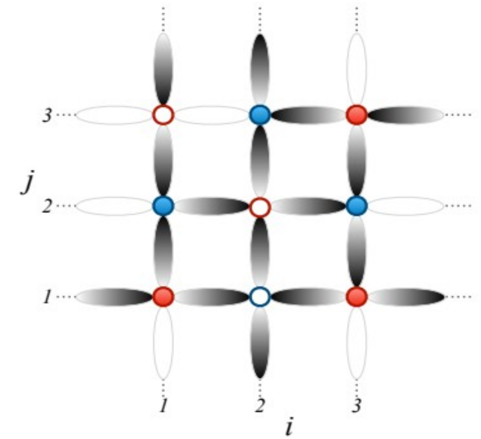
$$+m_e c^2 \sum_{\mathbf{j},\mu}(-1)^{j_x+j_y}\hat{\psi}_{\mathbf{j}}^{\dagger}\hat{\psi}_{\mathbf{j}}$$

$$+g^2\frac{c\hbar}{4a}\sum_{\mathbf{j},\mu}\left(\hat{S}^z_{\mathbf{j},+\mu}\right)^2 + \left(\hat{S}^z_{\mathbf{j},-\mu}\right)^2$$

$$-\frac{1}{g^2}\frac{c\hbar}{2as^4}\sum_{\mathbf{j}}\left(\hat{\xi}_{\mathbf{j},+\mu_x}\hat{\xi}^{\dagger}_{\mathbf{j},+\mu_y}\cdots\hat{\xi}_{\mathbf{j}+\mu_y,-\mu_y}\hat{\xi}^{\dagger}_{\mathbf{j}+\mu_y,+\mu_y} + \mathrm{H.c.}\right)$$

Fermion parity PROTECTED at every dressed site

Why all the fuss?



$$H_{\text{QED}} = \frac{c\hbar}{2as}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{\xi}_{\mathbf{j},+\mu}\hat{\xi}^{\dagger}_{\mathbf{j}+\mu,-\mu}\hat{\psi}_{\mathbf{j}+\mu} + \text{H.c.}\right)$$

$$+m_e c^2\sum_{\mathbf{j},\mu}(-1)^{j_x+j_y}\hat{\psi}^{\dagger}_{\mathbf{j}}\hat{\psi}_{\mathbf{j}}$$

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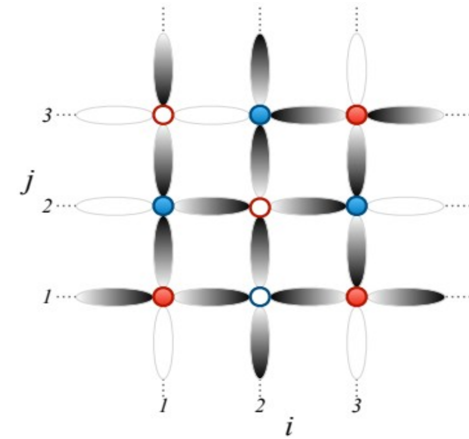
$$-\frac{1}{g^2}\frac{c\hbar}{2as^4}\sum_{\mathbf{j}}\left(\hat{\xi}_{\mathbf{j},+\mu_x}\hat{\xi}^{\dagger}_{\mathbf{j},+\mu_y}\cdots\hat{\xi}_{\mathbf{j}+\mu_y,-\mu_y}\hat{\xi}^{\dagger}_{\mathbf{j}+\mu_y,+\mu_y} + \text{H.c.}\right)$$

Fermion parity PROTECTED at every dressed site

→ we *Eliminated* fermionic matter

Why all the fuss?



$$H_{\text{QED}} = \frac{c\hbar}{2as}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{\psi}^\dagger_{\mathbf{j}}\hat{\xi}_{\mathbf{j},+\mu}\hat{\xi}^\dagger_{\mathbf{j}+\mu,-\mu}\hat{\psi}_{\mathbf{j}+\mu} + \text{H.c.}\right)$$

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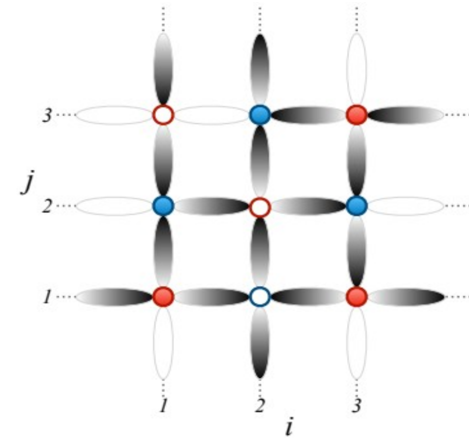
$$+g^2\frac{c\hbar}{4a}\sum_{\mathbf{j},\mu}\left(\hat{S}^z_{\mathbf{j},+\mu}\right)^2 + \left(\hat{S}^z_{\mathbf{j},-\mu}\right)^2$$

$$-\frac{1}{g^2}\frac{c\hbar}{2as^4}\sum_{\mathbf{j}}\left(\hat{\xi}_{\mathbf{j},+\mu_x}\hat{\xi}^\dagger_{\mathbf{j},+\mu_y}\cdots\hat{\xi}_{\mathbf{j}+\mu_y,-\mu_y}\hat{\xi}^\dagger_{\mathbf{j}+\mu_y,+\mu_y} + \text{H.c.}\right)$$

For better view, redefine

$$\hat{Q}_{\mathbf{j},\pm\mu} = \hat{\xi}^\dagger_{\mathbf{j},\pm\mu}\hat{\psi}_{\mathbf{j}} \qquad \hat{C}_{\mathbf{j},\pm\mu_1,\pm\mu_2} = \hat{\xi}_{\mathbf{j},\pm\mu_1}\hat{\xi}^\dagger_{\mathbf{j},\pm\mu_2}$$

Why all the fuss?



$$H_{\mathrm{QED}} = \frac{c\hbar}{2as}\sum_{\mathbf{j},\mu}\left(e^{i\phi_{\mathbf{j}\mu}}\hat{Q}^{\dagger}_{\mathbf{j},+\mu}\hat{Q}^{\dagger}_{\mathbf{j}+\mu,-\mu} + \mathrm{H.c.}\right)$$

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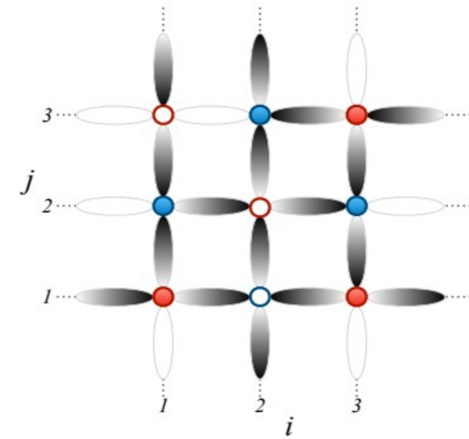
$$+g^2\frac{c\hbar}{4a}\sum_{\mathbf{j},\mu}\left(\hat{S}^z_{\mathbf{j},+\mu}\right)^2 + \left(\hat{S}^z_{\mathbf{j},-\mu}\right)^2$$

$$-\frac{1}{g^2}\frac{c\hbar}{2as^4}\sum_{\mathbf{j}}\left(\hat{C}_{\mathbf{j},+\mu_x,+\mu_y}\cdots\hat{C}_{\mathbf{j}+\mu_y,-\mu_y,+\mu_x} + \mathrm{H.c.}\right)$$

For better view, redefine

$$\hat{Q}_{\mathbf{j},\pm\mu} = \hat{\xi}^{\dagger}_{\mathbf{j},\pm\mu}\hat{\psi}_{\mathbf{j}} \qquad \hat{C}_{\mathbf{j},\pm\mu_1,\pm\mu_2} = \hat{\xi}_{\mathbf{j},\pm\mu_1}\hat{\xi}^{\dagger}_{\mathbf{j},\pm\mu_2}$$

Why all the fuss?



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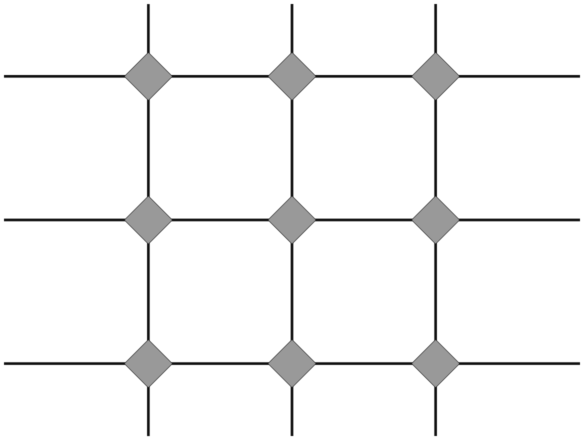
$$-\frac{1}{g^2}\frac{c\hbar}{2as^4}\sum_{\mathbf{j}}\left(\hat{C}_{\mathbf{j},+\mu_x,+\mu_y}\cdots\hat{C}_{\mathbf{j}+\mu_y,-\mu_y,+\mu_x} + \mathrm{H.c.}\right)$$

Now everything *mutually* commutes

$$\left[\hat{Q}_{\mathbf{j},\pm\mu_1}, \hat{Q}^{(\dagger)}_{\mathbf{j}'\neq\mathbf{j},\pm\mu_2}\right] = \left[\hat{C}_{\mathbf{j},\ldots}, \hat{C}^{(\dagger)}_{\mathbf{j}'\neq\mathbf{j},\ldots}\right] = \left[\hat{Q}_{\mathbf{j},\ldots}, \hat{C}^{(\dagger)}_{\mathbf{j}'\neq\mathbf{j},\ldots}\right] = 0$$

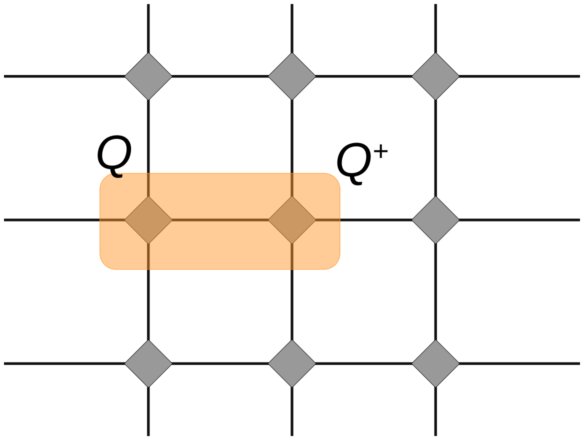
A simple square lattice (of dressed sites)

- Like a spin lattice (with large spins)



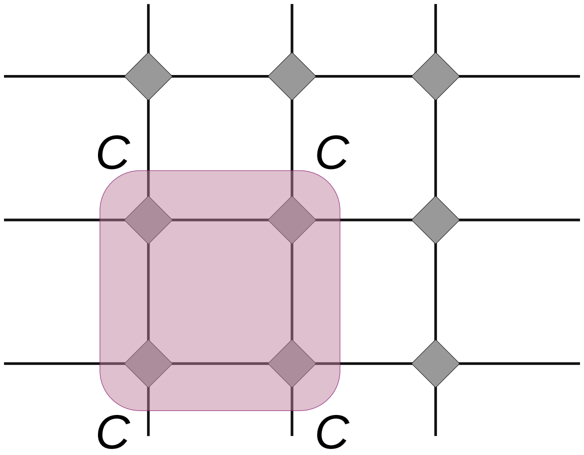
A simple square lattice (of dressed sites)

- Like a spin lattice (with large spins)
- Nearest Neighbor interaction



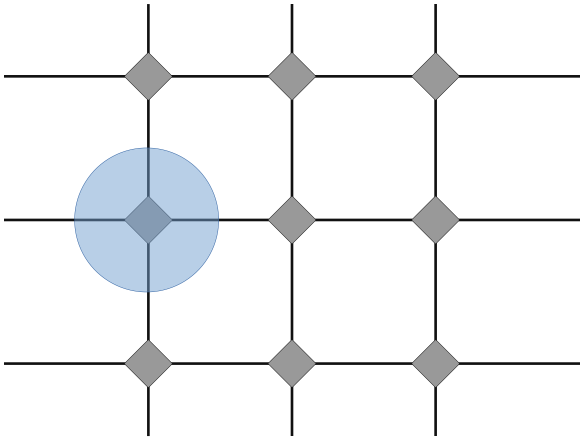
A simple square lattice (of dressed sites)

- Like a spin lattice (with large spins)
- Nearest Neighbor interaction
- Plaquette-type interaction



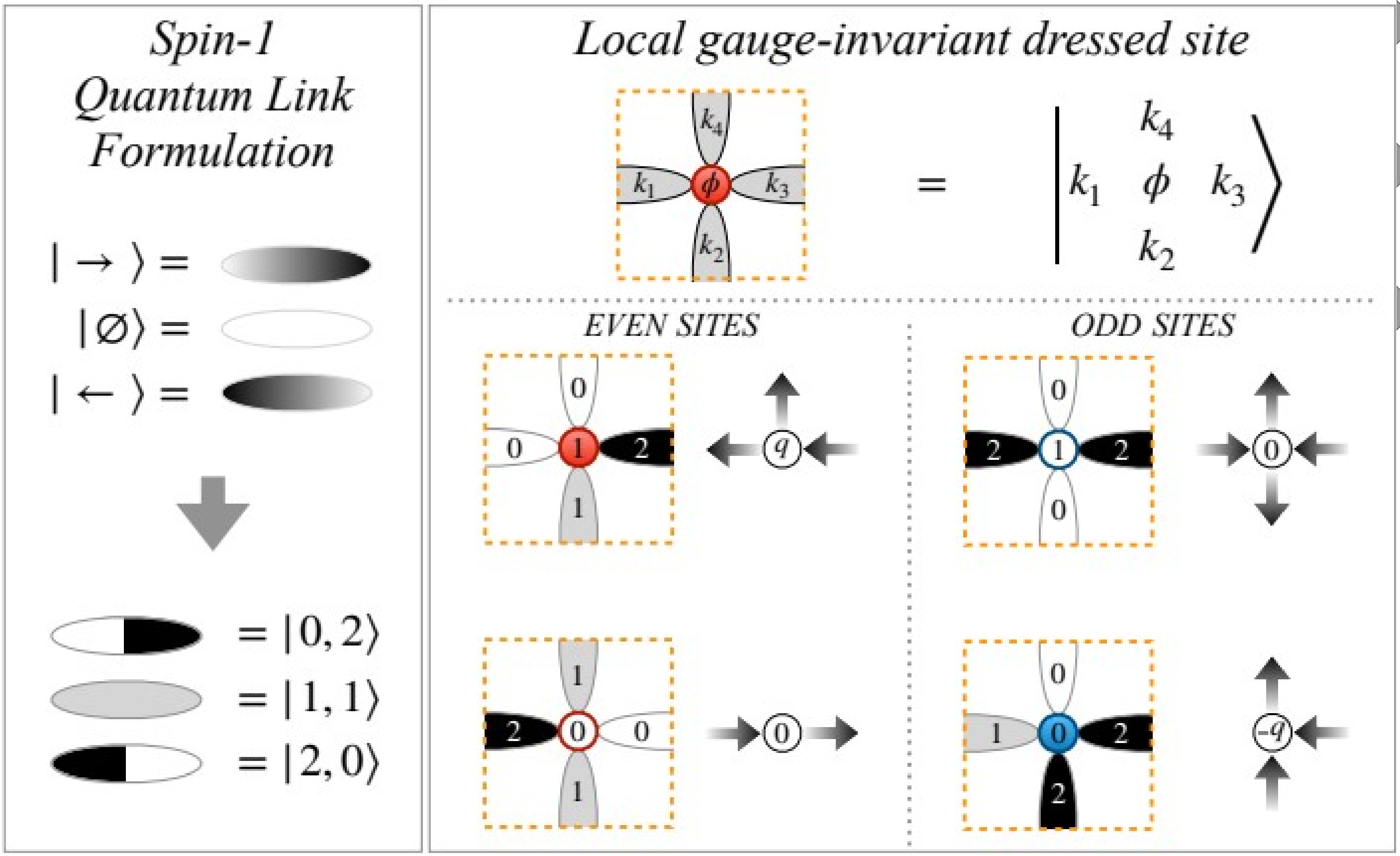
A simple square lattice (of dressed sites)

- Like a spin lattice (with large spins)
- Nearest Neighbor interaction
- Plaquette-type interaction
- Gauss’ Law is on-site: LOCAL BASIS FILTER



A simple square lattice (of dressed sites)

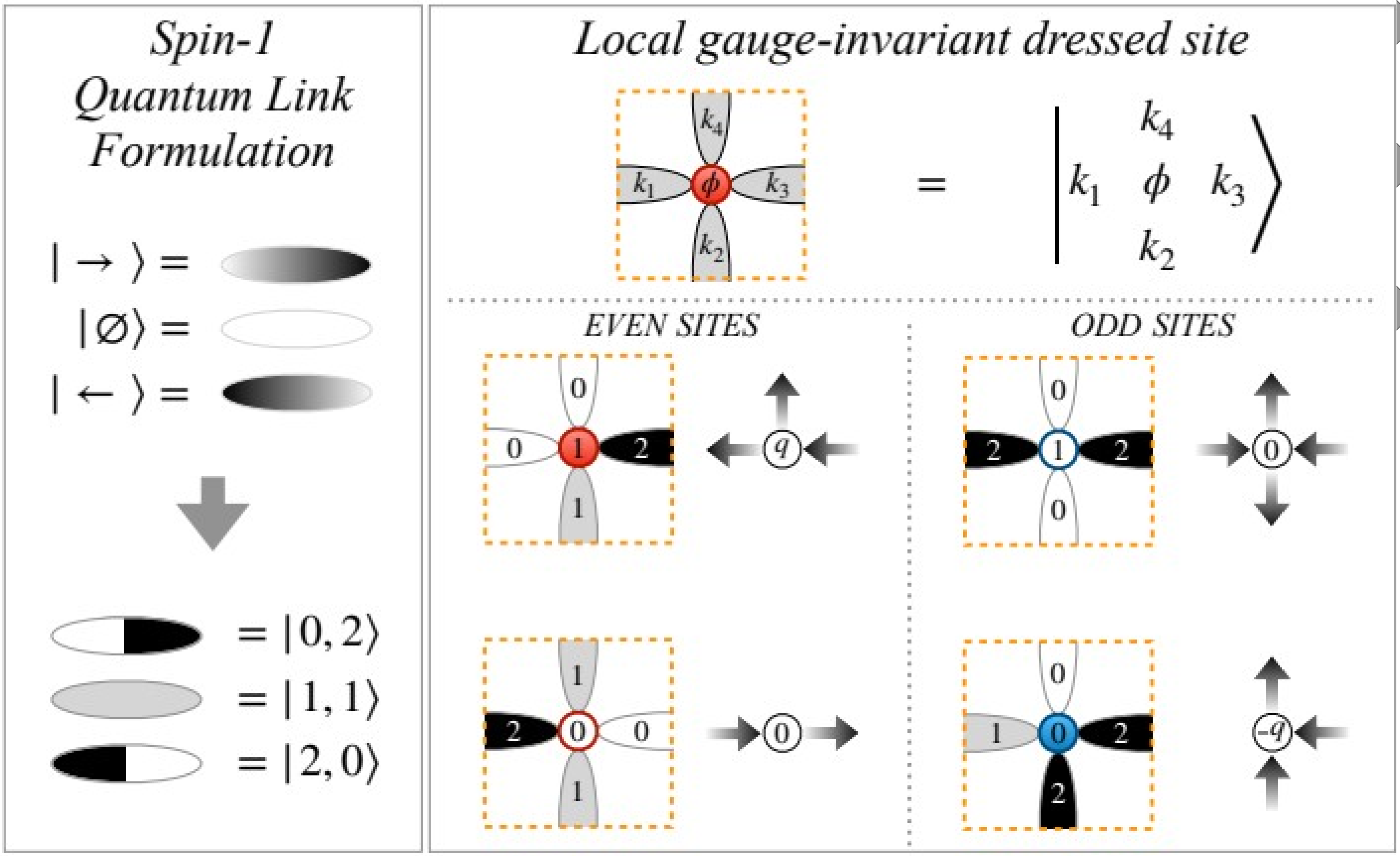
- L
- N
- F
- C
  E



Local dimension (2+1)D = 35

A simple square lattice (of dressed sites)

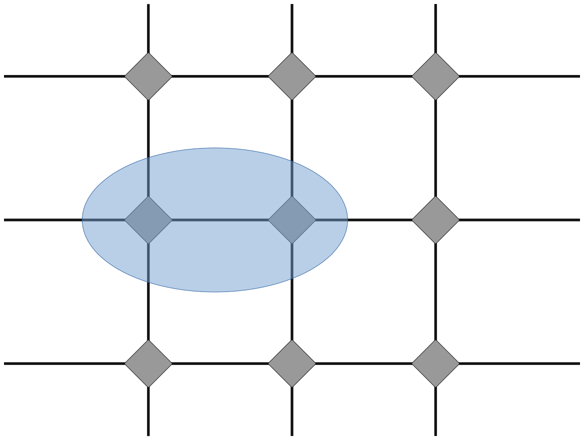
- L
- N
- F
- C
  E



Local dimension (3+1)D = 267

A simple square lattice (of dressed sites)

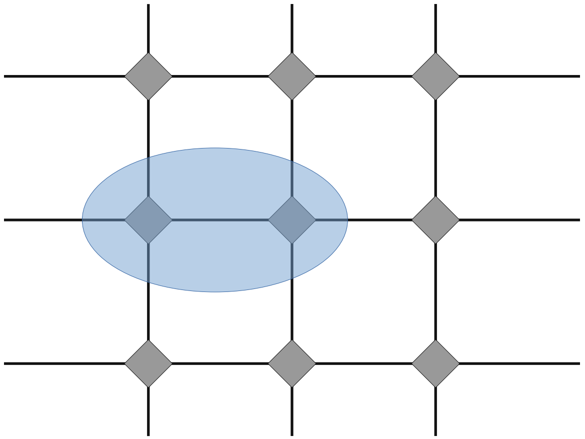
- Like a spin lattice (with large spins)
- Nearest Neighbor interaction
- Plaquette-type interaction
- Gauss' Law is on-site: LOCAL BASIS FILTER
- The Link symmetry is a nearest-neighbor selection rule (like a stabilizer)



$$\left(\hat{S}^z_{\mathbf{j},+\mu} + \hat{S}^z_{\mathbf{j}+\mu,-\mu}\right)|\Psi_{\text{phys}}\rangle = 0$$

A simple square lattice (of dressed sites)

- Like a spin lattice (with large spins)
- Nearest Neighbor interaction
- Plaquette-type interaction
- Gauss' Law is on-site: LOCAL BASIS FILTER
- The Link symmetry is a nearest-neighbor selection rule (like a stabilizer)



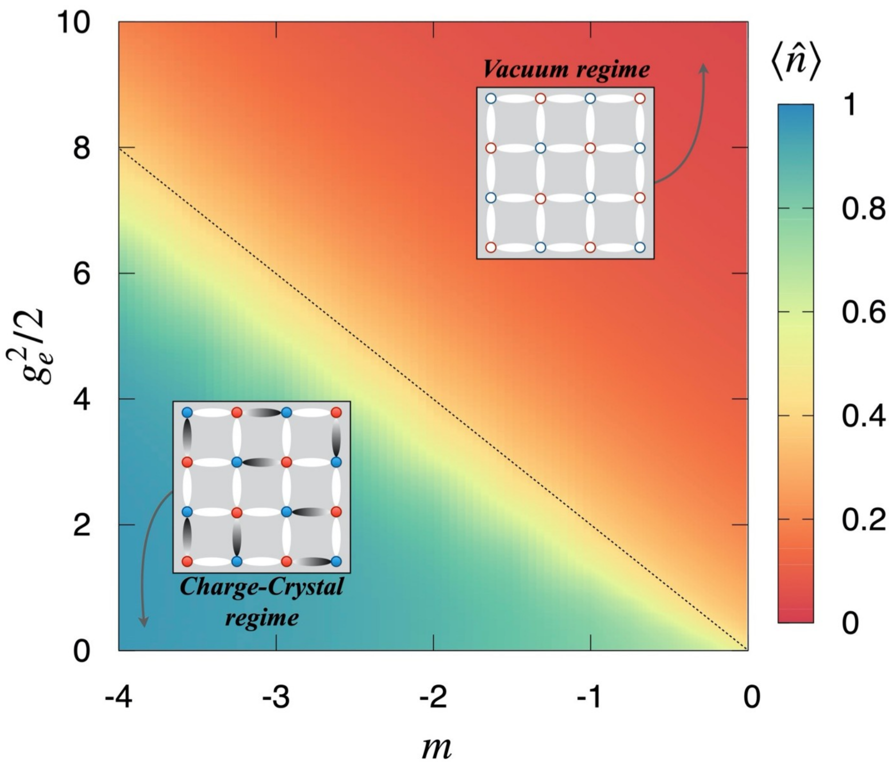
$$\left(\hat{S}^z_{\mathbf{j},+\mu} + \hat{S}^z_{\mathbf{j}+\mu,-\mu}\right) |\Psi_{\text{phys}}\rangle = 0$$

READY
For simulation

→ Tensor Networks

→ Quantum Simulator

(2+1)D: Ground-state properties as a function of and without magnetic terms

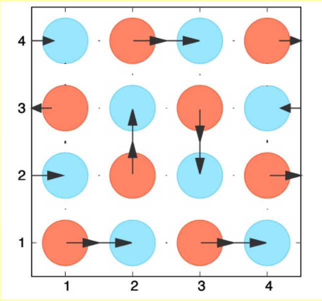


$$g_e^2/2 \gg 2|m|$$

Vacuum phase: no particles, no field excitations

$$-2m \gg g_e^2/2 > 0$$

Charge-Crystal Phase: particle-antiparticle dimers

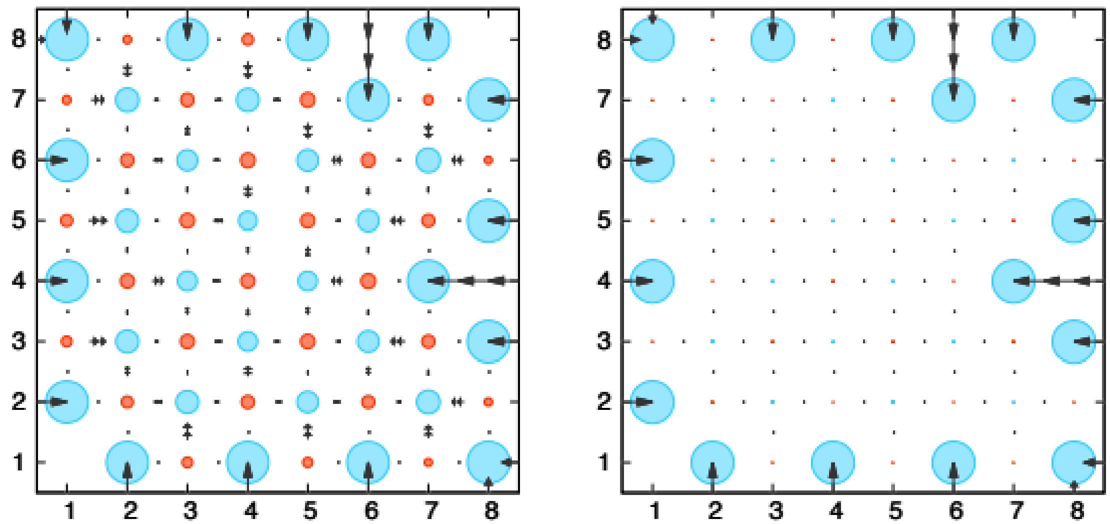


(2+1)D: Finite Charge Density Sector

Charge imbalance into the system: very challenging for Montecarlo techniques (sign problem)

Charges forced to reach the boundaries to minimise the electric energy

$$Q = -16$$



$g_e^2/2 = 2,\ m = -1$

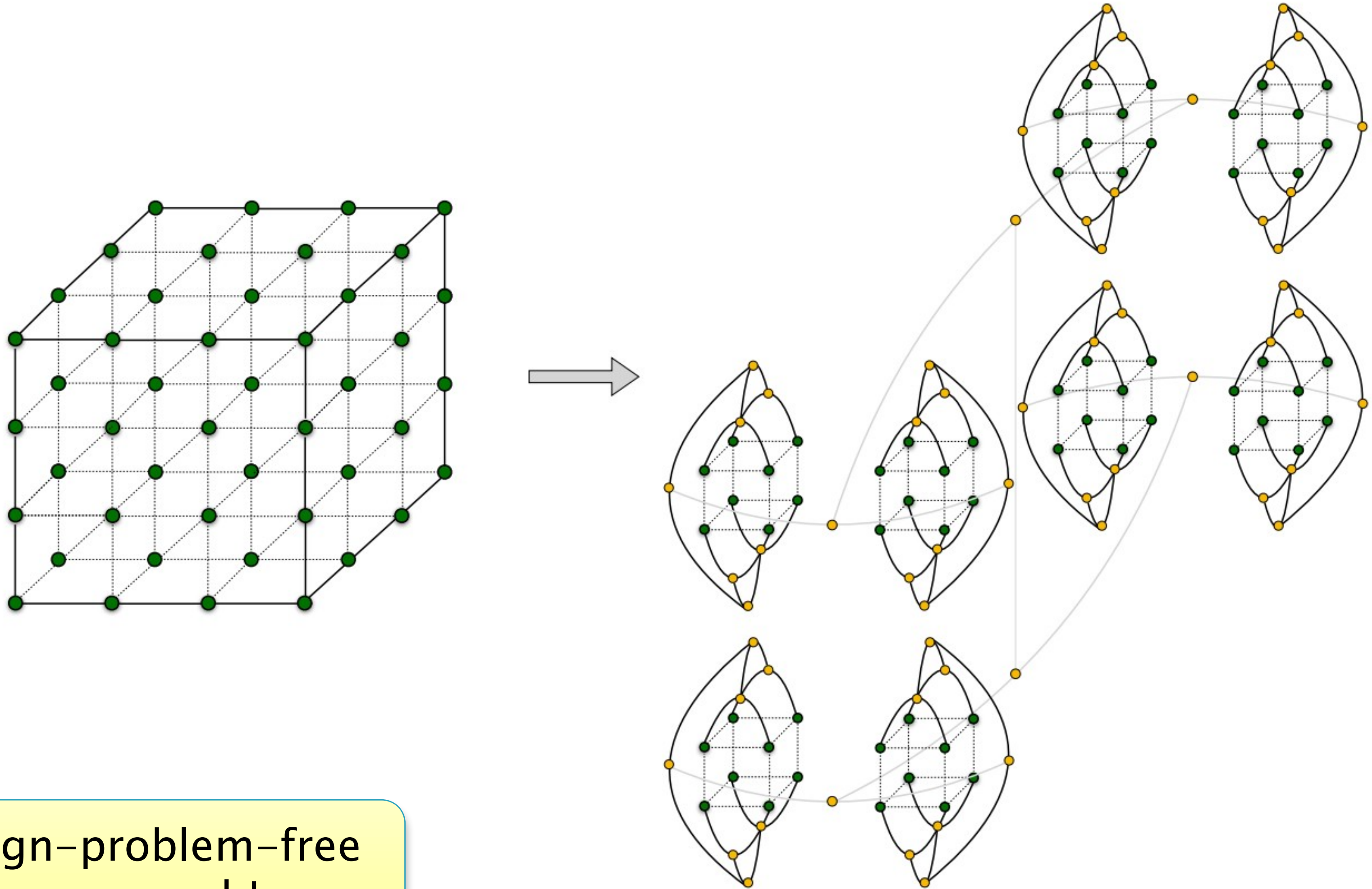
near the critical line

$g_e^2/2 = 2,\ m = 5$

deep in the vacuum phase

Phys. Rev. X 10, 041040 (2020)

Tree Tensor Networks in 3D



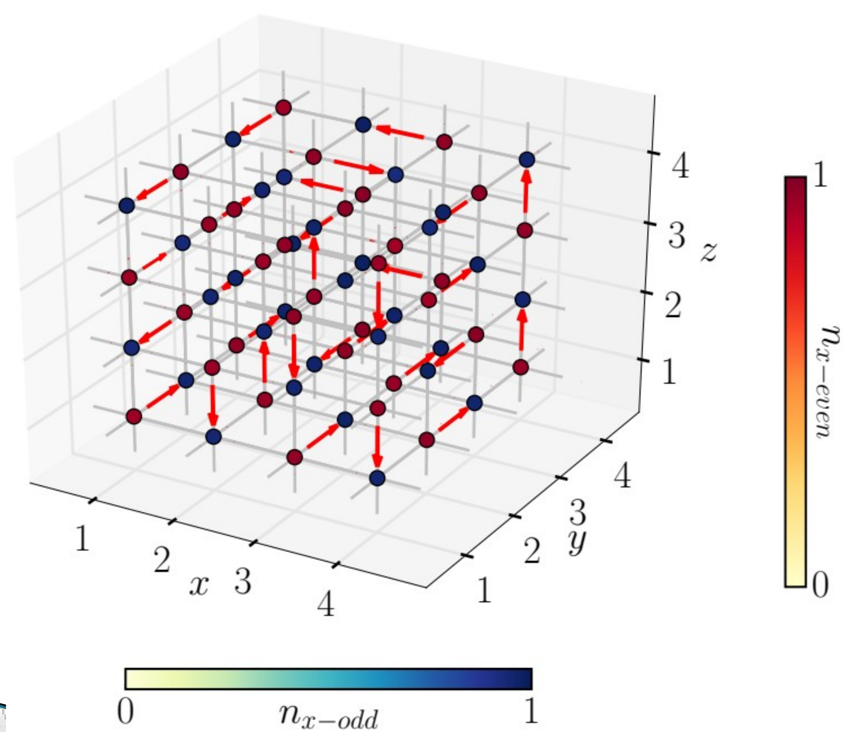
Sign-problem-free approach!

The optimization still has a complexity

(3+1)D: Local configurations of matter and gauge fields

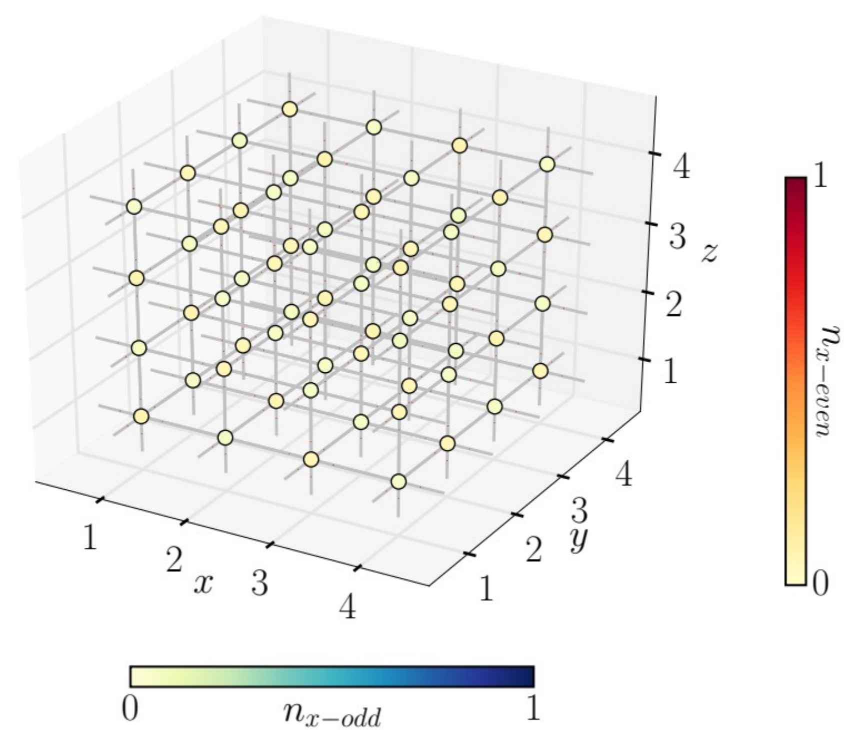
$$-2m \gg g_e^2/2 > 0$$

Charge-Crystal Phase: particle-antiparticle dimers



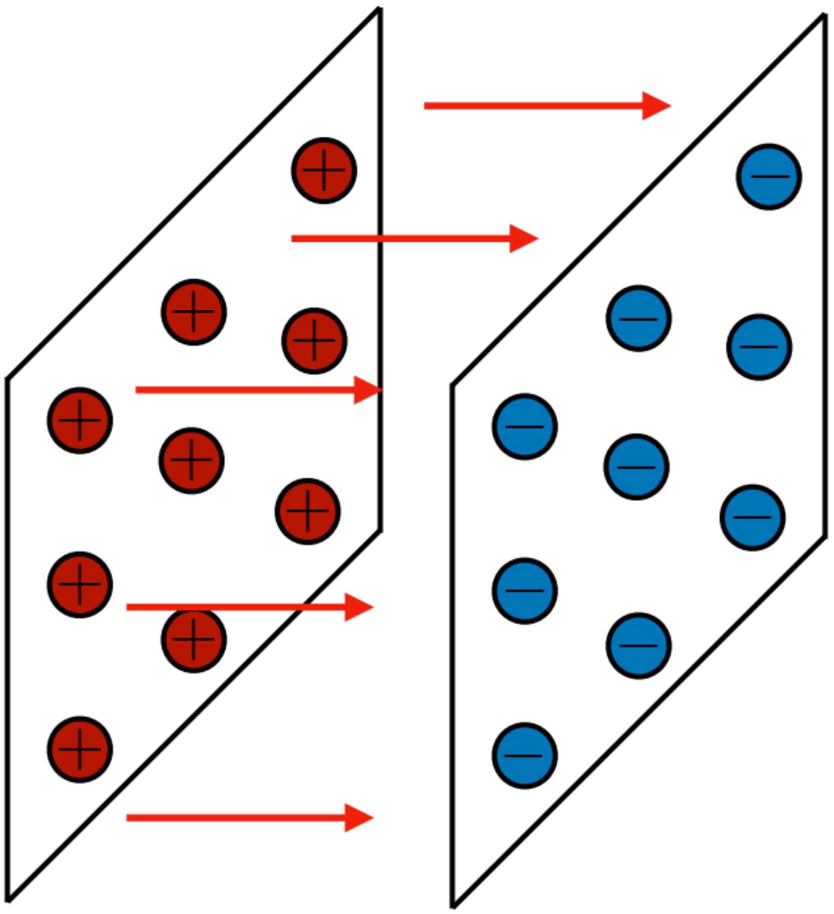
$$g_e^2/2 \gg 2|m|$$

Vacuum phase: no particles, no field excitations



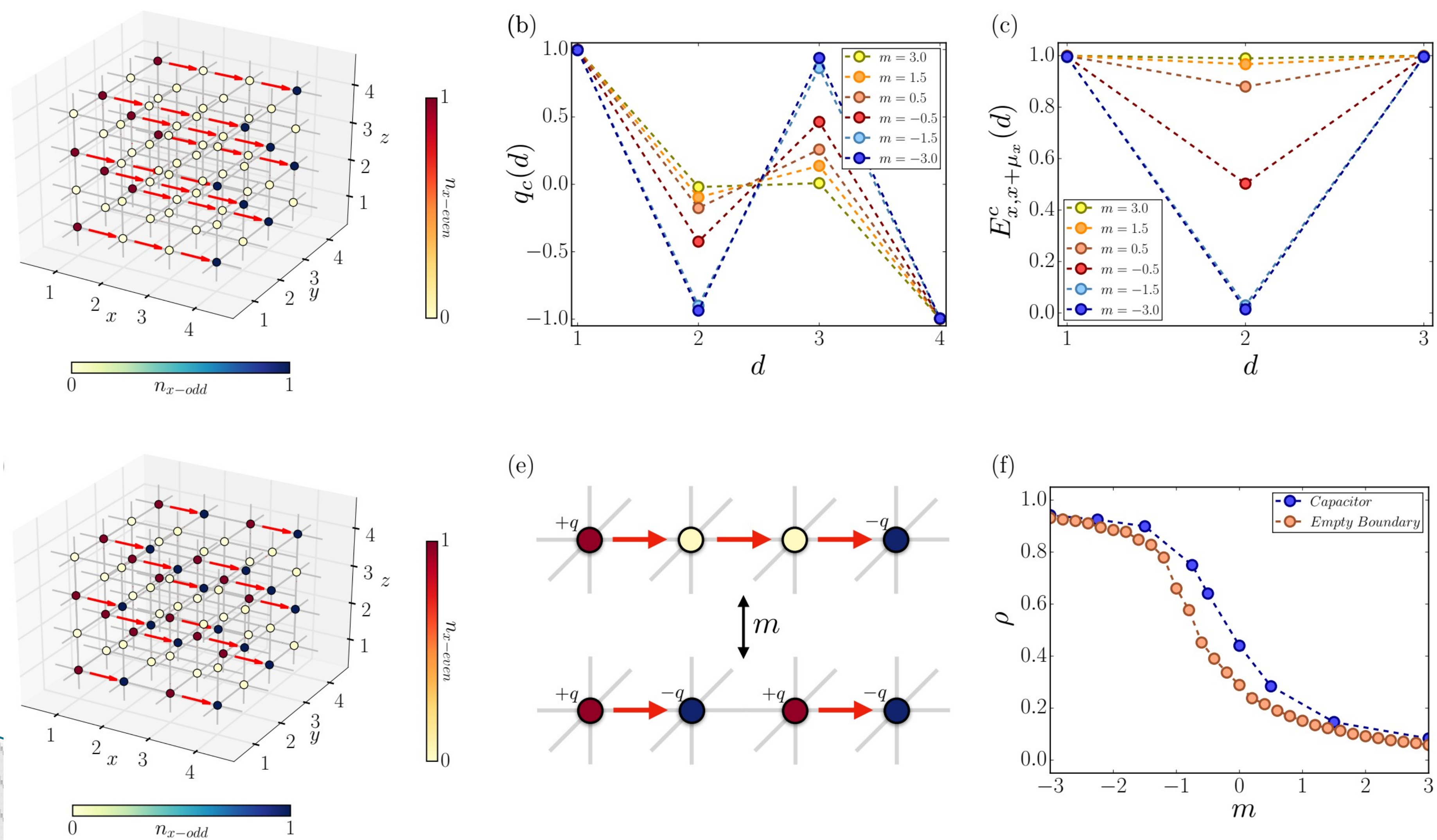
(3+1)D: Quantum Capacitor

Our approach is very flexible to simulate different geometries and charge-configurations.

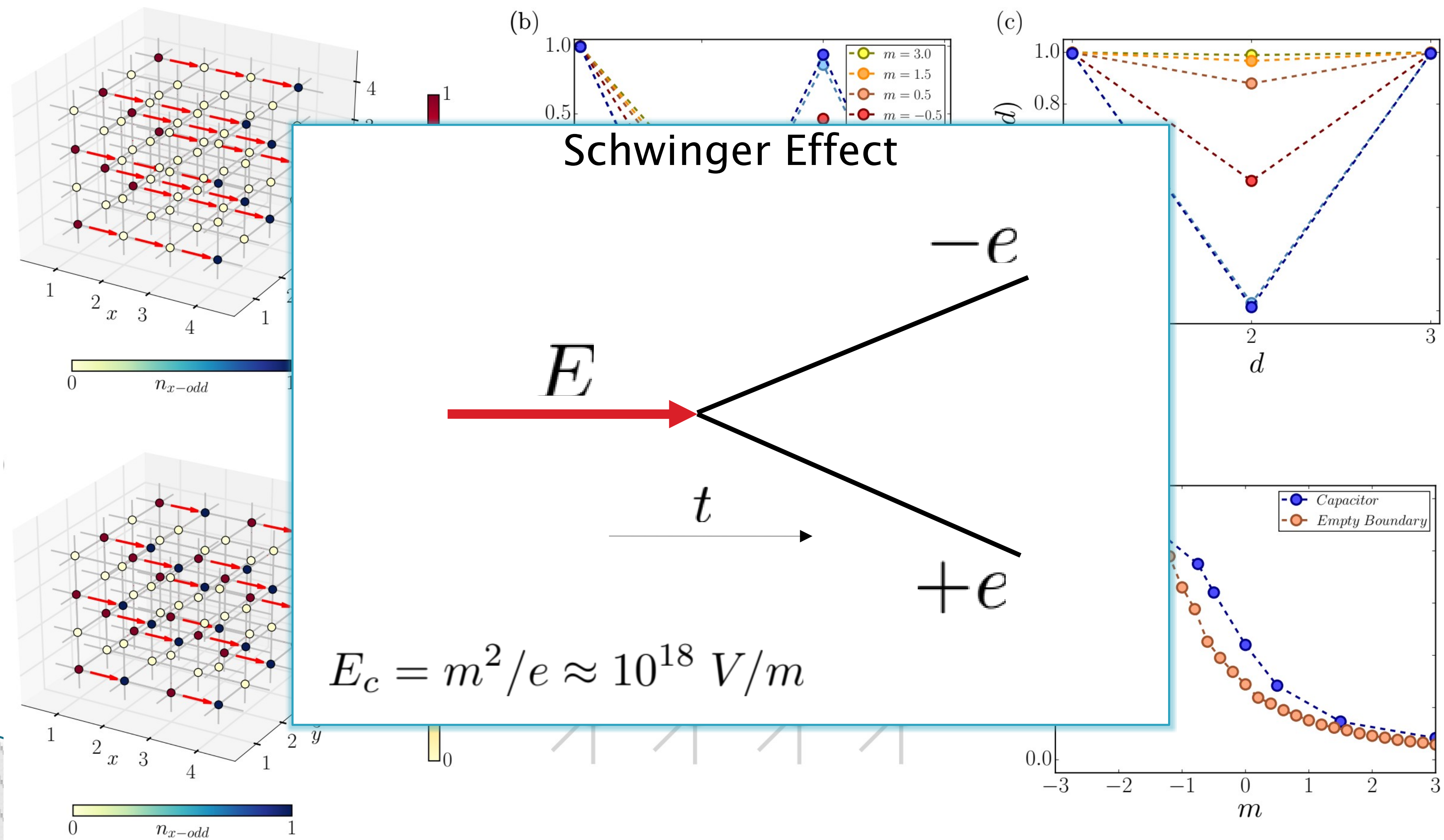


Field-screening and equilibrium string-breaking properties in presence of external field.

(3+1)D: Quantum Capacitor



(3+1)D: Quantum Capacitor



(3+1)D: Confinement Properties

$$g_m^2 = 8/g_e^2$$

$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$
$$+ m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$
$$- \frac{g_m^2}{2} \sum_x \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

Plaquette terms | Electric Field Energy

Weak-coupling | Strong-coupling g_e

Confinement Properties

Plaquette terms

Electric Field Energy

Weak-coupling

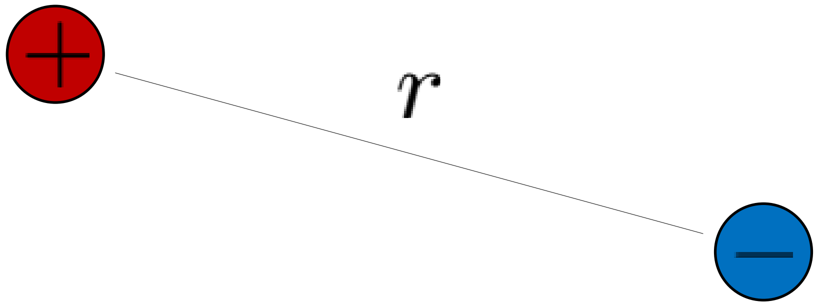
Strong-coupling g_e

$$V(r) \propto 1/r$$

Deconfined phase

$$V(r) \propto r$$

Confined phase



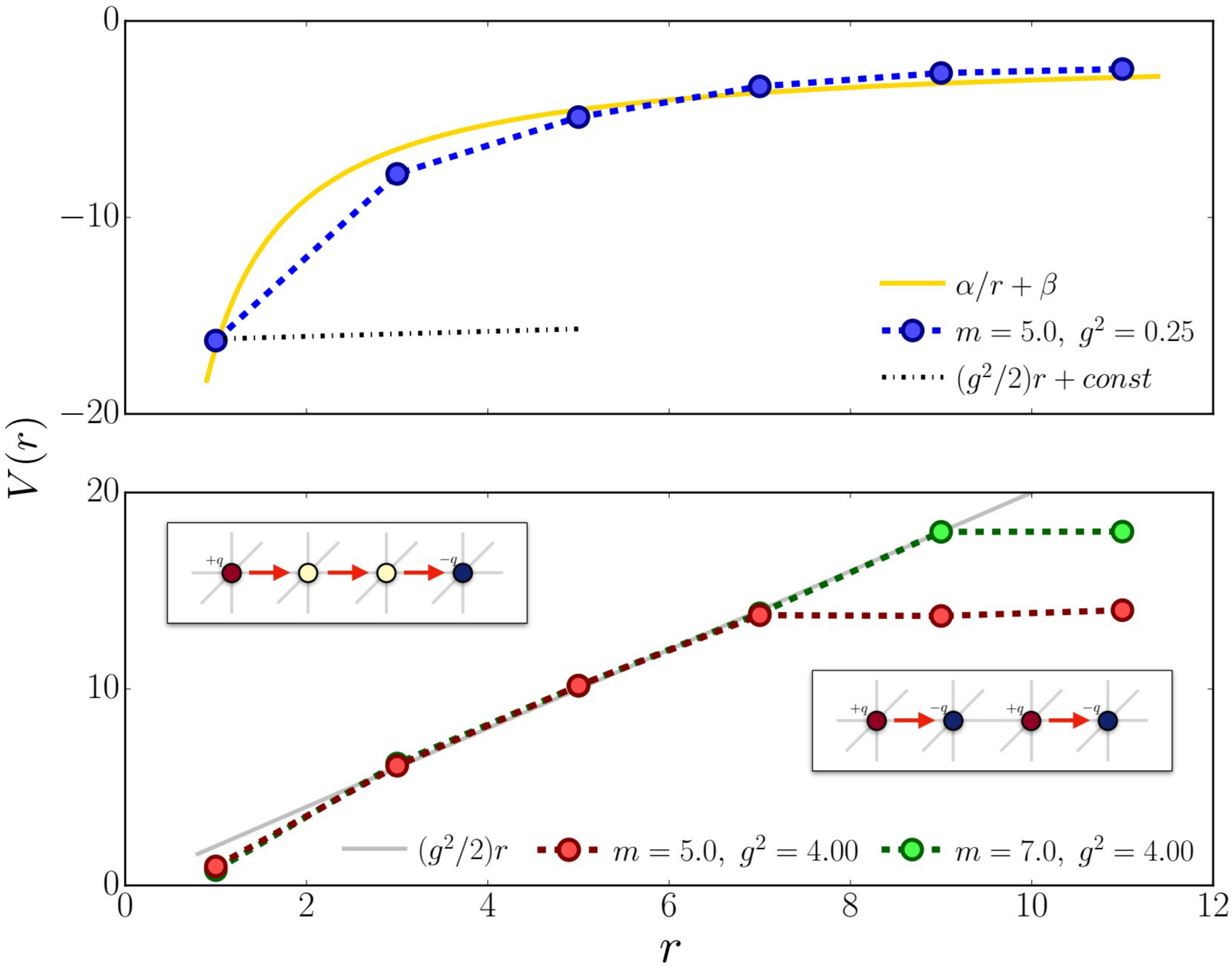
Kei-Ichi Kondo, Phys. Rev. D **58**, 085013 (1998)

Confinement Properties

$$V(r) = E(r) - E_0$$

Weak–coupling

Strong–coupling



Going non-Abelian: the Yang-Mills theory

$$H_{\text{QED}} = \frac{c\hbar}{2a} \sum_{\mathbf{j},\mu,a,a'} \left(e^{i\phi_{\mathbf{j}\mu}} \hat{\psi}^{\dagger}_{\mathbf{j},a} \hat{U}_{\mathbf{j},\mu;a,a'} \hat{\psi}_{\mathbf{j}+\mu,a'} + \text{H.c.} \right)$$

$$+ m_e c^2 \sum_{\mathbf{j},a} (-1)^{j_x + j_y} \hat{\psi}^{\dagger}_{\mathbf{j},a} \hat{\psi}_{\mathbf{j},a}$$

$$+ g^2_{\text{YM}} \frac{c\hbar}{2a} \sum_{\mathbf{j},\mu} \hat{C}^{(2)}_{\mathbf{j},\mu}$$

$$- \frac{1}{g^2} \frac{c\hbar}{2as^4} \sum_{\mathbf{j},a_1..a_4} \left(\hat{U}_{\mathbf{j},\mu_x;a_1,a_2} \ldots \hat{U}^{\dagger}_{\mathbf{j},\mu_y;a_1,a_4} + \text{H.c.} \right)$$

Matter has SU(N) color, the theory is color-invariant.

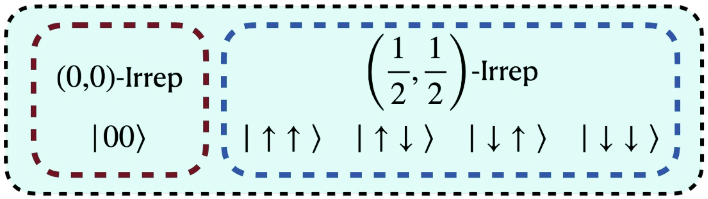
Similar manipulation as the QED case can be made.

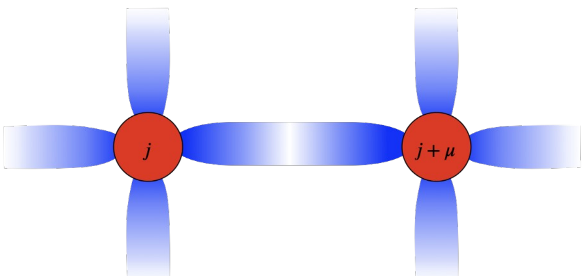
Giovanni's Talk

SU(2) Lattice Yang-Mills in (2+1)D

GIOVANNI CATALDI
University of Padua

$5d$ SU(2) Gauge Link Hilbert space $\mathscr{H}_G$



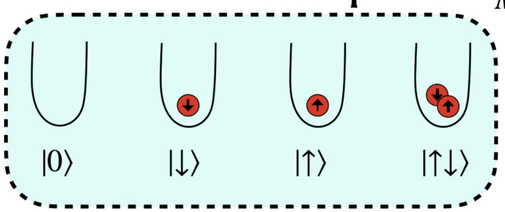


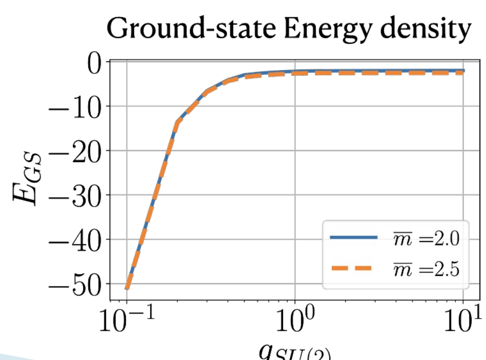
$$H^{2D}_{\text{LYM-SU(2)}} = \frac{c\hbar}{2a}\sum_j \left(-iQ^\dagger_{j,+\mu_x} Q_{j+\mu_x,-\mu_x} - (-1)^{j_x+j_y} Q^\dagger_{j,+\mu_y} Q_{j+\mu_y,-\mu_y} + \text{H.c.} \right) + mc^2 \sum_j (-1)^{j_x+j_y} D_j + H_{\text{pure}} + H_{\text{penalty}}$$

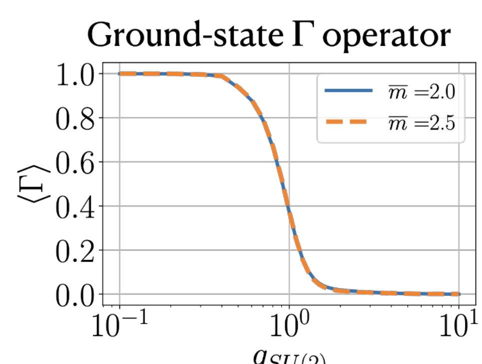
$$H_{\text{pure}} = \frac{3c\hbar g^2_{\text{SU(2)}}}{16a}\sum_j \Gamma_j - \frac{4c\hbar}{a g^2_{\text{SU(2)}}}\sum_j \left(C_{j;\mu_x,\mu_y} C_{j+\mu_x;\mu_y,-\mu_x} C_{j+\mu_x+\mu_y;-\mu_x,-\mu_y} C_{j+\mu_y;-\mu_y,\mu_x} + \text{H.c.} \right)$$

$$H_{\text{penalty}} = -\eta \sum_j \left[W_{j,+\mu_x} W_{j+\mu_x,-\mu_x} - W_{j,+\mu_y} W_{j+\mu_y,-\mu_y} \right]$$

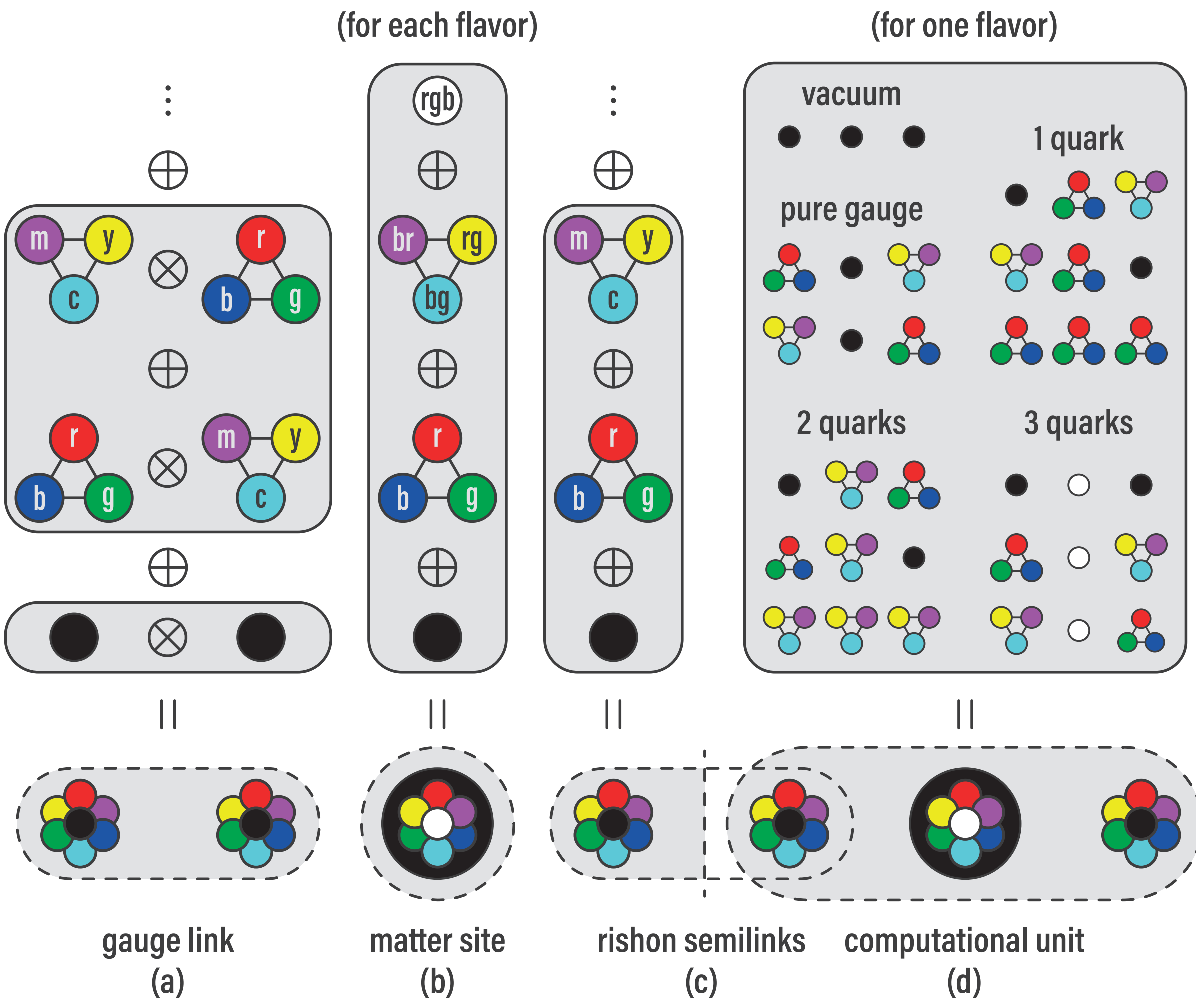
$4d$-Matter Hilbert space $\mathscr{H}_M$







MarcoRigo's Talk

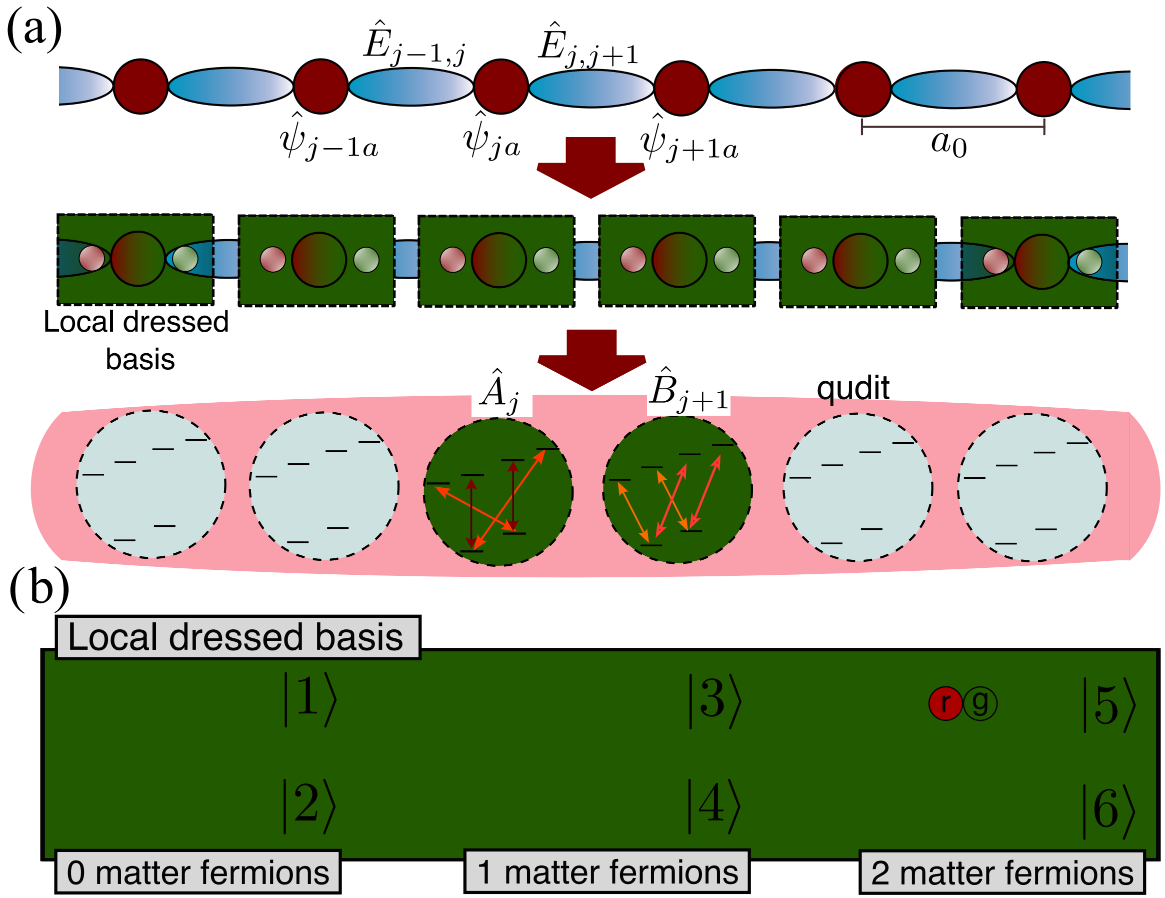


MARCO RIGOBELLO
University of Padua

SU(3) with 2 flavors in 1+1D

Work in progress

SU(2) Yang-Mills in 1+1D (Hardcore gluons) is mapped into a model of Qudits d = 6

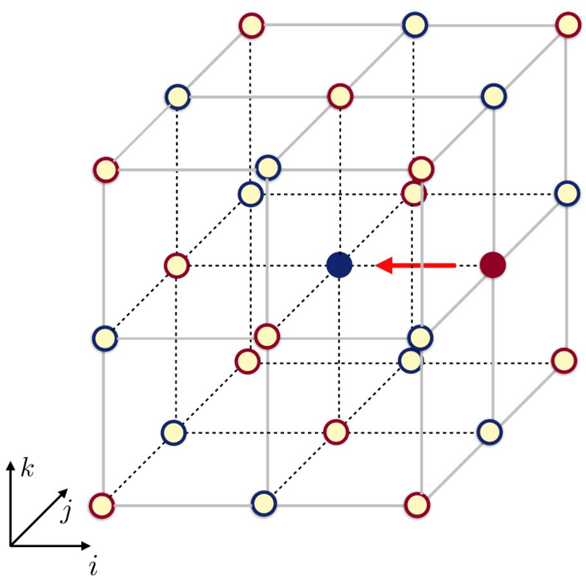


Perfect for trapped ion qudits (See Martin's Talk)

Conclusions

Tensor Network Simulation ⟷ Quantum Simulation

- Hamiltonian LGTs are an excellent formalism to complement MonteCarlo
- Several LGT models allow us to eliminate fermionic matter.
- Short-term goal: Real time simulation of scattering processes
- Long-term goal: tensor networks and quantum simulation of QCD.

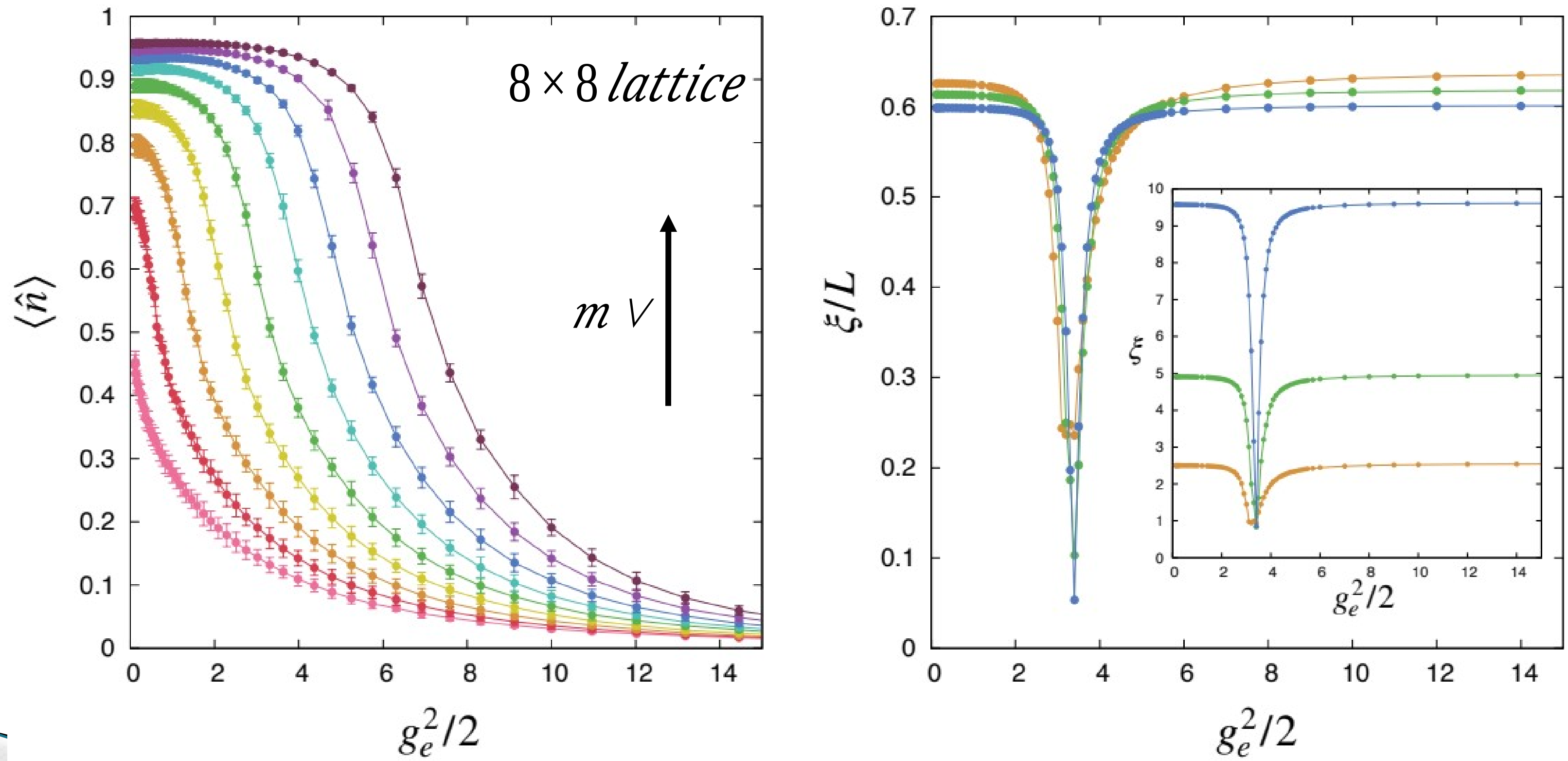


THANK YOU

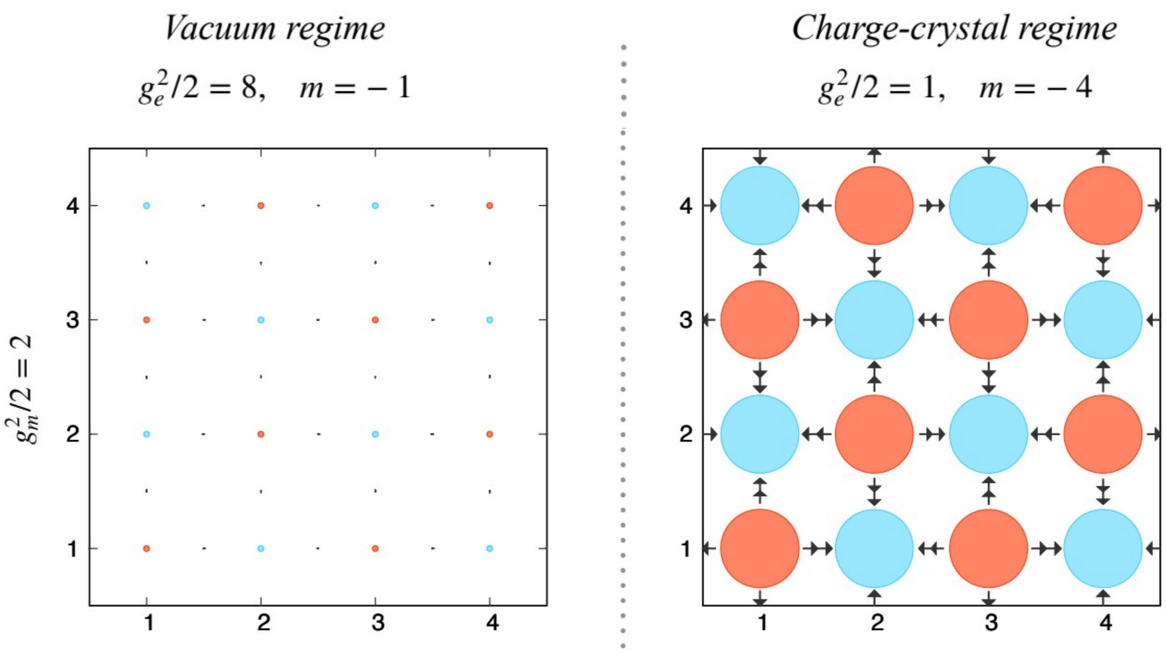
Ground-state properties as a function of and without magnetic terms

$m \in \{-0.01, -0.5, -1, -1.5, -2, -2.5, -3, -3.5, -4\}$

4×4 (*orange*), 8×8 (*green*), 16×16 (*blue*)



Finite magnetic-coupling effects



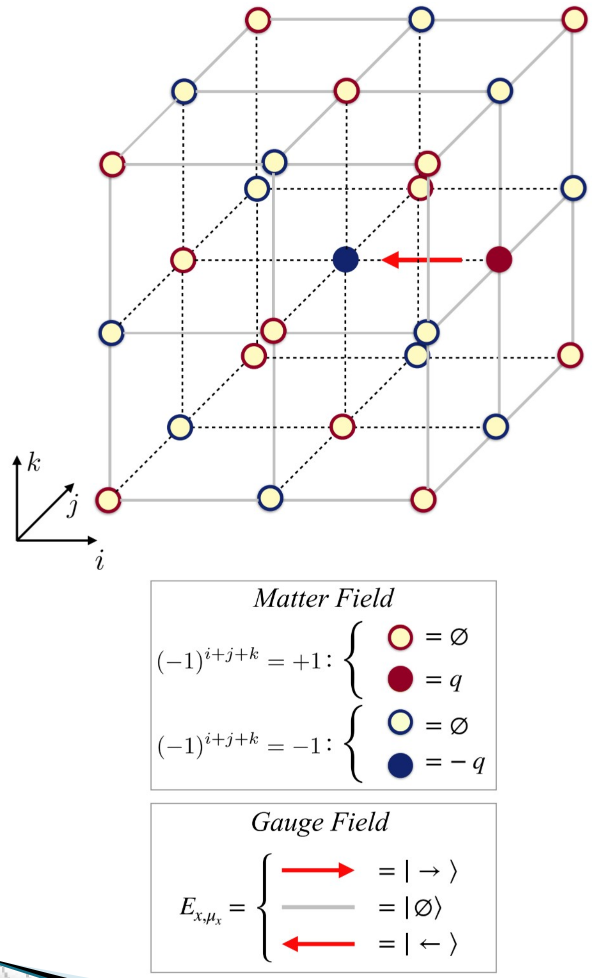
$$g_e^2/2 \gg 2|m|$$

No changes affecting the vacuum configuration

$$-2m \gg g_e^2/2 > 0$$

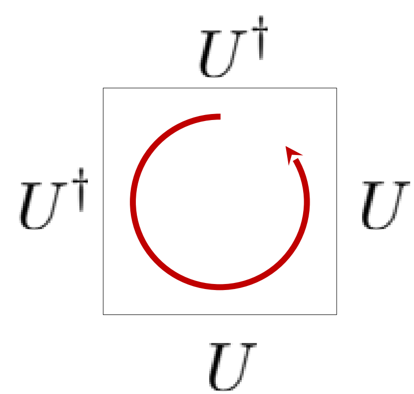
Nontrivial reorganisation of the electric fields, global entangled state of gauge fields

Lattice QED in (3+1)D

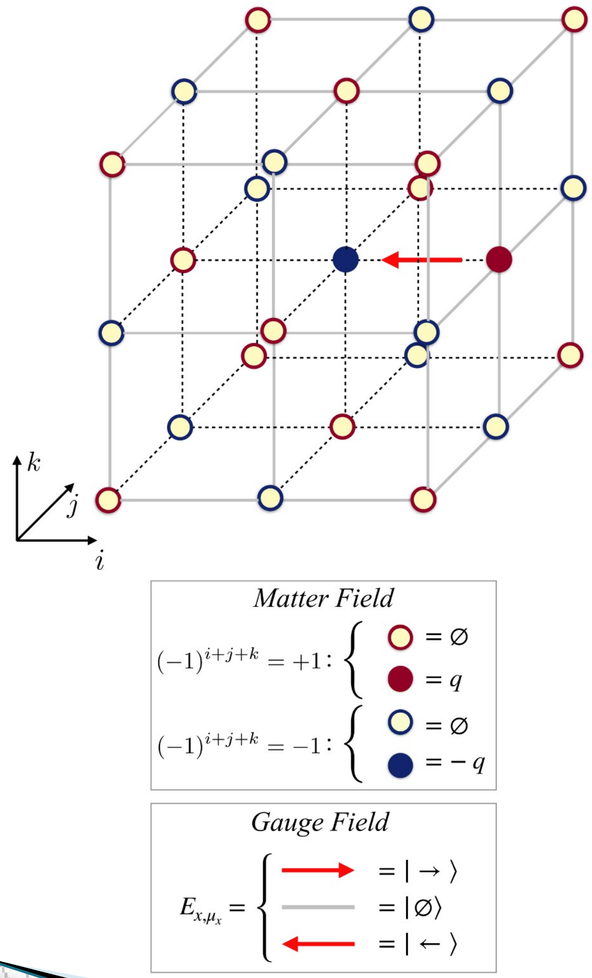


$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right) + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 - \frac{g_m^2}{2} \sum_x \left(\square_{\mu_x,\mu_y} + \square_{\mu_x,\mu_z} + \square_{\mu_y,\mu_z} + \text{H.c.} \right)$$

$$\square_{\mu_x,\mu_y} = U_{x,\mu_x} U_{x+\mu_x,\mu_y} U^\dagger_{x+\mu_y,\mu_x} U^\dagger_{x,\mu_y}$$



Lattice QED in (3+1)D

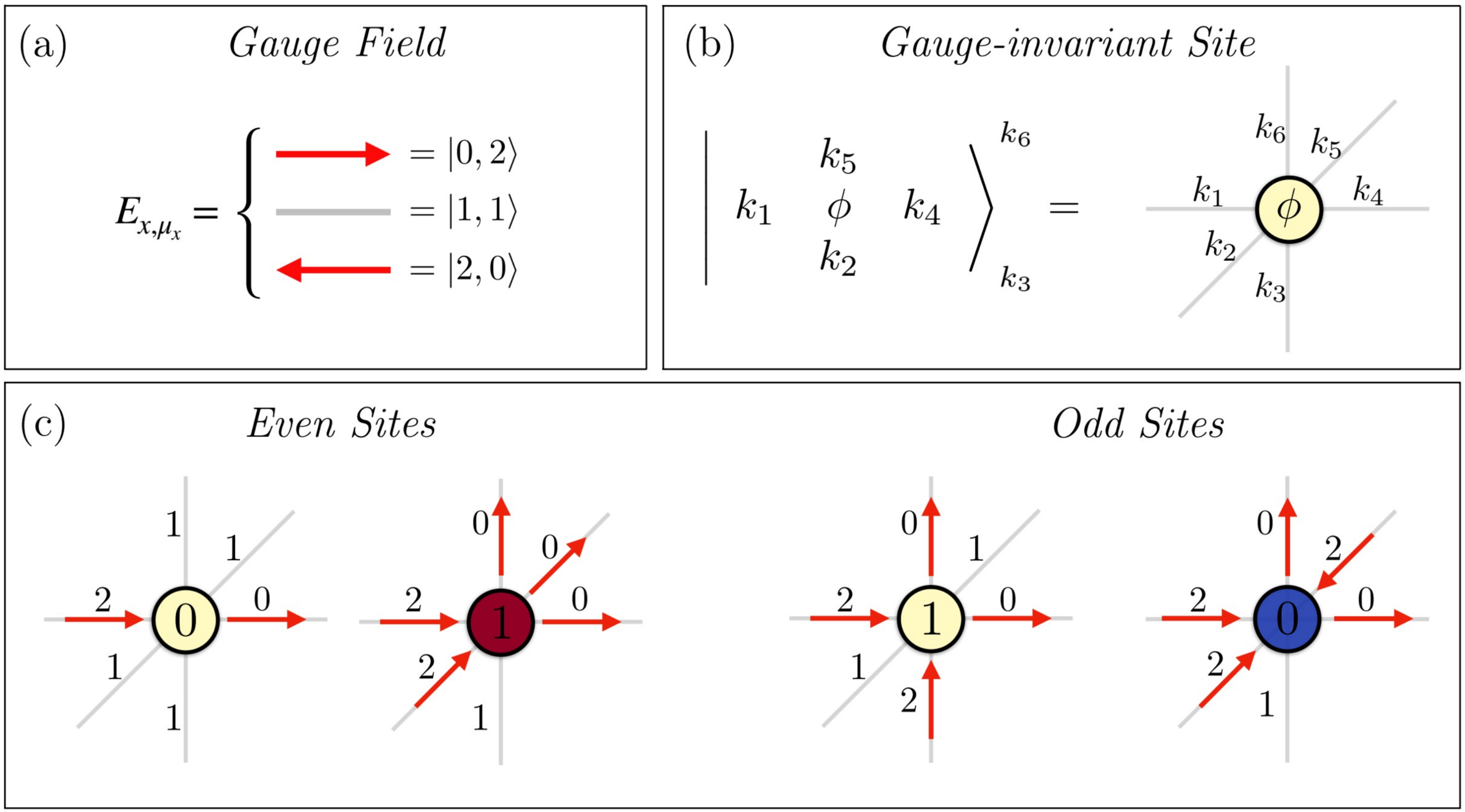


$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right) + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 - \frac{g_m^2}{2} \sum_x \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

Quantum Link Model discretization of Gauge Fields

$$\hat{E}_{x,\mu} \rightarrow \hat{S}^z_{x,\mu}$$
$$\hat{U}_{x,\mu} \rightarrow \hat{S}^+_{x,\mu}/s,$$

$$\hat{G}_x = \hat{\psi}_x^\dagger \hat{\psi}_x - \frac{1-p_x}{2} - \sum_\mu \hat{E}_{x,\mu} \qquad \hat{G}_x|\Phi\rangle = 0 \ \forall x$$



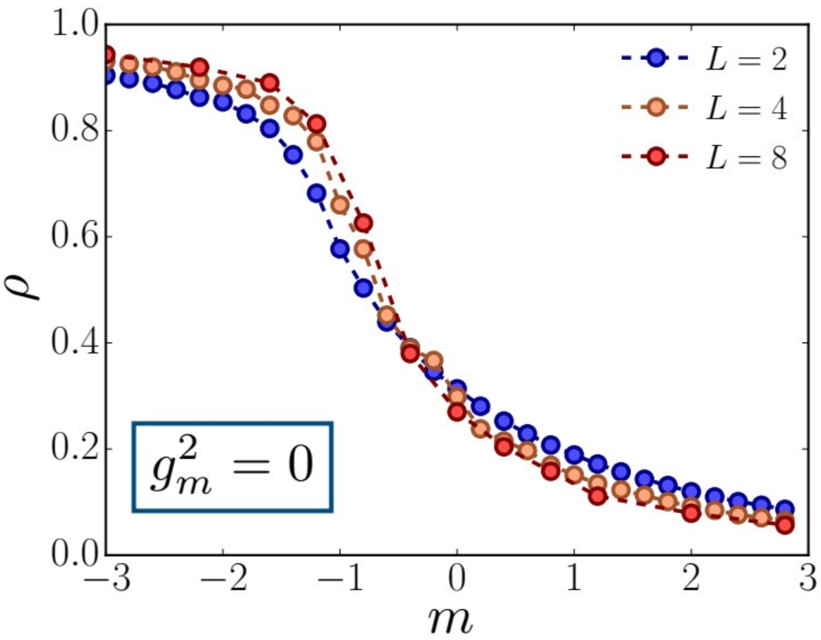
$$dimH_x = 267$$

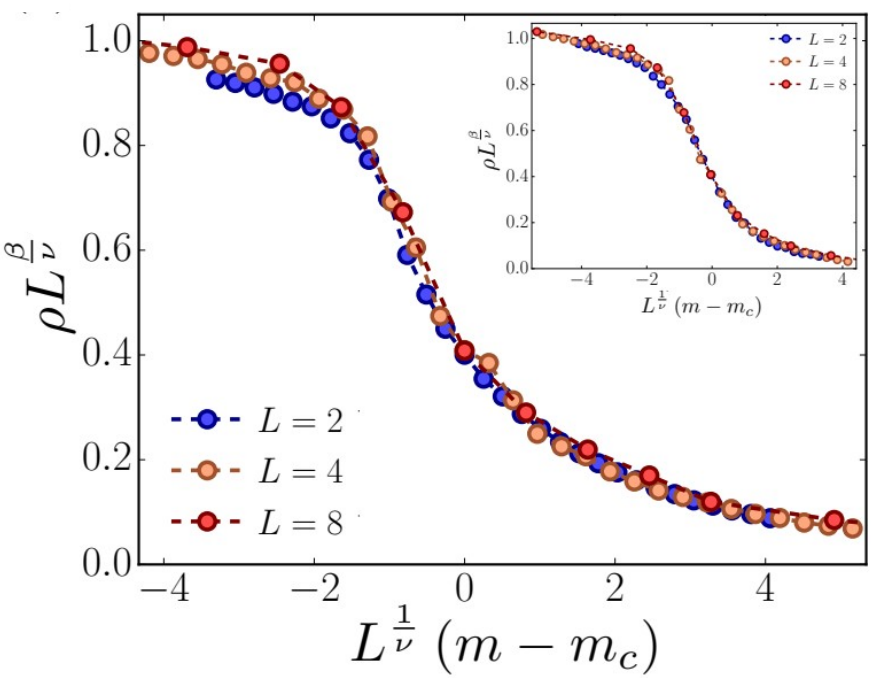
just for comparison, like
a spin system with

Ground state properties for

$$\rho = \frac{1}{L^3}\sum_x \langle GS|\hat{n}_x|GS\rangle$$

$$\rho L^{\frac{\beta}{\nu}} = \lambda\left(L^{\frac{1}{\nu}}(m - m_c)\right)$$





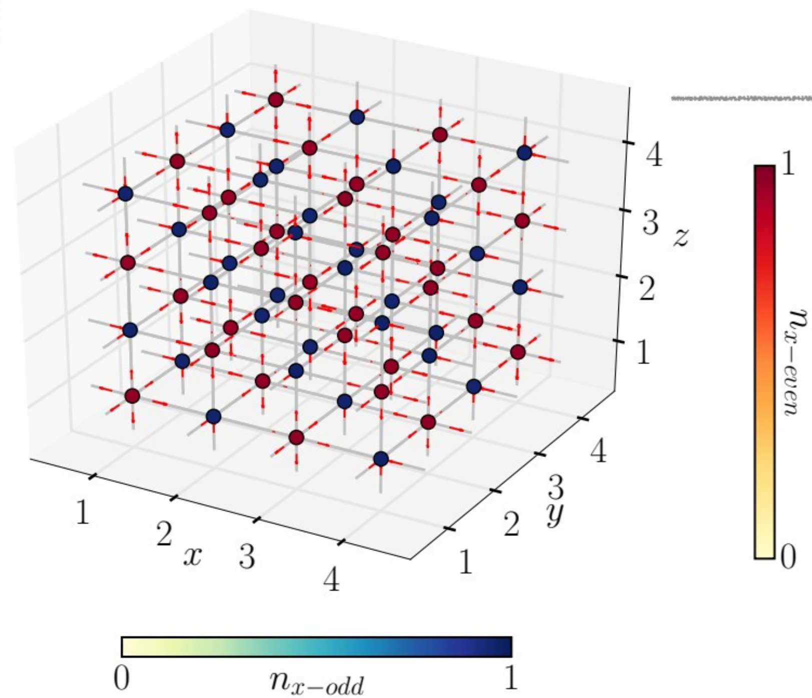
$$m_c = -0.39$$

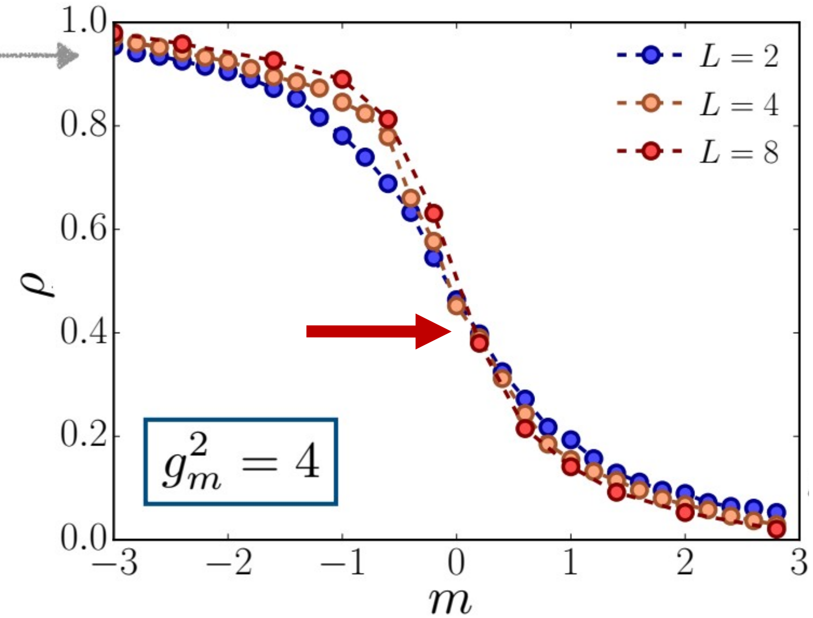
$$\beta = 0.16 \quad \nu = 0.22$$

Ground state properties for

$$\rho = \frac{1}{L^3} \sum_x \langle GS|\hat{n}_x|GS\rangle$$

$$\rho L^{\frac{\beta}{\nu}} = \lambda\left(L^{\frac{1}{\nu}}(m - m_c)\right)$$





$$m_c = +0.22$$
$$\beta = 0.16 \quad \nu = 0.22$$

Confinement Properties

$$g_m^2 = 8/g_e^2$$

$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \mathrm{H.c.} \right)$$
$$+ m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$
$$- \frac{g_m^2}{2} \sum_x \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \mathrm{H.c.} \right)$$

Plaquette terms

Electric Field Energy

Weak–coupling

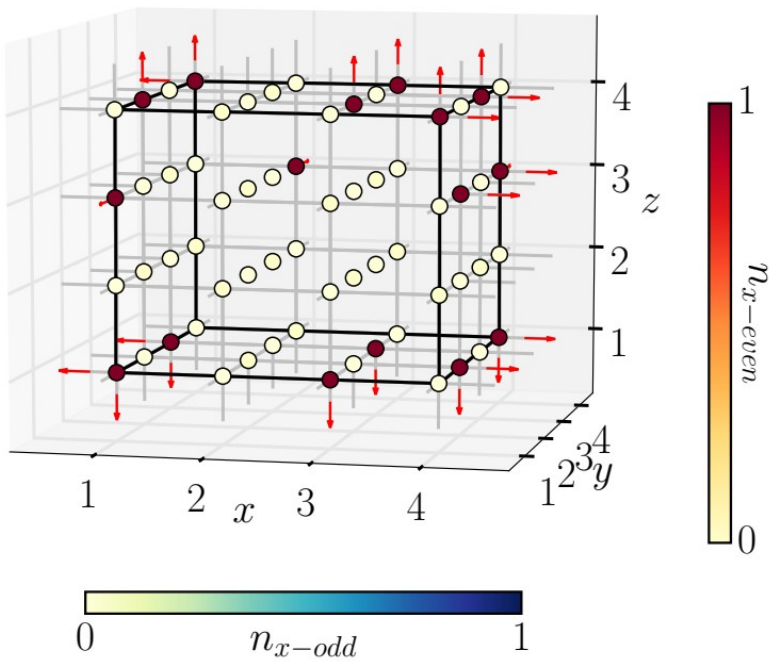
Strong–coupling

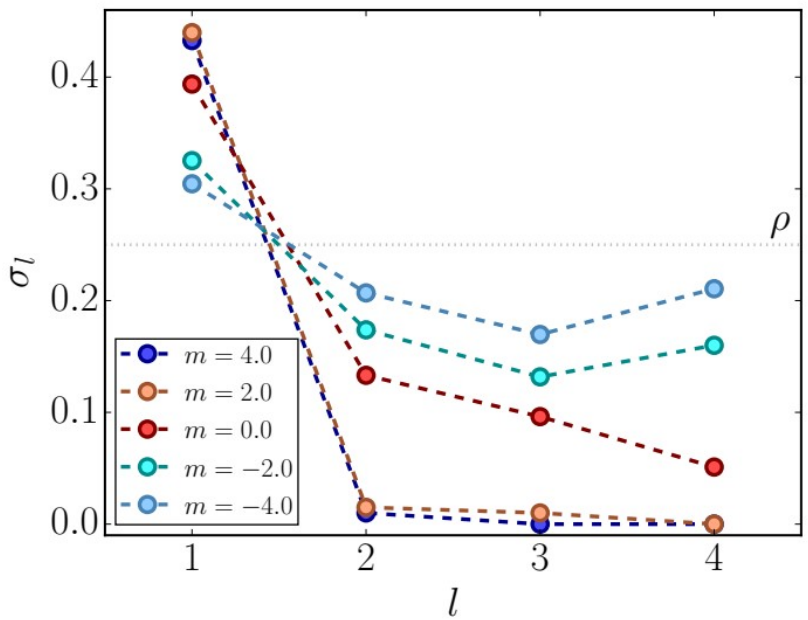
g_e

Finite Density

$L = 4\,, Q = 16\,, \rho = 1/4$

$L = 8\,, Q = 128\,, \rho = 1/4$

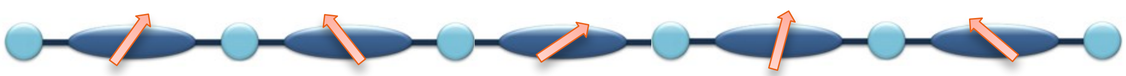




$$\sigma(l) = \frac{1}{A(l)} \sum_{x \in A(l)} \left\langle \hat{\psi}_x^\dagger \hat{\psi}_x \right\rangle$$

Tensor Networks – Scattering dynamics

$$\hat{H} = -t\sum_x \hat{\psi}_x^\dagger \hat{U}_{x,x+1}\hat{\psi}_{x+1} + \text{h.c.} + m\sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g^2}{2}\sum_x \hat{E}_{x,x+1}^2$$



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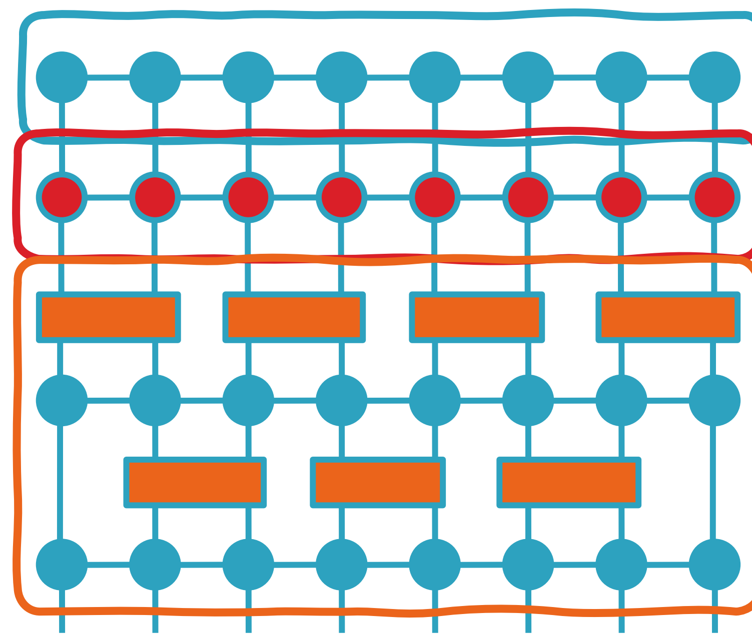


interacting vacuum MPS via DMRG

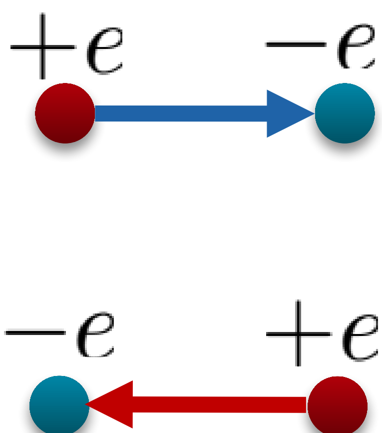
initial state via wave packet creation **MPOs**

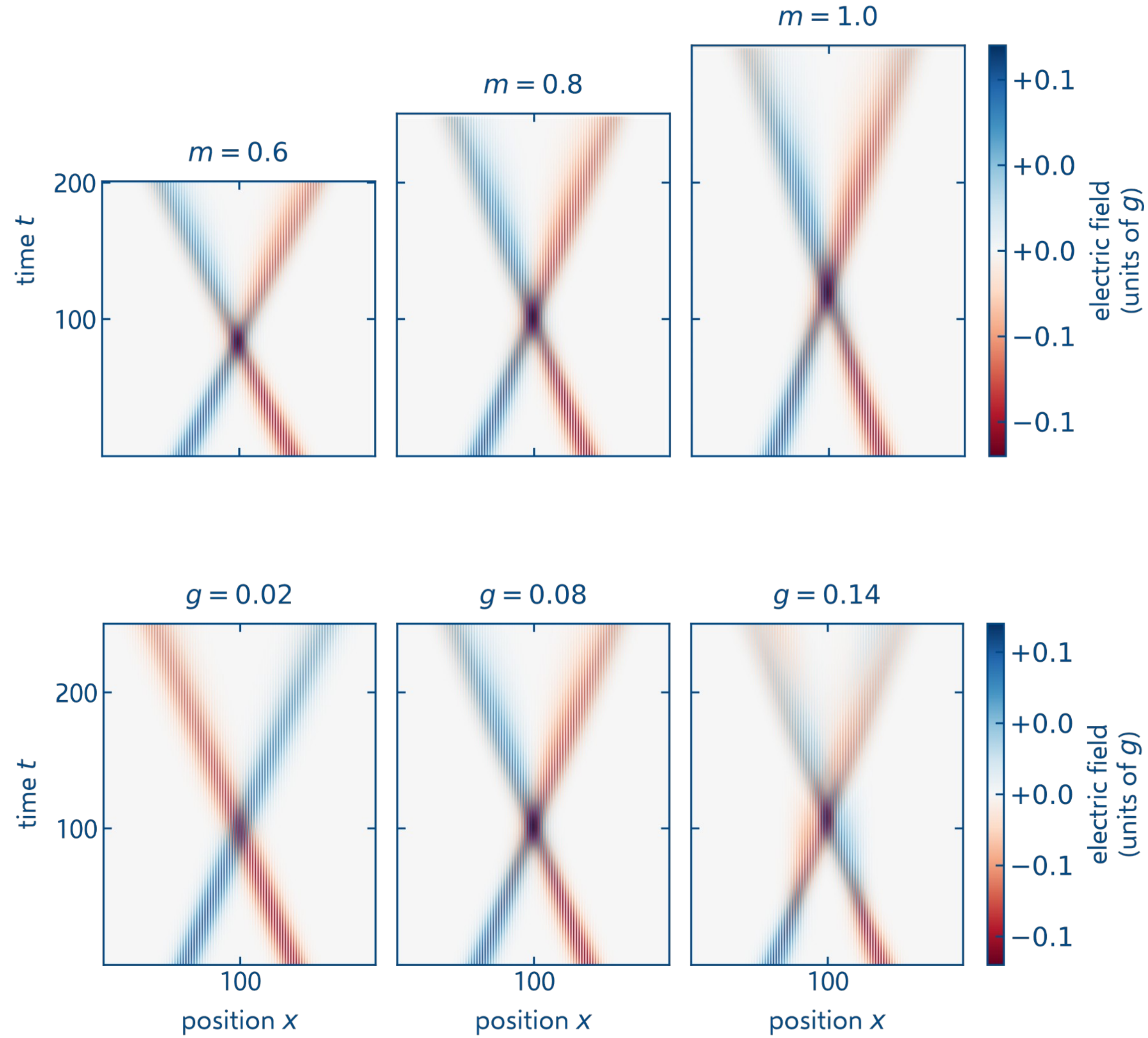
time evolution via TEBD & observables

Phys. Rev. D **104**, 114501 (2021)

Tensor Networks – Scattering dynamics

Mesons with opposite momenta and internal electric fields





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Tensor Networks – Scattering dynamics

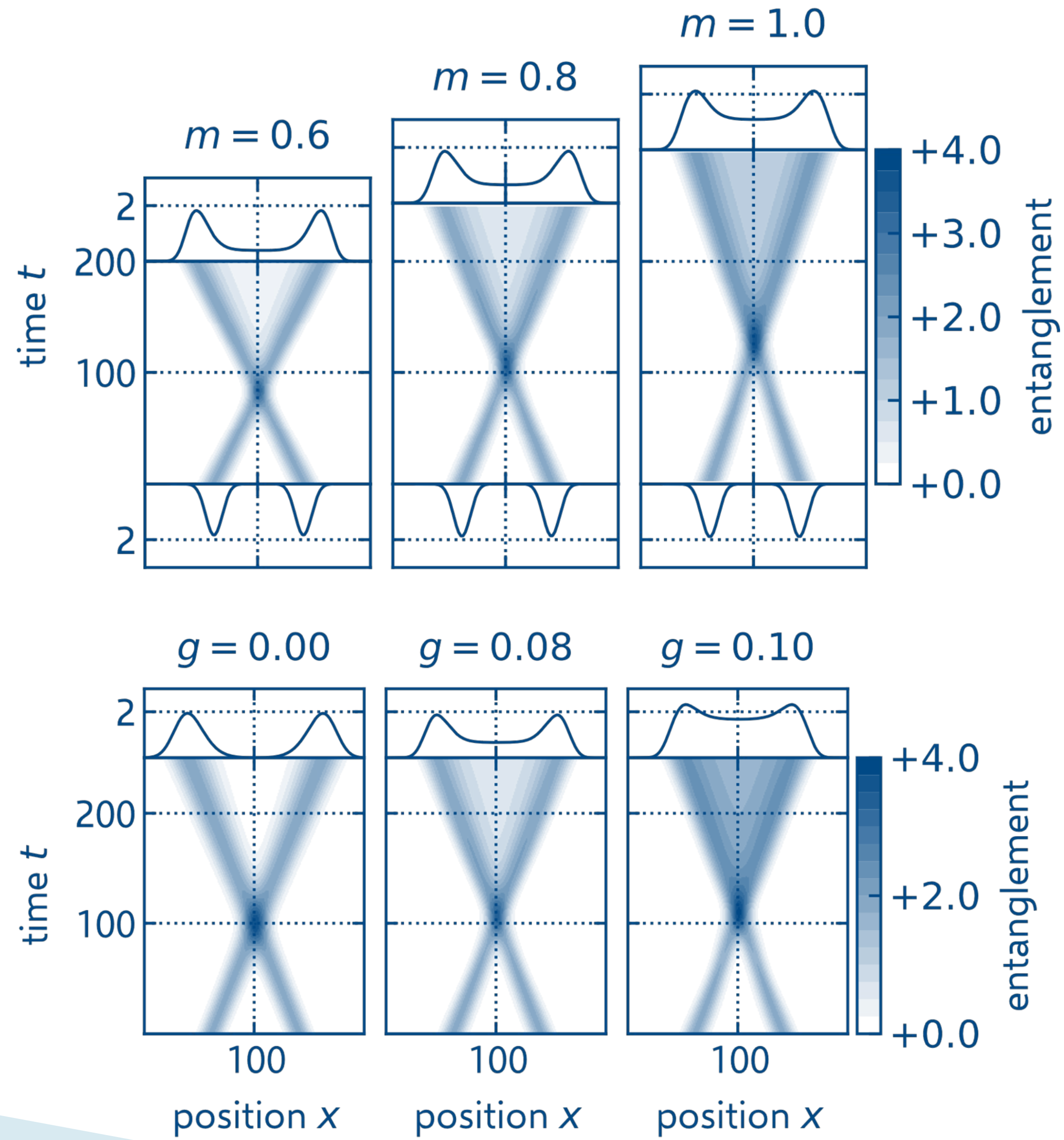
Vacuum

+

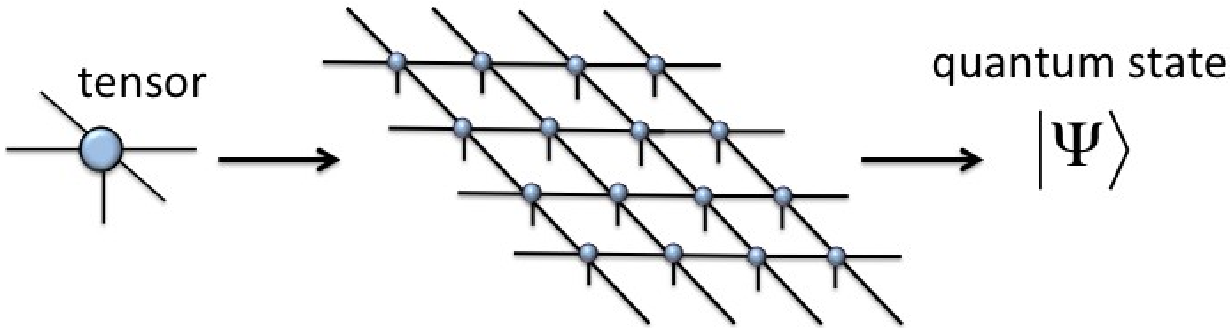
wave packets
intra-particle

+

interactions
intra-particle



Tensor Networks



The wave function is described by a network of interconnected tensors.

The network pattern represents directly the amount of entaglement of the state.

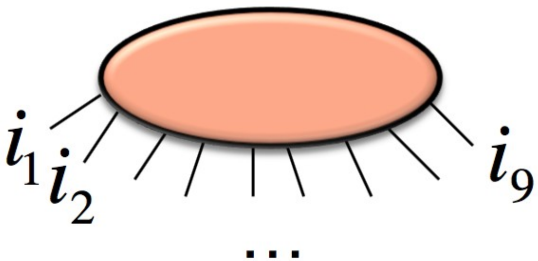
R. Orus, Annals of Physics 349 (2014) 117-158

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d-level systems

Tensor (multidimensional array of complex numbers)

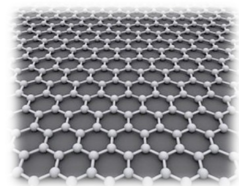
$C_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$

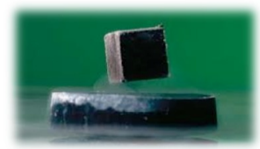


representation, exponentially large in the system size. Inefficient.

System of N spins 1/2

Everyday material, N ~ Avogadro number ~ $O(10^{23})$

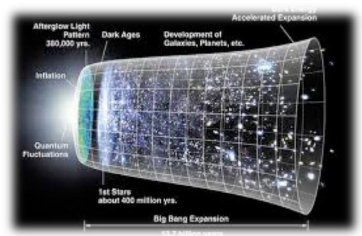




Number of basis states in the Hilbert space ~ $O(10^{10^{23}})$

Compare to…

Number atoms in the observable universe ~ $O(10^{80})$



$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d-level systems

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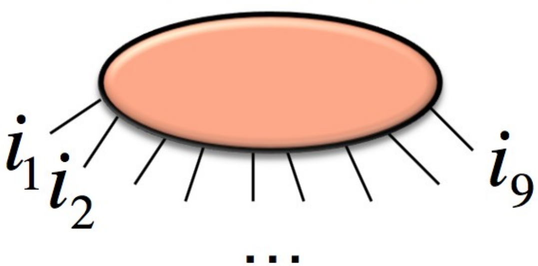
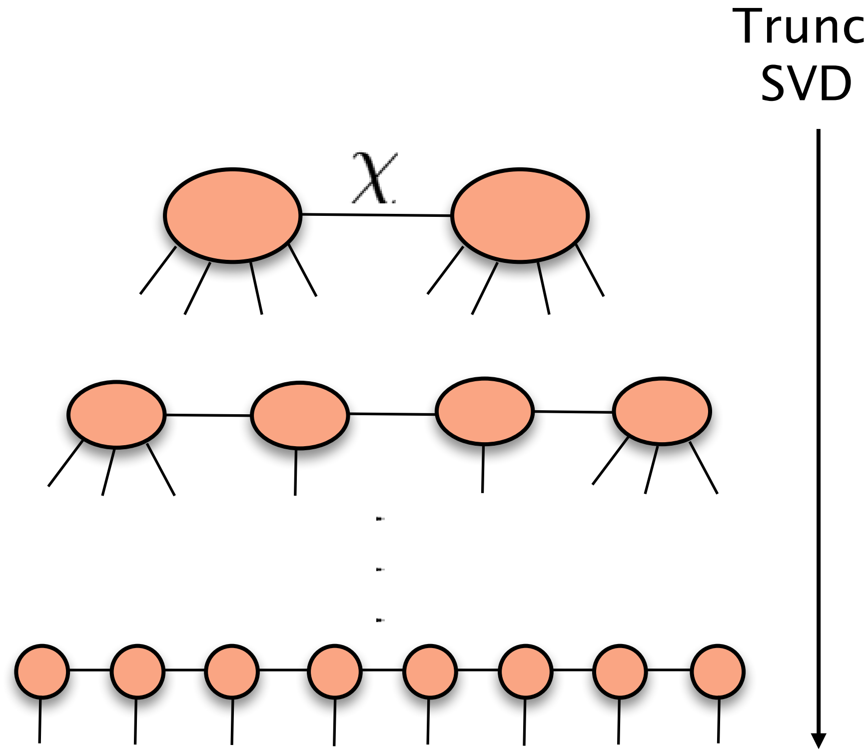


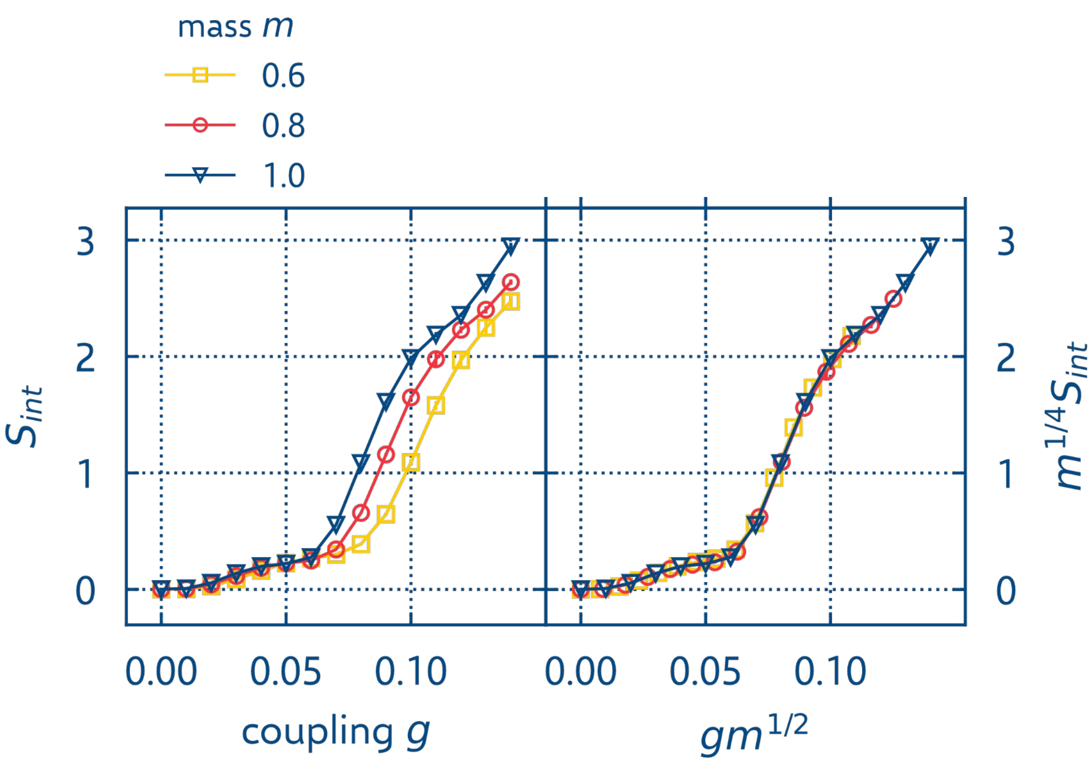


representation, exponentially large in the system size. Inefficient.

$N\chi^2 d$

$$|\psi\rangle = \sum_{\{s_i\},\{\alpha_i\}} A^{(s_1)}_{\alpha_1} A^{(s_2)}_{\alpha_1,\alpha_2} \ldots A^{(s_N)}_{\alpha_{N-1}} |s_1, s_2, \ldots, s_N\rangle$$

Tensor Networks – Scattering dynamics



scaling relation

is independent

two regimes

appearance of new effective d.o.f.

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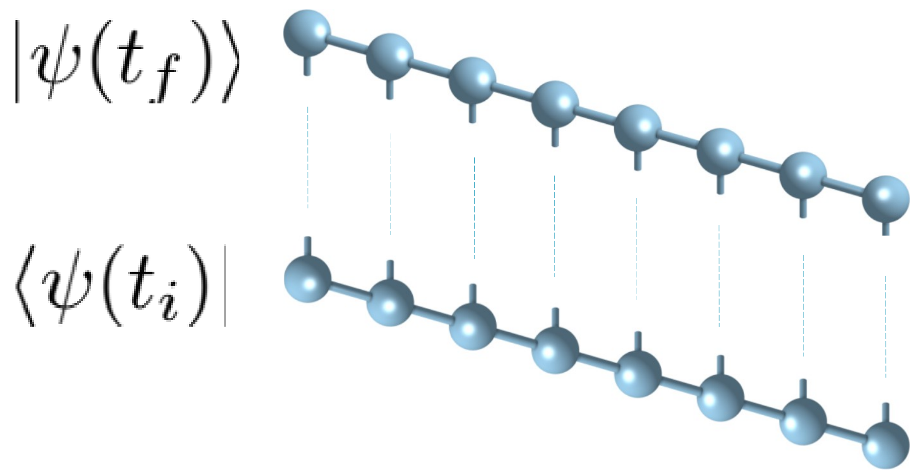
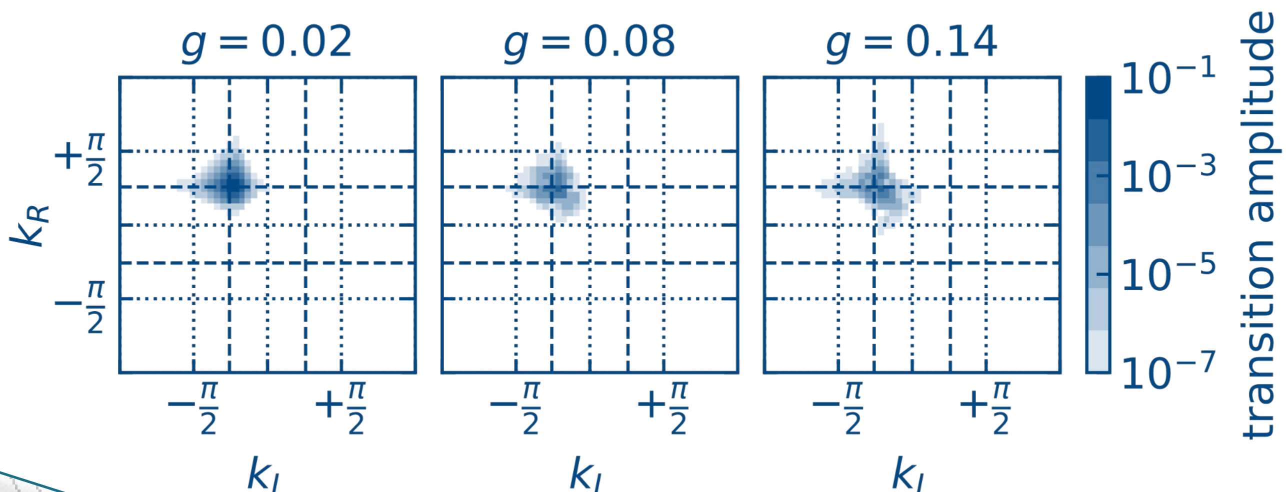
Tensor Networks – Scattering dynamics

overlap of final state with
pair of meson wave packet





~ S-matrix elements

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