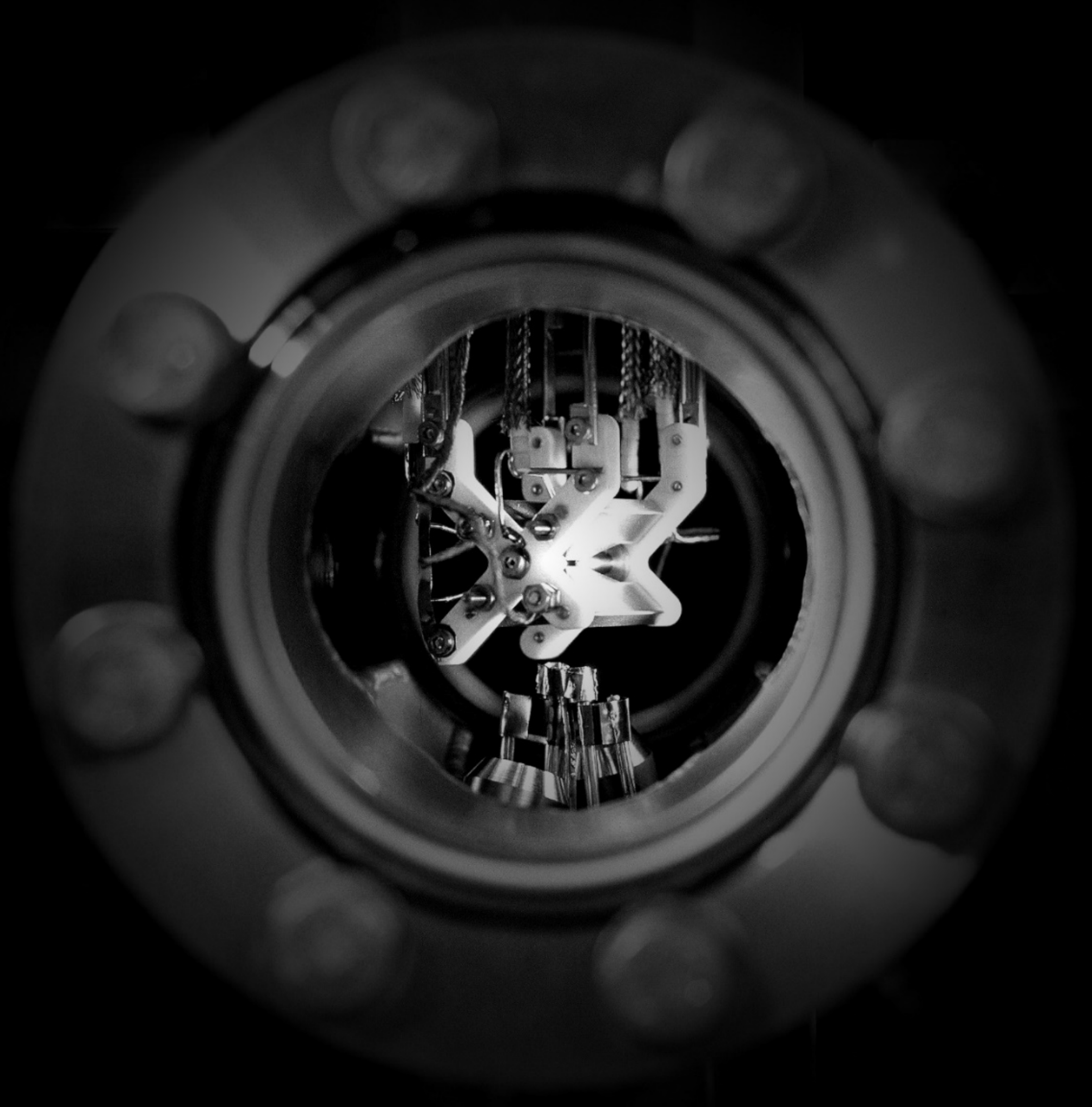
Simulating Lattice Gauge Theories with Trapped Ions



Martin Ringbauer
University of Innsbruck

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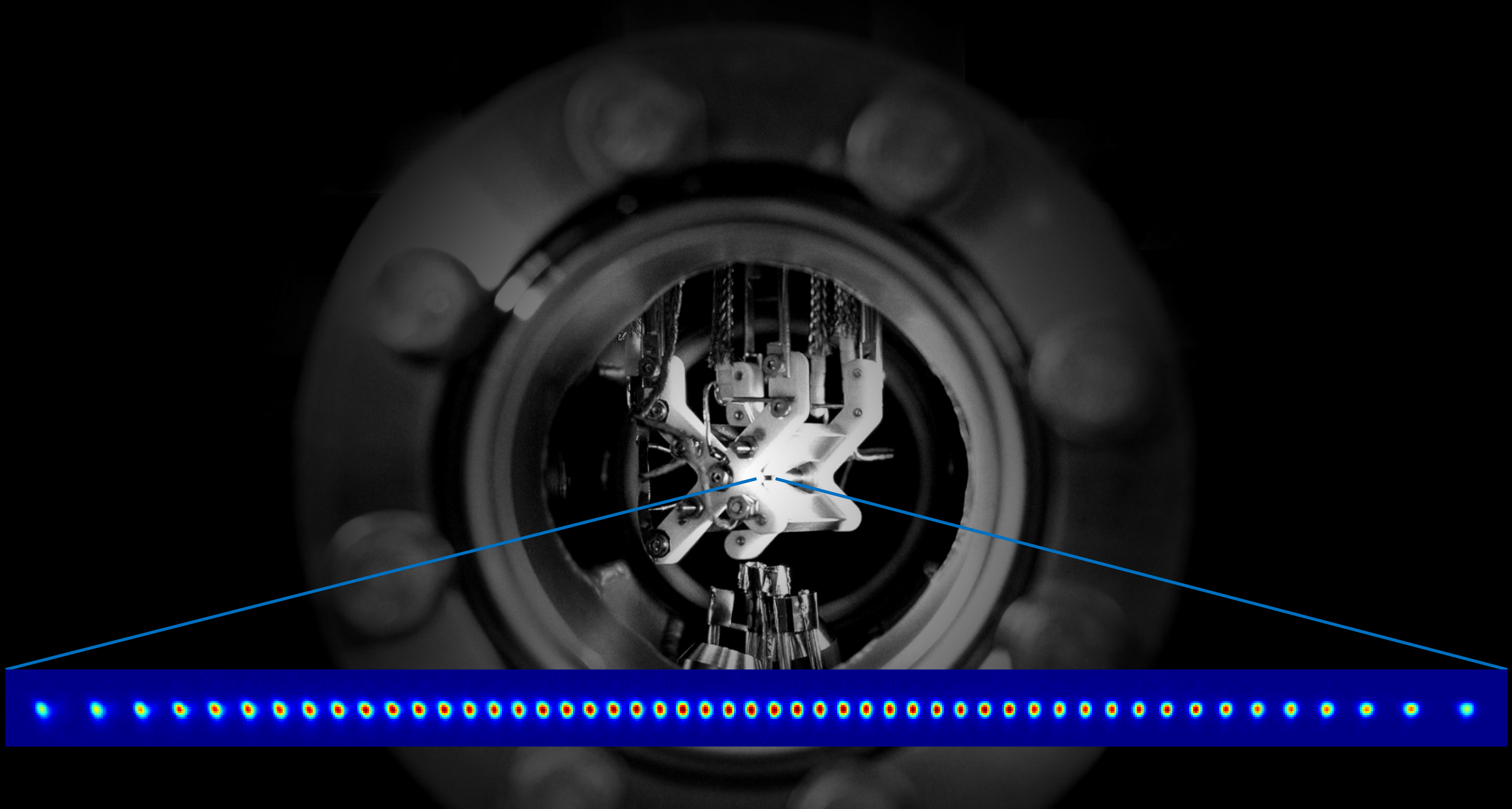

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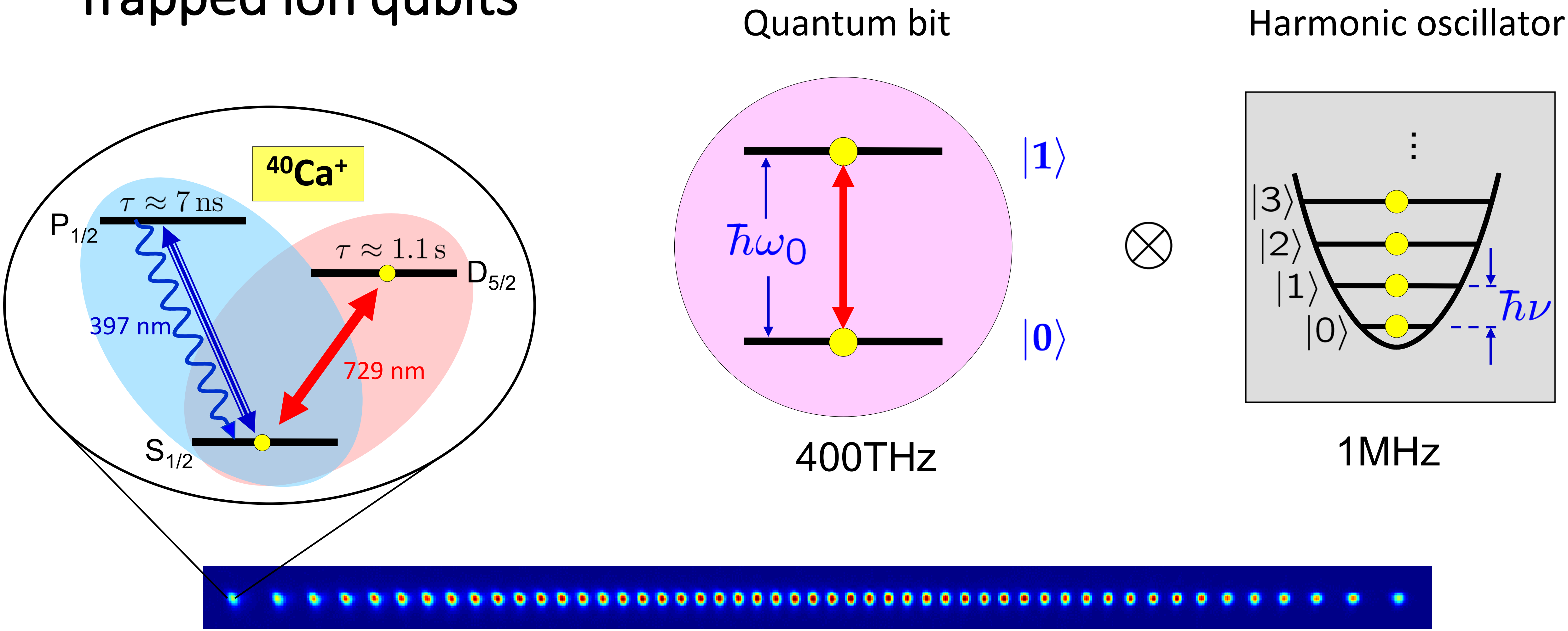
Trapped ion qubits



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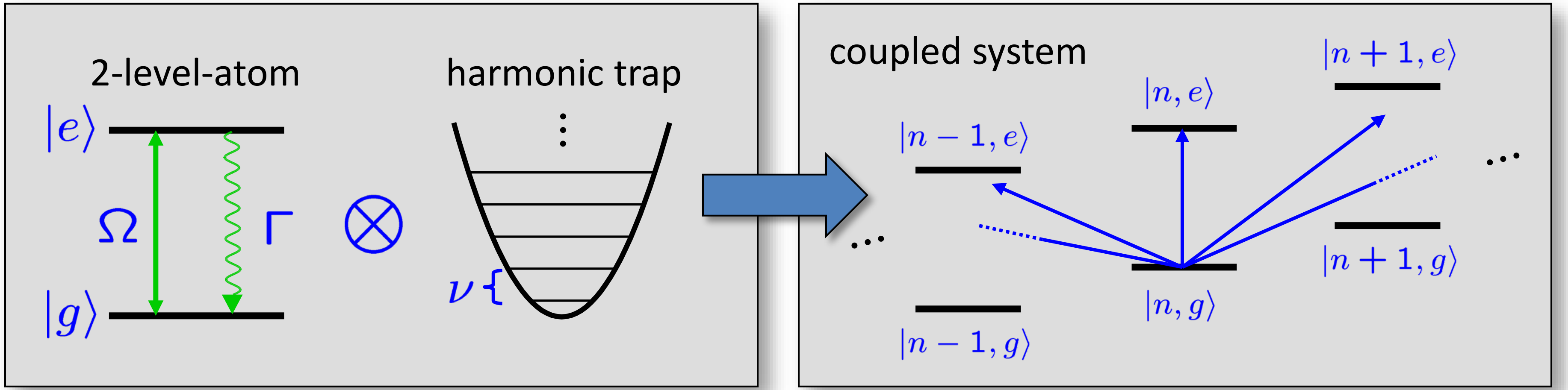
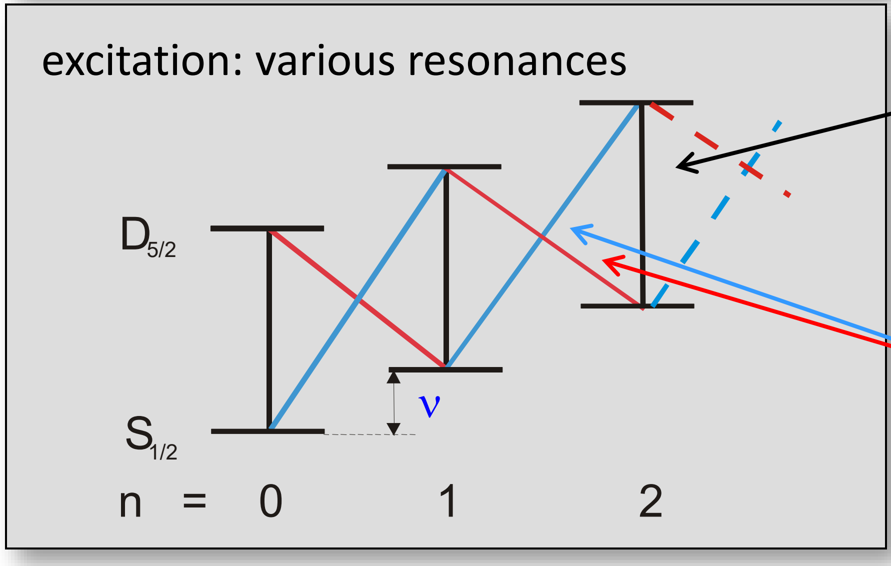
Quantum state manipulation: Carrier and Sidebands





Carier:
manipulate qubit
→ internal superpositions

Sidebands:
manipulate motion and qubit
→ create entanglement

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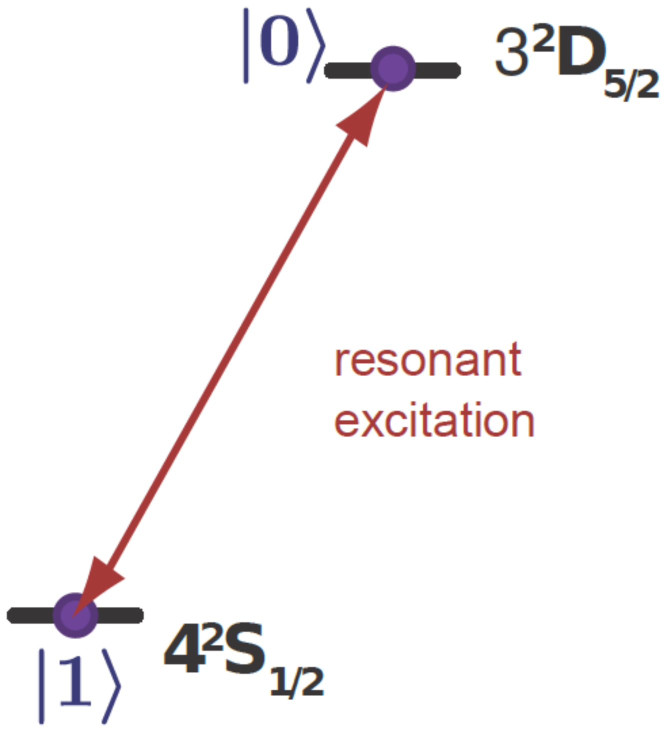
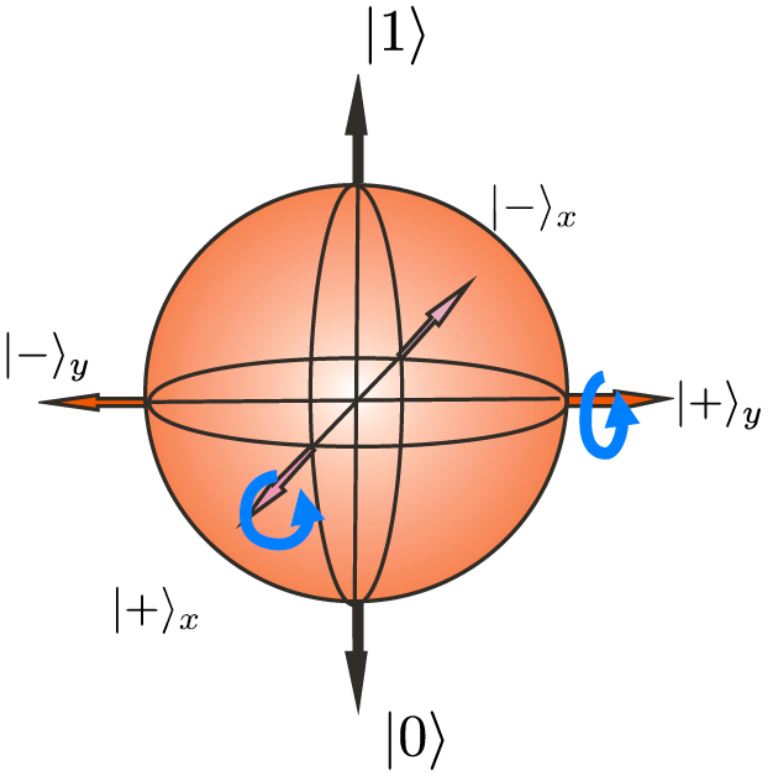
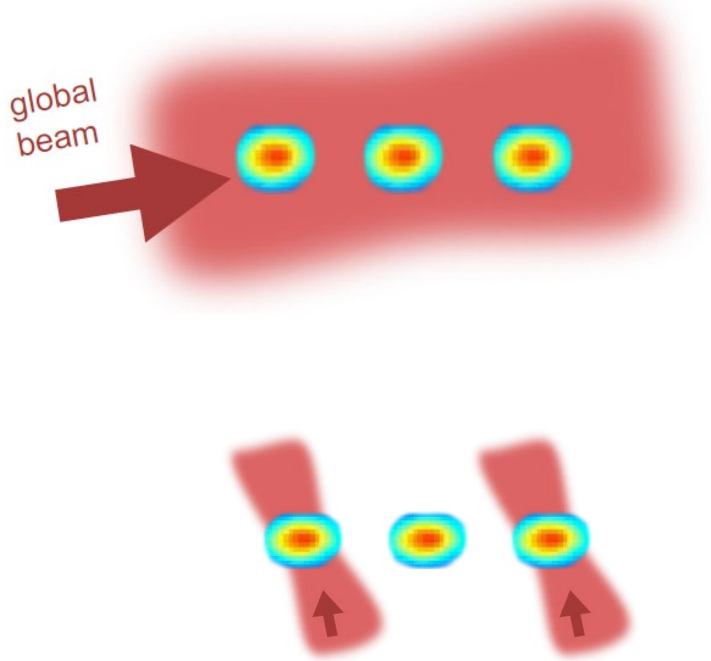
P. Schindler, at al., New. J. Phys. 15, 123012 (2013)

Resonant Operations



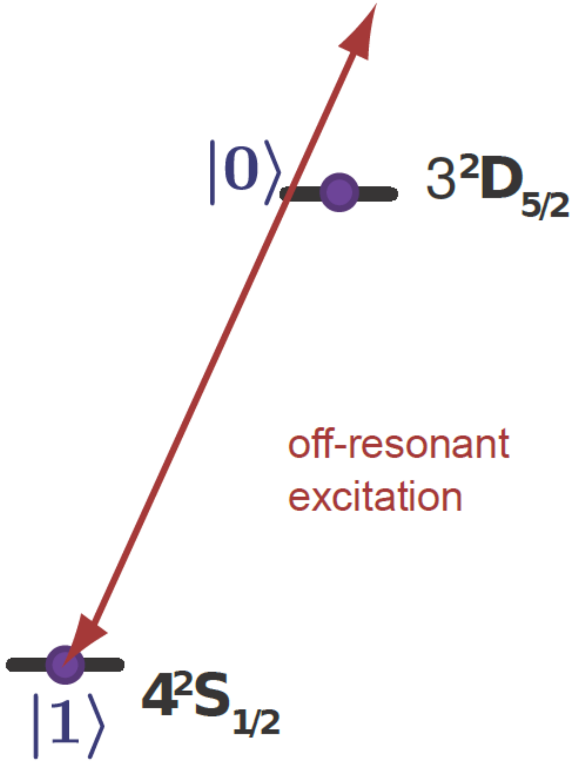


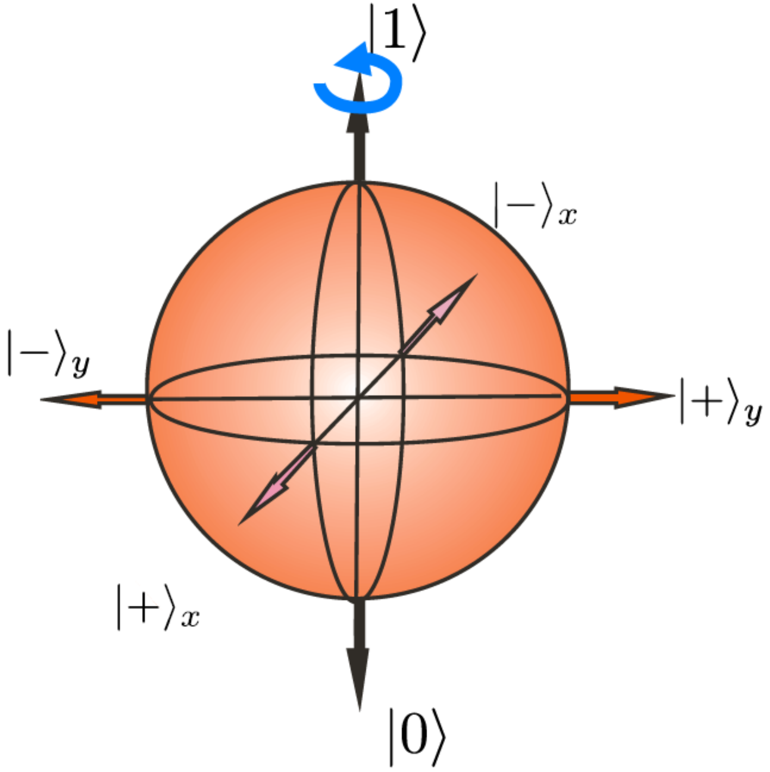


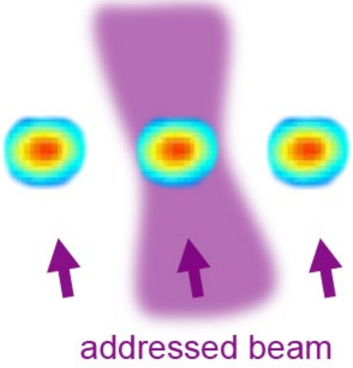
$$R(\theta,\phi) = e^{-i\theta/2(\sigma_x \cos\phi + \sigma_y \sin\phi)}$$

P. Schindler, at al., New. J. Phys. 15, 123012 (2013)

Off-resonant Operations







$$R_Z(\theta) = e^{-i\theta/2\sigma_z}$$

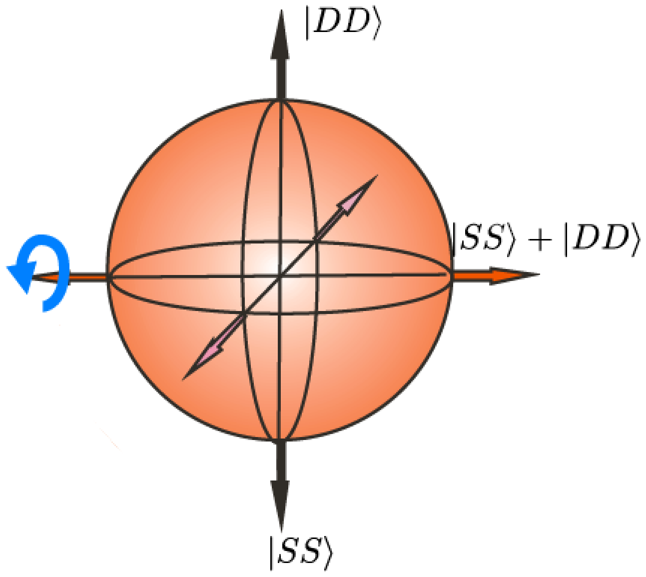
P. Schindler, at al., New. J. Phys. 15, 123012 (2013)

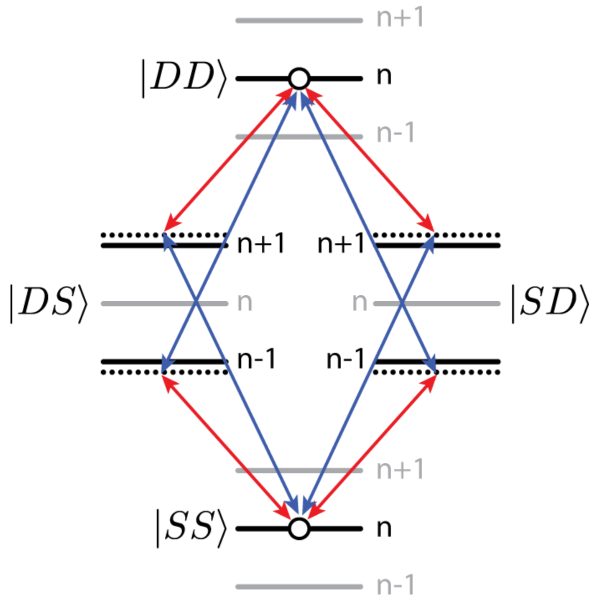
Mølmer-Sørensen Entangling Operation

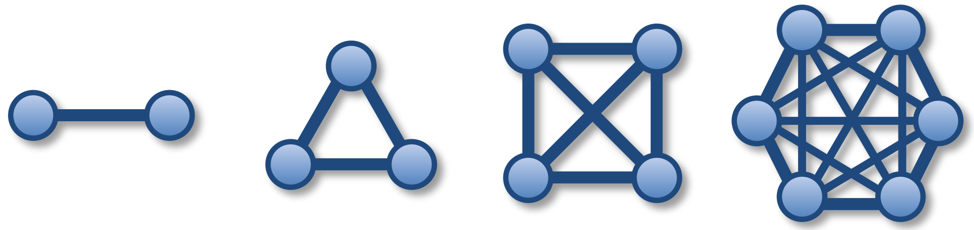
Works for any number of qubits

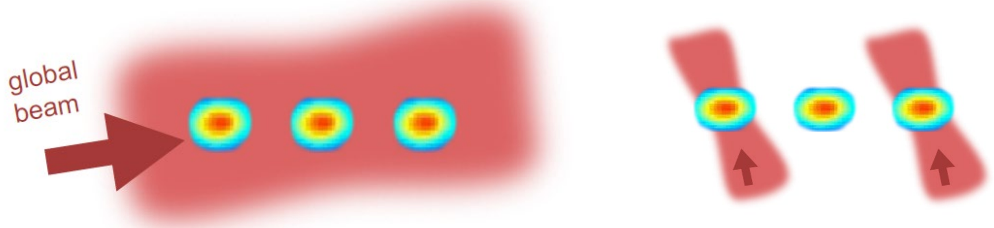
Effective infinite range 2-body interaction.

Enables arbitrary coupling graph









$$MS(\theta) \propto e^{-i\theta\sigma_x \otimes \sigma_x}$$

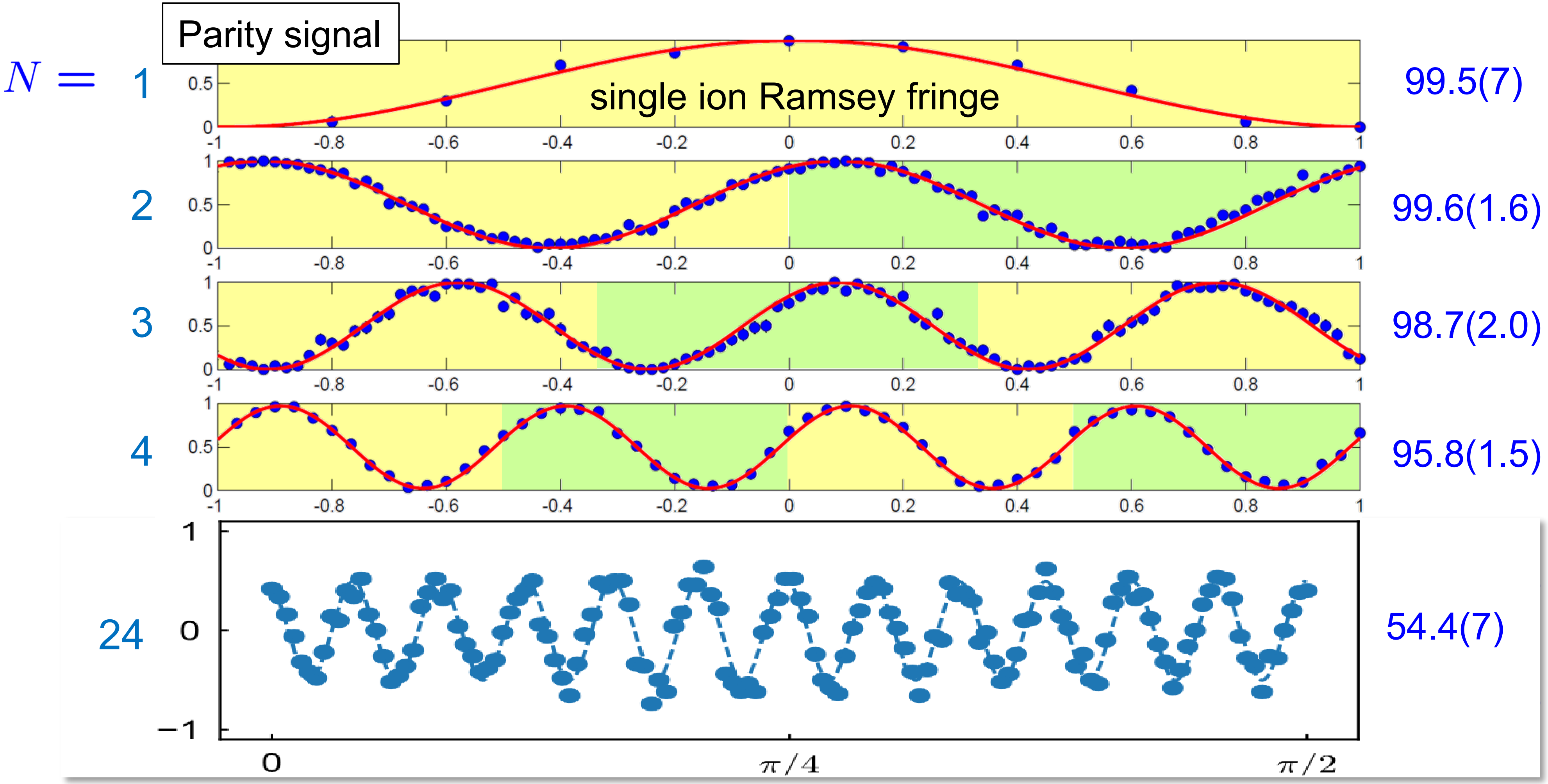
T. Monz et al., *PRL*. **106**, 130506 (2011).

K. Mølmer and A. Sørensen, PRL 82, 1835 (1999).

GHZ state fidelity vs register size

$$\Psi = \frac{1}{\sqrt{2}}(|SS\ldots S\rangle + |DD\ldots D\rangle)$$

Fidelity (%)

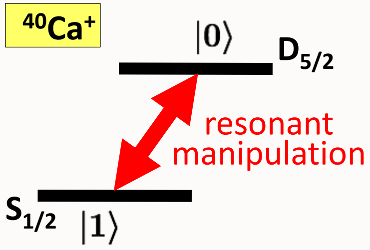


T. Monz et al., *PRL.* **106**, 130506 (2011).
V. Pogorelov et al., *PRX Quantum* **2**, 020343 (2021).

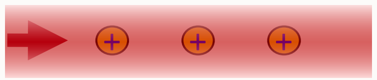
Quantum computing with global and local operations

Global

Collective Local Operations



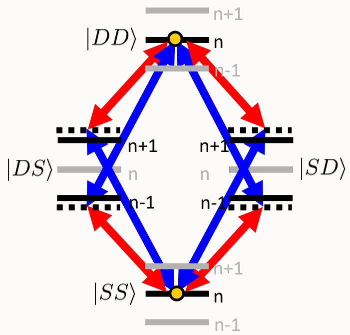
$S_{x,y}(\theta)$



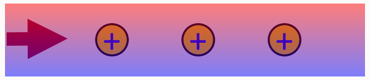
τ = 20µs

F > 99.9%

Global Mølmer- Sørensen entangling gate



$S^2_{x,y}(\theta)$

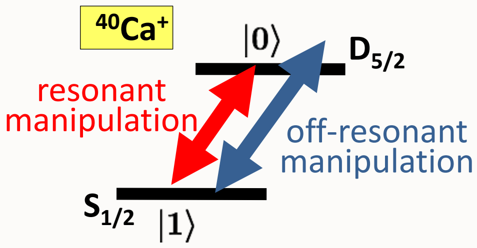


τ = 50µs

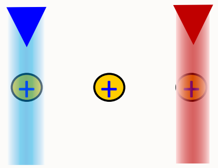
F_2 > 99%

Local

Individual (and parallel) local operations



$S_z(\theta)$ $S_{x,y}(\theta)$

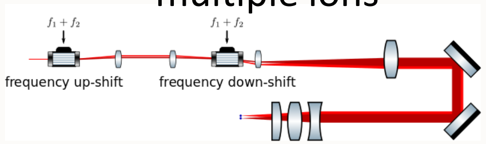


τ = 20µs

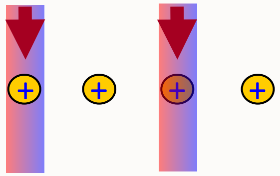
F > 99.9%

Local Mølmer- Sørensen entangling gate

Simultaneous addressing of multiple ions



$S^2_{x,y}(\theta)$

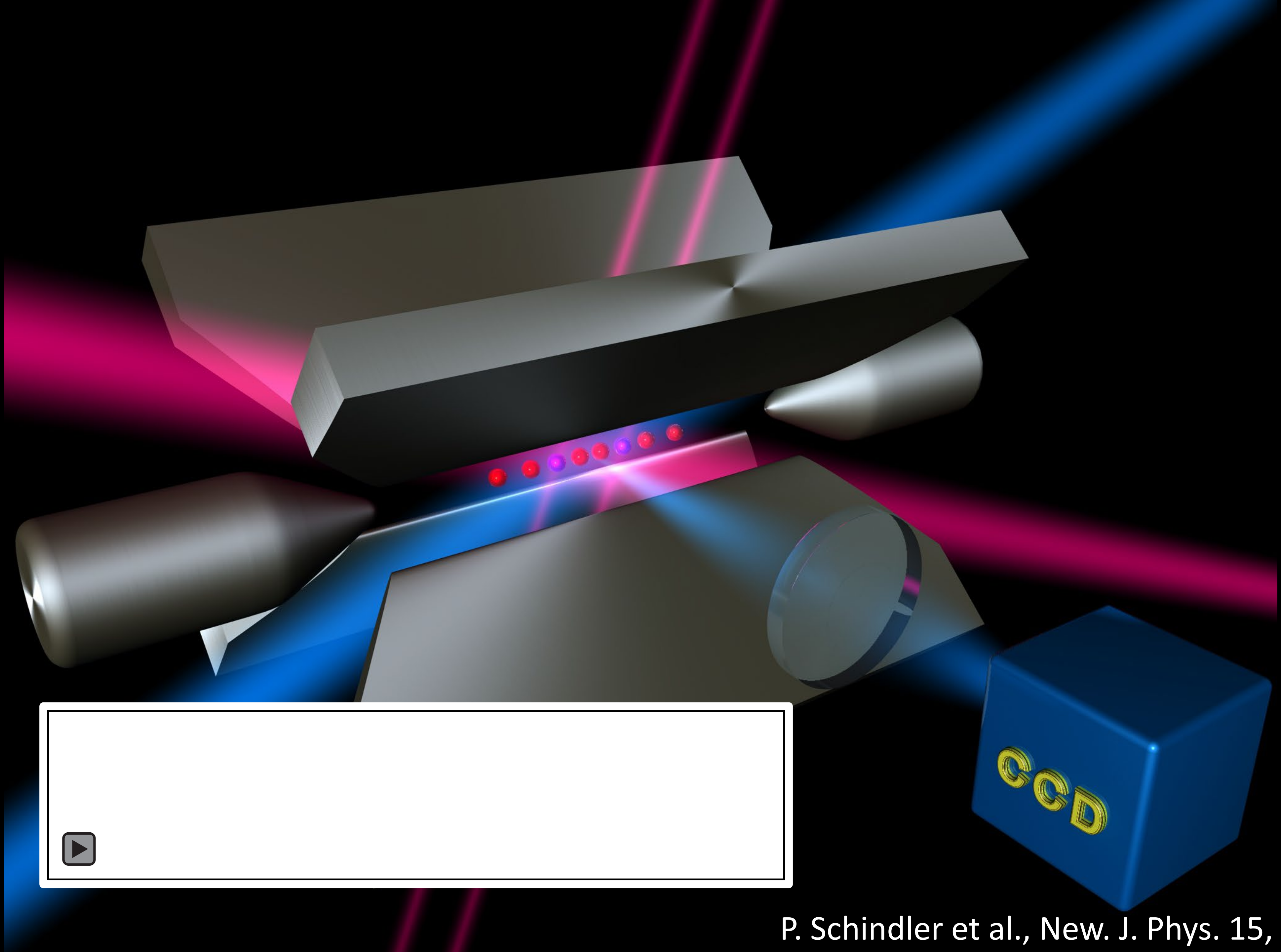


τ = 100µs

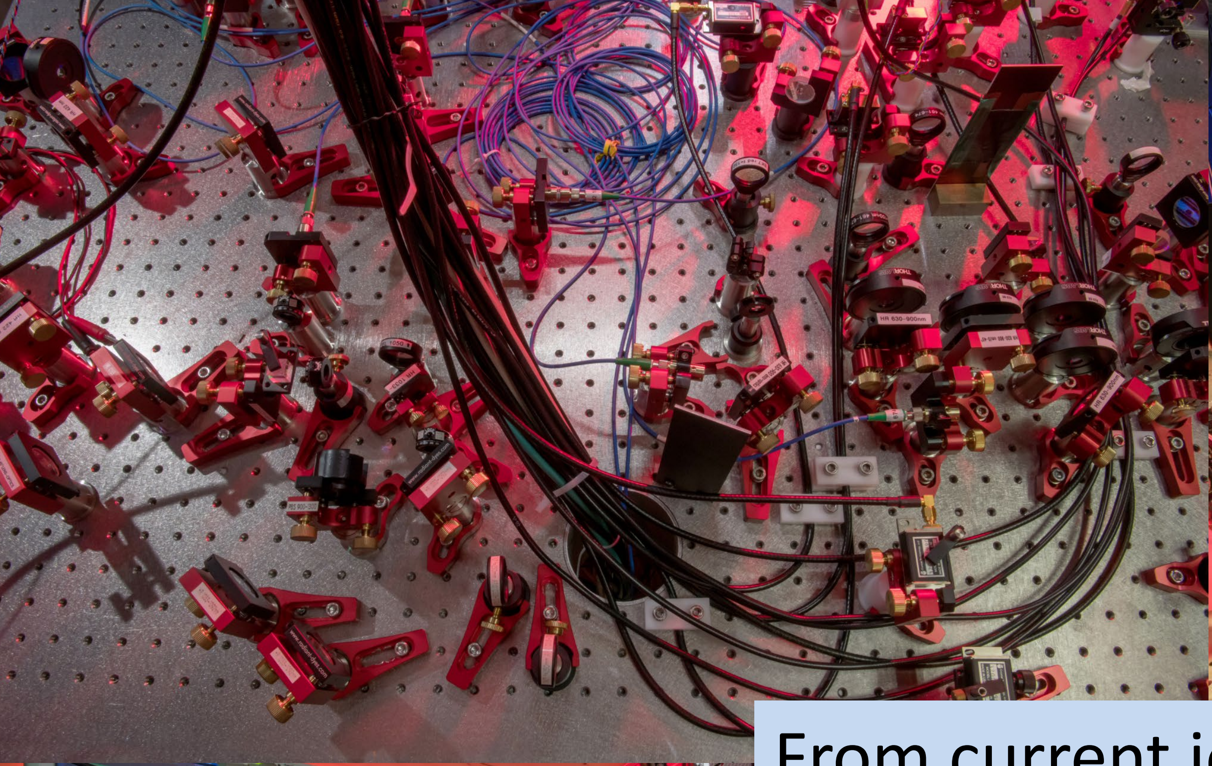
F_2 > 99%

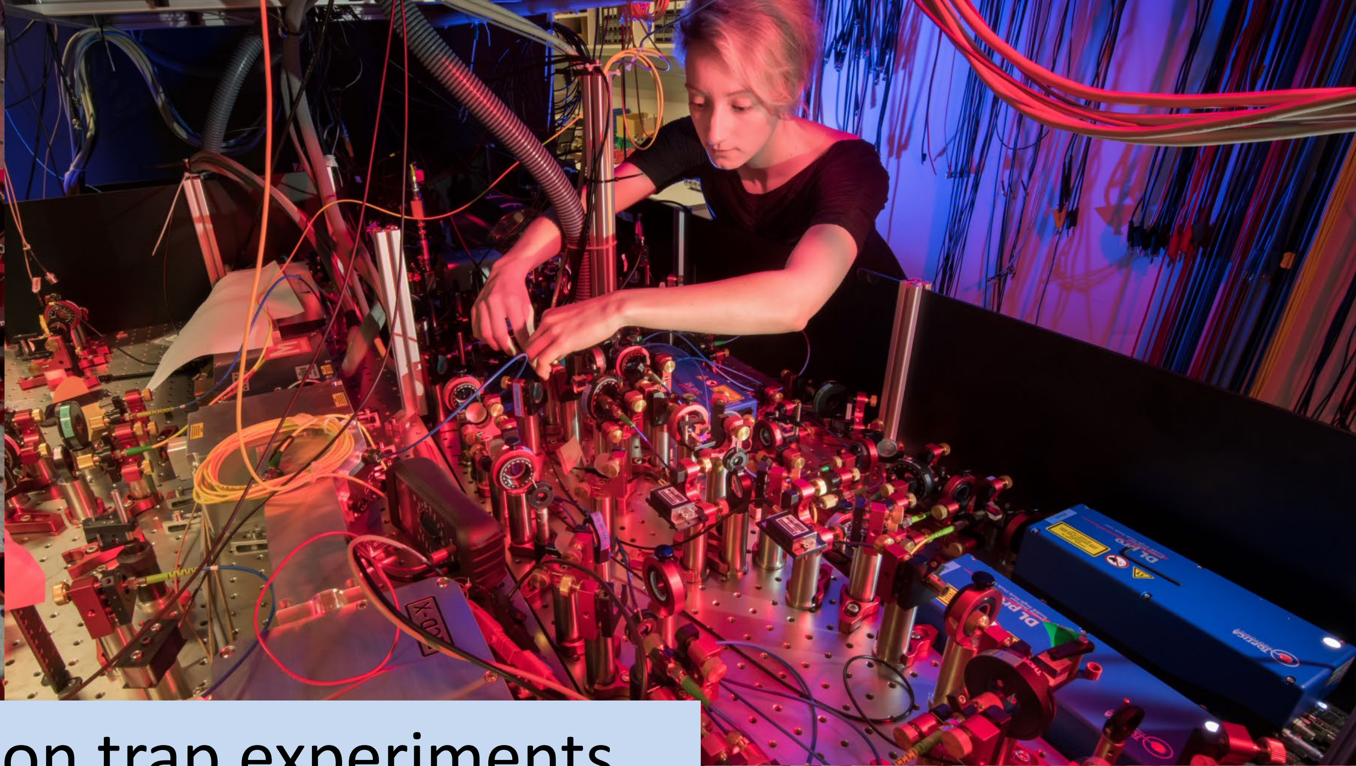
P. Schindler, at al., New. J. Phys. 15, 123012 (2013)

M. Ringbauer, et al., Nature Physics 18, 1053 (2022)

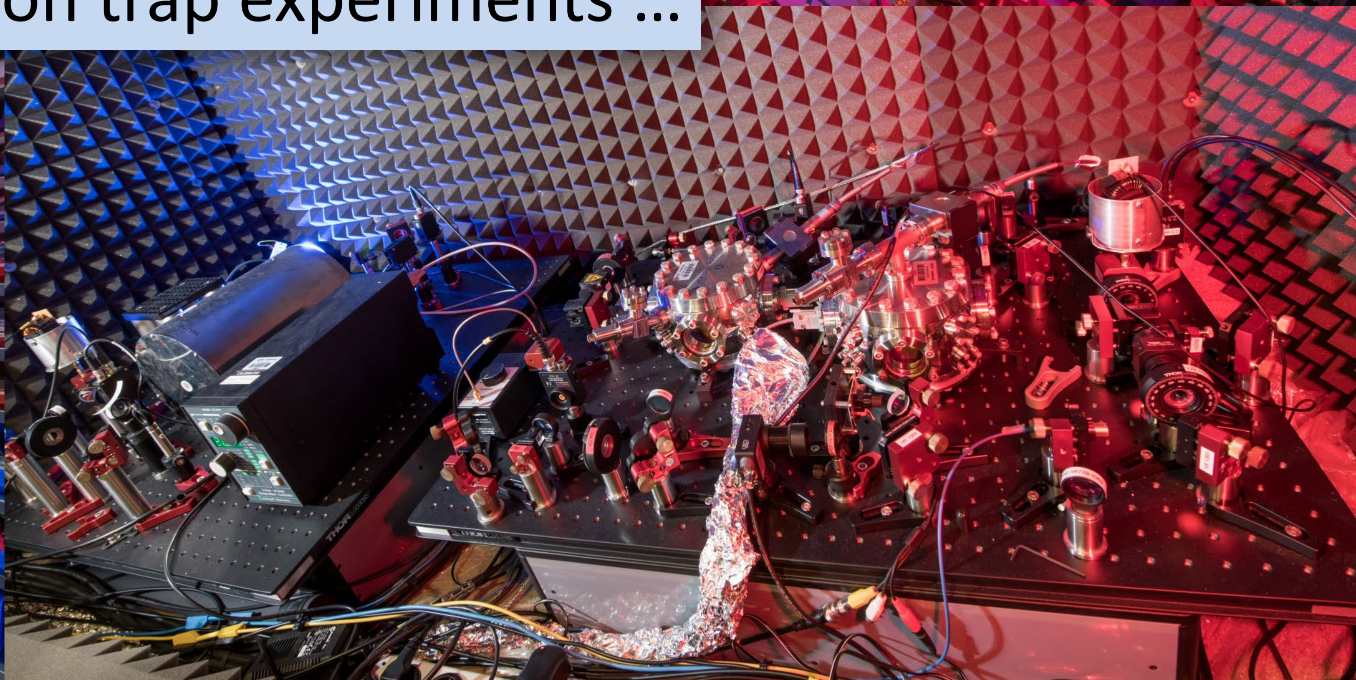


P. Schindler et al., New. J. Phys. 15, 123012 (2013)

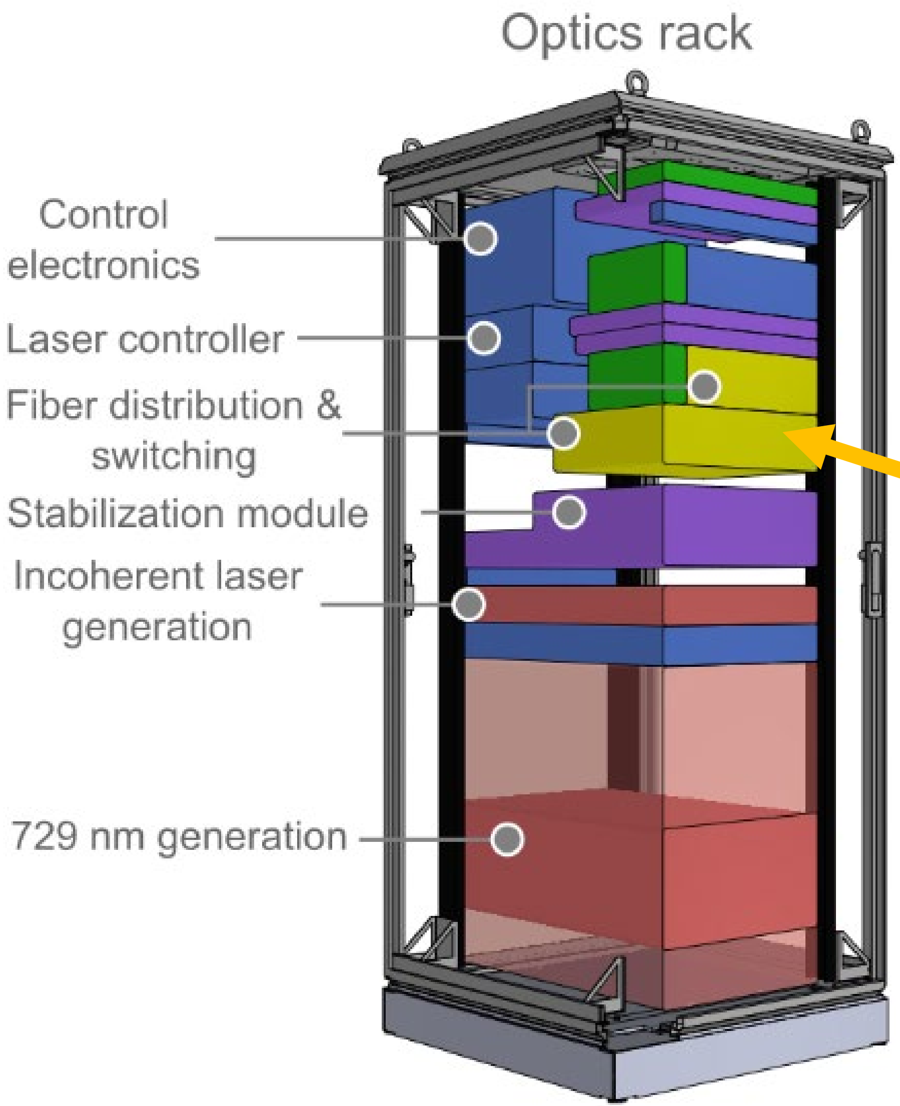


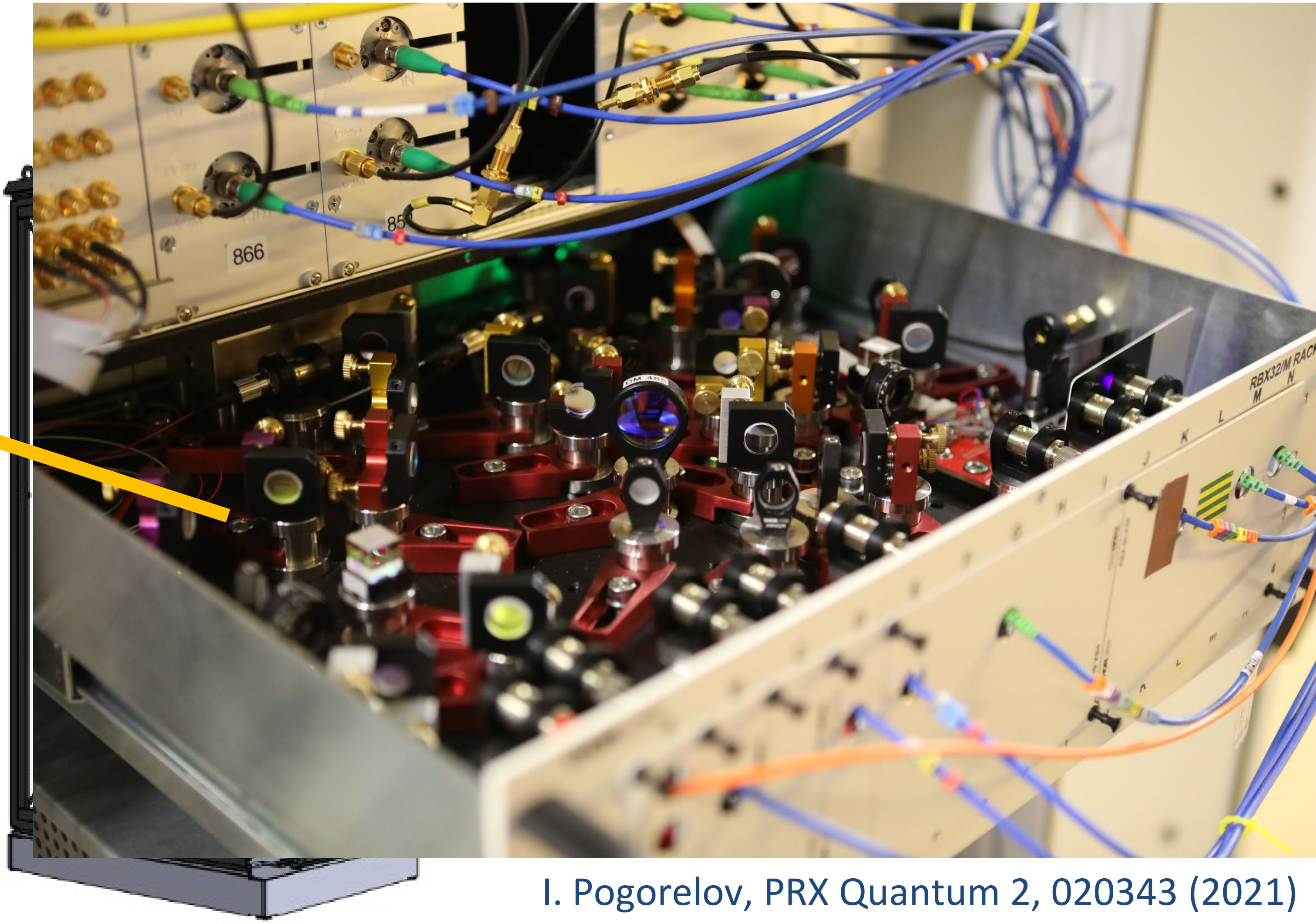


From current ion trap experiments …



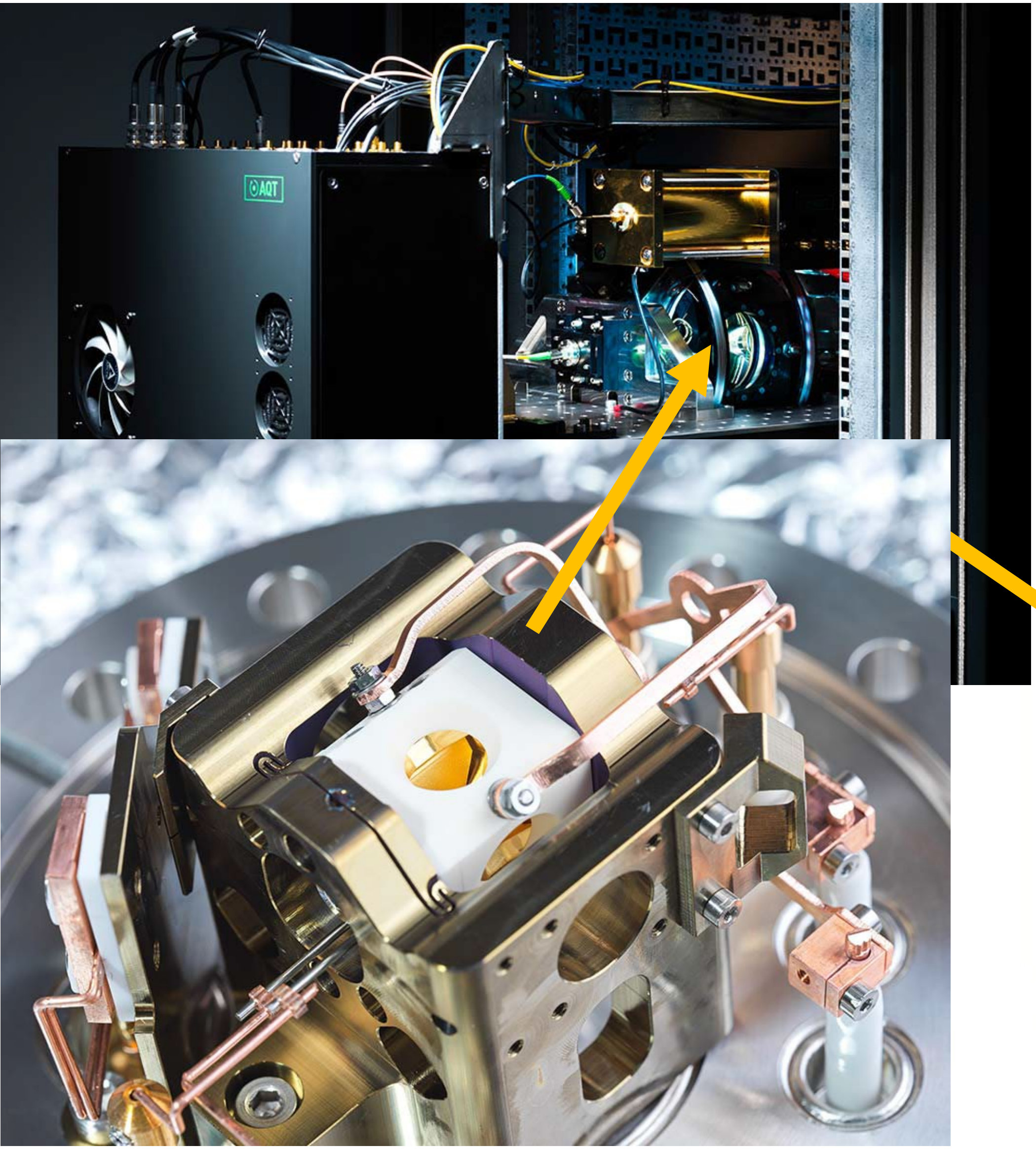
To a Compact, Modular System

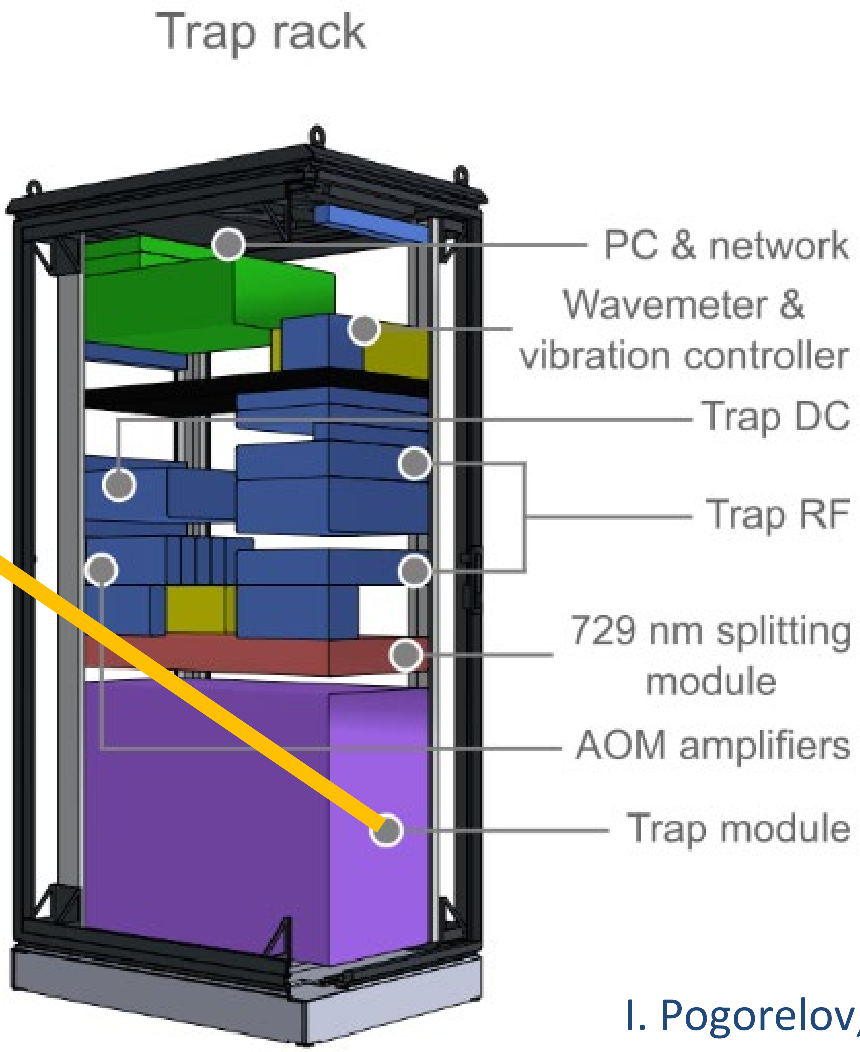




I. Pogorelov, PRX Quantum 2, 020343 (2021)

To a Compact, Modular System





I. Pogorelov, PRX Quantum 2, 020343 (2021)

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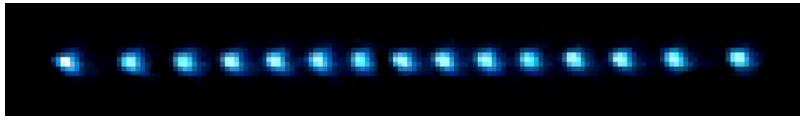


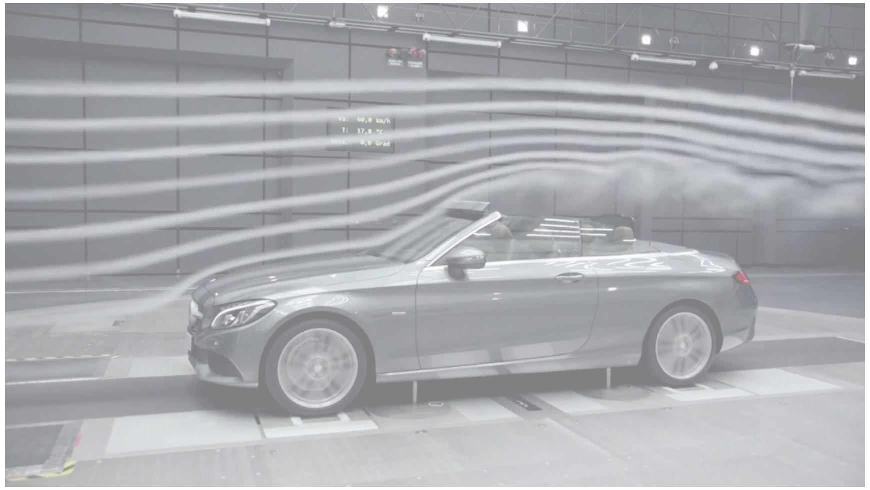
Performance

- 50 ion addressable
- Magnetic shielding
- Optical qubit T2 > 90ms
- Ground state qubit T2 > 18ms
- Local gate duration 10µs
- Entangling gate duration 100µs

- Entangling gate error < 2.5%
- Local gate error < 0.5%

I. Pogorelov, PRX Quantum 2, 020343 (2021)

Analog & Digital Quantum Simulation



Analog Simulation:

- Analog evolution
- Mimics the physics
- Special-purpose control
- Hard to verify
- Error mitigation
- > 50 qubits

Digital Simulation/Computing:

- Discrete evolution
- Mimics the "math/model"
- Universal computation
- Bounds on accuracy
- Quantum error correction
- > 10 qubits

Digital Simulation – Universal Quantum Simulator

$$H = \sum_k h_k$$ ⟵ model of some local system to be simulated for a time t

1) build each local evolution operator separately, for small time steps $u_k = e^{-ih_k t/n}$

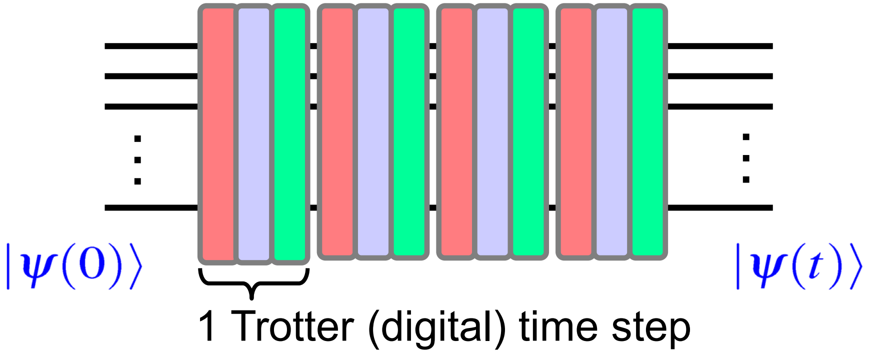
2) approximate global evolution operator using the Trotter approximation

$$U = e^{-iHt} \approx \left(e^{-ih_1 \frac{t}{n}} e^{-ih_2 \frac{t}{n}} e^{-ih_3 \frac{t}{n}} \dots e^{-ih_k \frac{t}{n}} \right)^n$$

"Efficient for local quantum systems"

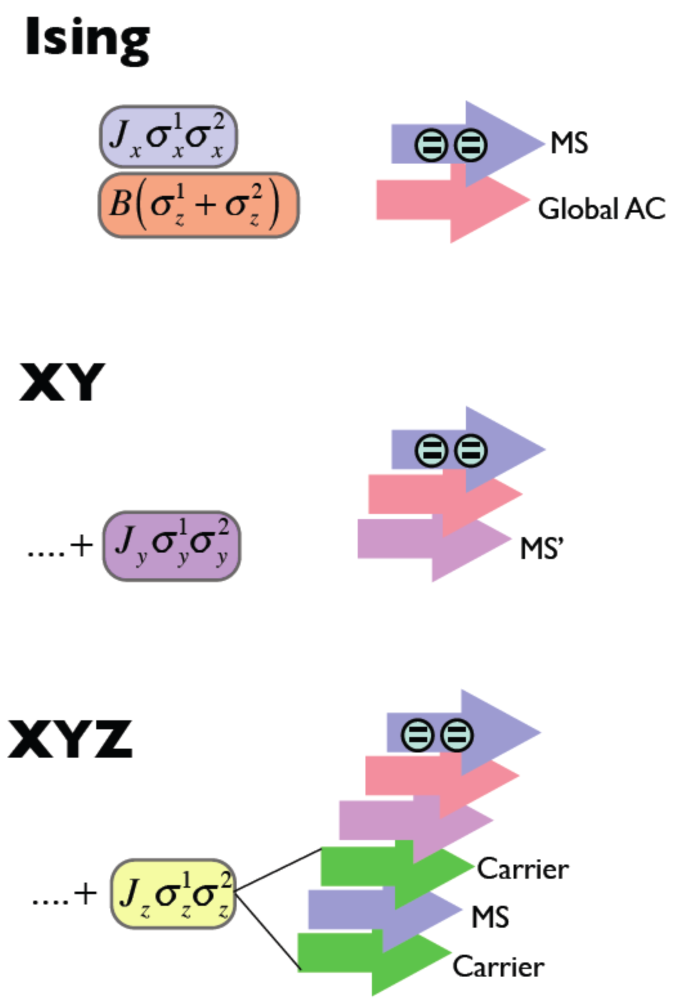
S. Lloyd,
Science **273**, 1073 (1996)

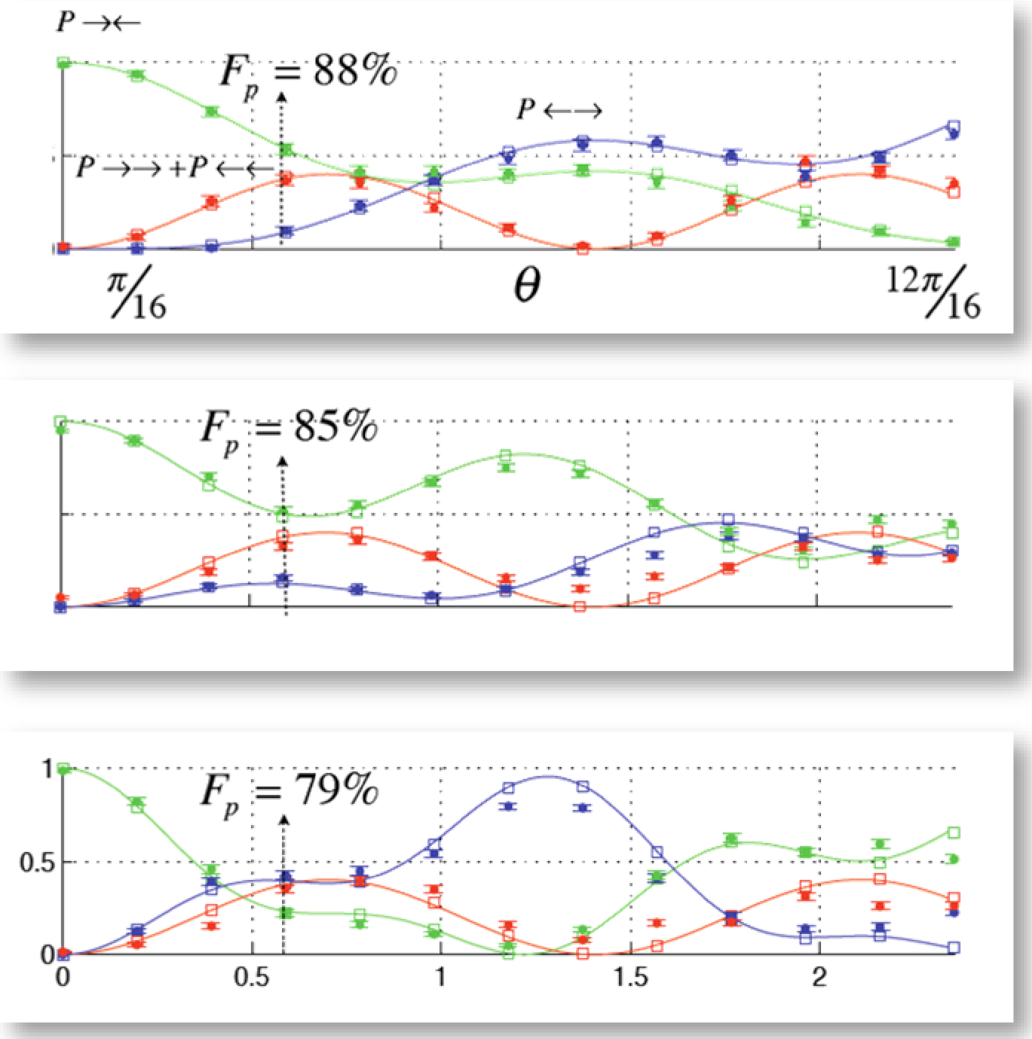
Discretization errors are well behaved
M. Heyl et al, Sci. Adv. **5**, eaau8342 (2019)



R. Blatt, C. Roos, Nat Phys 8, 277 (2012)

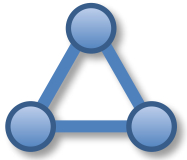
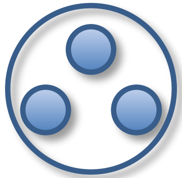
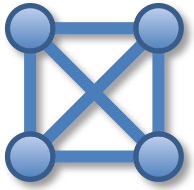
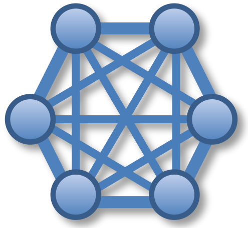
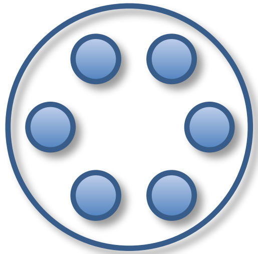
Digital Simulators are flexible





B. Lanyon et al., Science 334, 6052 (2011)

Some Examples

	Ising	XY	XYZ
2-spin simulations	 $J\sigma_x^1\sigma_x^2 + B\sum_{i=1}^{n}\sigma_z^i$	 $\cdots + J_y\sigma_y^1\sigma_y^2$	 $\cdots + J_z\sigma_z^1\sigma_z^2$
	Ising type 1	Ising type 2	n-body
3-spin simulations	 $J\sum_{i\neq j}\sigma_x^i\sigma_x^j + B\sum_{i=1}^{n}\sigma_z^i$	 $J_{12} = J_{23},\ J_{13} = 0$	 $H = \sigma_x^{\otimes n}$
>3-spin simulations			
	4 spins	6 spins	

B. Lanyon et al., Science 334, 6052 (2011)

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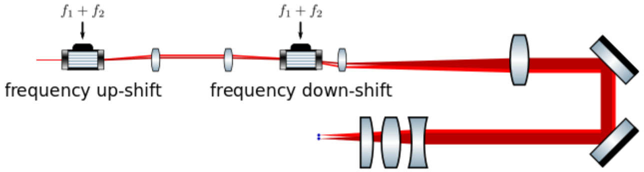
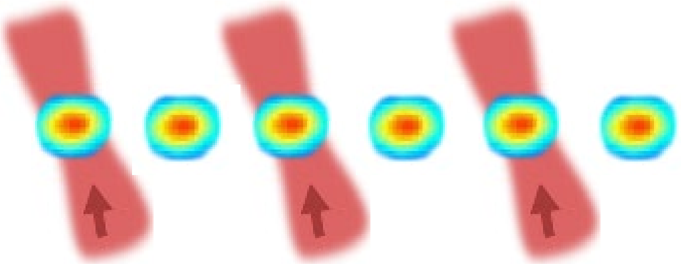
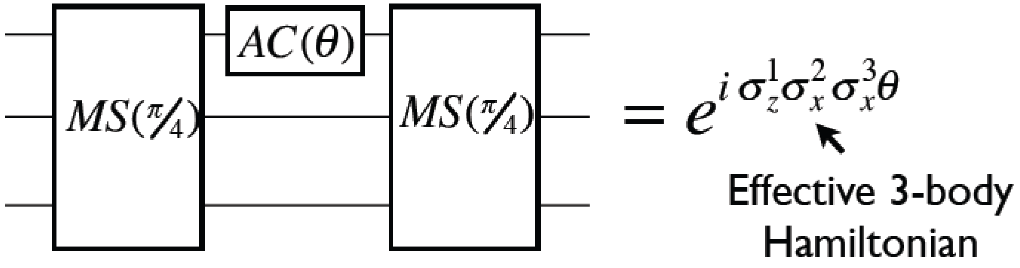
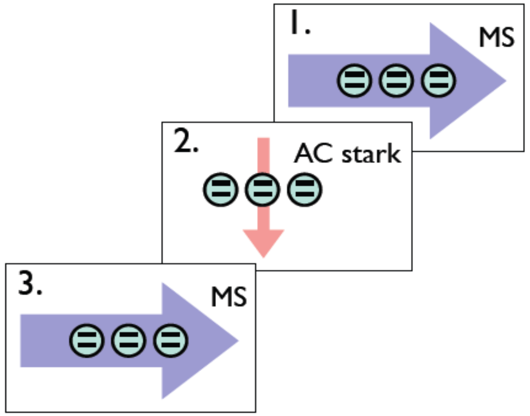
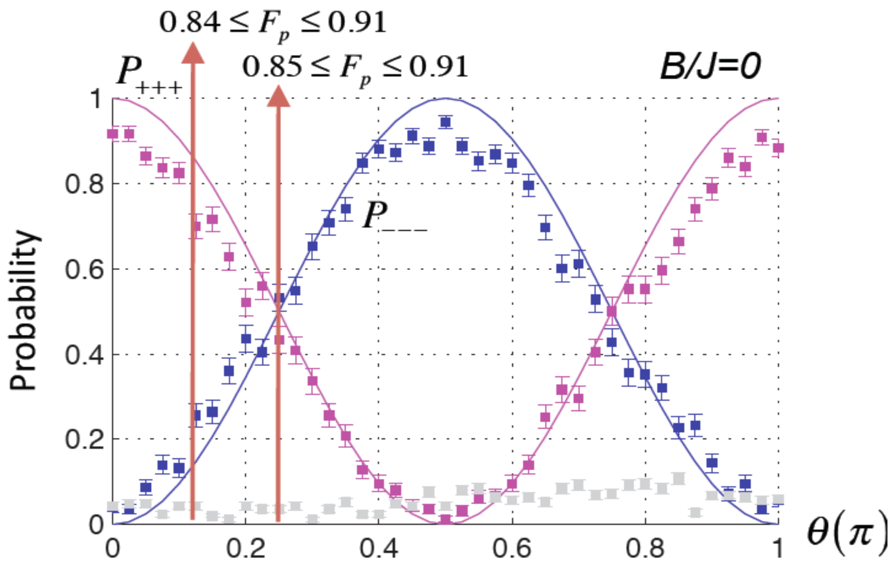
Many-body Interactions





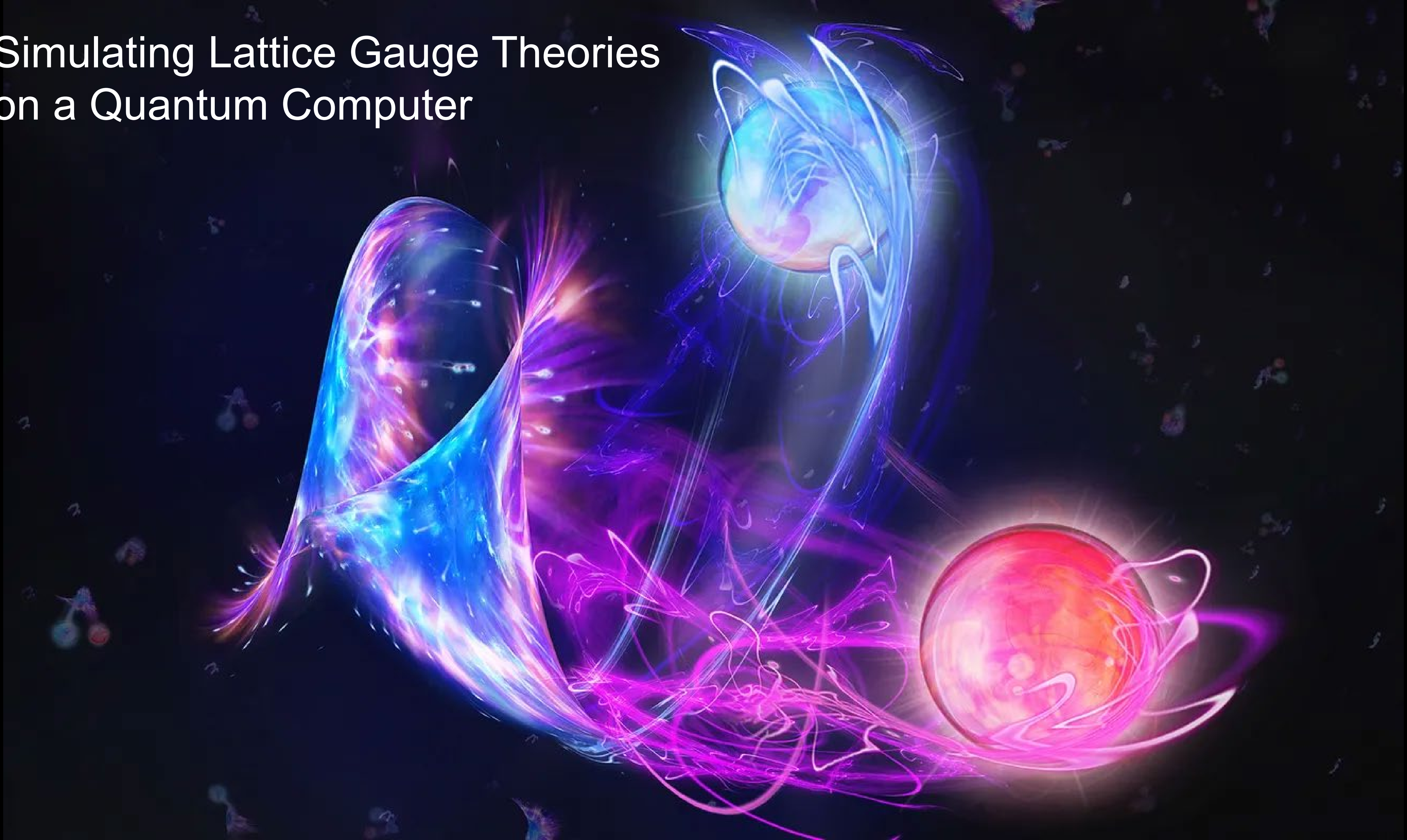






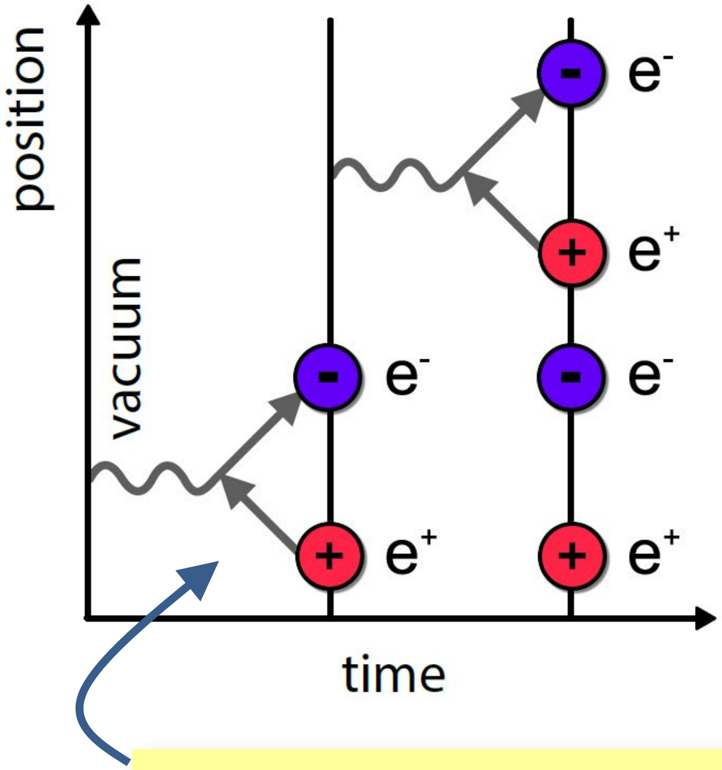
M. Müller et al, NJP 13, 085007 (2011)

Simulating Lattice Gauge Theories on a Quantum Computer

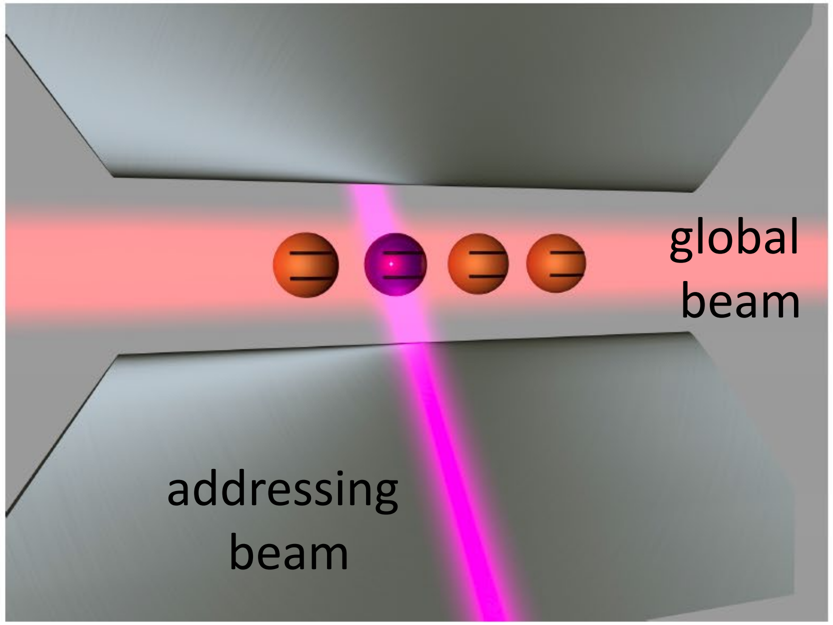


Simulating Lattice Gauge theories

- QED in one dimension on a lattice
- Particles (Fermions) are encoded spins (two-level systems of ions)
- Gauge fields are transformed to long-range interactions
- Time-Evolution is simulated stroboscopically (Trotter)

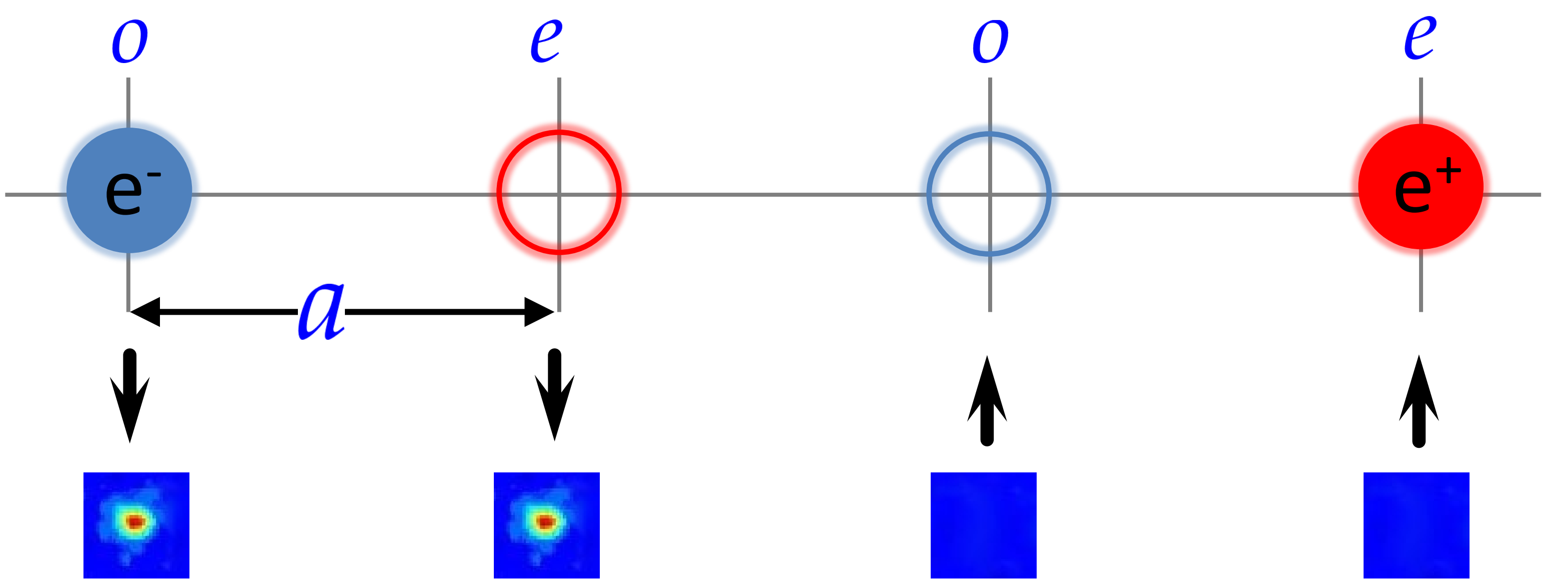


Schwinger-model: pair creation



Encoding Fermions into two-level systems

- Fermions (e⁻, e⁺) and holes are encoded in two-level systems (of ions)
- **Odd(o)** sites: e⁻, **even(e)** sites: e⁺



Hilbert space

$$|0000\rangle = |\uparrow\downarrow\uparrow\downarrow\rangle$$
$$|e^- e^+ 00\rangle = |\downarrow\uparrow\uparrow\downarrow\rangle$$
$$|0 e^+ e^- 0\rangle = |\uparrow\uparrow\downarrow\downarrow\rangle$$
$$|00 e^- e^+\rangle = |\uparrow\downarrow\downarrow\uparrow\rangle$$
$$|e^- 00 e^+\rangle = |\downarrow\downarrow\uparrow\uparrow\rangle$$
$$|e^- e^+ e^- e^+\rangle = |\downarrow\uparrow\downarrow\uparrow\rangle$$

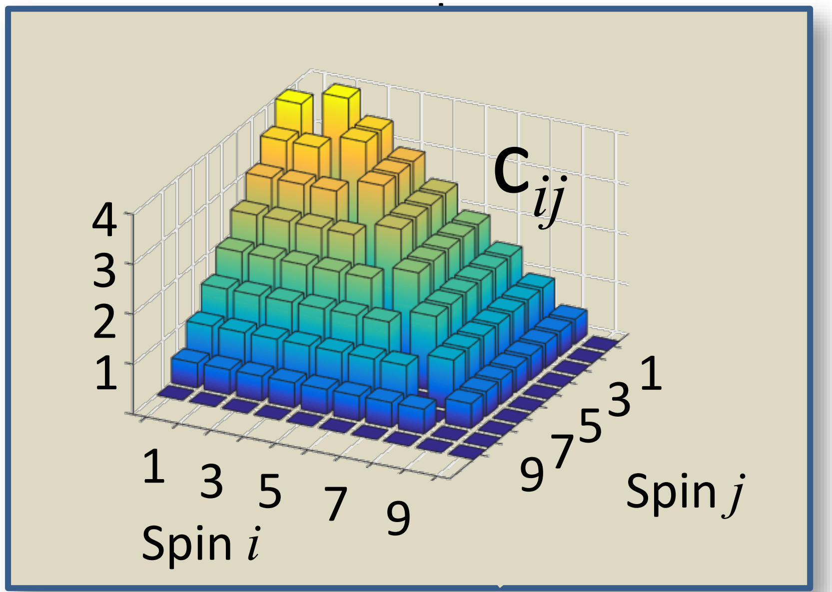
⇒ Error detection

C. Muschik et al., NJP 19, 103020 (2017)

Encoding gauge fields in interactions

Gauge fields are encoded in the interactions

$$H = J\sum_{i<j} c_{ij}\sigma_i^z\sigma_j^z + w\sum_i(\sigma_i^+\sigma_{i+1}^- + \sigma_{i+1}^+\sigma_i^-) + m\sum_i c_i\sigma_i^z + J\sum_i \tilde{c}_i\sigma_i^z$$



$H_\pm$ particle-antiparticle creation/annihilation

$$w = \frac{1}{2a}$$

a lattice spacing

H_z effective particle masses

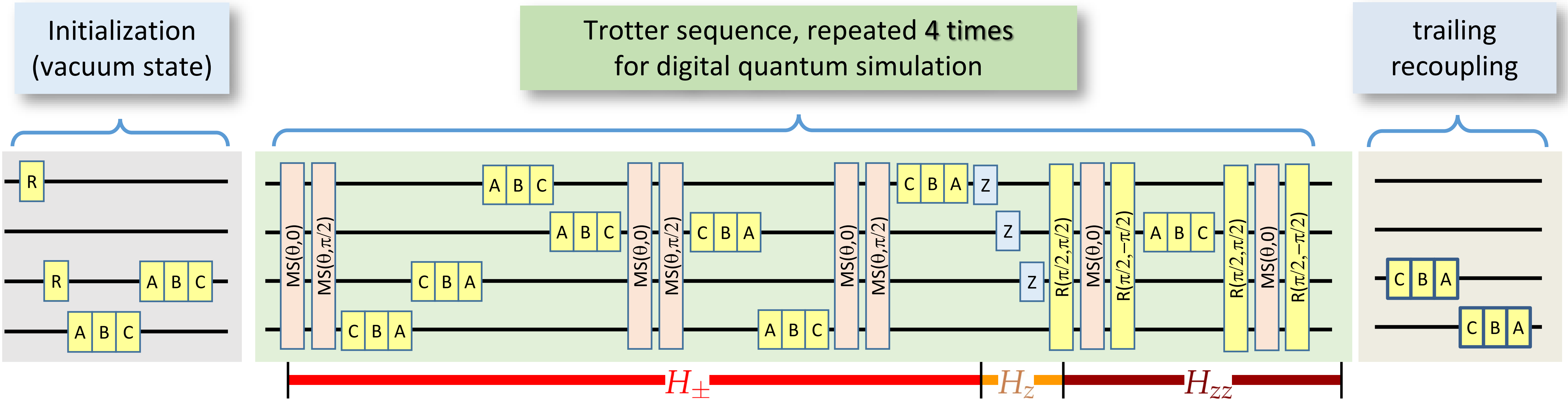
$$m$$

particle mass

E. Martinez et al., Nature 534, 516 (2016)

C. Muschik et al., NJP 19, 103020 (2017)

Compiled pulse sequence



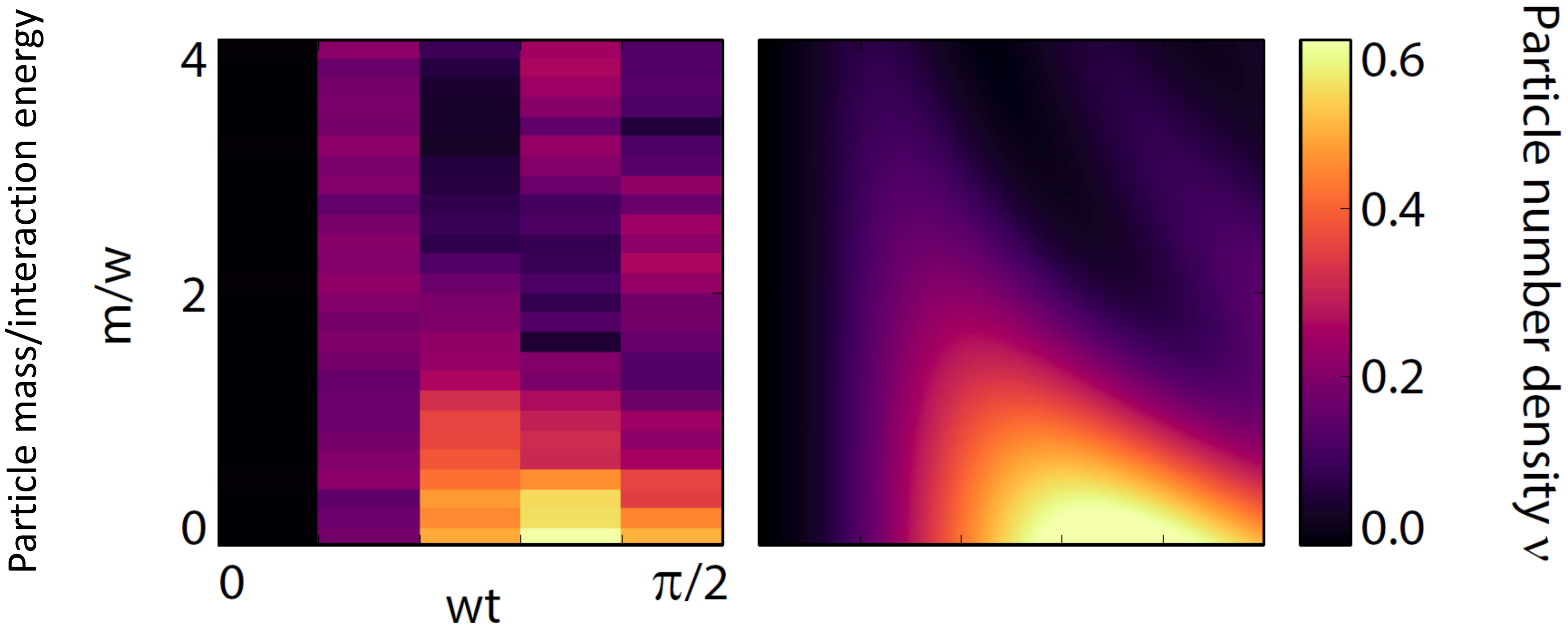
total sequence
for 4 sites

222
gate
operations

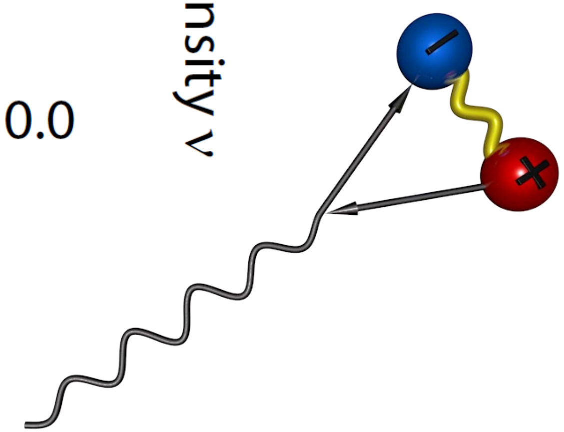
E. Martinez et al., Nature 534, 516 (2016)

See also: 6-site digital time evolution; N. Nguyen, et al., PRX Quantum 3, 020324 (2022)

Schwinger mechanism: Particle-Antiparticle creation



- Mass is a tunable parameter
- Interaction w is taken to be constant (sets the timescale)
- Particle number density ν defined: ν=0.5 corresponds to one pair

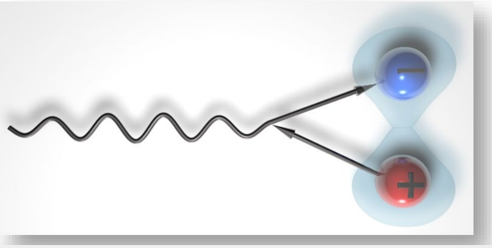


E. Martinez et al., Nature 534, 516 (2016)

Entanglement dynamics

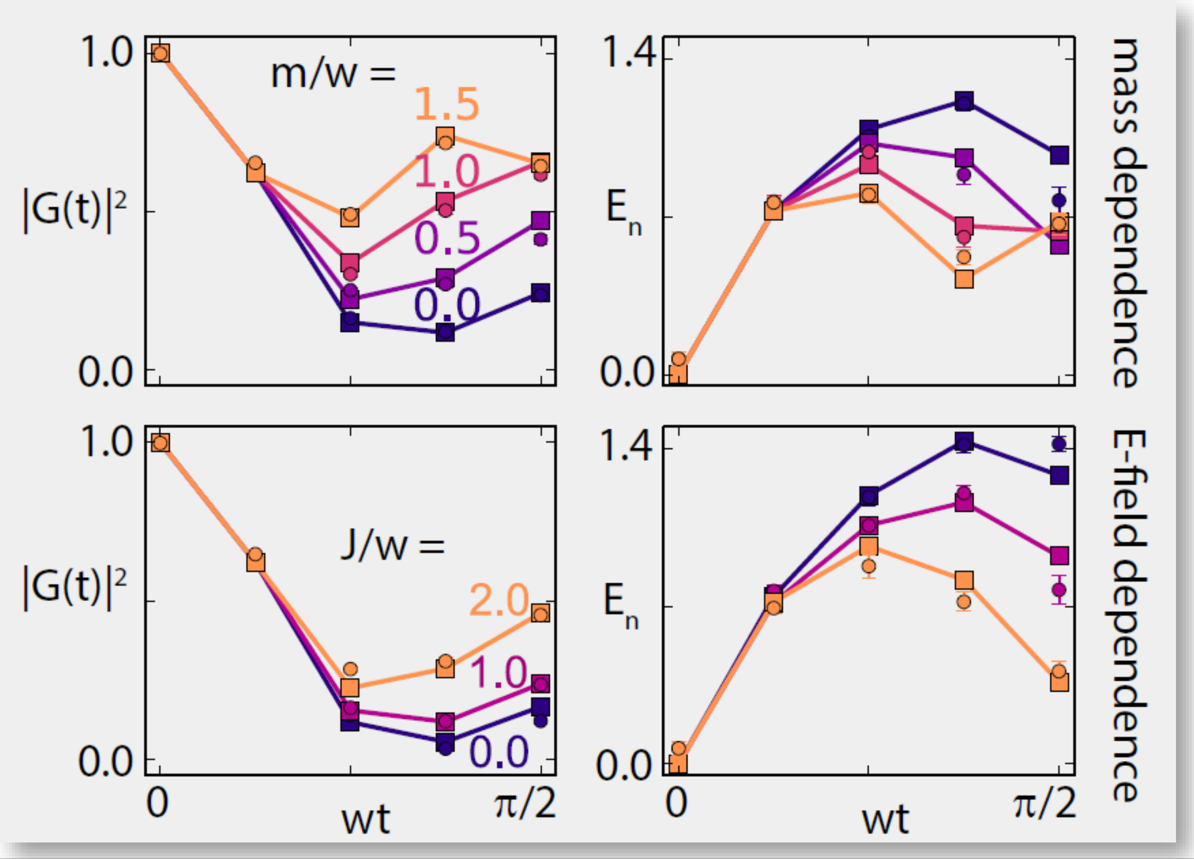
Vacuum persistence:

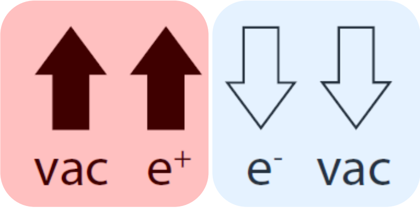
$$G(t) = \langle vac|e^{iHt}|vac\rangle$$



Entanglement:

E_n : log. negativity





bipartition

experimental data
error model

E. Martinez et al., Nature 534, 516 (2016)

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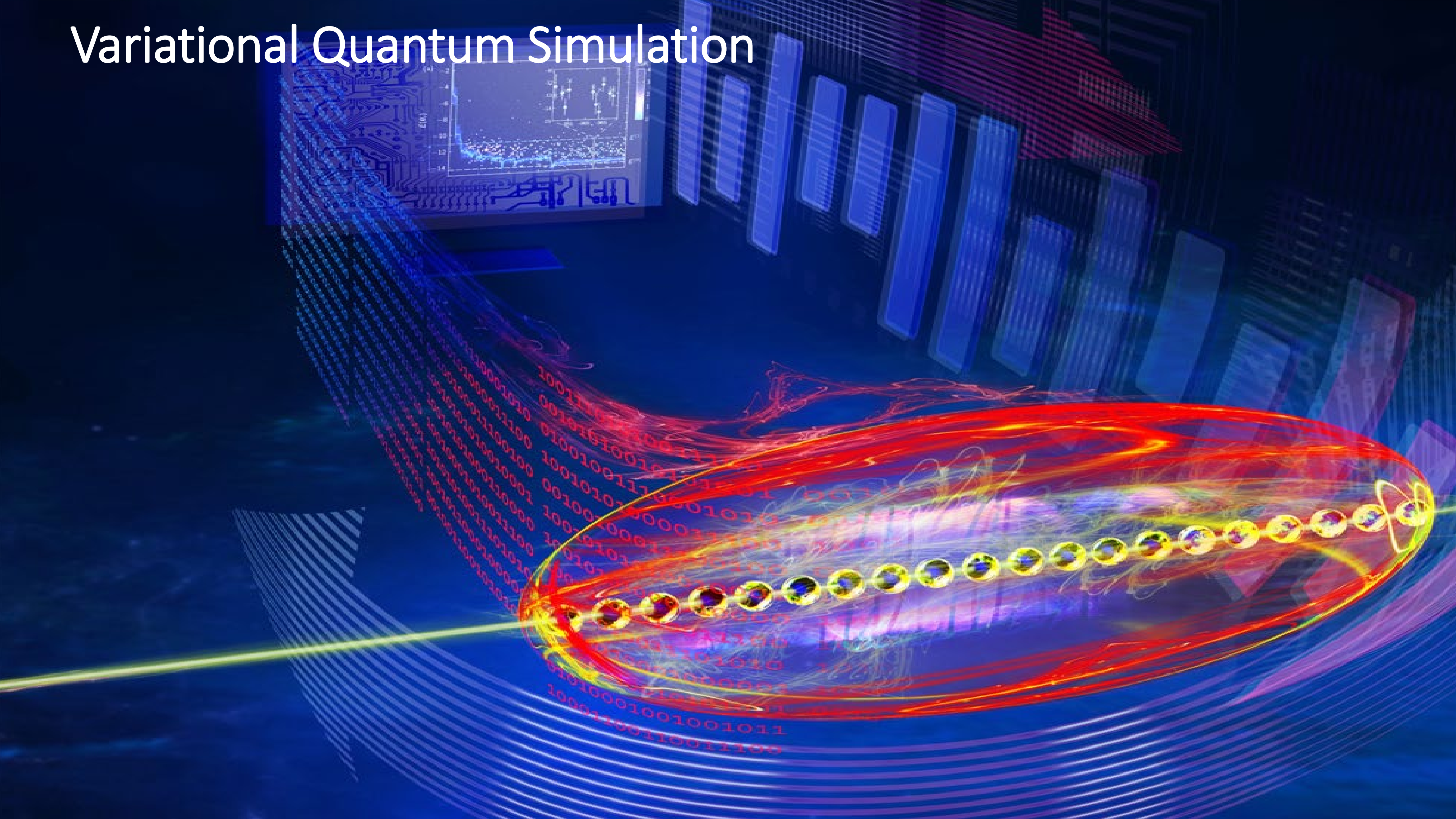
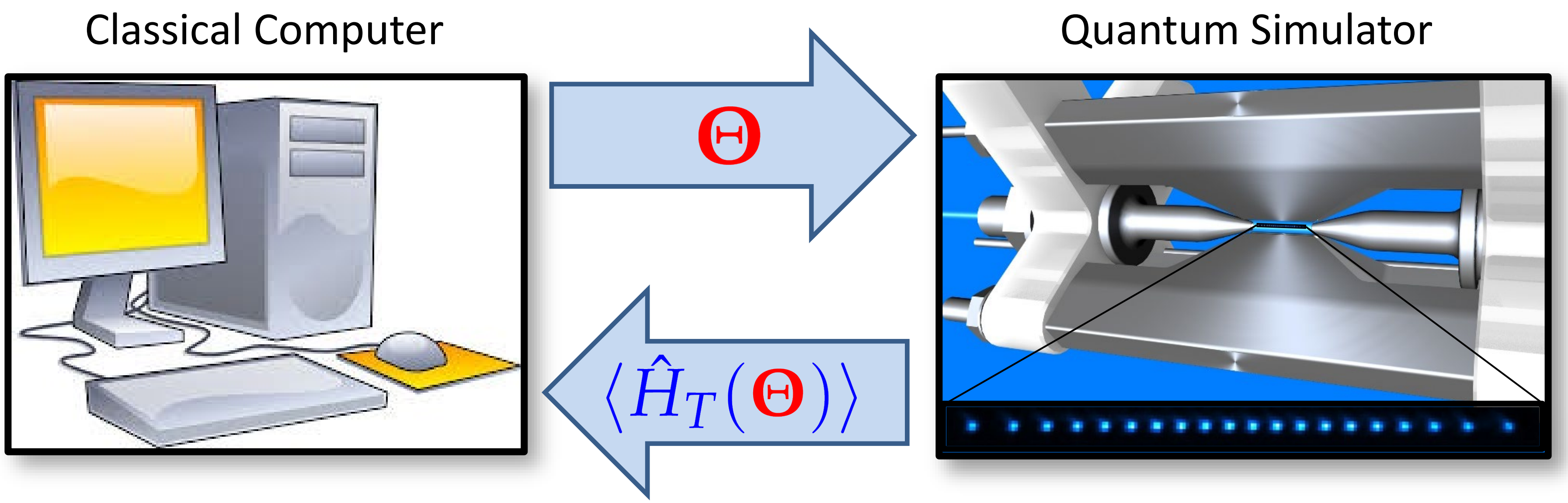
Variational Quantum Simulation



Variational Quantum Simulation

Goal: prepare groundstate of $\hat{H}_T$ by minimizing $\langle \psi(\boldsymbol{\Theta}) | \hat{H}_T | \psi(\boldsymbol{\Theta}) \rangle$

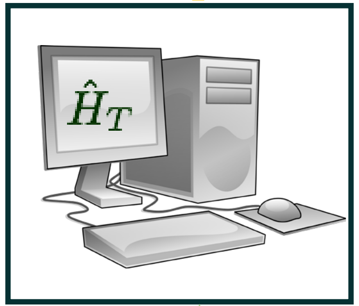


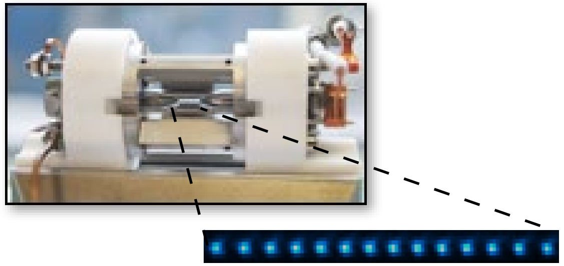
- Target - Hamiltonian "lives" only in the classical computer
- Feedback loop between classical computer and quantum co-processor

Perùzzo et al., Nature Comm. 5, 4213 (2014), Farhi et al., arXiv:1411.4028 (2014), McClean et al., NJP 18, 023023 (2016)
C. Kokail, et al., Nature 569, 355 (2019)

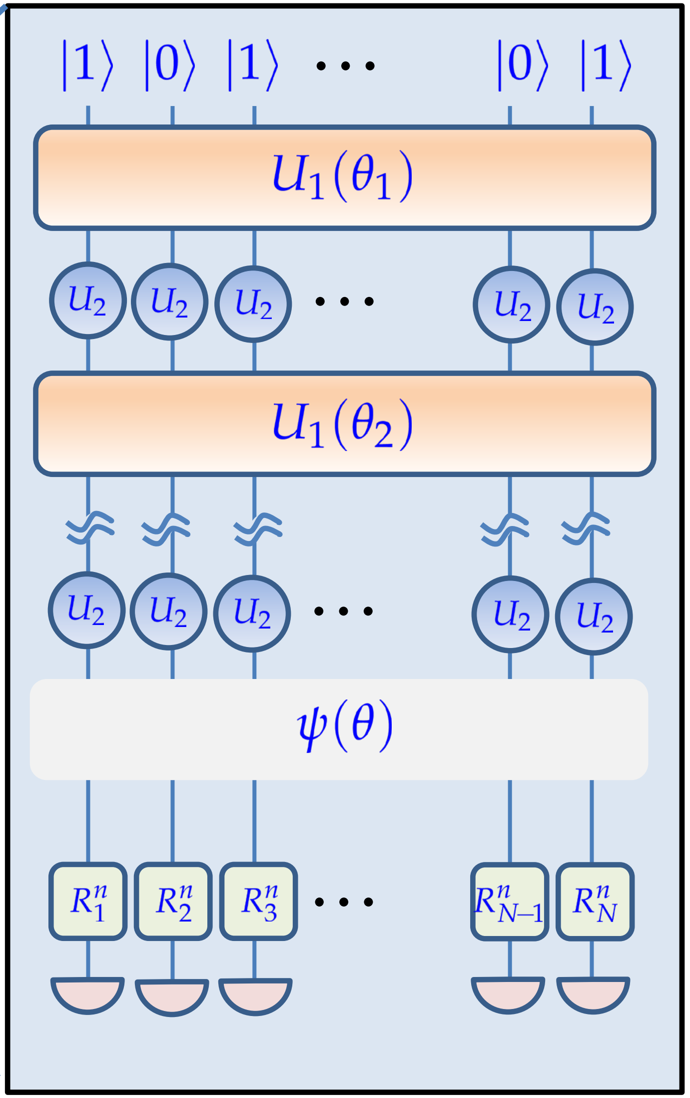
Variational Quantum Simulation

parameters Θ





measurements



Target Hamiltonian

$$\hat{H}_T = \sum_{n=1}^{M} \hat{h}_n$$

e.g. $\hat{H}_T = A \cdot \hat{\sigma}_1^x + B \cdot \hat{\sigma}_3^z \hat{\sigma}_4^z + \quad \ldots$

Chooses Θ to minimize $\langle \hat{H}_T \rangle$

Generate states using available resources:

$$U_1(\Theta) = \exp\Big(-\mathrm{i}\Theta \big(\sum_{i<j} J_{ij} \sigma_i^+ \sigma_j^- + h.c. \big) \Big)$$

$$U_{2,i}(\Theta) = \exp\left(-\mathrm{i}\Theta \sigma_i^z \right)$$

$\langle \hat{\sigma}^x \rangle, \langle \hat{\sigma}^y \rangle, \langle \hat{\sigma}^z \rangle$ measurements

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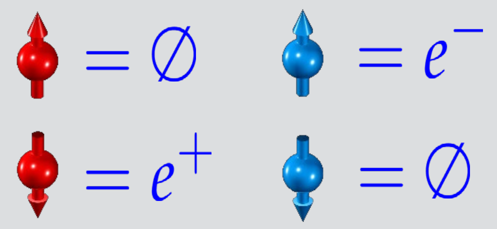
C. Kokail, et al., Nature **569**, 355-360 (2019)

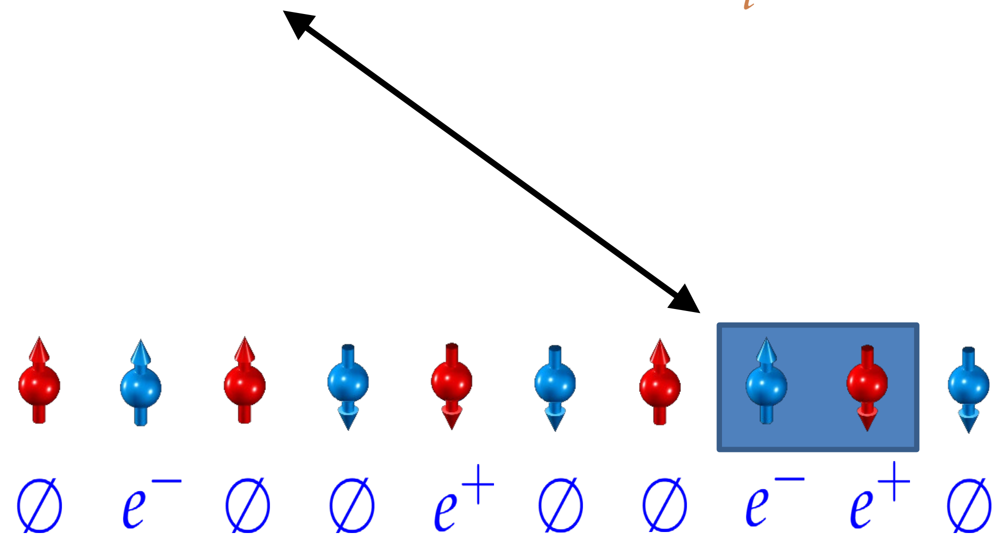
Generalization to time evolution P. Popov, et al., arxiv:2307.15173

Target Hamiltonian: Lattice Schwinger Model

$$H = J\sum_{i<j} c_{ij}\sigma_i^z\sigma_j^z + w\sum_i(\sigma_i^+\sigma_{i+1}^- + \sigma_{i+1}^+\sigma_i^-) + m\sum_i c_i\sigma_i^z + J\sum_i \tilde{c}_i\sigma_i^z$$

Kogut-Susskind encoding





E. A. Martinez et al., Nature **534**, 516 (2016)
C. Muschik et al., NJP **19**, 103020 (2017)

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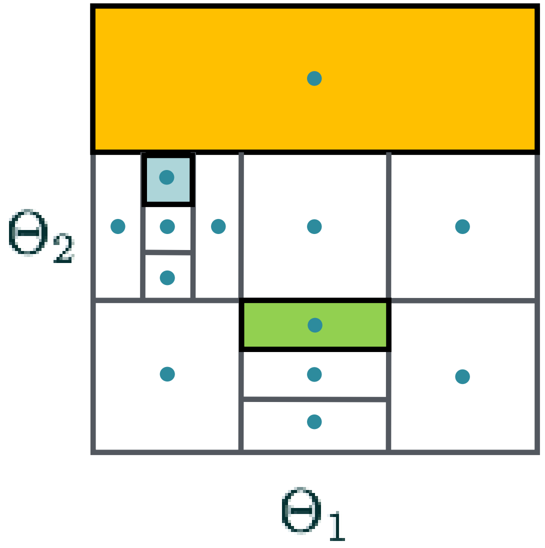
Classical search algorithm in parameter space

- Global optimization problem with many local minima
- Noisy problem (energies are estimated by measuring a finite number of quantum states)
- No gradients available
- Finite number of energy evaluations

Chosen Classical Algorithm:

DIviding **RECT**angles (DIRECT) global optimisation algorithm

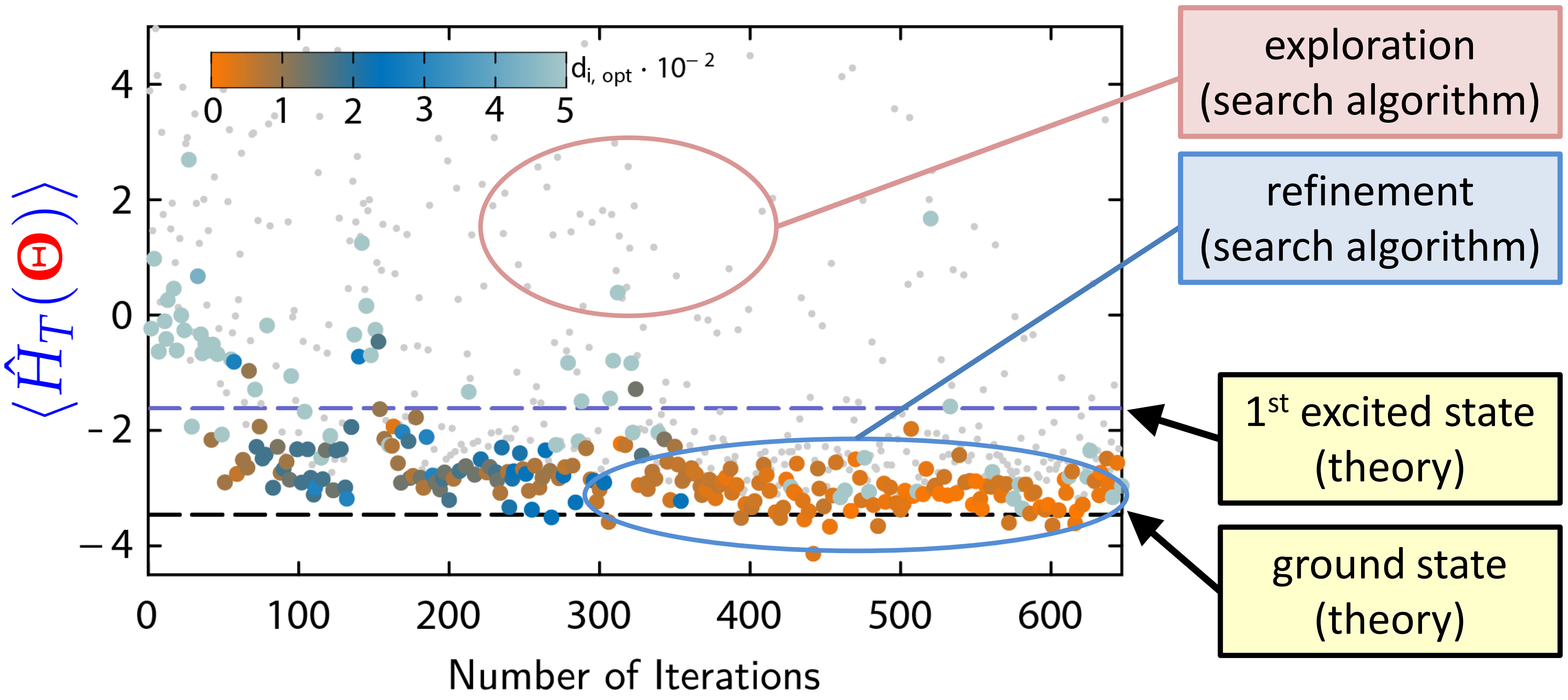


Identifying promising regions in a 2D search space

Jones et.al., "Lipschitzian optimization without the Lipschitz constant." Journal of Optimization Theory and Applications, **79**(1), 157 (1993)

Finding the ground state of a Hamiltonian

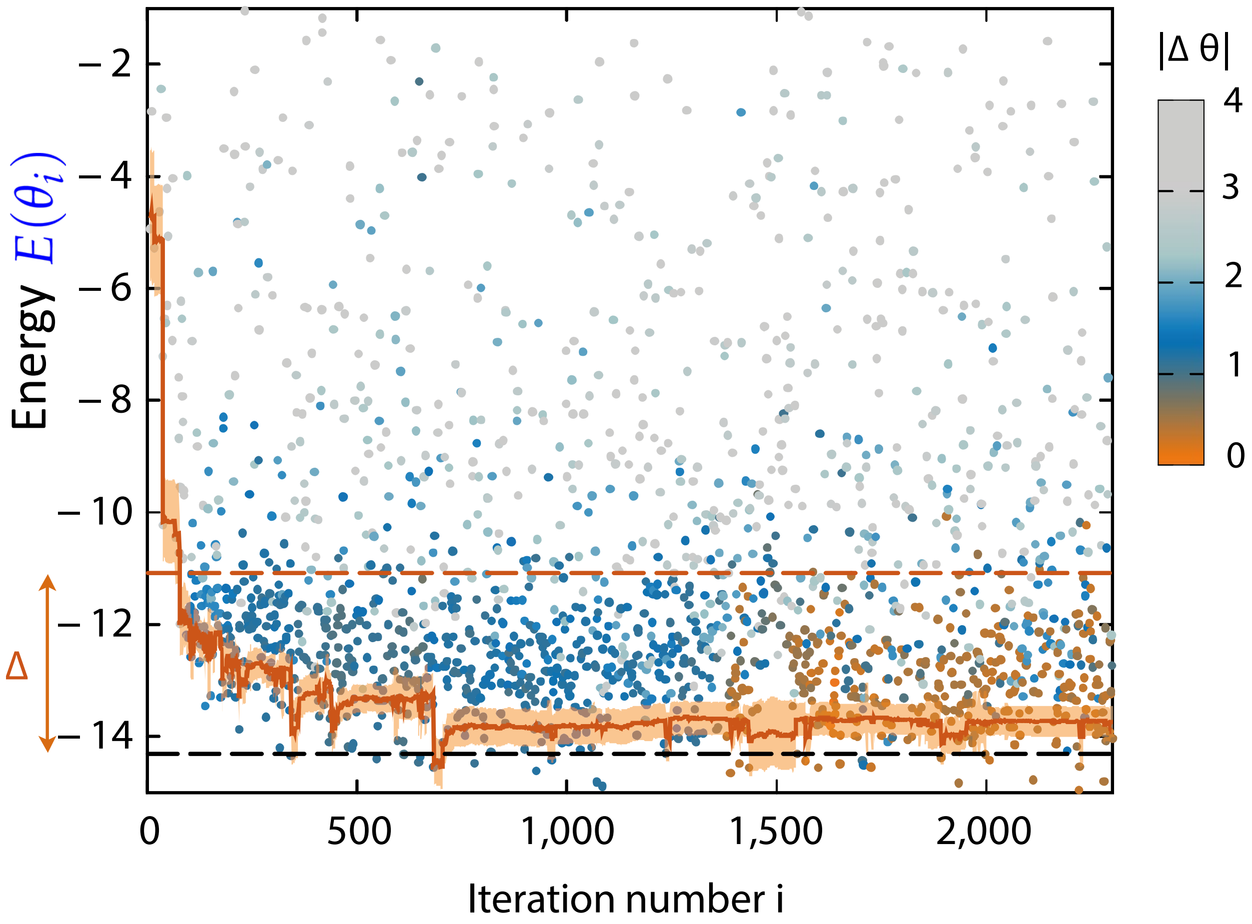
8 ions, 10 parameters Θ, example: **Lattice Schwinger Model Hamiltonian**



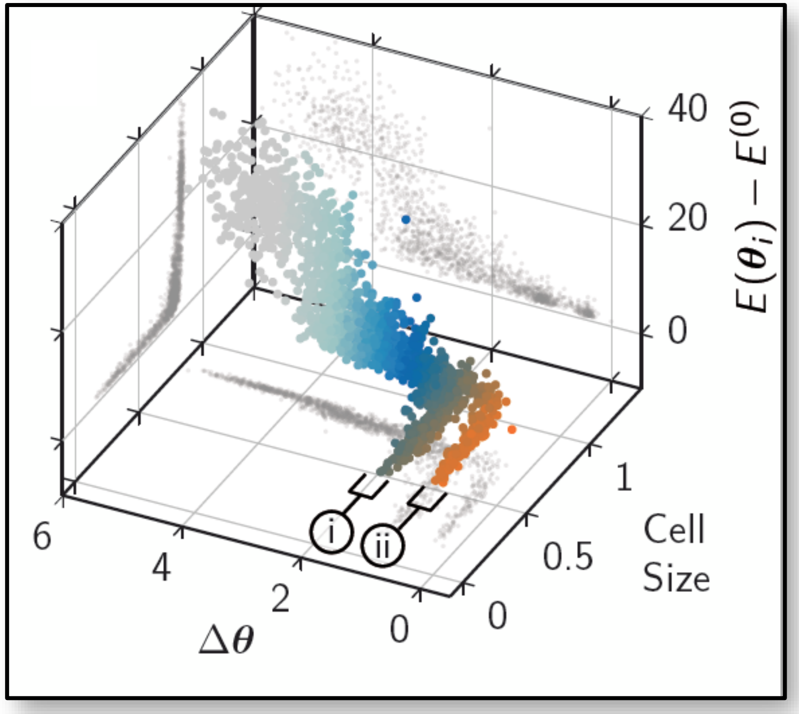
C. Kokail, et al., Nature **569**, 355-360 (2019)

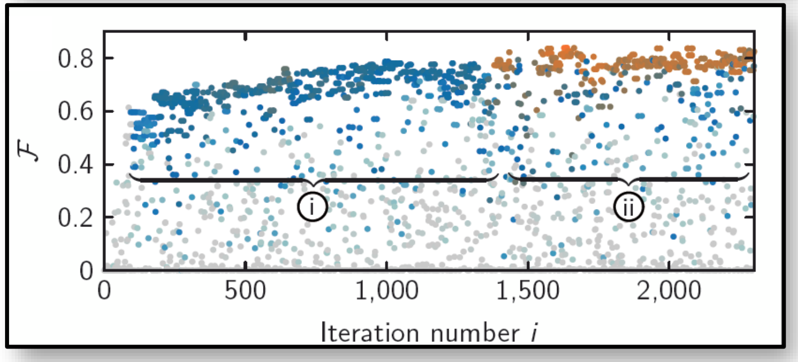
Search Space Landscape

20 ions



Color indicates distance of Θ_i from Θ_{opt}





C. Kokail, et al., Nature **569**, 355-360 (2019)

Verification of the experimental results

8 ions

How much can we trust the experimentally determined energy?

Variance of the Schwinger Model:

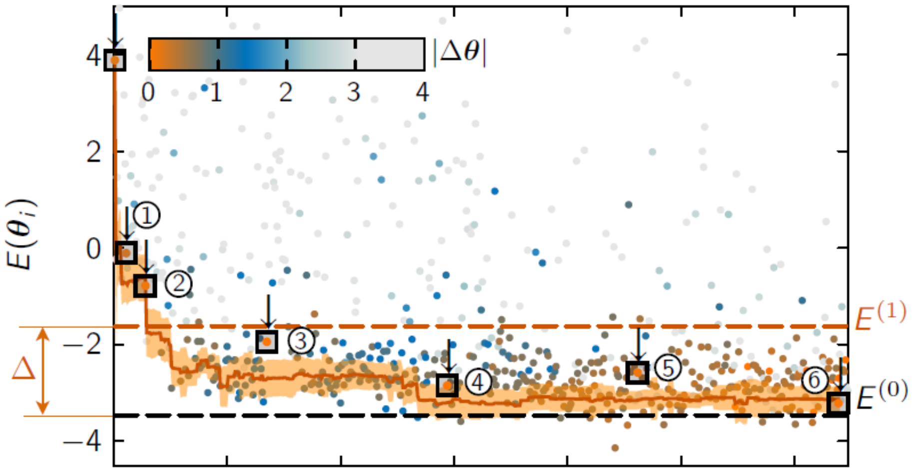
$$\mathrm{var}(\hat{H}_S) = \langle(\hat{H}_S - \langle\hat{H}_S\rangle)^2\rangle_\Theta$$

Variance measures closeness to an eigenstate

Measurement in <u>3*N* different bases</u>

➡ Can be reduced to 1 measurement basis

R. Stricker, et al., PRX Quantum **3**, 040310 (2022)



C. Kokail, et al., Nature **569**, 355-360 (2019)

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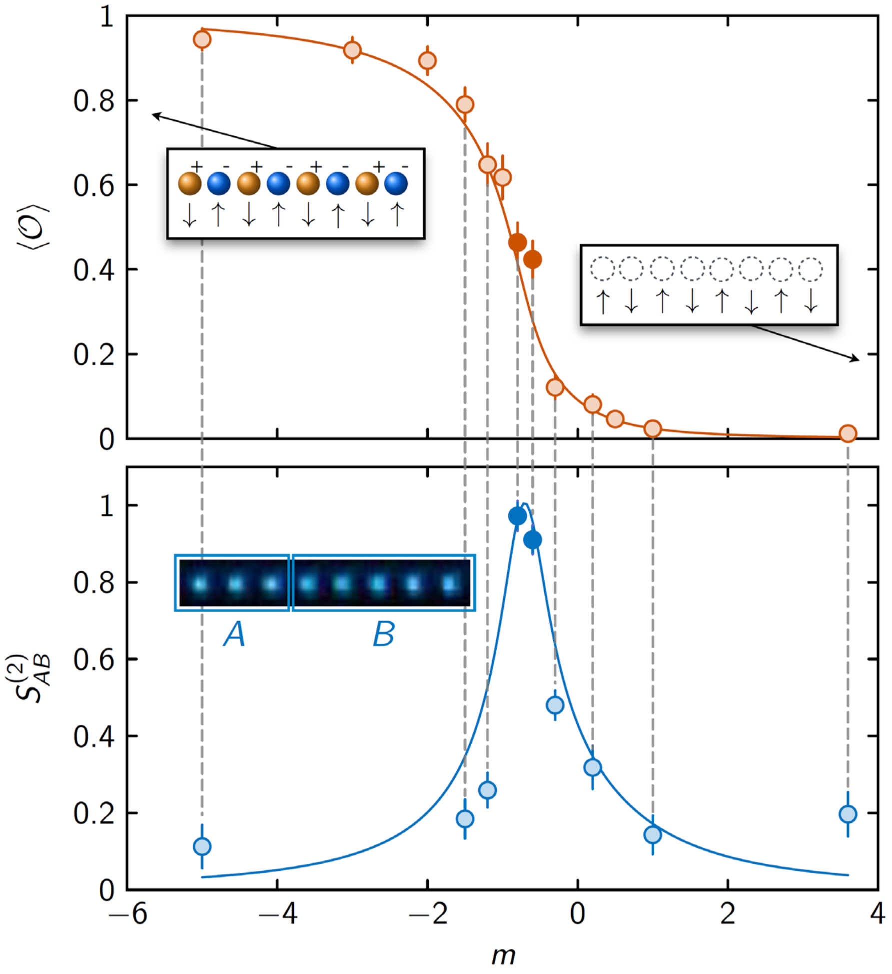
Ground-state properties



Phase transition in order parameter

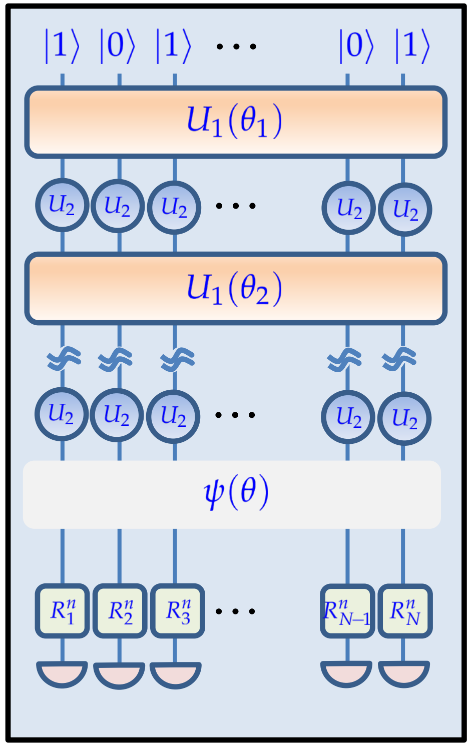
$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{2N(N-1)} \sum_{i,j>i} \langle (1 + (-1)^i \hat{\sigma}_i^z)(1 + (-1)^j \hat{\sigma}_j^z) \rangle$$

2nd order Renyi entropy

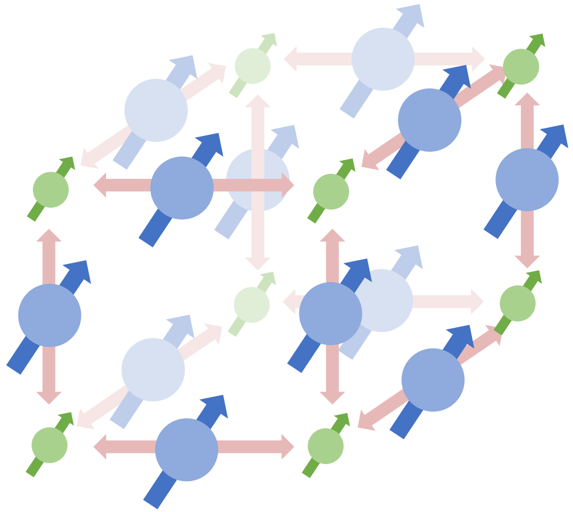
C. Kokail, et al., Nature **569**, 355-360 (2019)

Where to from here?

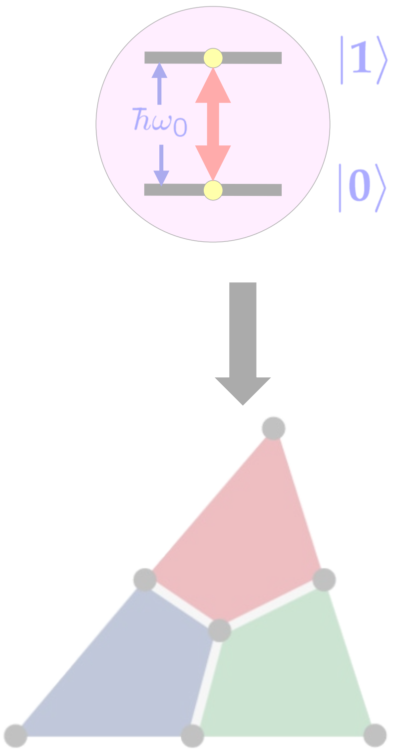
Performance and efficiency



Beyond 1D QED



Quantum Error Correction



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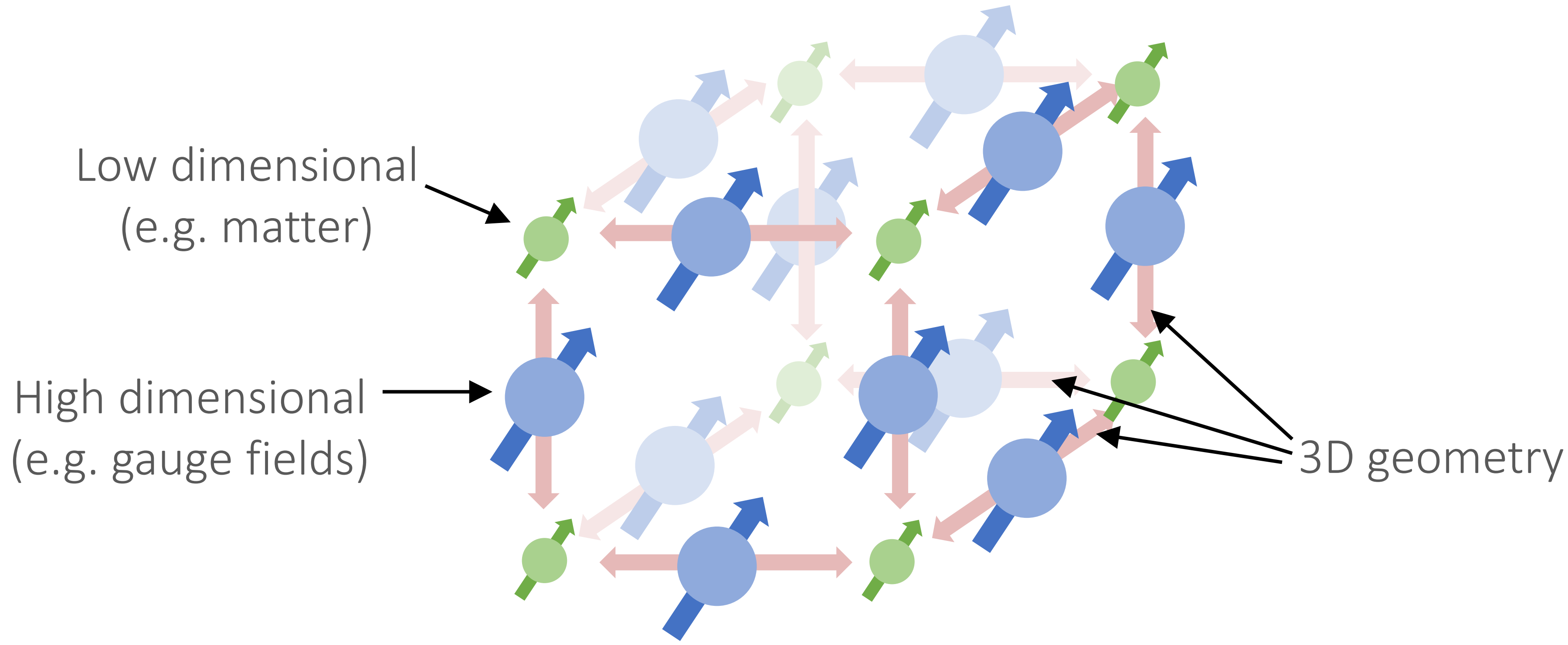
Quantum Simulating Lattice Gauge Theories



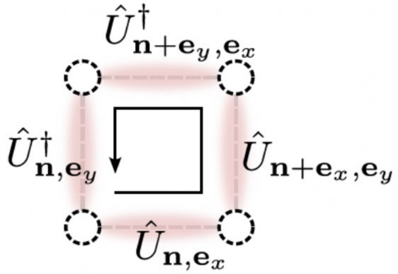
Encoding Gauge Fields

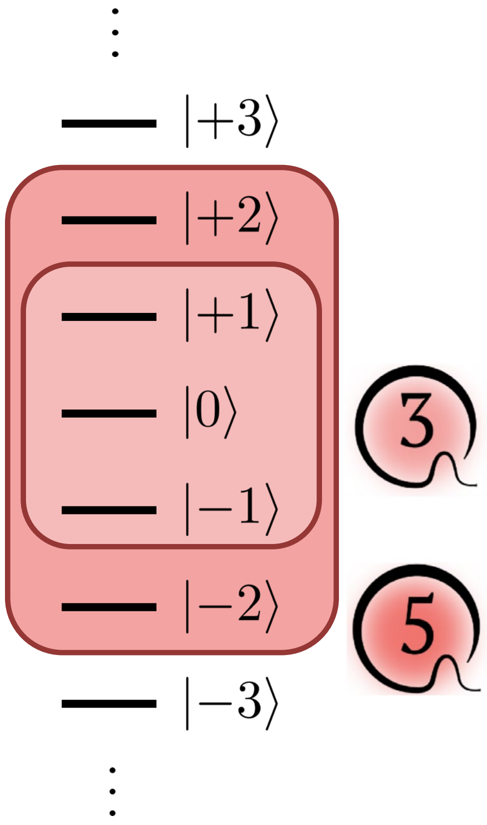
Example: 1D QED

- Gauge fields can be eliminated

Beyond 1D QED

- “dynamical” gauge fields
- magnetic field effects





In classical and quantum simulation:

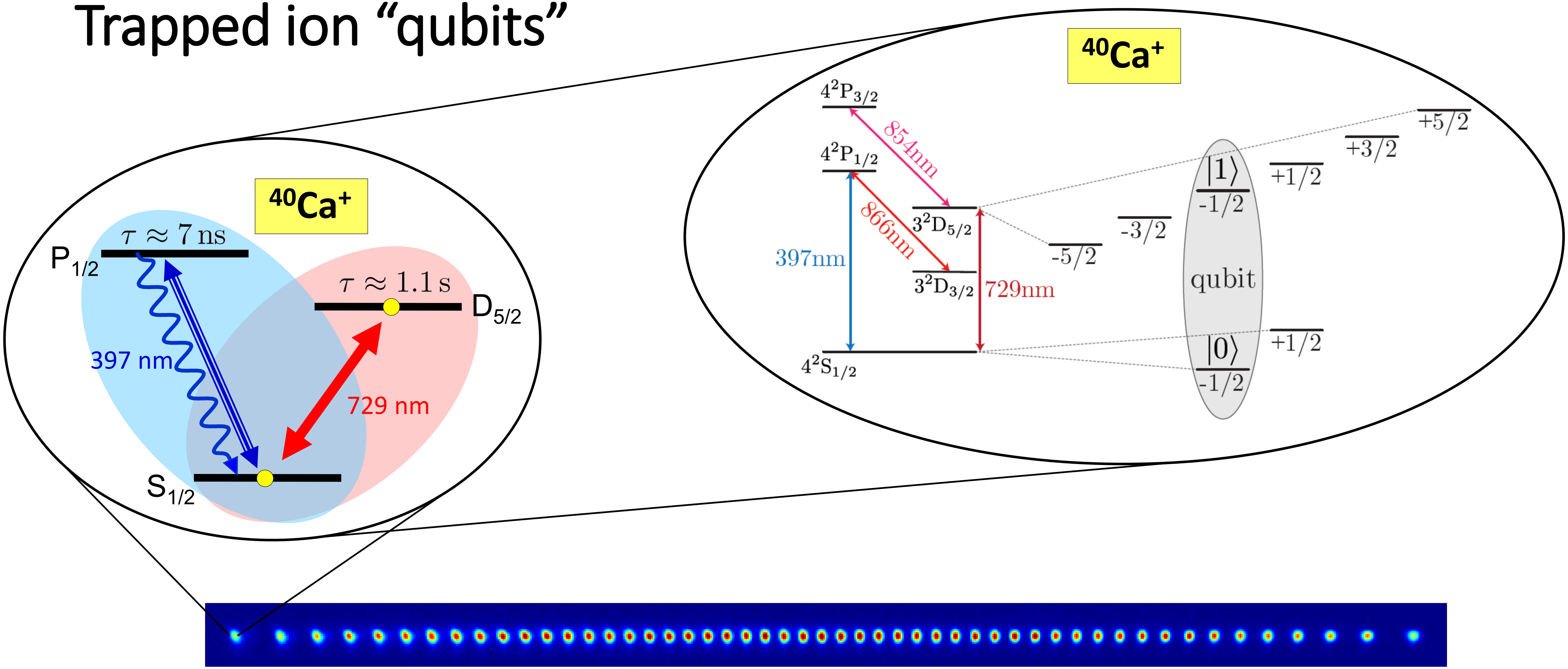
Gauge fields must be truncated

Minimal truncation: d=3

- field in pos direction
- zero
- field in neg direction

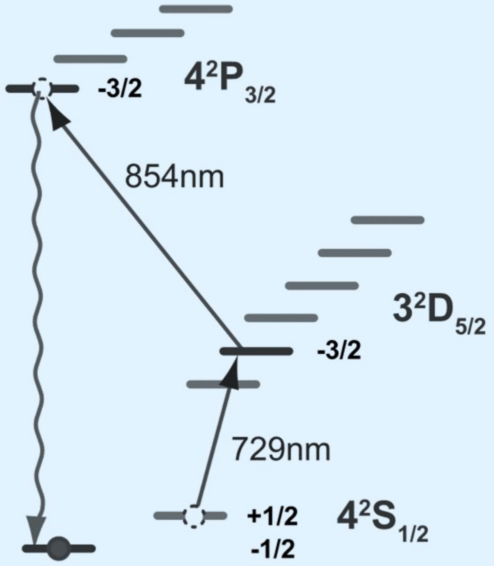
Better truncation: d=5

Trapped ion “qubits”



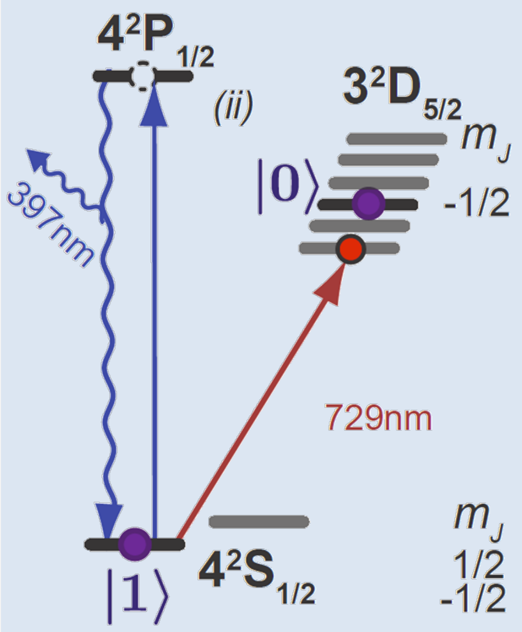
Capabilities beyond Qubits

Optical pumping

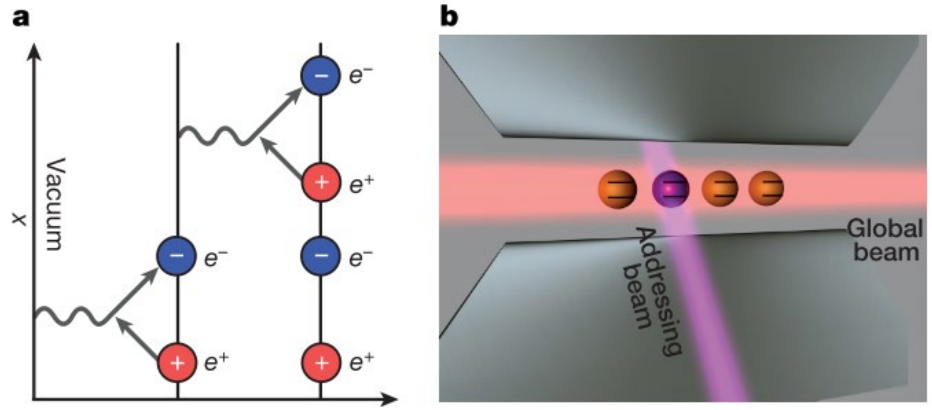


Initializes qubit in one Zeeman state

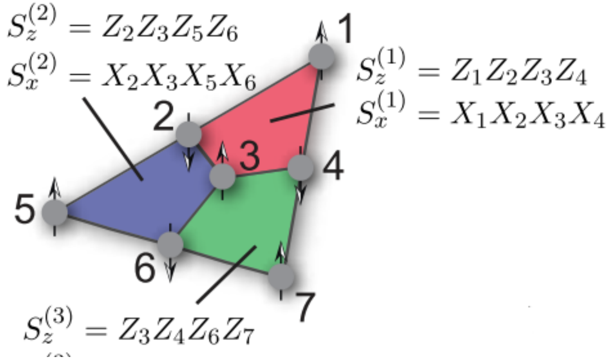
Decoupling



reduces, enlarges the computational subspace



E. Martinez, et al., Nature **534**, 516 (2016)

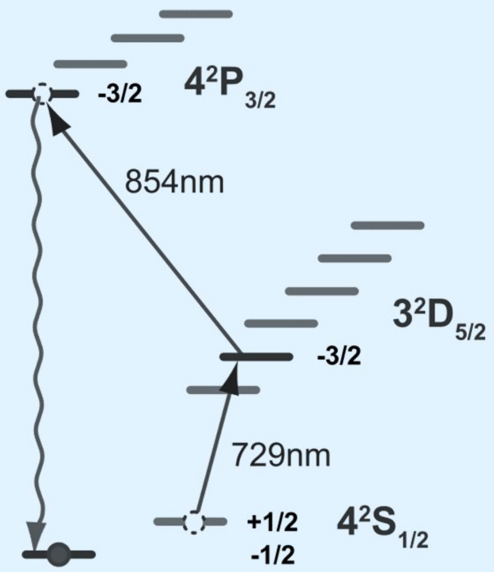


D. Nigg, et al., Science **345**, 302 (2014)

P. Schindler et al., New. J. Phys. 15, 123012 (2013)

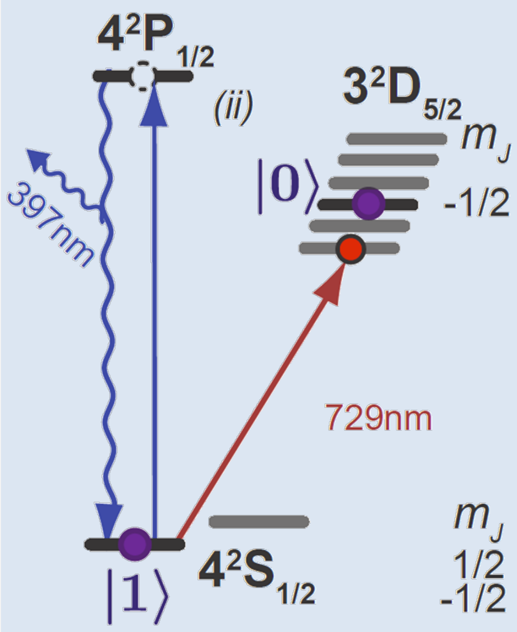
Capabilities beyond Qubits

Optical pumping



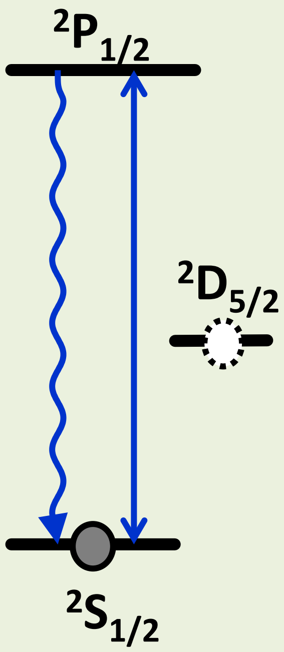
Initializes qubit in one Zeeman state

Decoupling

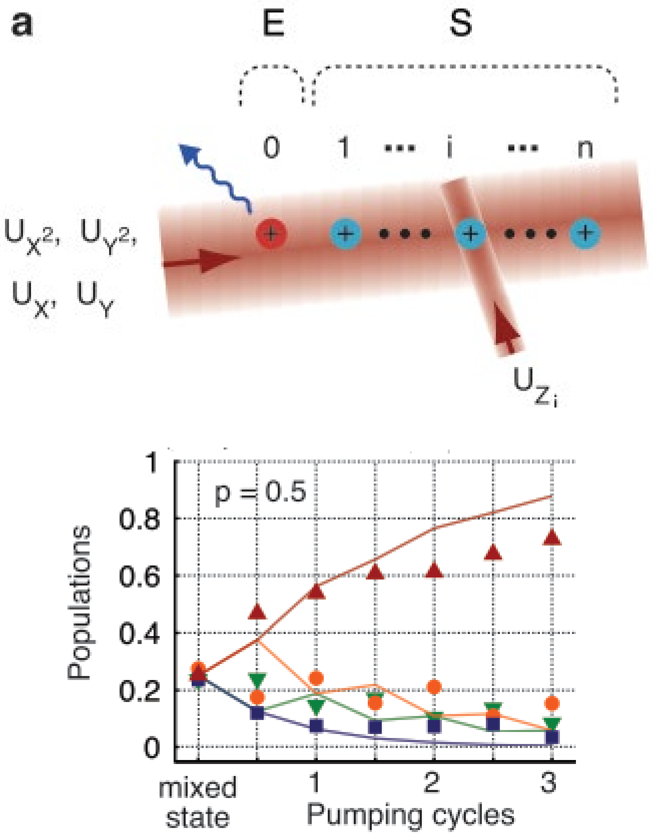


reduces, enlarges the computational subspace

Dephasing



controlled dissipation

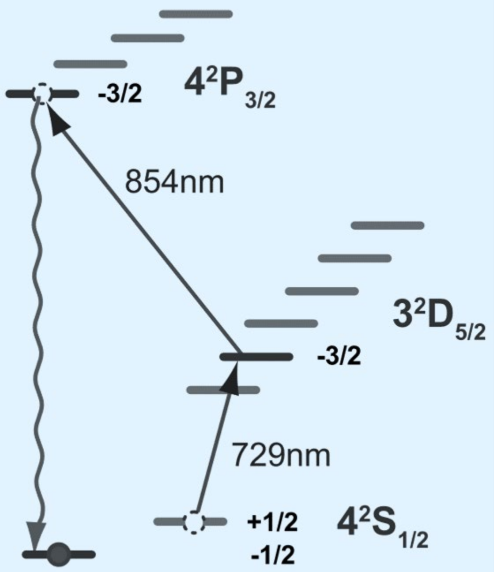


J. Barreiro, et al., Nature **470**, 486 (2011)

P. Schindler et al., New. J. Phys. 15, 123012 (2013)

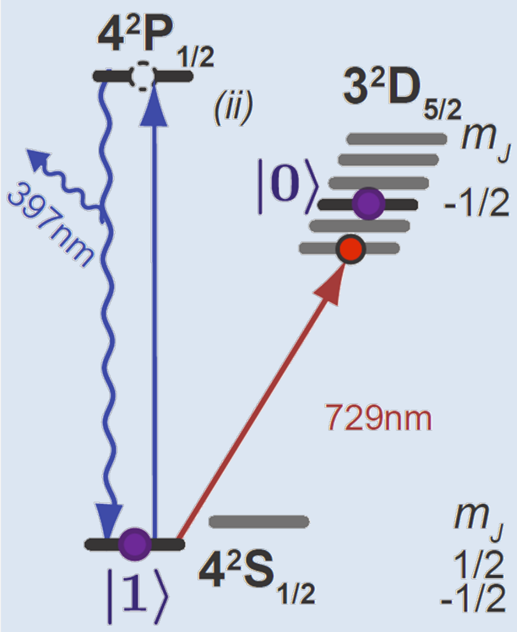
Capabilities beyond Qubits

Optical pumping



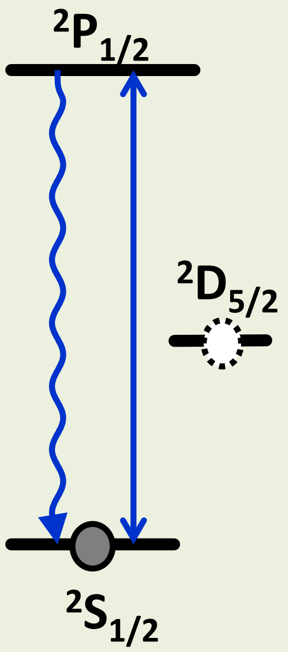
Initializes qubit in one Zeeman state

Decoupling



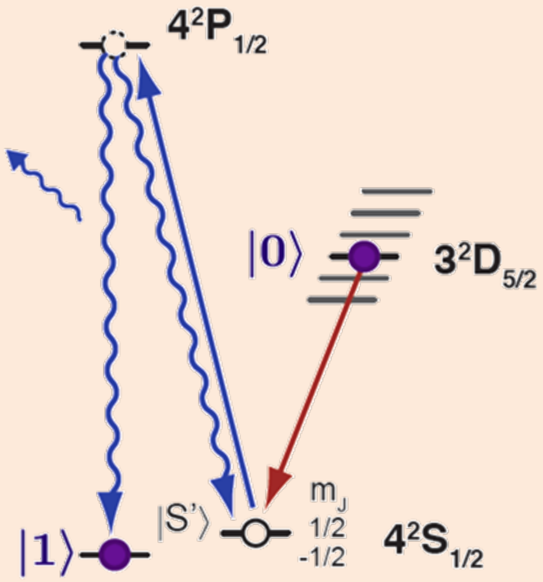
reduces, enlarges the computational subspace

Dephasing



controlled dissipation

Resetting



initializes the qubit

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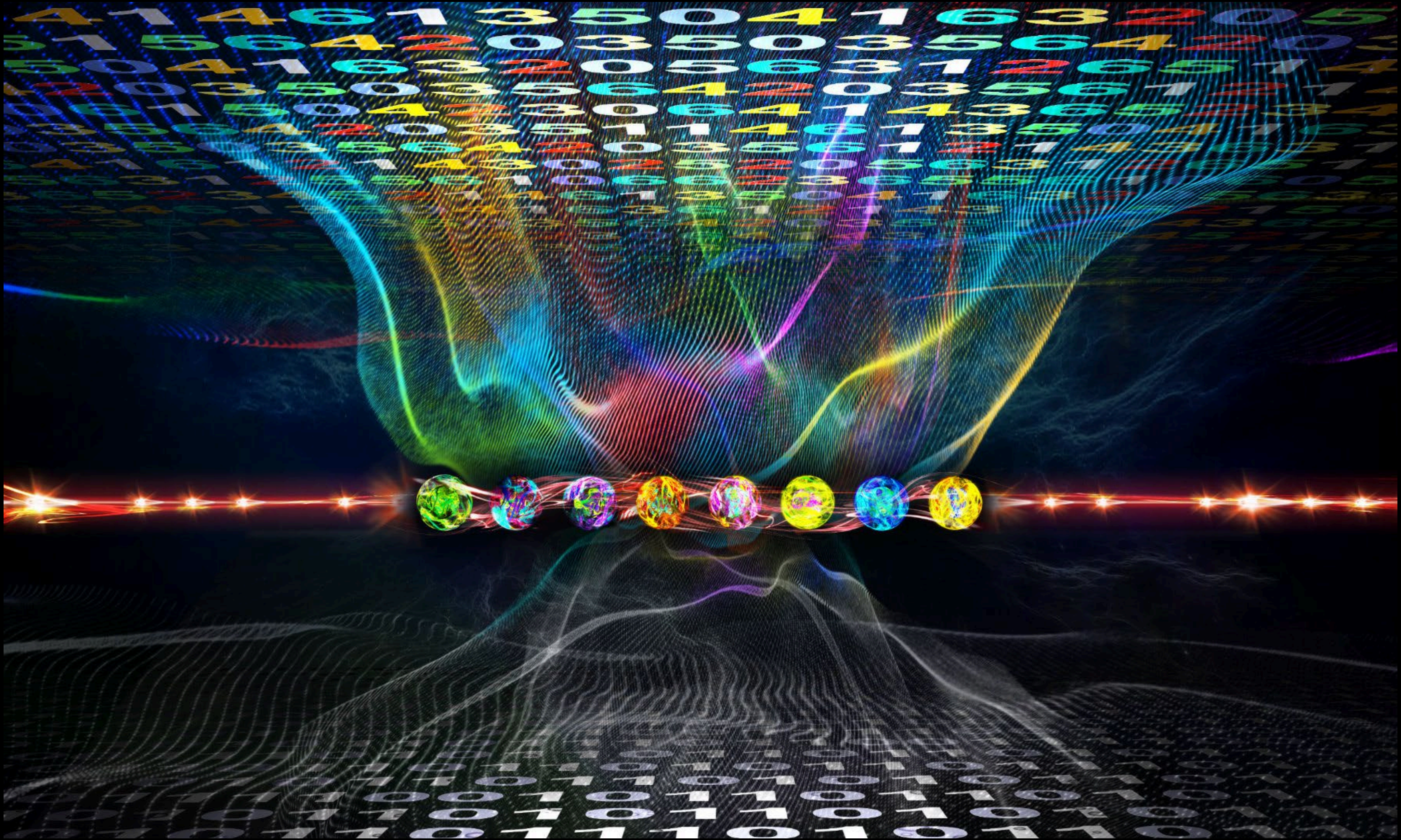
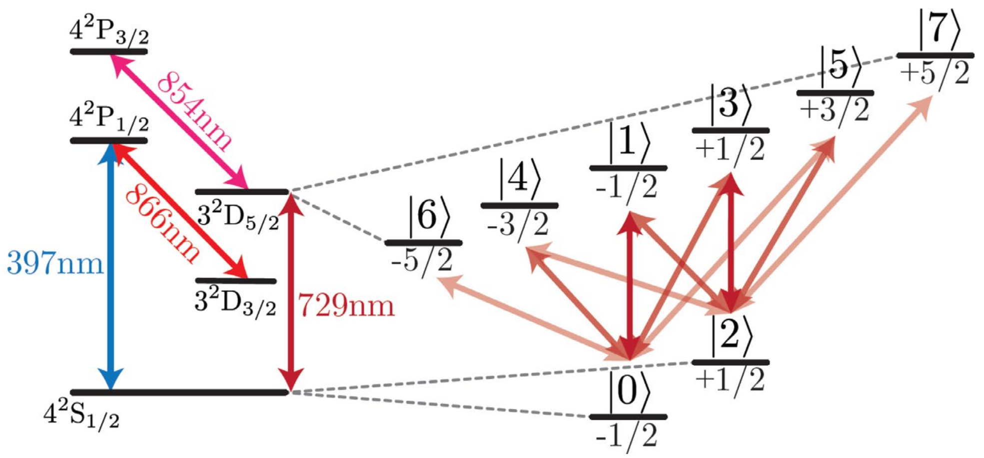
P. Schindler et al., New. J. Phys. 15, 123012 (2013)

Towards QIP with trapped-ion qudits

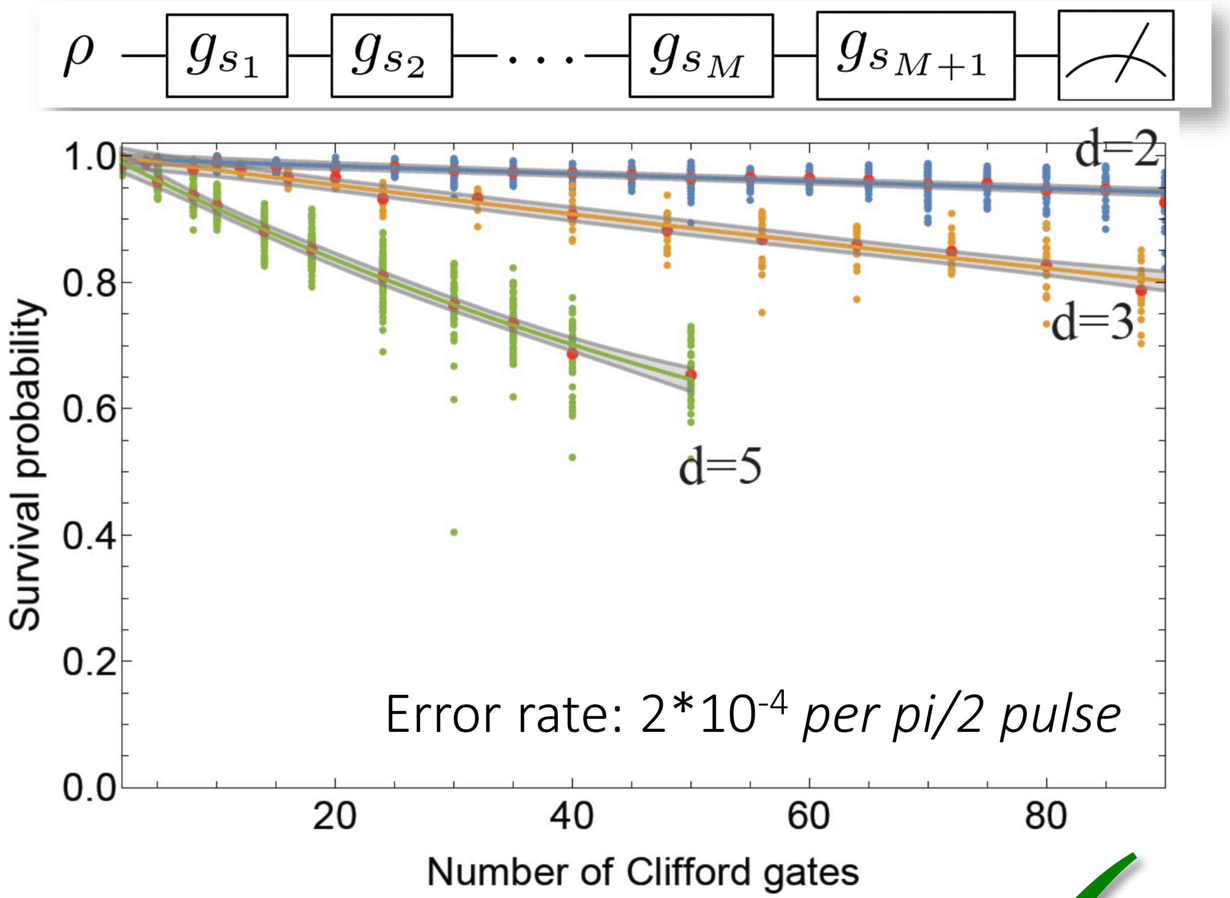


Single Qudit Operations



Trapped ions naturally encode qudits

Universal QC requires only "Clifford+T"

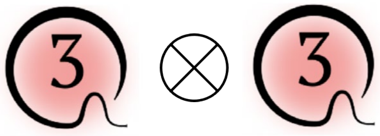


Consistent performance for all *d*!

MR et al., Nature Physics **18**, 1053 (2022)

Qudit entangling gates

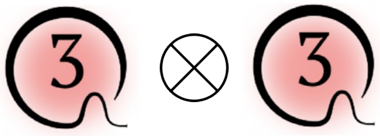
Embedded qubit gates



Creates the state

$$|00\rangle + |11\rangle$$

Genuine qudit gates

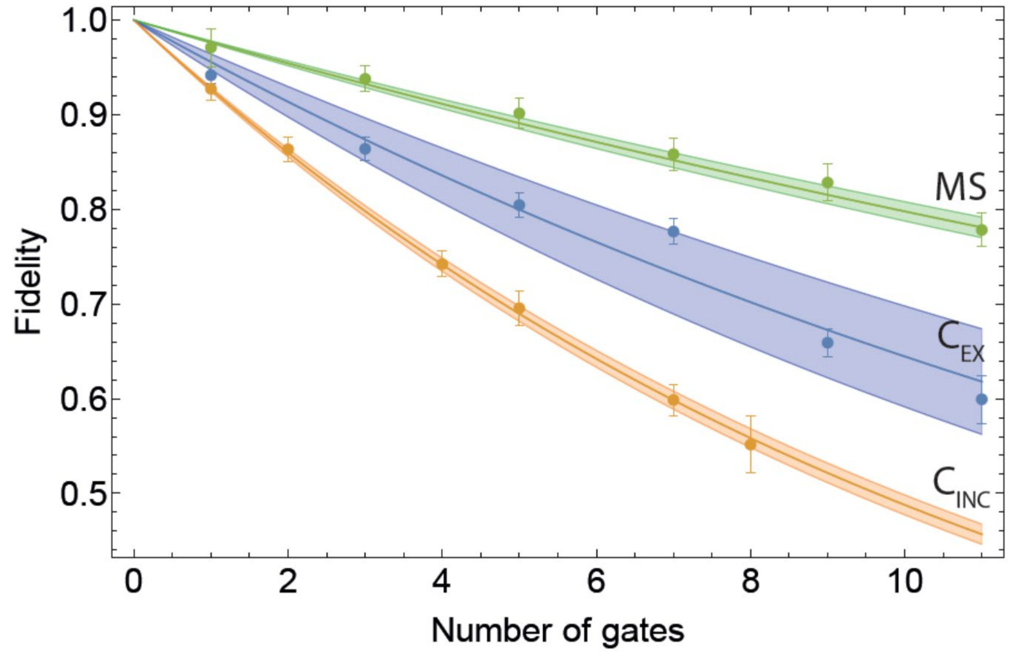


Creates the state

$$|00\rangle + |11\rangle + |22\rangle$$

Qudit entangling gates

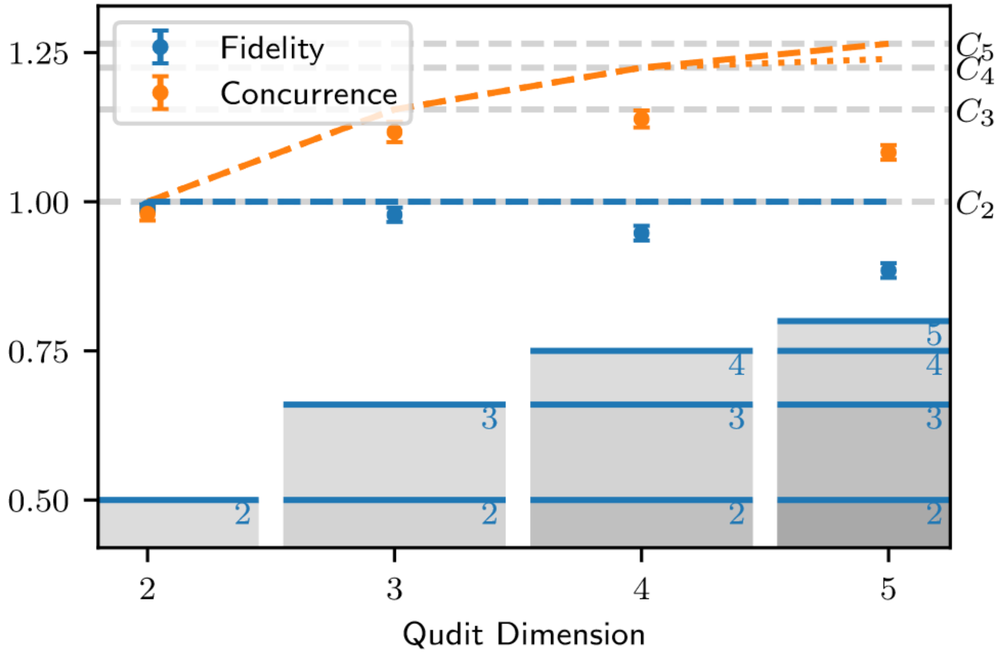
Embedded qubit gates



Two-level entanglement in qudit Hilbert space

→ No drop in fidelity due to larger Hilbert space ✓

Genuine qudit gates



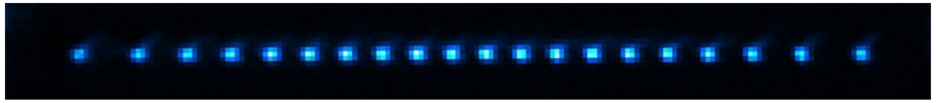
Genuine qutrit gate with F=0.990(4) %

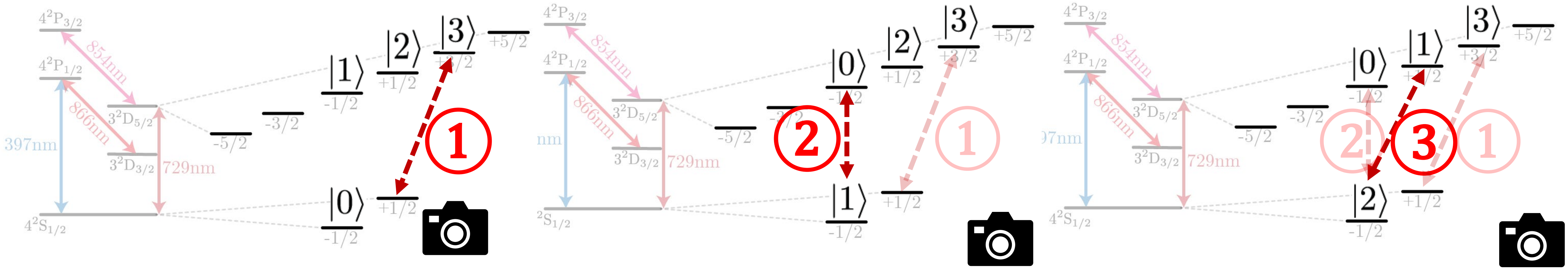
→ One control parameter independent of dim ✓

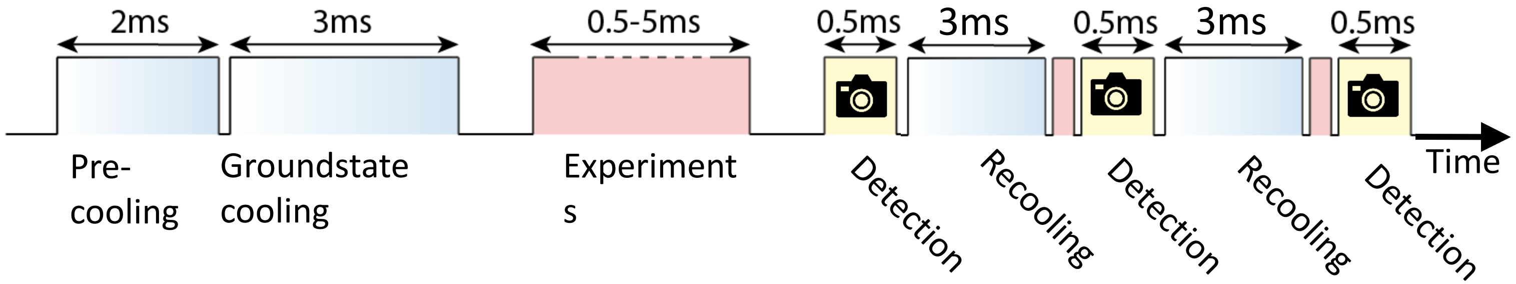
MR et al., Nature Physics **18**, 1053 (2022)

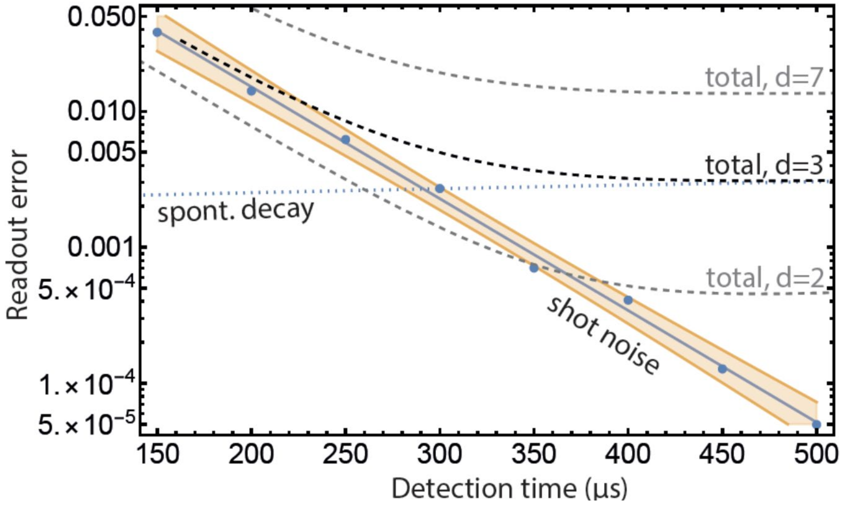
P. Hrmo, et al., Nature Commun. **14**, 2242 (2023)

Qudit Measurement





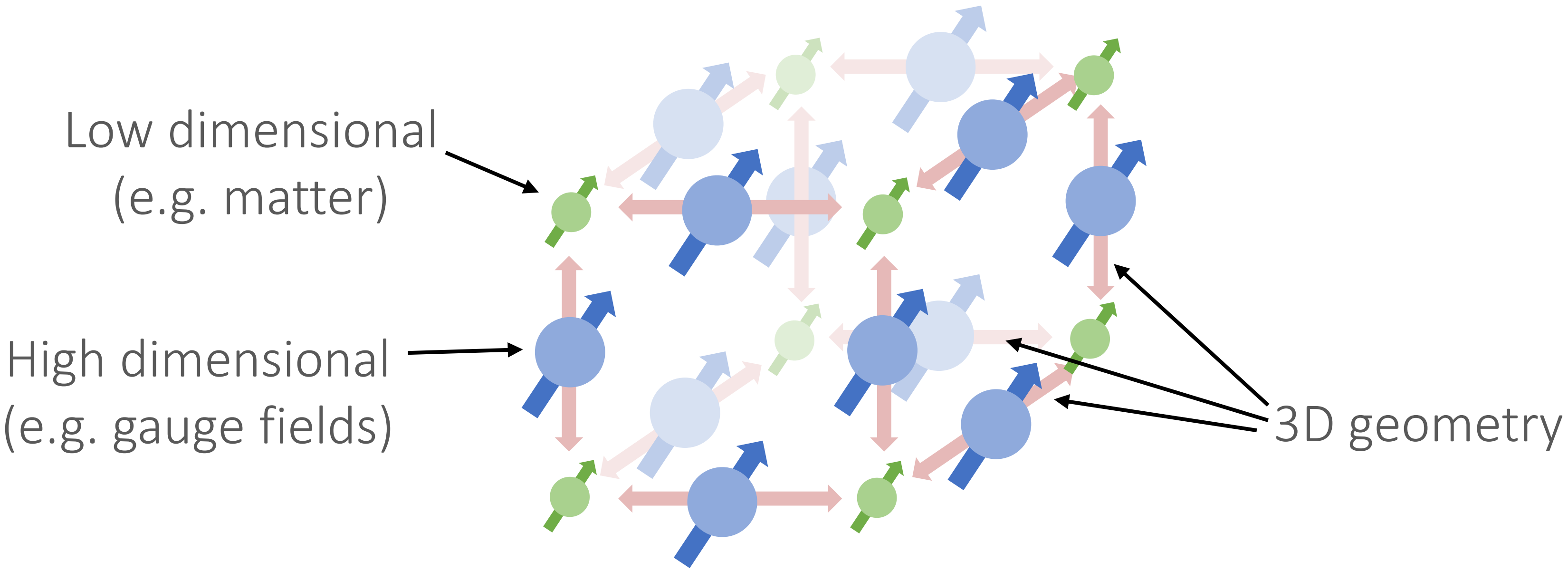




→ Full qudit readout with competitive performance ✓

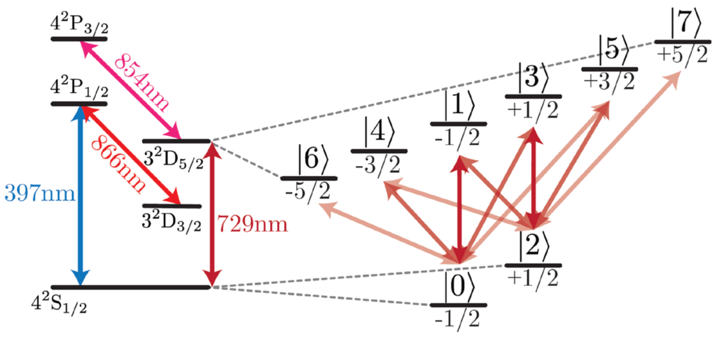
MR et al., Nature Physics **18**, 1053 (2022)

Natural Platform for Quantum Simulations



- ✓ Native support for mixed-dimensional systems w/o loss of fidelity
- ✓ Arbitrary geometries through all-to-all connectivity
- ✓ Fully compatible with quantum error correction methods

Take home message



✔ Universal qudit quantum computing

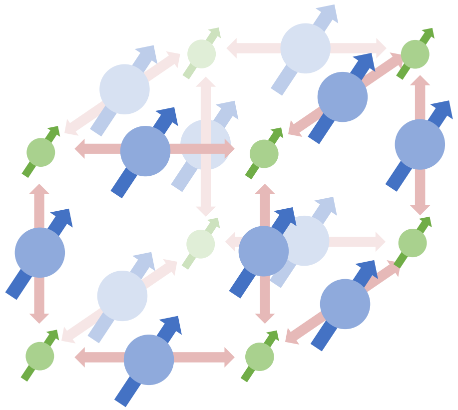
MR et al., Nature Physics **18**, 1053 (2022)

✔ Enables optimal measurements

R. Stricker, et al., PRX Quantum **3**, 040310 (2022)

✔ Natural platform for LGT simulations

- Scalable pathway towards Abelian LGT simulations in 3D
- Can be generalized to non-Abelian LGTs
  (recall e.g. talk by Daniel González-Cuadra and Pietro Silvi)
- Custom gates & higher dimensions available upon request



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The Innsbruck Ion Trappers 2023

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