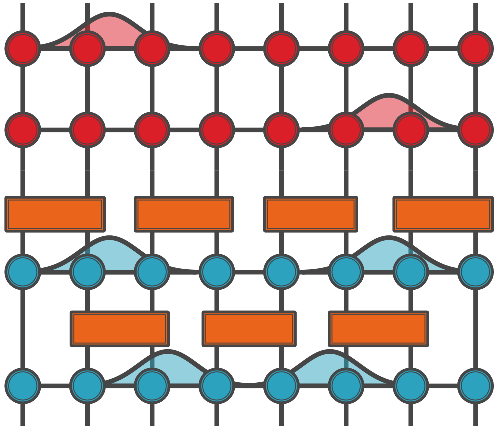
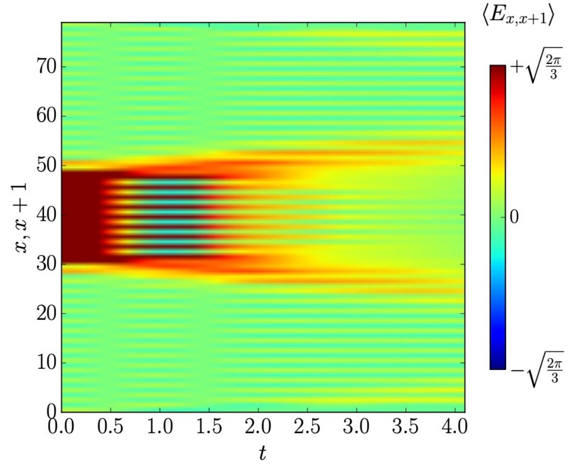
Tensor Network methods for real-time dynamics of lattice gauge theories





Giuseppe Magnifico

Department of Physics - University of Bari

Istituto Nazionale di Fisica Nucleare (INFN) - Bari

supercomputing-icsc.it

quantumbari.com

QuantHEP Conference
Bari, 25-27 September 2023

Real Time Dynamics and Confinement in the $\mathbb{Z}_n$ Schwinger-Weyl lattice model for 1+1 QED

Giuseppe Magnifico[1,2], Marcello Dalmonte[3,4], Paolo Facchi[5,6], Saverio Pascazio[5,6,7], Francesco V. Pepe[5,6], and Elisa Ercolessi[1,2]

[1]Dipartimento di Fisica e Astronomia dell'Università di Bologna, I-40127 Bologna, Italy
[2]INFN, Sezione di Bologna, I-40127 Bologna, Italy
[3]Abdus Salam ICTP, Strada Costiera 11, I-34151 Trieste, Italy
[4]SISSA, Via Bonomea 265, I-34136 Trieste, Italy
[5]Dipartimento di Fisica and MECENAS, Università di Bari, I-70126 Bari, Italy
[6]INFN, Sezione di Bari, I-70126 Bari, Italy
[7]Istituto Nazionale di Ottica (INO-CNR), I-50125 Firenze, Italy

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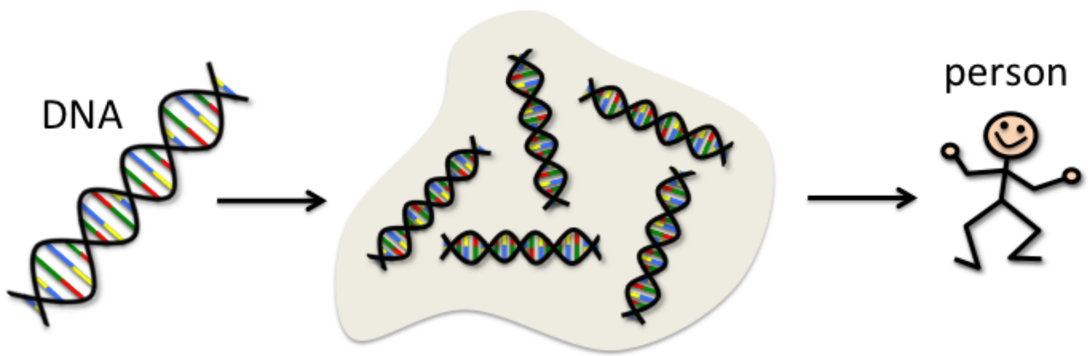
PHYSICAL REVIEW D **104,** 114501 (2021)

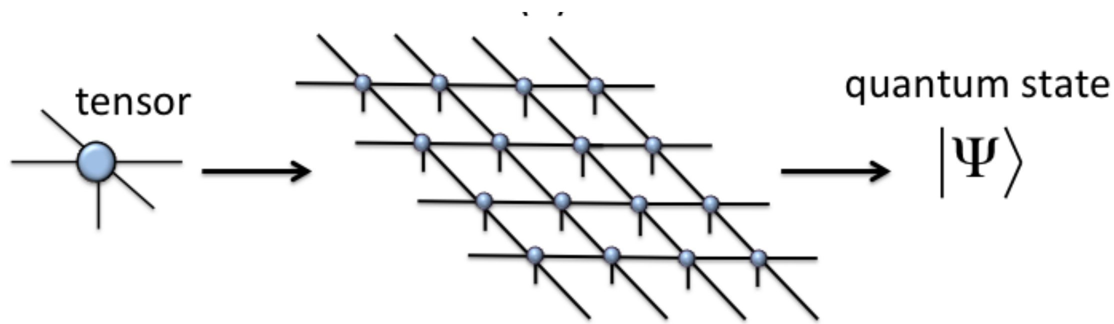
Entanglement generation in (1+1)D QED scattering processes

Marco Rigobello,* Simone Notarnicola, Giuseppe Magnifico, and Simone Montangero

Dipartimento di Fisica e Astronomia "G. Galilei", via Marzolo 8, I-35131 Padova, Italy and Padua Quantum Technologies Research Center, Università degli Studi di Padova and INFN, Sezione di Padova, via Marzolo 8, I-35131 Padova, Italy

Tensor Networks





Wave function is described by a network of interconnected tensors

Network pattern directly represents the amount of entaglement of the state

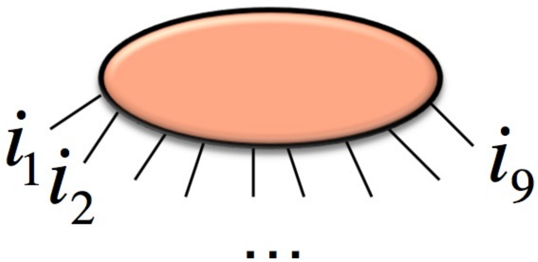
Annals of Physics 349 (2014) 117-158
SciPost Phys. Lect. Notes 8 (2019)

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d-level systems

Tensor (multidimensional array of complex numbers)

$$C_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$$

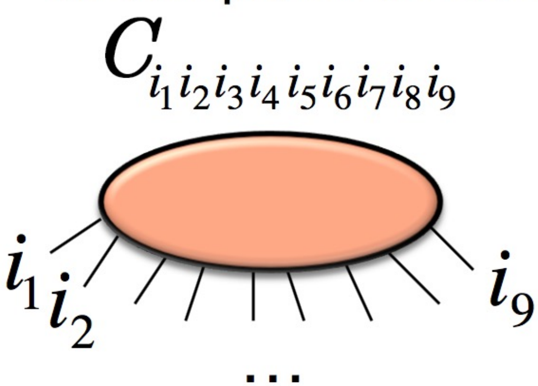


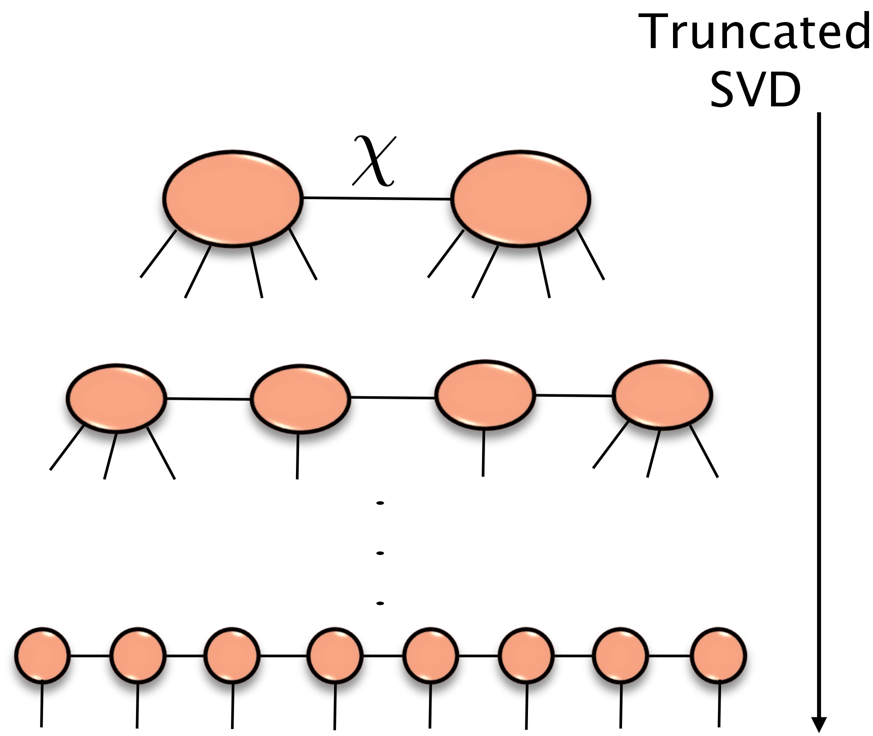
$O(d^N)$ representation, exponentially large in the system size. Inefficient.

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d-level systems

Tensor (multidimensional array of complex numbers)





$O(d^N)$ representation, exponentially large in the system size. Inefficient.

$$N\chi^2 d$$

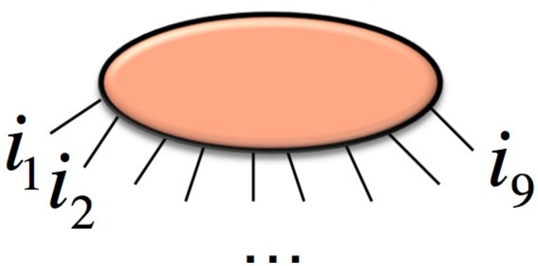
$$|\psi\rangle = \sum_{\{s_i\},\{\alpha_i\}} A^{(s_1)}_{\alpha_1} A^{(s_2)}_{\alpha_1,\alpha_2} \ldots A^{(s_N)}_{\alpha_{N-1}} |s_1, s_2, \ldots, s_N\rangle$$

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d−level systems

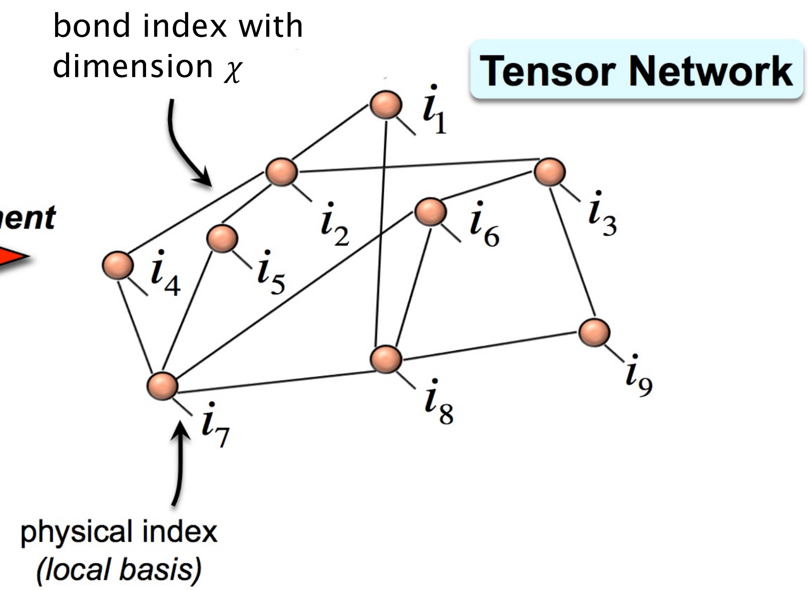
Tensor (multidimensional array of complex numbers)

$C_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$



replacement

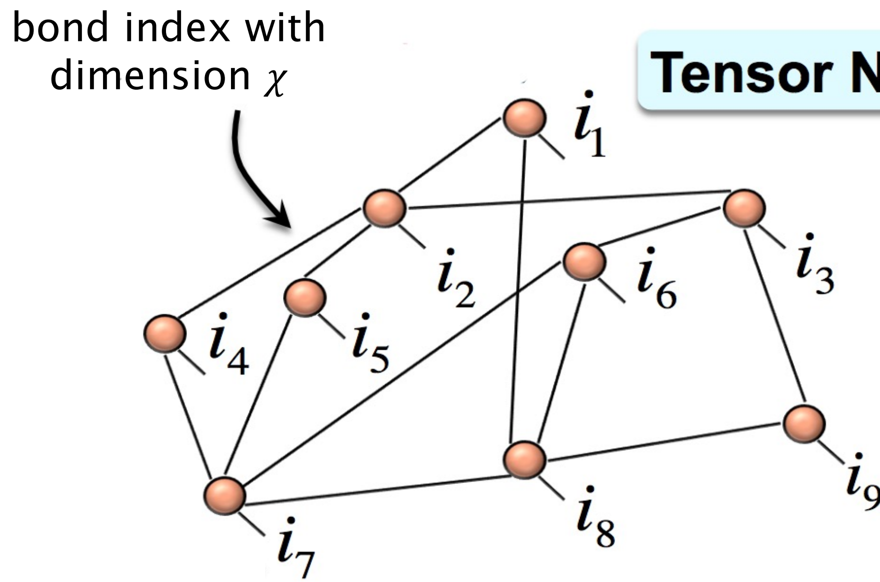
multiple SVD

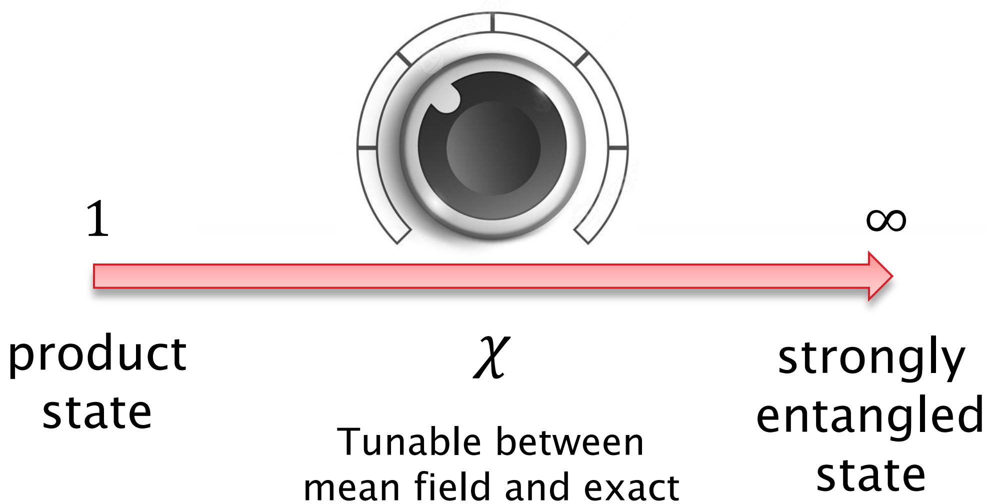


$O(d^N)$ representation, exponentially large in the system size. Inefficient.

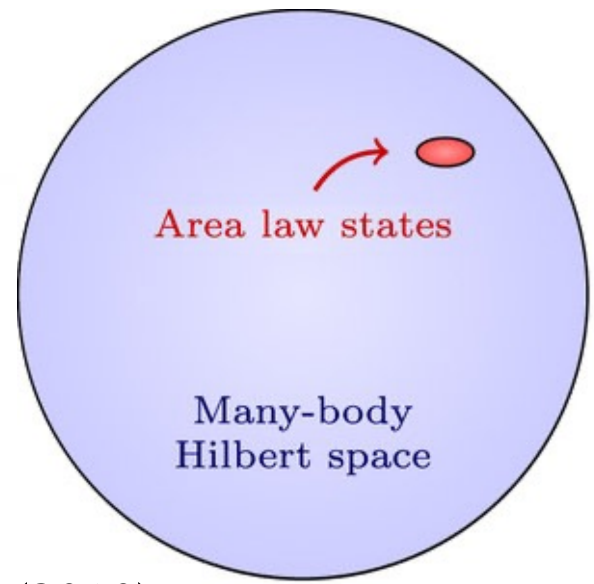
The number of parameters is $O(poly(N)poly(\chi))$. Efficient!

Tensor Network





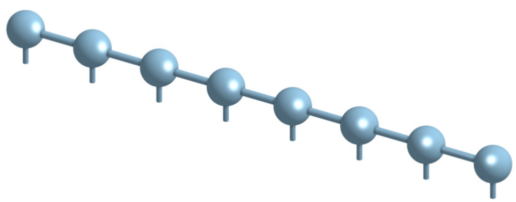
Low-Energy States



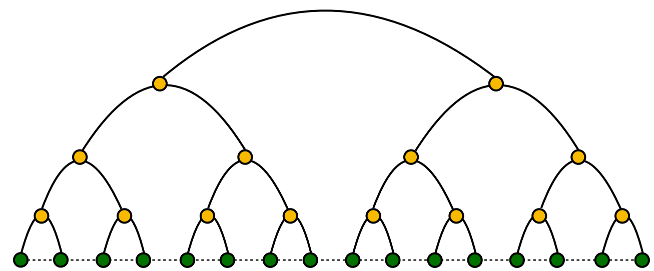
Rev. Mod. Phys. 82, 277 (2010)

Tensor Network structures

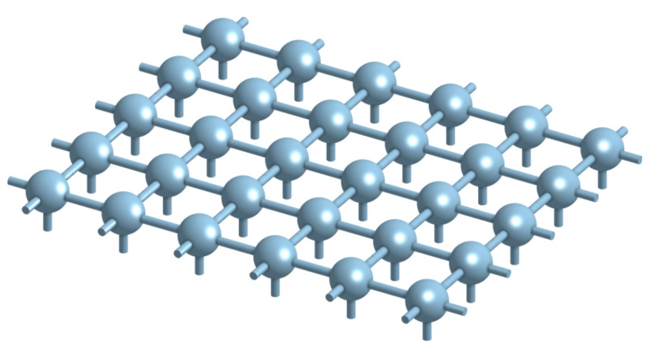
Matrix Product States (MPS)

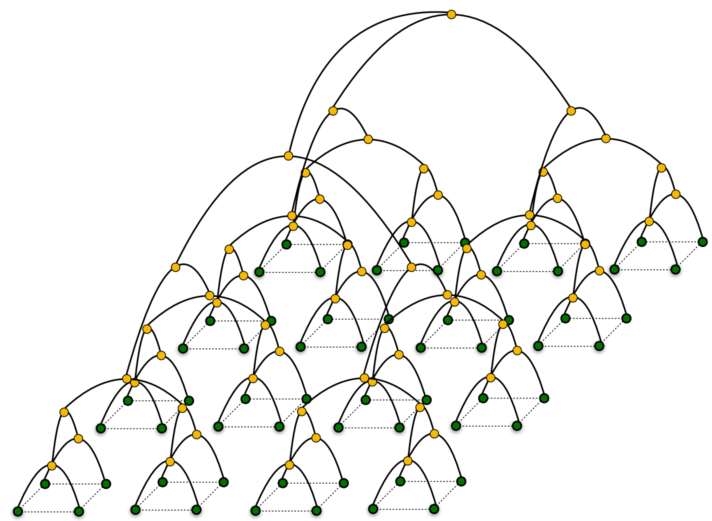


Tree Tensor Networks (TTN)



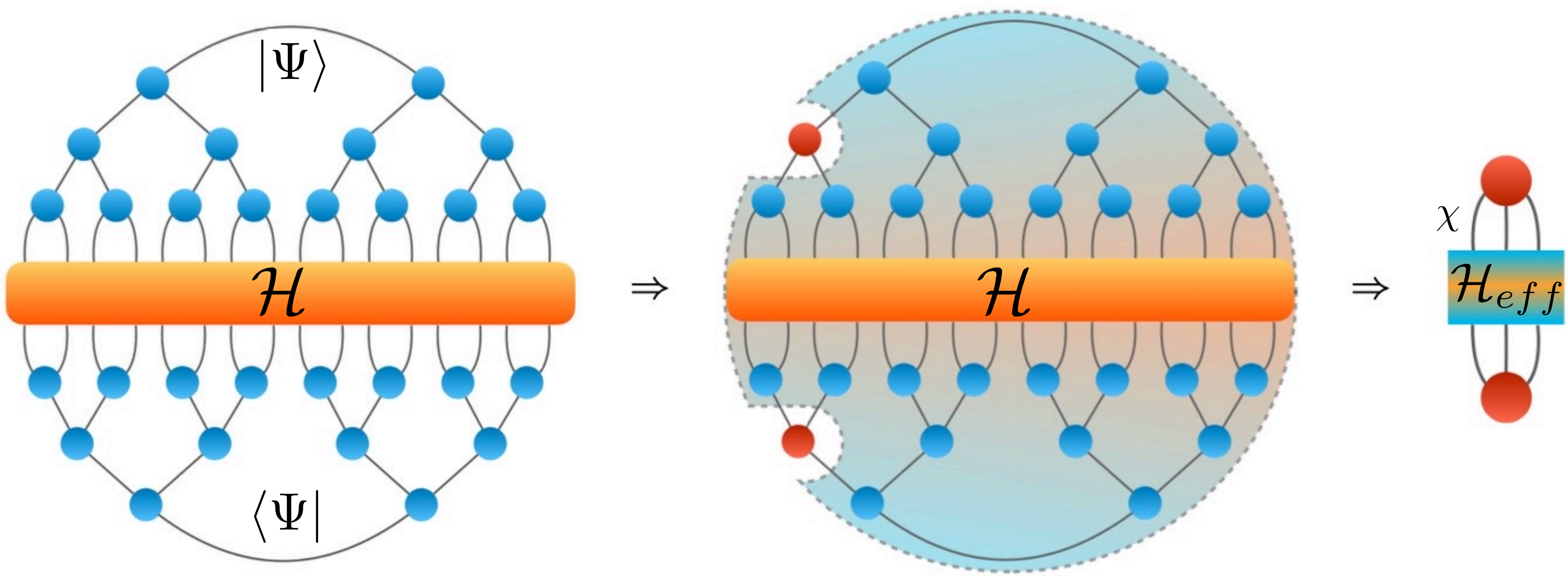
Projected Entangled Pair States (PEPS)





Tensor Network algorithms

Energy minimisation: $\min_{\Psi}\{E(\Psi)\} = \min_{\Psi}\{\langle\Psi|\mathcal{H}|\Psi\rangle\}$



At the end: E_{gs} , $|\Psi_{gs}\rangle$ and low-energy states

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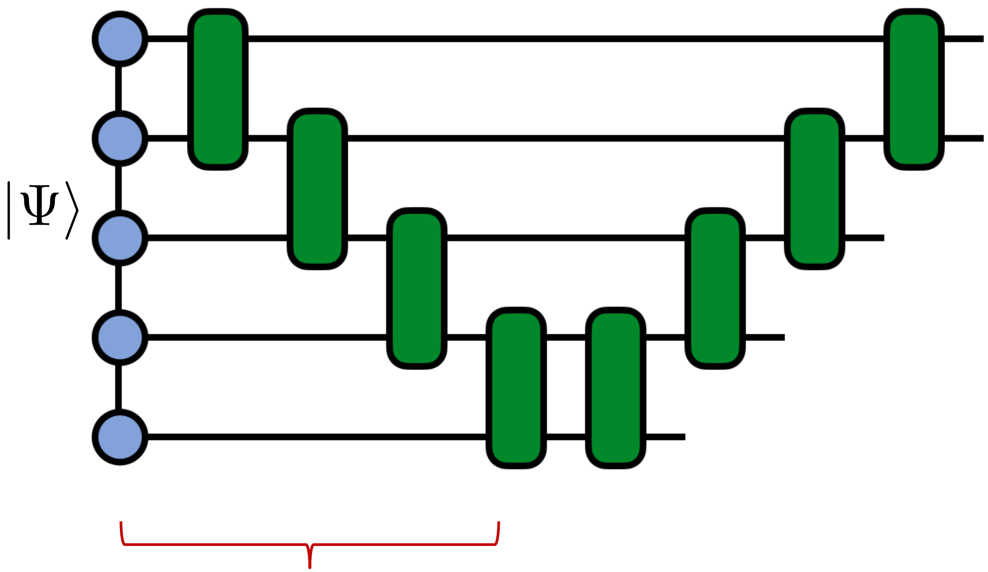
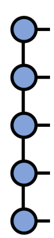
Tensor Network algorithms

<u>Real-time dynamics</u> (and simulation of quantum circuits) :

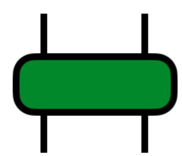


compression
to truncated
MPS form $\approx$



$$\mathcal{H} = \sum_j h_{j,j+1}$$

$$e^{-i\tau H} \approx e^{-ih_{1,2}\tau/2} e^{-ih_{2,3}\tau/2} \cdots e^{-ih_{N-1,N}\tau/2}$$

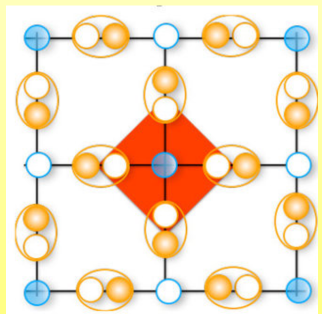


$$= e^{-i\tau h_{j,j+1}/2}$$

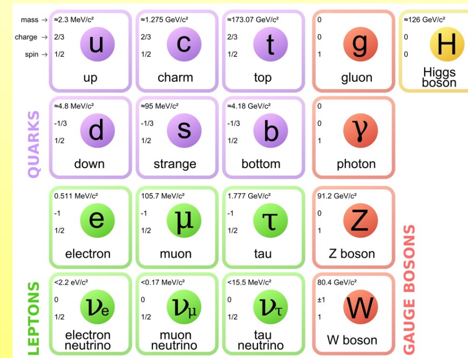
Direct access to real-time evolution:
observables and entanglement dynamics

LGT are almost everywhere in physics!

As emergent theories in condensed matter: **high-Tc superconductors, frustrated systems, spin liquids.**



As fundamental description in particle physics: **Standard Model**



- They are <u>**extremely demanding**</u> from a numerical point of view (quantum matter + quantum fields + gauge symmetries).
- Powerful numerical methods, such as Monte Carlo, fail in several regimes of finite-density or for non-equilibrium phenomena (**sign-problem**).
- Ideal goal for quantum-inspired efficient algorithms and quantum simulation/computation: **no sign-problem**!

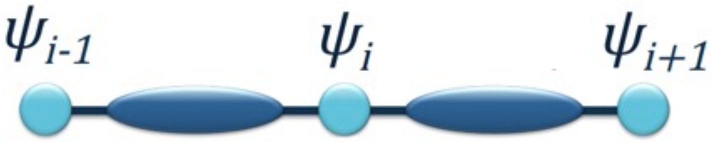
The Schwinger model: 1+1D QED

(1+1)D

$$H = \bar{\psi}(i\gamma^x D_x + m)\psi + \frac{1}{2}(E^2)$$

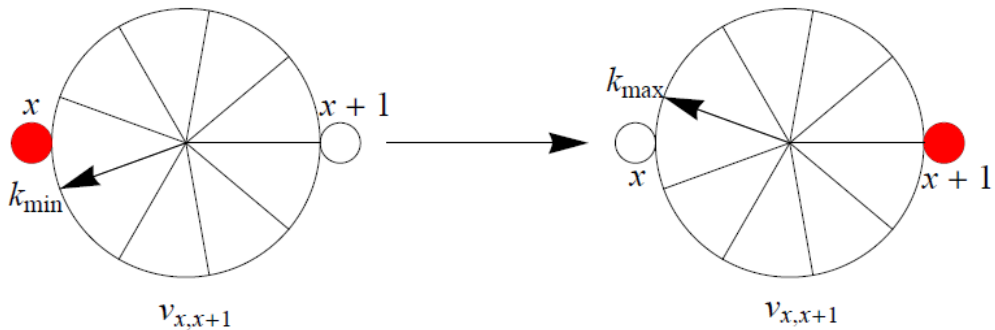
We have to perform a discretization on a 1d chain. How?

- Matter on lattice sites;
- Field on the links;

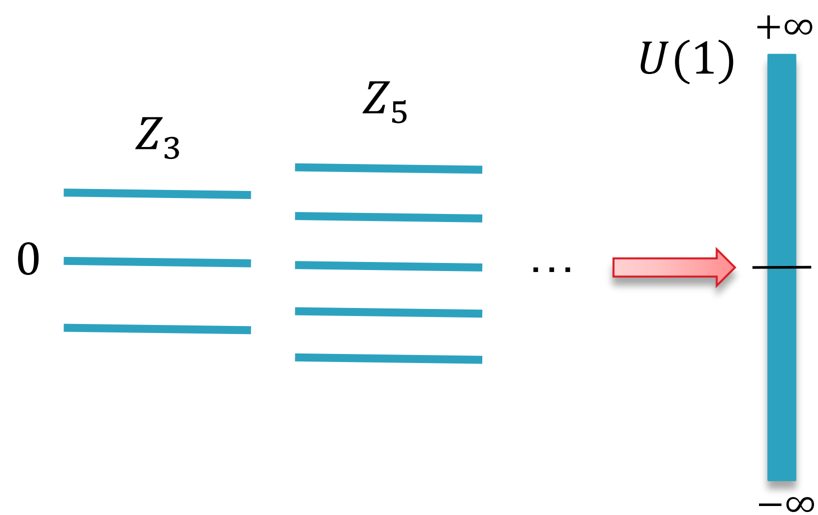


The $U(1)$ original symmetry is replaced by a Z_n discrete symmetry

$$\hat{H} = -t\sum_x \hat{\psi}_x^\dagger \hat{U}_{x,x+1}\hat{\psi}_{x+1} + \text{h.c.} + m\sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g^2}{2}\sum_x \hat{E}^2_{x,x+1}$$

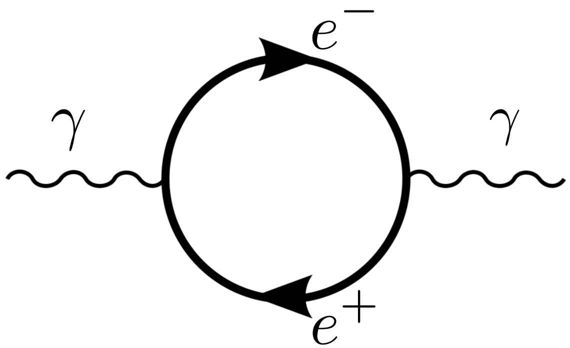


Discretization of the electric field levels with spacing $\epsilon = \sqrt{2\pi/n}$.



Real–time dynamics of pair production

Dynamical creation of matter,
particle–antiparticle pairs,
from the vacuum.



- Paradigmatic phenomenon in QFT.
- Non–perturbative effect in the coupling expansion.

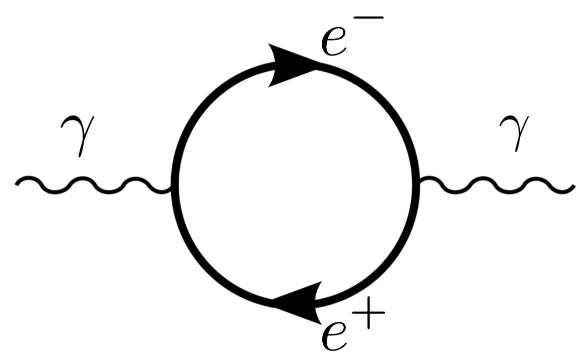
Typically studied in the presence of a **strong external electric field.** Pair proliferation is expected at

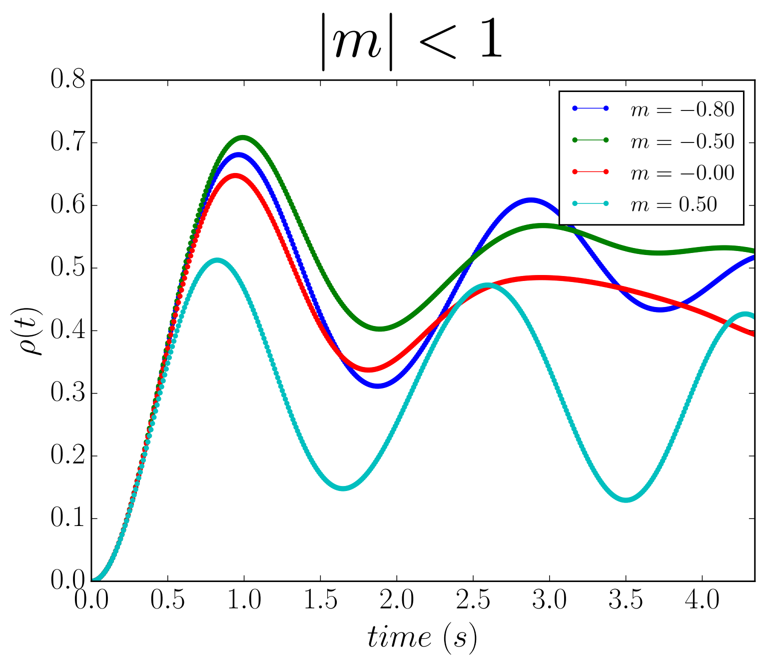
$$E_{ext} = m^2/e \approx 1.3 \times 10^{18} \ V/m$$

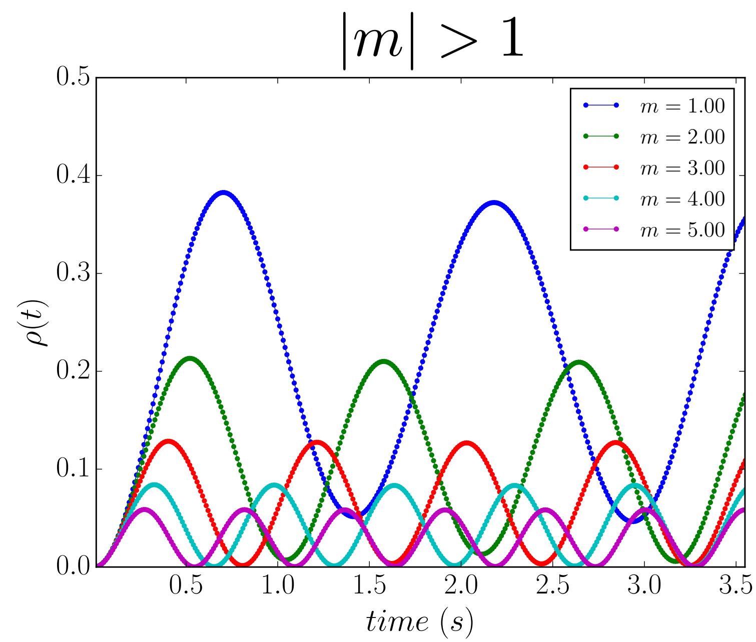
out of reach of the most powerful lasers, such as the extreme light infrastructure ELI.

Real-time dynamics of pair production

Dynamical creation of matter, particle-antiparticle pairs, from the vacuum.





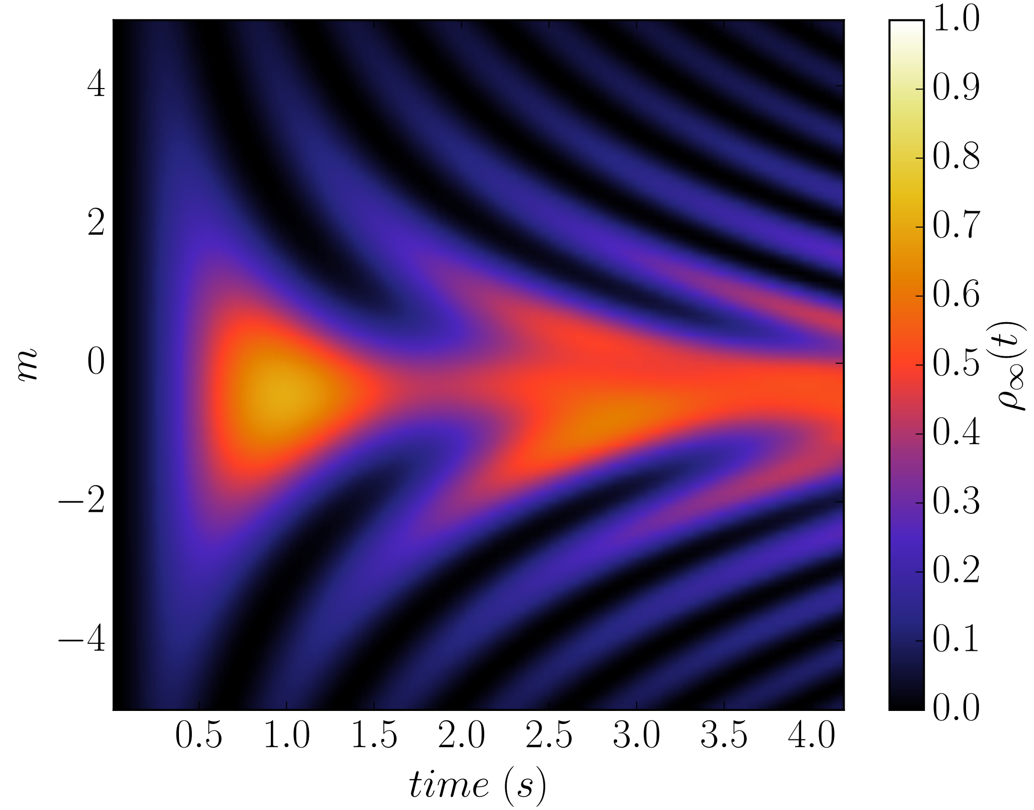


Quantum 4, 281 (2020)

Spontaneous Pair Production

Density profile for each value of m.

In the region around $m \approx -0.50$, pair production is clearly dominant in the dynamics



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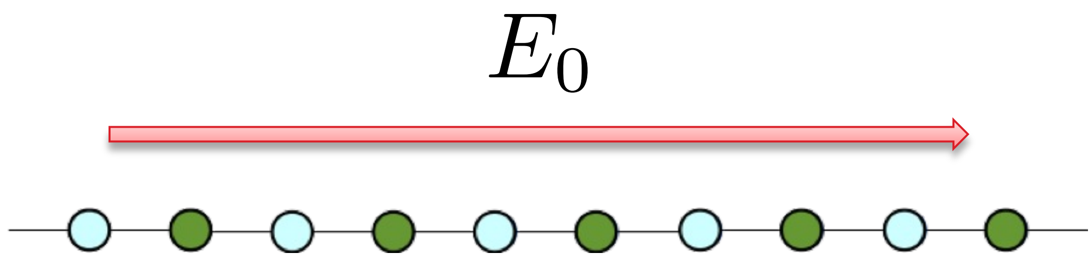
Pair Production in external field

constant external field



From the theory we have the *Schwinger's formula*.

$$\dot{\rho} = \frac{eE_0}{2\pi} \exp\left(-\frac{\pi m^2}{eE_0}\right) = \frac{m^2}{2\pi} \epsilon \exp\left(-\frac{\pi}{\epsilon}\right)$$

$$\epsilon = E_0/E_c$$

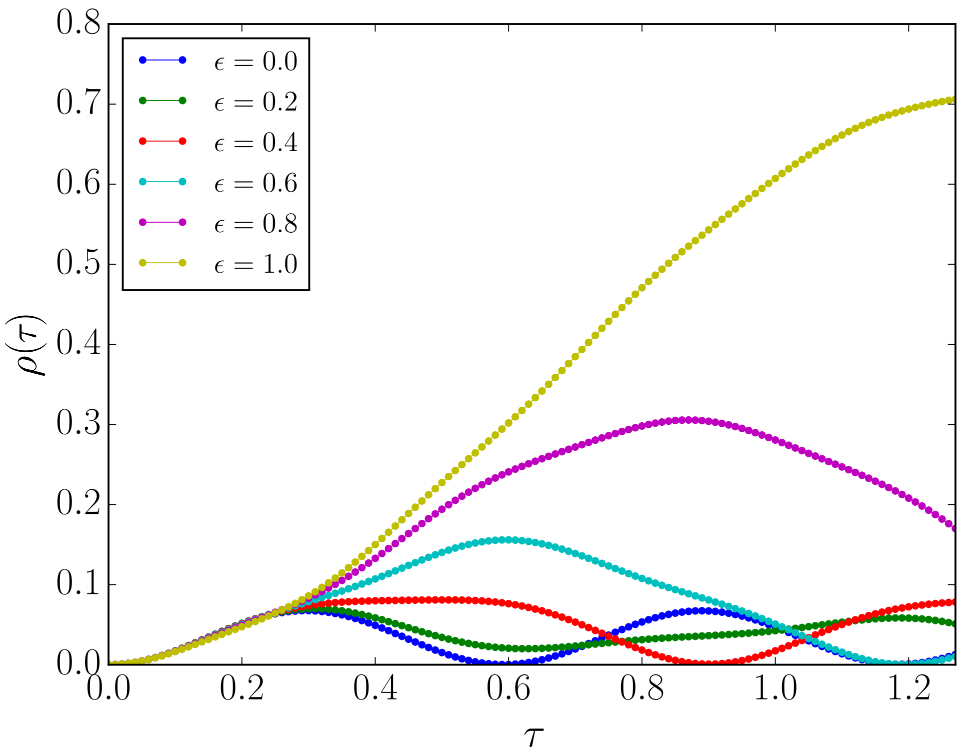
$$E_c = m^2/e$$

Z_3 – model: Pair Production

$$\dot{\rho} = \frac{eE_0}{2\pi} \exp\left(-\frac{\pi m^2}{eE_0}\right) = \frac{m^2}{2\pi} \epsilon \exp\left(-\frac{\pi}{\epsilon}\right) \qquad \epsilon = E_0/E_c$$

We choose a point in which the bare vacuum is stable, $m = 4.5$.

Evolution for different values of the external field (i.e. the ratio ϵ): qualitative agreement with the *Schwinger's formula!*



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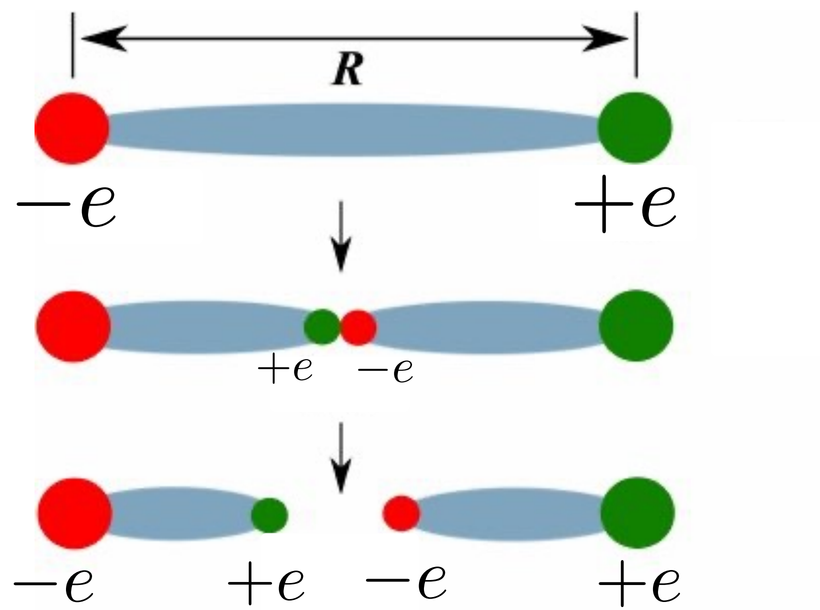
Real-time dynamics of String Breaking

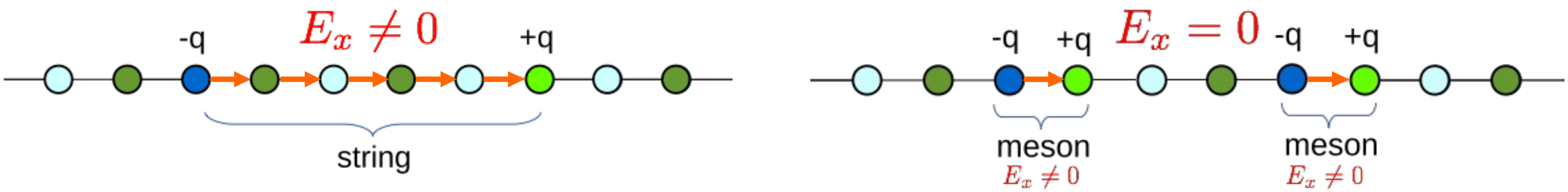
String breaking:
a consequence of confinement, and one fundamental aspects of gauge theories

1+1 QED:
the potential increases linearly with charge's distance, as for quarks in higher dimensional QCD

$$V(R) \propto R$$



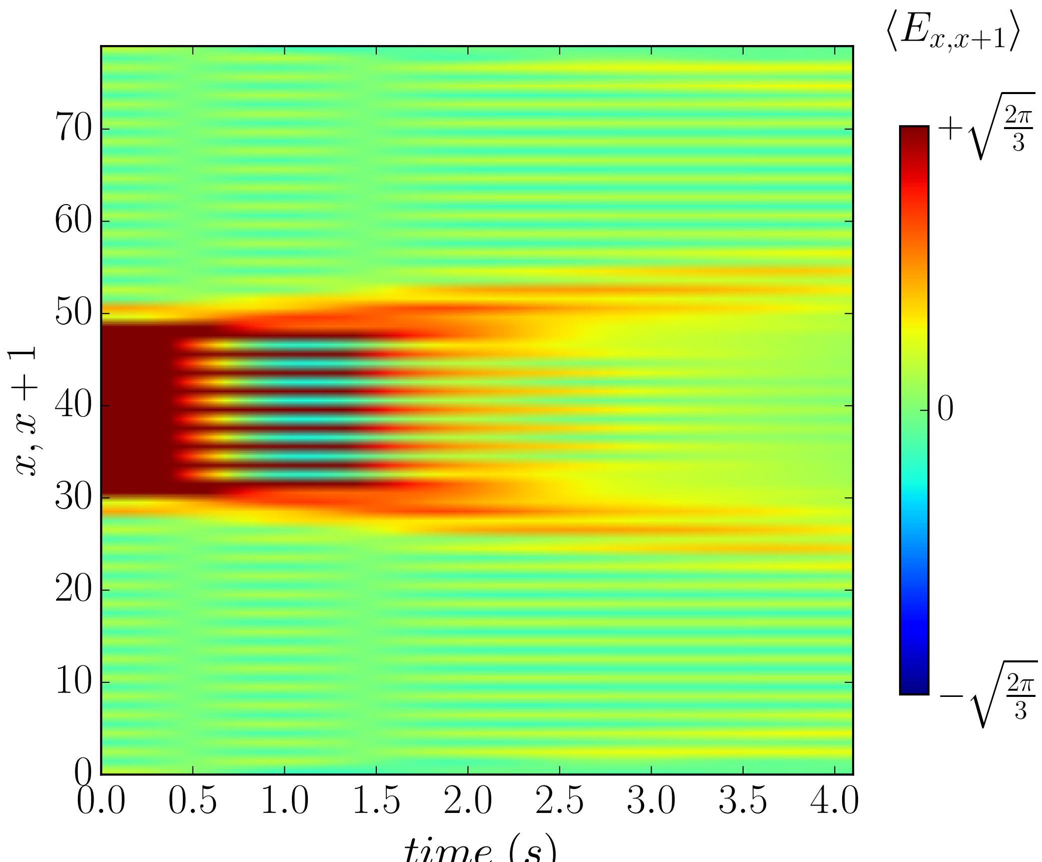
Real-time dynamics of String Breaking



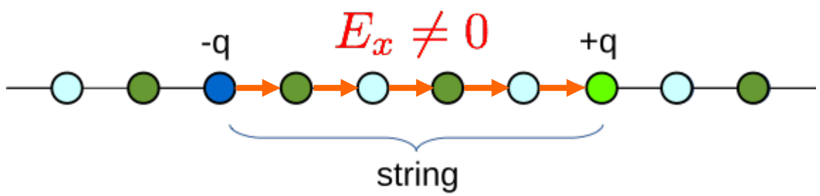
$$m = g = 0.1$$

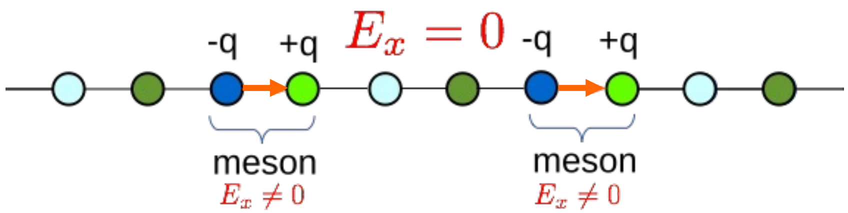
Confinement of charges

Quantum 4, 281 (2020)



Real-time dynamics of String Breaking

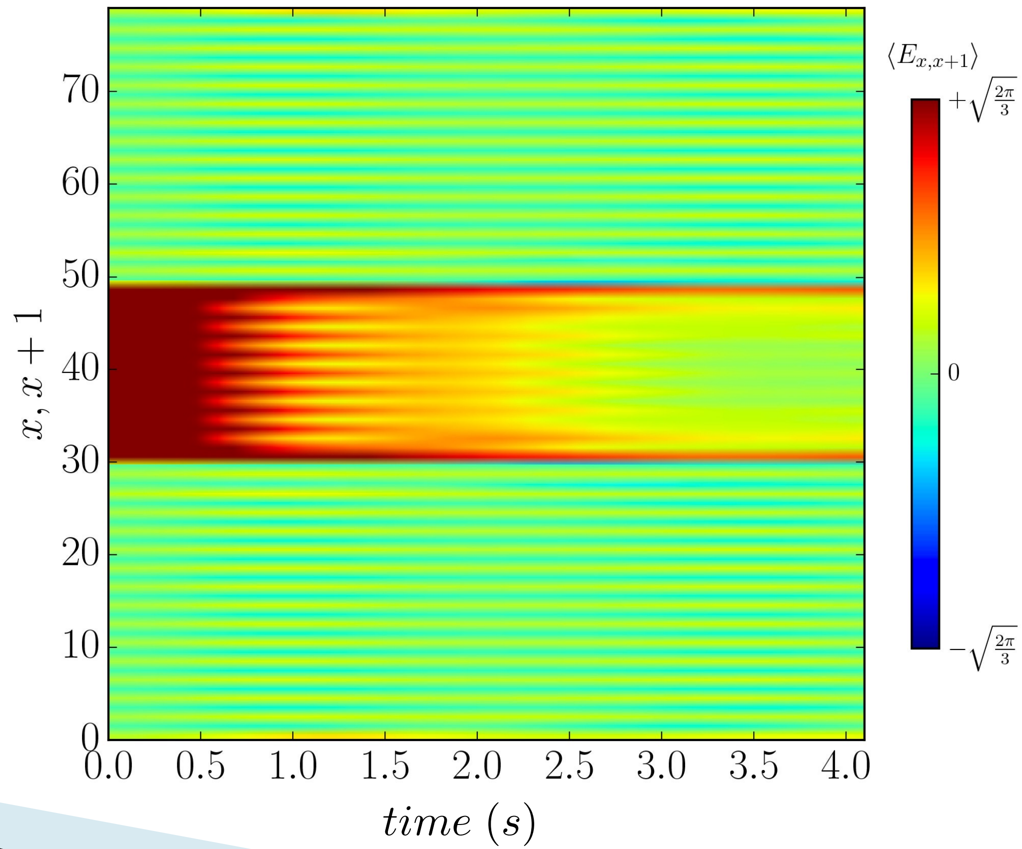




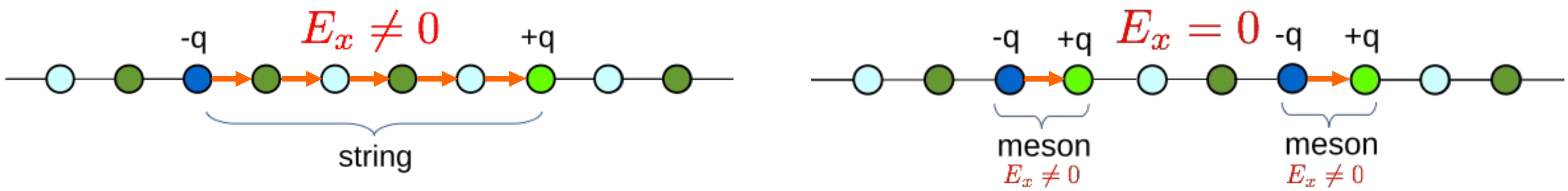
$$m = 0.3 \quad g = 0.8$$

Confinement of charges

Quantum 4, 281 (2020)

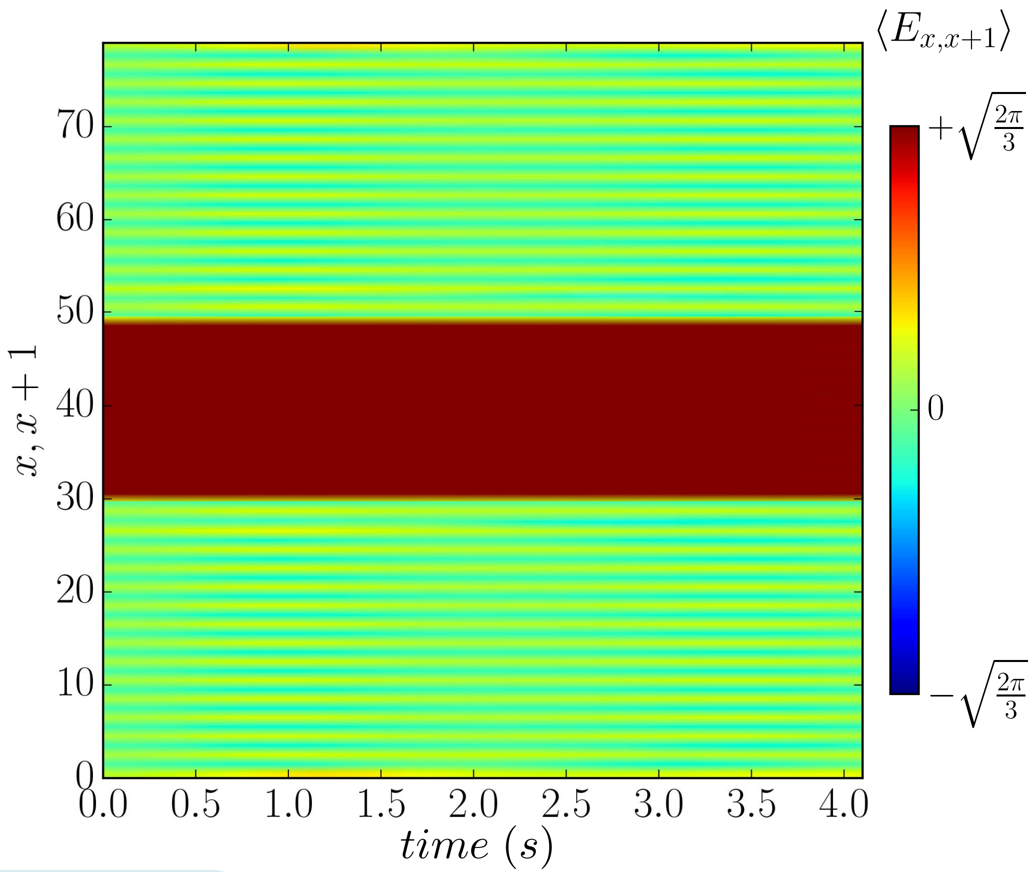


Real-time dynamics of String Breaking



$$m = 3.0 \quad g = \sqrt{2}$$

The string remains stable

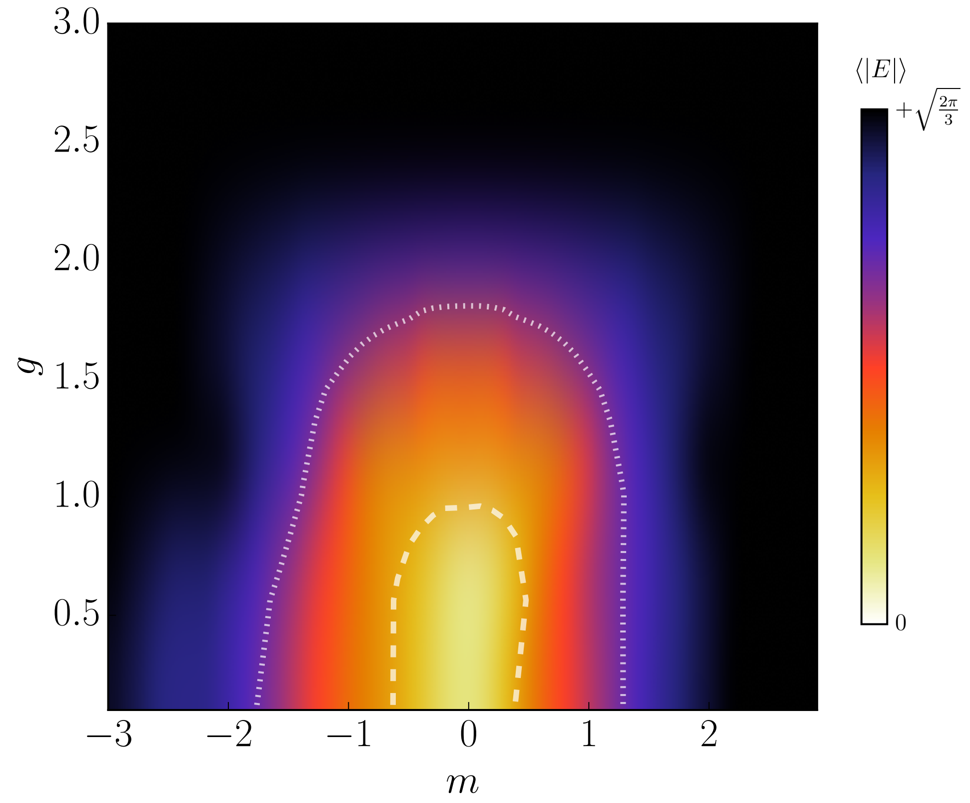


Quantum 4, 281 (2020)

String Breaking

Considering the minimum value reached by the electric field of the string, a "phase-diagram» of the string breaking effect is obtained.

Lines correspond to 10% and 50% of the initial value.



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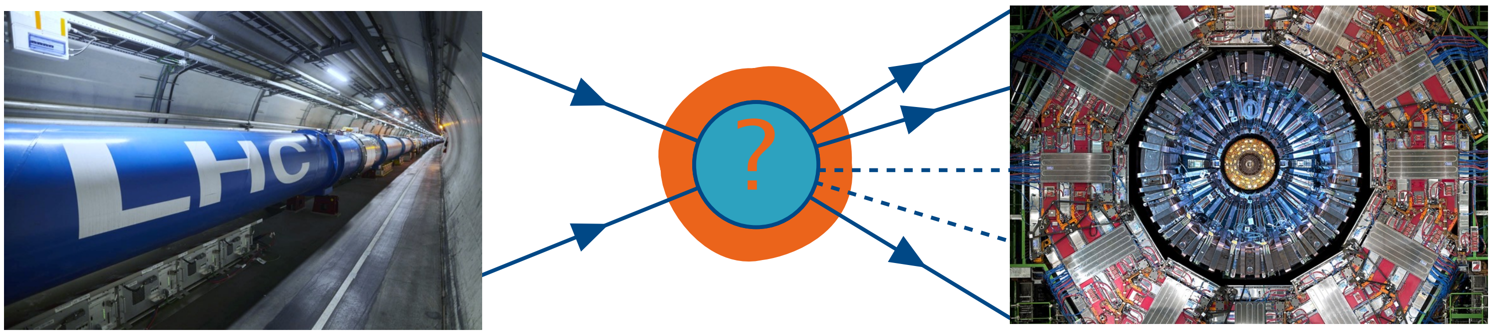
Scattering

incoming ‘free’ particles

outgoing ‘free’ particles



experiment

S-matrix

experiment

perturbative
expansion
Monte Carlo

TN **real-time** dynamics

Scattering: simulation scheme with MPS

variational vacuum MPS via DMRG

⬇

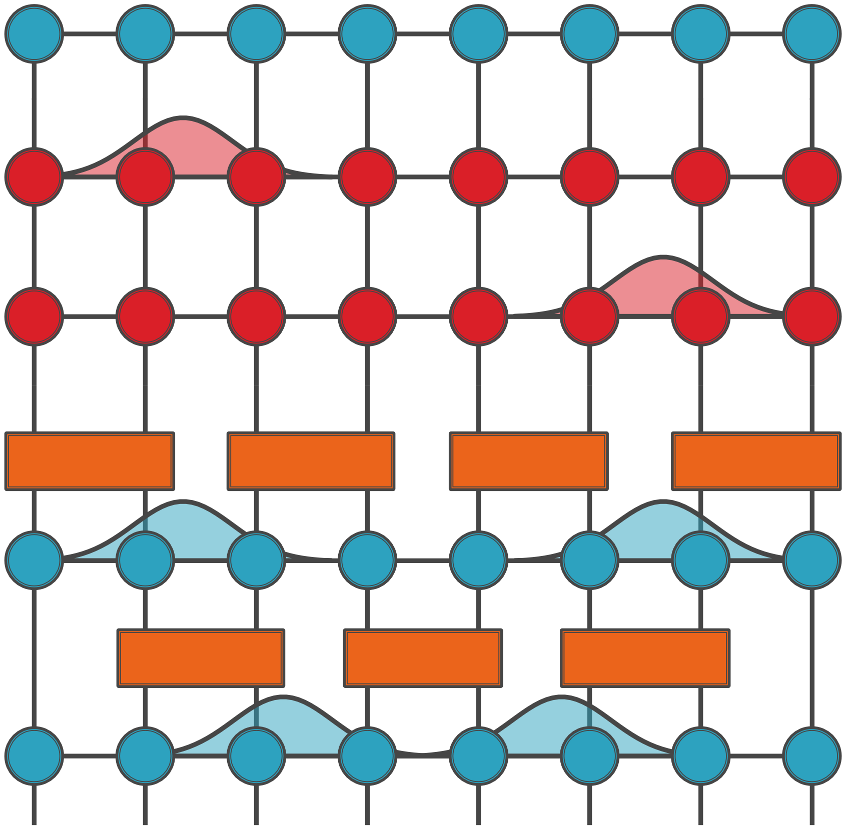
initial state via

wave packet creation MPOs

⬇

time evolution via TEBD

& monitor observables

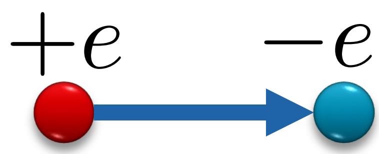


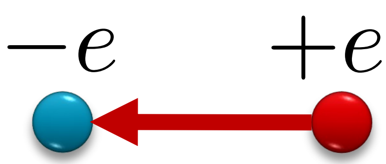
Phys. Rev. D **104**, 114501 (2021)

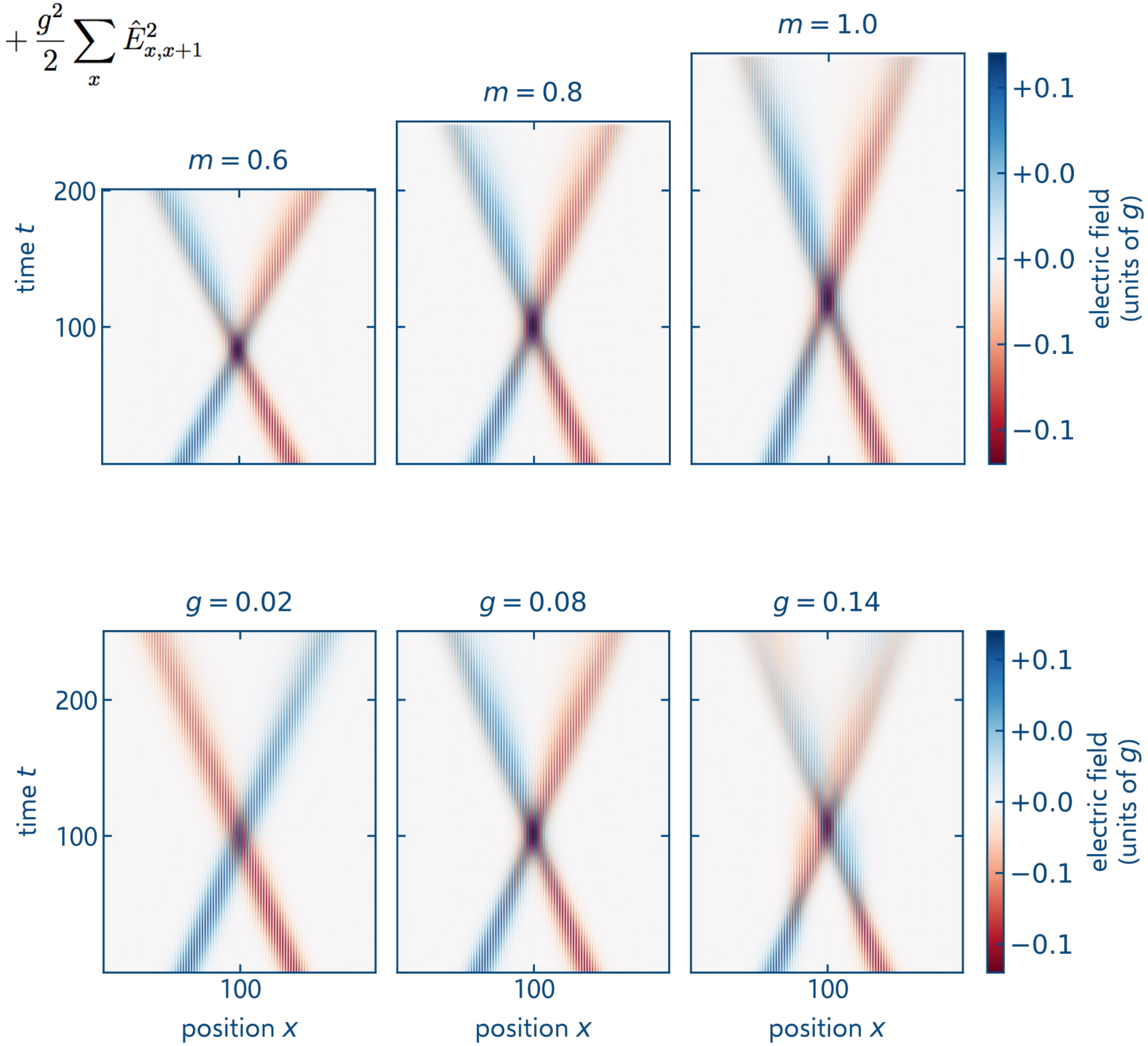
Tensor Networks – Scattering dynamics

$$\hat{H} = -t\sum_x \hat{\psi}_x^\dagger \hat{U}_{x,x+1}\hat{\psi}_{x+1} + \text{h.c.} + m\sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g^2}{2}\sum_x \hat{E}_{x,x+1}^2$$

Mesons with opposite momenta and internal electric fields







Phys. Rev. D **104**, 114501 (2021)

Tensor Networks – Scattering dynamics

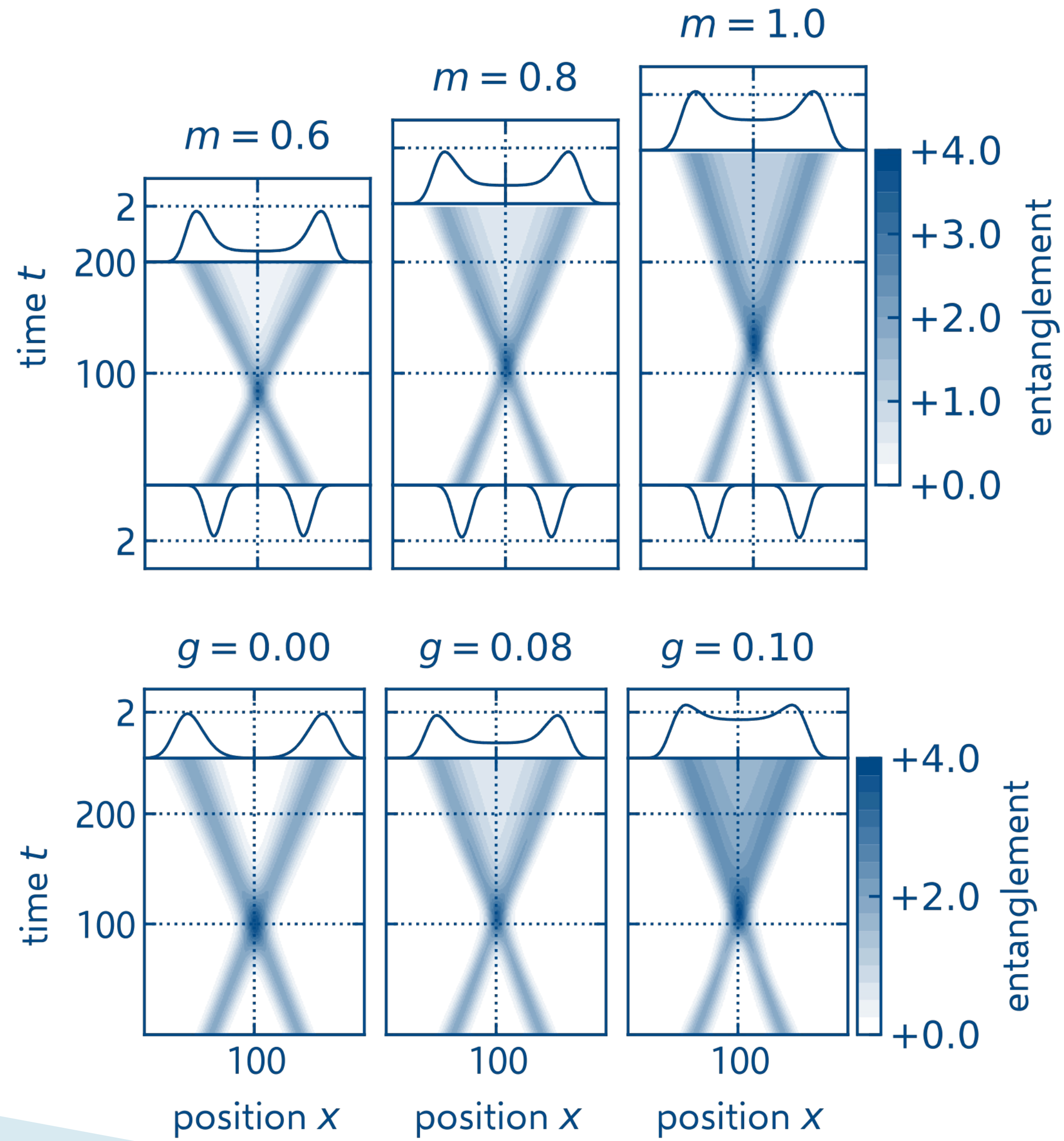
Vacuum

+

wave packets
intra–particle

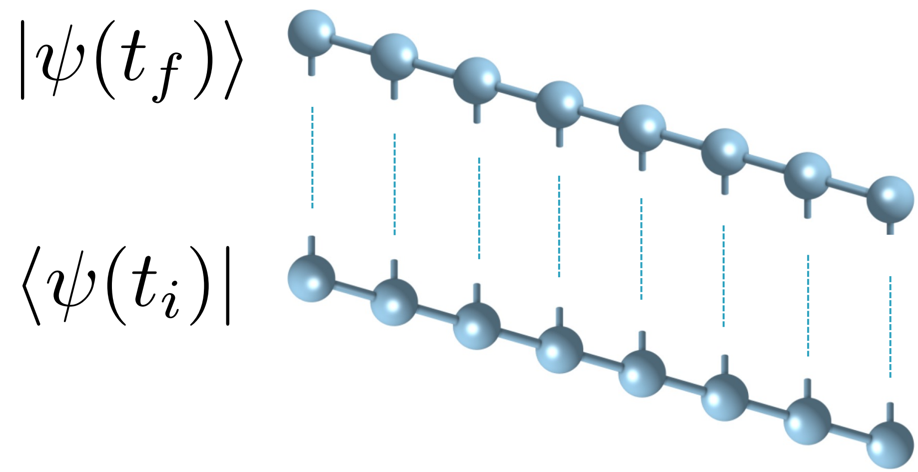
+

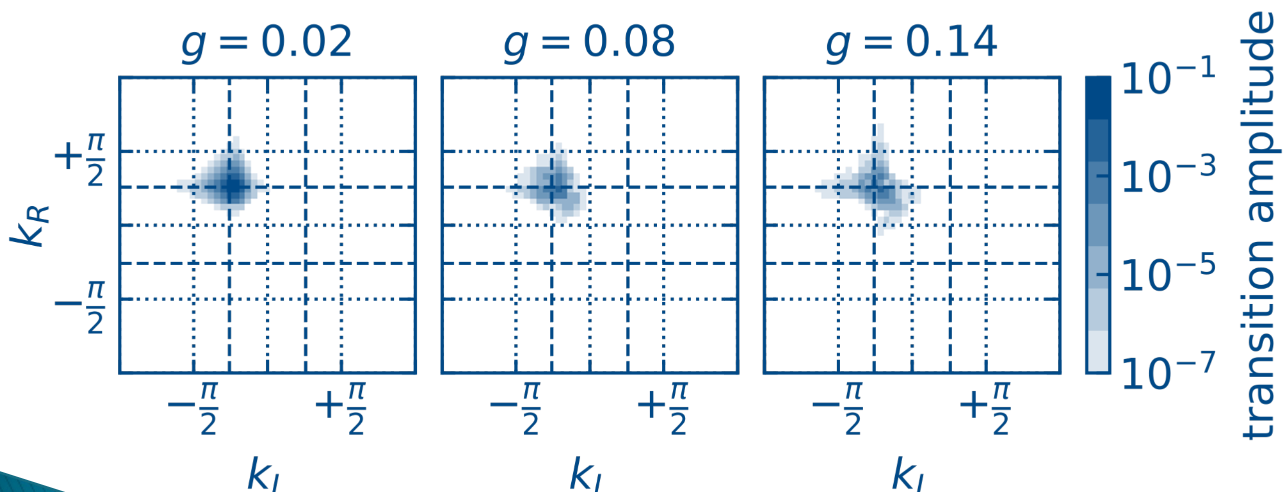
interactions
inter–particle



Tensor Networks – Scattering dynamics

overlap of final state with
pair of meson wave packet





~ S-matrix elements

Phys. Rev. D **104**, 114501 (2021)

Talks on TN at QuantHEP 2023

PIETRO SILVI
University of Padua

Advances in Lattice Gauge Theories with Tensor Networks

26 Sept, 10:30

GIOVANNI CATALDI
University of Padua

(2+1)D SU(2) Yang-Mills Lattice Gauge Theory at finite density via tensor networks

26 Sept, 12:00

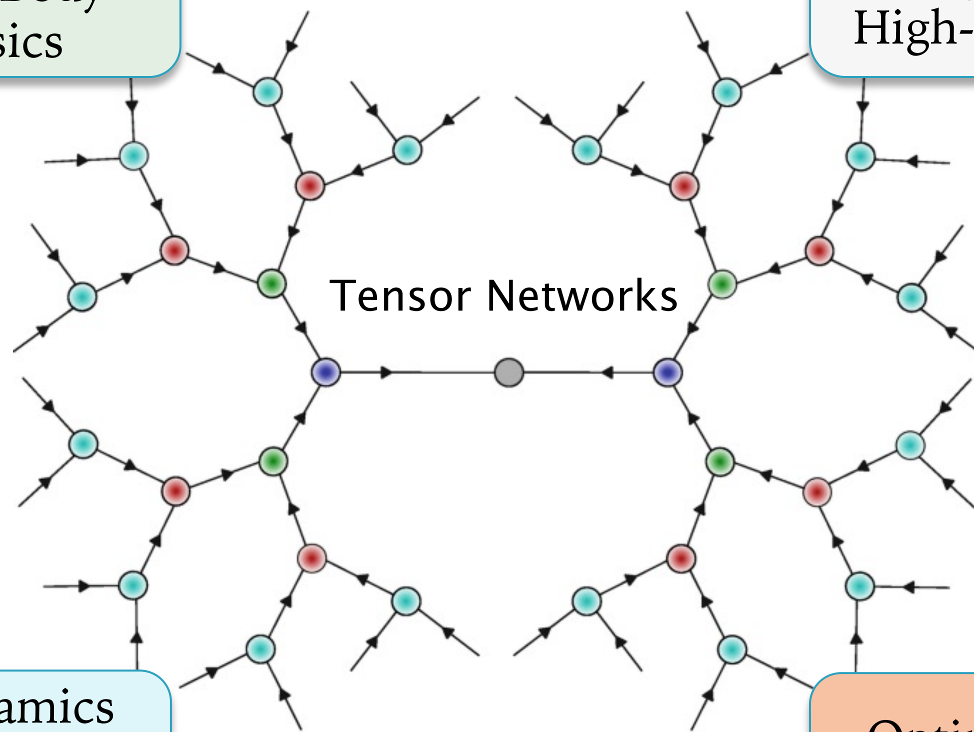
MARCO RIGOBELLO
University of Padua

Hadrons in (1+1)D Hamiltonian hardcore lattice QCD

27 Sept, 10:30

Quantum Many-Body Physics

Sign-problem-free algorithms for High-Energy Physics



Real-time dynamics of non-perturbative phenomena

Optimization problems

Thank You!