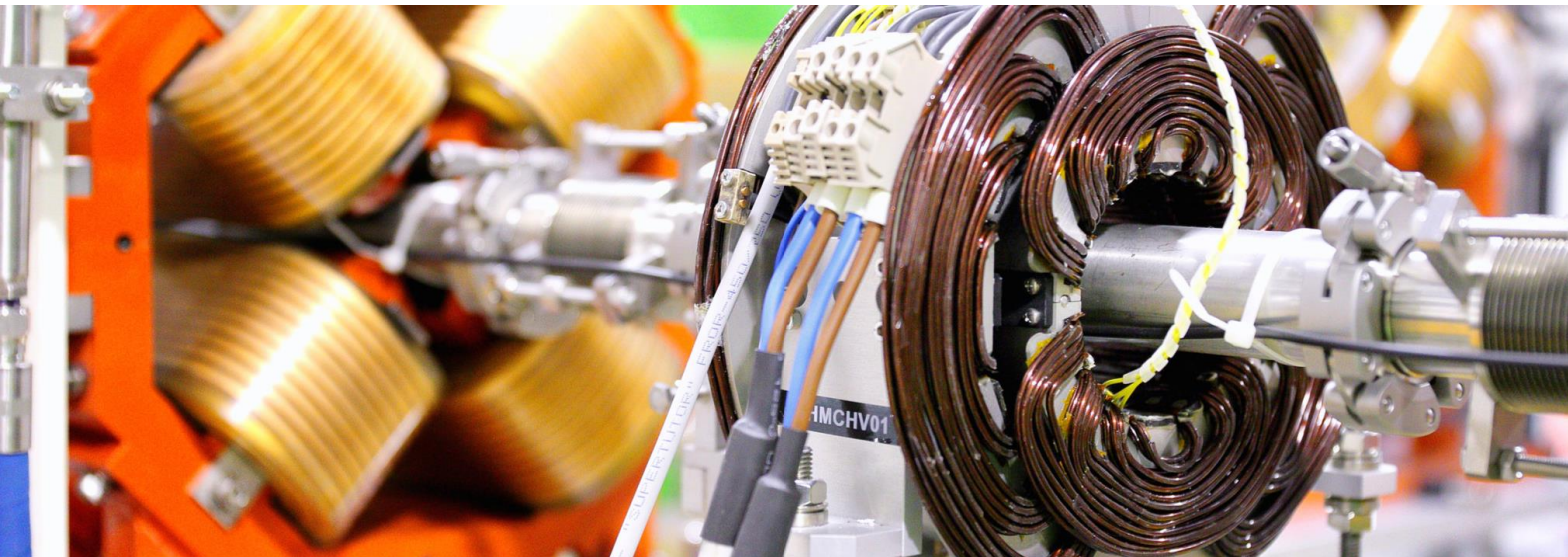# Magnetic Measurements

Particle Accelerators Physics PhD –Università di Roma La Sapienza

Istituto Nazionale di Fisica Nucleare – Laboratori Nazionali di Frascati

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Divisione Acceleratori - Servizio Ingegneria Elettrotecnica

References

- Transverse Beam Dynamics (A. Latina) lectures of JUAS 2016 Course1

- Magnets Course (T. Zickler) JUAS 2016 Course2

- Magnets Course (D. Tommasini) JUAS 2016 Course2

- Superconducting Magnets (T. Wilson) JUAS 2015 Course 2

- CERN Accelerator School (CAS) – Magnets 2006 (CERN–2010–004)

Outline

- Recall: Maxwell equations
- What are the Magnets Tasks in Particle Accelerators
- Magnet Types
- Air Dominated vs Iron Dominated Magnets
- Iron Dominated Magnets Iron Yoke
- Field Description in Air Gap
- Magnetic Length
- Relevant Magnetic Parameters in Magnets
- Magnetic Field Quality
- Magnetic Measurements

SPark_Magete
da INFN - Laboratori Nazionali di Frascati
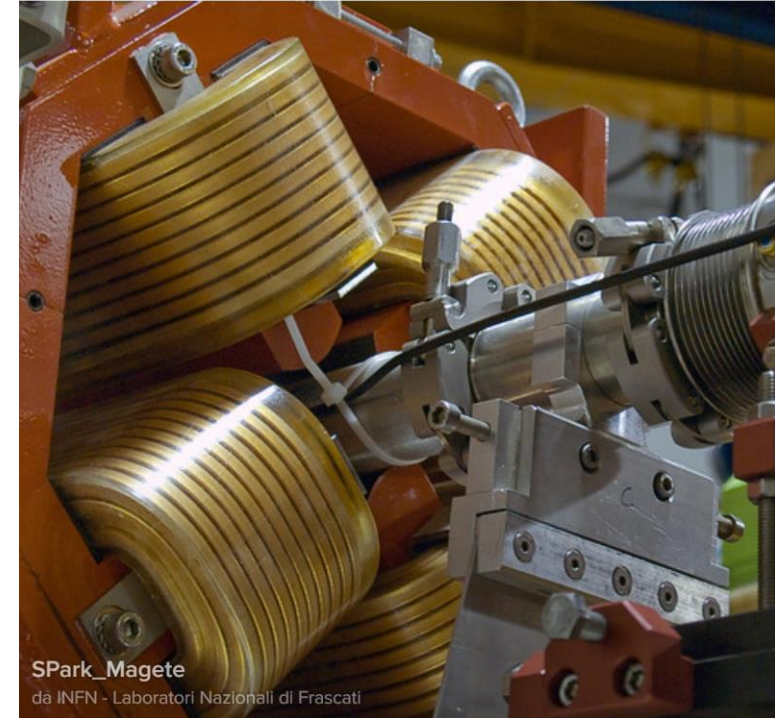
Maxwell's equations

In 1873, Maxwell published "Treatise on Electricity and Magnetism" in which he summarized the discoveries of Coulomb, Oersted, Ampere, Faraday, et. al. in four mathematical equations:

Gauss' law for electricity:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Gauss' law of flux conservation:

$$\nabla \cdot \vec{B} = 0$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

Faraday's law of induction:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}$$

Ampere's circuital law:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$
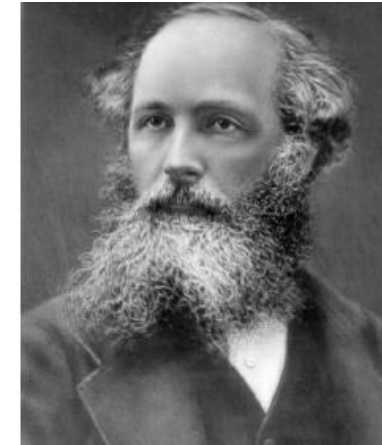
$$\oint_{\partial A} \vec{B} \cdot d\vec{s} = \int_A \mu_0 \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_A \mu_0 \varepsilon_0 \vec{E} \cdot d\vec{A}$$

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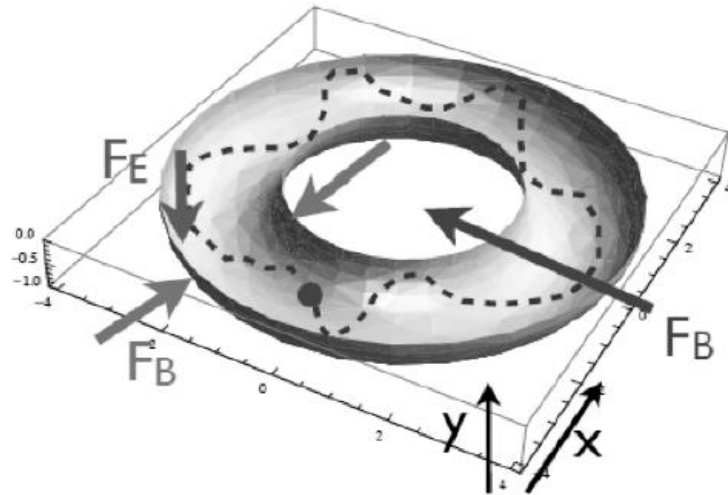
What are the Magnets Tasks in Particle Accelerators

They generate a magnetic field (B) that interacts with the particle beam by means of **Lorentz Force:**

$$\overline{F} = q\,(\overline{E} + \overline{v} \times \overline{B})$$

At each applications of the Lorentz force, the particles experienced a transverse "**kick**" in a defined direction. Aiming to keep the beam on a trajectory (or orbit) avoiding a particle spread on the vacuum pipe, several kicks (**magnets!**) are needed along their path in the accelerator.

Remember the 1d harmonic oscillator: $F = -k\,x$
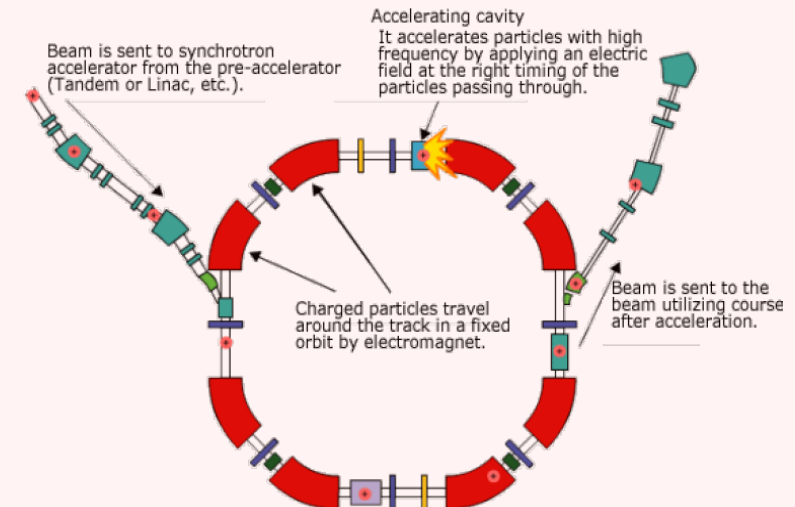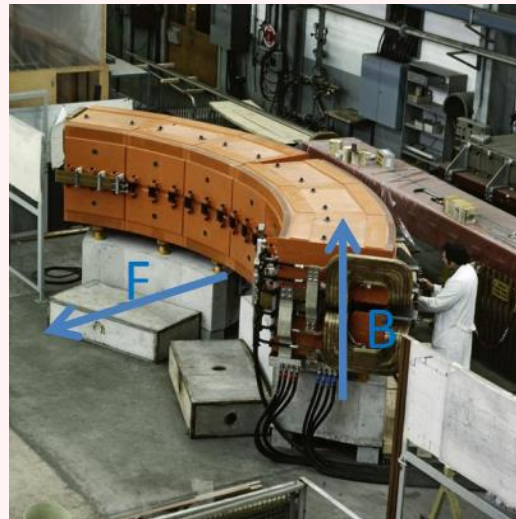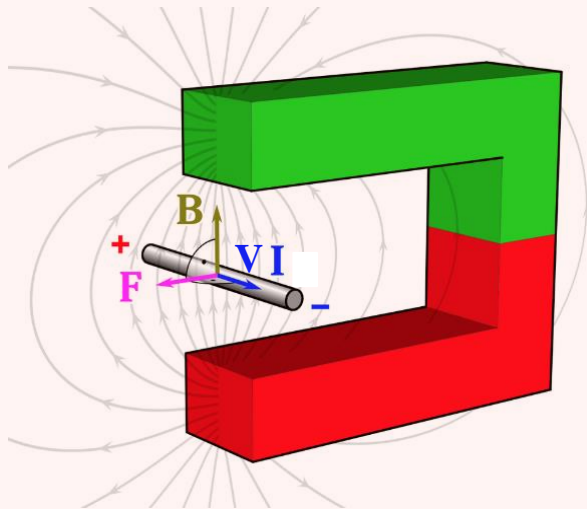

Circular Accelerator

Linear Accelerator
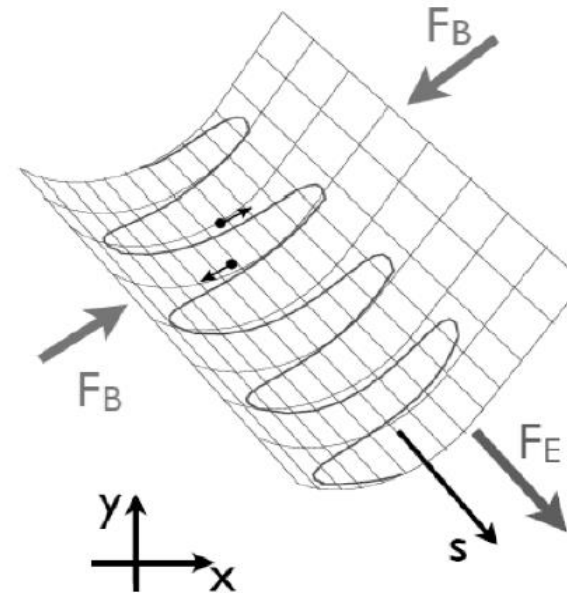
What are the Magnets Tasks in Particle Accelerators
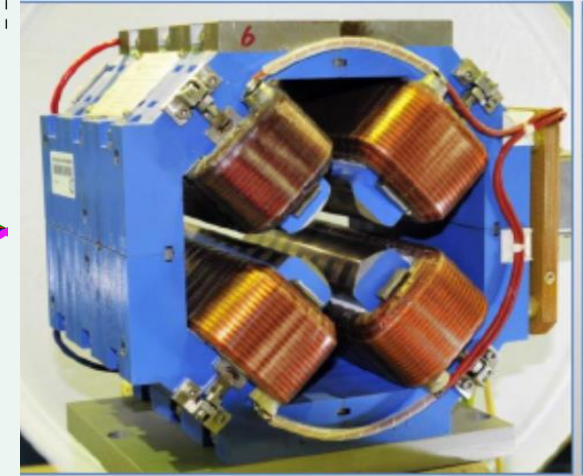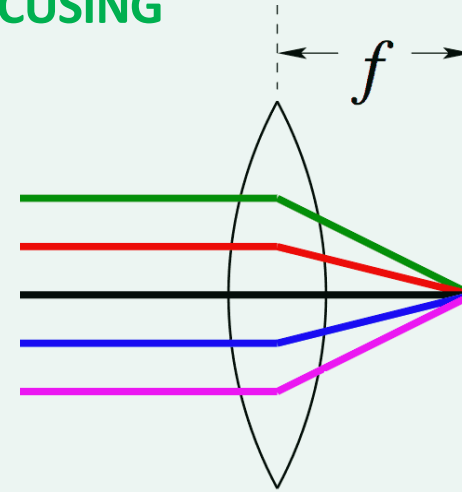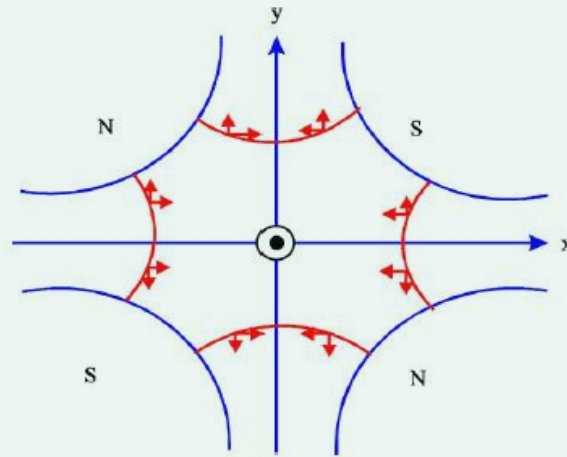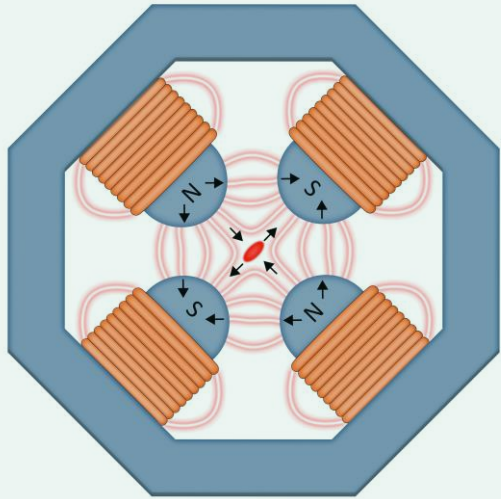
The main functions of the magnets are:

— **Guide** the beam to keep it on an orbit (circular) or a trajectory (linac) → Dipoles and Steerers

— **Focus the beam** (like an optic lens) → Quadrupoles and solenoids

— Other corrections → Sextupoles, Octupoles, etc.

DIPOLES: BEAM DEFLECTION
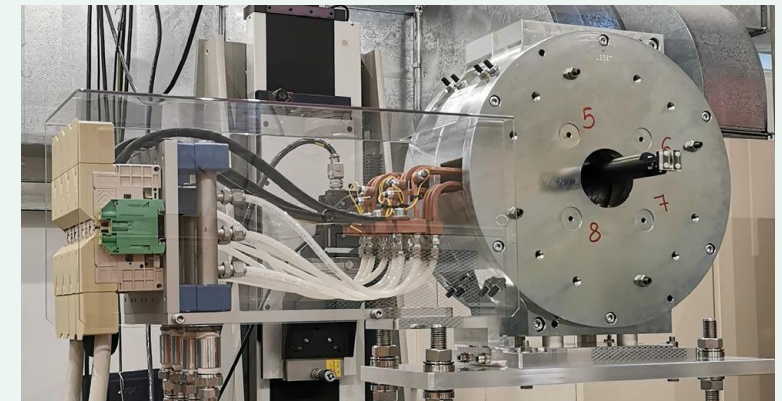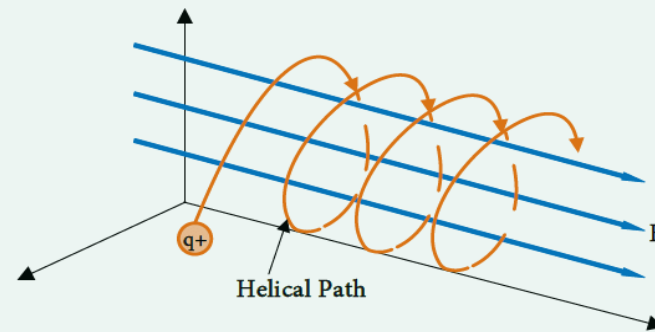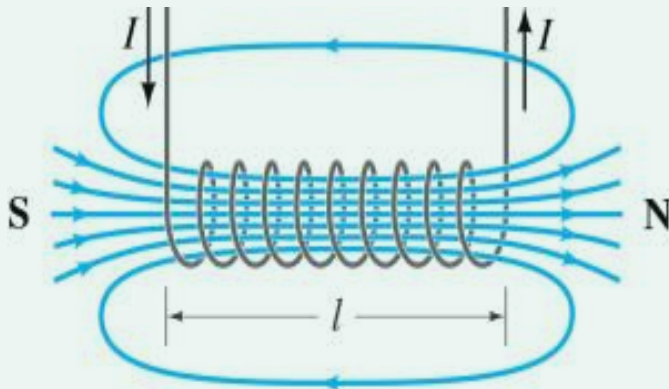
What are the Magnets Tasks in Particle Accelerators

QUADRUPOLES: BEAM FOCUSING

SOLENOIDS: BEAM FOCUSING

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Dipole Magnets – The Magnetic Guide

$$\text{Lorentz force} \quad F_L \quad = qvB$$

$$\text{Centrifugal force} \quad F_{centr} \quad = \frac{\gamma m_0 v^2}{\rho}$$

$$\frac{\gamma m_0 v\!\!\!/}{\rho} \quad = q\!\!\!/B$$

$$P = m_0 \gamma v = mv \; \text{"momentum"}$$

$$\frac{P}{q} = B\rho$$

$$B\rho = \text{"beam ridigity"}$$

$$\frac{1}{\rho \, [m]} \approx 0.3 \frac{B \, [T]}{P \, [GeV/c]}$$

- The ratio of p to q describes the 'stiffness' of a beam, it can be considered as a measure of how much angular deflection results when a particle travels through a given magnetic field.

- For a specific magnetic field, the greater the momentum of a particle, the less it will be bent as it travels through that field. The greater the charge of a particle, the more it will be bent as it travels through a given magnetic field.

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Quadrupole magnets are required to keep the trajectories in vicinity of the ideal orbit
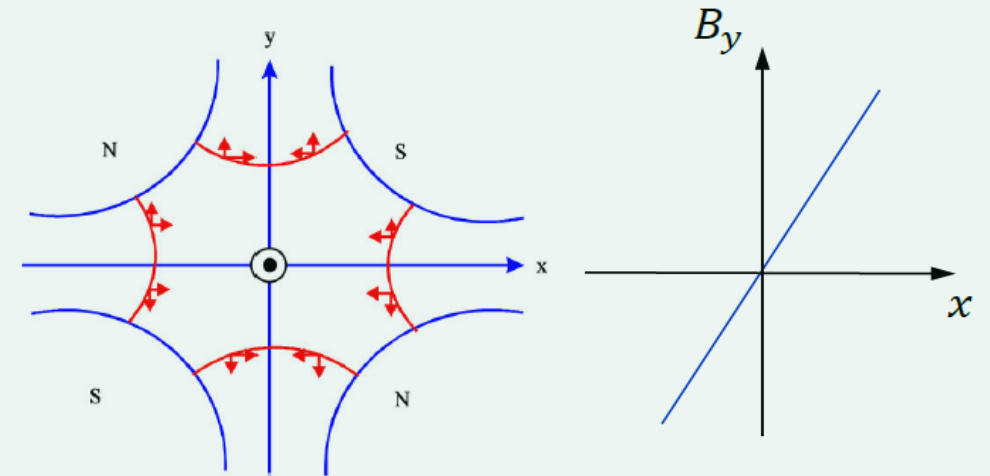
They exert a linearly increasing Lorentz force, thru a linearly increasing magnetic field:

$$\begin{aligned} B_x &= gy \\ B_y &= gx \end{aligned} \Rightarrow \begin{aligned} F_x &= -qv_z gx \\ F_y &= qv_z gy \end{aligned}$$

Gradient of a quadrupole magnet:

$$g = \frac{2\mu_0 nI}{r^2} \left[\frac{T}{m}\right] = \frac{B_{poles}}{r_{aperture}} \left[\frac{T}{m}\right]$$

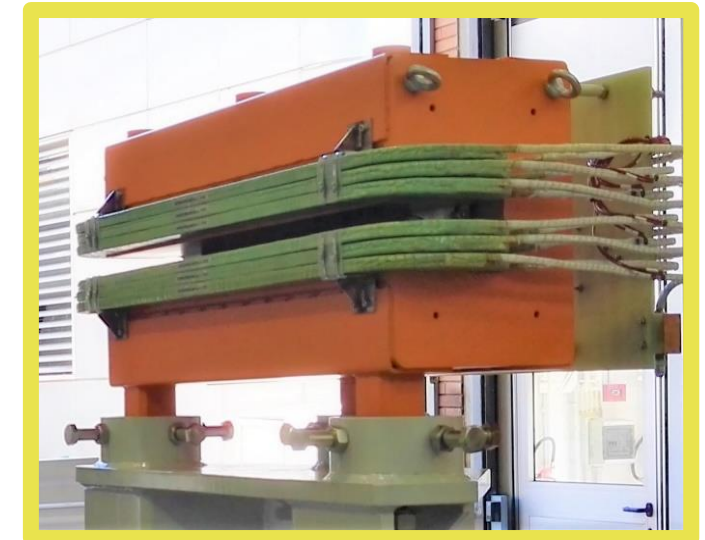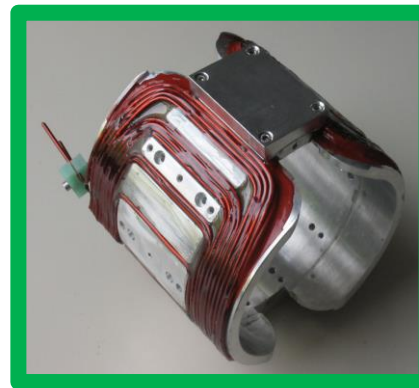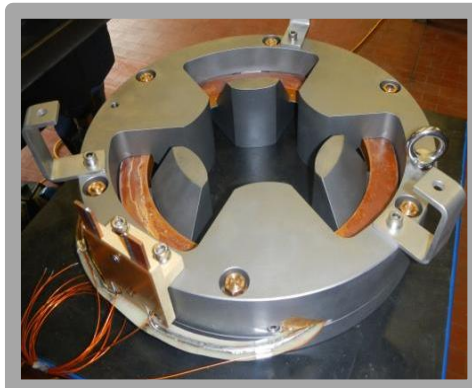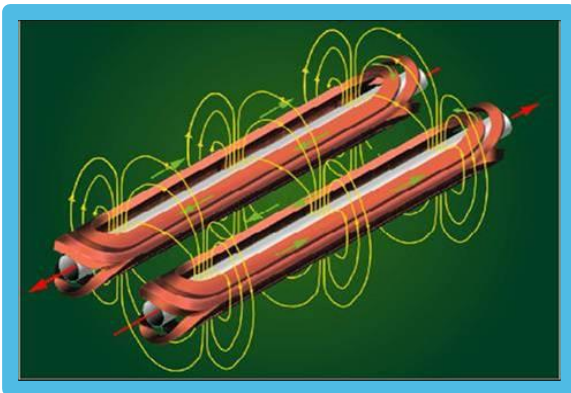▶ LHC main quadrupole magnets:
$g \approx 25 \ldots 235$ T/m

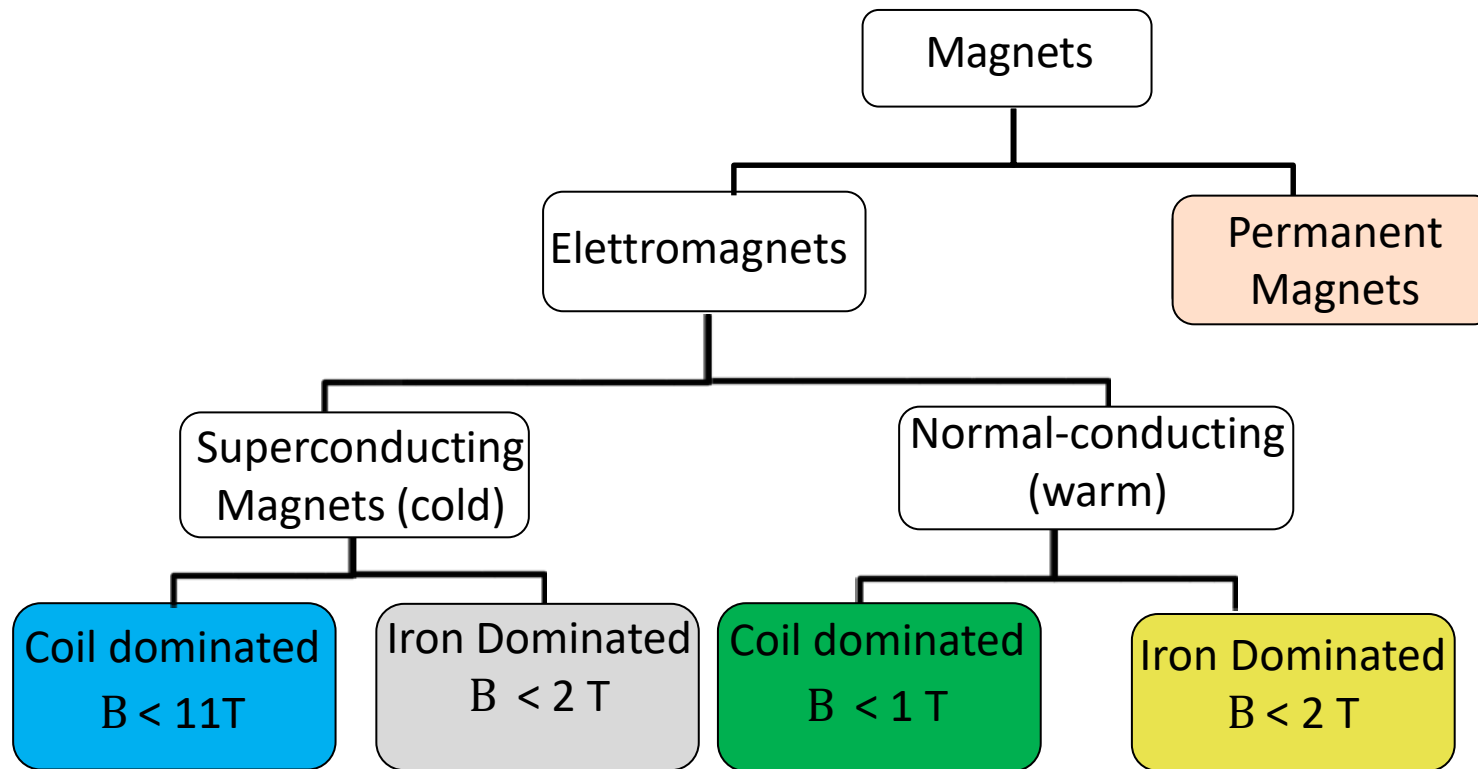the arrows show the force exerted
on a particle

Divide by p/q to find the nornalised focusing strength, k:

$$k = \frac{g}{P/q} \; [m^{-2}]; \quad \Rightarrow \quad g = \left[\frac{T}{m}\right]; \quad q = [e]; \quad \frac{P}{q} = \left[\frac{GeV}{c \cdot e}\right] = \left[\frac{GV}{c}\right] = [T \; m]$$

A simple rule: $k \left[m^{-2}\right] \approx 0.3 \dfrac{g \; [T/m]}{P/q \; [GeV/c/e]}$.

Magnet Types

Magnets
- Elettromagnets
  - Superconducting Magnets (cold)
    - Coil dominated $B < 11T$
    - Iron Dominated $B < 2 T$
  - Normal-conducting (warm)
    - Coil dominated $B < 1 T$
    - Iron Dominated $B < 2 T$
- Permanent Magnets

Air Dominated vs Iron Dominated Electromagnets

- In all the magnets for particle accelarators there is an air region (**air gap**) between the poles where the beam experiences the magnetic field.
- In electromagnets the magnetic field source is a current carried by coil made of electrical conductor materials.
- The flux density lines have a close path that could include:
  - Only air → **Coil dominated magnet**
  - Air + Iron → **Iron dominated magnet**
- Applying Ampere's law to a iron dominated magnet (i.e. Dipole) we have a strict correlation between current and magnetic field and the bigger part of the «magnetizing force» (NI) is needed to create the flux density in the air gap where $\mu_r=1$ **(if Iron not saturated → Biron<1,8T)**
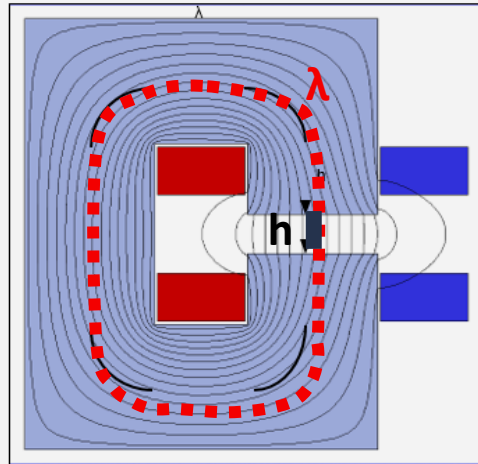
Iron Dominated

Ampere's law $\oint \bar{H} \cdot \bar{dl} = \bar{NI}$ and $\bar{B} = \mu \bar{H}$

leads to $NI = \oint \frac{\bar{B}}{\mu} \cdot \bar{dl} = \int_{gap} \frac{\bar{B}}{\mu_{air}} \cdot \bar{dl} + \int_{yoke} \frac{\bar{B}}{\mu_{iron}} \cdot \bar{dl} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$

assuming, that B is constant along the path.

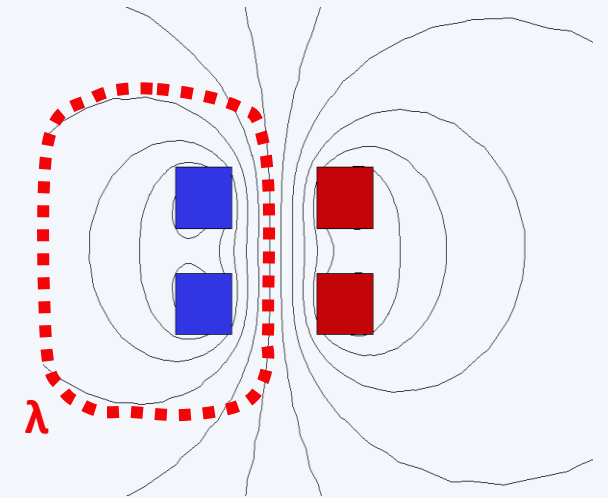If the iron is not saturated: $\frac{h}{\mu_{air}} \gg \frac{\lambda}{\mu_{iron}}$

then: $NI_{(per pole)} \approx \frac{Bh}{2\mu_0}$

Coil dominated

$NI_{(per pole)} = \oint_{\lambda} H \cdot dl = \frac{\bar{B}\lambda}{2\mu_0}$

$h \ll \lambda$

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- The goal of magnetic designer is to obtain a **desired field distribution (Field Quality)** in a region of the air gap where there is *a high probability* (usually few σ for the envelope) *to contain the beam envelope* **(Good Field Region)**.

- In this region the field has to be within certain tolerancies.

- **Iron pole dimensions and shapes** define the field distribution and therefore the field quality for Iron dominated magnets.

- **Currents geometric displacement** define the field distribution and therefore the field quality for coil dominated magnets
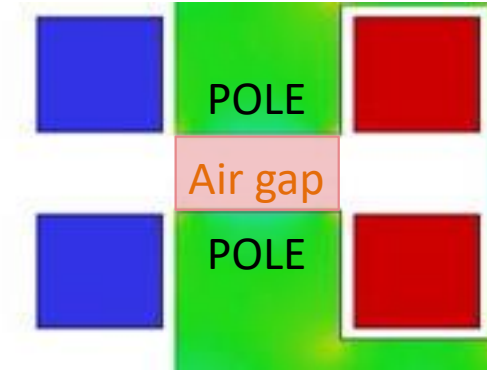
$$\sigma = \sqrt{\varepsilon\beta + \left(D\frac{\Delta p}{p}\right)^2}$$
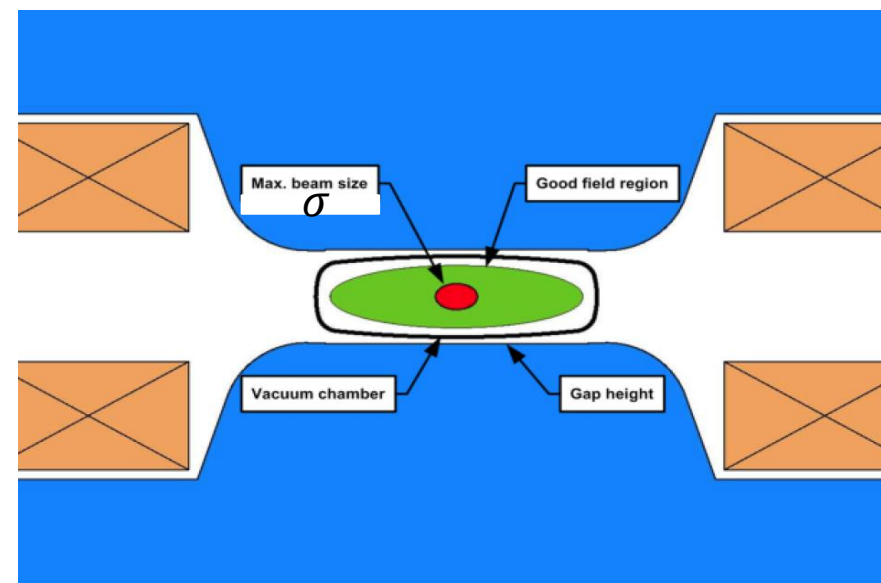
General Field Description in Air Gap
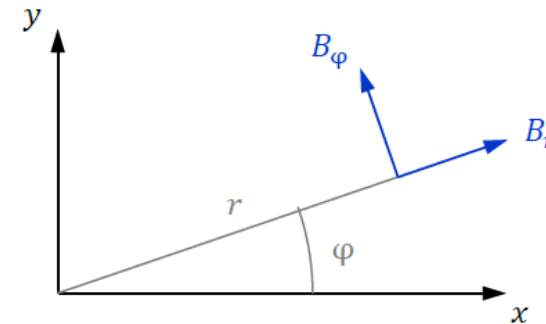
In the air gap region between poles we assume:

- No currents
- No magnetic materials ($\mu_r = 1$)
- B_z is constant (2D case!)

The 2D vector field of B can be expressed as a series of multipole coefficients $B_n(r_0)$, $A_n(r_0)$ with r_0 being the reference radius (this is a solution of Maxwell equation inside the air gap) [1] :

$$B_r(r_0, \varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} [B_n \sin(n\varphi) + A_n \cos(n\varphi)]$$

$$B_\varphi(r_0, \varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} [B_n \cos(n\varphi) - A_n \sin(n\varphi)]$$

[1] *Maxwell's equations for magnets - A. Wolski Cern Accelerating School (CAS) CERN–2010–004*

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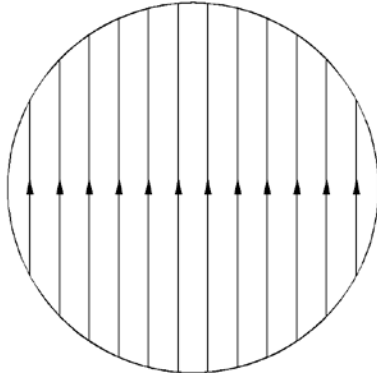# General Field Description in Air Gap

- For simplicity let's write these series with complex notation definining a *z complex variable*. In this way we avoid sin and cos.

- Note that the field distrbution is distinguished between two components : **normal** (**Bn**) and **skew** (**An**)
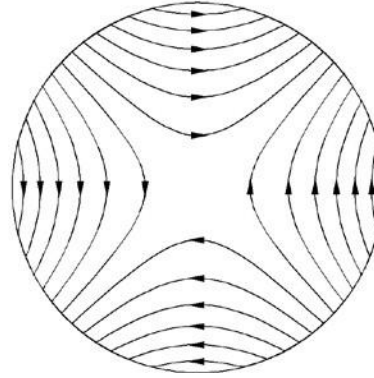
$$z = x + iy = re^{i\varphi}$$

$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{r_0}\right)^{n-1}$$
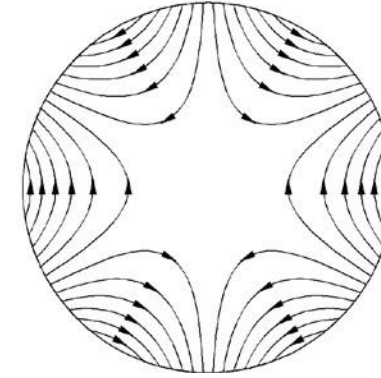
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General Field Description in Air Gap
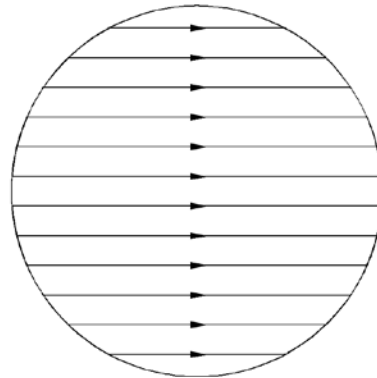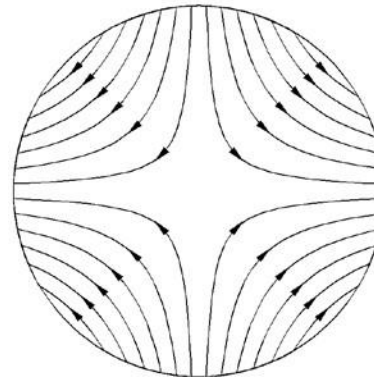
B_1: normal dipole
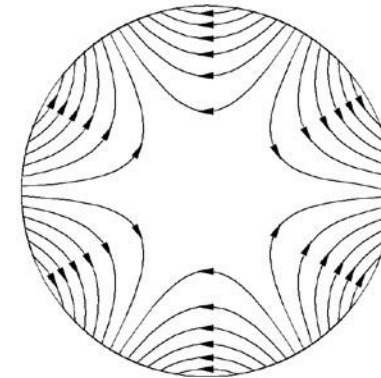
B_2: normal quadrupole

B_3: normal sextupole
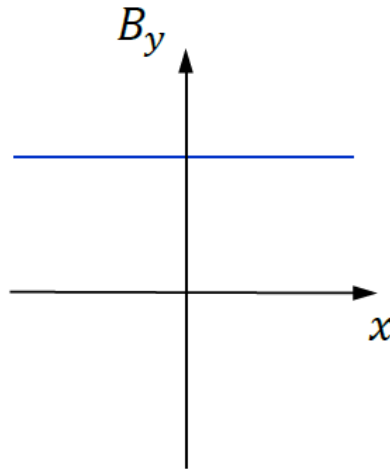
A_1: skew dipole

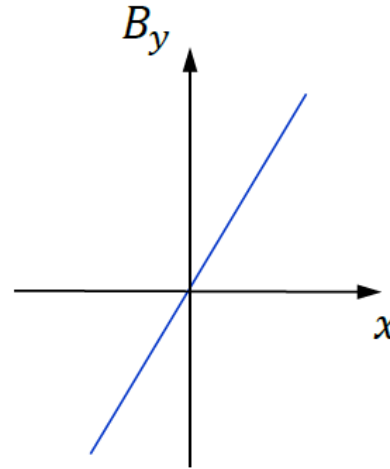A_2: skew quadrupole

A_3: skew sextupole

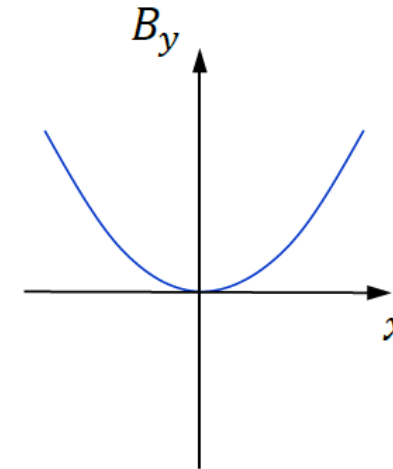Each multipole term has a corresponding magnet type

$$B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{r_0}\right)^{n-1} = B_1 + B_2 \frac{x}{r_0} + B_3 \frac{x^2}{r_0^2} + \cdots$$

B_1: dipole

B_2: quadrupole

B_3: sextupole

$$G = \frac{B_2}{r_0} = \frac{\partial B_y}{\partial x}$$

The field profile in the horizontal plane follows a polynomial expansion
The ideal poles for each magnet type are lines of constant scalar potential

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Aspetti Costruttivi dei Magneti Permanenti

Nucleo
in ferro

Blocchi in
Sm_2Co_{17}

Poli in
ferro

- Forza magneto-motrice (N*I) generata non più da correnti ma da blocchi magneti permanenti.
- Blocchi magneti permanenti sono composti da **terre rare sinterizzate (samario, cobalto, neodimio, ecc.)** che vengono magnetizzate in fase di produzione con un campo magnetico esterno
- E' presente anche un nucleo in ferro per ridurre la riluttanza del circuito magnetico (stessa funzione degli elettromagneti)
- In alcune configurazioni è possibile regolare il campo magnetico modificando la geometria del magnete (ad es. spostando blocchetti magnetizzati).
- Grande interesse per ridurre impatto energetico acceleratori (resta problema terre rare…)
- Vantaggi: no consumo energia, no raffreddamento bobine
- Svantaggi: difficile regolazione del campo magnetico, magnetizzazione blocchi legata a irraggiamento e a temperatura.
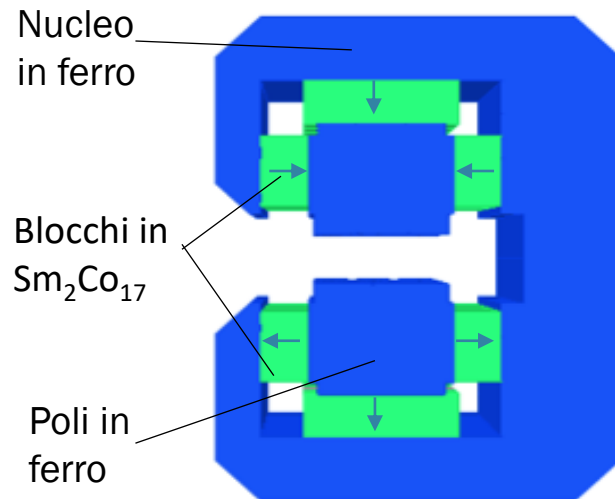
Halbach Array

Configurazione di magneti permanenti con orientamento polarizzazione magnetica periodico che **massimizza il campo magnetico in una data regione dello spazio** (i.e. su un lato dell'array)


Halbach Array

Alternating Polarity
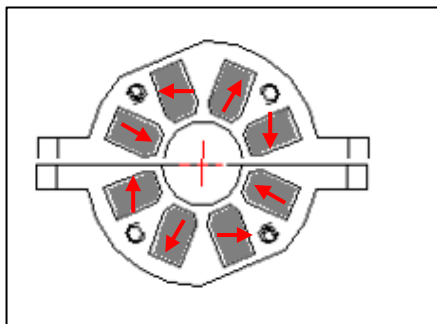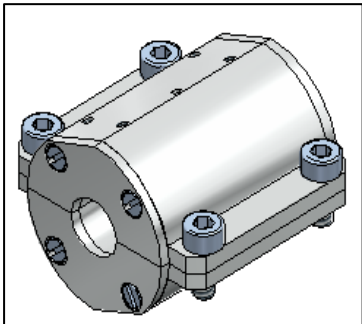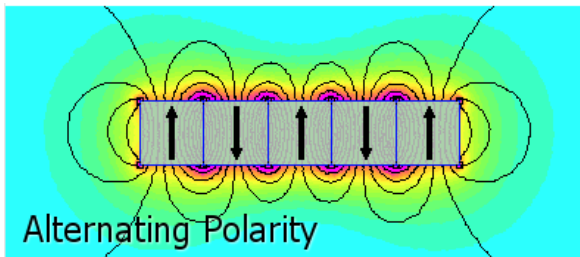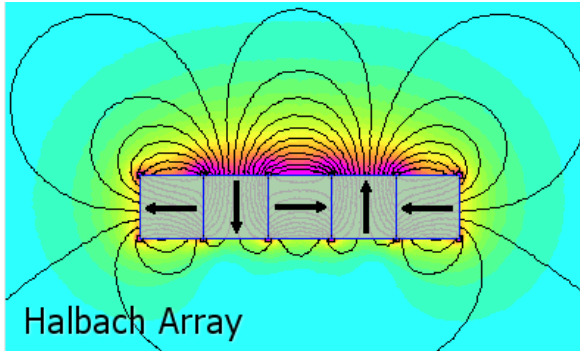
- Nati dall'esigenza di massimizzare la focalizzazione di quadrupoli a magneti permanenti
- Una tipica configurazione per acceleratori di particelle è quella di un cilindro con vettore di magnetizzazione che varia l'orientamento in modo periodico. A seconda del periodo si può ottenere un **multipolo** differente (i.e. dipolo, quadrupolo, sestupolo, ecc.)
- Nella pratica si hanno blocchetti magnetizzati ciascuno con un proprio orientamento del vettore magnetizzazione.
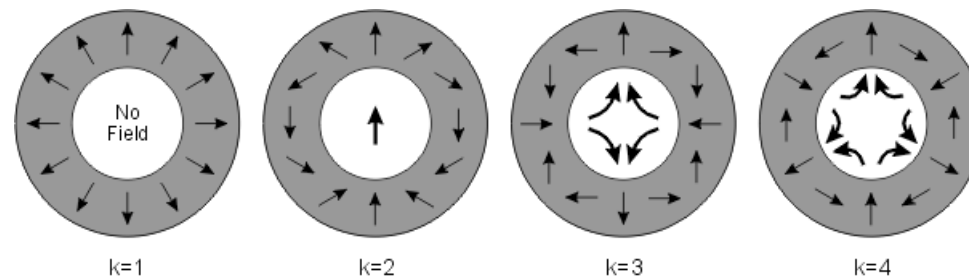
$$M = M_r \left[\cos\left((k-1)\left(\varphi - \frac{\pi}{2}\right)\right)\hat{\rho} + \sin\left((k-1)\left(\varphi - \frac{\pi}{2}\right)\right)\hat{\varphi} \right]$$

Magnetic Length

- For all types of magnets, the lognitudinal filed profile is almost similar (for quadrupole the off-axis profile).
- It shows two «tails» that is the stray field and a «flat top» where the field is almost constant
- It is possible to define a «Magnetic length» (effective length) that is always larger than the actual iron
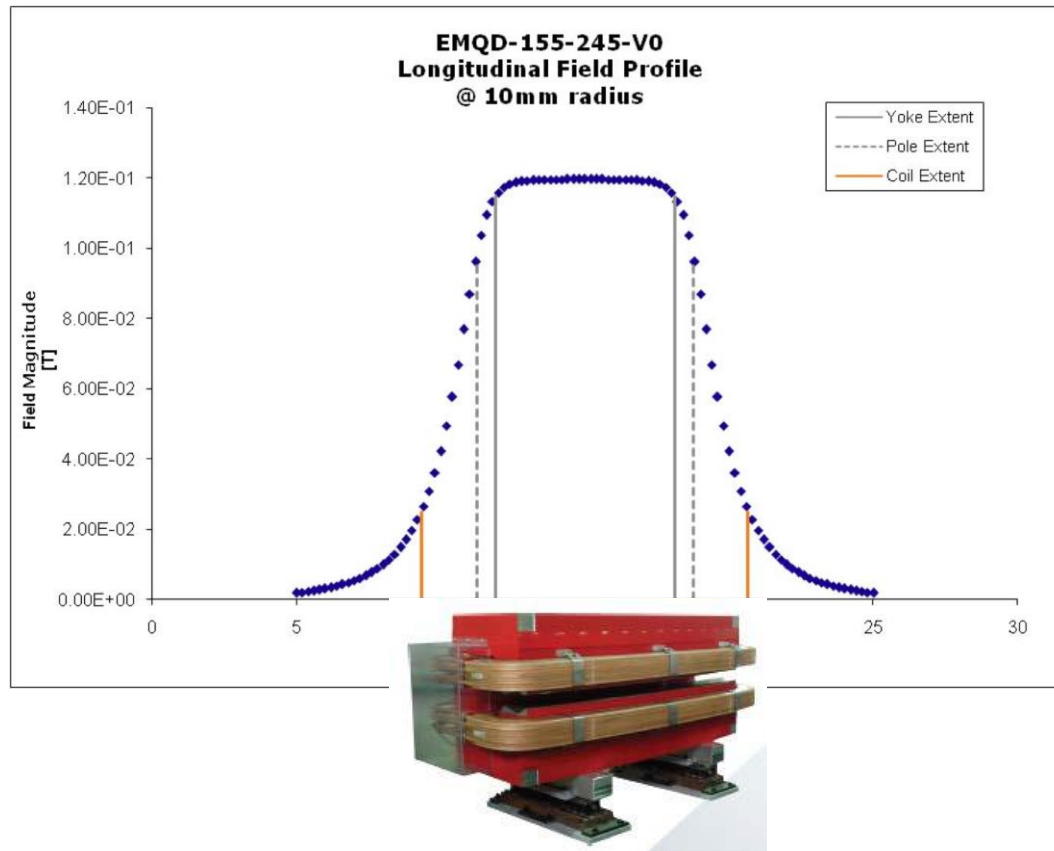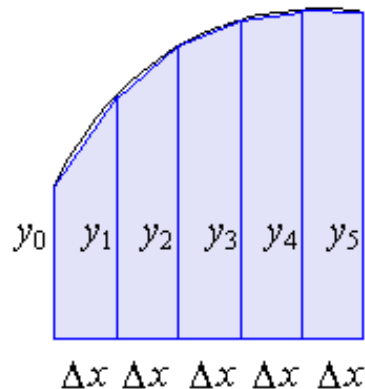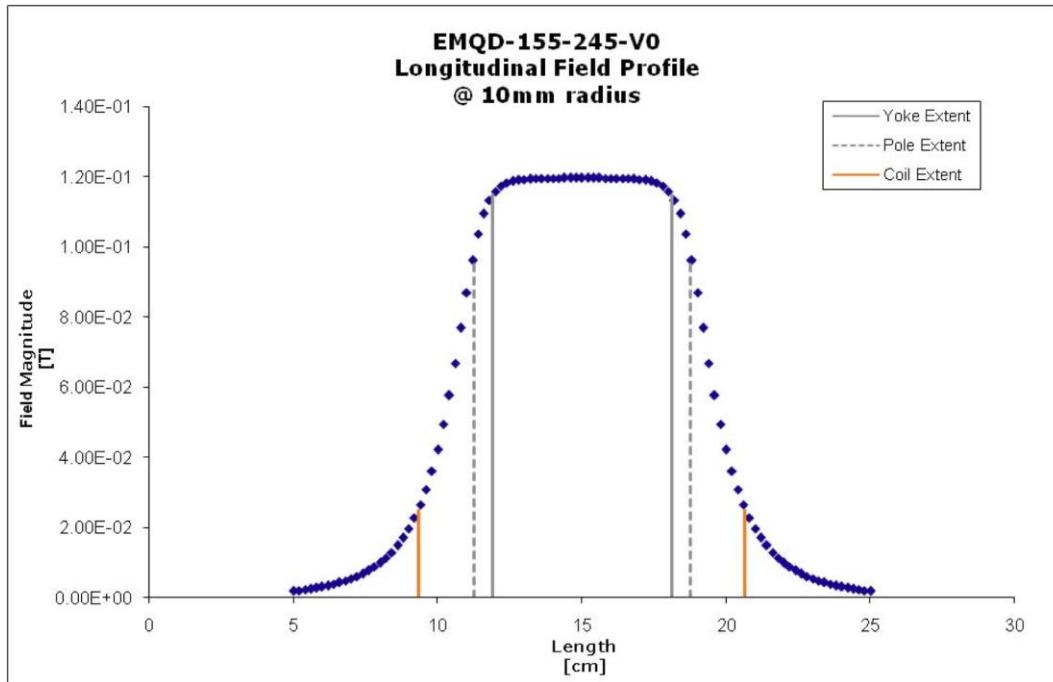

$$L_{mag} = \frac{\int_{-\infty}^{+\infty} B_y(z)\,dz}{B_0}$$

Dipole: $l\_mag \approx l\_iron + 2hk$ h: magnet gap
k: geometrical factor, typically 0.3÷0.6

Quadrupole: $l\_mag \approx l\_iron + 2rk$ r: aperture radius
k: geometrical factor, typically ~0.45)

Magnetic Length

EMQD-155-245-V0
Longitudinal Field Profile
@ 10mm radius

Dipole

$$IntB = B_{max} \cdot L_{mag} = B \cdot \rho \cdot \theta = \frac{E \; [MeV]}{300} \cdot \theta$$

Quadrupole

$$IntG = \int_{-\infty}^{+\infty} Grad_z(y) dy = G \cdot Lmag = \frac{E \; [MeV]}{300} \cdot K \cdot Lmag$$

Trapezoidal rule to evaluate a general integral
$$\int y dx = \frac{\Delta x}{2}(y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-2} + 2y_{n-1} + y_n)$$

Magnetic Field Quality – Field Uniformity (+ integrated)

Defined as:

- Field or (integrated field) uniformity
- Relarive field harmonic

A field quality figure of merit is also the relative difference between the central field (for dipoles) or gradient (for quadrupoles) w.r.t. the field or gradient in another position within good field region
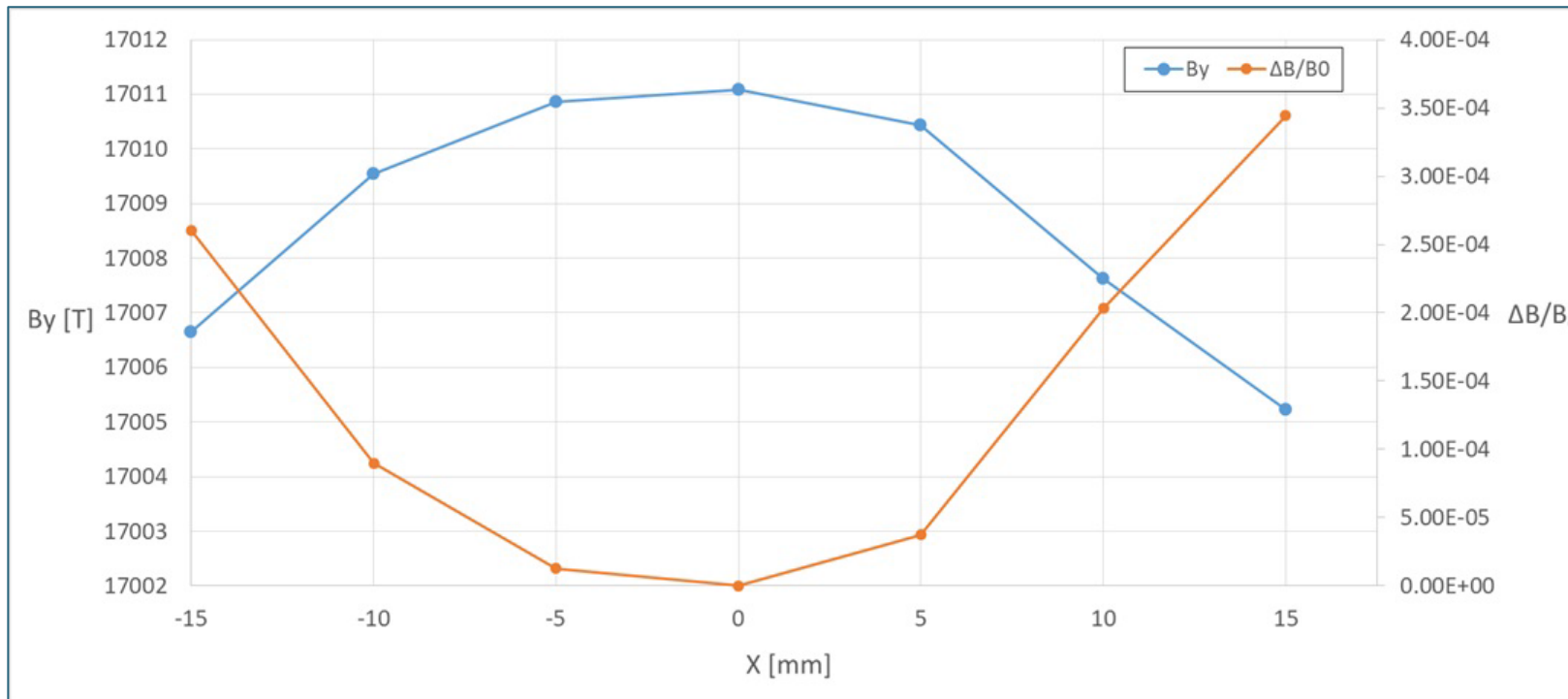
$$\frac{\Delta B}{B_0} = \frac{B(x,y) - B(0,0)}{B(0,0)}$$

$$\frac{\Delta IB}{IB_0} = \frac{IB(x,y) - IB(0,0)}{IB(0,0)}$$

Dipoles

$$\frac{\Delta G}{G_0} = \frac{G(x,y) - G(0,0)}{G(0,0)}$$

$$\frac{\Delta IG}{IG_0} = \frac{IG(x,y) - IG(0,0)}{IG(0,0)}$$

Quad

Magnetic Field Quality – Field Multiples

Defined as:

- **Field or (integrated field) uniformity**
- **Relarive field harmonic**

$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n)\left(\frac{z}{r_0}\right)^{n-1}$$

$$z = x + iy = re^{i\varphi}$$

$$b_i = \frac{B_i}{B_{ref}} 10^4$$

- When you have a real magnet of a specific type you wish to produce that given type of harmonic only, but you will get also other harmonics, more or less large.
- We define «relative field harmonic» the ratio between that field harmonic and the reference field harmonic expressed in units of $10{-}4$ of the main harmonic, at a reference radius $r0$.

- Harmonic Analysis performed measuring the field on a close path (circular usually).
- FFT of the signal, defining all the series coefficents that corrispond to a multiple

Normal and skew higher harmonic content

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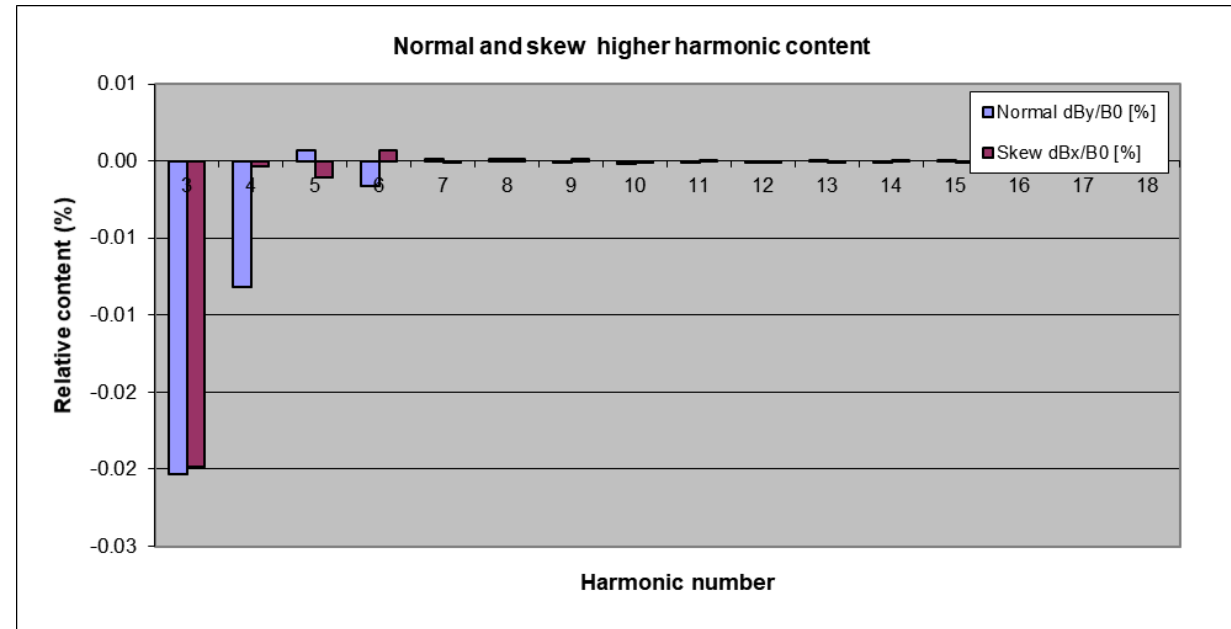
1. **How is the magnetic distribution in the good field region:**
   - Solenoids: In an air volume near the solenoid axis
   - Other Magners: air gap between the poles
2. Magnets electrical parameters (R and L) and check of turn to turn and coil to ground insulation
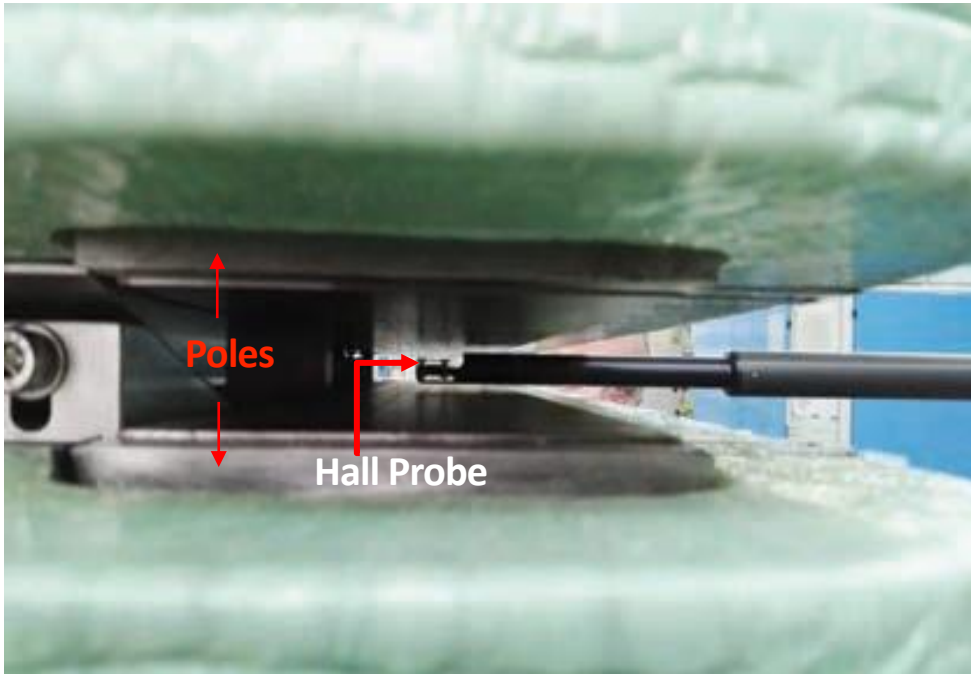
Two magnetic measurements families:

1. Point by point → we move a probe sensible to the magnetic field (**Hall probe**) with a micrometric precision movement system (coordinatometer) in the good field region. We got a 3D field map with the flux density point by point.
2. Integrated → moving coils, that could have several geometries, within magnetic field we've an induced voltage due to the flux variation in the coil (Faraday-Neumann-Lenz law).
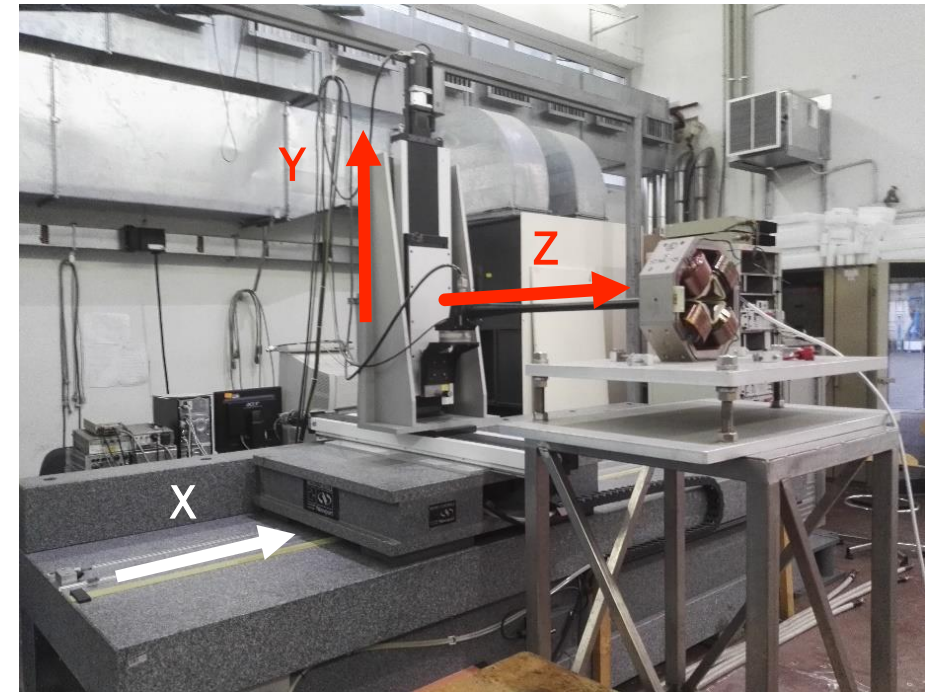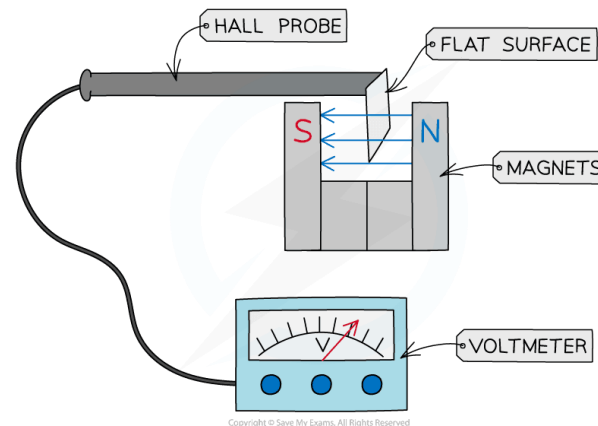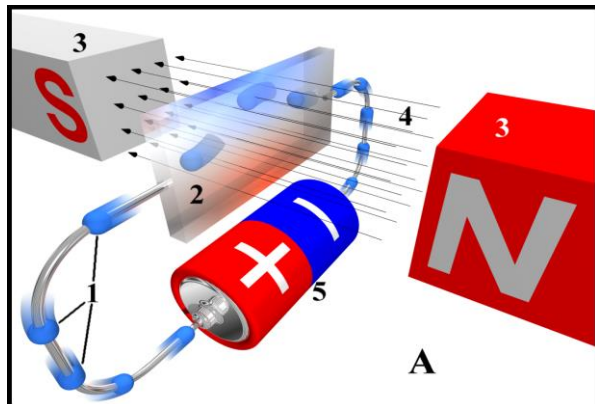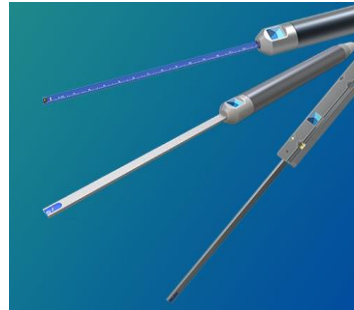
$$e.m.f. = -\frac{d\Phi}{dt}$$

We don't measure the field point by point but a field induced in the coil area.

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- The Hall probe is a B→V active transductor.
- Reading the voltage generated by Hall effect, we can define the magnetic field thanks to a calibration.
- The high precision coordinatometer allows to move the hall probe in every desired position.
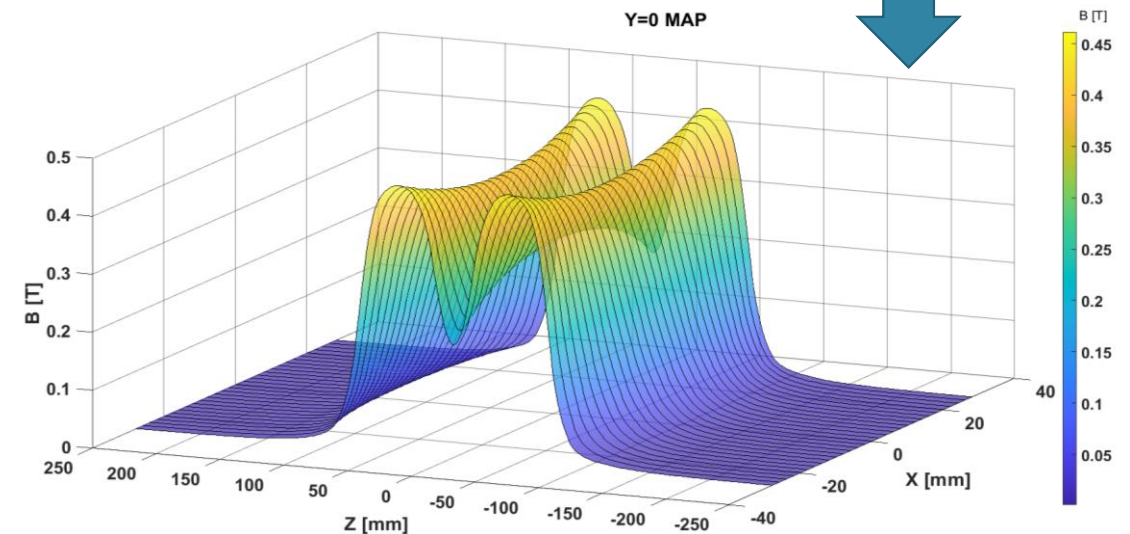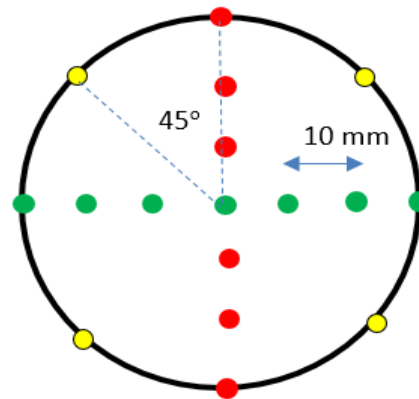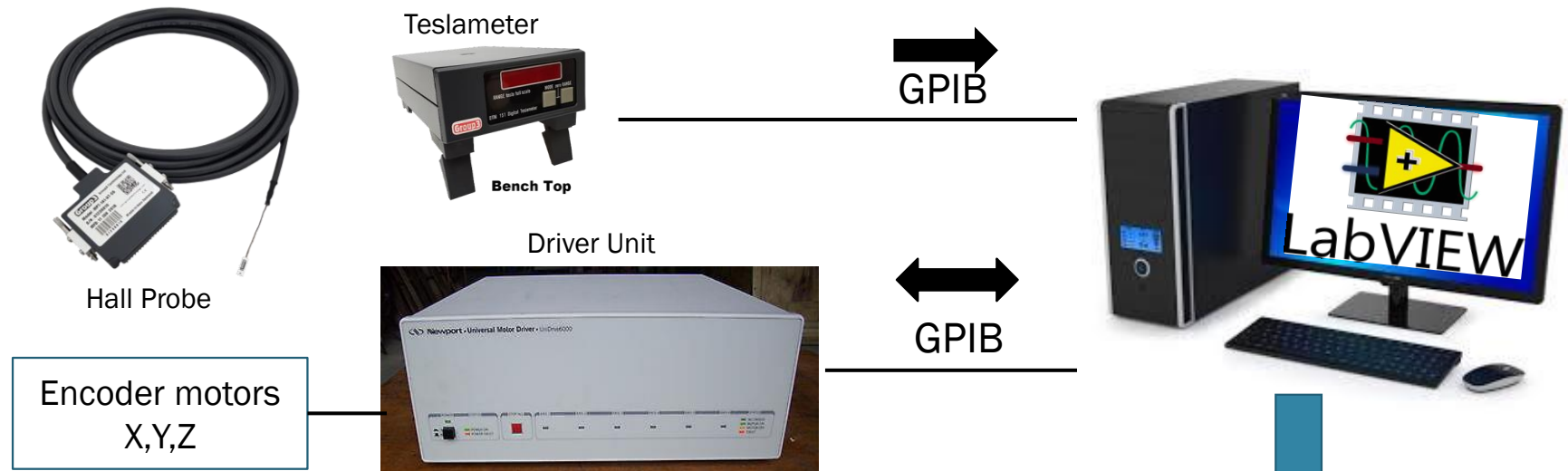

Coordinatometer

Point by Point measurement : Hall Probe

The typical measurement setup foreseen Labview VI installed on a PC equipped with DAQ boards and a driver unit.

This setup allows to:

- Setup all the desired movements of the probe
- Acquire all the position and magnetic field data
- Write file output (txt format) with position and related field.

Teslameter

GPIB

Hall Probe

Driver Unit

Encoder motors X,Y,Z

GPIB

LabVIEW

Bz

Y=0 MAP

45°

10 mm

B [T]

B [T]
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05

Z [mm]

X [mm]

Campo magnetico in direzione longitudinale (Bz) lungo le traiettorie rappresentate dai punti verdi

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Integrated Magnetic Measurement : Rotating Coil

- The coil periodic signal is acquired (sinusoidal coil), elaborated with a FFT
- All the series cofficients are divided by the main component (i.e. quadrupolar) deriving the relative content for all the harmonics and checking that it is within a required range.
- Check of integrated field.
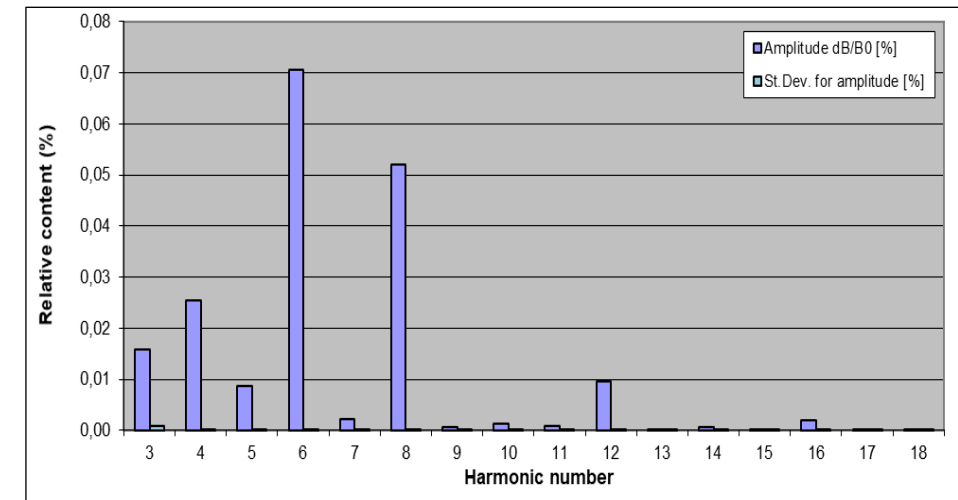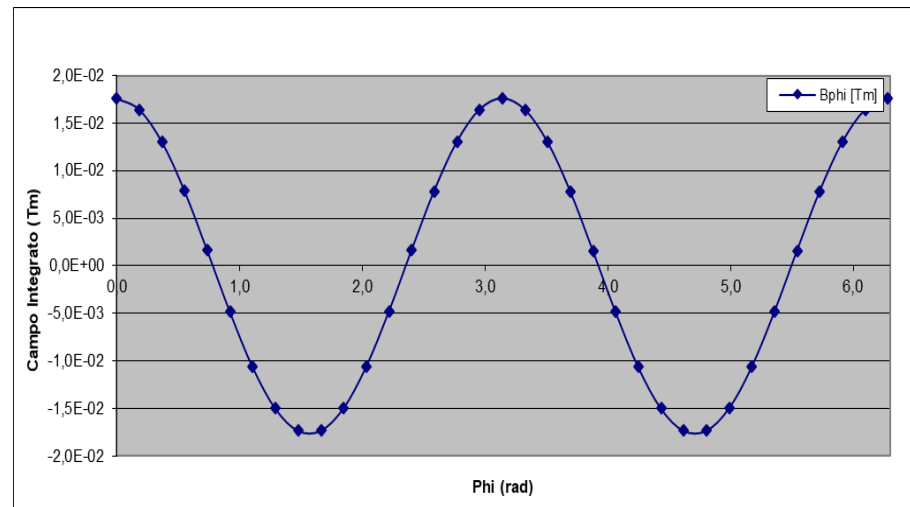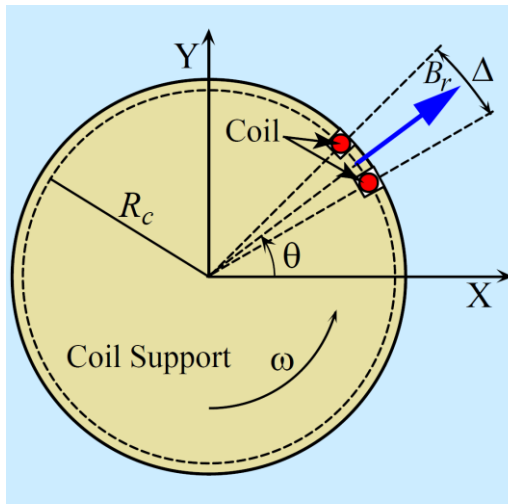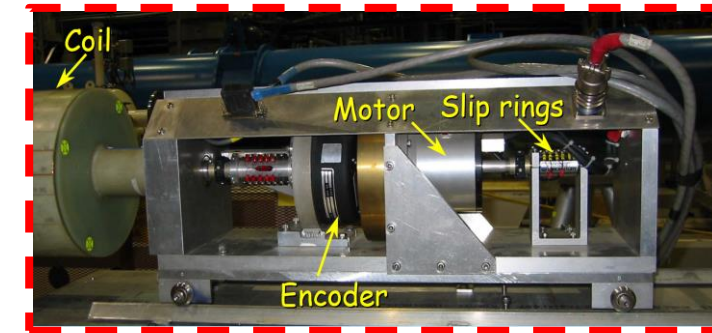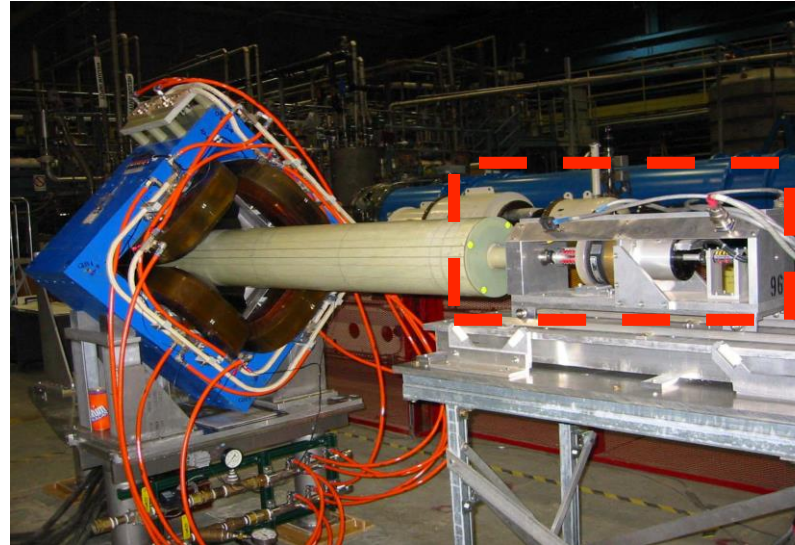
Integrated Magnetic Measurement : Rotating Coil

Integrated signal is not affected by rotation speed variations
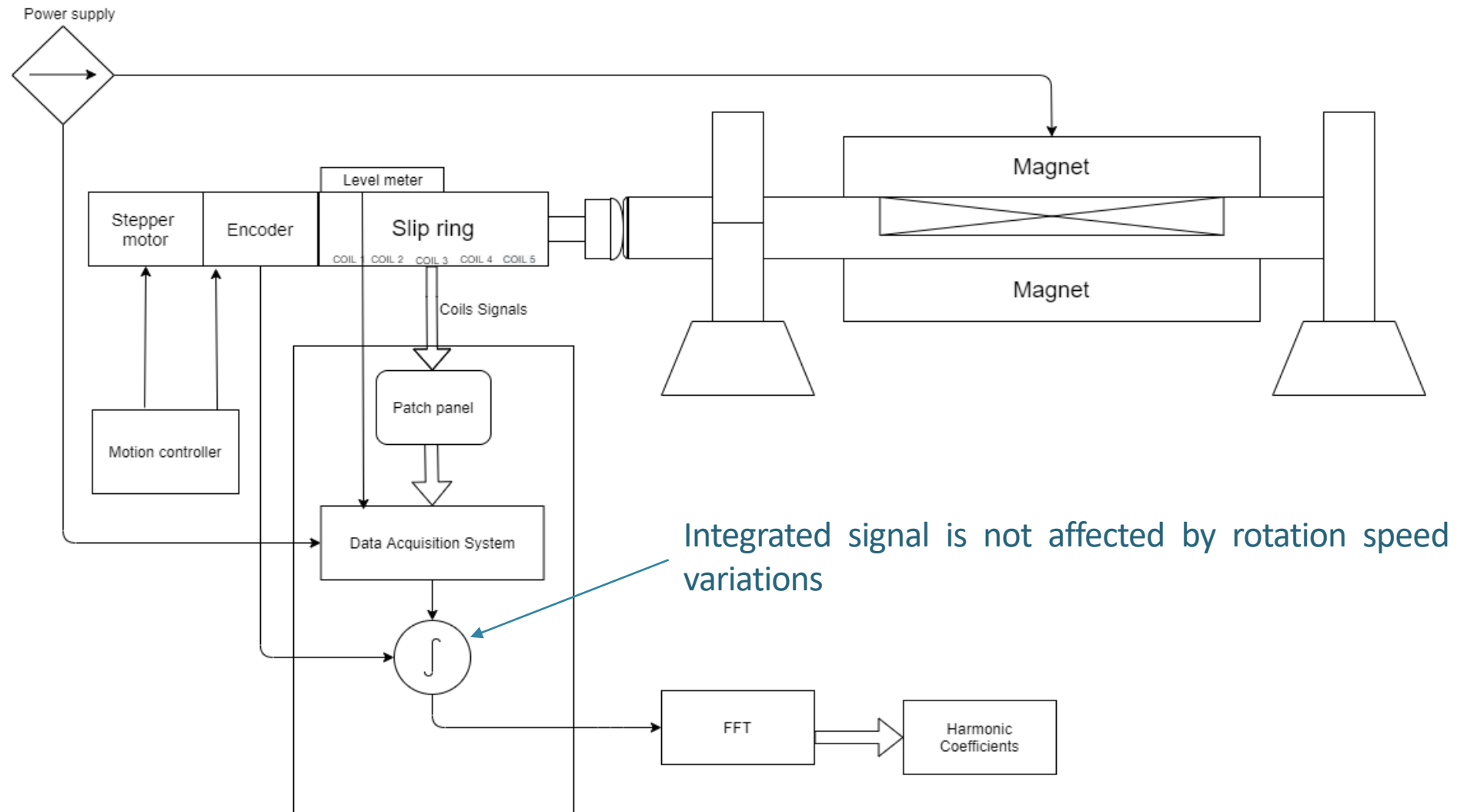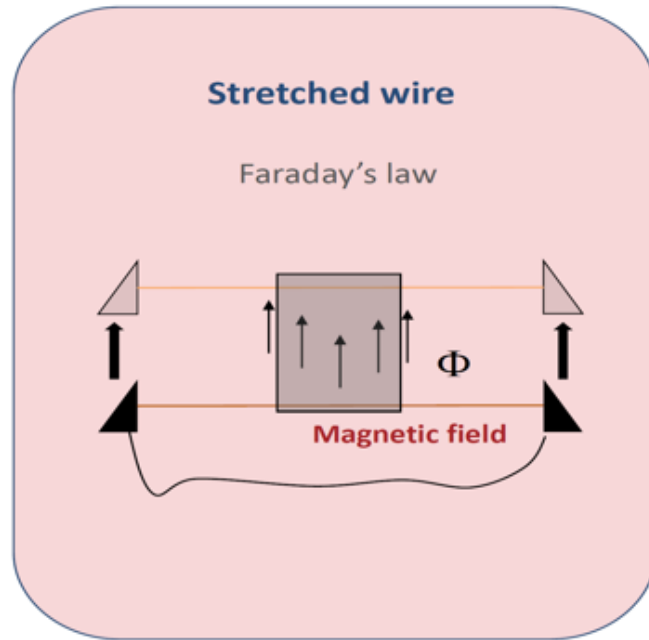
- We have a coil with a fixed part and a part that could be moved (the wire), both of electrical conductor.
- The wire experenced the magnetic field of the tested magnet
- Moving the wire with a high precision stage, it varies the coil area.
- The variation of the area correspond to a magnetic flux variation, there fore to an induced voltage on the coil terminals:

$$e.m.f. = -\frac{d\Phi}{dt}$$

- It's possible to define the integrated magnetic field along the wire length by a correlation between the induced voltage and the positon of the wire.
- Moving the wire on a circular trajectories it's possible to perform the same harmonic analysis of a rotating coil
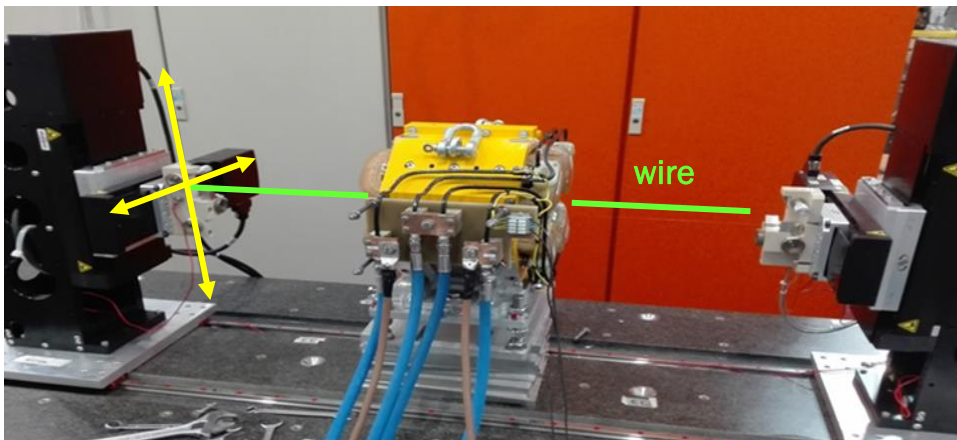
Relevant Magnetic Parameters in Magnets

DIPOLO MISURATO CON SONDA HALL

Parametro da misurare	Grandezze ricavate
Profilo longitudinale di Campo su più tratiettorie (direzione del fascio)	Campo magnetico integrato
	Angolo di deflessione (conoscendo Energia fascio)
	Qualità integrata di campo
Profilo trasverso di Campo	Qualità di campo «puntuale»
Corrente vs Induzione magnetica	Funzione di Trasferimento I ->B

QUADRUPOLO MISURATO CON SINGLE STRETCHED WIRE (SSW)

Parametro da misurare	Grandezze ricavate
Tensione indotta filo SSW su traiettoria circolare	Gradiente integrato
	Armoniche integrate -> Qualità di campo

Grazie per l'attenzione

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