Finanziato dall'Unione europea
NextGenerationEU

Italiadomani
PIANO NAZIONALE DI RIPRESA E RESILIENZA

NATIONAL BIODIVERSITY FUTURE CENTER

On-off intermittency in a host-parasitoid model with a deterministic chaotic driver

Angela Monti

Istituto per le Applicazioni del Calcolo ‘M. Picone’, CNR, Bari

Workshop MathAIEOapp

Bari, 5-7 July 2023

Ministero dell’Università e della Ricerca

Istituto per le Applicazioni del Calcolo "Mauro Picone"

Outline

1. The Beddington-Free-Lawton model
   - Fixed points
   - Stability & bifurcation analysis
2. Types of intermittency
   - Chaotic intermittency
   - On-off intermittency
3. Deterministic chaotic driver
   - δ, grazing parameter
   - r, host growth rate

Joint work with:

- **Deborah Lacitignola** (Università di Cassino e del Lazio Meridionale)
- **Fasma Diele** (IAC-CNR, Bari)

The Beddington-Free-Lawton model

$$\begin{cases} N_{t+1} = \delta\, N_t \exp\left[r \left(1 - \frac{N_t}{K} \right) - a\, P_t \right] \\ P_{t+1} = b\, N_t \left[1 - \exp\left(-\, a\, P_t \right) \right] \end{cases}$$

with N host and P parasitoid biomass, δ grazing intensity, r host growth rate, K carrying capacity, b parasitoid growth rate, a searching efficiency. Set $Y = a\, P$ and $X = a\, b\, N$ and $k = a\, b\, K$

$$\begin{cases} X_{t+1} = \delta\, X_t \exp\left[r \left(1 - \frac{X_t}{k} \right) - Y_t \right] \\ Y_{t+1} = X_t \left[1 - \exp\left(-\, Y_t \right) \right] \end{cases}$$

- J. Beddington, C. Free, J. Lawton, *The Journal of Animal Ecology* (1976)
- T. Azizi, *International Journal of Modern Nonlinear Theory and Application* (2020)
- G. Vissio, A. Provenzale, *Journal of Theoretical Biology* (2022)

Fixed points

Set $\rho := r + log(\delta)$ and $k_r := k/r$

- extinction $P_0 = (0, 0)$
- free-parasitoid $P_1 = (\rho\, k_r, 0)$
- coexistence $P^* = (X^*, Y^*)$

Let $\widetilde{J}$ be the Jacobian matrix evaluated at the fixed point $\widetilde{P}$.

$$\eta(t) = \widetilde{J}\eta(t-1) \implies \eta(t) = \widetilde{J}^t\, \eta(0)$$

Stability & bifurcation

$$|\lambda_i(\widetilde{J})| < 1 \text{ for } i = 1, 2$$

- **saddle-node**, when $\widetilde{J}$ has an eigenvalue equal to 1;
- **period-doubling**, when $\widetilde{J}$ has an eigenvalue equal to -1;
- **Neimark-Sacker**, when $\widetilde{J}$ has two complex conjugate eigenvalues on the unit circle.

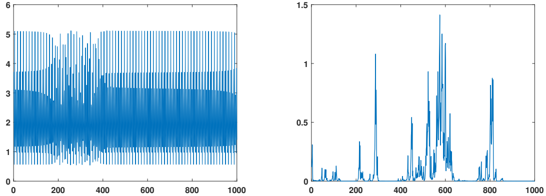
Chaotic intermittency

Switching behavior between periodic, quasi periodic and chaotic regimes.

- Type I $\rightarrow$ saddle-node;
- Type II $\rightarrow$ period-doubling;
- Type III $\rightarrow$ Neimark-Sacker.

On-off intermittency

The alternation between **laminar (off)** phases and sudden **bursts (on phases)**, generated by the temporally-varying stability of an invariant set.



- Y. Pomeau, P. Manneville, *Communications in Mathematical Physics* (1980)

On-off intermittency $\rightarrow$ stochastic drivers

Deterministic driving process

$$\delta_t = d + (D - d)\,\epsilon_t \text{ , for all } t \geq 0$$

where $0 < d \leq D$.

$$\begin{cases} X_{t+1} &= \delta_t X_t \exp\left[r \left(1 - \frac{X_t}{k} \right) - Y_t \right] \\ Y_{t+1} &= X_t \left[1 - \exp\left(- Y_t \right) \right] \\ \epsilon_{t+1} &= s\,\epsilon_t \left(1 - \epsilon_t \right) \end{cases}$$

Logistic map fixed points $\rightarrow \epsilon^0 = 0, \epsilon^{(s)} = 1 - \frac{1}{s} < 1$

- N. Platt, E. Spiegel, C. Tresser, *Physical Review Letters* (1993)

- **Extinction**

$$P_l^{(0)} = (0, 0, \epsilon^{(l)})$$

- **free-parasitoid**

$$P_l^{(1)} = (X_l^{(1)}, 0, \epsilon^{(l)})$$

where $X_l^{(1)} = \dfrac{k}{r}\,\rho_r(\epsilon^{(l)})$ and $\rho_r(\epsilon^{(l)}) := r + log(\delta(\epsilon^{(l)}))$

- **coexistence**

$$P_l^* = \left(X_l^*, Y_l^*, \epsilon^{(l)}\right)$$

where (X_l^*, Y_l^*) are the positive solutions of the system

$$\begin{cases} X_l^* = \dfrac{k}{r}\left(\rho_r(\epsilon^{(l)}) - Y_l^*\right) \\ Y_l^* = X_l^*\left[1 - \exp\left(-Y_l^*\right)\right] \end{cases}$$

$l = 0,\ s.$

$P_I^{(0)}$ Stability analysis

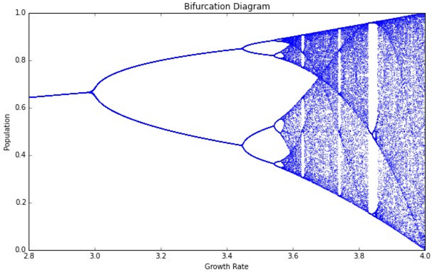
Theorem

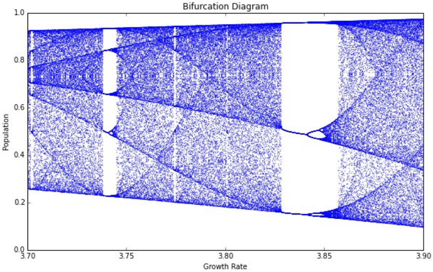
1. The extinction fixed point $P_0^{(0)} = (0,0,0)$ is asymptotically stable if

$$\begin{cases} 0 < s < 1 \\ 0 \leq d < \exp(-r) \end{cases}$$

2. The extinction fixed point $P_s^{(0)} = \left(0,0,\epsilon^{(s)}\right)$ is asymptotically stable if

$$\begin{cases} 1 < s < 3 \\ 0 \leq \delta(\epsilon^{(s)}) < \exp(-r) \end{cases}$$





- **saddle-node** bifurcation at $\bar{s} = 3.82843$;
- **period doubling** bifurcation at $\tilde{s} = 3.8415$

- R. M. May, *Nature* (1976)

On-off intermittency $\rightarrow$ "blowout bifurcation"

Response system: determines the number of invariant subspaces (quiescent phases)

$$S = \{(X, Y, \epsilon) : X = Y = 0\}$$

Drive system: determines the dynamics of the chaotic attractor in the invariant subspace

$$f(\epsilon) = s\, \epsilon_t\, (1 - \epsilon_t)$$

- Y. Nagai, Y.C. Lai, *Physical Review E* (1997)

Transverse Lyapunov exponent

Question: The chaotic attractor in S is also an attractor in the full 3D phase space?

Answer: It depends on the sign of the largest transverse Lyapunov exponent

$$\mu_{t+1} = J_t \, \mu_t = \begin{pmatrix} exp(\rho_r(\epsilon_t)) & 0 \\ 0 & 0 \end{pmatrix} \mu_t$$

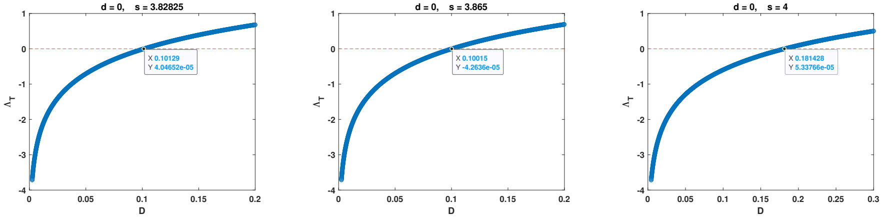
$$\Lambda_T = \lim_{t \to \infty} \frac{1}{t} \ln \left(\max_{\|\mu_0\| \neq 0} \frac{\|\mu_t\|}{\|\mu_0\|} \right) = \lim_{t \to \infty} \frac{1}{t} \ln |||\overset{\curvearrowleft}{\prod_{i=0}^{t-1}} J_i \, |||$$

$$= \lim_{t \to \infty} \frac{1}{t} \ln \sigma_{max} \left(\overset{\curvearrowleft}{\prod_{i=0}^{t-1}} J_i \right) = \lim_{t \to \infty} \frac{1}{t} \ln \left| \prod_{i=0}^{t-1} \exp(\rho_r(\epsilon_i)) \right|$$

- Q. Zhou, Z. Chen, Z. Yuan, *Physica A: Statistical Mechanics and its Applications* (2007)

- $\Lambda_T < 0 \rightarrow$ the chaotic attractor in S is also an attractor in the 3D space;
- $\Lambda_T > 0 \rightarrow$ trajectories in the neighborhood of S are repelled away from it.

Blowout bifurcations $\rightarrow \Lambda_T$ moves from negative to slightly positive values.



The onset of on-off intermittency

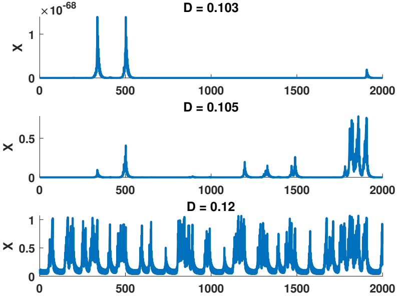


Figure: Intermittent host dynamics for $s = 3.82825$ (left) and $s = 3.865$ (right).

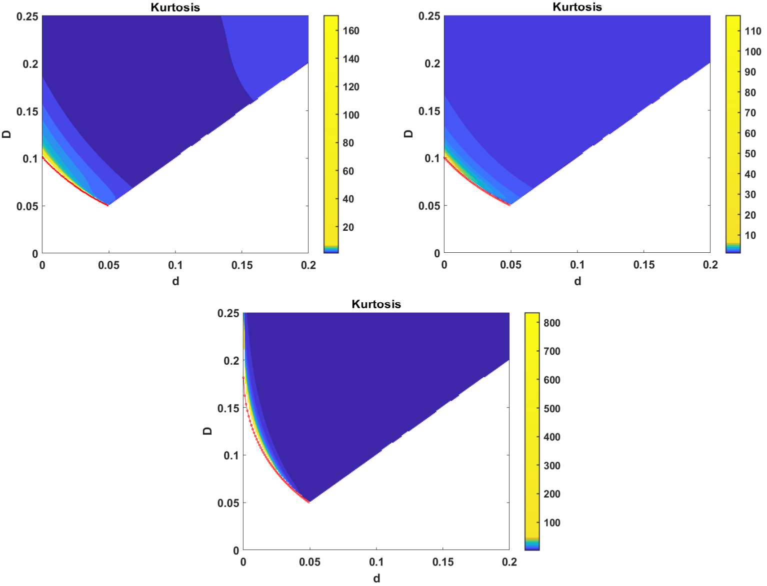
Statistical tool

The **kurtosis** of the time series

s	D	kurtosis
3.82825	0.103	146.7690
	0.105	37.1694
	0.120	4.2875
3.865	0.102	232.1845
	0.104	12.1260
	0.120	3.5081

- J . Heagy, N. Platt, S. Hammel, *Physical Review E* (1994)
- C. Toniolo, A. Provenzale, E. A. Spiegel, *Physical Review E* (2002)
- S. Metta, A. Provenzale, E. A. Spiegel, *Chaos, Solitons & Fractals* (2010)

Kurtosis & critical curve



Deterministic driving process

$$r_t = r_{min} + (R - r_{min})\,\epsilon_t \text{ , for all } t \geq 0$$

where $0 < r_{min} \leq R$

$$\begin{cases} X_{t+1} &= X_t \exp\left[r_t\left(1 - \frac{X_t}{k}\right) - Y_t\right] \\ Y_{t+1} &= X_t\left[1 - \exp(-Y_t)\right] \\ \epsilon_{t+1} &= s\,\epsilon_t\,(1 - \epsilon_t) \end{cases}$$

- **Extinction**

$$P_l^{(0)} = (0, 0, \epsilon^{(l)})$$

- **free-parasitoid**

$$P_l^{(1)} = (k, 0, \epsilon^{(l)})$$

- **coexistence**

$$P_l^* = \left(X_l^*, Y_l^*, \epsilon^{(l)} \right)$$

where (X_l^*, Y_l^*) are the positive solutions of the system

$$\begin{cases} X_l^* & = k - \dfrac{k}{r(\epsilon^{(l)})} Y_l^* \\ Y_l^* & = X_l^* \left[1 - \exp\left(-Y_l^* \right) \right] \end{cases}$$

$l = 0,\ s.$

$P_I^{(1)}$ Stability analysis

Theorem

1. The free-parasitoid fixed point $P_0^{(1)} = (k, 0, 0)$ is asymptotically stable if
$$\begin{cases} 0 < s < 1 \\ 0 < r_{min} < 2 \\ k < 1 \end{cases}$$
2. The free-parasitoid fixed point $P_s^{(1)} = \left(k, 0, \epsilon^{(s)}\right)$ is asymptotically stable if
$$\begin{cases} 1 < s < 3 \\ 0 \leq r\left(\epsilon^{(s)}\right) < 2 \\ k < 1 \end{cases}$$

Invariant subspace: $S_1 = \{(X, Y, \epsilon) : X = k, Y = 0\}$

$$\mu_{t+1} = J_t^{(1)} \mu_t = \begin{pmatrix} 1 - r(\epsilon_t) & -k \\ 0 & k \end{pmatrix} \mu_t$$

$$\Lambda_T = \lim_{t\to\infty} \frac{1}{t} \ln \sigma_{max} \left(\overset{\curvearrowleft}{\prod_{i=0}^{t-1}} J_i^{(1)} \right)$$

$$\tilde{\Lambda}_T = \lim_{t\to\infty} \frac{1}{t} \ln \max_i \left| \lambda_i \left(\overset{\curvearrowleft}{\prod_{i=0}^{t-1}} J_i^{(1)} \right) \right|$$

Suppose $0 < k < r_{min} - 1 \leq 1$,

$$\tilde{\Lambda}_T = \lim_{t\to\infty} \frac{1}{t} \ln \prod_{i=0}^{t-1} (r(\epsilon_i) - 1) = \lim_{t\to\infty} \frac{1}{t} \sum_{i=0}^{t-1} \ln (r(\epsilon_i) - 1).$$

Reactivity of a fixed point

Discrete map $\mathbf{x}(t+1) = \mathbf{f}(\mathbf{x}(t))$

$$\eta(t) = J_{\bar{\mathbf{x}}}^{(t-1)}\eta(t-1) = \ldots = \prod_{i=0}^{t-1} J_{\bar{\mathbf{x}}}^{(i)}\,\eta(0) = H_{t,\bar{\mathbf{x}}}\,\eta(0)$$

Amplification envelope $\phi(t) := \max_{\|\eta(0)\|\neq \mathbf{0}} \dfrac{\|\eta(t)\|}{\|\eta(0)\|} = |||H_{t,\bar{\mathbf{x}}}|||$

$$\phi(t) = \phi_2(t) := \sqrt{\psi(H_{t,\bar{\mathbf{x}}}^T\, H_{t,\bar{\mathbf{x}}})} = \sigma_{max}(H_{t,\bar{\mathbf{x}}})$$

Resilience $L_{1,\bar{\mathbf{x}}} := \lim_{t\to\infty} \dfrac{1}{t} \ln \sigma_{max}(H_{t,\bar{\mathbf{x}}})$

Reactivity $\nu := \log \phi_2(1) = \ln\left[\sigma_{max}(J_{\bar{\mathbf{x}}}^{(0)})\right]$

$$\nu > 0 \implies \sigma_{max}(J_{\bar{\mathbf{x}}}^{(0)}) > 1 \implies \bar{x}\ \textit{reactive}$$

- H. Caswell, M. G. Neubert, *Journal of Difference Equations and Applications (2005)*

$$\begin{cases} X_{t+1} & = X_t \exp\left[r(\epsilon_t)\left(1 - \frac{X_t}{k}\right) - Y_t\right] \\ Y_{t+1} & = X_t\left[1 - \exp\left(- Y_t\right)\right] \end{cases}$$

Stability of $P_1 = (k, 0) \rightarrow S_1 = \{(X, Y, \epsilon) : X = k, Y = 0\}$

Reactivity of $P_1 = (k, 0) \rightarrow \log|\sigma_{max}(J_0^{(1)})| > 0$

$$J_0^{(1)} = \begin{pmatrix} 1 - r(\epsilon_0) & -k \\ 0 & k \end{pmatrix} \rightarrow \textit{depends on } \epsilon_0$$

"Average" reactivity $\lim_{t\to\infty} \frac{1}{t} \log|\sigma_{max}(J_t^{(1)})| > 0.$

S_1 stability & reactivity

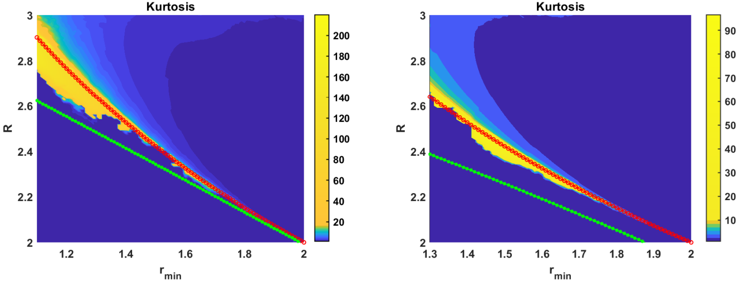
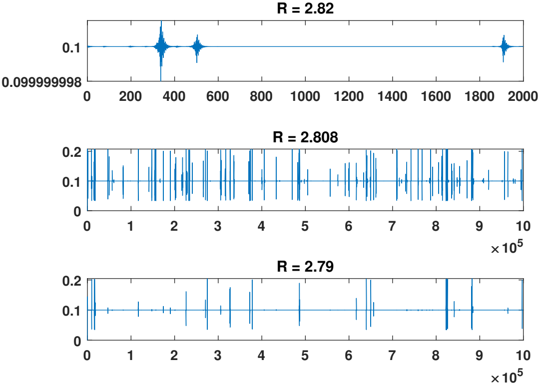


Figure: Kurtosis values for $s = 3.82825$ and $k = 0.1$ (left), $k = 0.3$ (right). Red: stability curve. Green: reactivity curve.



R	kurtosis
2.82	105.46
2.808	226.7
2.79	236.64

Other parameters: $r_{min} = 1.16$, $k = 0.1$, $s = 3.82825$.

Conclusions

- On-off intermittency in a **deterministic scenario**
  - δ, grazing parameter
  - r, growth rate
- The onset of on-off intermittency in the reactivity region of S_1.

Work in progress

- On-off intermittency in a **randomic scenario**

Future work

- The transmission of intermittency in host-parasitoid coupled systems.

References

- T. Azizi, Local stability analysis and bifurcations of a discrete-time host-parasitoid model, *International Journal of Modern Nonlinear Theory and Application* (2020)
- J. Beddington, C. Free, J. Lawton, Concepts of stability and resilience in predator-prey models, *The Journal of Animal Ecology* (1976)
- J. Heagy, N. Platt, S. Hammel, Characterization of on-off intermittency, *Physical Review E* (1994)
- **D. Lacitignola, F. Diele, A. Monti, On-off intermittency in a host-parasitoid model with a deterministic chaotic driver (in preparation)**
- S. Metta, A. Provenzale, E. A. Spiegel, On–off intermittency and coherent bursting in stochastically-driven coupled maps, *Chaos, Solitons & Fractals* (2010)
- **A. Monti, F.Diele, C.Marangi, A. Provenzale, The onset of intermittency in the Beddington-Free-Lawton model (in preparation)**
- Y. Nagai, Y.C. Lai, Characterization of blowout bifurcation by unstable periodic orbits, *Physical Review E* (1997)
- Y. Pomeau, P. Manneville, Intermittent transition to turbulence in dissipative dynamical systems, *Communications in Mathematical Physics* (1980)
- C. Toniolo, A. Provenzale, E. A. Spiegel, Signature of on-off intermittency in measured signals, *Physical Review E* (2002)
- G. Vissio, A. Provenzale, On-off intermittency and irruptions in host- parasitoid dynamics, *Journal of Theoretical Biology* (2022)
- Q. Zhou, Z. Chen, Z. Yuan, On–off intermittency in continuum systems driven by Lorenz system, *Physica A: Statistical Mechanics and its Applications* (2007)

Thank you for your attention!

a.monti@ba.iac.cnr.it