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Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

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Models for the spread and control of invasive species

Fasma Diele

Carmela Marangi
Angela Martiradonna

Istituto per le Applicazioni del Calcolo "Mauro Picone"

Ailanthus Altissima and wild boars in Alta Murgia National Park

In collaboration with:

Chris Baker (University of Melbourne, AU)
Antonello Provenzale (CNR-IGG)
Deborah Lacitignola (Università di Cassino)

Workshop MathAIEOapp 5 -7 July 2023 Dipartimento di Scienze del Suolo, della Pianta e degli Alimenti, Univ. Bari, Italy

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EU projects context

BIOSOS (FP7-SPACE) BIO_SOS

- BIOdiversity multi-SOurce monitoring System: from Space TO Species
- FP7-SPACE-2010-1. Collaborative Project, addressing topic SPACE.2010.1.1-04 Stimulating the development of GMES services in specific areas with application to BIODIVERSITY G.A. 263435
- Coordinatore Palma Blonda IIA (già ISSIA)-CNR.
- Responsabilità di Unità operativa Carmela Marangi, IAC-CNR.
- Dal: 01/12/2010 Al: 30/11/2013.
- Activity: Development of Innovative Ecological (Environmental) Models for the Management of Natura 2000 Sites, incorporating Monitoring Information

ECOPOTENTIAL (H2020)

- Improving Future Ecosystem Benefits Through Earth Observations
- H2020 research and innovation programme. GA.641762,
- Coordinatore Antonello Provenzale IGG-CNR.
- Responsabilità di Unità operativa Carmela Marangi, IAC-CNR.
- Dal 1-6-2015 Al: 31-5-2019
- Activity: Mathematical modeling of the spread of invasive species with assimilation of satellite data. Optimal control of invasive species in support of Protected Area management activities.

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National Biodiversity Future Center-ongoing project

On May 22, 2023, in celebration of the World Biodiversity Day, the National Biodiversity Future Center (NBFC) was presented in Rome.

The National Biodiversity Future Center is one of the five national centers dedicated to frontier research funded by the "Piano Nazionale di Ripresa e Resilienza (PNRR)"

It is the first Italian research center dedicated to biodiversity and is coordinated by the National Research Council (CNR).

The center involves 2,000 scientists and 49 institutions who are committed to studying and preserving ecosystems and biological diversity.

The National Biodiversity Future Center has the responsibility of conserving, restoring, monitoring, and enhancing Italian and Mediterranean biodiversity.

the NBFC strives to bridge the gap between scientific knowledge and practical conservation actions.

The HUB & Spoke Model

The center is structured into six thematic spokes dedicated to the sea, land, wetlands, and cities.

Spoke 4 – Task 4.3

Biodiversity and Ecosystem Modeling

Coordinator: Antonello Provenzale (CNR-IGG)

Soil Organic Carbon Dynamics

Control of Invasive Species

CNR-IAC Staff:

Carmela Marangi, Fasma Diele,

Angela Martiradonna (TD-PNRR)

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Natura2000 site IT9120007 Murgia Alta

Wild boar in Alta Murgia National Park

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Wild boar emergency, hectares of crops destroyed in Puglia

We need a plan of interventions for truly sustainable coexistence between agricultural activities, environmental and animal protection, biodiversity.

Il Cinghiale e la Biodiversità

Scillitani L. [1], Monaco A. [2], Bertolino S. [3]

1: collaboratrice presso Parco Nazionale del Gran Sasso e Monti della Laga
2: Agenzia Regionale per I Parchi – Regione Lazio
3: DISAFA - Università degli Studi di Torino

Cronaca

PARCO DELL'ALTA MURGIA, CINGHIALE AGGREDISCE UN PASTORE 40ENNE E LO FERISCE AD UNA MANO

20 Novembre 2021 Francesco Marinelli 0 commenti News

Coldiretti: "In Puglia 300 accidents a year due to wild boars"

The alarm was launched by Coldiretti Puglia, in reference to the number of road accidents caused by wild animals.

di Simone Ricci — December 20 2020 in Puglia, latest Reading time: 2 minutes of reading AA 0

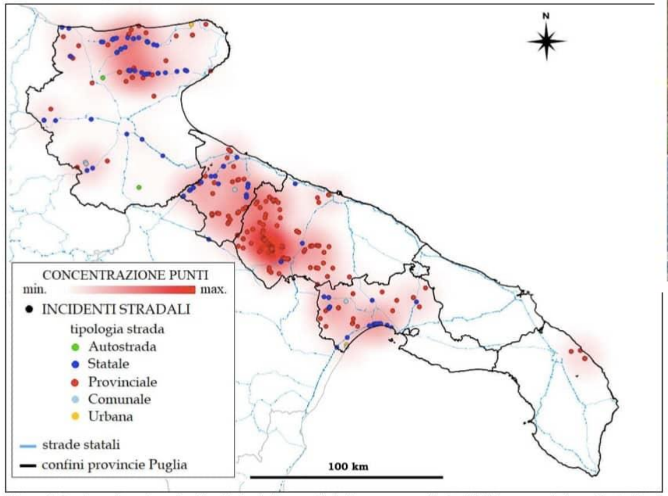

Figura 1.B – Localizzazione incidenti stradali causati dalla presenza di cinghiali in careggiata e tipologia di strada su cui si è verificato. Gradiente di densità degli incidenti sul territorio regionale

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Wolf in Alta Murgia National Park

- Wolves, as well as other large carnivores, e.g. coyotes and dingoes, are considered keystone species
- The legal status of wolves in the European Union countries is directly specified in the Habitats Directive (92/43/EEC). The Directive requires strict protection, prohibiting any destruction or damage to the wolf population
- The challenge for the regional authorities is to plan actions capable of limiting the presence of wild boars while maintaining the small population of wolves in the park
- The crucial point is: to what extent could one *control* wild boars without affecting the wolves' survival?

Puglia: allevatori difendono il lupo "ci aiuta a controllare i cinghiali nell'Alta Murgia"

21 Agosto 2020

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Mathematical tools

- qualitative analysis
- Z-control
- Optimal control

Ecological Modelling 316 (2015) 28–40

Contents lists available at ScienceDirect

Ecological Modelling

journal homepage: www.elsevier.com/locate/ecolmodel

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Dynamical scenarios from a two-patch predator–prey system with human control – Implications for the conservation of the wolf in the Alta Murgia National Park

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Mathematical Biosciences 280 (2016) 10–23

Contents lists available at ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

ELSEVIER

On the dynamics of a generalized predator–prey system with Z-type control

CrossMark

Deborah Lacitignola[a], Fasma Diele[b,*], Carmela Marangi[b], Antonello Provenzale[c]

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Nonlinear Analysis: Real World Applications 49 (2019) 45–70

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

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ELSEVIER

Optimal control of invasive species through a dynamical systems approach

Check for updates

Christopher M. Baker[a,b], Fasma Diele[c,*], Deborah Lacitignola[d], Carmela Marangi[c], Angela Martiradonna[c]

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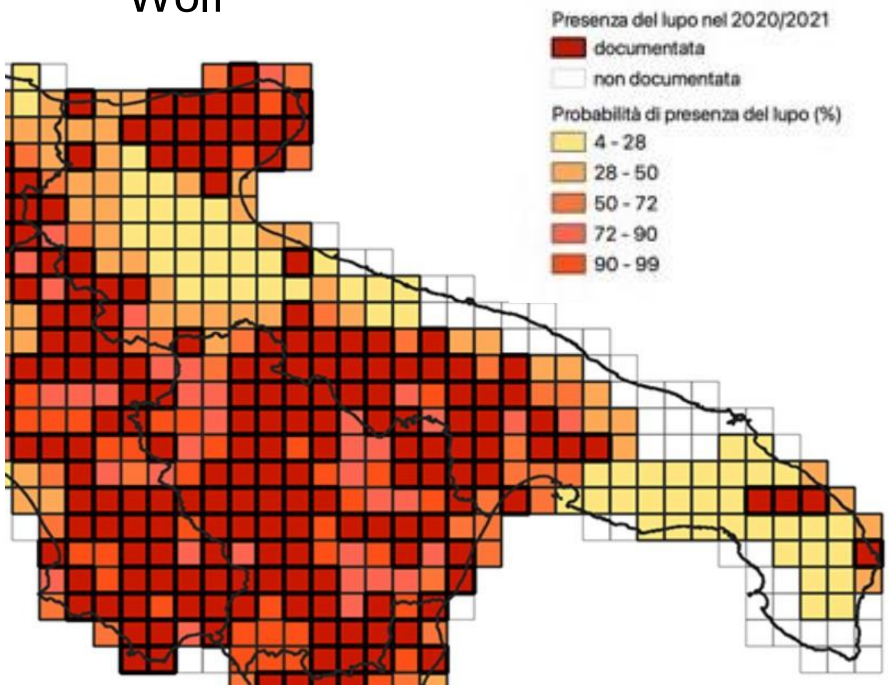
Wolf-wild boar metapopulation dynamics

Wolf



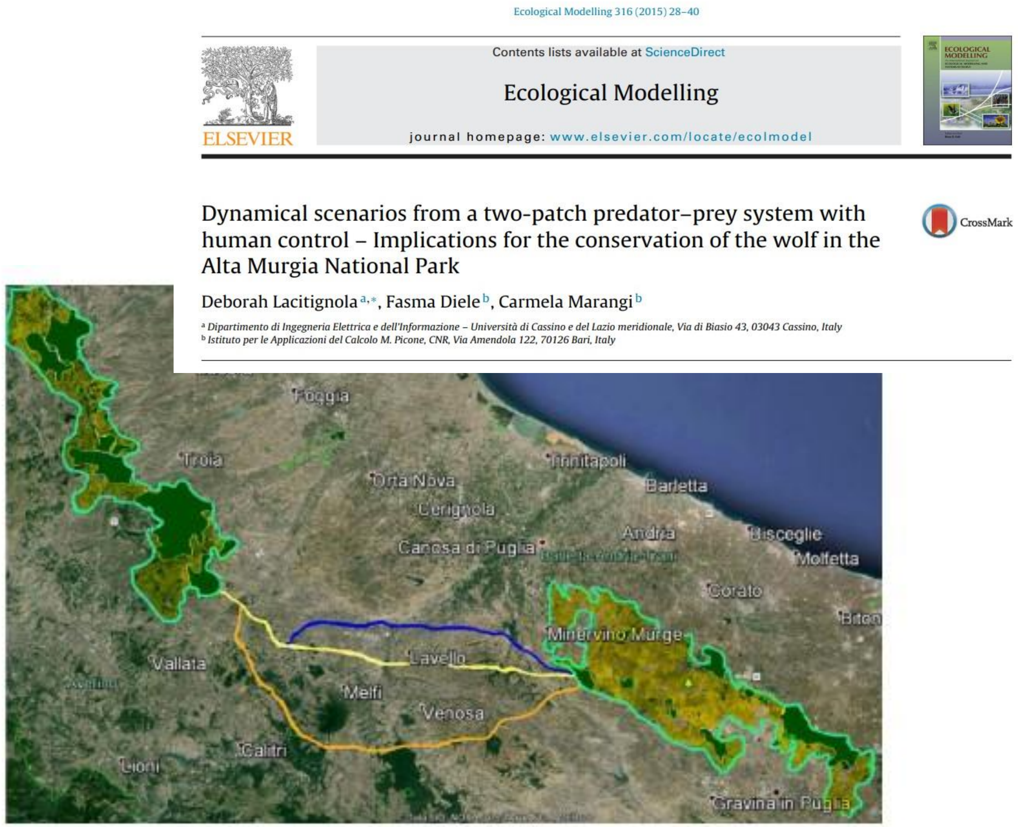

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Fig. 1. Representation of the two areas (Alta Murgia and Monti Dauni) and of the three possible corridors that represent, from top to bottom, three alternative hypotheses of wolf colonization of the Alta Murgia from Monti Dauni: (top) the hypothesis that they have crossed the river Ofanto; (middle) the hypothesis of the shortest path; (bottom) the hypothesis of the path of natural areas (Pennacchioni, 2010). In our modeling framework, the two areas are razionalized as two patches.

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The model

a_1	Predation coefficient
r	Wild boar growth rate
e	Conversion coefficient
μ	Wolf death rate
k	Carrying capacity
A	Allee threshold
q	Catchability coefficient

$$\frac{dn}{d\tau} = \epsilon\left[rn\left(1-\frac{n}{k}\right)\left(\frac{n}{A}-1\right) - a_1 n p_1 - qEn\right]$$

$$\frac{dp_1}{d\tau} = d_2 p_2 - d_1 p_1 + \epsilon[-(\mu + qE)p_1 + e a_1 n p_1]$$

$$\frac{dp_2}{d\tau} = d_1 p_1 - d_2 p_2 + \epsilon[-\mu p_2],$$

- Allee effect: fast growth only above a threshold A
- Two time scales: τ and t= τϵ, ϵ <<1

The aggregated model

Auger et al. 2007, Lecture Notes in Mathematics, Math. BioSciences Subseries, 1936, pp. 209-264

$$p = p_1 + p_2$$

$$v_1 = p_1/p$$

$$v_2 = p_2/p$$

$$\frac{dn}{d\tau} = \epsilon n\left[r\left(1-\frac{n}{k}\right)\left(\frac{n}{A}-1\right) - a_1 v_1 p - qE\right]$$

$$\frac{dv_1}{d\tau} = d_2 - v_1(d_1+d_2) + \epsilon v_1(1-v_1)(e a_1 n - qE)$$

$$\frac{dp}{d\tau} = \epsilon p[-\mu + e a_1 v_1 n - qE v_1].$$

Aggregation method equilibrium of the fast variable:

$$v_1^* = d_2/(d_1+d_2) = d$$

substituted in the first and third eqs to reduce the number of the variables

.

$$\frac{dn}{dt} = n\left[r\left(1-\frac{n}{k}\right)\left(\frac{n}{A}-1\right) - a_1 d p - qE\right]$$

$$\frac{dp}{dt} = p[-\mu + e a_1 d n - qEd]$$

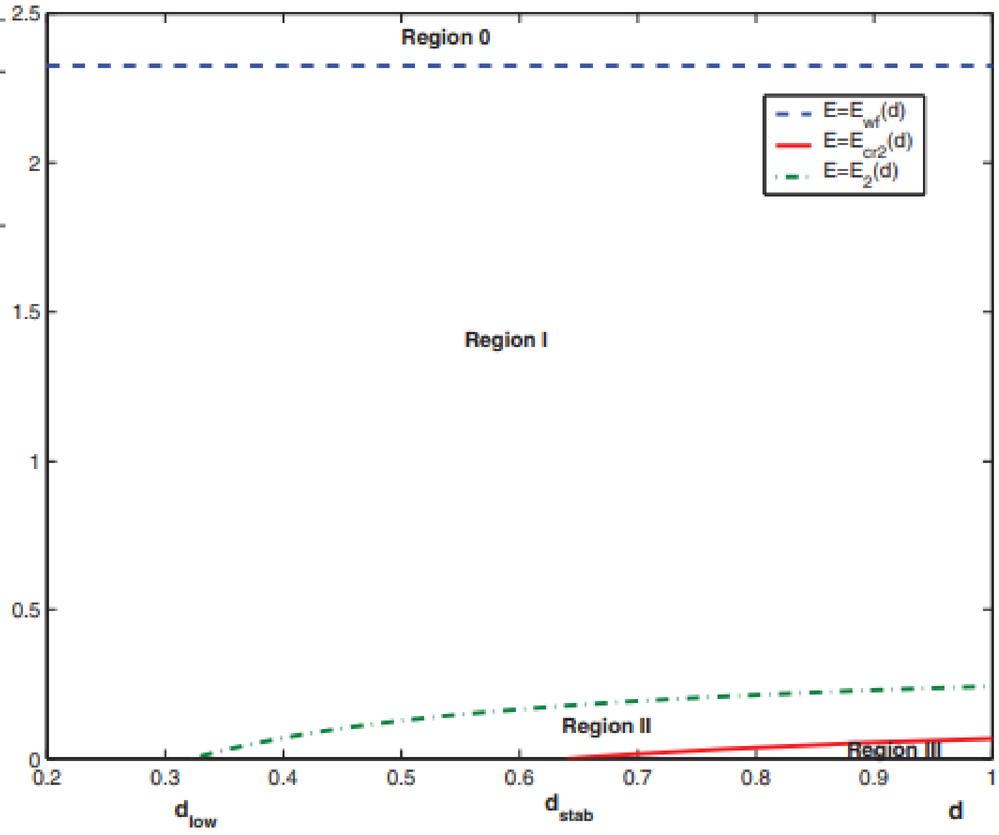
Dynamical scenario

Control parameters of the models with the related description.

Control parameter	Description
d_1	Wolf migration rate from patch1 to patch2
d_2	Wolf migration rate from patch2 to patch1
E	Effort of the control
d	Relative migration rate from patch2 to patch1 ($d = d_2/(d_1 + d_2)$)

The aggregated model (5). Existence and stability properties for the equilibria of the aggregated system (5) in the four regions of the two-parameter bifurcation diagram in Fig. 5.

Region 0	Region I	Region II	Region III
P_0 stable	P_0 stable P_1 unstable P_2 stable	P_0 stable P_1 unstable P_2 unstable P^* stable	P_0 stable P_1 unstable P_2 unstable P^* unstable



Conservations of the wolf in the Alta Murgia Park

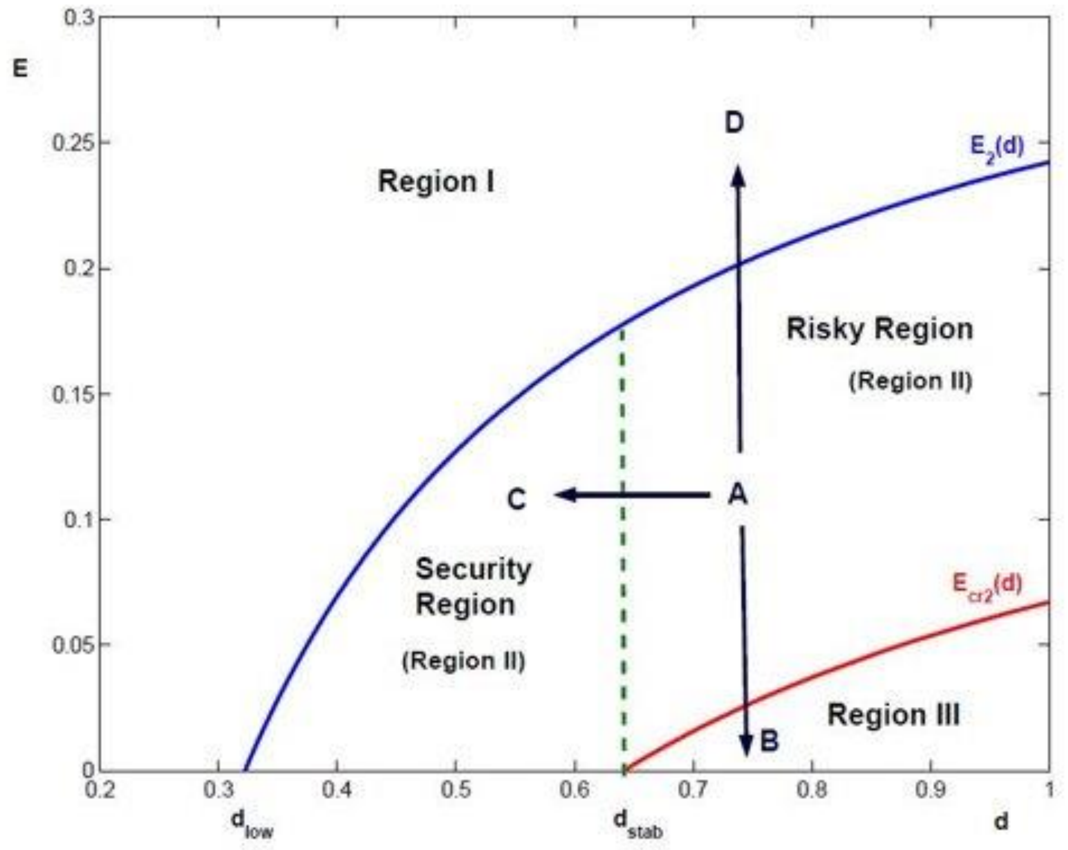


Fig. 7. A detail of the (d, E) bifurcation diagram in Fig. 5 which specifically focuses on Region II. The bifurcation parameter d is related to the wolf migration processes between patches whereas the bifurcation parameter E represents the effort of the human control on the wild boar population. In Region II coexistence between wolf and wild boar population can be ensured through a stable equilibrium. Theoretical results suggest that, from the point of view of the control strategies, such region can be divided in two different sub regions: a *Security Region* and a *Risky Region*. (For

Table 1
Parameter values used in simulations and the related sources.

Parameter	Description	Value	Units	Source
a_1	Predation coefficient	0.1108	$km^2\ year^{-1}\ N^{-1}$	Nores et al. (2008)
r	Wild boar growth rate	0.0484	$year^{-1}$	Census data
e	Conversion coefficient	0.0280	Dimensionless	Knickerbocker (2013)
μ	Wolf death rate	0.12	$year^{-1}$	Knickerbocker (2013)
k	Carrying capacity	120	$N\ km^{-2}$	Leaper et al. (1999)
A	Allee threshold	0.6182	$N\ km^{-2}$	Leaper et al. (1999) Howells and Edward-Jones (1997)
q	Catchability coefficient	1	$year^{-1}$	Assumed

Table 2
Variables of the models with initial values and related sources.

Variable	Description	Initial value	Source
p_1	Wolf density in the Alta Murgia Park (patch1)	0.0363	PGC (2012)
p_2	Wolf density in the Dauni Mountains site (patch2)	0.02	Pennacchioni (2010)
n	Wild boar density in the Alta Murgia Park	14	PGC (2012)
p	Total number of wolves	0.0563	Pennacchioni (2010) PGC (2012)

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Z-control: basic idea

Y. Zhang and Z. Li. Zhang neural network for online solution of time-varying convex quadratic program subject to time-varying linear-equality constraints. Physics Letters A, 373(18):1639–1643, 2009.

Generalized Lotka-Volterra model

$$\dot{n} = n[H(n) - \beta p] \qquad \dot{p} = pF(n)$$

$$\Rightarrow \quad \dot{n} = n[H(n) - \beta p] \qquad \dot{p} = p[F(n) - u_{pred}]$$

$$\Rightarrow \quad \dot{n} = n[H(n) - \beta p] \qquad \dot{p} = f(t; n, p)$$

The goal of Z-control is to design an analytical expression for the variable $u_{pred} = u_{pred}(t)$ so that the solution is forced to achieve a desidered state $n_d = n_d(t)$

$$e(t) = n(t) - n_d(t) \to 0$$

$$\dot{e} = -\lambda e(t)$$

λ is the *design* parameter

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Z-control of the wolf-wild boar dynamics

Mathematical Biosciences 280 (2016) 10–23

Contents lists available at ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

On the dynamics of a generalized predator–prey system with Z-type control

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By applying the indirect Z-control of the prey population acting on predator to the aggregated model of wolf-wild boar

$$\dot{n} = n\left[r\left(1-\frac{n}{k}\right)\left(\frac{n}{a}-1\right) - a_1\, dp - qE\right]$$

$$\dot{p} = p[-\mu + ea_1\, dn - qEd],$$

$$H(n) = r(1-\tfrac{n}{k})(\tfrac{n}{a}-1) - qE$$

$$u_{pred} = -\mu + e\, a_1\, d\, n - qEd - \frac{f(n,p)}{p(t)}$$

$$\dot{n} = n[H(n) - \beta p]$$

$$\dot{p} = f(t; n, p)$$

$$f(n,p) = \frac{n\left[\left(r\left(1-\frac{n}{k}\right)\left(\frac{n}{a}-1\right) - qE - a_1\, d\, p + \lambda\right)^2 + \dot{H}(n)\right] - \lambda^2 n_d}{a_1\, d\, n}$$

with

$$\dot{H}(n) = \frac{n\, r}{k\, a}(k + a - 2n)\left[r\left(1-\frac{n}{k}\right)\left(\frac{n}{a}-1\right) - qE - a_1\, d\, p\right].$$

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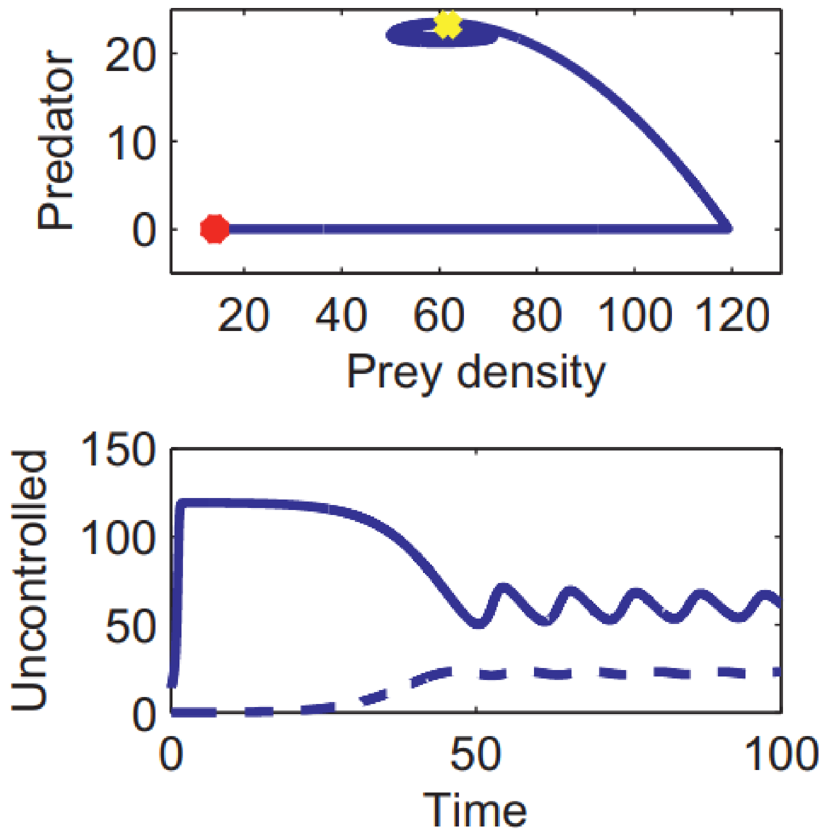
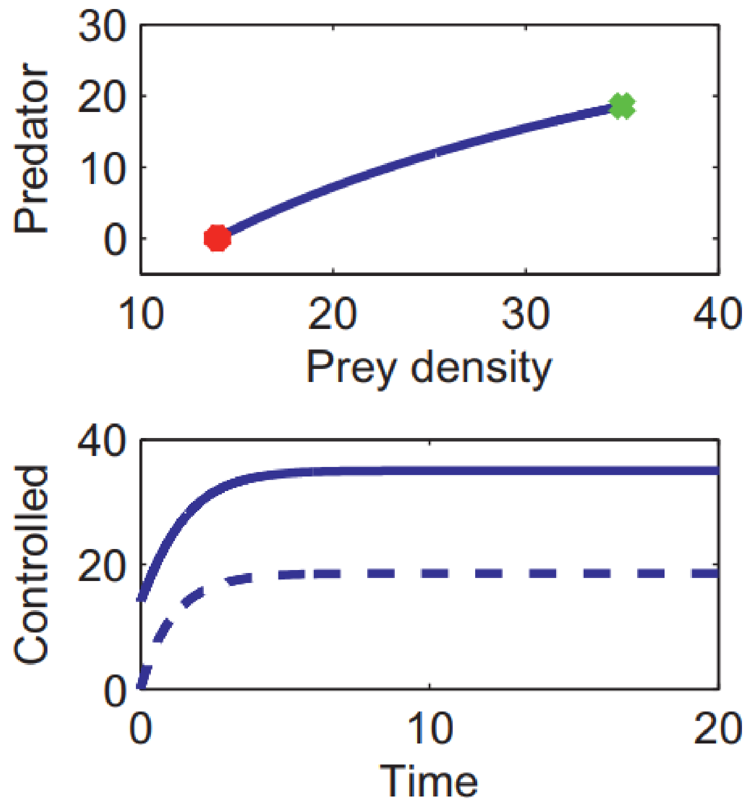
Coexistence via self-sustained oscillations



$$P_e = (60.268, 22.766)$$



$$P^* = (n_d, \frac{H(n_d)}{\beta}) = (35,\ 18.577)$$

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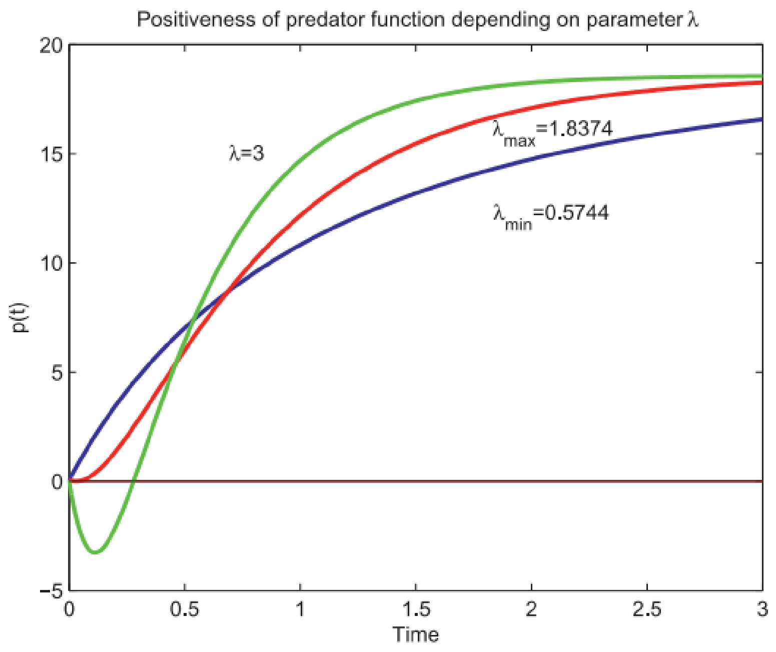
Thresholds for positiveness

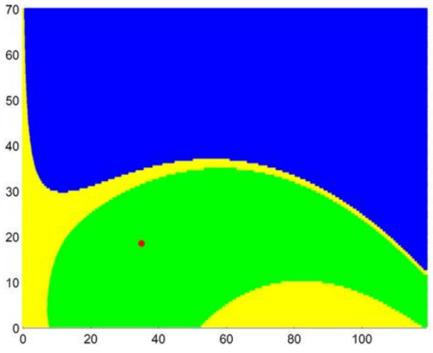


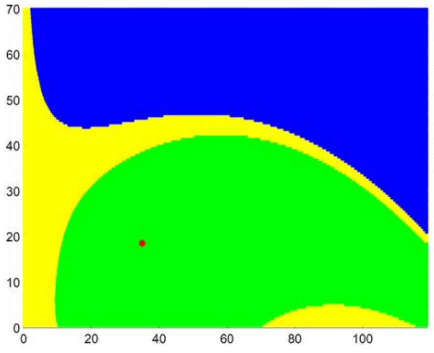
$n(0) = 14,\ p(0) = 0.0563.$ → $0.5774 < \lambda < 1.8374$

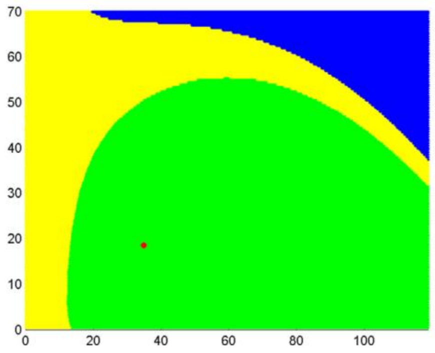
Basin of attraction

$\lambda = 0.6$

$\lambda = 1$







$\lambda = 1.8$

Optimal control

Dynamics of the invasive species

$$\dot{u} = f(u) - u(\mu E)^q$$

μ scaling parameter, q diminishing return parameter, $0 < q < 1$

Goal: reduce the density from $u(0) = u_0$ to $u(T) = u_T$ with a minimal cost

$$\min_{E(t)} \int_0^T \mu(t)\, E(t)\, dt$$

Optimal control: Pontryagin's maximum principle applied to

$$H(t, u, \lambda, E) = \mu(t)E + \lambda(f(u) - u\mu(t)^q E^q)$$

$$\dot{u} = f(u) - u\,(q\,u\,\lambda)^{\frac{q}{1-q}}$$

$$\dot{\lambda} = -\lambda\left(f'(u) - (q\,u\,\lambda)^{\frac{q}{1-q}}\right)$$

State-costate system

$$\dot{u} = f(u) - u\,\mu^q E^q$$

$$\dot{E} = \frac{1}{1-q}\left(\frac{f(u)}{u} - f'(u)\right)E$$

State-control system

Nonlinear Analysis: Real World Applications 49 (2019) 45–70

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Nonlinear Analysis: Real World Applications

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Optimal control of invasive species through a dynamical systems approach

Christopher M. Baker [a,b], Fasma Diele [c,*], Deborah Lacitignola [d], Carmela Marangi [c], Angela Martiradonna [c]

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The case of wild-boars

$$H(u(t), E(t), \lambda(t)) = E + \lambda u \left[r \left(\frac{u}{k_0} - 1 \right) \left(1 - \frac{u}{k} \right) - E^q \right]$$

State-costate system

$$\dot{u} = u\, r \left(\frac{u}{k_0} - 1 \right) \left(1 - \frac{u}{k} \right) - u \ (E^*_{u,\lambda})^q$$
$$\dot{\lambda} = r\,\lambda \left(\frac{3\,u^2}{k_0 k} - \frac{2\,u}{k_0} - \frac{2\,u}{k} + 1 \right) + \lambda \ (E^*_{u,\lambda})^q$$

⟺

State-control system

$$\dot{u} = r\,u \left(\frac{u}{k_0} - 1 \right) \left(1 - \frac{u}{k} \right) - u \ \min\{\max\{E^q, a^q\}, b^q\}$$
$$\dot{E} = A_1 \ (2u - k - k_0) \ u\, E,$$

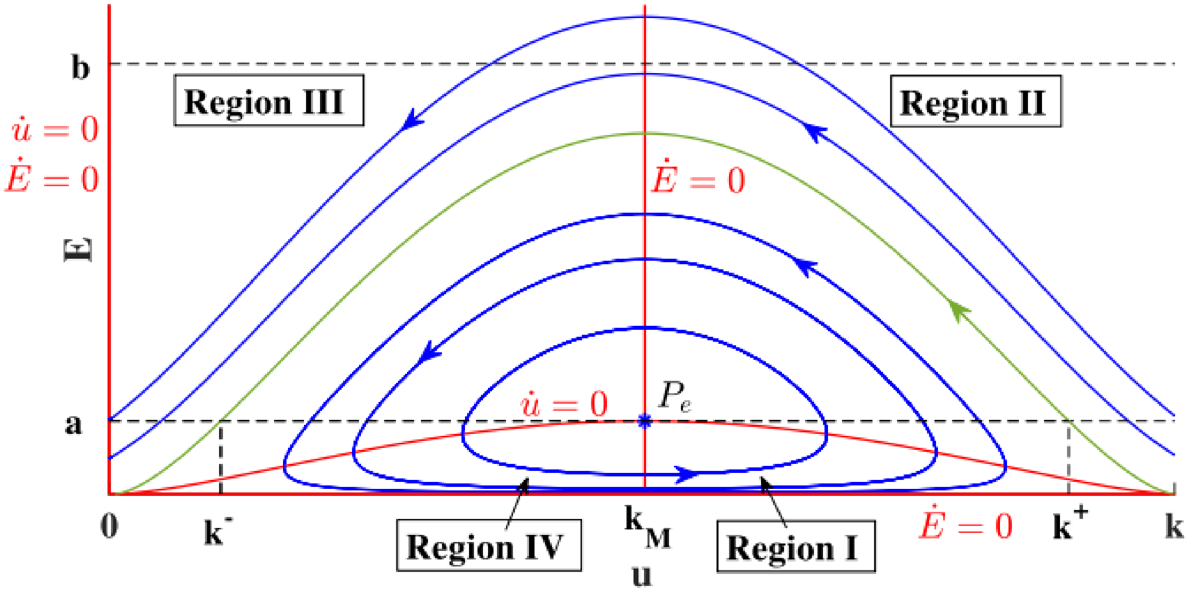
$$E^*_{u,\lambda}(t) = \min\left\{\max\left\{(q\,u(t)\,\lambda(t))^{\frac{1}{1-q}}, a\right\}, b\right\}$$

$$A_1 = \frac{r}{k\,k_0(1-q)}$$

Phase plane analysis of state-control system

$$\dot{u} = r\,u\left(\frac{u}{k_0}-1\right)\left(1-\frac{u}{k}\right) - u\,E^q$$
$$\dot{E} = A_1\,(2u-k-k_0)\,u\,E,$$

$$P^* = (k,0),\ \widetilde{P} = (k_0,0),\ P_e = \left(\frac{k+k_0}{2},\ \left[\frac{r(k-k_0)^2}{4\,k_0\,k}\right]^{1/q}\right)\quad P_E = (0,E)$$

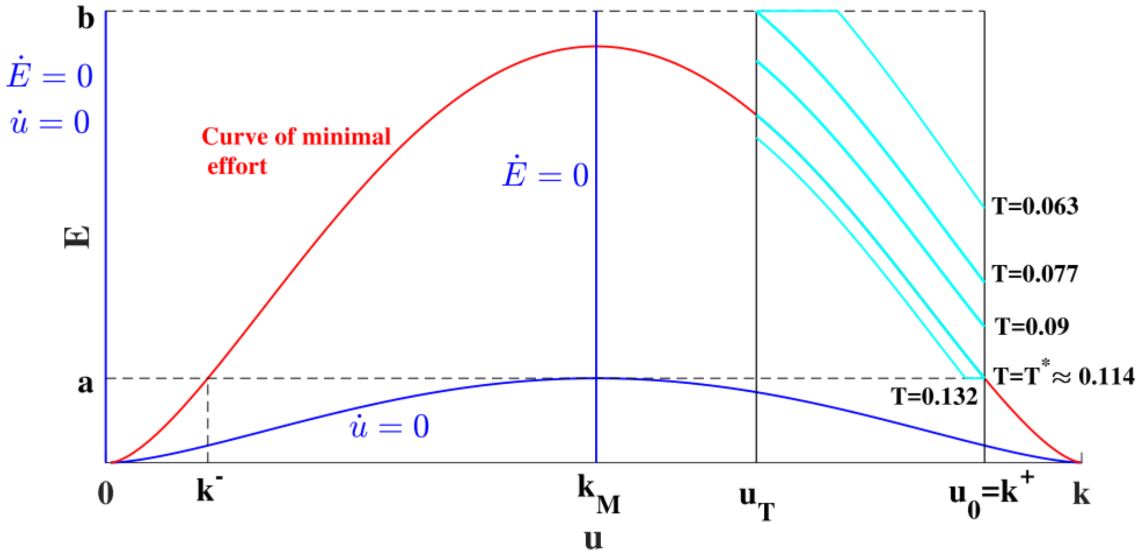


$$\psi(u) = \left[\frac{r}{1-q}\left(\frac{u}{k_0}-1\right)\left(1-\frac{u}{k}\right)\right]^{1/q}$$

the unstable manifold of the hyperbolic equilibrium P^*

Optimal paths for the state-control optimality system

The case $u_0 = k^+$ *with* $u_T \geq k_M$



$$\min_{(E,T)\in U_{a,b}\times[0,\bar{T}]} \int_0^T E(t)\,dt$$

$$T^* = \frac{1}{q\,A_1}\log\left[\left(\frac{u_T}{u_0}\right)^B \left(\frac{u_0-k_0}{u_T-k_0}\right)^C \left(\frac{k-u_T}{k-u_0}\right)^D\right]$$

$$E(t;T^*) = \left[\frac{r}{1-q}\left(\frac{u(t;T^*)}{k_0}-1\right)\left(1-\frac{u(t;T^*)}{k}\right)\right]^{1/q}$$

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Main result

The optimal value of the time horizon T* corresponding to the minimal management effort E*

Aanalytic expressions of population density u*

Understanding the optimal time horizon of an abatement or eradication project is of utmost importance in terms of strategic planning, since it improves the long term sustainability of nature conservation actions.

Theorem 7.3. *Consider the control set* $U_{a,b} = \{E \in L^1(0,T) : a \le E \le b,\ 0 \le T \le \bar{T}\}$, *with* $b \ge \left(\frac{r\,(k-k_0)^2}{(1-q)\,4\,k_0\,k}\right)^{1/q} > \left(\frac{r\,(k-k_0)^2}{4k_0\,k}\right)^{1/q} = a$, *and* $0 < q < 1$. *Let*

$$T^* = \frac{1}{q\,A_1} \log\left[\left(\frac{u_T}{u_0}\right)^B \left(\frac{u_0-k_0}{u_T-k_0}\right)^C \left(\frac{k-u_T}{k-u_0}\right)^D\right].$$

Provided that $\max(k^-, u_b(T^*)) < u_T < u_a(\bar{T}) \le u_0 \le k^+$ *and*

$$\left(\frac{u_T}{u_0}\right)^B \left(\frac{u_0-k_0}{u_T-k_0}\right)^C \left(\frac{k-u_T}{k-u_0}\right)^D \le e^{q\,A_1\,\bar{T}}$$

then, the minimization problem

$$\min_{(E,T)\in U_{a,b}\times[0,\bar{T}]} \int_0^T E(t)\,dt$$

subject to

$$\dot{u} = r\,u\left(\frac{u}{k_0}-1\right)\left(1-\frac{u}{k}\right) - u\,E^q, \quad 0 \le t \le T,\ u(0) = u_0,\ u(T) = u_T,$$

has the optimal solution (E^*, T^*), *where*

$$E^*(t) = \left[\frac{r}{1-q}\left(\frac{u^*(t)}{k_0}-1\right)\left(1-\frac{u^*(t)}{k}\right)\right]^{1/q}, \quad t \in [0, T^*].$$

Moreover, the optimal density solution $u^*(t)$ *is defined implicitly by*

$$\frac{(u^*(t)-k_0)^C}{(u^*(t))^B\,(k-u^*(t))^D} = \frac{(u_T-k_0)^C}{u_T^B\,(k-u_T)^D}\,e^{qA_1(T^*-t)}, \quad t \in [0, T^*].$$

Finanziato dall'Unione europea NextGenerationEU

Ministero dell'Università e della Ricerca

Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

NATIONAL BIODIVERSITY FUTURE CENTER

However, time is not the only relevant dimension of the problem...

It's time for Angela Martiradonna's talk!

Thank you all for your kind attention!

Received: 12 February 2018 | Accepted: 9 July 2018

DOI: 10.1111/nrm.12190

WILEY Natural Resource Modeling

Optimal spatiotemporal effort allocation for invasive species removal incorporating a removal handling time and budget

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