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GravityShapePisa: New Frontiers in Gravity Phenomenology

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The effects of orbital precession on hyperbolic encounters
[arXiv:2307.00915 [gr-qc]]

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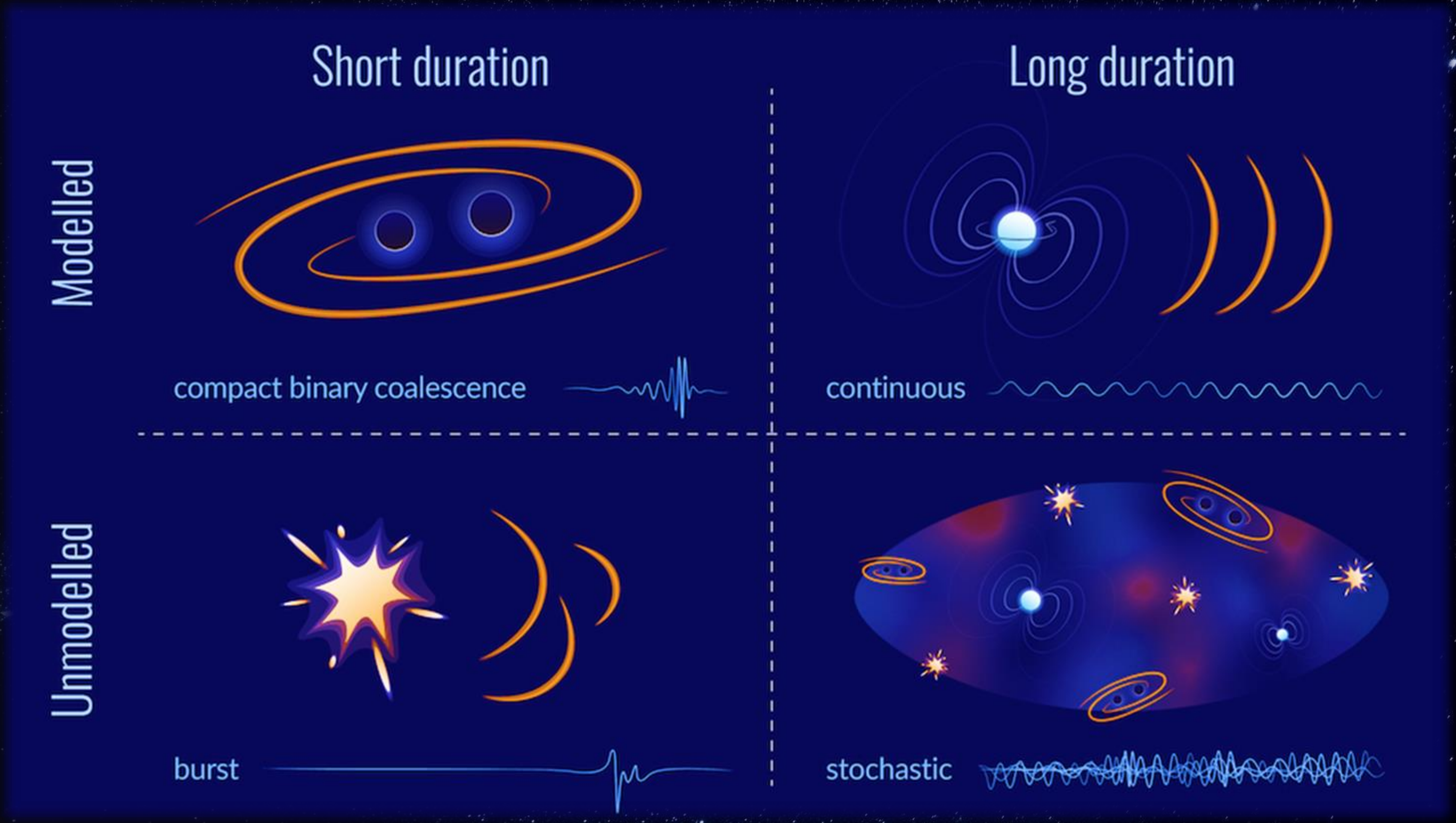
Excelencia Severo Ochoa

Different types of gravitational-wave sources



Credit: spaceaustralia.com

The features of the emitted GW peak signal depend on different parameters, so it may be possible to shed light on the properties of objects and the dynamics of the encounter.

They can produce interesting consequences (spin induction, subsequent mergers, generation of a stochastic GW background, etc).

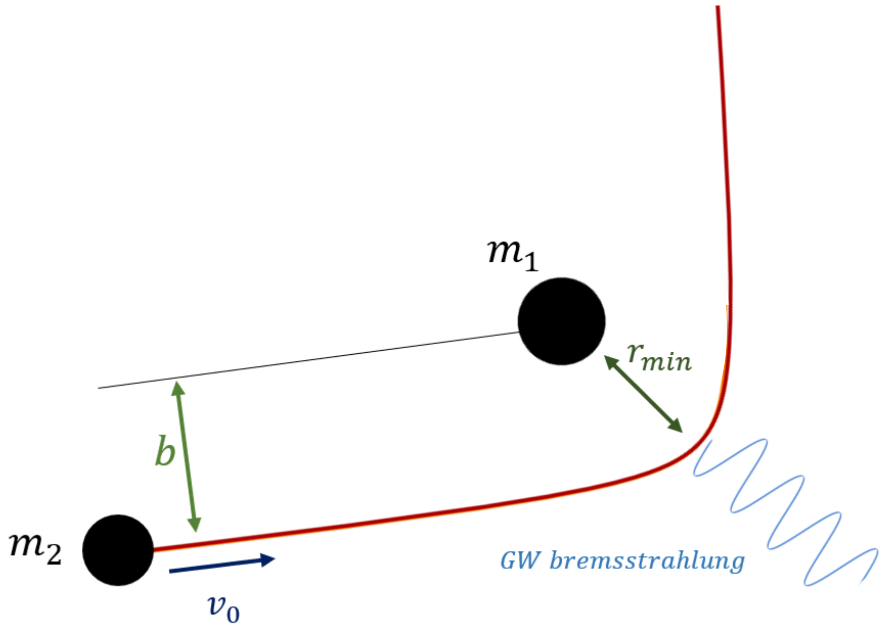
Orbital precession:

- encapsulates some general relativistic effects between two bodies.
- crucial to properly study the physics of such encounters and so for the analysis of GWs signals, as it corresponds to higher post-Newtonian (PN) corrections, needed for more accurate waveforms.
- susceptible to future observations: experimental and analysis challenge.

Why hyperbolic encounters?

M.C., S. Kuroyanagi, S. Nesseris, J. Garcia-Bellido
arXiv: 2307.00915 [gr-qc]

A. without orbital precession



Eccentricity

$$e \equiv \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{b^2 v_0^4}{G^2 M^2}} > 1$$

Semi-major axis and the impact parameter

$$a = \frac{r_{\rm min}}{e-1} \qquad b = r_{\rm min}\sqrt{\frac{e+1}{e-1}}$$

Distance of minimum approach

$$r_{\rm min} = a\,(e-1) = b\,\sqrt{\frac{e-1}{e+1}} > R_s \equiv \frac{2GM}{c^2}$$

Asymptotic velocity

$$v_0 = \sqrt{\frac{(e-1)GM}{r_{\rm min}}}$$

Trajectory

$$r(\varphi) = \frac{a\,(e^2-1)}{1 + e\cos(\varphi)}$$

GW emission

- Quadrupole moment $Q_{ij} = M_{ij} - \frac{1}{3}\delta_{ij} M_{kk}$

$$= \mu\, r^2(\varphi) \begin{pmatrix} \frac{1}{6}(1+3\cos 2\varphi) & \cos\varphi\sin\varphi & 0 \\ \cos\varphi\sin\varphi & \frac{1}{6}(1-3\cos 2\varphi) & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

$$r(\varphi) = \frac{a\,(e^2-1)}{1+e\cos(\varphi)}$$

- GW strain $h_{ij} = \frac{2G}{rc^4}\ddot{Q}_{ij}$

$$h_+ = -\frac{G\mu v_0^2}{rc^4(e^2-1)}\left[4\cos(2\varphi) + e\Big(2e + 5\cos(\varphi) + \cos(3\varphi)\Big)\right]$$

$$h_\times = -\frac{G\mu v_0^2}{rc^4(e^2-1)}\left[4\sin(2\varphi) + e\Big(5\sin(\varphi) + \sin(3\varphi)\Big)\right]$$

- Power emitted $P = \frac{dE}{dt} = \frac{G}{5c^5}\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\rangle$

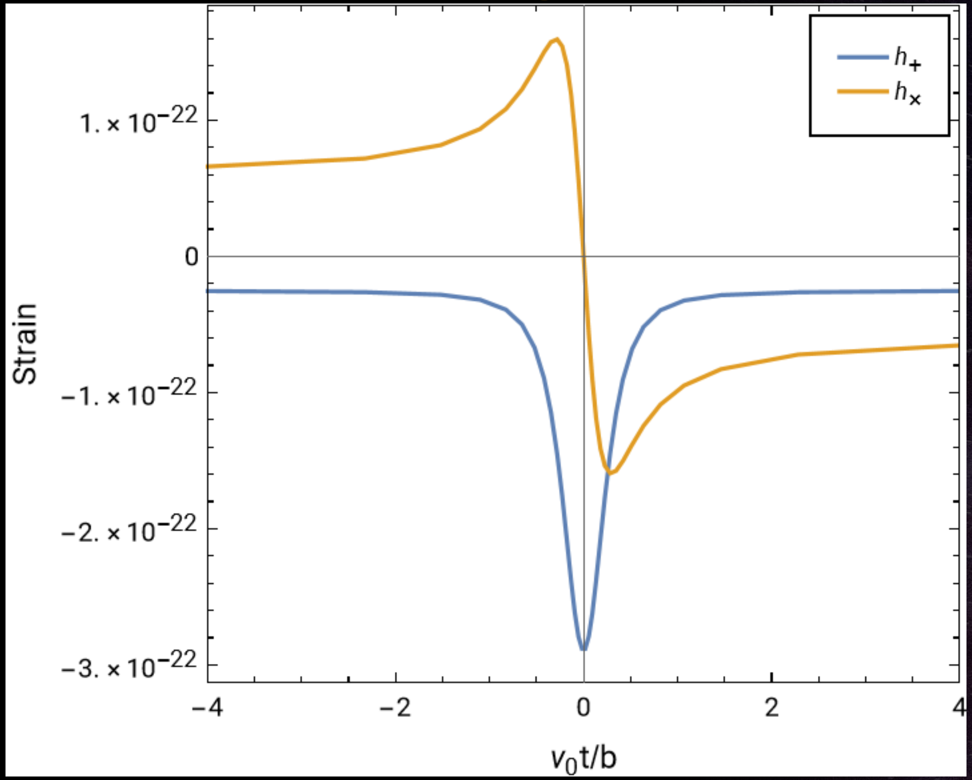
$$P = \frac{32 G\mu^2 v_0^6}{45 c^5\, b^2} f(\varphi, e)$$

$$f(\varphi, e) = \frac{3\,(1+e\cos(\varphi))^4}{8(e^2-1)^4}\left[24 + 13e^2 + 48e\cos(\varphi) + 11e^2\cos(2\varphi)\right]$$

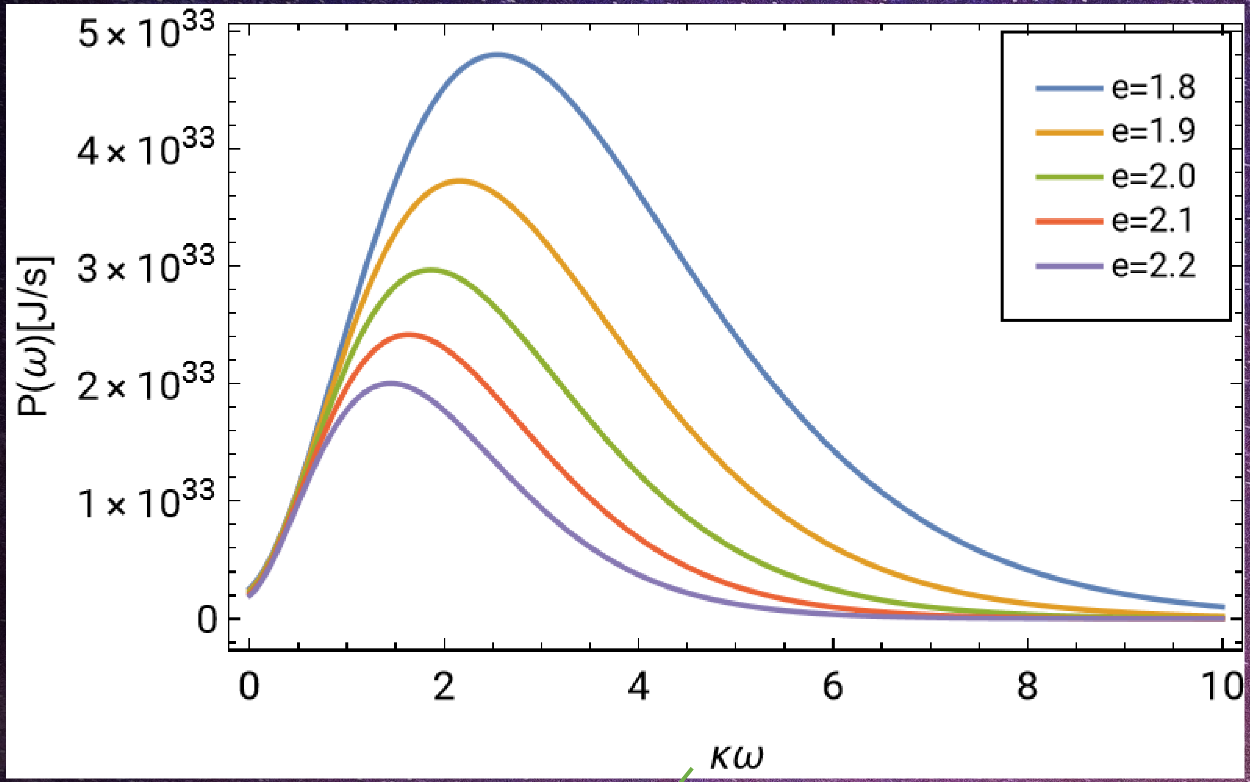
- Fourier transform $P(\omega) = \frac{G}{5c^5}\sum_{i,j}|\widehat{\dddot{Q}_{ij}}|^2 = \frac{G}{5c^5}\,\omega^6 \sum_{i,j}|\widehat{Q_{ij}}|^2$

Results

GW polarizations

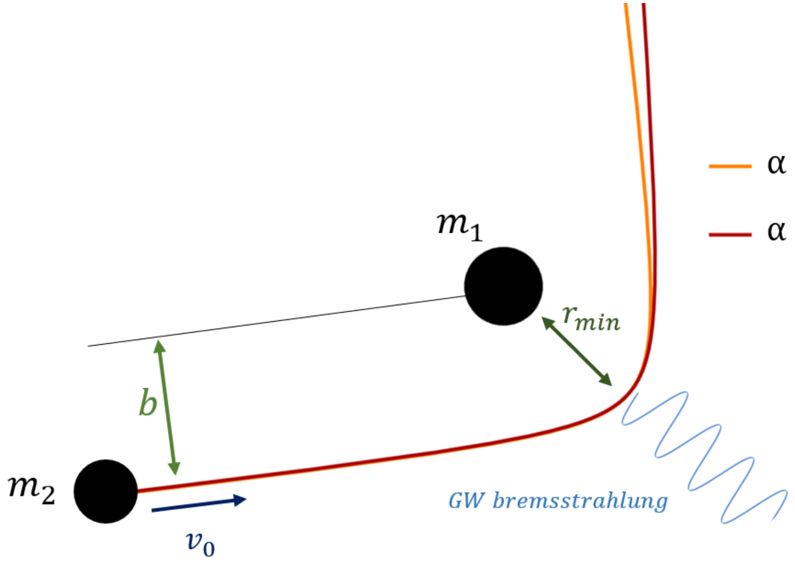


GW power spectrum



$\kappa = \sqrt{a^3/GM}$

B. with orbital precession



Trajectory in polar coordinates

$$r_{\rm pr}(\varphi) = \frac{a\,(e^2-1)}{1+e\cos[(1-\alpha)\varphi]}$$

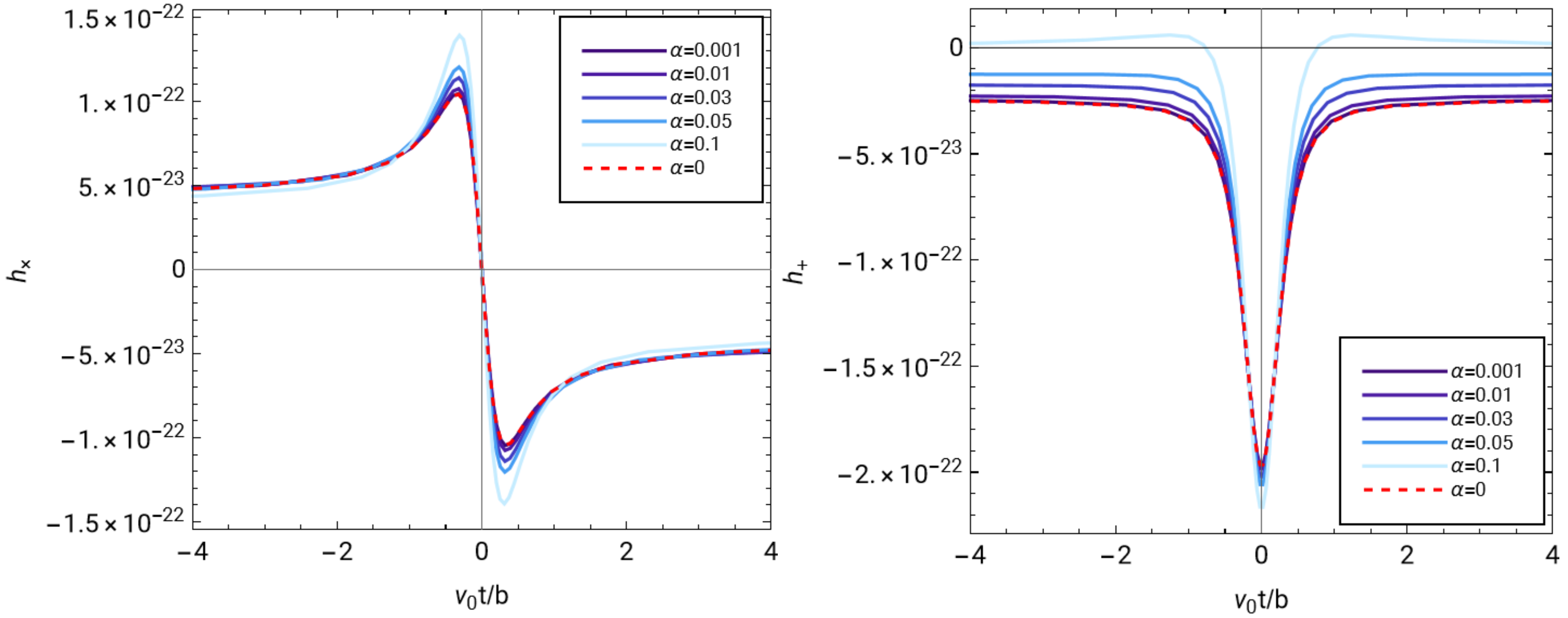
$$\alpha = \frac{3G^2M^2}{c^2L^2} = \frac{3R_s}{2(e+1)r_{\rm min}}$$

GW strain

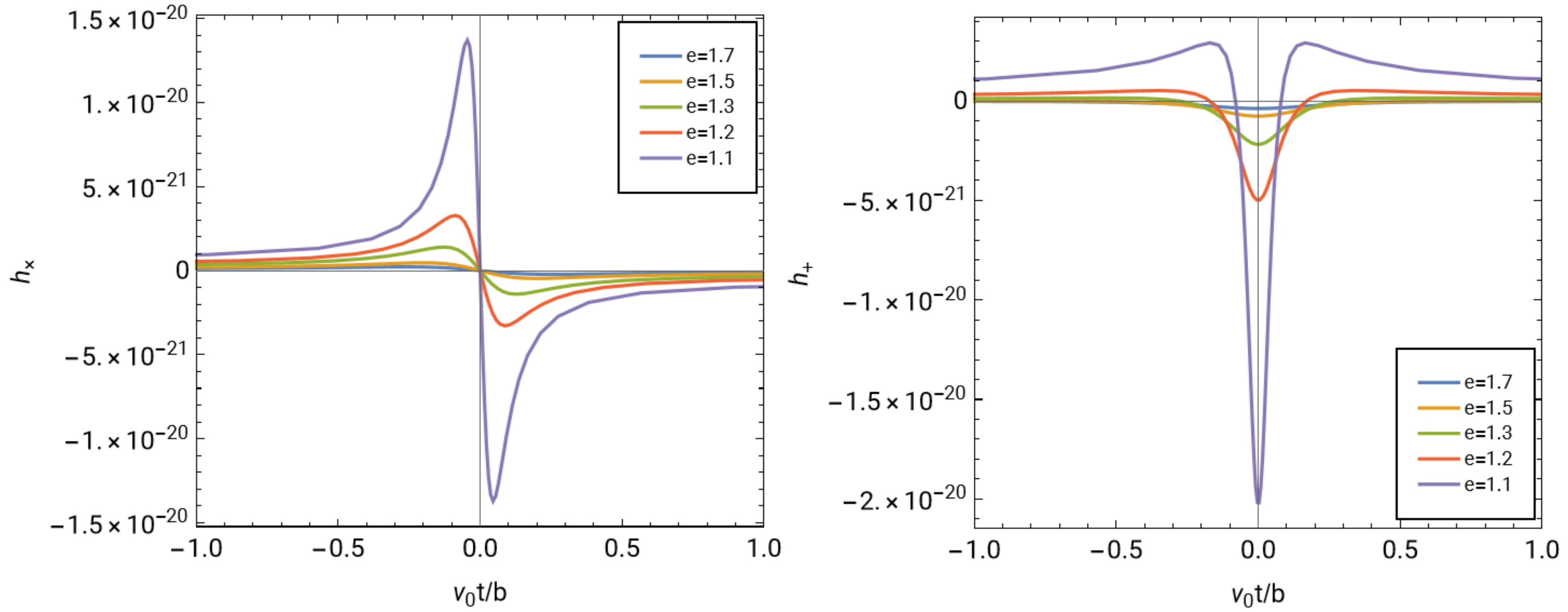
$$h_+ = \frac{G\mu v_0^2}{rc^4(e^2-1)}\Big[2\big(-2+e^2(-2+\alpha)\alpha\big)\cos(2\varphi) + e\Big(\big(-1+(-4+\alpha)\alpha\big)\cos((-3+\alpha)\varphi) - e\,\alpha\cos(2(-2+\alpha)\varphi) + e(-2+\alpha)\cos(2\alpha\varphi) + (-5+\alpha^2)\cos((1+\alpha)\varphi)\Big)\Big]$$

$$h_\times = \frac{G\mu v_0^2}{rc^4(e^2-1)}\Big[2\big(-2+e^2(-2+\alpha)\alpha\big)\sin(2\varphi) + e\Big(\big(1-(-4+\alpha)\alpha\big)\sin((-3+\alpha)\varphi) + e\,\alpha\sin(2(-2+\alpha)\varphi) + e(-2+\alpha)\sin(2\alpha\varphi) + (-5+\alpha^2)\sin((1+\alpha)\varphi)\Big)\Big]$$

Strain for different values of α



Strain for different values of eccentricity



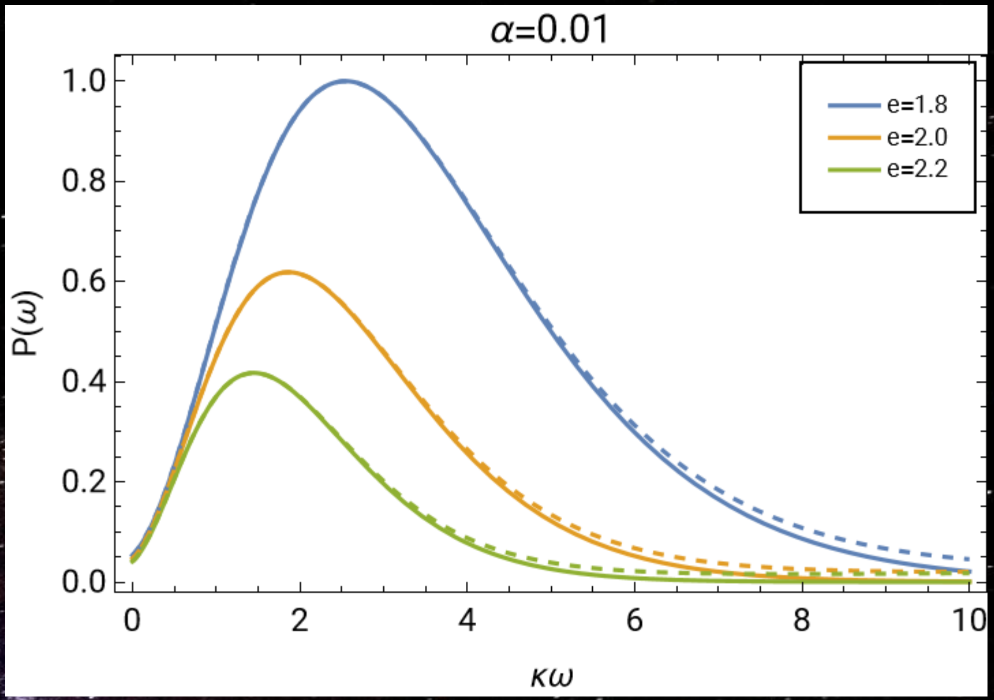
Power spectra

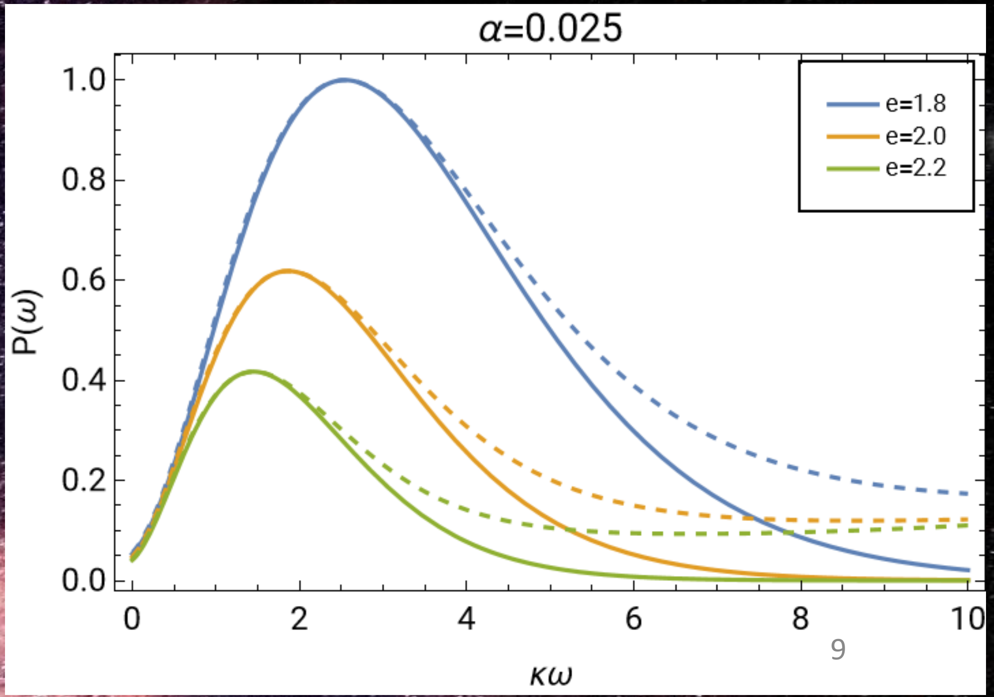
Power spectra variation in terms of the α parameter and the eccentricity e, after numerically integrating.

Solid line: case without orbital precession (agreement with previously obtained results)

- $\alpha = 0$: the result is analytic in terms of Hankel functions – i.e., arXiv:1711.09702v2 [astro-ph.HE]).

Dashed line: comparison with the $\alpha \neq 0$ case.





Memory effect

Long time scale difference in the values of the observed metric perturbation associated with the GW, due to non-linearities in GR, and may be detectable by the Einstein Telescope

$$\Delta h_{+,\times} = \lim_{t\to+\infty} h_{+,\times} - \lim_{t\to-\infty} h_{+,\times} \neq 0$$

- Without considering orbital precession

$$\Delta h_+ = 0\,,$$

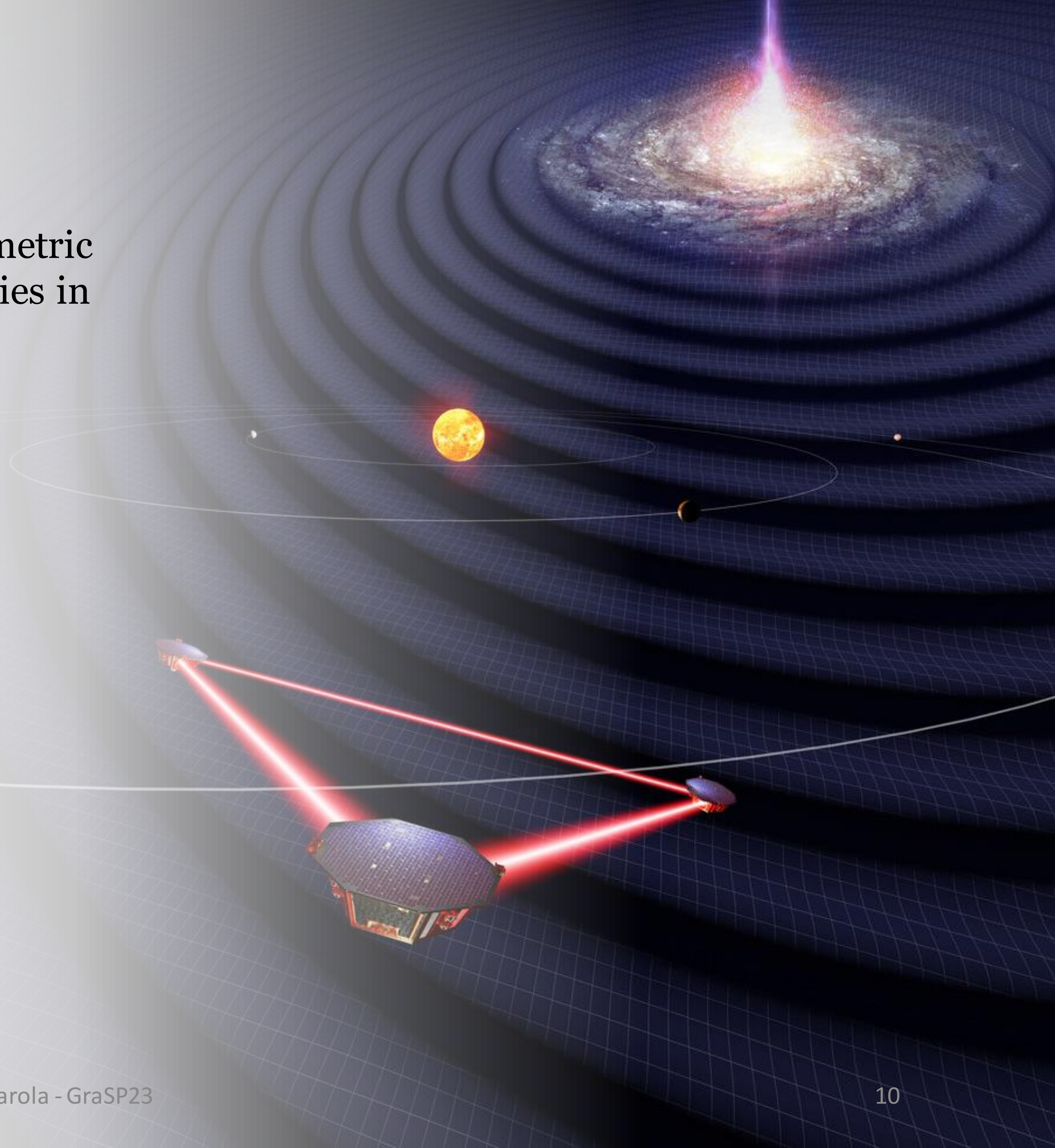
$$\Delta h_\times = \frac{8G\mu v_0^2\sqrt{e^2-1}}{Rc^4e^2}$$

- Considering orbital precession

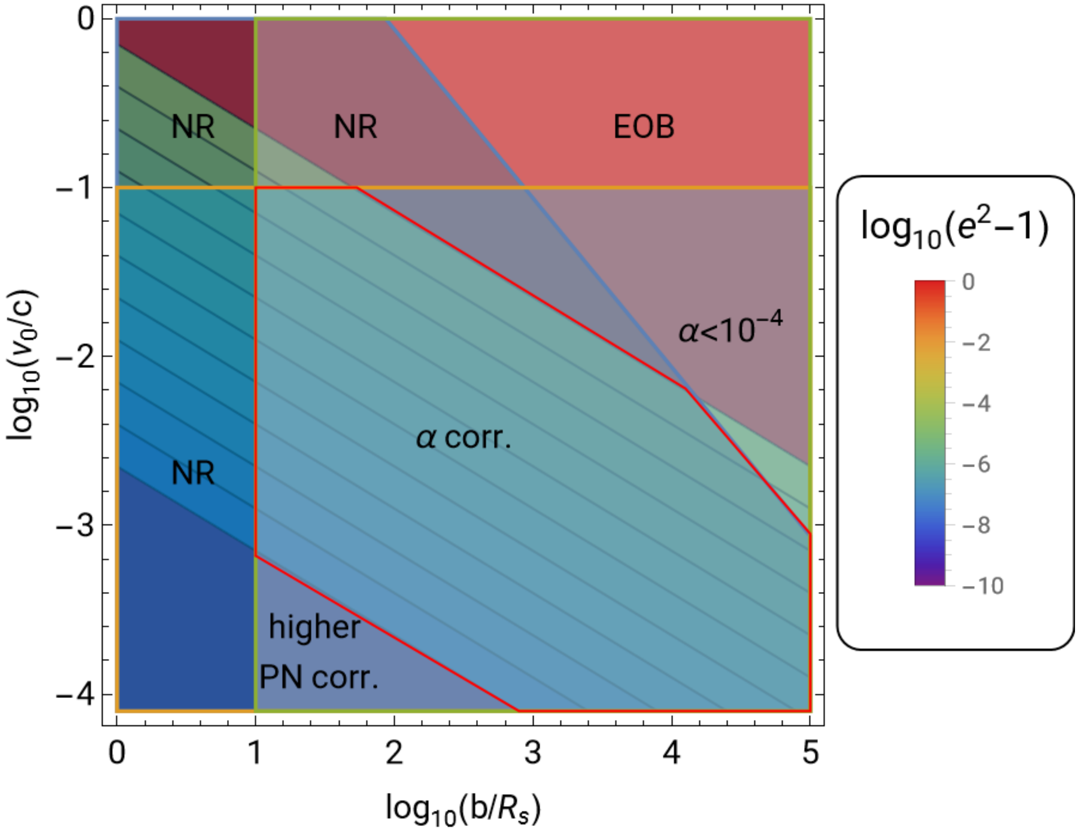
$$\Delta h_+ = 0\,,$$

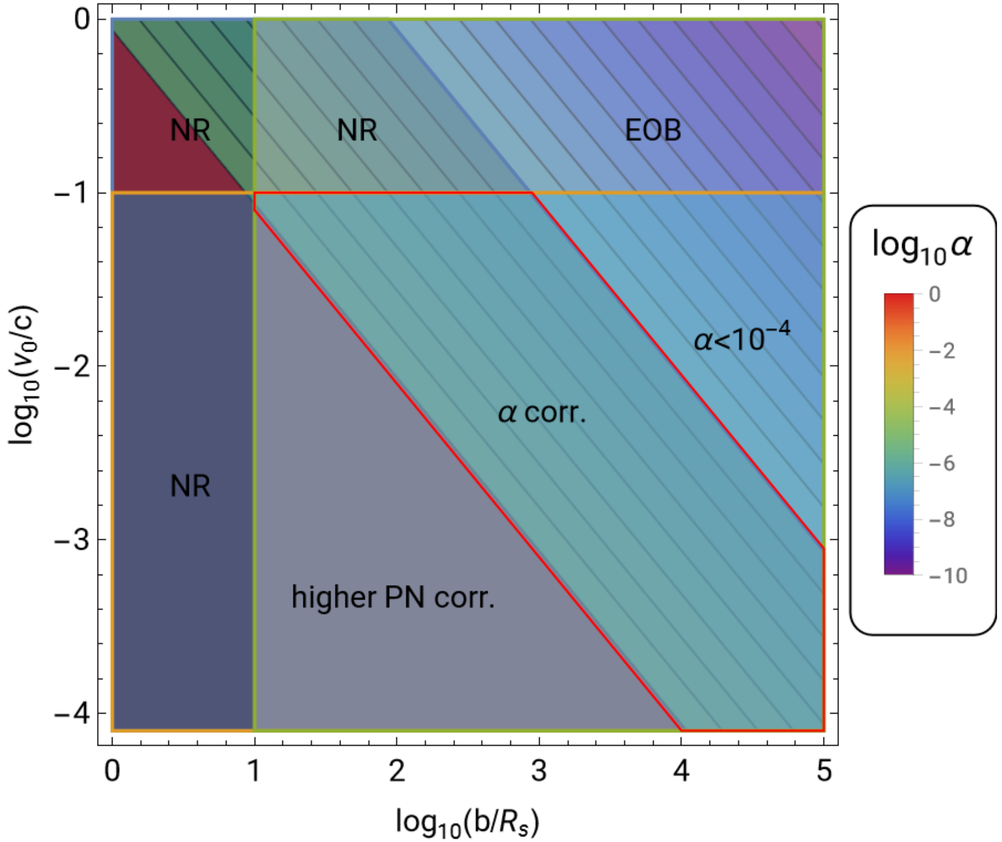
$$\begin{aligned}\Delta h_\times = \frac{2G\mu v_0^2}{rc^4(e^2-1)}\Big[&2\big(-2+e^2(-2+\alpha)\alpha\big)\sin(2\varphi_0)\\ &+e\Big((1-(-4+\alpha)\alpha)\sin((-3+\alpha)\varphi_0)\\ &+e\alpha\sin(2(-2+\alpha)\varphi_0)+e(-2+\alpha)\sin(2\alpha\varphi_0)\\ &+(-5+\alpha^2)\sin((1+\alpha)\varphi_0)\Big)\Big]\end{aligned}$$

$$\varphi_0 = \frac{\arccos(-\frac{1}{e})}{\alpha-1}$$



Viable range of parameters





More about hyperbolic encounters

Next-to-leading order corrections to gravitational wave emission from close hyperbolic encounters

A. Roskill, M. C., S. Kuroyanagi, S. Nesseris

arXiv: 2310.07439

$$h_{ij}^{\mathrm{TT}} = \frac{1}{D}\frac{4G}{c^4}\Lambda_{ij,kl}\left(S^{kl} + \frac{n_m}{c}\dot{S}^{kl,m} + \dots\right)$$

$$\left(h_{ij}^{\mathrm{TT}}\right)_{\mathrm{quad}} = \frac{1}{D}\frac{2G}{c^4}\Lambda_{ij,kl}\ddot{M}^{kl}$$

$$\left(h_{ij}^{\mathrm{TT}}\right)_{\text{next-to-leading}} = \frac{1}{D}\frac{4G}{c^5}\Lambda_{ij,kl}n_m\dot{S}^{kl,m}$$

$$= \frac{1}{D}\frac{2G}{3c^5}\Lambda_{ij,kl}n_m\dddot{M}^{klm} + \frac{1}{D}\frac{4G}{3c^5}\Lambda_{ij,kl}n_m\ddot{\mathcal{Z}}^{klm}$$

$$\left(h_{ij}^{\mathrm{TT}}\right)_{\mathrm{quad}} = \frac{1}{D}\frac{2G}{c^4}\Lambda_{ij,kl}\ddot{Q}^{kl}$$

$$\left(h_{ij}^{\mathrm{TT}}\right)_{\mathrm{oct}} = \frac{1}{D}\frac{2G}{3c^5}\Lambda_{ij,kl}n_m\dddot{O}^{klm}$$

$$\left(h_{ij}^{\mathrm{TT}}\right)_{\mathrm{quad.cur.}} = \frac{1}{D}\frac{4G}{3c^5}\Lambda_{ij,kl}n_m\ddot{\mathcal{Z}}^{klm}$$

RESULTS

$$(h_+)_{\mathrm{quad}} = -\frac{G\mu}{8c^2D(e+1)R}\Big[8e^2\cos^2\iota + e(7\cos 2\iota + 13)\cos\phi + (\cos 2\iota + 3)(e\cos 3\phi + 4\cos 2\phi)\Big],$$

$$(h_\times)_{\mathrm{quad}} = -\frac{G\mu}{c^2D(e+1)R}\cos\iota\sin\phi\Big[e\cos 2\phi + 3e + 4\cos\phi\Big],$$

$$(h_+)_{\mathrm{quad.cur.}} = \frac{\sqrt{2}G\mu\sin\iota}{3c^2D(e+1)^{3/2}R^{3/2}}\cos\phi\,(1+e\cos\phi)^2,$$

$$(h_\times)_{\mathrm{quad.cur.}} = \frac{G\mu\sin 2\iota}{3\sqrt{2}c^2D(e+1)^{3/2}R^{3/2}}\sin\phi\,(1+e\cos\phi)^2$$

$$(h_+)_{\mathrm{oct}} = \frac{G\mu\sin\iota}{192\sqrt{2}c^2D(e+1)^{3/2}R^{3/2}}\Big[4e\Big\{(3-24e^2)\cos 2\iota - 24e^2 + 1\Big\} - 9e^2(\cos 2\iota + 3)\cos 5\phi + 2\Big\{(6-111e^2)\cos 2\iota - 177e^2 + 2\Big\}\cos\phi - \Big\{3(19e^2+36)\cos 2\iota + 131e^2 + 324\Big\}\cos 3\phi - 80e(3\cos 2\iota + 7)\cos 2\phi - 60e(\cos 2\iota + 3)\cos 4\phi\Big],$$

$$(h_\times)_{\mathrm{oct}} = -\frac{G\mu\sin 2\iota\sin\phi}{48\sqrt{2}c^2D(e+1)^{3/2}R^{3/2}}\Big[9e^2\cos 4\phi + 4(14e^2+27)\cos 2\phi + 95e^2 + 260e\cos\phi + 60e\cos 3\phi + 52\Big].$$

Next-to-leading order corrections to gravitational wave emission from close hyperbolic encounters

A. Roskill, M. C., S. Kuroyanagi, S. Nesseris

arXiv: 2310.07439

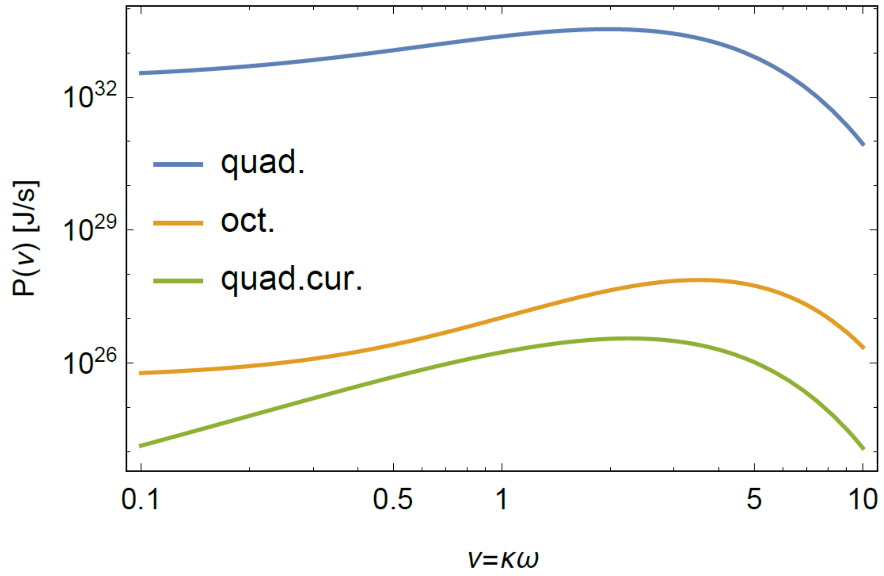
$$P = \frac{c^3 D^2}{32\pi G}\int d\Omega\, \langle \dot{h}_{ij}\dot{h}^{ij}\rangle$$

$$P = \frac{G}{c^5}\left[\frac{1}{5}\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\rangle + \frac{16}{45c^2}\langle \dddot{\mathcal{J}}_{ij}\dddot{\mathcal{J}}^{ij}\rangle + \frac{1}{189c^2}\langle \ddddot{O}_{ijk}\ddddot{O}^{ijk}\rangle + \dots\right],$$

$$P_{\rm quad} = \frac{G\,\mu^2\,c}{30\,(1+e)^5\,R^5\,r_s^2}\Big[1+e\,\cos\phi\Big]^4 \times \left(11\,e^2\,\cos 2\phi + 13e^2 + 48\,e\,\cos\phi + 24\right)$$

$$P_{\rm oct} = \frac{G\,\mu^2\,c}{20160\,(1+e)^6\,R^6\,r_s^2}\Big[1+e\cos\phi\Big]^4 \times \Big[3101e^4 + 5e\Big\{8\left(653e^2+1012\right)\cos\phi + e\{\left(796e^2+5624\right)\cos 2\phi + 5e(39e\cos 4\phi + 344\cos 3\phi)\}\Big\} + 28088e^2 + 10936\Big],$$

$$P_{\rm quad.cur.} = \frac{G\mu^2 c}{180(1+e)^6\,R^6\,r_s^2}\Big[1+e\cos\phi\Big]^6 \times \left(-3e^2\cos 2\phi + 5e^2 + 4e\cos\phi + 2\right)$$



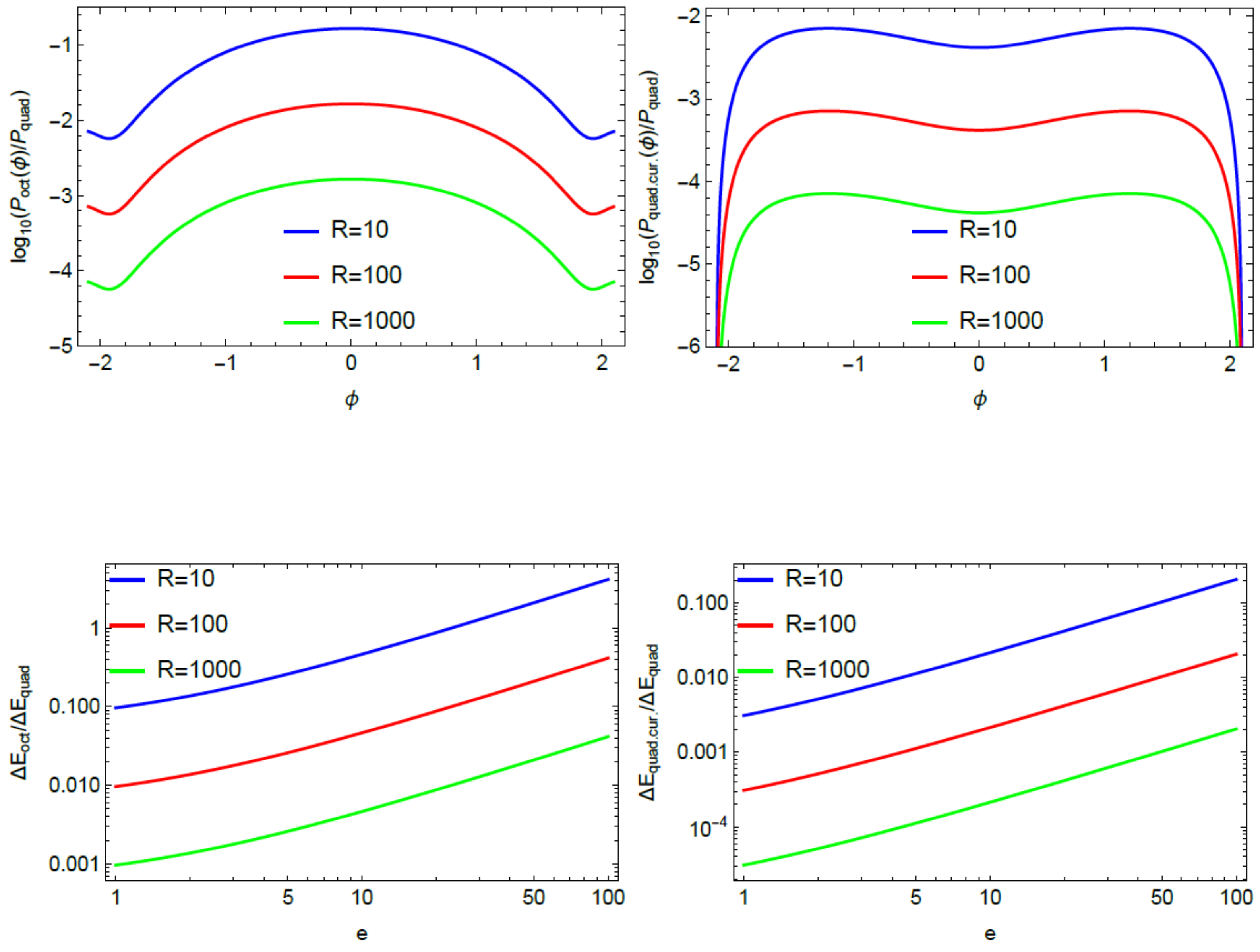
Frequency domain

$$P_{\rm quad}(\omega) = \frac{G}{5\,c^5}\sum_{i,j}|\widehat{\dddot{Q}_{ij}}|^2 = \frac{G}{5\,c^5}\,\omega^6\sum_{i,j}|\widehat{Q_{ij}}|^2$$

$$P_{\rm oct}(\omega) = \frac{G}{189\,c^7}\sum_{i,j,k}|\widehat{\ddddot{O}_{ijk}}|^2 = \frac{G}{189\,c^7}\,\omega^8\sum_{i,j,k}|\widehat{O_{ijk}}|^2,$$

$$P_{\rm quad.cur.}(\omega) = \frac{16\,G}{45\,c^7}\sum_{i,j}|\widehat{\dddot{\mathcal{J}}_{ij}}|^2 = \frac{16\,G}{45\,c^7}\,\omega^6\sum_{i,j}|\widehat{\mathcal{J}_{ij}}|^2.$$

Next-to-leading order corrections to gravitational wave emission from close hyperbolic encounters



A. Roskill, M. C., S. Kuroyanagi, S. Nesseris

arXiv: 2310.07439

Black hole induced spins from hyperbolic encounters in dense clusters

S. Jaraba, J. Garcia-Bellido
arXiv:2106.01436 [gr-qc]

- Proposal of a mechanism that can occur in dense clusters of BHs: spin up of primordial BHs when they are involved in close hyperbolic encounters.
- Exploration of this effect numerically with the Einstein Toolkit for different initial conditions, including variable mass ratios.
- Induced spins in two initially non-spinning equal-mass BHs are larger for higher initial velocities and smaller values of the impact parameters.
- For different-mass BHs, for a given impact parameter and initial velocity, the highest spin is induced on the most massive BH.

Black hole induced spins from hyperbolic encounters in dense clusters

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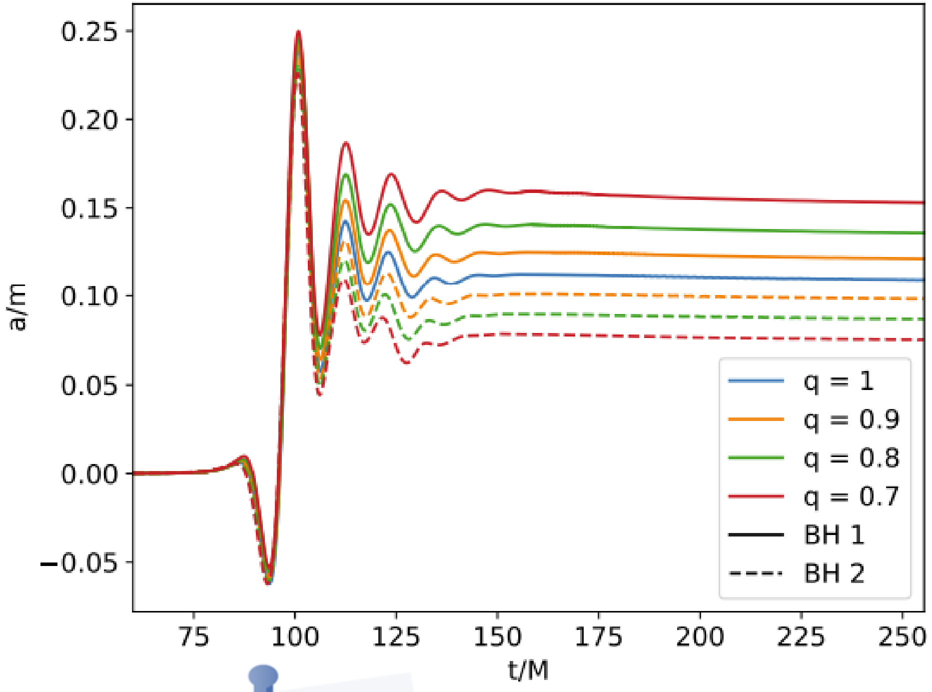
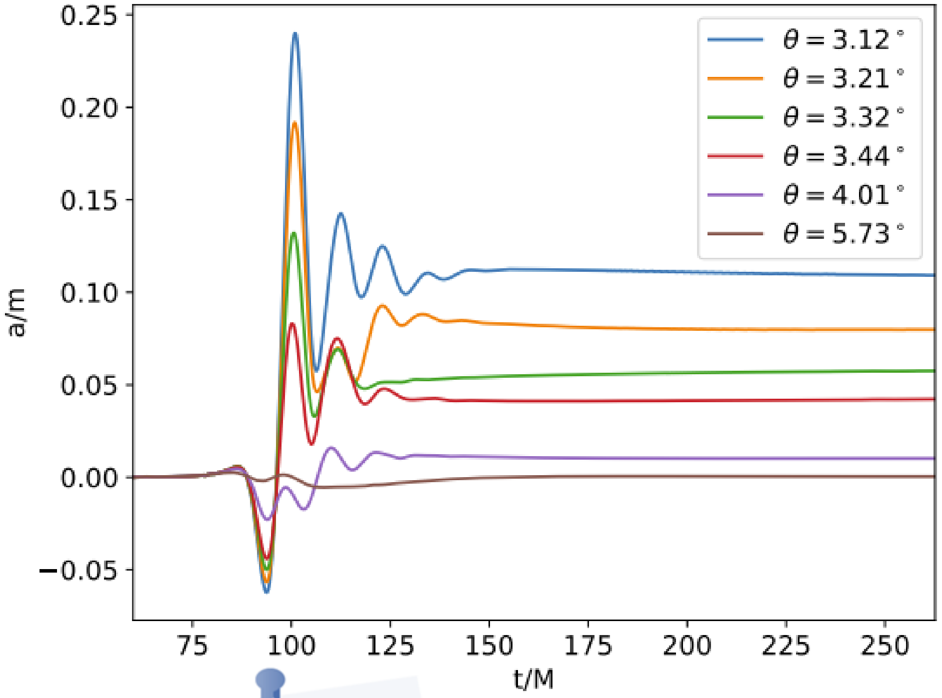


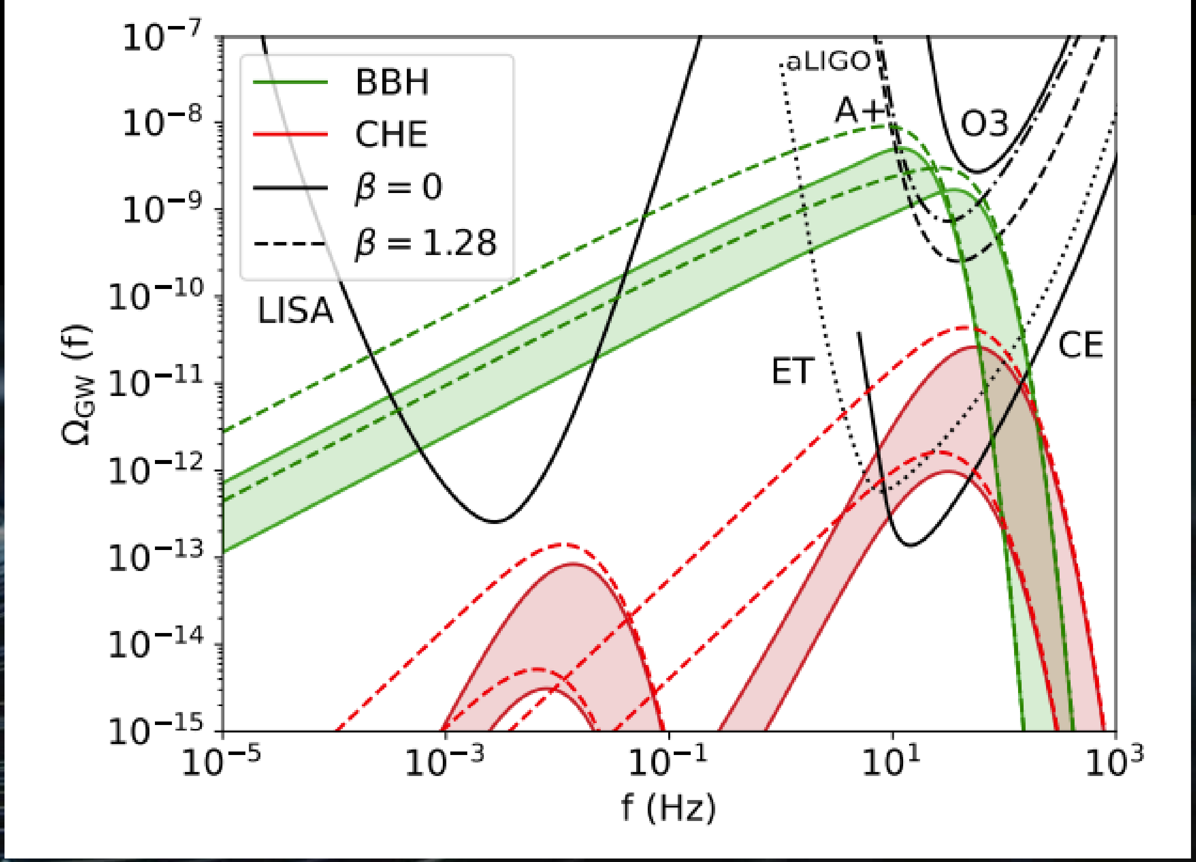
SPIN EVOLUTION FOR DIFFERENT VALUES OF $q = m_2/m_1$



SPIN EVOLUTION FOR DIFFERENT VALUES OF θ

The stochastic gravitational wave background (SGWB) from close hyperbolic encounters (CHE) of primordial black holes in dense clusters

- Computation of the SGWB spectrum from a superposition of GWs from CHE events and comparison of the amplitude with the one from BBHs.
- Different frequency dependencies of the spectra, which would help to distinguish the two different origins when detection of SGWB is made.
- There exist combinations of parameter values that can make the CHE contribution detectable by future GW interferometers, especially with ET, CE or LISA.



J. Garcia-Bellido, S. Jaraba, S. Kuroyanagi
arXiv:2109.11376 [gr-qc]

Conclusions

- General theoretical study of hyperbolic encounters between massive compact objects, expected to happen in dense BHs clusters when two objects gravitationally scatter with each other.
- Exploration of the influence of orbital precession at Newtonian order providing templates for GW amplitudes and power spectra, including the precession of the orbit.
- Evaluation of the linear GW memory effect, only present in the cross polarization state for non-spinning compact binaries in hyperbolic orbits.
- The GW signatures from hyperbolic encounters could provide valuable information that can help in estimating parameters and broaden our knowledge of these intriguing phenomena and the nature of the objects that originated them. This is also a challenge from an experimental point of view (need to disentangle these signals from typical interferometer noise bursts).

Thank you for your attention!