A gravitational-wave perspective on neutron-star seismology

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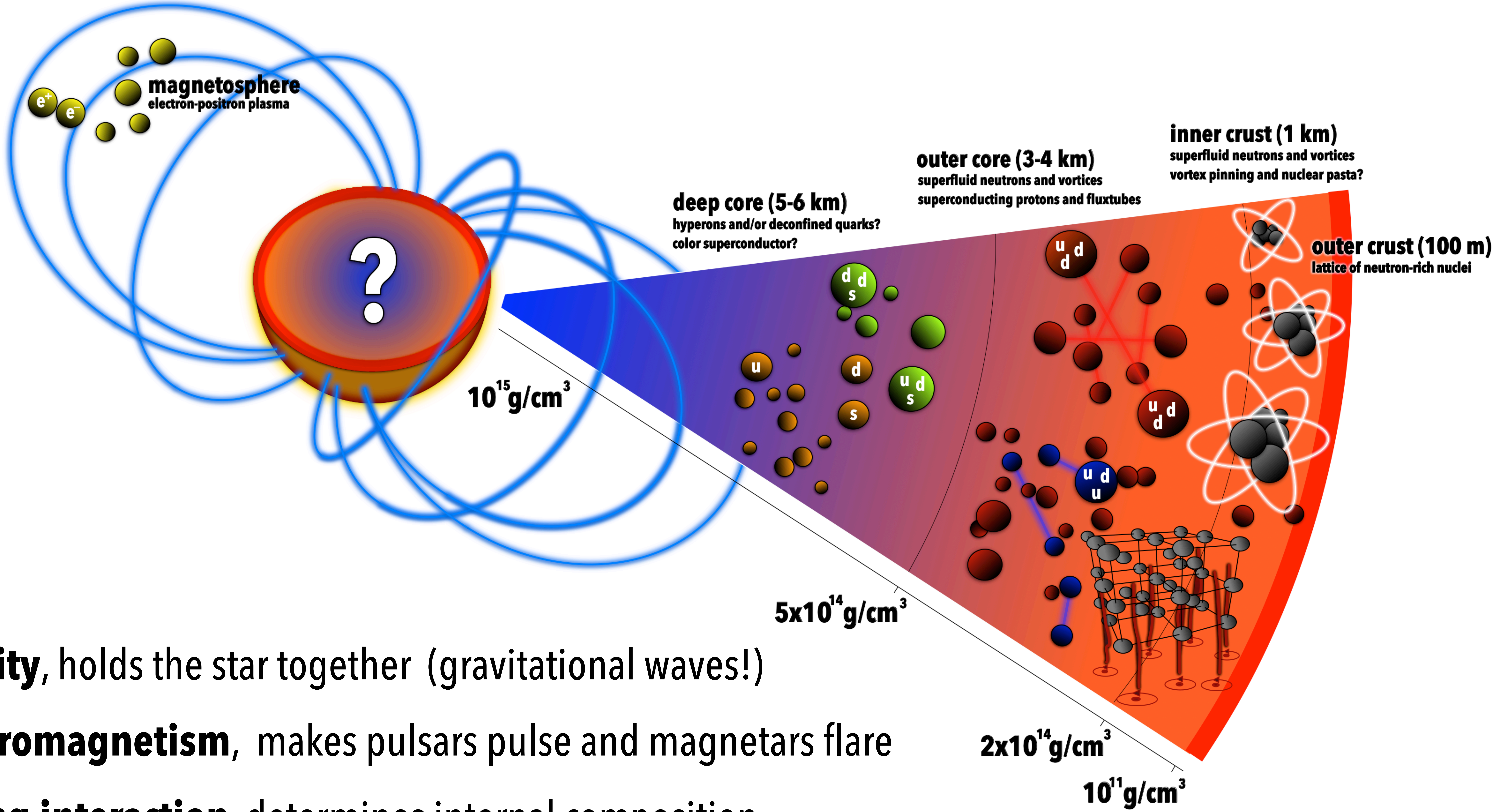


Gravity, holds the star together (gravitational waves!)

Electromagnetism, makes pulsars pulse and magnetars flare

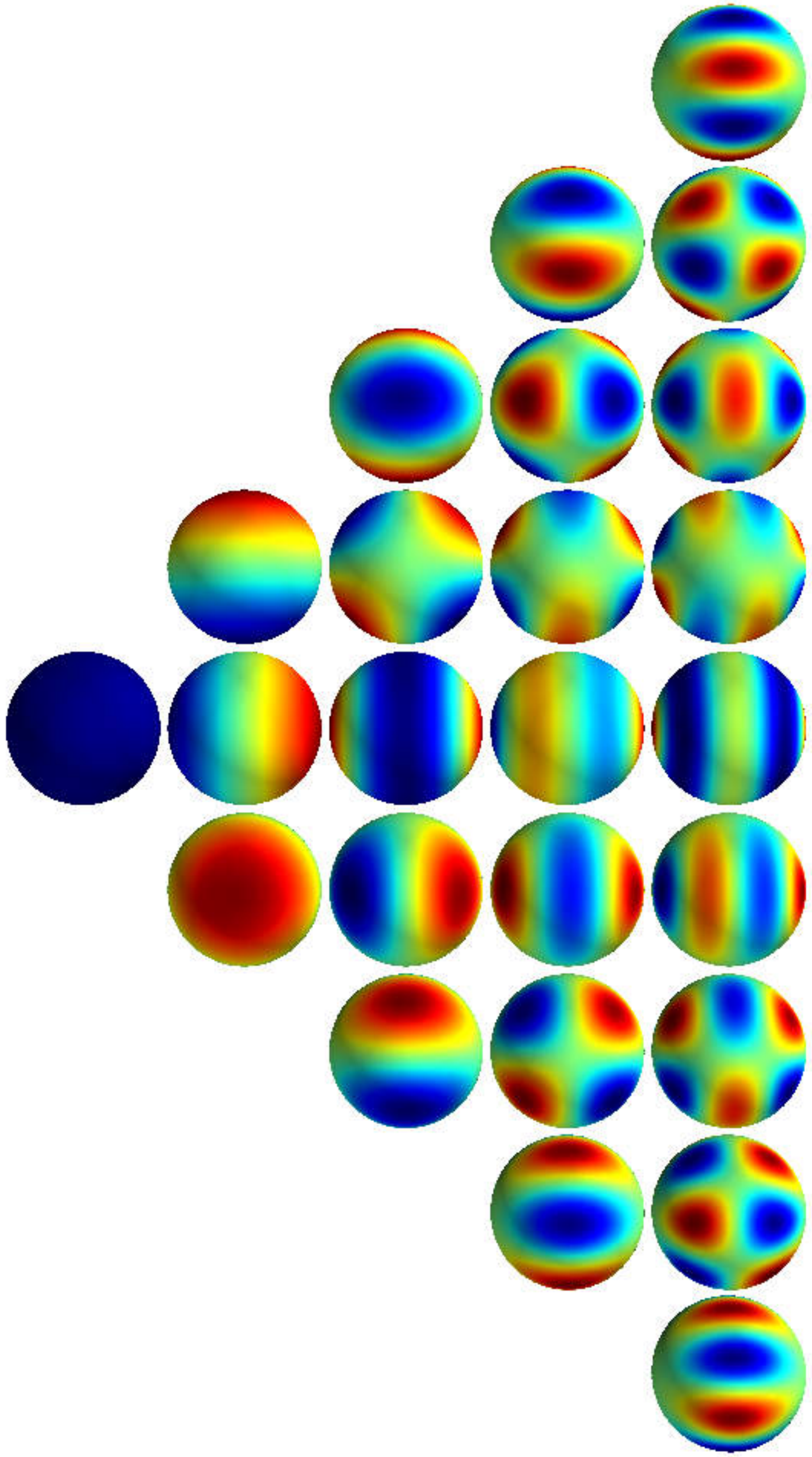
Strong interaction, determines internal composition

Weak interaction, affects reaction rates - cooling and internal viscosity

The main idea of **asteroseismology** is to match observed stellar oscillations against theory to gain insight into the involved physics.

- solar oscillations observed in 1960s and identified as modes in the mid 1970s (5 minute range)
- helioseismology: GONG network and SOHO satellite in the 1990s (note: Rossby waves in the Earth's ocean)
- space-based photometry with CoRoT and Kepler in the 2000s (high-quality seismology data for hundreds of main-sequence and subgiant stars and 10,000s of red giants)
- NASAs TESS mission and ESAs PLATO mission take this further (characterise host stars in exoplanet systems)

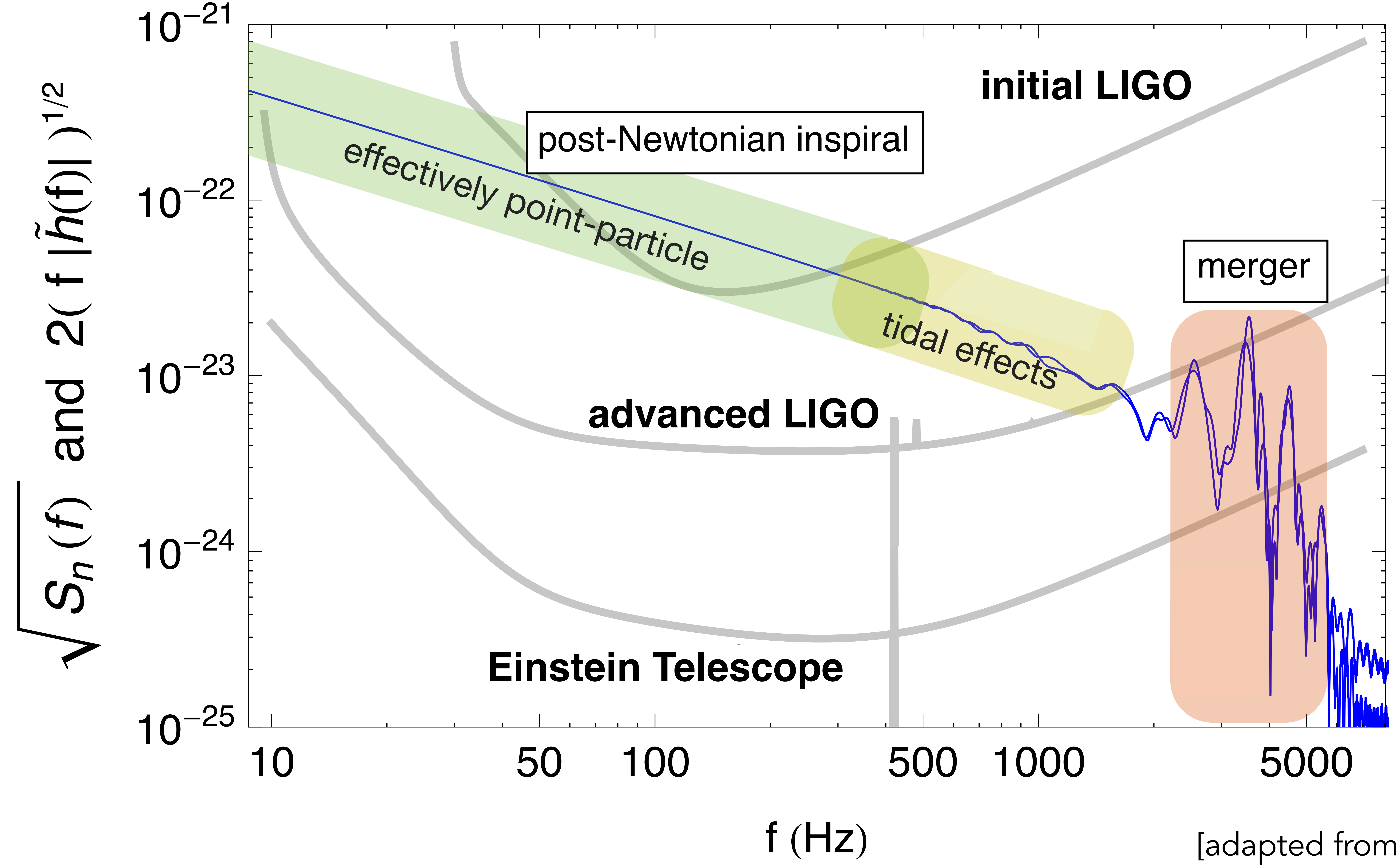
Want to use seismology strategy to probe neutron stars using gravitational-wave data.



From the GW perspective we need global modes which involve significant density variations.

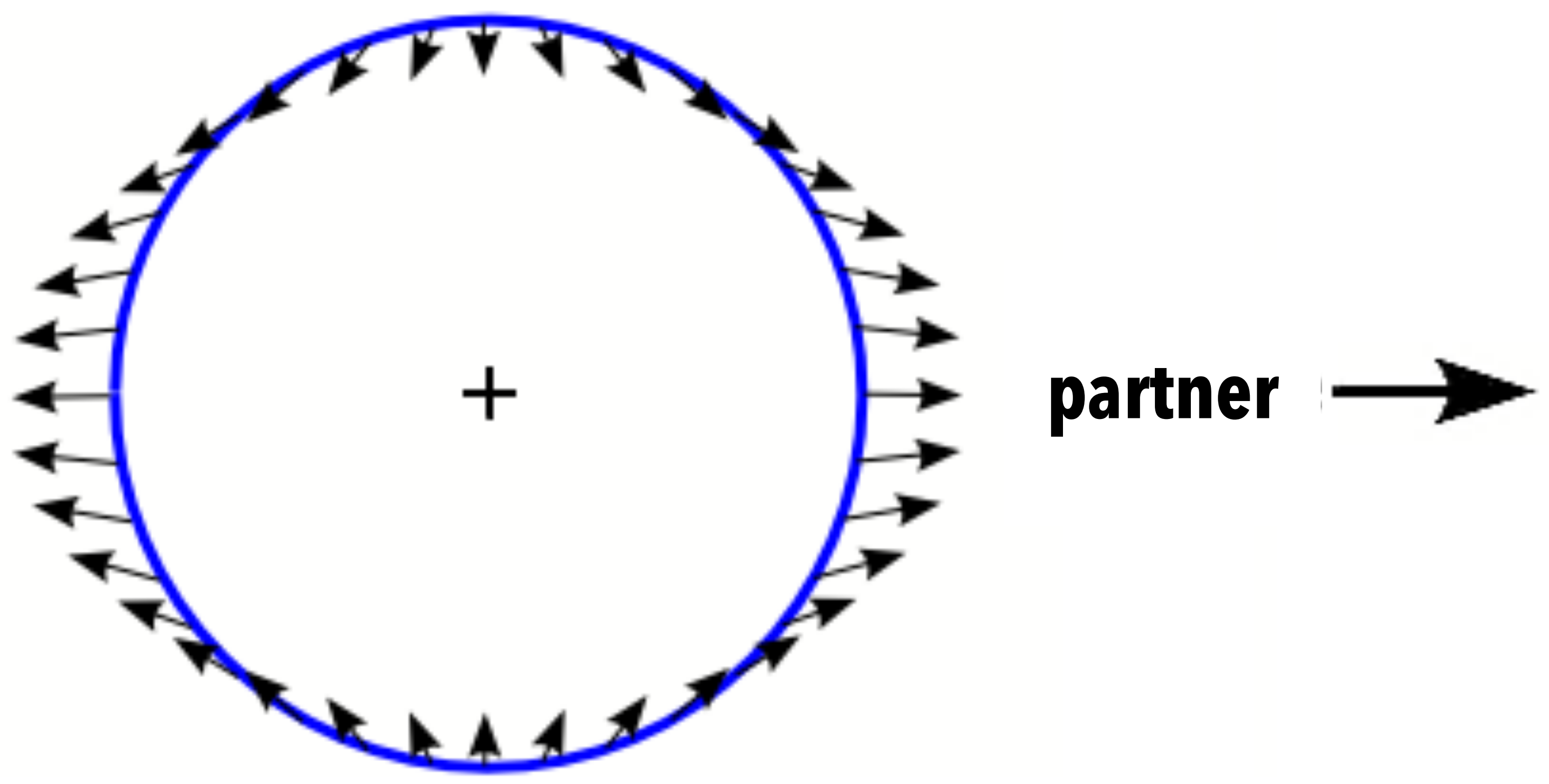
- ***f*-mode**: Fundamental oscillation of the star; scales with the average density, $\omega_\alpha/(2\pi) \sim \sqrt{GM/R^3} \sim 1 - 2\,\mathrm{kHz}$.
- ***p*-modes**: Restored by the pressure of the fluid (speed of sound); higher frequencies
- ***g*-modes**: Restored by buoyancy associated with temperature/composition gradients; lower frequencies, $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.
- **inertial modes** (including the ***r*-mode**): Restored by rotation; may be driven unstable by GW emission; $\omega_\alpha \sim \Omega$.
- ***i*-modes**: Oscillation feature associated with the core-crust interface; may induce crust fractures during binary inspiral and trigger short gamma-ray bursts; $\omega_\alpha/(2\pi) \sim 100\,\mathrm{Hz}$.

binary inspiral



[adapted from Read]

tidal deformability

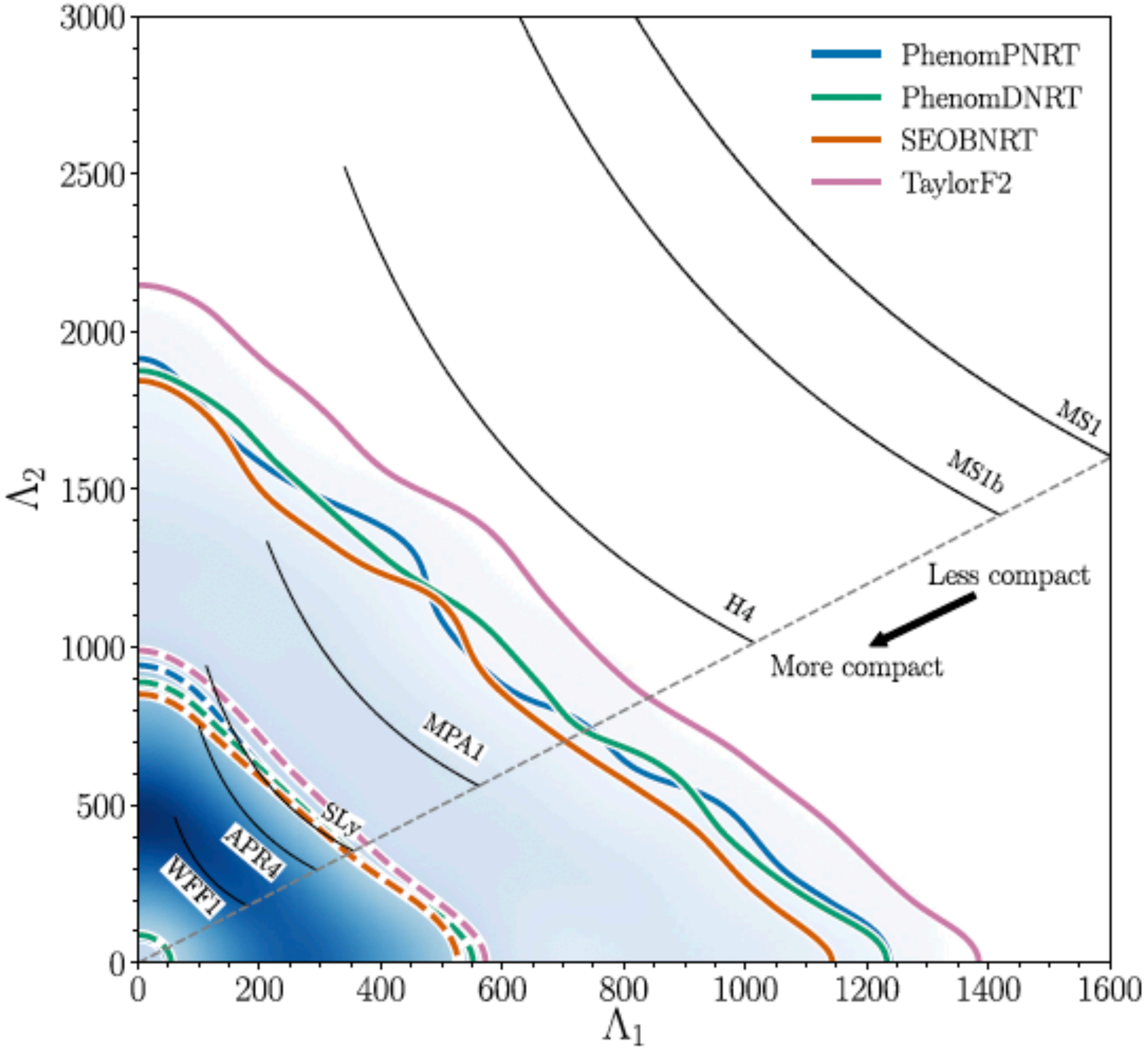


GW signal from binary neutron stars differs from that of black holes due to the **tidal deformability**.

Effect of static tide enters at 5PN order through the induced quadrupole moment.

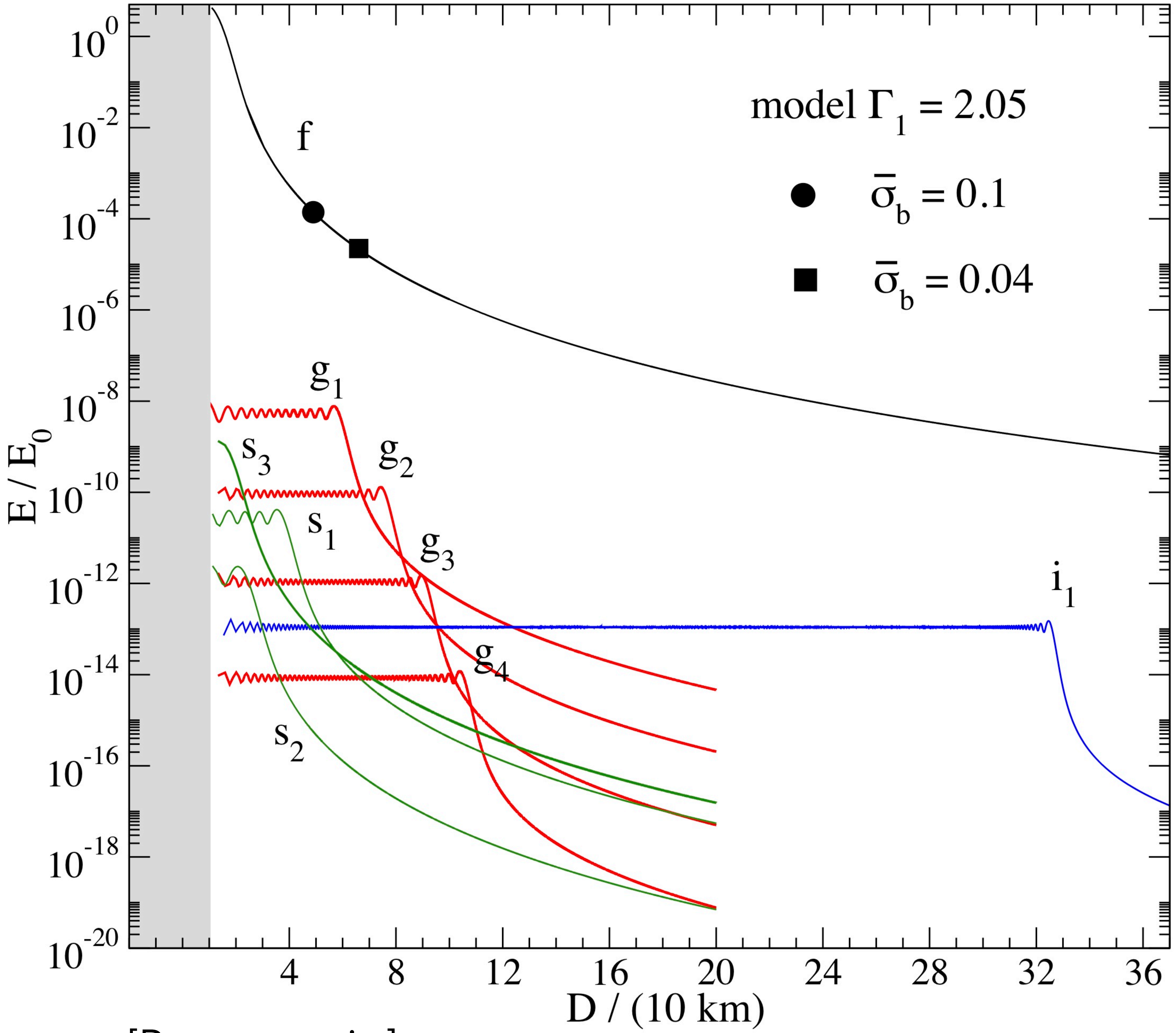
Characterised by the **Love numbers**

$$\Lambda_A = \frac{2}{3} k_{2A} \left(\frac{c^2 R_A}{G M_A} \right)^5$$



[GW170817: Abbott+]

dynamical tides



[Passamonti+]

The **dynamical tide** is represented by resonances with individual oscillation modes.

Need global modes which involve significant density variations. Overlap integral

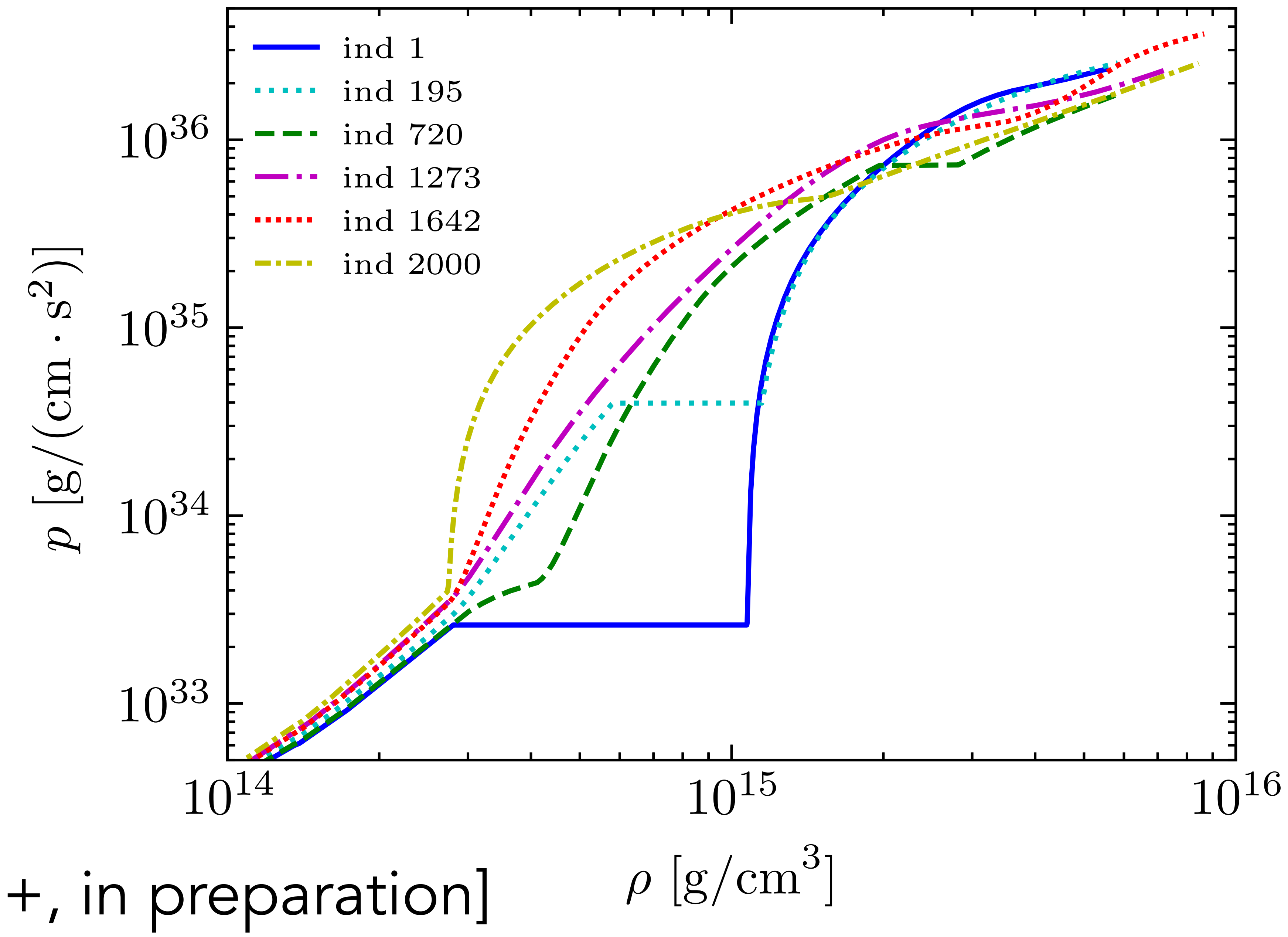
$$I_\alpha \equiv \int_0^R \delta\rho_\alpha(r) r^{l+2} dr$$

leads to the effective Love number:

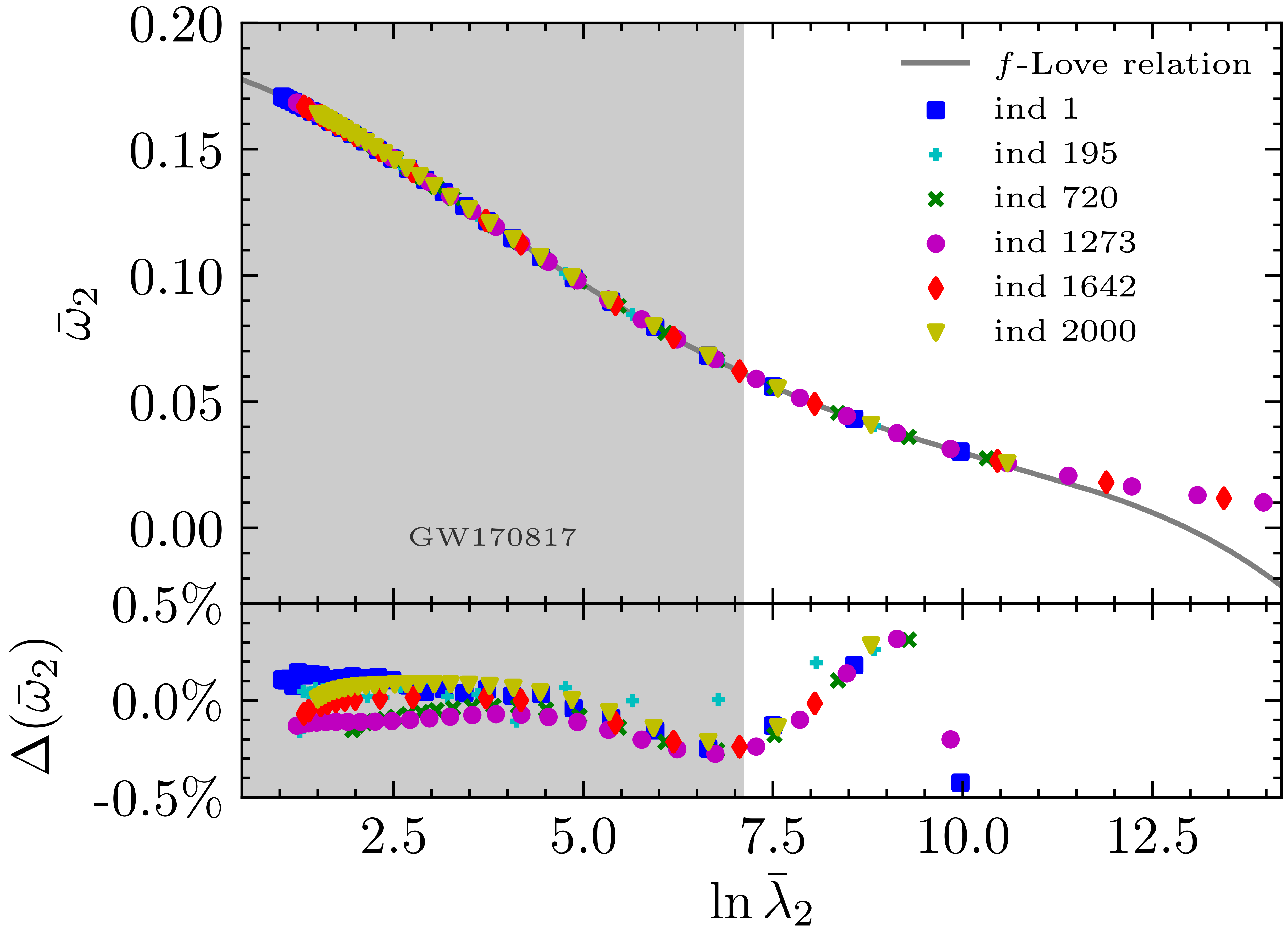
$$k_{lm} = \frac{2\pi G}{(2l+1)R^{2l+1}} \sum_{\alpha'} \frac{I_\alpha^2}{\mathcal{A}_\alpha^2[\omega_\alpha^2 - (m\Omega)^2]}$$

Results in significant enhancement of tidal imprint near merger.

universal f-Love relation



[Chen+, in preparation]



In the static limit $(\Omega \to 0)$ we get

$$k_l = \frac{2\pi G}{(2l+1)R^{2l+1}} \sum_{\alpha'} \frac{I_\alpha^2}{\mathcal{A}_\alpha^2 \omega_\alpha^2}$$

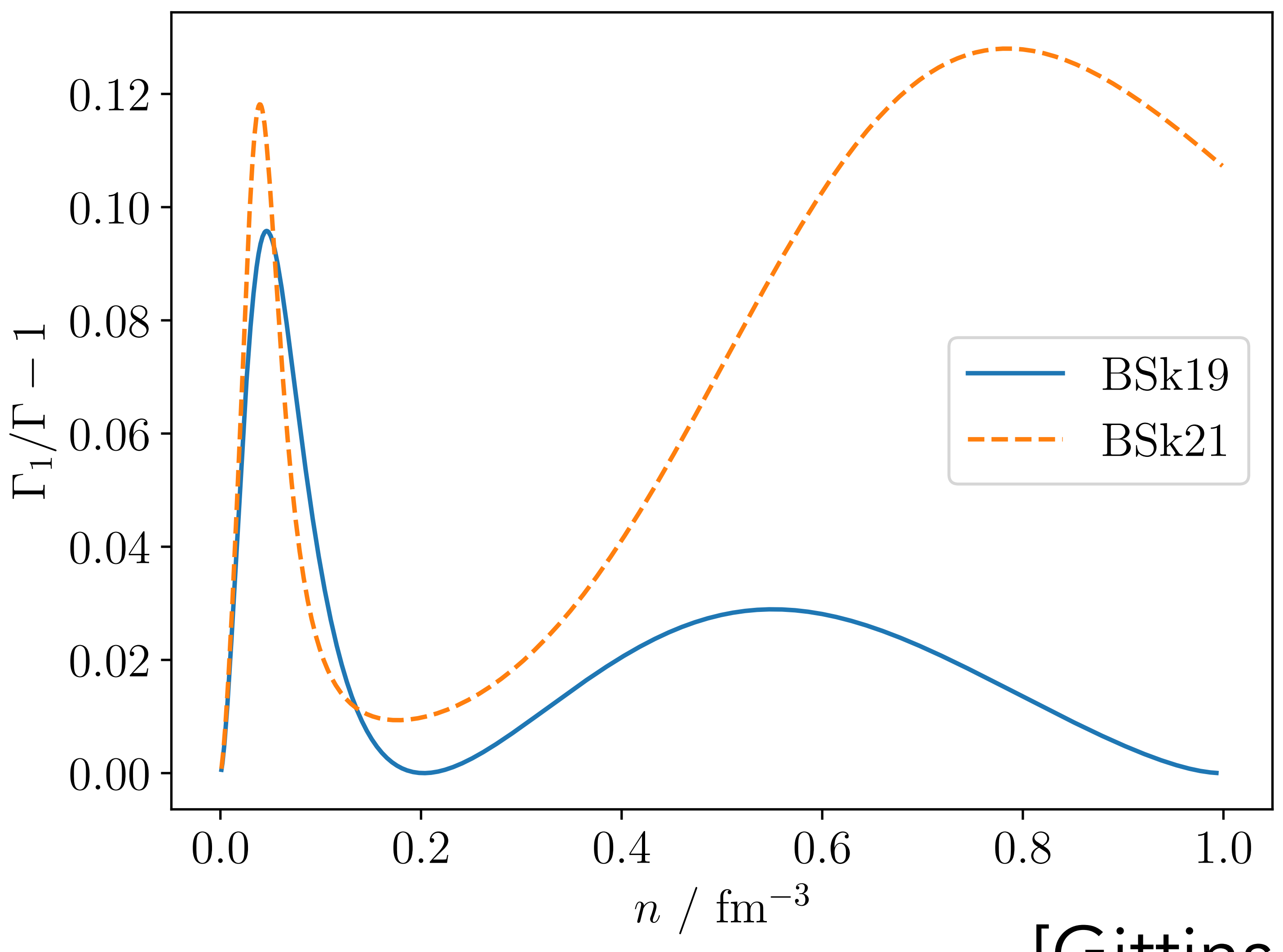
If the fundamental mode dominates the sum, we expect a universal relation between mode frequency and tidal deformability. Numerical evidence that this relation is very robust.

beyond mass and radius

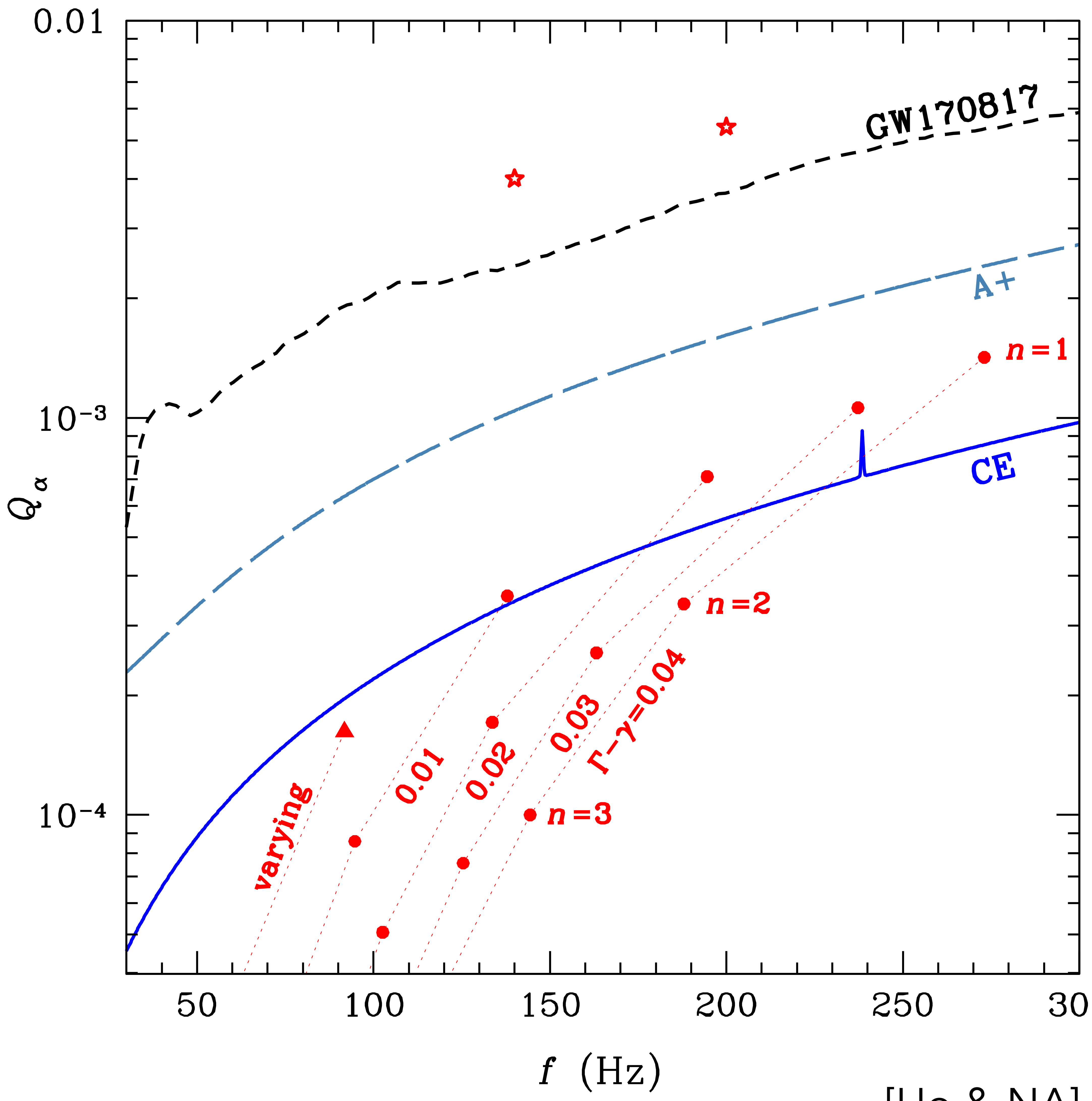
The g-modes carry information about the internal matter composition.

Sensitive to deviation from chemical equilibrium, e.g. the (local) Brunt-Väisälä frequency

$$N^2 = \frac{\rho g^2}{p}\left(\frac{1}{\Gamma} - \frac{1}{\Gamma_1}\right).$$



[Gittins & NA]



[Ho & NA]

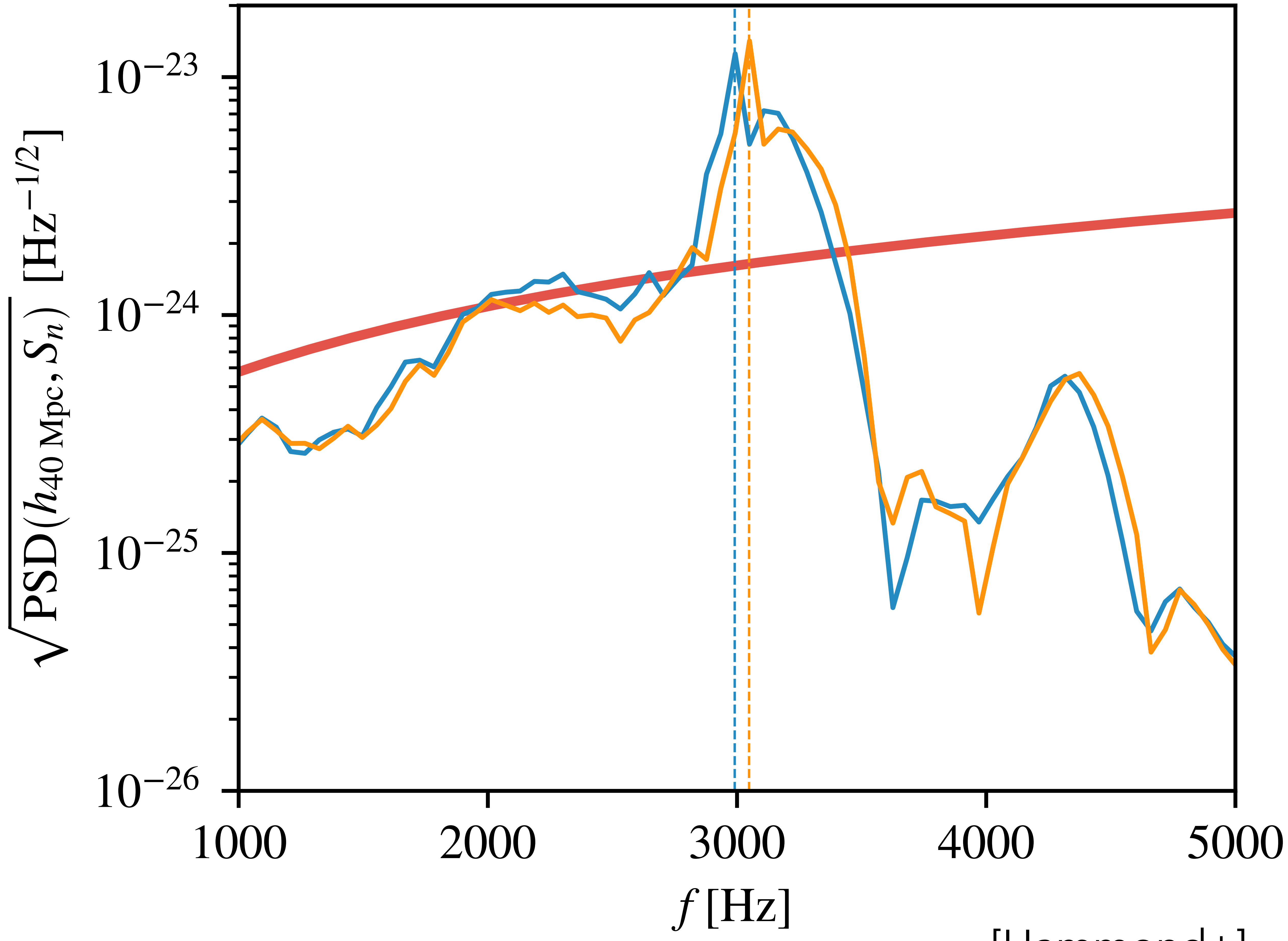
mergers

Merger dynamics should be within reach of next-generation detectors.

Requires **robust nonlinear simulations** with a reliable physics implementation

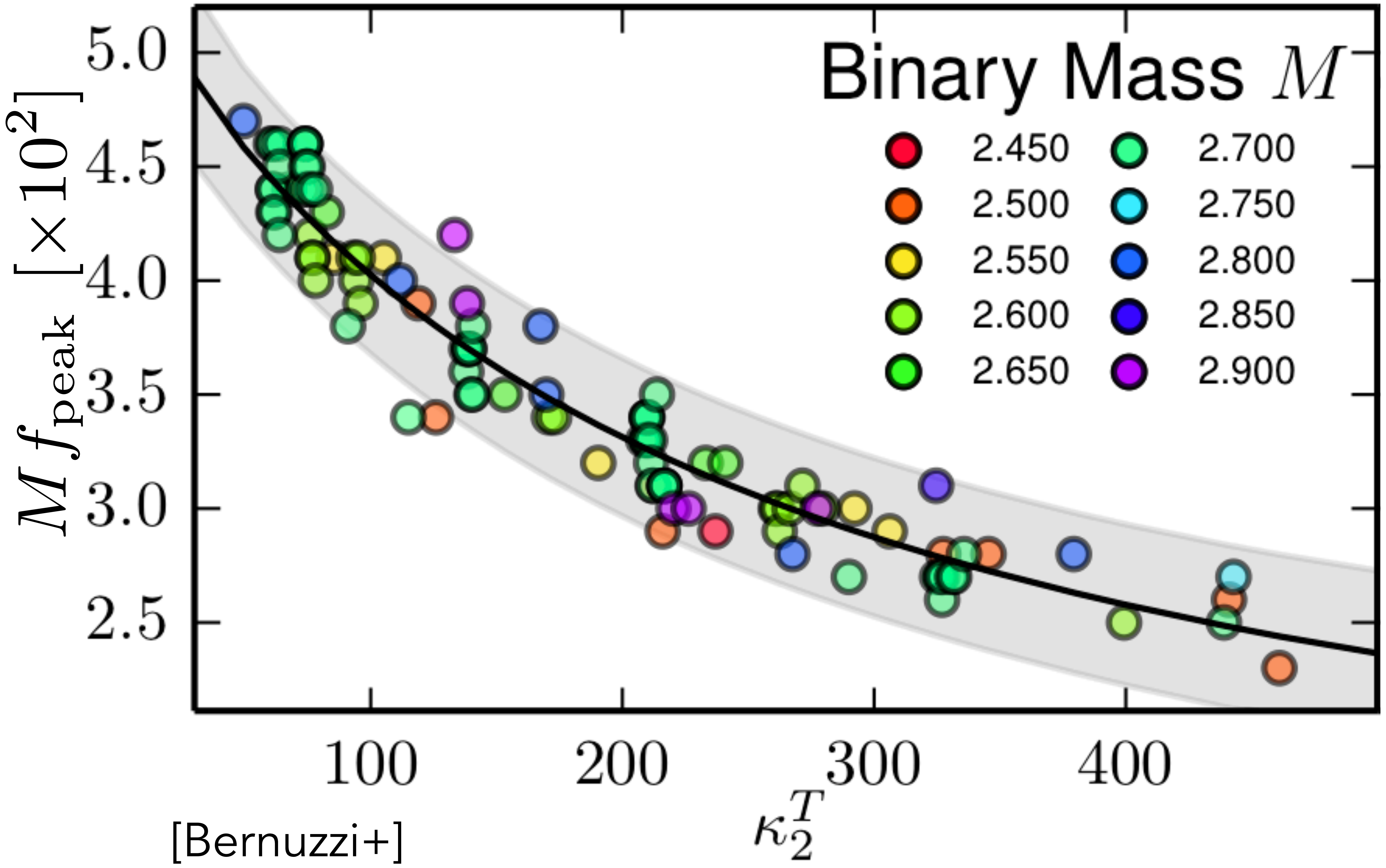
Assuming a 3-parameter model $p = p(n, \varepsilon, Y_\mathrm{e} = n_\mathrm{e}/n_\mathrm{b})$ and stepping up the complexity, we may

- assume that reactions are fast enough that the matter remains in equilibrium, or
- slow enough that the composition is frozen, or
- add whatever other physics we may be interested in…



[Hammond+]

another "universal relation"



[Bernuzzi+]

Numerical simulations suggest a more "surprising" universal relation, linking the tidal deformability (=cold EoS) to the peak frequency from the merger dynamics (=hot EoS).

The origin of this relation is not well understood.

Also do not (yet) know how "robust" it is…

I have outlined:

- the main idea behind asteroseismology and why it is relevant for GW astronomy (now and in the future)

I have not talked about:

- the technical state of the art (Newtonian vs relativity/ phenomenology vs precision)
- nonlinear tides (p-g instability?)
- other scenarios, e.g. core collapse supernovae or the gravitational-wave driven instability (f-mode/r-mode) in (isolated) spinning neutron stars
- starquakes/glitches/GW searches
- neutron star ocean modes/crust seismology/X-rays