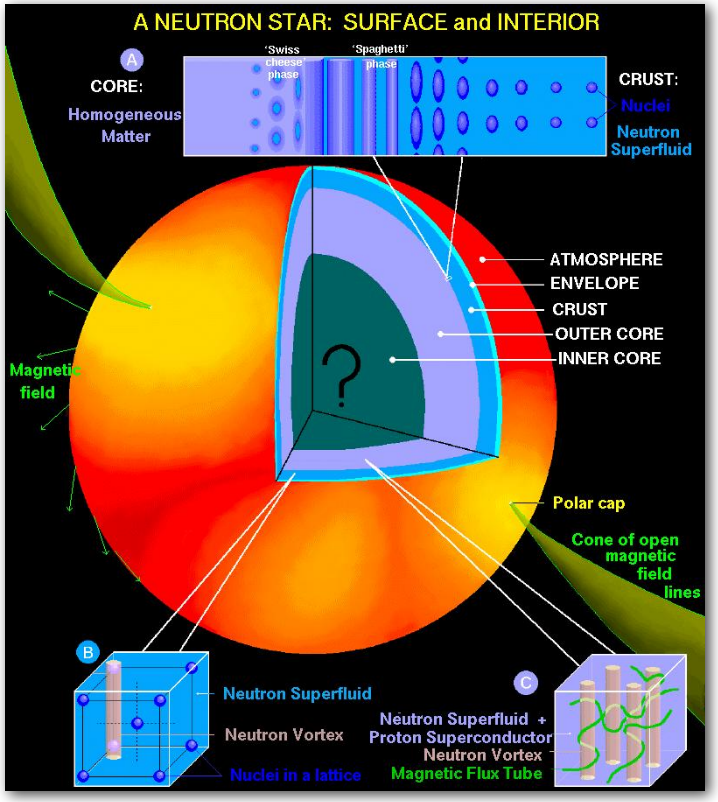
GraSP23 || GravityShapePisa 2023



THE NEUTRON STAR MATTER EQUATION OF STATE : A CHALLENGE FOR NUCLEAR PHYSICS THEORY

Fiorella Burgio

INFN Sezione di Catania

INFN
CATANIA
Istituto Nazionale di Fisica Nucleare
Sezione di Catania

OUTLINE

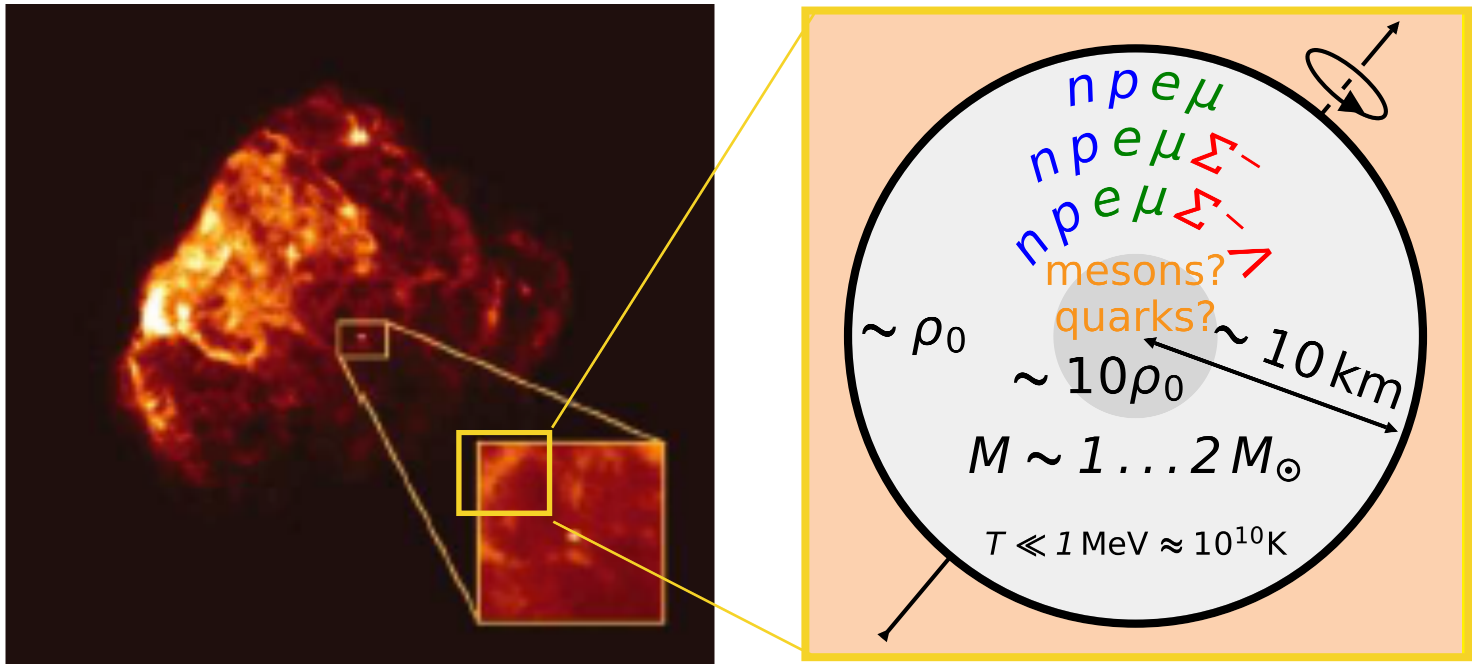
- Motivation
- The NS matter EoS
- Theoretical approaches to the EoS of the core
- Check of the EoS wrt terrestrial laboratory data and NS observations
- One of the major open problems : the onset of strange baryonic matter
- Conclusions - a (partial) list of unsolved issues

MOTIVATION

A theorist's view of a neutron star

A huge nucleus: ~ 10^{57} nucleons

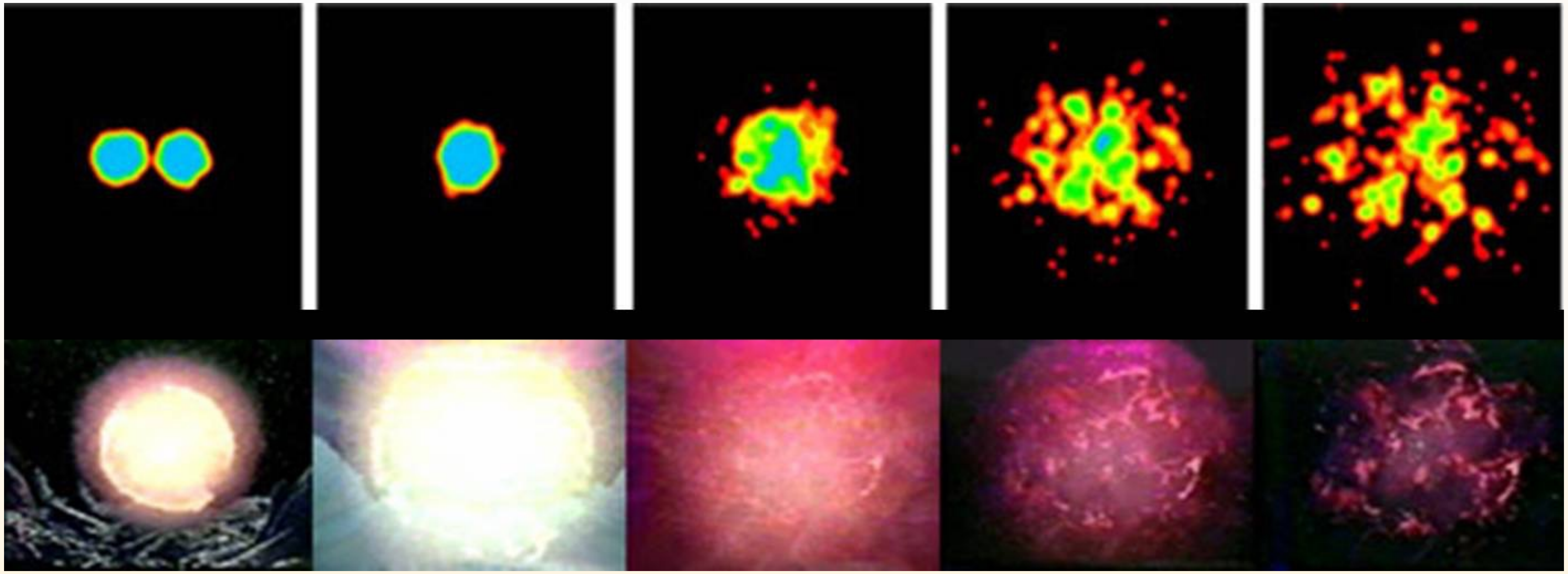
ROSAT image of Puppis A



The only "laboratory" for ρ_B ~ $(8\text{ -}10)\rho_0$ in the Universe !

Need EOS of nuclear matter including hyperons and quarks.

Relevance of the EoS



1. Heavy ion collisions (small N/Z, high T)

2. Supernovae and Neutron Stars
   (high N/Z, high (small) T in SN (NS))

3. Binary NS merger and GW emission
   (high density, high N/Z and T)

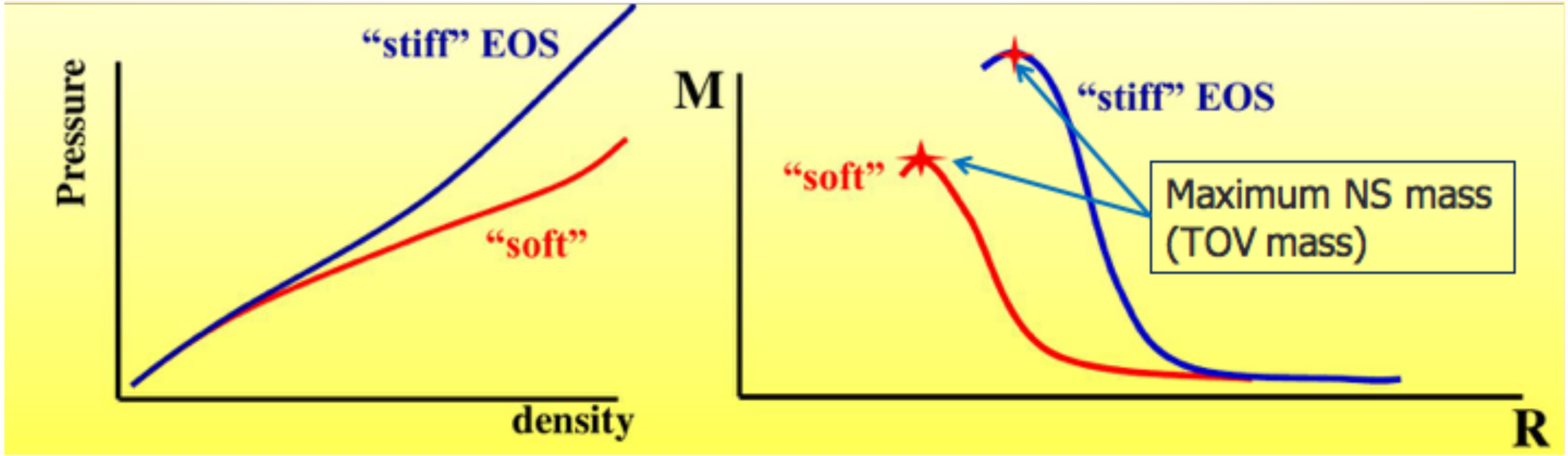
Quite different physical conditions in each case !

A nuclear matter theory must be able to treat all these physical situations.

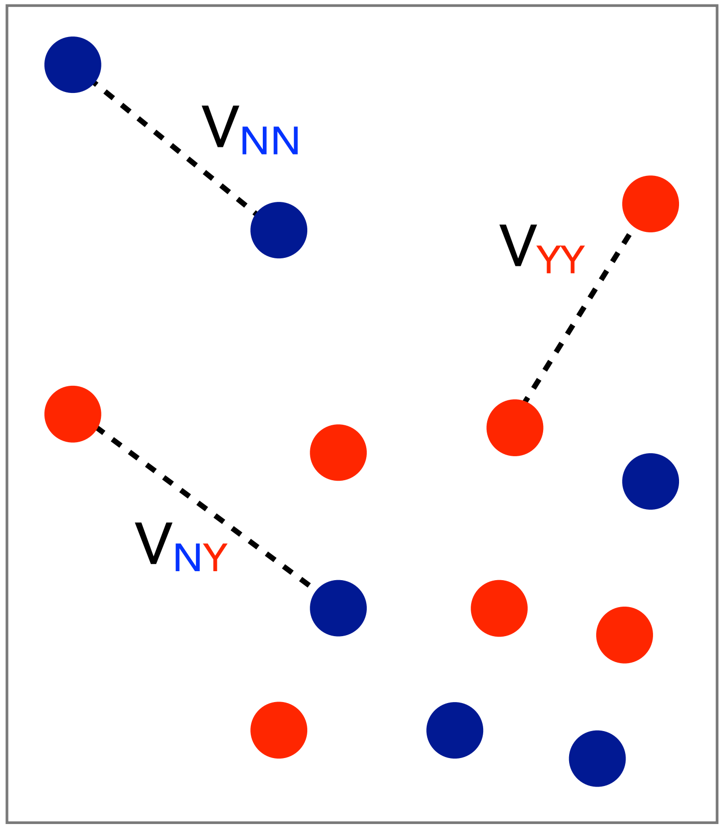
Need an Equation of State $P = P(\varrho, x_i, T)$

Soft or Stiff ?

Given an EoS, compute the sequence of equilibrium models :
The Mass-Radius relation



COMPOSITION



$\rho = \rho_N + \rho_Y$

$N = n, p$ (939 MeV)

$Y = \Lambda^0$ (1116 MeV)

Σ^{-0+} (1193 MeV)

Ξ^{-0} (1318 MeV)

To obtain the equation of state we need

- ✳ A theory of strong interaction for nucleons and hyperons

V_{NN}: Argonne, Bonn, Paris, *chiral* potentials

V_{NY}: Nijmegen (NSC89,NSC97,ESC08…)

V_{YY}: ? (no scattering data)

- ✳ A theoretical method to solve the many-body Schrödinger equation

We need to compute the energy density of this system !

THEORETICAL METHODS FOR (HYPER)NUCLEAR MATTER

- ✓ Ab-initio microscopic methods: (Dirac)BHF, Variational, SCGF, Monte-Carlo, χEFT

Start from bare two- and three-nucleon interactions able to reproduce nucleon scattering data and properties of bound few-nucleon systems.

- ✓ Phenomenological: Non-relativistic EDF, Relativistic mean-field (RMF) models

Use effective interactions with a simple structure dependent on a limited number of parameters, which are fitted to different properties of finite nuclei in their g.s.

CAVEAT !

Nuclear interaction unknown at large density —-> extrapolation !

CHECK *WRT* NUCLEAR PHYSICS CONSTRAINTS

Non-relativistic

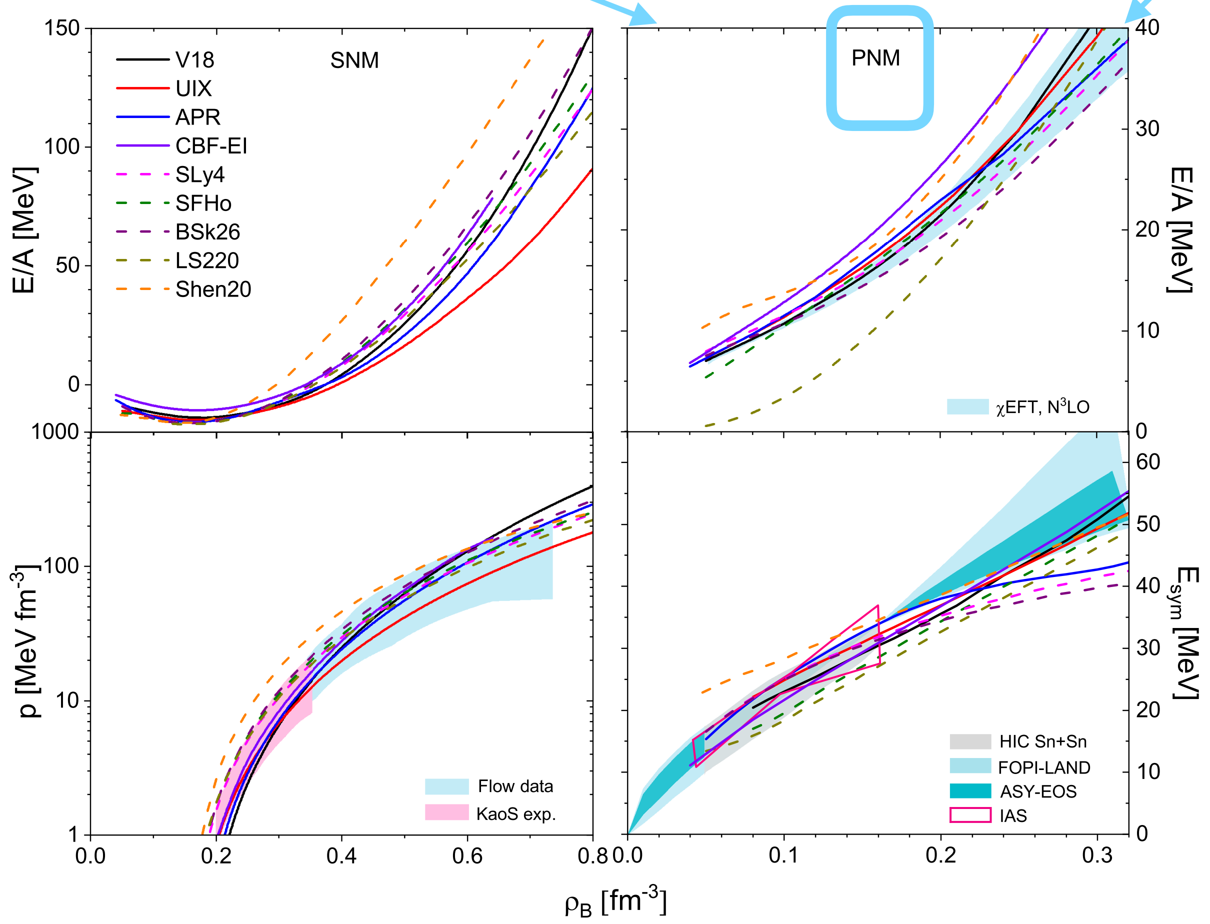
Microscopic EoS (solid lines)

- V18. BHF with Argonne V18 supplemented by a microscopic TBF or phenomenological Urbana IX TBF (UIX).
- APR .Variational EoS with Argonne V18 and Urbana IX TBF.
- CBF-EI. Correlated Basis Function- Effective Interaction with simplified Av6' potential and Urbana IX TBF.

Phenomenological EoS (dashed lines)

- SLy4. EoS built with Skyrme forces.
- BSk26. Unified EoS by the Brussels-Montreal group.
- SFHo. Nuclear Statistical Equilibrium (NSE) for an ensemble of nuclei and nucleons with RMF.
- LS220. Mixture of heavy nuclei, α particles, free neutrons and protons. Nuclei described within the LDM, and a Skyrme NN interaction employed for nucleons.
- Shen20. Similar to LS220, but RMF model for nucleons.

Light blue : predictions from the χEFT up to N3LO order
C. Drischler et al., PRL 125, (2020) 202702



- Check *wrt* to χEFT : CBF-EI, Shen20 and LS220 not compatible.
- Any reliable EoS model should pass through the region defined by the expt data.
- Most of the EoSs are compatible with expt. data
- Constraints inferred from HICs to be taken with care, since dependent on transport models.

Flow data P. Danielewicz, et al. *Science*, 298:1592, 2002.
KaoS exp. D. Misťkowiec et al. *Phys. Rev. Lett.*, 72:3650, 1994.
HIC Sn+Sn G Verde et al J. Phys.: Conf. Ser. 312 082044, 2011.
FOPI-LAND J. L. Ritman et al. *Z. Phys. A*, 352:355, 1995.
ASY-EOS P. Russotto et al. *Phys. Rev. C*, 94:034608, 2016.
IAS P. Danielewicz et al., *Nucl. Phys. A*, 922:1, 2014.

G.F. Burgio, H.-J. Schulze, I. Vidaña, and J.-B. Wei, Prog. Part. Nucl. Phys., **120**:103879, 2021.

CHECK *WRT* NUCLEAR PHYSICS & OBSERVATIONAL CONSTRAINTS

Around saturation density and isospin asymmetry **β**=0 →

$$\frac{E}{A}(\rho,\beta) = E_0 + \frac{1}{2}K_0x^2 + \frac{1}{6}Q_0x^3 + \left(S_0 + Lx + \frac{1}{2}K_{sym}x^2 + \frac{1}{6}Q_{sym}x^3\right)\beta^2 + O(\beta^4) \qquad x = (\rho-\rho_0)/3\rho_0$$

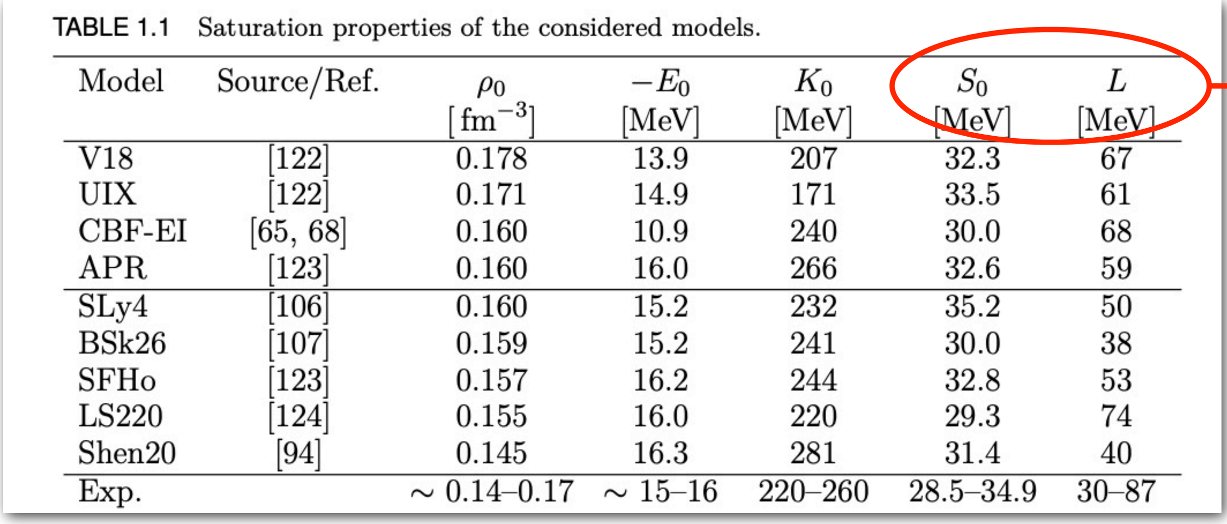


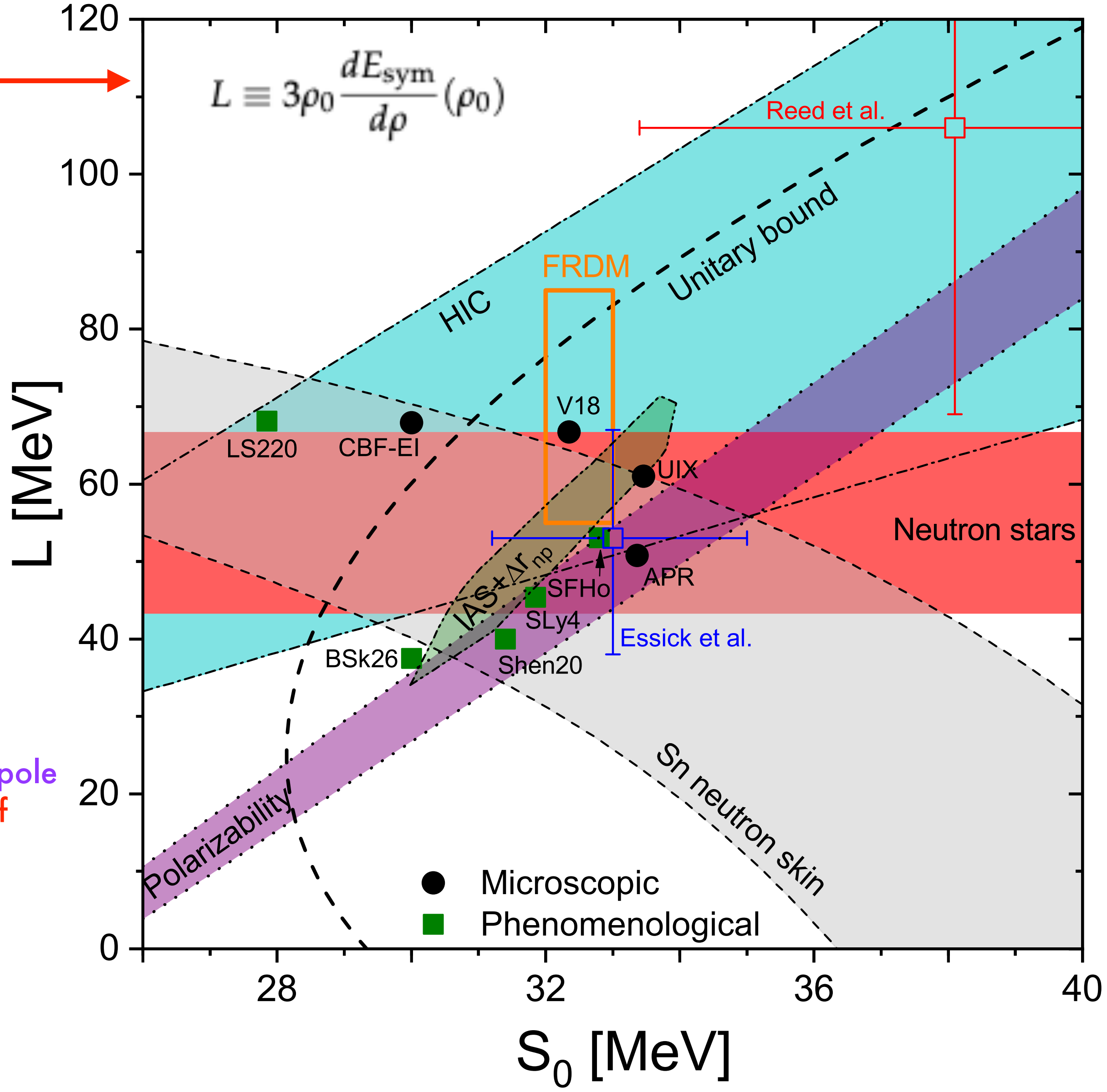
TABLE 1.1 Saturation properties of the considered models.

Model	Source/Ref.	ρ_0 [fm^{-3}]	$-E_0$ [MeV]	K_0 [MeV]	S_0 [MeV]	L [MeV]
V18	[122]	0.178	13.9	207	32.3	67
UIX	[122]	0.171	14.9	171	33.5	61
CBF-EI	[65, 68]	0.160	10.9	240	30.0	68
APR	[123]	0.160	16.0	266	32.6	59
SLy4	[106]	0.160	15.2	232	35.2	50
BSk26	[107]	0.159	15.2	241	30.0	38
SFHo	[123]	0.157	16.2	244	32.8	53
LS220	[124]	0.155	16.0	220	29.3	74
Shen20	[94]	0.145	16.3	281	31.4	40
Exp.		∼ 0.14–0.17	∼ 15–16	220–260	28.5–34.9	30–87

L correlated with radius and tidal deformability of a 1.4 M⊙ NS.

Data from neutron-skin thickness in Sn isotopes, isospin diffusion in HIC, electric dipole polarizability, IAS combined with the skin-width data , FRDM, Bayesian analysis of mass and radius measurements in NSs.

Recent PREX-II measurement of the ^{208}Pb neutron skin (Reed) and its more recent analysis (Essick).



Most of the models (full points) fulfill all these constraints.

«Recipe» for Neutron Star Structure Calculation:

Brueckner results: $\boldsymbol{\epsilon}(\{\boldsymbol{\rho_i}\})\ ;\ i = n, p, e, \mu, \Lambda, \Sigma, u, d, s, \ldots$

Chemical potentials: $\mu_i = \dfrac{\partial \epsilon}{\partial \rho_i}$

Beta-equilibrium: $\mu_i = b_i \mu_n - q_i \mu_e$

Charge neutrality: $\sum_i x_i q_i = 0$

$\mu_e = \mu_\mu = \mu_n - \mu_p$

$\mu_{\Sigma^-} = 2\mu_n - \mu_p$

$\mu_{\Sigma^0} = \mu_\Lambda = \mu_n$

$\mu_{\Sigma^+} = \mu_p$

Composition: $x_i(\rho)$

Equation of state: $\boldsymbol{p}(\boldsymbol{\rho}) = \rho^2 \dfrac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$

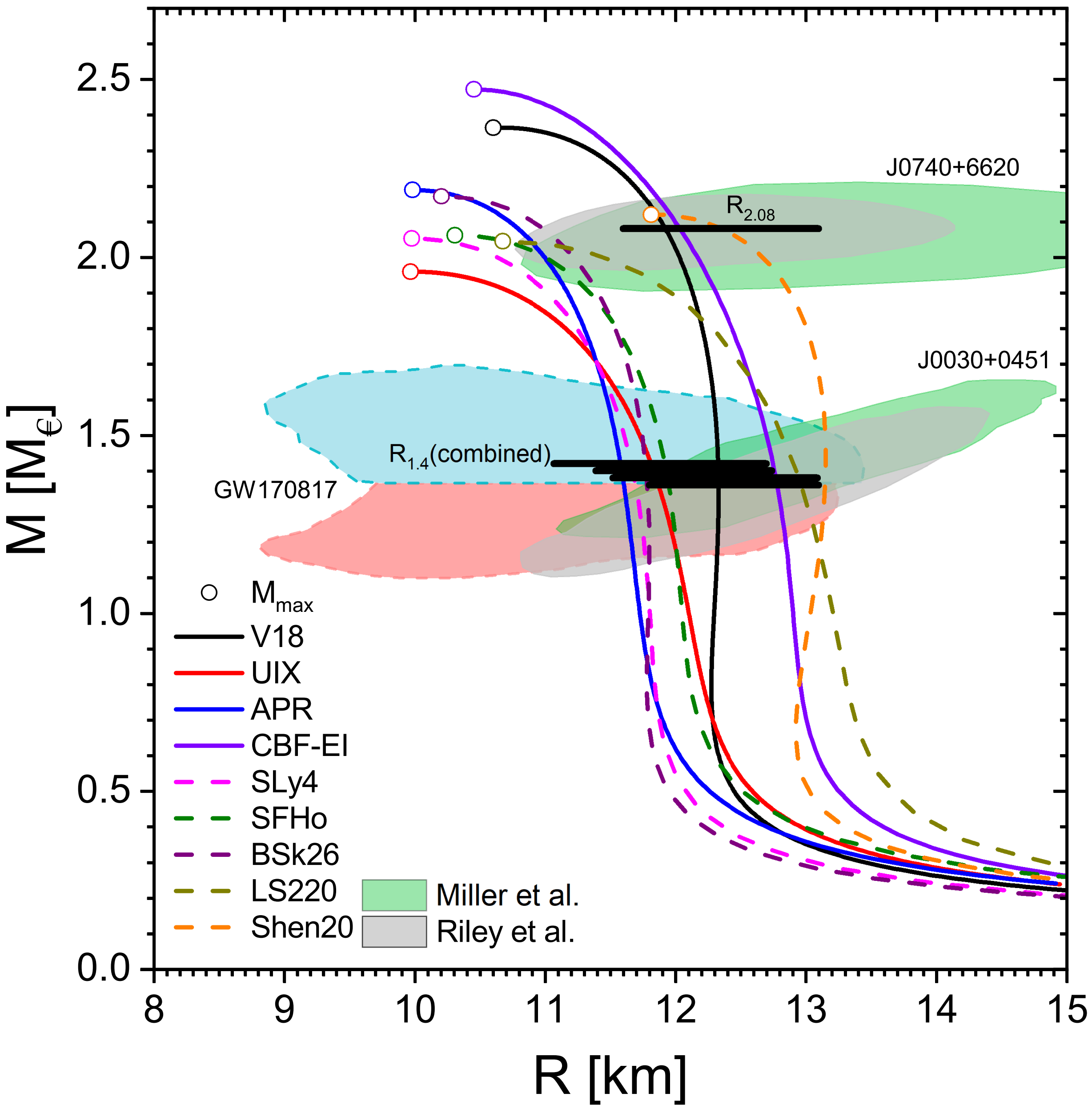
TOV equations:

$$\frac{dp}{dr} = -\frac{Gm\epsilon}{r^2}\frac{(1+p/\epsilon)(1+4\pi r^3 p/m)}{1-2Gm/r}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

Structure of the star: $\rho(r)$, $\boldsymbol{M}(\boldsymbol{R})$ etc.

CHECK *WRT* NS MASS-RADIUS OBSERVATIONS



The simultaneous measurement of both mass and radius of the same NS would provide the most definite observational constraint on the nuclear EoS.

NICER mission : J0740+6620 and J0030+0451.

Green and grey → two different data analysis

Black bars results from the combination of NICER and GW170817

$R_{2.08} = 12.35 \pm 0.75$ km

$R_{1.4} = 12.45 \pm 0.65$ km (M. C. Miller et al., *Astrophys. J. Lett.*, 918(2):L28, 2021.)

$R_{1.4} = 11.94^{+0.76}_{-0.87}$ km (P. T. H. Pang, et al., *Astrophys. J.*, 922(1):14, 2021.)

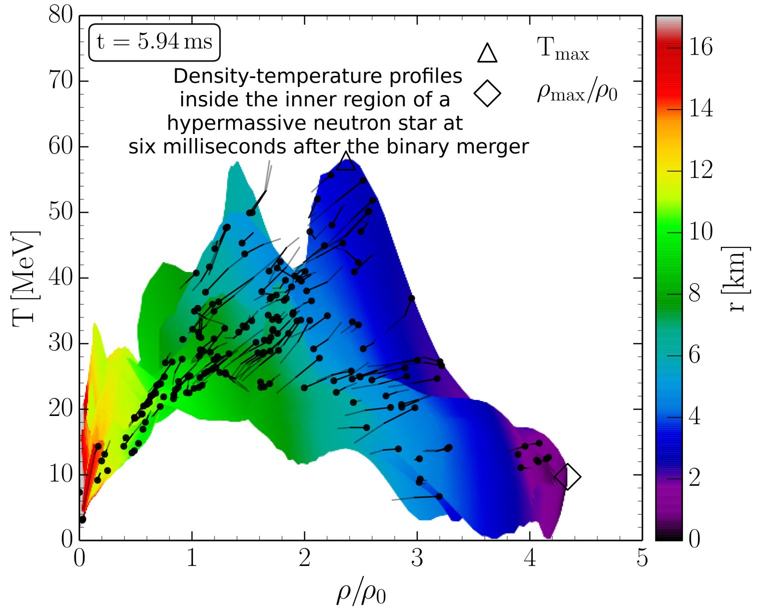
$R_{1.4} = 12.33^{+0.76}_{-0.81}$ km (G. Raaijmakers et al. *Astrophys. J. Lett.*, 918(2):L29, 2021.)

What do we learn about the EoS ?

- UIX is too soft … problem of TBF at high densities
- NICER+GW170817 might exclude CBF-EI, LS220 and Shen20
- $R_{2.08}$ would admit V18, CBF-EI and Shen20

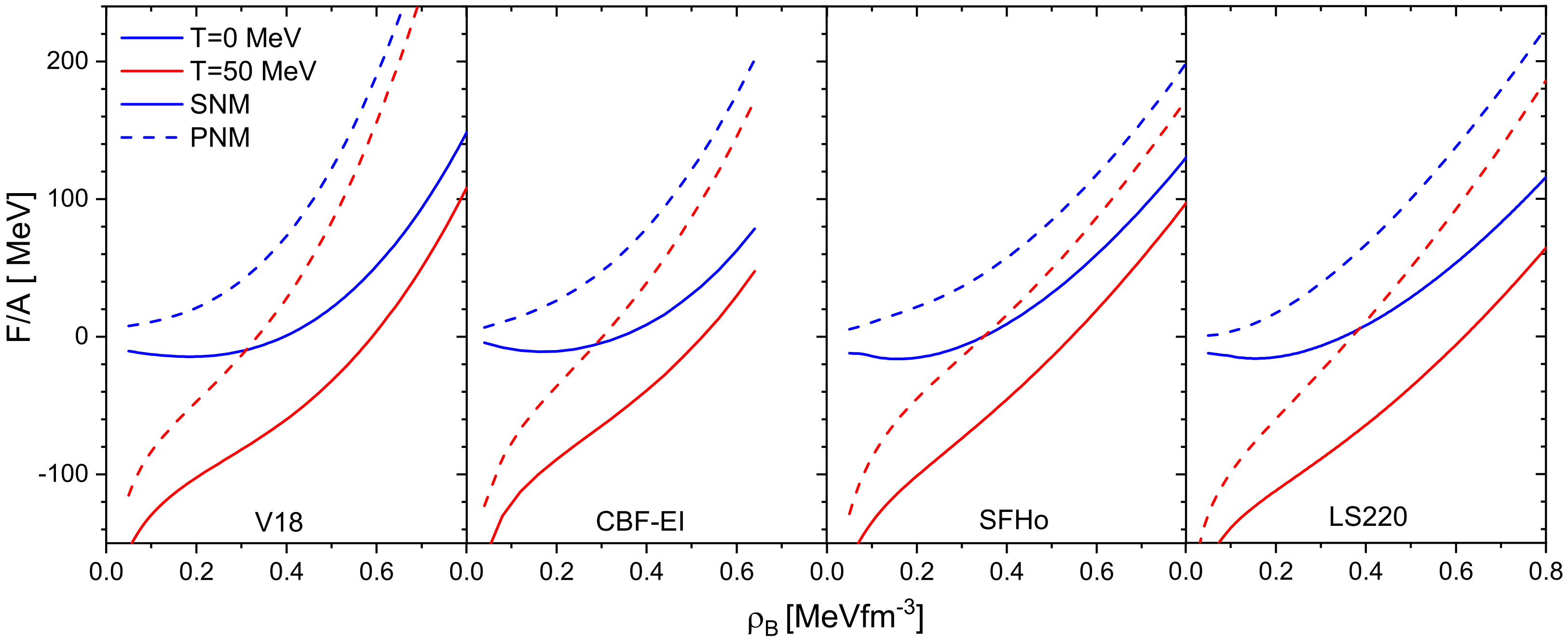
Future refinements of these radii limits will allow to pin down the EoS very well.

THERMAL EFFECTS ON THE EOS



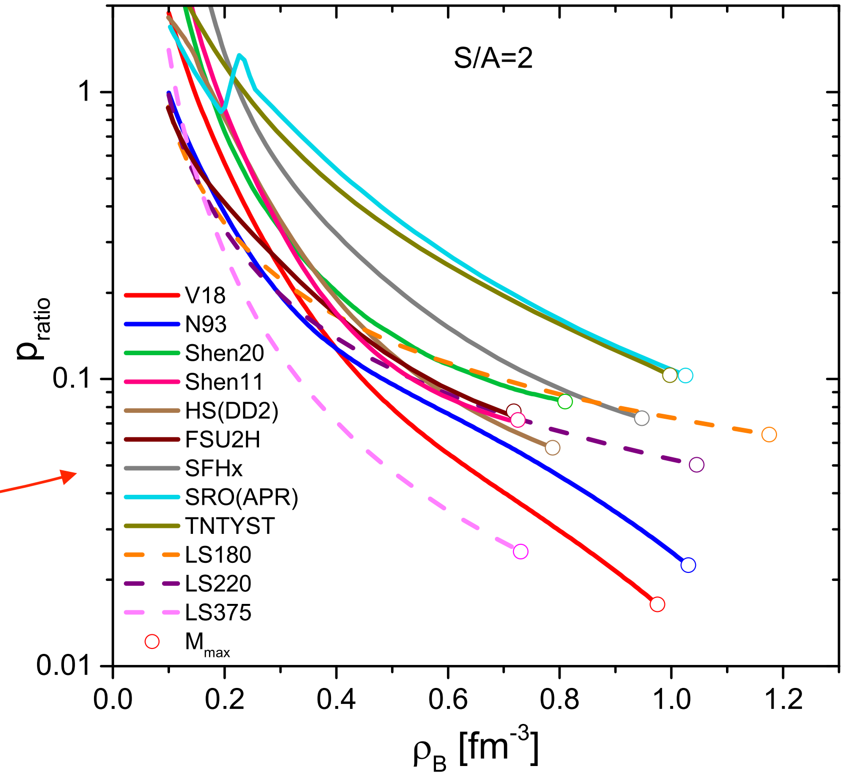
Density-temperature profiles inside the inner region of a hypermassive neutron star at six milliseconds after the binary merger

Hanauske, Steinheimer et. al,
Particles **2019**, 2, 44-56



The thermal pressure

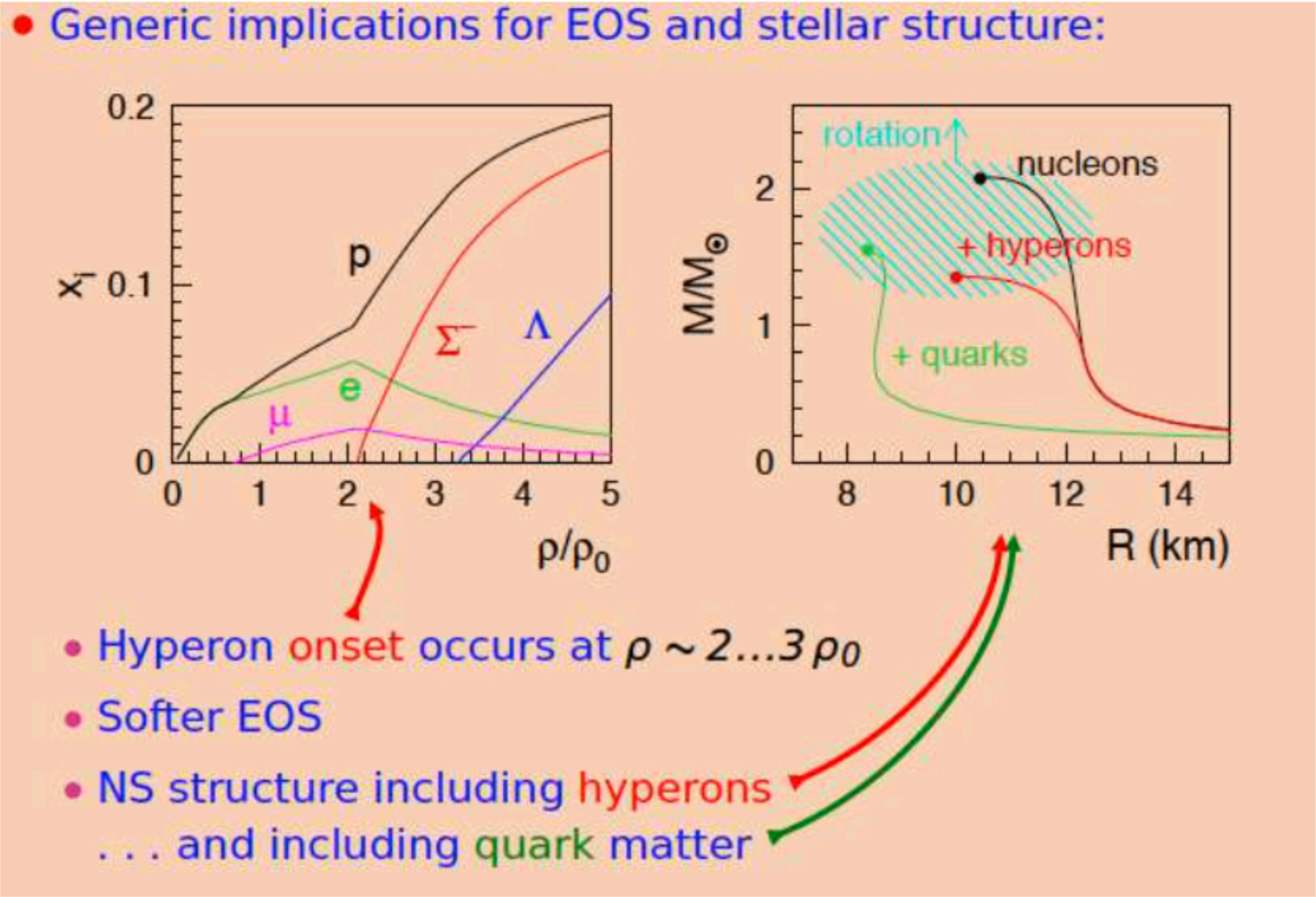
$$p_{ratio} = \frac{p_{th}}{p_0}(\rho,T) = \frac{p(\rho,x_T,T)-p(\rho,x_0,0)}{p(\rho,x_0,0)}$$



At the central density corresponding to the maximum mass, thermal effects on the pressure are below 3% for the microscopic models, and can reach 10% for the phenomenological ones.

ONSET OF BARYONIC STRANGE MATTER : THE HYPERON PUZZLE

Unsolved … after almost 25 years !

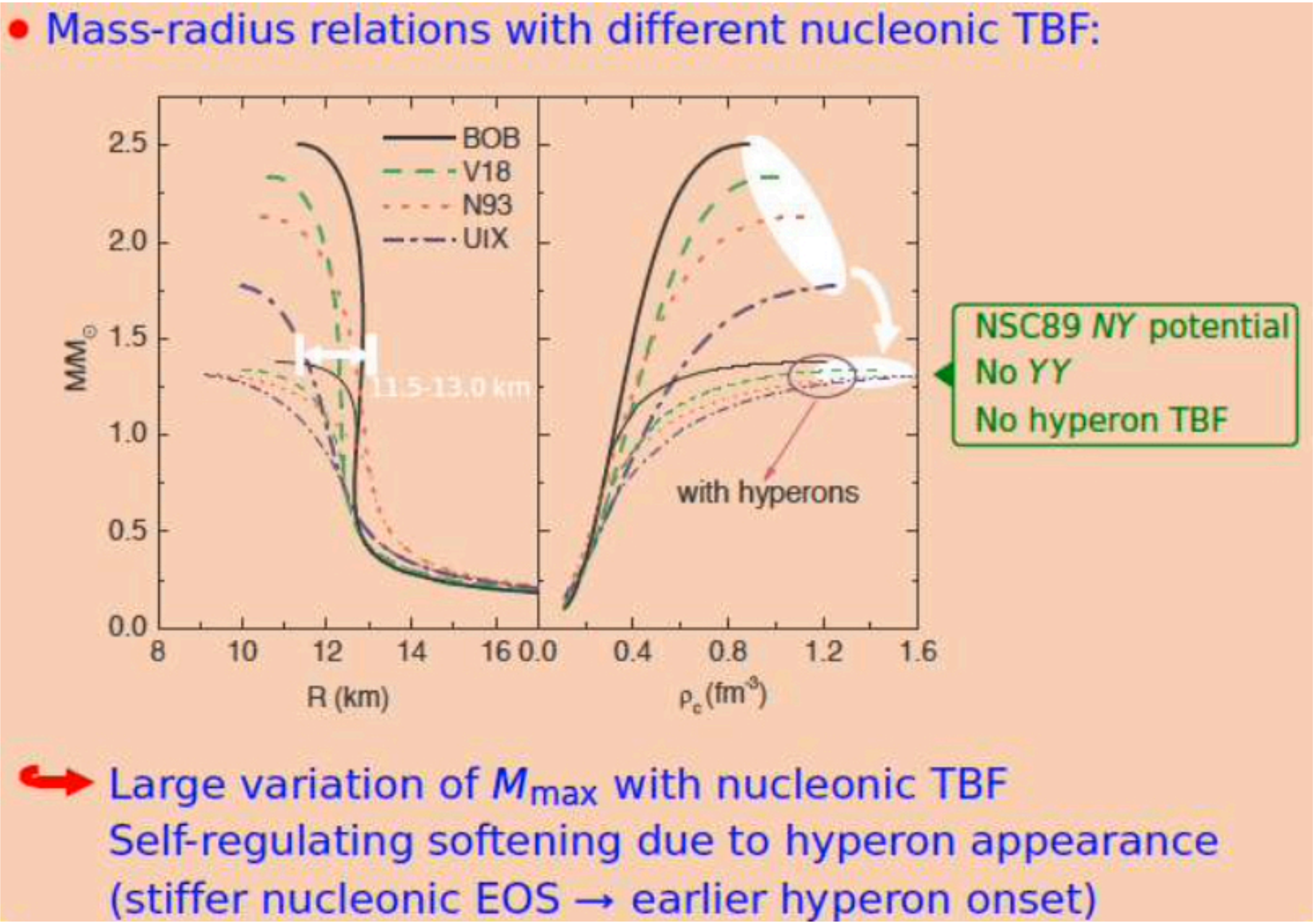


- Generic implications for EOS and stellar structure:

- Hyperon onset occurs at $\rho \sim 2\ldots3\,\rho_0$
- Softer EOS
- NS structure including hyperons
  . . . and including quark matter

Courtesy : Hans-Josef Schulze (2019)

THE HYPERON PUZZLE



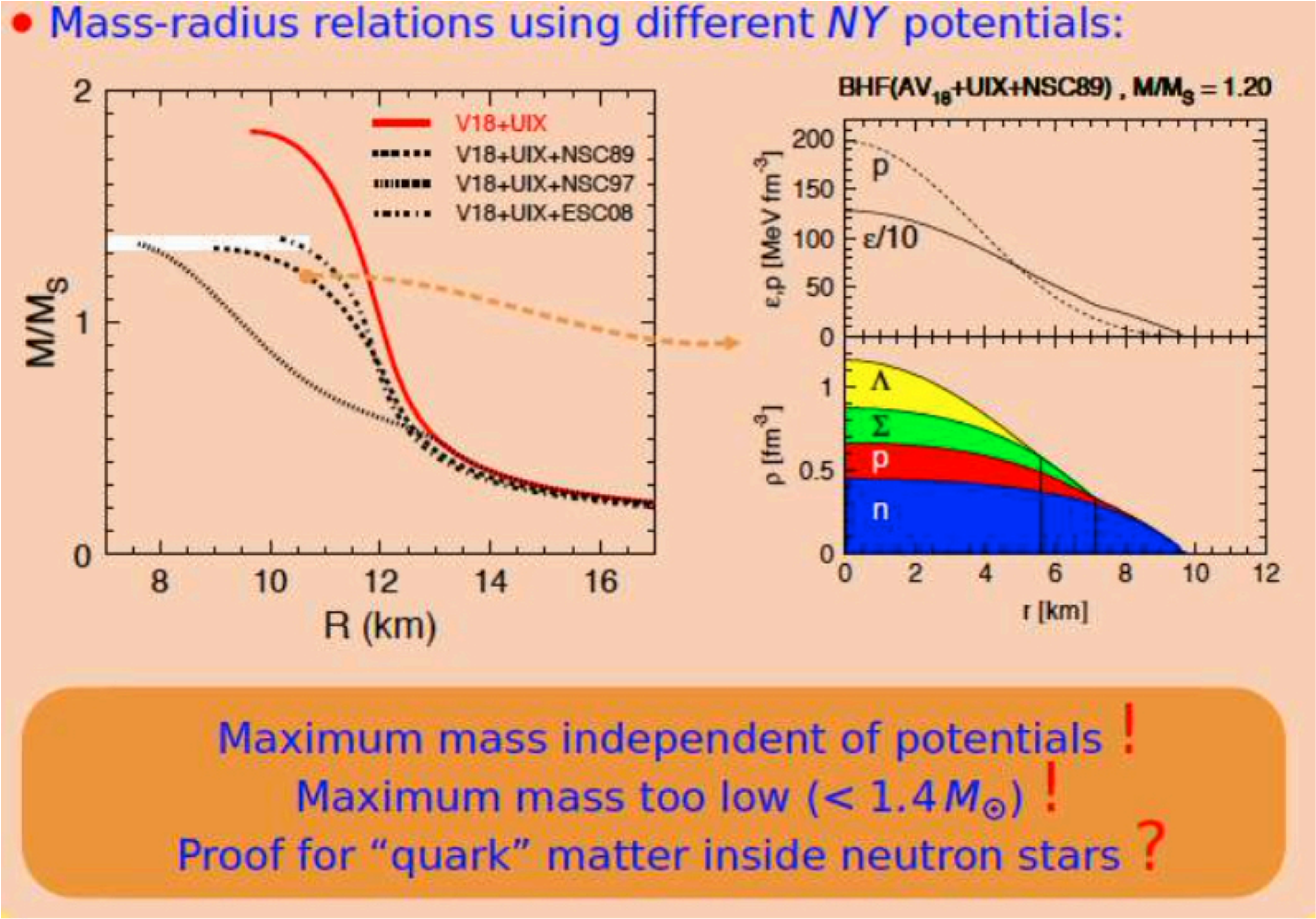
- Mass-radius relations with different nucleonic TBF:

NSC89 *NY* potential
No *YY*
No hyperon TBF

➡ Large variation of M_{max} with nucleonic TBF
Self-regulating softening due to hyperon appearance
(stiffer nucleonic EOS → earlier hyperon onset)

Courtesy : Hans-Josef Schulze (2019)

THE HYPERON PUZZLE



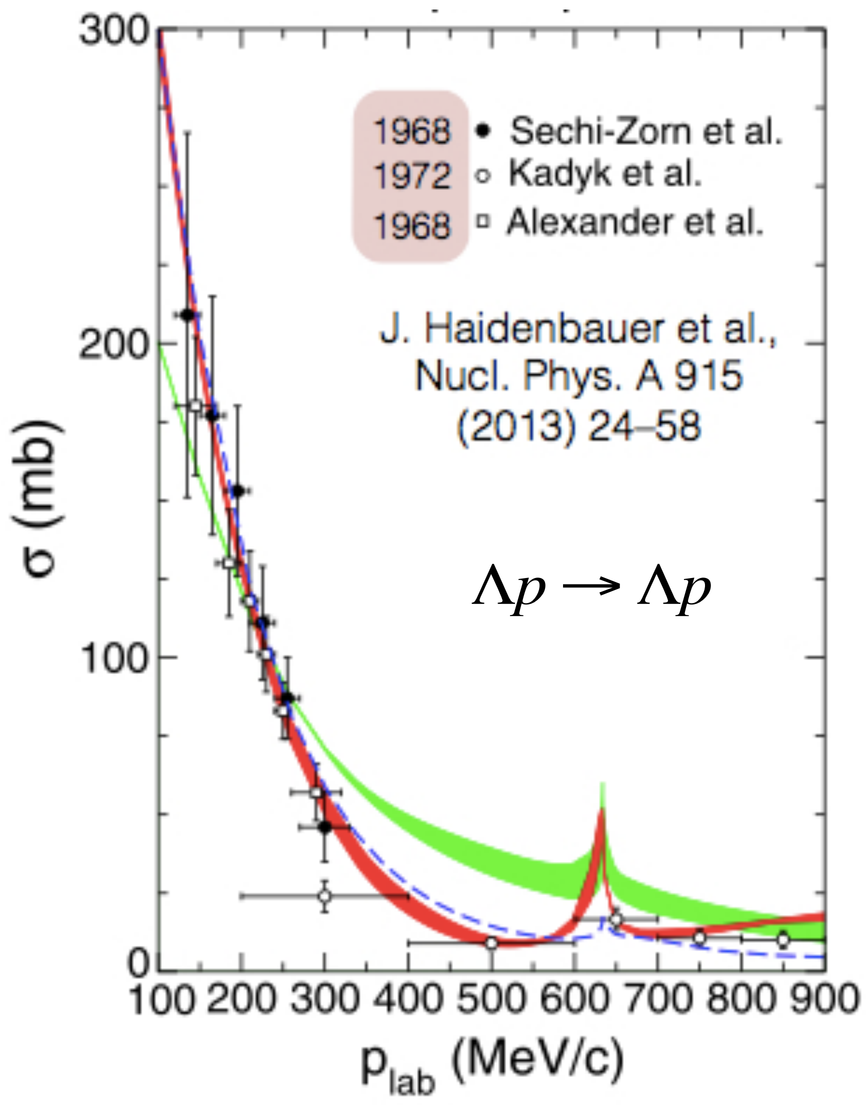
- Mass-radius relations using different *NY* potentials:

Maximum mass independent of potentials !
Maximum mass too low ($< 1.4 M_\odot$) !
Proof for "quark" matter inside neutron stars ?

Courtesy : Hans-Josef Schulze (2019)

What do we know to include hyperons in the EoS ?

Unfortunately, much less than in the pure nucleonic sector
to put stringent constraints on the YN & YY interactions



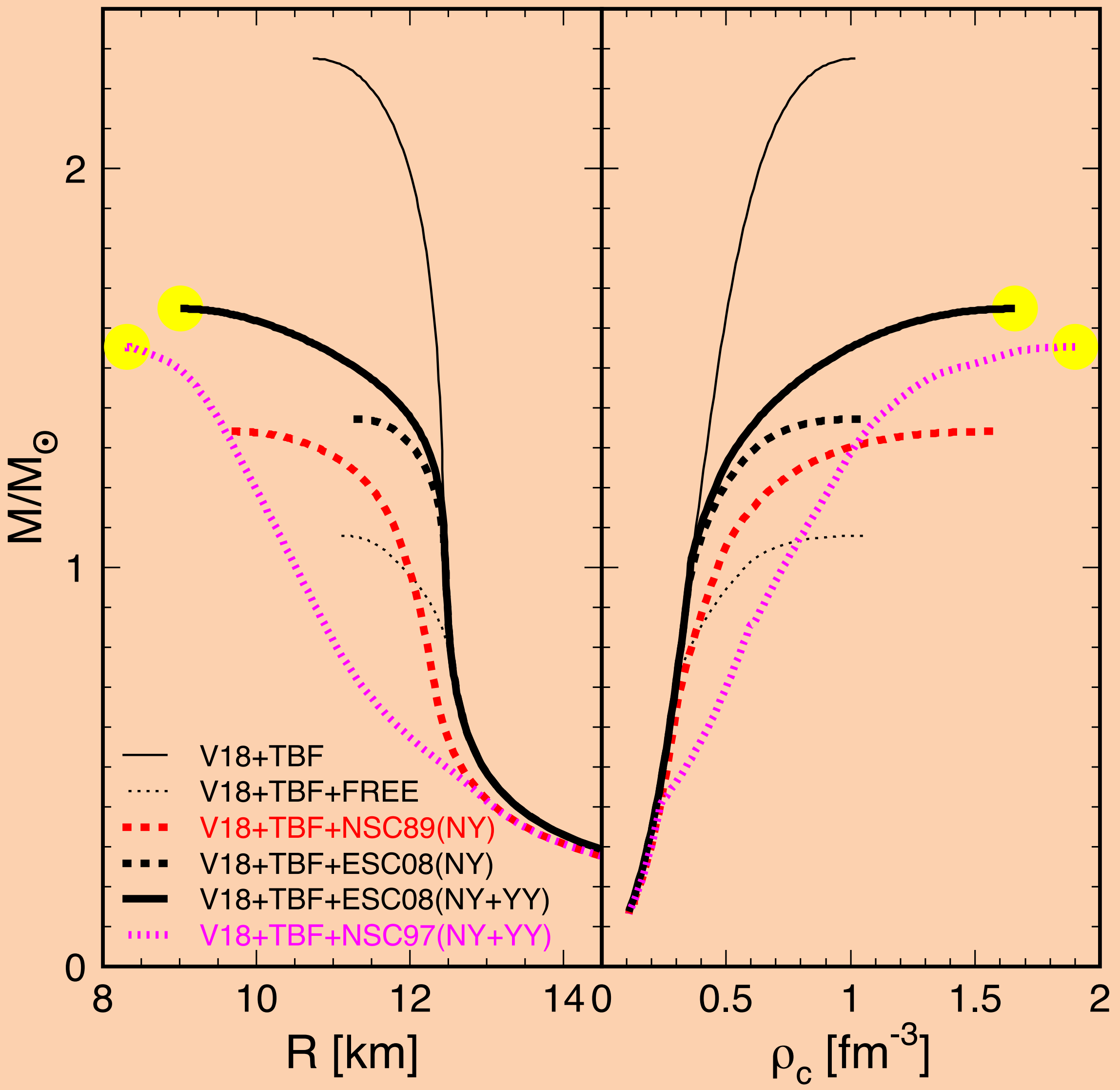
- Very few YN scattering data due to short lifetime of hyperons & low intensity beam fluxes
  - ~ 35 data points, all from the 1960s
  - 10 new data points, from KEK-PS E251 collaboration (2000)
- No YY scattering data exists

(cf. > 4000 NN data for $E_{lab} < 350$ MeV)

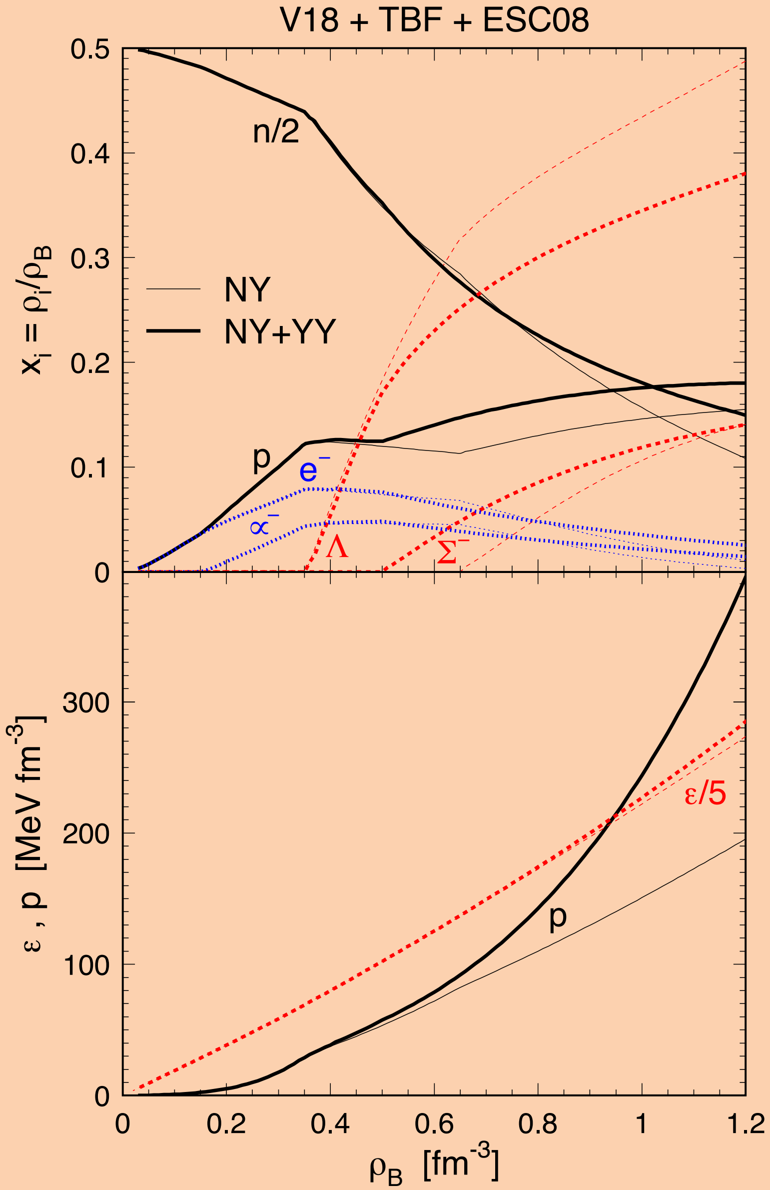
Courtesy by I. Vidaña

EFFECT OF YY INTERACTION

- Effect of YY Interactions:



Mass increase to $\lesssim 1.7 M_\odot$



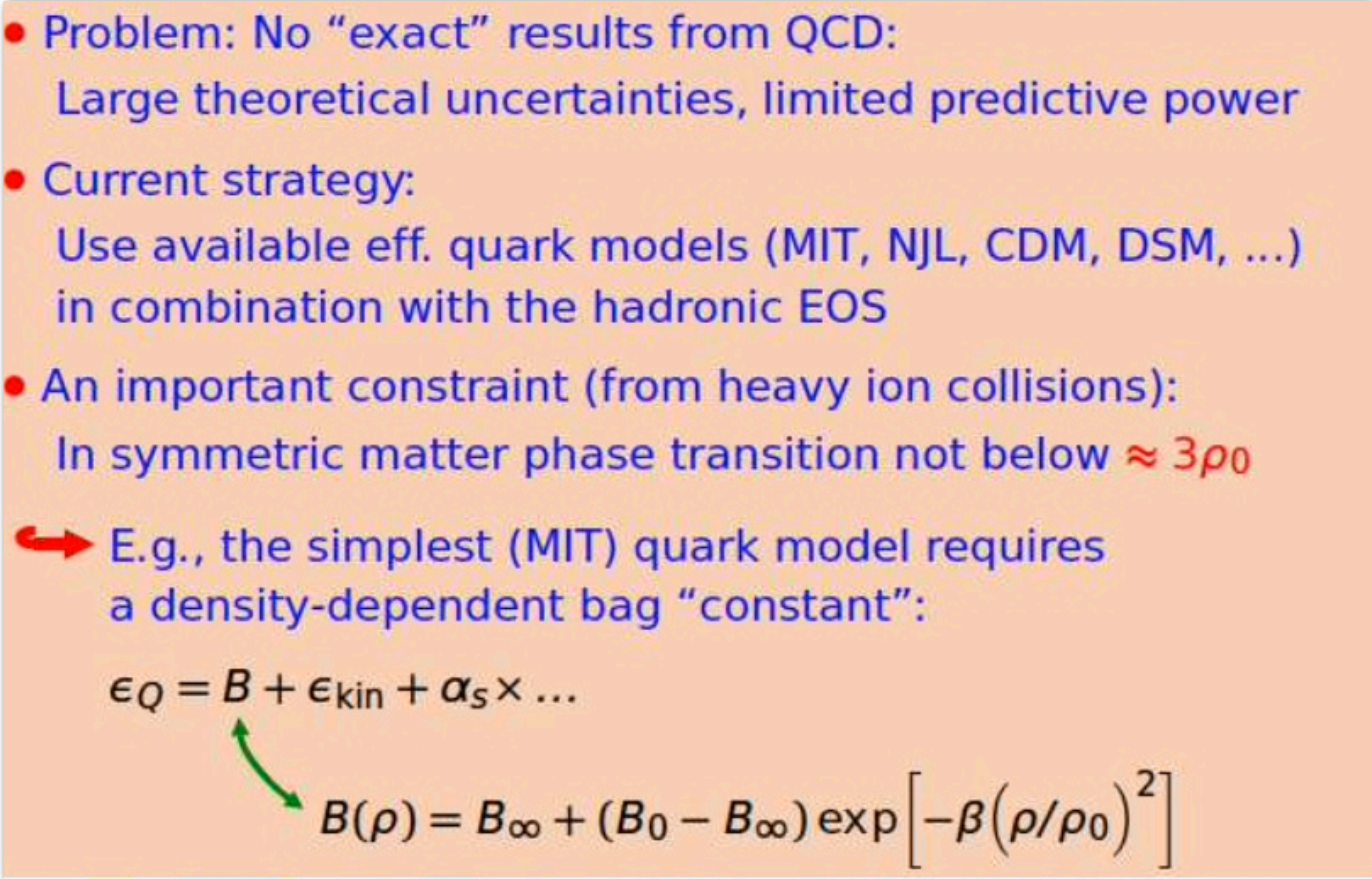
$\Lambda\Lambda$, $\Sigma^-\Sigma^-$ repulsive

$\Lambda\Sigma^-$ attractive !

- Hyperon TBF (YNN, YYN, YYY) unknown (exp. and theor.) !

Courtesy : Hans-Josef Schulze (2019)

QUARK MATTER EOS OF DENSE MATTER



- Problem: No "exact" results from QCD:
  Large theoretical uncertainties, limited predictive power
- Current strategy:
  Use available eff. quark models (MIT, NJL, CDM, DSM, ...)
  in combination with the hadronic EOS
- An important constraint (from heavy ion collisions):
  In symmetric matter phase transition not below $\approx 3\rho_0$

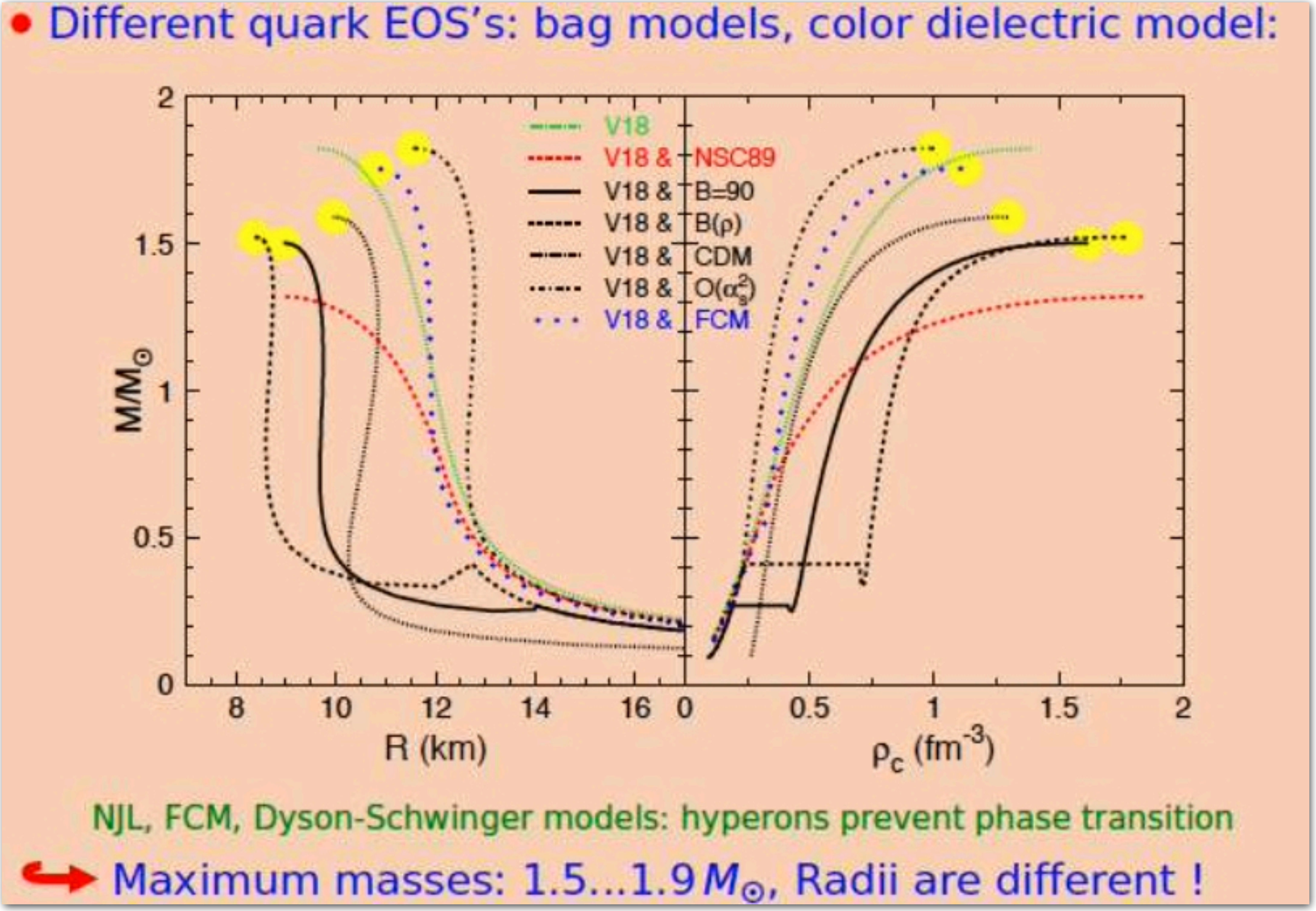
➡ E.g., the simplest (MIT) quark model requires
a density-dependent bag "constant":

$$\epsilon_Q = B + \epsilon_{kin} + \alpha_s \times \dots$$

$$B(\rho) = B_\infty + (B_0 - B_\infty)\exp\left[-\beta\left(\rho/\rho_0\right)^2\right]$$

Courtesy : Hans-Josef Schulze (2019)

QUARK MATTER EOS OF DENSE MATTER



- Different quark EOS's: bag models, color dielectric model:

NJL, FCM, Dyson-Schwinger models: hyperons prevent phase transition

→ Maximum masses: 1.5...1.9 $M_\odot$, Radii are different !

Courtesy : Hans-Josef Schulze (2019)

A PARTIAL LIST OF OPEN PROBLEMS …

- Dependence on the theoretical many-body methods
- Relevance of the nucleonic three-body forces at high density : no complete theory available yet !
- Onset of baryonic strange matter : the hyperon puzzle
- Deconfinement phase transition : at which baryon density ?
- NS Cooling, DURCA processes and nuclear matter superfluidity gaps
- …

After many years the shopping list is still very rich !