Study of the scaling properties in SU(2) gauge theory with eight flavors

Ref. arXiv:1011.0373
arXiv:1011.0516 and work in progress

Hiroshi Ohki
KMI, Nagoya University

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Collaborators

- Tatsumi Aoyama a
- Masafumi Kurachi a
- C. –J. David Lin e
- Hideo Matsufuru f
- Tetsuya Onogi g
- Etsuko Itou g
- Eigo Shintani h
- Takeshi Yamazaki a

Numerical simulation was carried out on the vector supercomputer
NEC SX-8 in YITP, Kyoto University
and RCNP, Osaka University
SR and BlueGene in KEK

a : Nagoya University
b : University of Graz
c : YITP, Kyoto University
d : Tohoku University
e : National Chio-Tung University, and National Center for Theoretical Science
f : KEK
g: Osaka University
h: RIKEN
i: Tsukuba University

outline

- Introduction
- Calculation of gauge coupling in lattice theory
- Simulation of pure SU(3)
- Results
- Large flavor simulation
- summary

Introduction

Introduction

- **The physics of the conformal gauge theory is not well known.**

Theoretical interest

Phase structure of the non-SUSY gauge theories

(gauge group, representation, # of flavor)

Phenomenology

Test of the models of physics beyond the SM

(ex. Walking technicolor model)

-> SU(2) gauge theory is very important.

Phenomenology

Technicolor model [Weinberg(1979), Susskind(1979)]

- **One candidate of the physics beyond the standard model**

Electro-weak symmetry breaking
-> techniquark condensation Q
(Ex. gauge hierarchy
-> dimensional transmutation Λ_{QCD} in QCD)

Ex. $SU(N)_{TC} \times SU(3)_{color} \times SU(2)_L \times U(1)_Y$

$$\langle \bar{Q} Q \rangle = \Lambda_{TC}^3 \qquad \Lambda_{TC} \sim 250 \text{ GeV}$$

Extended technicolor model (ETC)

[S. Dimopoulos and L. Susskind (1979), E. Eichten and K. Lane (1980)]

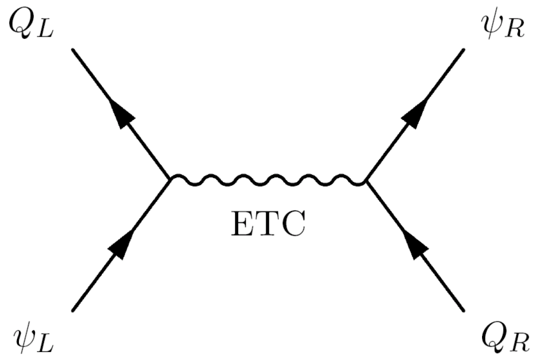
The origin of the quark/lepton mass
(spontaneous chiral symmetry breaking)
-> Extended technicolor interaction

ETC quark $$(\underbrace{Q, \cdots, Q}_{\text{techni fermion}}, \underbrace{\psi, \cdots, \psi}_{\text{quark}})$$

$$\psi_L \cdot H\psi_R \quad \Rightarrow \quad \psi_L \cdot \frac{\langle \bar{Q}Q \rangle}{\Lambda_{ETC}^2}\psi_R$$

Extended technicolor

Ref. [S. Sannino, arXiv[0804.0182]]



$$\alpha_{ab}\frac{\bar{Q}T^aQ\bar{Q}T^bQ}{\Lambda^2_{ETC}} + \beta_{ab}\frac{\bar{Q}_LT^aQ_R\bar{\psi}_RT^b\psi_L}{\Lambda^2_{ETC}} + \gamma_{ab}\frac{\bar{\psi}_LT^a\psi_R\bar{\psi}_RT^b\psi_L}{\Lambda^2_{ETC}} + \dots$$

PNG Masses

SM-Fermion Masses

FCNC Operators

Problems in Extended technicolor

FCNC

$$\frac{g_{ETC}^2}{\Lambda_{ETC}^2}(\bar{s}\gamma d)(\bar{s}\gamma d) + \frac{g_{ETC}^2}{\Lambda_{ETC}^2}(\bar{s}\gamma d)(\bar{e}\gamma\mu) + \cdots$$

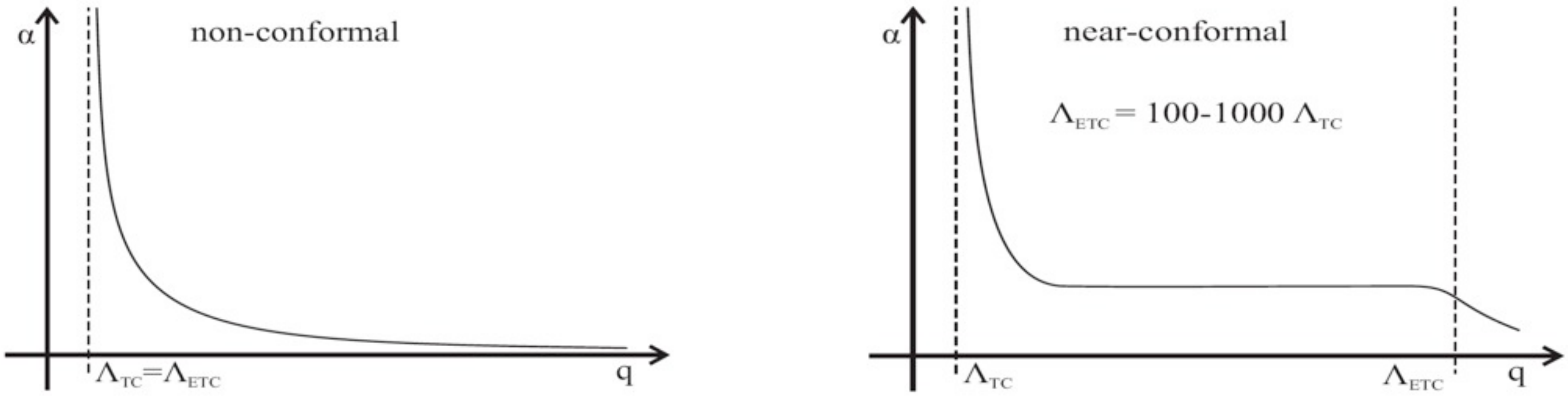
$$\Delta_K^{exp} \longrightarrow \Lambda_{ETC} \geq 10^{2\sim 3}\Lambda_{TC}$$

Quark mass $m_f \sim g_{\mathrm{ETC}}^2 \frac{\langle\bar{Q}Q\rangle}{\Lambda_{\mathrm{ETC}}^2}$

$$\langle\bar{Q}Q\rangle_{ETC} \sim \langle\bar{Q}Q\rangle_{TC} \sim \Lambda_{TC}^3$$

$\Rightarrow$ **Small quark mass & NG boson mass**

walking (conformal) dynamics



$$\langle \bar{Q}Q \rangle_{ETC} = \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} d(\ln\mu)\gamma(\alpha(\mu)) \right) \langle \bar{Q}Q \rangle_{TC}$$

QCD like

$$\alpha(\mu) \propto \frac{1}{\ln\mu} \qquad \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} d(\ln\mu)\gamma(\alpha(\mu)) \right) = \ln\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma}$$

Conformal

$$\exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} d(\ln\mu)\gamma(\alpha(\mu)) \right) = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma(\alpha^*)}$$

Quark mass enhancement

large anomalous dimension $\gamma \sim \mathcal{O}(1)$

$$m_f \sim \frac{g^2_{ETC}}{\Lambda^2_{ETC}} \langle \bar{Q}Q \rangle_{ETC} \sim \frac{g^2_{ETC}}{\Lambda^2_{ETC}} \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma(\alpha^*)} \langle \bar{Q}Q \rangle_{TC}$$

may resolve these problems (quark mass & FCNC)

Test of Walking (conformal) technicolor

Walking technicolor model requires

1. "Walking" coupling : approximate IR fixed point
2. Spontaneous chiral symmetry breaking
3. Asymptotic freedom
4. Large anomalous dimension of QQ: $\gamma(g^*) \geq 1$

For realistic model

Derive quark/lepton mass hierarchy and mixing
Calculation of S parameter
etc…

running coupling constant
(non-perturbative)

Renormalized coupling constant in lattice gauge theory

necessary condition to calculation of non-perturbative coupling constant

- Possible to simulation of gauge invariant operator
- Extraction of gauge coupling in finite volume
- Perturbative matching

Running coupling constant ⇨ step scaling method

- Wilson loop (arXiv:0902.3768)
- Polyakov loop (NPB433:390, NPB437:447)
- Schrodinger functional gauge coupling

  (Luscher et al. NPB359:221)

Nonperturbative renormalized coupling constant (generic)

$$< \mathcal{O} >^{NP} \equiv Z_{\mathcal{O}} < \mathcal{O} >^{tree}$$

If $< \mathcal{O} >^{tree} = k g_0^2$, the above equation gives the nonperturbative coupling constant

$$\begin{aligned} g_R^2 &\equiv Z_{\mathcal{O}} g_0^2 \\ &= < \mathcal{O} >^{NP} / k \end{aligned}$$

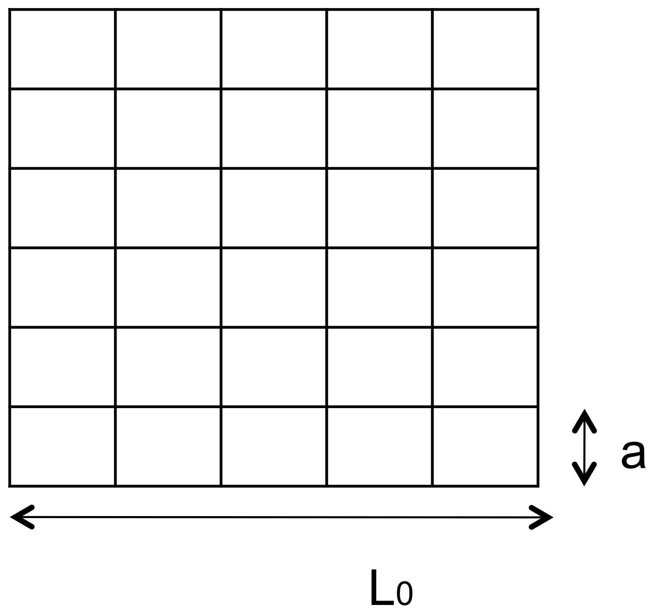
On the lattice study, we can calculate nonperturbatively the VEV on the right hand side.

In the lattice gauge theory,

$$< \mathcal{O} > = \int D\Phi \mathcal{O} e^{-S_E}$$

The measurement value has a discritization error.

- Lattice size (L0)
- Lattice spacing (a)
- bare coupling constant $\beta = 2N/g_0^2$



Step scaling method

g_R^2 non-perturbative coupling on finite volume L^4

$$g_R^2(\mu) = kA(\mu) \quad \text{at} \quad \mu = 1/L$$

On lattice simulation $g_R^2(\mu) \to kA(a/L, \beta(a/L))$

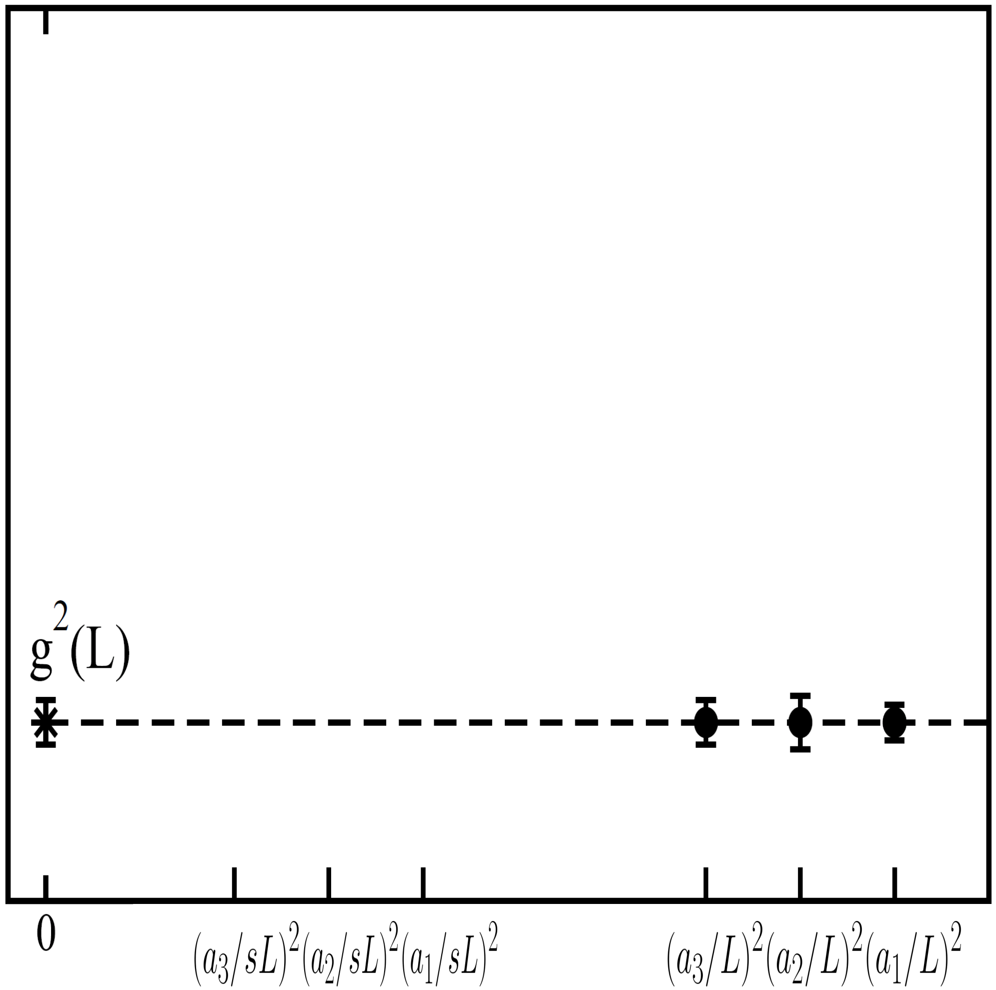
$$\beta = 1/g_0^2$$

Taking the continuum limit $a \to 0$ on a constant physics (fixed L)

step scaling

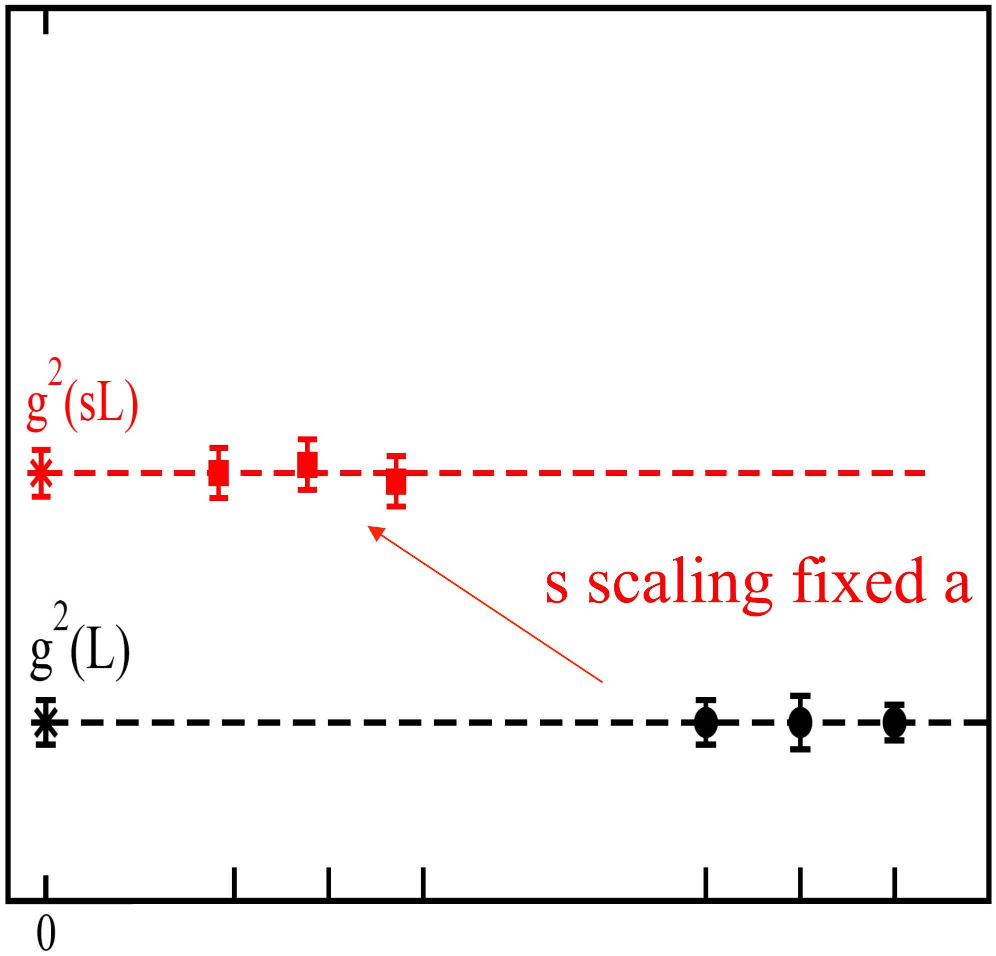
$$\mu \to \mu/s \quad \Longleftrightarrow \quad L \to sL$$

Step scaling



Calculate $g_R^2(L)$ with an input on each $(a/L, \beta)$
for small lattice size

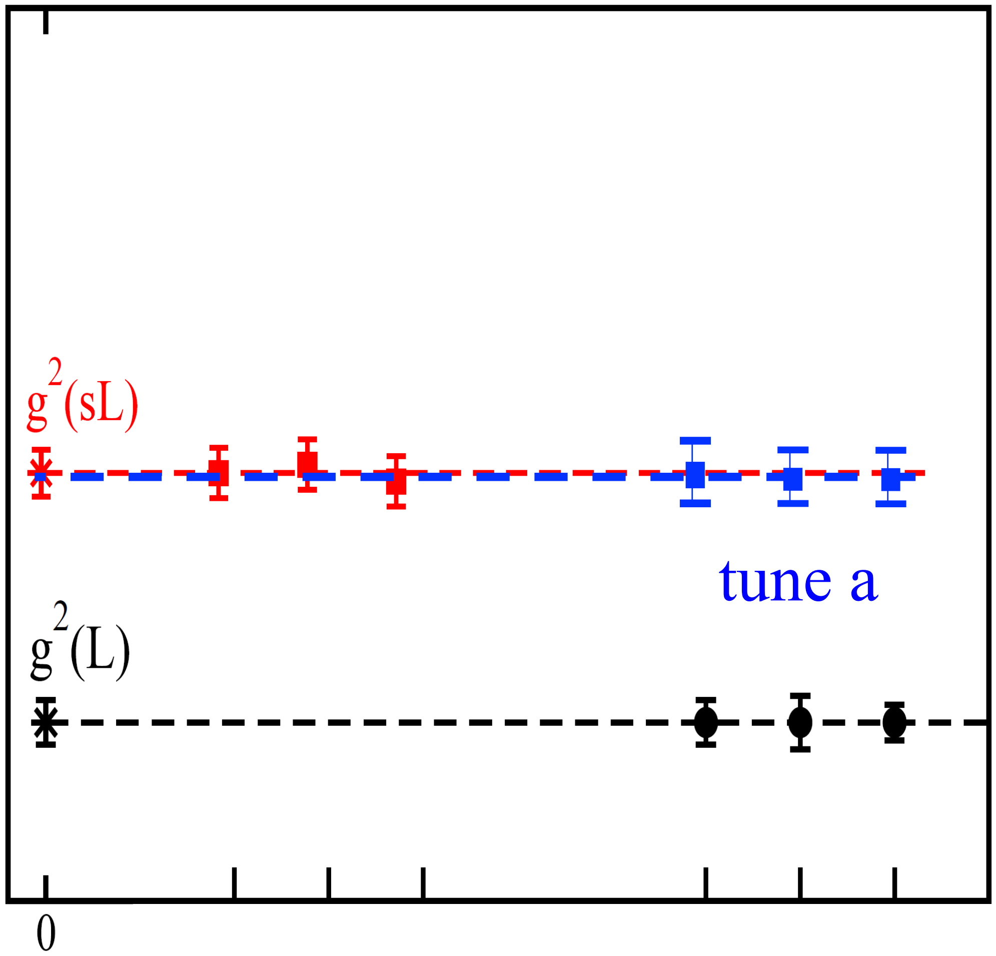
Step scaling



Calculate $g_R^2(sL)$ fixed a

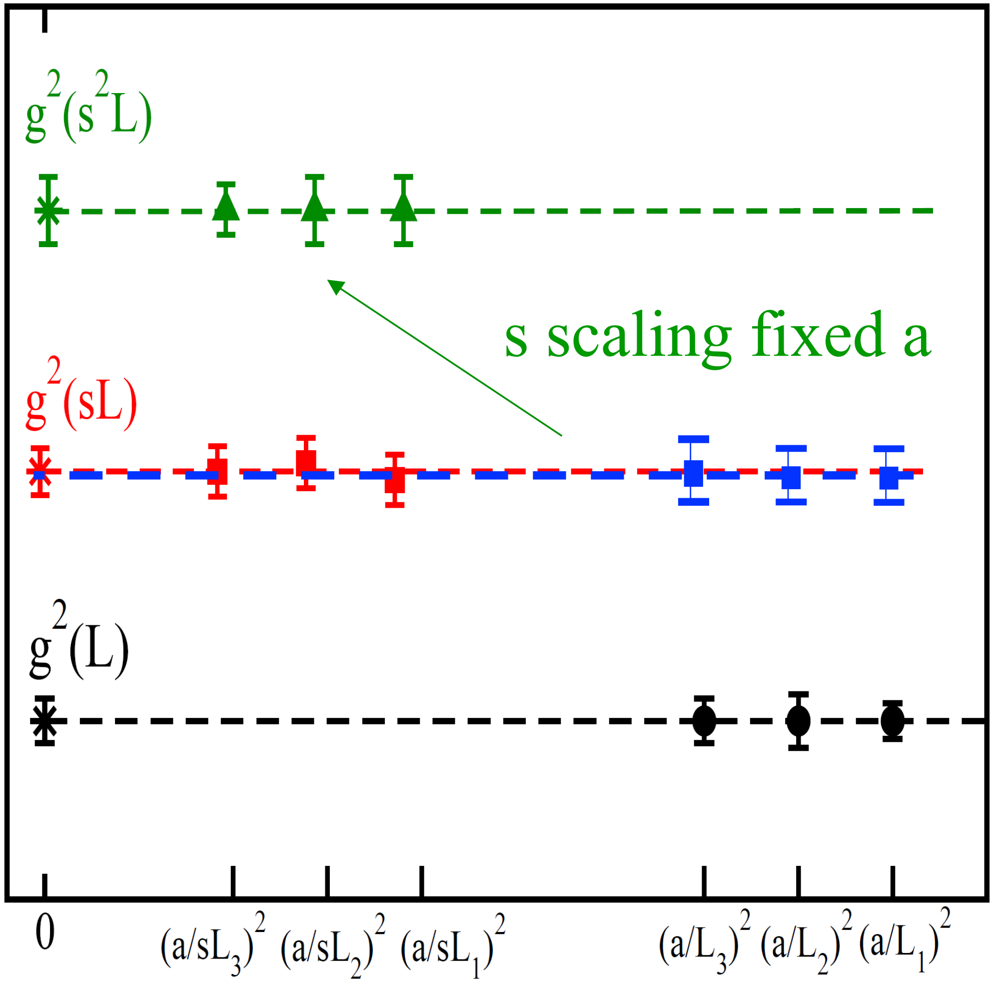
then, take continuum limit a → 0

Step scaling



Tune a on L/a to get same $g_R^2(sL)$

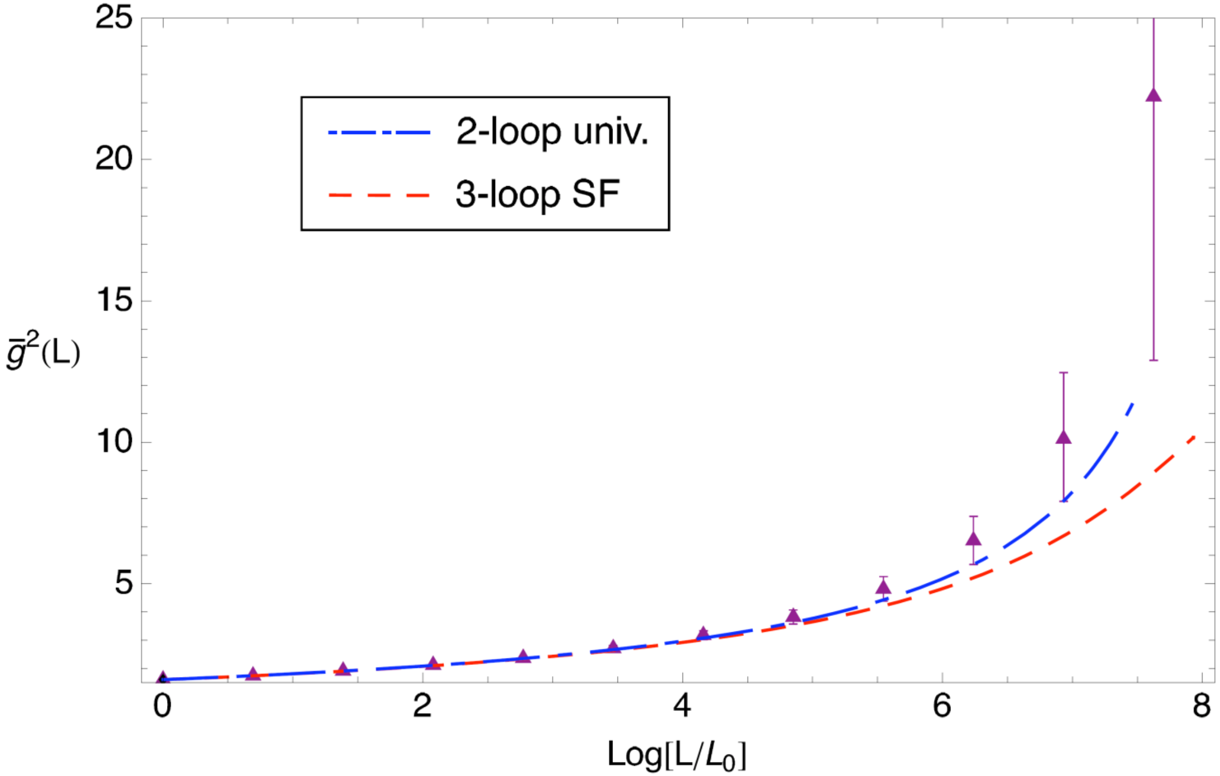
Step scaling



Calculate $g_R^2(s^2L)$ fixed a

Each step take continuum limit a $\rightarrow$ 0

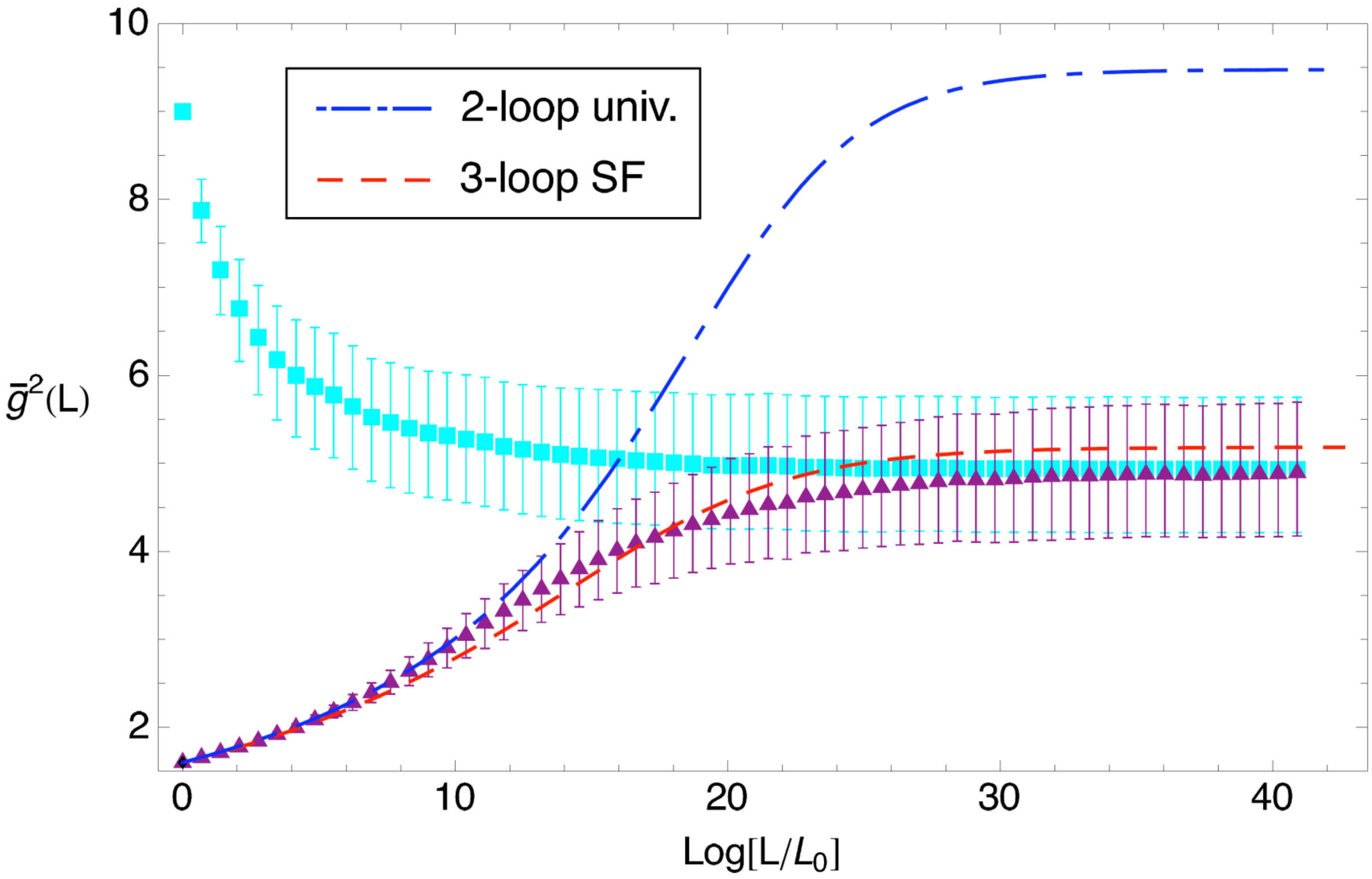
Appelquist et al. arXiv:0901.3766 $N_f = 8$



High energy $\Longleftrightarrow$ low energy

Similar to QCD

Appelquist et al. arXiv:0901.3766 $N_f = 12$



High energy $\Longleftrightarrow$ low energy

However there exists systematic error of Schrodinger functional scheme.
O(a) discretization error of boundary counter term

Our work

- We use the methods for extracting the gauge coupling with twisted boundary conditions
  (alternative method without O(a) error) .

- We study the scaling properties of running gauge coupling in SU(2) with 8 flavor.

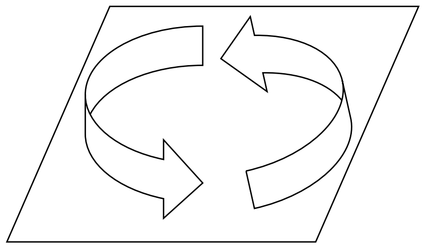
The 2-loop perturbative results suggest that the SU(2) with 8 flavor theory is near the conformal window.

Therefore this model may be a candidate for walking technicolor scenario.

Twisted boundary condition ('t Hooft NPB153:131)

Eliminate degenerate Z_N vacua
Kill the zero mode contributions

$$A_\mu(x + L\hat{\mu}) = \Omega_\nu A_\mu(x) \Omega_\nu^\dagger$$



$$\Omega_0 = \Omega_3 = 1 \qquad \Omega_1 \Omega_2 = e^{2\pi i/N} \Omega_2 \Omega_1$$

$$\Omega_\mu^N = 1 \qquad \Omega_\mu \Omega_\mu^\dagger = 1$$

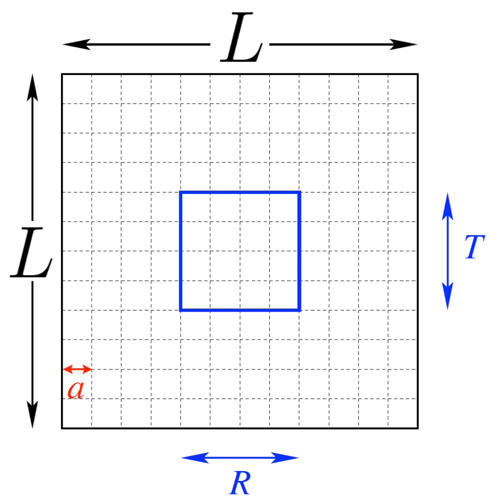
typical twist matrix for N=2

$$\Omega_x = i\sigma_1 \qquad \Omega_y = i\sigma_2$$

(twisted) Wilson loop scheme (1)

[Bilgici et. al. Phys.Rev.D80:034507,2009]

1. **Definition of the gauge coupling in finite volume**



$$kg_w^2 \equiv kg_0^2 + \mathcal{O}(g_0^4) = -R^2 \frac{\partial^2}{\partial R \partial T} \log \langle W(R,T) \rangle |_{T=R}$$

where k is the tree level matching factor.

In lattice we use the Creutz ratio

$$\chi(R+1/2, L/a) = -\log \left(\frac{W(R+1, T+1; L/a) W(R,T; L/a)}{W(R+1, T; L/a) W(R, T+1, L/a)} \right) \Bigg|_{T=R}$$

$$g_w^2 \left(L, r, \frac{a}{L} \right) = r^2 \chi(r; L/a)/k$$

$$r \equiv (R + a/2)/L$$

On the lattice study, we can calculate $r^2\chi(r; L/a)$ non-perturbatively.

2. **Fixing the scheme** **We take r= 0.25, (R=T)**

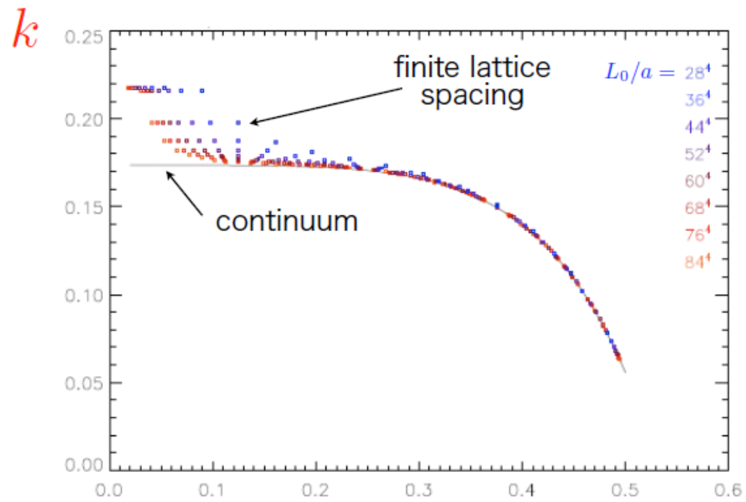
3. **Running coupling by finite scaling method**

-> Quenched test is succeeded.

Wilson loop scheme

3. Improvement by using discretized $k(r, L/a)$

$$g_w^2\left(L, r, \frac{a}{L}\right) = r^2\chi(r; L/a)/k(r, L/a)$$



- Cancellation of the huge discretization error between the finite lattice calculation of the Creutz ratio and k(r).
- g^2(r) becomes smooth function of r.

k(r) has large discrepancy between discrete and continuum one.

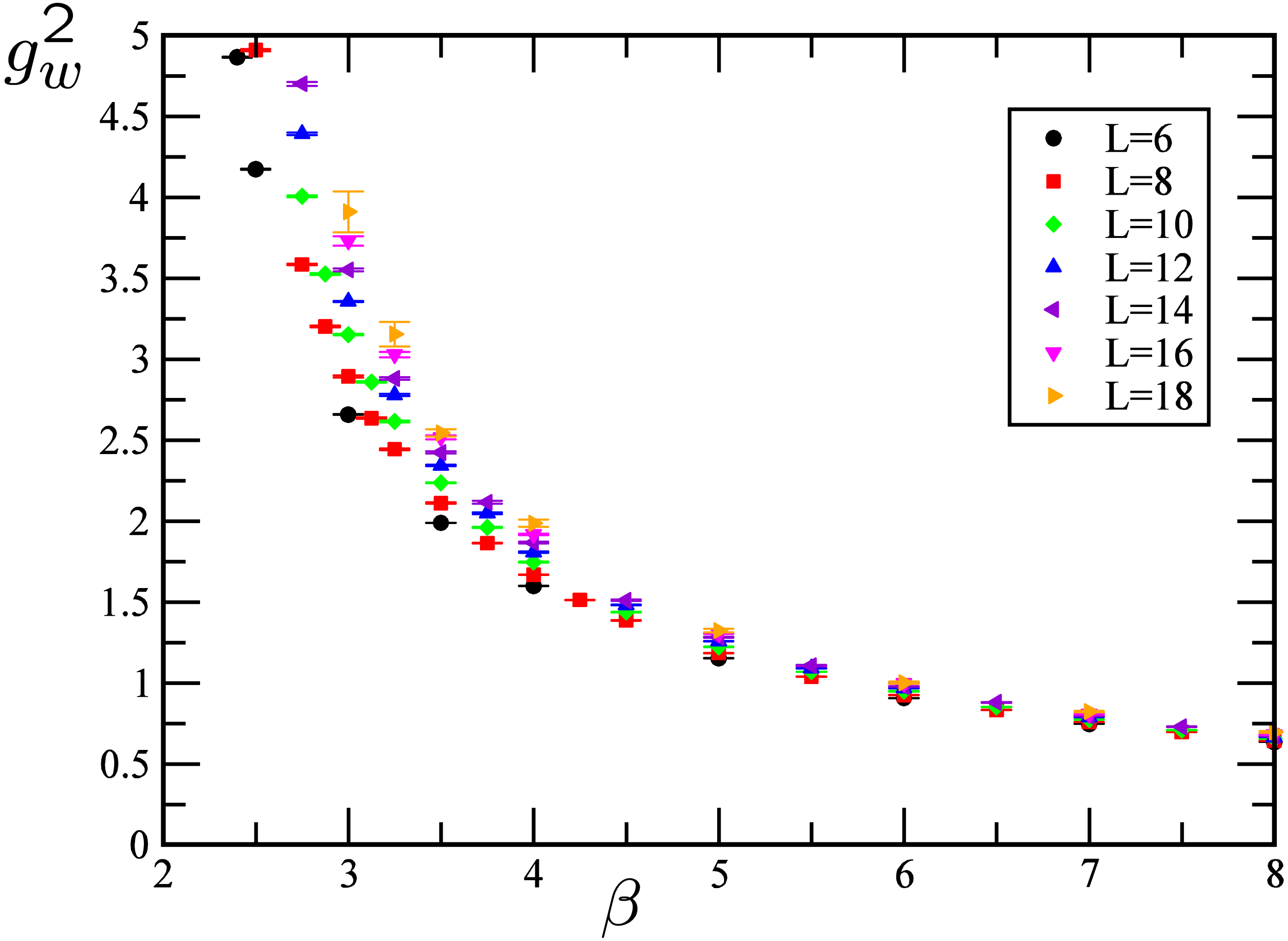
4. Fixing the scheme

We take r= 0.25, (R=T)

5. Running coupling by finite scaling method

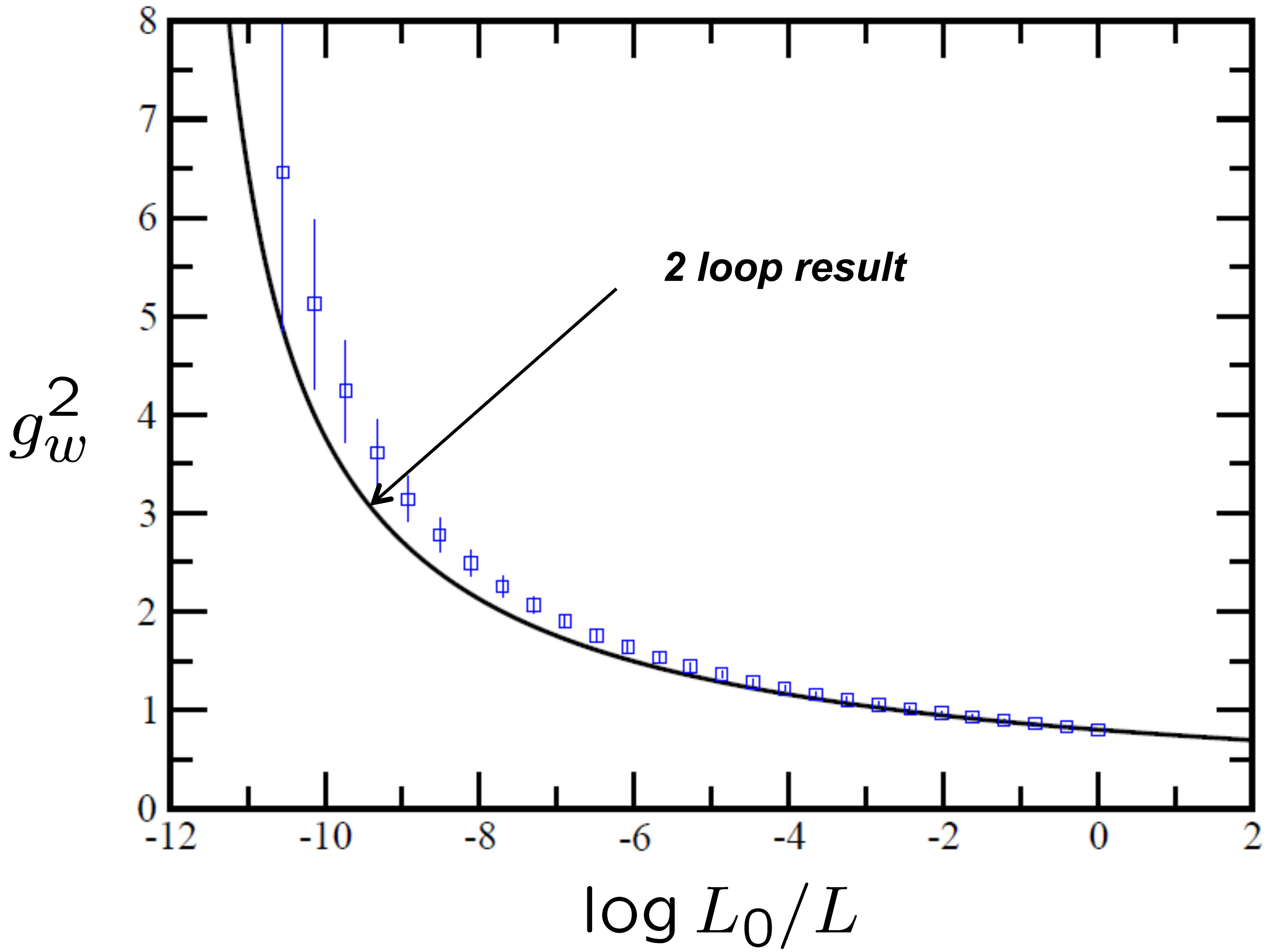
-> Quenched test is succeeded.

Pure SU(2) results



Pure SU(2) results

Good agreement with perturbative 2 loop result in the weak coupling region



8 flavor SU(2) case

Twisted boundary condition for fermions

Introduce "smell" $\to \psi^a_\alpha(x)$: $N_c \times N_s (= N_c)$ matrix

Parisi, 1983(Unpublished)

$$\psi^a_\alpha(x + \hat{\nu}L/a) = e^{i\pi/3}\Omega^{ab}_\nu \psi^b_\beta(x)(\Omega^\dagger_\nu)_{\beta\alpha} \quad (\nu = 1, 2)$$

Ensure gauge invariance of twisted boundary condition

$\Downarrow$

Smell degree is extra flavor degree

$$N_f = n_f \times N_s \qquad n_f = \begin{cases} n & \textbf{Wilson type fermion} \\ 4n & \textbf{Staggered fermion} \end{cases}$$

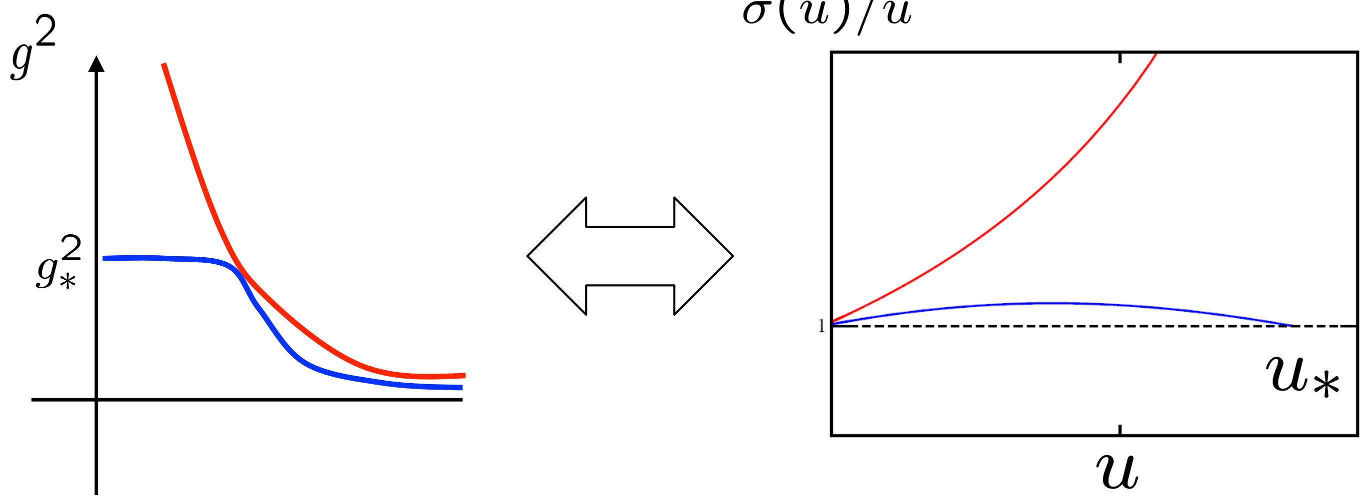
Staggered fermion requires $\quad N_f = 4 \times N_s$

Theoretical expectation of the conformal fixed point

- **study the scaling properties of SU(2) with 8 flavor gauge theory.**
  - **We use following functions**

the relative step scaling function $\sigma(u)/u$

$u = g_w^2(L) \qquad \sigma(u) = g_w^2(sL)$ **S=1.5**



2 loop perturbative prediction

- **In the SU(2) with 8flavors theory, it has a IR fixed point u*~ 15.8.**
  - **Then a inflection point is around u~7.9.**

We explore such a behavior beyond perturbation.

simulation setup

- **Wilson action**
- **Staggerd fermion with twisted boundary condition**

Introduce "smell" $\rightarrow \psi^a_\alpha(x)$: $N_c \times N_s (= N_c)$ matrix

Parisi, 1983(Unpublished)

$$\psi^a_\alpha(x + \hat{\nu}L/a) \;=\; e^{i\pi/3}\Omega^{ab}_\nu \psi^b_\beta(x)(\Omega^\dagger_\nu)_{\beta\alpha} \quad (\nu = 1, 2)$$

Smell degree is extra flavor degree

Staggered fermion requires $N_f = 4 \times N_s$

- **HMC (Omelyan integrator)**
- **Every sweep measurement of Wilson loop.**
- **Numerical calculation by NEC SX-8@YITP, @RCNP**
  KEKSR-11000@KEK
- **Simulation parameters**
  L =6,8,10,12,14,16,18, Beta= 1.375 ~ 15

  #config 10000~80000 for L=6~18

analysis step

1. **Calculation of the gauge coupling by Wilson loop of each L, beta.**
   -> global fit of g^2 as a function of beta for each L
   (Interpolation to odd lattice for step scaling)

2. **Step scaling by continuum extrapolation**

$$u = g_w^2(L)$$

$$\sigma(u) = g_w^2(sL) = \lim_{a \to 0} g_w^2(\beta, \frac{a}{sL})$$

3. **Studying the running of the gauge coupling we use the relative step scaling function**

$$\sigma(u)/u$$

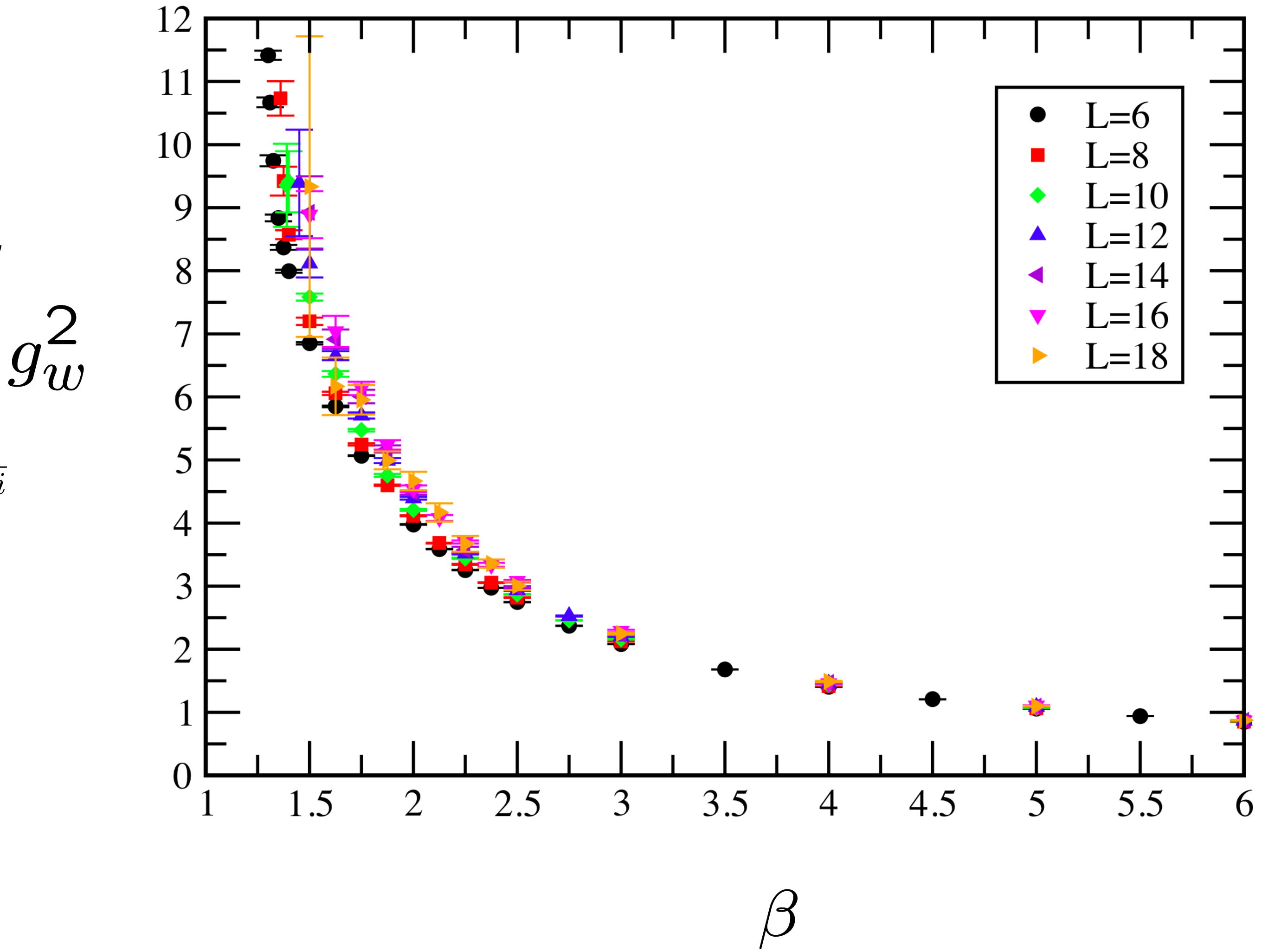
Preliminary Results

stage 1. Plot of the coupling for L=6~18, beta=1.375~15

Interpolation fit function

$$g_w^2(\beta) = \sum_{i=1}^{n} \frac{A_i}{(\beta - B)^i}$$

n=7,8



stage2, Step Scaling

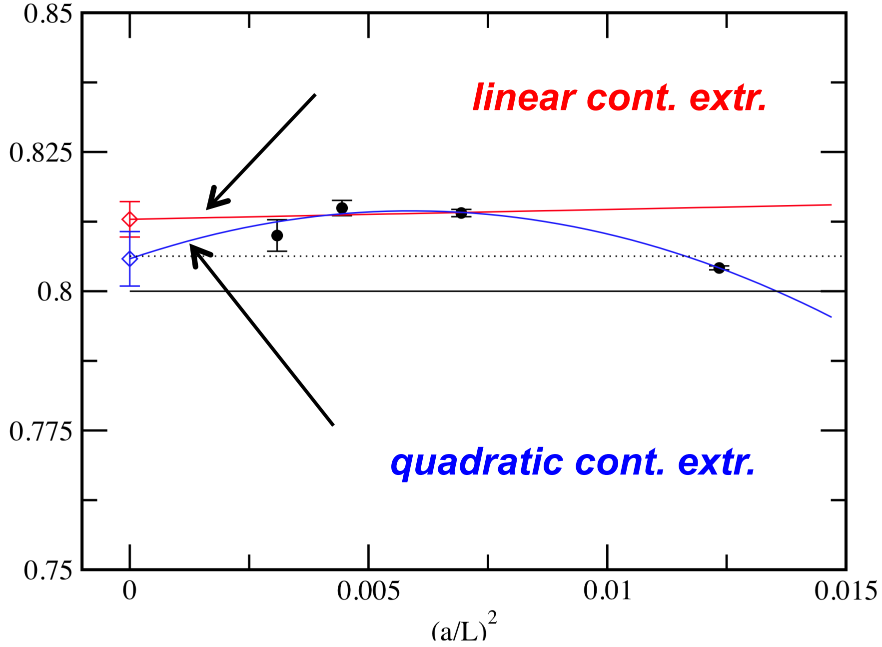
Running coupling : finite scaling method

Scaling step s=1.5

We use linear and quadratic of (a/L)^2 continuum extrapolation for the estimation of the systematic uncertainty of discretization error.

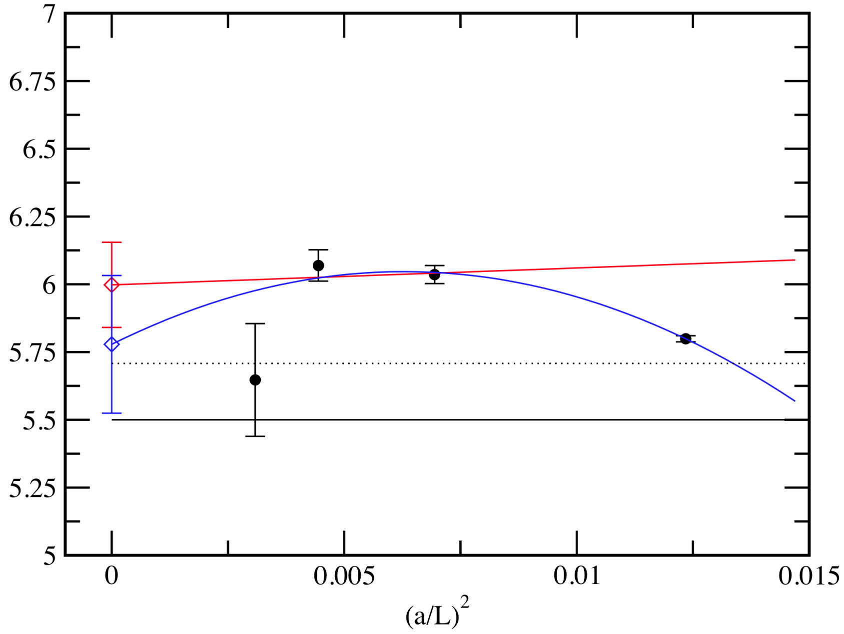
Constant extrap. (3pt) L/a= 8, 10, 12 -> sL/a= 12, 15, 18
Linear extrap. (4pt) L/a=6, 8, 10, 12 -> sL/a=9, 12, 15, 18

Stage 2. continuum extraploration



Weak coupling region (g^2=0.8)

The data is reasonably fitted.
Linear (3pt.) and quadratic (4pt.) extrapolations are consistent each other.

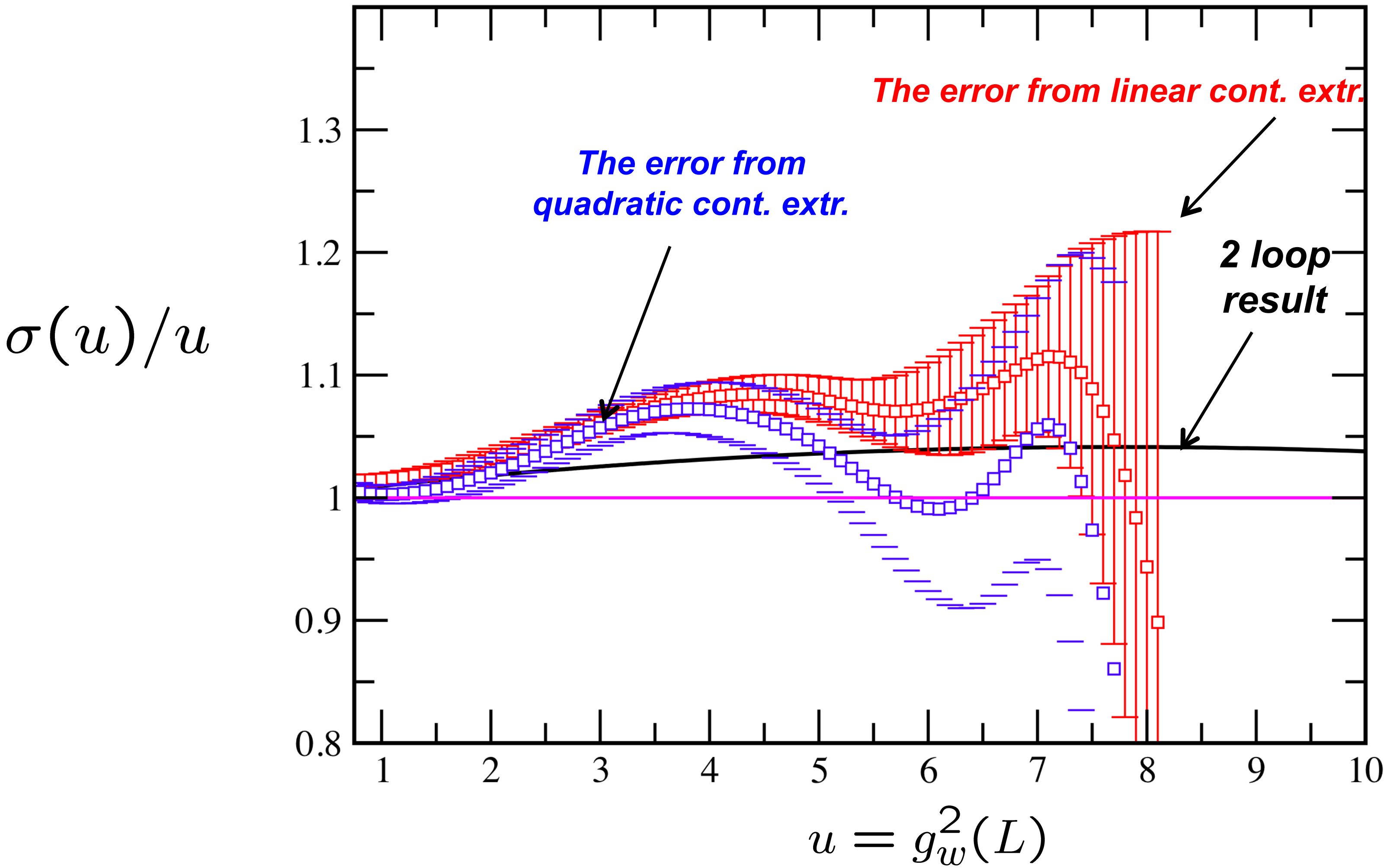


Strong coupling region (g^2=5.5)

Linear (3pt.) and quadratic (4pt.) extrapolations have large uncertainty

Stage3. The relative step scaling function

$$u = g_w^2(L) \qquad \sigma(u) = g_w^2(sL)$$



Summary

- We have calculated the running coupling in SU(2) 8 flavors
- IF behavior is different from QCD like theories
- We obtain consistent result with IRFP appear above g^2~6 but large uncertainty.
- We need more statistics for larger lattice and to take continuum limit carefully.

Future prospects

- We survey the running of the coupling in the large coupling region
- Measurement of the anomalous dimension at the IR point the composite operator of fermions is interesting (universality)