

Early Dark Energy in Type IIB String Theory

Matteo Licheri

University of Bologna
INFN, Bologna

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*Based on 2303.03414 with M. Cicoli, ML, R. Mahanta, E. McDonough,
F. G. Pedro, M. Scalisi*

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Hubble tension

5σ discrepancy between Planck¹ and SH0ES² inference of H_0 .
Planck measured $100\theta_s = 1.0411 \pm 0.0003$ with

$$\theta_s = \frac{r_s(z_*)}{D_A(z_*)}, \quad (1)$$

having defined

$$r_s(z_*) = \int_{z_*}^{z_{re}} \frac{dz}{H(z)} c_s(z), \quad D_A(z_*) = \int_0^{z_*} dz \frac{1}{H(z)}. \quad (2)$$

The 0.03% measurement of the angle θ_s can accommodate the $\approx 10\%$ increase in H_0 if there is a commensurate increase in $H(z \sim z_*)$.

¹Planck collaboration, 2018

²A. G. Riess et al., 2021

Early Dark Energy

Proposal: reduce the sound horizon with an ultralight scalar axion³ satisfying

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0 \quad (3)$$

where the best fitting potential is given by

EDE potential

$$\begin{aligned} V(\varphi) &= V_0 \left[1 - \cos\left(\frac{\varphi}{f}\right) \right]^3 \\ &= V_0 \left[\frac{5}{2} - \frac{15}{4} \cos\left(\frac{\varphi}{f}\right) + \frac{3}{2} \cos\left(\frac{2\varphi}{f}\right) - \frac{1}{4} \cos\left(\frac{3\varphi}{f}\right) \right]. \end{aligned} \quad (4)$$

with $V_0 = m^2 f^2$. This is such at the minimum $V \propto \phi^6$ and thus the energy density redshifts as $a^{-9/2}$.

³V. Poulin, T. L. Smith, T. Karwal and M. Kamionkowski, 2019

The parameter at which we are aiming are

- Recombination of electron and protons when $T \sim \text{eV}$ giving us $V_0 \sim \text{eV}^4$
- mass comparable to the Hubble scale at recombination $H \sim T^2/M_P$ giving $m \sim 10^{-27} \text{ eV}$ and from fit one singles out $f \sim 0.2M_P$ as the preferred value⁴

We will construct our models in the context of moduli stabilisation of [Type IIB String Theory](#).

⁴E. McDonough, M.-X. Lin, J. C. Hill, W. Hu and S. Zhou, 2022

Useful equations

Given the superpotential W and the Kähler potential K , in the SUGRA approximation of Type IIB we compute the scalar potential as

$$V = e^K \left(D_I W K^{I\bar{J}} D_{\bar{J}} \bar{W} - 3|W|^2 \right), \quad (5)$$

with $D_I W = W_I + W K_I$.

Then, the gravitino mass is given by

$$m_{3/2} = e^{K/2} |W|. \quad (6)$$

Chiral field T and nilpotent field X

$$W = W_0 + MX + Ae^{-aT}, \quad (7)$$

$$K = -3 \ln(T + \bar{T}) + 3 \frac{X\bar{X}}{T + \bar{T}}, \quad (8)$$

leading to

$$V_{\text{KKLT}} = \frac{\alpha^2 A^2 e^{-2aT}}{6T} + \frac{\alpha A^2 e^{-2aT}}{2T^2} - \frac{\alpha A |W_0| e^{-aT}}{2T^2} + \frac{M^2}{12T^2}. \quad (9)$$

Imposing a Minkowski minimum we find

$$\begin{aligned} |W_0| &= \frac{2}{3} A \alpha \tau e^{-\alpha \tau} \left(1 + \frac{5}{2\alpha \tau} \right), \\ M &= \sqrt{2\alpha} A e^{-\alpha \tau} \sqrt{\alpha \tau + 2}. \end{aligned} \tag{10}$$

Thus, the gravitino mass is

$$m_{3/2} = \frac{A \alpha}{3\sqrt{2\tau}} e^{-\alpha \tau}. \tag{11}$$

$$K = -2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) + \frac{\bar{X}X}{\mathcal{V}^{2/3}}, \quad (12)$$

having defined the volume as

$$\text{Swiss-Cheese CY:} \quad \mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2} \quad (13)$$

$$\text{Fibrated CY:} \quad \mathcal{V} = \sqrt{\tau_1 \tau_2} - \tau_s^{3/2} \quad (14)$$

with chiral fields $T_k = \tau_k + i\theta_k$ ($k = b, s$ or $k = 1, 2, s$ respectively). Moreover the superpotential is

$$W = W_0 + MX + \sum_k A_k e^{-a_k T_k}, \quad (15)$$

obtaining

$$V_{\text{LVS}} = \frac{8a_s^2 A_s^2 e^{-2a_s \tau_s} \sqrt{\tau_s}}{3\mathcal{V}} - \frac{4a_s A_s \tau_s |W_0| e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{3|W_0|^2 \hat{\xi}}{4\mathcal{V}^3} + \frac{M^2}{\mathcal{V}^{4/3}}.$$

The potential admits a global Minkowski minimum at

$$\mathcal{V} \simeq \frac{3|W_0|\sqrt{\tau_s}}{4a_s A_s} e^{a_s \tau_s} \simeq |W_0| e^{\frac{a_s}{g_s} \left(\frac{\xi}{2}\right)^{2/3}}, \quad (16)$$

$$\tau_s = \left(\frac{\xi}{2}\right)^{2/3} \frac{1}{g_s}, \quad (17)$$

$$M^2 = \frac{27}{20} \frac{|W_0|^2}{a_s} \frac{\sqrt{\tau_s}}{\mathcal{V}^{5/3}}. \quad (18)$$

Moreover, the gravitino mass turns out to be

$$m_{3/2} = \frac{|W_0|}{\mathcal{V}} \simeq e^{-\frac{a_s}{g_s} \left(\frac{\xi}{2}\right)^{2/3}}. \quad (19)$$

Odd axions

We define the 4-dimensional axion fields as,

$$b^a = \int_{\Sigma_a} B_2, \quad c^a = \int_{\Sigma_a} C_2, \quad \theta^\alpha = \int_{D_\alpha} C_4, \quad (20)$$

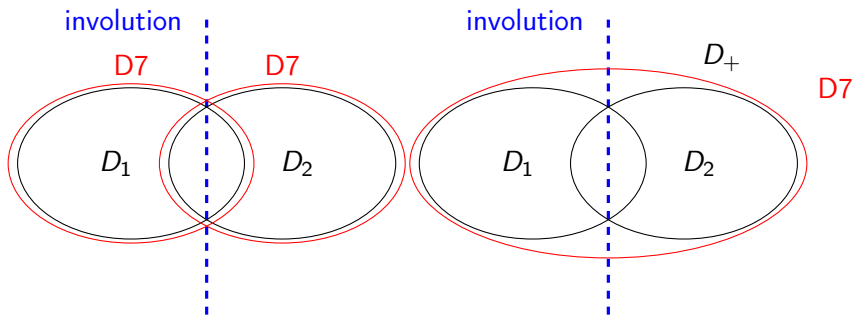
giving the chiral coordinates

$$G^a = \bar{S}b^a + ic^a = \frac{b^a}{g_s} + i(c^a - C_0 b^a), \quad \tau_\alpha = \frac{1}{2} k_{\alpha\beta\gamma} t^\beta t^\gamma, \quad (21)$$

$$T_\alpha = \tau_\alpha + i\theta_\alpha - \frac{1}{4} g_s k_{\alpha ab} G^a (G + \bar{G})^b. \quad (22)$$

In order to introduce the axions in the EFT we need
non-perturbative effects.

D7-branes



$$W_{D7} = A e^{-\frac{2\pi}{N} f_{D7}}, \quad (23)$$

where f_{D7} reads

$$\text{D7 on } D_1: \quad f_{D7} = T + k(f_+ + f_-) G + \frac{1}{2} (k f_-^2 + \tilde{k} f_+^2 + 2k f_+ f_-) \bar{S},$$

$$\text{D7 on } D_+: \quad f_{D7} = T + k f_- G + \frac{1}{2} (k f_-^2 + \tilde{k} f_+^2) \bar{S}.$$

ED1-instantons and gaugino condensation on D5-branes

We propose for ED1 instantons

$$\mathcal{K}_{\text{ED1}} = -3 \ln \left(\text{Re}(T) - \frac{2\gamma}{g_s^2} b^2 + \dots + \sum_{\substack{n \in \mathbb{N} \\ \hat{f}_- \in \mathbb{Z}}} A_{n, \hat{f}_-} e^{-2\pi n \left(\frac{t}{\sqrt{g_s}} + k \hat{f}_- G \right)} \right), \quad (24)$$

with $t = \sqrt{\frac{2}{k} \left(\text{Re}(T) - \frac{2\gamma}{g_s^2} b^2 \right)}$.

Furthermore, we propose for gaugino condensation on D5-branes

$$\mathcal{K}_{\text{D5}} = -3 \ln \left(\text{Re}(T) - \frac{2\gamma}{g_s^2} b^2 + \dots + \sum_{i=1}^P A_i e^{-\frac{2\pi}{N_i} \left(\frac{t}{\sqrt{g_s}} + k f_i G \right)} \right). \quad (25)$$

EDE in KKLT

We need three contributions to construct the $(1 - \cos)^3$ potential

$$K = -3 \ln [T + \bar{T} - \gamma(G + \bar{G})^2] + 3 \frac{\bar{X}X}{T + \bar{T}}, \quad (26)$$

$$W = W_0 + MX + A e^{-\alpha T} + \sum_{n=1}^3 A_n e^{-\tilde{\alpha}(T + nk_f G)}, \quad (27)$$

with $\alpha = \frac{2\pi}{N} < \tilde{\alpha} = \frac{2\pi}{M} \Leftrightarrow M < N$.

Computing we reproduce the EDE potential with the field defined as

$$\varphi = \sqrt{\frac{6\gamma}{\tau}} c, \quad (28)$$

with $\gamma = -\frac{1}{4}g_s k$, and having defined $A_1 = 15 \tilde{A}/4$, $A_2 = -3 \tilde{A}/2$ and $A_3 = \tilde{A}/4$.

The important quantities are

EDE in KKLT with C_2 axion

$$f = \sqrt{\frac{6\gamma}{\tau}} \frac{1}{\tilde{a}|k|f} = \sqrt{\frac{3g_s}{2|k|}} \frac{1}{\tau \tilde{a}f}, \quad (29)$$

$$V_0 = \frac{N\tilde{A}}{\sqrt{2}\tau^{3/2}} \left(\frac{2}{M} - \frac{3}{N} \right) \left(\frac{m_{3/2}}{M_P} \right) e^{-\frac{3}{4\pi} \frac{g_s M}{|k|f^2} \left(\frac{M_P}{f} \right)^2} M_P^4. \quad (30)$$

Imposing $f = 0.2M_P$, $V_0 \sim eV^4$ and $m_{3/2} > \text{TeV}$

g_s	\tilde{A}	M	N	τ	$V_0 10^{108} M_P^{-4}$	$m_{3/2}$ (TeV)	m_τ (TeV)
0.1	1	340	3200	10980.7	1.4	4.6	155.5
0.3	1	114	1000	3703.4	2.0	4.6	155.2
0.3	10^{-11}	100	750	2849.7	1.6	3.8	129.6
0.3	10^{-27}	80	470	1823.8	0.9	4.6	154.8
0.3	10^{-61}	36	85	369.3	2.2	3.0	104.0

Ranks of the condensing gauge group are too high.

EDE in LVS: C₄ axion

Taking a Swiss-Cheese CY we have

$$K = -2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) + \frac{\bar{X}X}{\mathcal{V}^{2/3}}, \quad (31)$$

$$W = W_0 + MX + A_s e^{-\alpha_s T_s} + \sum_{n=1}^3 A_n e^{-n\alpha T_b}, \quad (32)$$

with $\alpha_s = 2\pi/N_s$ and $\alpha = 2\pi/N_b$.

We reproduce the EDE potential with the field

$$\varphi = \sqrt{\frac{3}{2}} \frac{\theta_b}{\tau_b}, \quad (33)$$

and introduced $A_1 = 15 A_b/4$, $A_2 = -3 A_b/2$ and $A_3 = A_b/4$.

EDE in LVS with C₄ axion

$$f = \sqrt{\frac{3}{2}} \frac{1}{\alpha_b \tau_b} \simeq 0.2 \frac{N_b}{\tau_b} \quad \text{for} \quad \alpha_b = \frac{2\pi}{N_b}, \quad (34)$$

$$V_0 = \frac{64\pi^3}{3} \left(\frac{|W_0| A_b}{N_b^3} \right) \left(\frac{f}{M_P} \right)^2 M_P^4 e^{-\sqrt{\frac{3}{2}} \frac{M_P}{f}}. \quad (35)$$

This model requires an **heavy fine tuning**

N_b	N_s	τ_b	$ W_0 $	A_b	A_s	$V_0 10^{108} M_P^{-4}$
100	3	97.5	6.0×10^{-10}	5×10^{-92}	0.29	1.8
1000	4	974.6	1.1×10^{-7}	2×10^{-91}	0.28	1.3

Similar (fine-tuned) results for a fibrated CY.

EDE in LVS: C₂ axion

First model: gaugino condensation on D7-branes (Swiss-Cheese CY)

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2} = \frac{1}{2\sqrt{2}} \left[(T_b + \bar{T}_b - \gamma(G + \bar{G})^2)^{3/2} - (T_s + \bar{T}_s)^{3/2} \right],$$

with

$$K = -2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) + \frac{\bar{X}X}{\mathcal{V}^{2/3}}, \quad (36)$$

$$W = W_{\text{LVS}} + \sum_{n=1}^3 A_n e^{-\tilde{\alpha}(T_b + nkfG)}. \quad (37)$$

We reproduce the EDE potential with the field

$$\varphi = \sqrt{\frac{6\gamma}{\tau_b}} c. \quad (38)$$

EDE in LVS with C₂ axion: D7 gaugino condensation

$$f = \frac{1}{\tilde{\alpha}|k|f} \sqrt{\frac{6\gamma}{\tau_b}} = \sqrt{\frac{3g_s}{8\pi^2|k|f^2}} \frac{M}{\sqrt{\tau_b}} \quad \text{for} \quad \tilde{\alpha} = \frac{2\pi}{M}, \quad (39)$$

$$V_0 = \frac{16(2\pi)^5 |k|^2 f^4}{9g_s^2 M^5} |W_0| \tilde{A} \left(\frac{f}{M_P} \right)^4 e^{-\frac{3}{4\pi} \frac{g_s M}{|k|f^2} \left(\frac{M_P}{f} \right)^2} M_P^4. \quad (40)$$

Setting $\tilde{A} = 1$ we get

g_s	N_s	M	$ W_0 $	$\mathcal{V} = \tau_b^{3/2}$	$V_0 10^{108} M_P^{-4}$	A_s	m_ν (TeV)
0.3	1	128	1	3.2×10^5	2.6	1.70	3.2×10^7
0.3	1/2	121	2.8×10^{-6}	2.7×10^5	2.7	1.51	50
0.1	2	362	3.3×10^{-5}	1.4×10^6	2.2	2.96	50

Similar results for a fibred CY.

Second model: gaugino condensation on D5-branes.

One takes standard Swiss-Cheese LVS with substitution

$$\tau_b \rightarrow \tau_b - \gamma (G + \bar{G})^2 + e^{-\tilde{\alpha} t_b / \sqrt{g_s}} \sum_{n=1}^3 A_n \operatorname{Re} [e^{-n\tilde{\alpha} k_f G}], \quad (41)$$

reproducing again the EDE potential with axion field

$$\varphi = \sqrt{\frac{6\gamma}{\tau_b}} c. \quad (42)$$

EDE in LVS with C₂ axion: D5 gaugino condensation

$$f = \frac{1}{\tilde{\alpha} f} \sqrt{\frac{3g_s}{2|k|\tau_b}} = \frac{1}{f} \sqrt{\frac{3g_s}{8\pi^2|k|}} \frac{M}{\sqrt{\tau_b}}, \quad (43)$$

$$V_0 = \frac{3\tilde{A}|W_0|^2\tilde{\alpha}^2}{2g_s^2\mathcal{V}^2} e^{-\frac{1}{f} \sqrt{\frac{3}{|k|}} \left(\frac{M_P}{f}\right)} M_P^4. \quad (44)$$

Imposing the right values of f and V_0 we find the necessity of an **exponentially small fine tuning** of \tilde{A} .

Conclusions

Best model:

LVS with C_2 axion from D7 gaugino condensation

Achievements

- Controlled de Sitter moduli stabilization
- Decoupling of non-EDE modes
- Absence of fine-tuning
- Explicit Calabi-Yau realization

Outlooks

- Gravitational Waves from coupling with gauge⁵
- Incorporate with Inflation and/or Dark Matter

⁵Z. J. Weiner, P. Adshead and J. T. Giblin,,2021

*Thanks for the
attention!*