Staging in Plasma Accelerators

International School of Particle Accelerators – ERICE 2023

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U.S. DEPARTMENT OF ENERGY

Outline

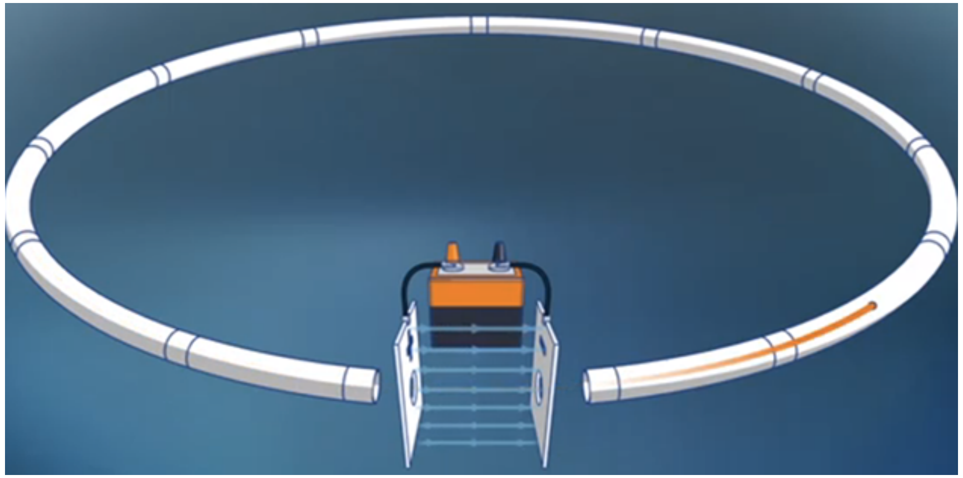
- Why is staging important?
- Challenges and things to consider between stages
- Elements of a complete solution so far considered
- Benefits of having multiple stages
- Staging demonstrations
- Conclusions & Outlook

Note – although this is a school, to cover the topic in one hour, there will not be many derivations. Rather I will highlight topics, give examples of scalings and supply plenty of references to help you get more information and dig deeper

Why Aren't Electrons Accelerated in Circular Machines?

- High energy (multi-GeV) electron beams have many applications in HEP (Colliders) and Photon Science (X-ray Lasers)

So why don't we just make all accelerators circular?



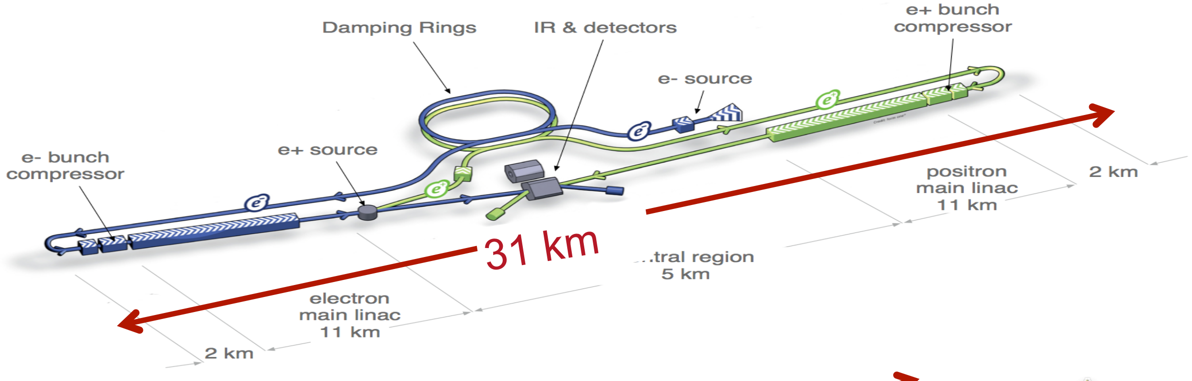
- A charged particle emits radiation when accelerated.
- **The good**: allows devices like synchrotron light sources and free electron lasers to work, and can be used to cool beams to make them brighter
- **The bad**: radiating can degrade the beam (especially coherent radiation)
- **The ugly**: power lost per revolution in a circular machine

$$P_\gamma = \frac{1}{6\pi\varepsilon_0}\frac{q^2 a^2}{c^3}\gamma^4 \approx \frac{\gamma^4}{m^2}$$

→ low-mass electrons radiate too much!

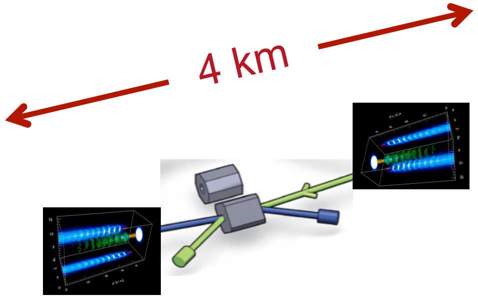
The Scale for a TeV Linear Collider

Today's technology LC – a 31km tunnel:



Plasma Wakefield Technology LC:

➔ GeV/m (TeV/km) accelerating gradient



The Luminosity Challenge:

➔ High-efficiency

$$\mathcal{L} = \frac{P_b}{E_b}\left(\frac{N}{4\pi\sigma_x\sigma_y}\right)$$

…and must do it for positrons too!

The Electron Beam Driven Plasma Wakefield Accelerator

VOLUME 43, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1979

Laser Electron Accelerator

T. Tajima and J. M. Dawson
Department of Physics, University of California, Los Angeles, California 90024
(Received 9 March 1979)

VOLUME 54, NUMBER 7 PHYSICAL REVIEW LETTERS 18 FEBRUARY 1985

Acceleration of Electrons by the Interaction of a Bunched Electron Beam with a Plasma

Pisin Chen[a]
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

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Department of Physics, University of California, Los Angeles, California 90024
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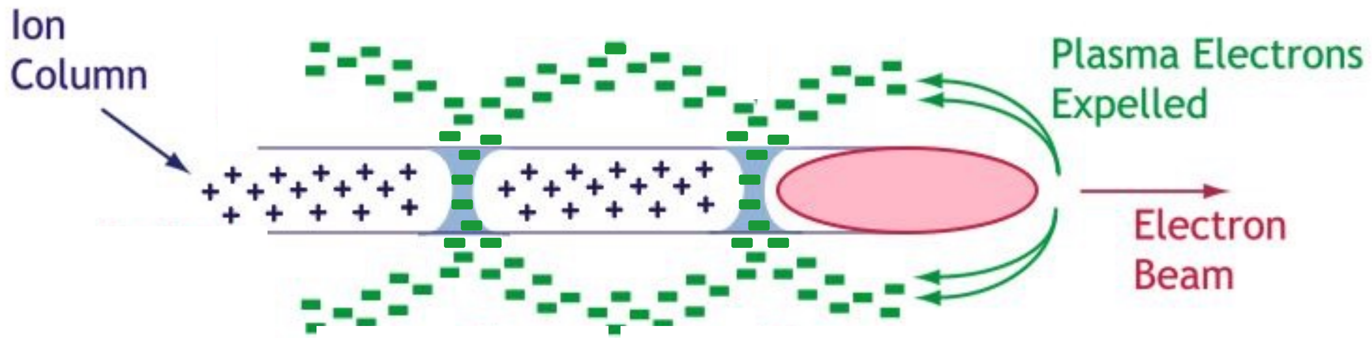

- Blow-out when $n_b >> n_p$
- Large accelerating gradients ~ GeV/m
- Strong ideal focusing ~ MT/m
- Relativistic driver, no de-phasing

$$E_0 \sim 10\sqrt{\frac{n_0}{1\times 10^{16}[\mathrm{cm}^{-3}]}}[\mathrm{GeV/m}]$$

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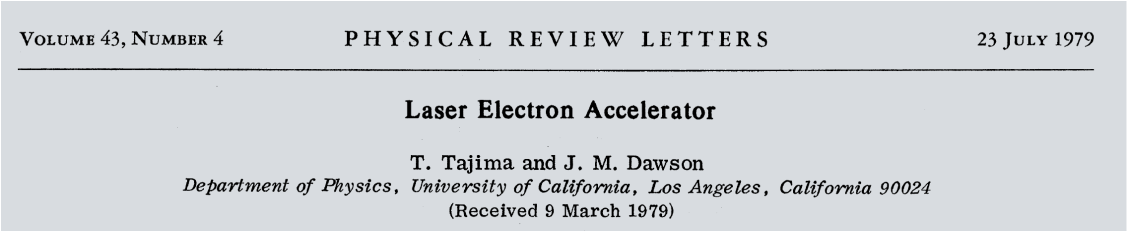
Accelerating Particles to Accelerating Beams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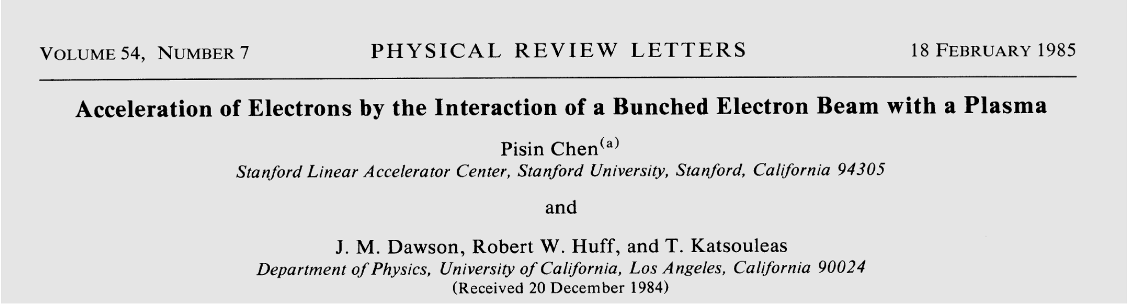
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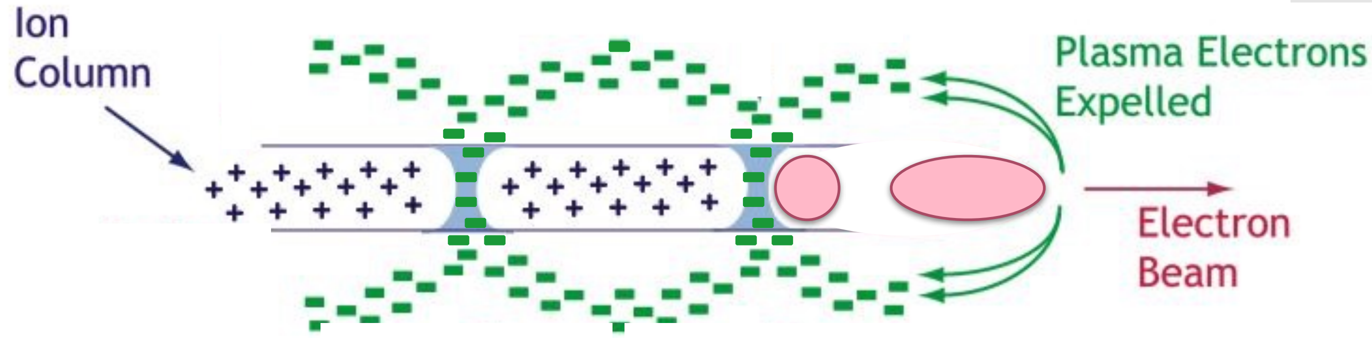


- Two bunches externally injected
- Dimensions and spacing ~ c/w_p ~ 20µm
- Blow-out when $n_b >> n_p$
- Plasma = highly efficient transformer

How much energy do we need?

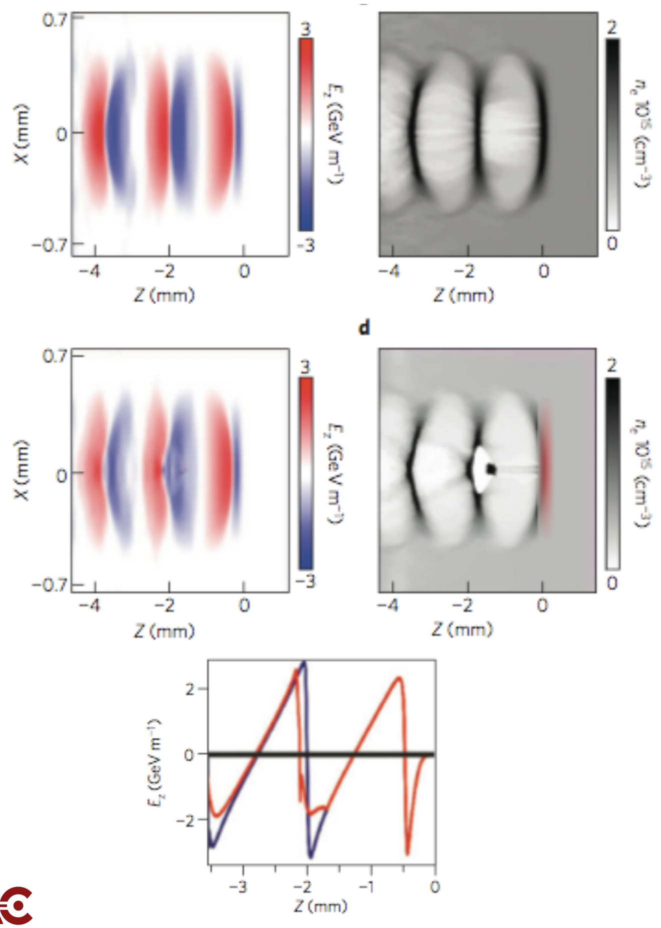
- 1nC * 1GeV = 1J
- For Higgs ~ 1nC * 250GeV = 250J @ 40kHz for 10MW beam power for Luminosity
- What if XFEL as driver? ~ 1nC * 10GeV = 10J/bunch so need 25 stages
- *Note: SPS/LHC have 20kJ/300kJ/bunch but long bunches at low rep rate (see AWAKE)*

AWAKE Collaboration is Studying Proton Driven PWFA

nature physics
ARTICLES
PUBLISHED ONLINE: 12 APRIL 2009; CORRECTED ONLINE: 24 APRIL 2009 | DOI: 10.1038/NPHYS1248

Proton-driven plasma-wakefield acceleration

Allen Caldwell[1]★, Konstantin Lotov[2,3], Alexander Pukhov[4] and Frank Simon[1,5]



Idea to Harness the Large Stored Energy in Proton Bunches to make High Energy Electrons

Goals of the AWAKE Collaboration:

- >500 GeV e- in single long plasma cell (400m)!
- Requires short proton bunches (100µm vs 10 cm)
- Study physics of self-modulation of long p bunches
- Probe wakefields with externally injected e-
- Study injection dynamics for multi-GeV e-
- Develop long, scalable and uniform plasma cells
- Develop schemes for production and acceleration of short p bunches

HIgh energy…but low rep rate (Luminosity)

For context - what might a plasma based collider look like?

One of the earliest examples:
"Towards a Plasma Wake-field Acceleration-based Linear Collider", J.B. Rosenzweig, et al., Nuclear Instruments and Methods A 410 532 (1998).

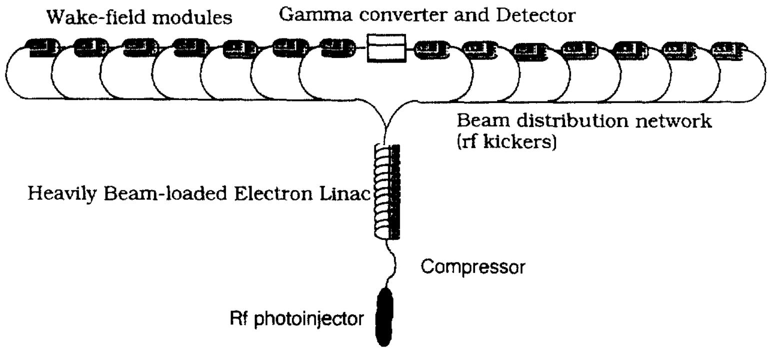


Fig. 6. Schematic of a γ–γ collider using a hardware transformer scheme. A large number of bunches are created in h
linac fed by an RF photoinjector followed by a compressor. Seperate wake modules are driven by the beams, whi
a binary RF spliting scheme.

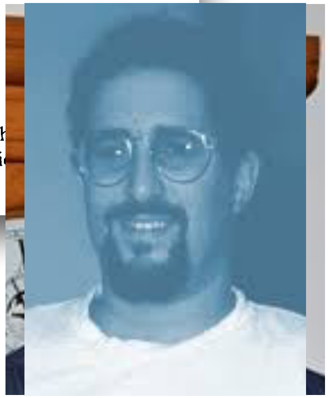


Table 1
Nominal drive beam and accelerating module parameters for the plasma wake-field accelerator-based collider shown in Fig. 4

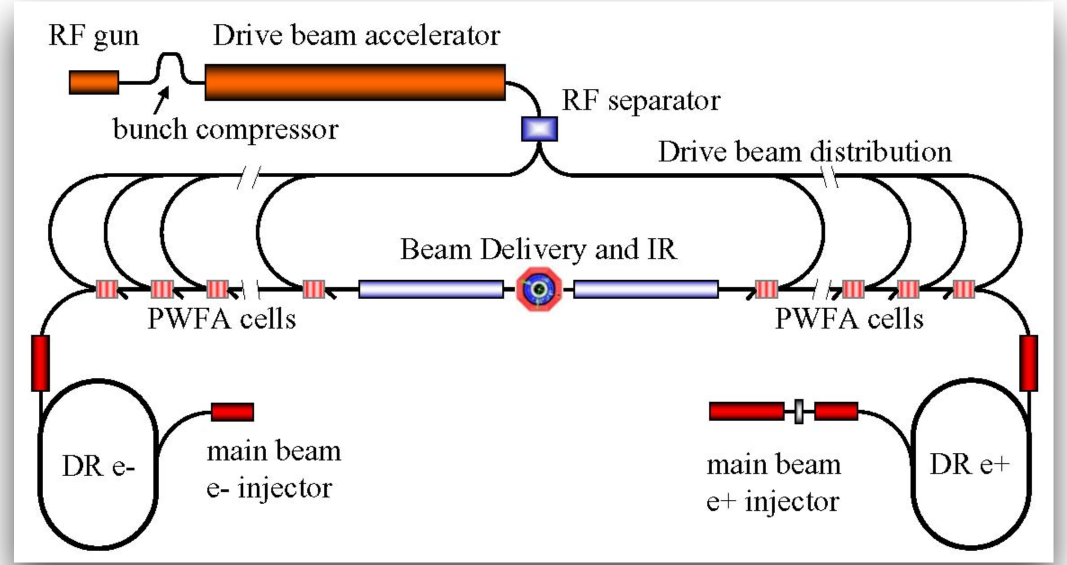
	L-band case	S-band case
Beam energy	3 GeV	3 GeV
Beam charge	20 nC	9 nC
Stored energy/bunch	60 J	27 J
Bunch length	0.8 mm	0.36 mm
Norm.emittance	50 mm mrad	23 mm mrad
Plasma density	2×10^{14} cm^{-3}	10^{15} cm^{-3}
Plasma wavelength	2.2 mm	1 mm
Deceleration wake	500 MeV/m	1.1 GeV/m
Accelerating wake	1 GeV/m	2.2 GeV/m
Wake module length	5.7 m	2.6 m
Intermodule drift	2.66 m	1.21 m

First SLAC Concept Developed with FACET Proposal < 2009

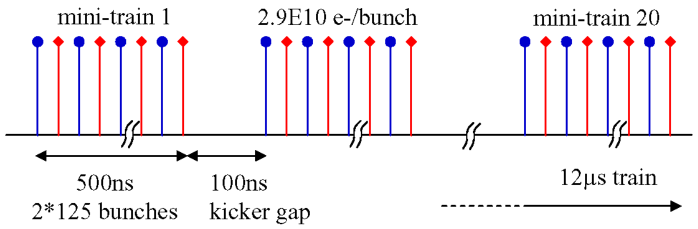
SLAC-PUB-13766

A CONCEPT OF PLASMA WAKE FIELD ACCELERATION LINEAR COLLIDER (PWFA-LC)*

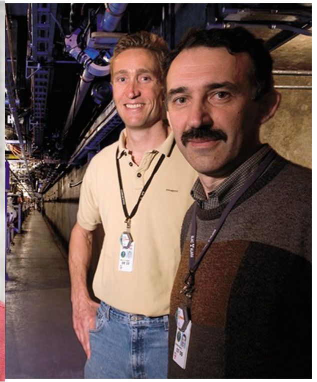
Andrei Seryi, Mark Hogan, Shilun Pei, Tor Raubenheimer, Peter Tenenbaum (SLAC), Tom Katsouleas (Duke University), Chengkun Huang, Chan Joshi, Warren Mori (UCLA, California), Patric Muggli (USC, California).



- 'Warm' Drive Linac
- 4ns bunch spacing
- Many turnarounds



Main beam: bunch population, bunches per train, rate	1×10^{10}, 125, 100 Hz
Total power of two main beams	20 MW
Drive beam: energy, peak current and active pulse length	25 GeV, 2.3 A, 10 μs
Average power of the drive beam	58 MW
Plasma density, accelerating gradient and plasma cell length	1×10^{17} cm^{-3}, 25 GV/m, 1 m
Power transfer efficiency drive beam=>plasma =>main beam	35%
Efficiency: Wall plug=>RF=>drive beam	50% × 90% = 45%
Overall efficiency and wall plug power for acceleration	15.7%, 127 MW
Site power estimate (with 40MW for other subsystems)	170 MW
Main beam emittances, x, y	2, 0.05 mm-mrad
Main beam sizes at Interaction Point, x, y, z	0.14, 0.0032, 10 μm
Luminosity	3.5×10^{34} cm^{-2}s^{-1}
Luminosity in 1% of energy	1.3×10^{34} cm^{-2}s^{-1}



Alternative Conceptual Layout for TeV PWFA Linear Collider Developed prior to CSS2013

- Efficient drive beam generation from recirculating superconducting linacs
- Rapid & Efficient acceleration in meter long plasma cells
- Illustrates R&D challenges for next decade: beam quality, positrons, staging

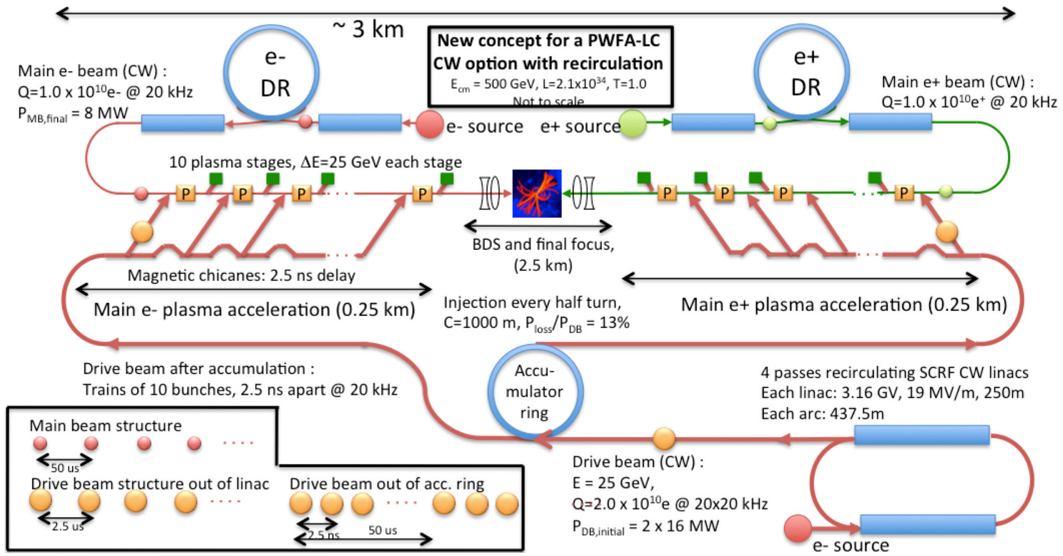


Figure 1: Layout of a 500 GeV PWFA Linear Collider. Each main bunch is accelerated by 25 GeV in each of ten plasma stages. The plasma is driven by e^- bunches, generated by a SCRF CW recirculating linac, and distributed co-linearly with the main beams.

- 'Cold' Drive Linac
- 100µs bunch spacing
- Tricky delay chicanes

E. Adli et al., Proceedings of IPAC2013
E. Adli et al., ArXiv 1308.1145
J. P. Delahaye et al., Proceedings of IPAC2014

Table 1: PWFA-LC parameters for 500 and 3,000 GeV. Parameters are also available for 250 and 1,000 GeV [9].

Main parameters		
E_{CM} [GeV]	500	3,000
Effective gradient [MV/m]	1,000	1,000
Number of bunches [1×10^{10}]	1	1
Bunch spacing (CW) [μs]	50	100
Main beam power per beam [MW]	8	24
Linac length [km]	0.25	1.5
Overall facility length [km]	3	8
IP parameters		
σ_x [μm]	0.47	0.19
σ_y [nm]	2.7	1.1
β_x [cm]	1.1	1.1
β_y [cm]	0.01	0.01
σ_z [μm]	20	20
Total L [10^{34}/cm$^2/s$]	2.1	6.3
$L_{1\%}$ [10^{34}/cm$^2/s$]	1.3	3.8
Efficiency and power		
Drive to main bunch efficiency [%]	50	50
# of plasma stages per linac	10	60
Drive linac bunch rep. freq. [kHz]	400	1200
Drive beam power per beam [MW]	16.2	48.6
Total wall plug power [MW]	150	297
Beam acceleration efficiency [%]	21	23
Wall plug to main beam efficiency [%]	11	16

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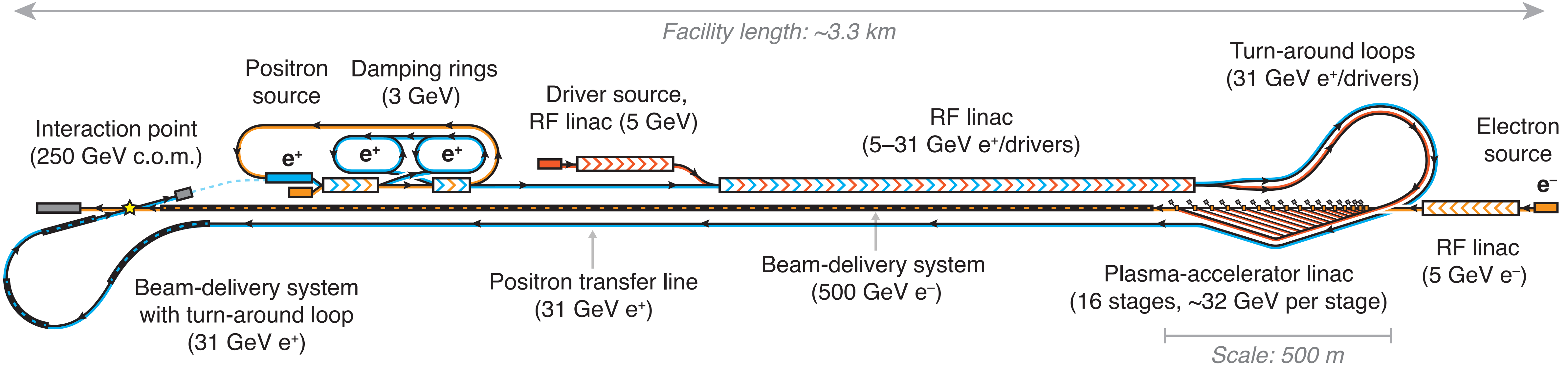
HALHF

A hybrid, asymmetric, linear Higgs factory based on plasma-wakefield and radio-frequency acceleration





https://arxiv.org/abs/2303.10150

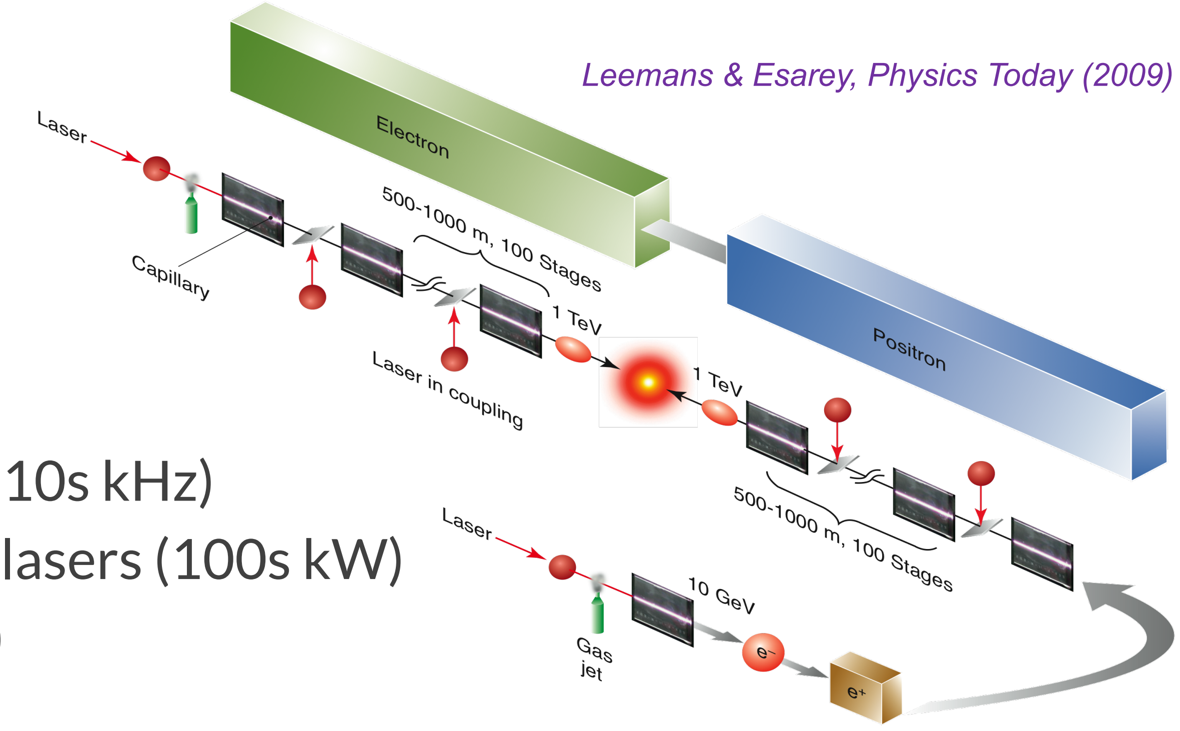
Laser-plasma collider concept

Basic concept: Staged laser-plasma accelerators:

- Plasma density scalings indicates operation at n~10^{17} cm^{-3} [high average gradient and low wall plug power]
- Quasi-linear regime (a~1): e+ and e- focusing and acceleration; focusing control
- Staging & laser coupling into plasma channels (for laser guiding):
  - Tens of J laser/energy per stage
  - Energy gain/stage ~ few GeV in < 1m

C. B. Schroeder et al., PR ST-AB (2010)
C. B. Schroeder et al., NIMA (2016)

Leemans & Esarey, Physics Today (2009)



Laser technology development required:

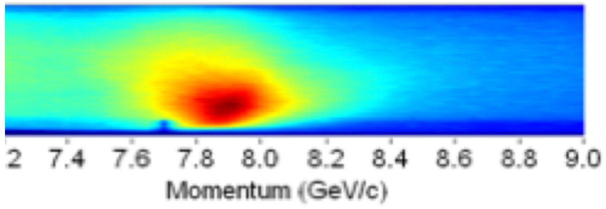
- High luminosity requires high rep-rate lasers (10s kHz)
- Requires development of high average power lasers (100s kW)
- High laser efficiency (~tens of % wall-to-laser)

Rapid Experimental Progress Since Last Snowmass

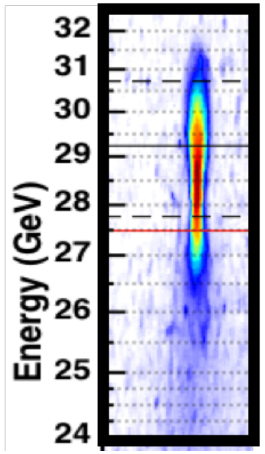
8 GeV **energy gain** in 20 cm stage using BELLA PW laser

9 GeV in 1.3 m using SLAC beam driver at FACET

New: >10 GeV from U. Texas laser

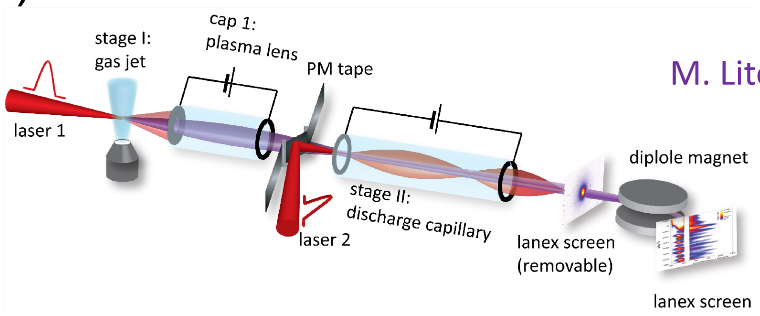


A. J. Gonsalves et al. PRL (2019)



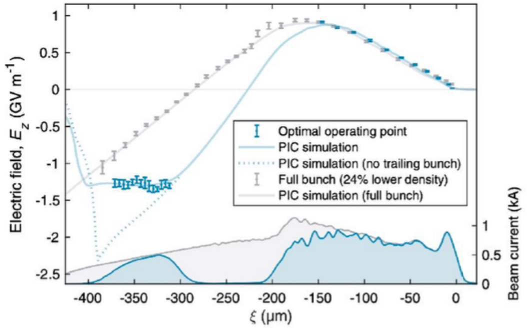
M. Litos et al. PPCF (2015)

Proof-of-principle **staging** (~100 MeV energy gain) using laser drivers, high gradient plasma-lenses



S. Steinke et al. Nature (2016)

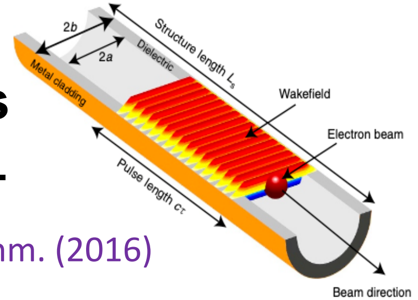
Optimized plasma beam loading enables uniform, **high-efficiency** acceleration.



42% transfer efficiency with 0.2% energy spread

C. A. Lindstrom et al. PRL (2021)

Demonstration >1**GeV/m gradients** dielectric structures.



B. O'Shea et al. Nature Comm. (2016)

Positron acceleration

A. Doche et al., Nature Sc. Rep. (2017)

Plasma recovery at high rep-rate

R. D'Arcy et al., Nature (2022)

Demonstration 0.5 **GW power** SWFA structures.

Driver Technology:

Superconducting XFELs, New laser technology (fibers, Thulium) promise high average power at high efficiency

Also: hollow channels for low emittance growth, 0.1 micron emittance

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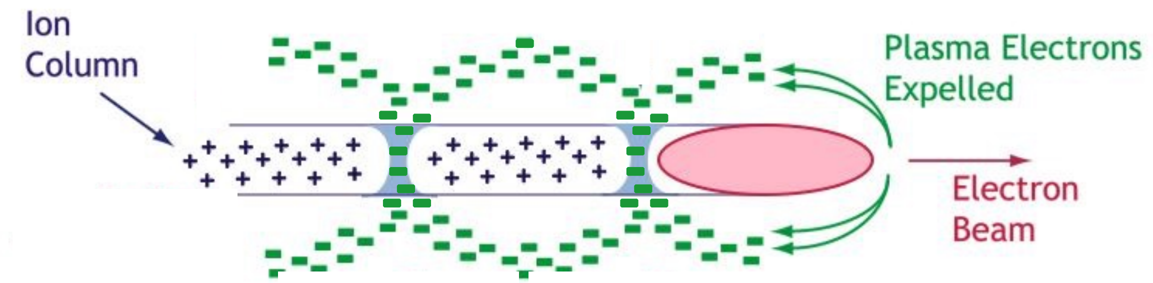
Transverse Forces: Focusing in the Ion Column

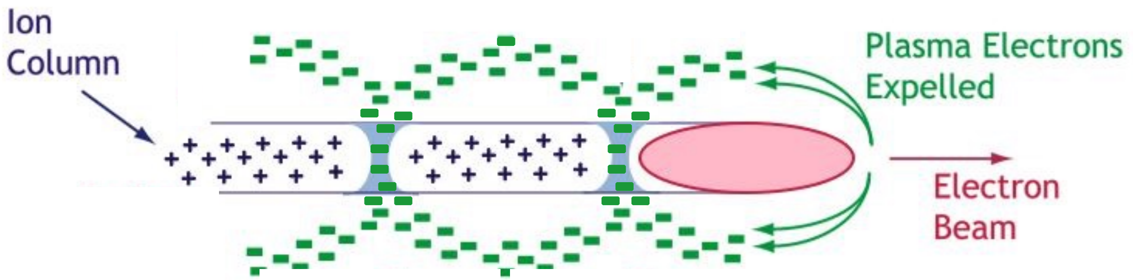


- Uniform ion density n_i = initial plasma density n_{e0}
- Focusing is balance between radial E and v x B ~ $E_r - cB_{phi}$
- Assume $n_b/n_p > 1$ and fully blown-out ion column
  - no plasma return currents within the beam (CFI)
  - In beam frame then no currents to drive B_{phi}
- Focusing then simply obtained from Gauss law for an infinite cylinder (approximation)

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad 2\pi r dz E_r = \frac{\pi r^2 e n_i}{\varepsilon_0} \quad \Rightarrow \quad E_r = \frac{1}{2}\frac{e n_{e0}}{\varepsilon_0} r$$

  - linear in r (ideal lens, no geometric aberration)
  - May preserve incoming emittance

Propagation in the Ion Column – Single Electron



$$E_r = \frac{1}{2}\frac{en_{e0}}{\varepsilon_0} r$$

- Motion of a single electron in the ion column:

$$\gamma m \frac{dv_\perp}{dt} = F_\perp \quad \Rightarrow \quad \gamma mc^2 \frac{d^2 r}{dz^2} = e\frac{1}{2}\frac{en_{e0}}{\varepsilon_0} r \quad \Rightarrow \quad \frac{d^2 r}{dz^2} = \frac{1}{2\gamma c^2}\frac{e^2 n_{e0}}{m\varepsilon_0} r = \frac{\omega_{pe}^2}{2\gamma c^2} r = \frac{k_{pe}^2}{2\gamma} r = k_\beta^2 r$$

- Harmonic motion as long as no energy gain or loss:

$$\frac{d^2 r}{dz^2} = k_\beta^2 r \quad \Rightarrow \quad r(z) = r_0 e^{ik_\beta z}$$

- Relativistic electrons though, so will get synchrotron (betatron) radiation

- Particles oscillate at: $k_\beta^2 = \frac{k_p^2}{2\gamma}$ or $\omega_\beta = \omega_{pe}/\sqrt{2\gamma} << \omega_{pe}$

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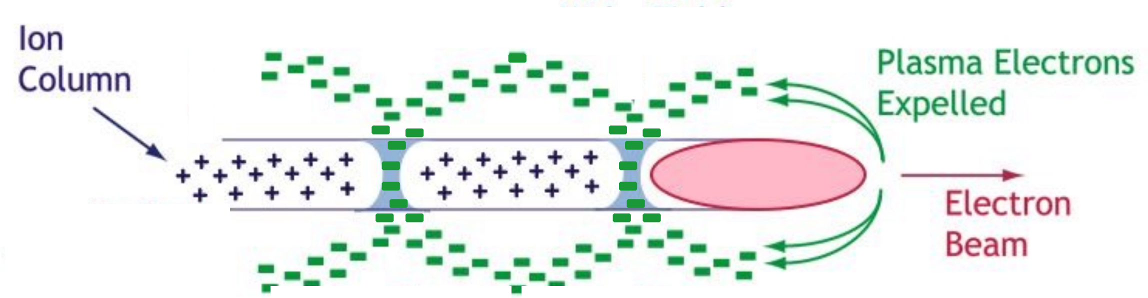
Propagation in the Ion Column for a Beam of Electrons



$$E_r = \frac{1}{2}\frac{en_{e0}}{\varepsilon_0}r$$

- Beam evolution described by the envelope equation:

$$\frac{d^2\sigma}{dz^2} + K\sigma = \frac{\varepsilon^2}{\sigma^3} \quad \text{with} \quad K = \frac{k_p^2}{2\gamma} = k_\beta^2$$

- No evolution of spot size (sigma) when have matched condition:

$$\frac{d^2\sigma}{dz^2} = 0 \Rightarrow K = \frac{\varepsilon^2}{\sigma^4} = \frac{1}{\beta^2} \quad \text{or} \quad \beta_{matched} = \frac{\sqrt{2\gamma}}{k_p} = \sqrt{2\gamma}\frac{c}{\omega_p} \qquad \text{recalling} \quad \sigma^2 = \beta\varepsilon$$

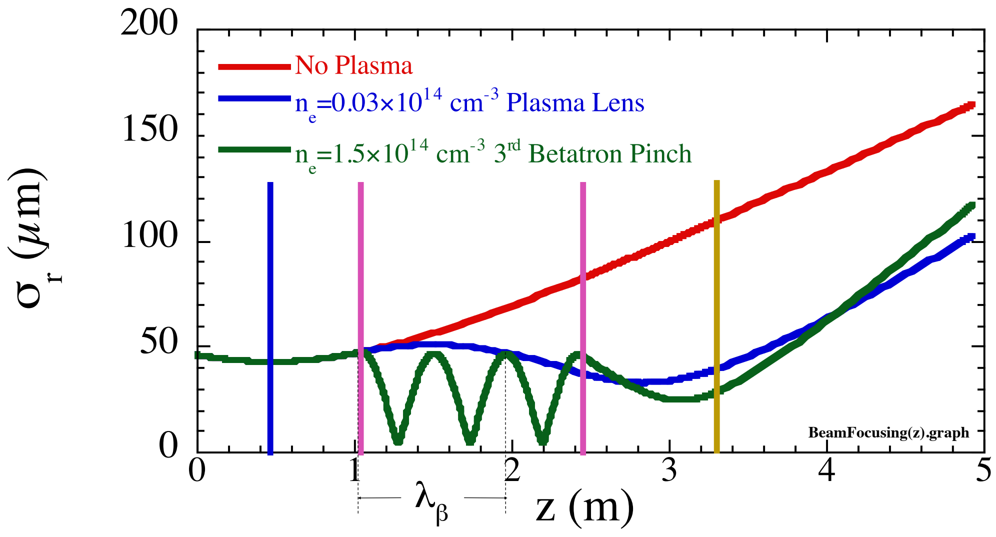
- There is a matched beta (n_p dependent) – not a matched spot size (ε_n dependent), e.g. $n_p = 10^{17}$, $c/w_p = 17\mu m$ and Beta matched = 1mm (<<L_p!). For $\varepsilon_n = 1\mu m$, E = 1GeV get a matched sigma = 0.7µm

Measured Plasma Focusing for Matched & Mismatched Beams

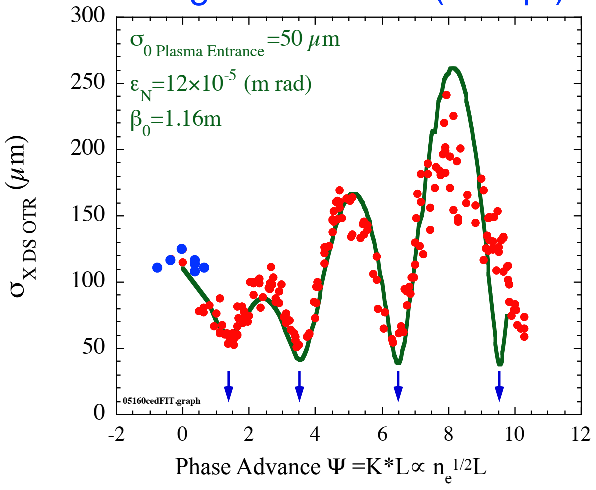
- Start with beam evolution in vacuum

$$\sigma_r(z) = \sigma_{r0}\left(1 + \frac{\varepsilon^2 z^2}{\sigma_0^4}\right)^{1/2} = \sigma_{r0}\left(1 + \frac{\varepsilon^2}{\beta_0^2}\right)^{1/2}$$

- Increase the density/focusing
  - Can't always measure in plasma
  - Look on profile monitor downstream
  - Sigma(z) at fixed np same as sigma(np) at fixed z

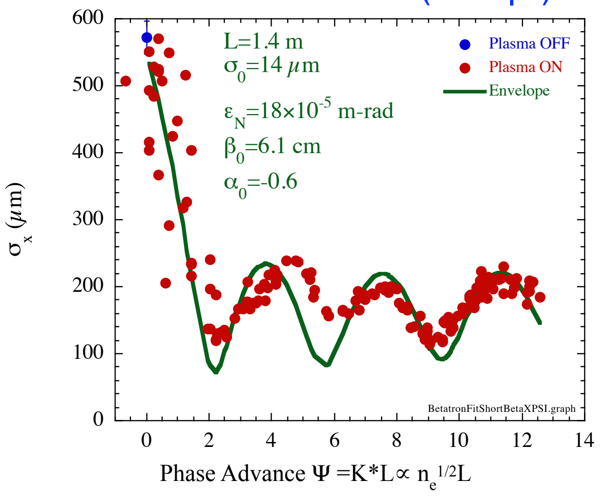


Large Beam Size ($K>1/\beta_0$)



Phys. Rev. Lett. **88**, 154801 (2002)

Small Beam Size ($K\leq 1/\beta_0$)



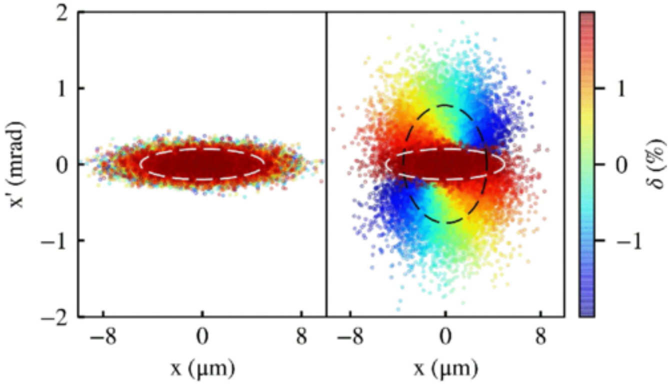
Phys. Rev. Lett. **93**, 014802 (2004)

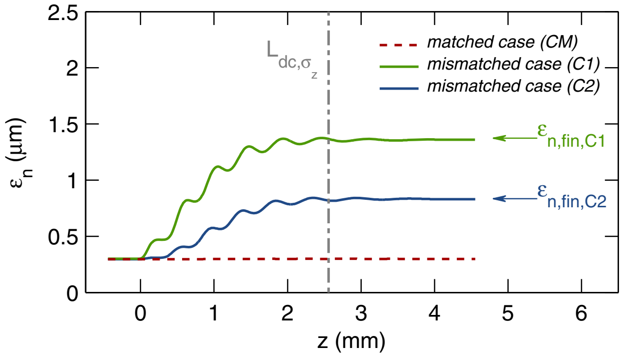
- Focusing orders of magnitude larger than beamline quadrupoles
- Well described by simple model
- Enables high density beam propagation over long distances

Matching and Emittance Preservation

- Need small emittance for Luminosity in a collider (as low as 0.01 mm-mrad) and/or for beam brightness in a free electron laser (0.1-1 mm-mrad)
- Need to limit emittance growth in and in between stages to less than above
- Mismatching bunches with a finite energy spread leads to emittance growth, because the phase space ellipses of different energy slices rotate at different rates
- Saturates after a distance L_{sat}
- Avoid emittance growth from mismatch by ensuring that $\beta = \beta_m$ and $\alpha = 0$

$$\mathcal{L} = H_D \frac{N^2 f \gamma}{4\pi \sqrt{\beta_x \epsilon_{nx}} \sqrt{\beta_y \epsilon_{ny}}}, \qquad B_{6D} = \frac{N}{\epsilon_{nx}\epsilon_{ny}\epsilon_{nz}},$$





$$\frac{\epsilon_{sat}}{\epsilon} = \frac{1}{2}\left((1+\alpha^2)\frac{\beta_m}{\beta} + \frac{\beta}{\beta_m}\right), \qquad L_{sat} \approx \beta_m/\sigma_\delta,$$

Complete decoherence occurs after $1/\sigma_\delta$ betatron oscillations. e.g. density 10^{17}/cc, 1 GeV, 3% energy spread, 5 mm beta, alpha −1; matched beta is 1 mm, resulting in a saturated emittance growth of 250% after a decoherence length of 0.2 m (33 betatron oscillations).

T. Mehrling, J. Grebenyuk, F. S. Tsung, K. Floettmann, and J. Osterhoff, Transverse emittance growth in staged laser-wakefield acceleration, Phys. Rev. Accel. Beams 15, 111303 (2012).

Chromaticity & Chromatic Amplitude

- Strong focusing in the plasma results in small beam sizes and highly diverging beams
- Difficult to capture and refocus without degrading the beam quality
- Different energy slices are not all focused in the same way – an effect known as chromaticity.
- Defined in terms of the chromatic amplitude which measures (to first order) the combined mismatch of the Twiss parameters α and β, for a relative energy offset δ = ΔE/E

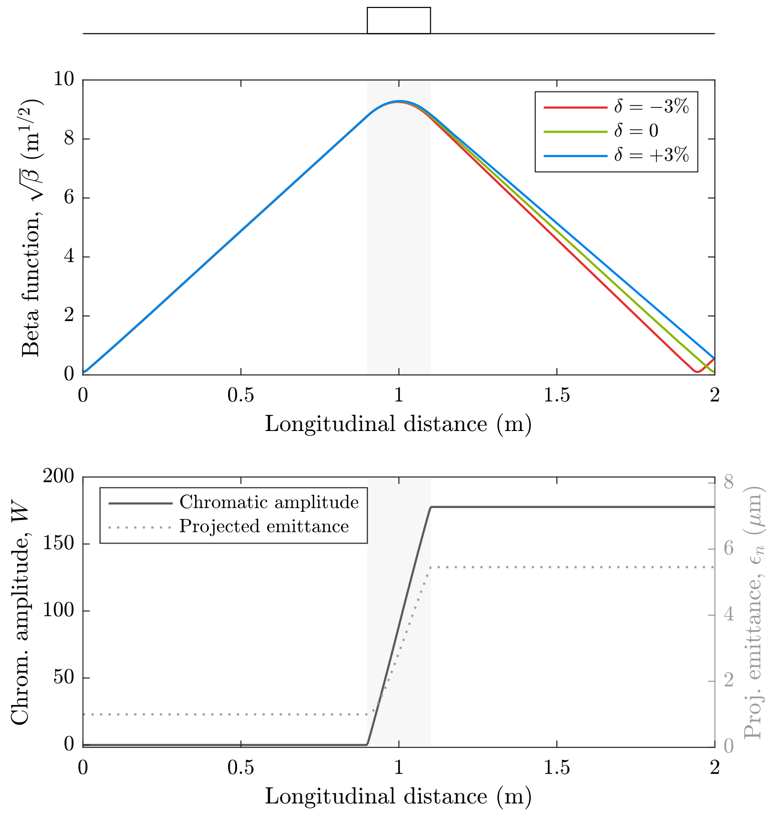


FIG. 3. Example of emittance growth due to chromaticity. A 10 GeV beam with 3% rms energy spread diverges from a plasma accelerator of density 10^{16} cm^{-3} ($\beta_m \approx 10$ mm). A simple beam optics lens captures and refocuses the beam into the next stage, which introduces significant chromaticity. As a result, the projected (energy-averaged) emittance increases by more than a factor of 5.

$$\xi = \frac{1}{2\pi}\frac{\partial \mu}{\partial \delta}$$

$$W = \sqrt{\left(\frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta}\frac{\partial \beta}{\partial \delta}\right)^2 + \left(\frac{1}{\beta}\frac{\partial \beta}{\partial \delta}\right)^2}$$

$$\frac{\Delta \epsilon^2}{\epsilon_0^2} = W^2 \sigma_\delta^2 + \mathcal{O}(\sigma_\delta^4)$$

$$W = \beta K l \qquad F = \frac{L}{2} = (Kl)^{-1}$$

$$\beta \approx \frac{L^2}{\beta_m} \qquad \frac{\Delta \epsilon^2}{\epsilon_0^2} \approx \frac{4L^2}{\beta_m^2}\sigma_\delta^2$$

10GeV, 10^{16}/cc with matched Beta 10mm, L = 1m, limit emittance growth to 1% then max dE/E = 0.07% so severe limit if uncorrected

Carl A. Lindstrøm, Staging of plasma-wakefield accelerators , Phys. Rev. Accel. Beams 24, 014801 (2021) .

SLAC

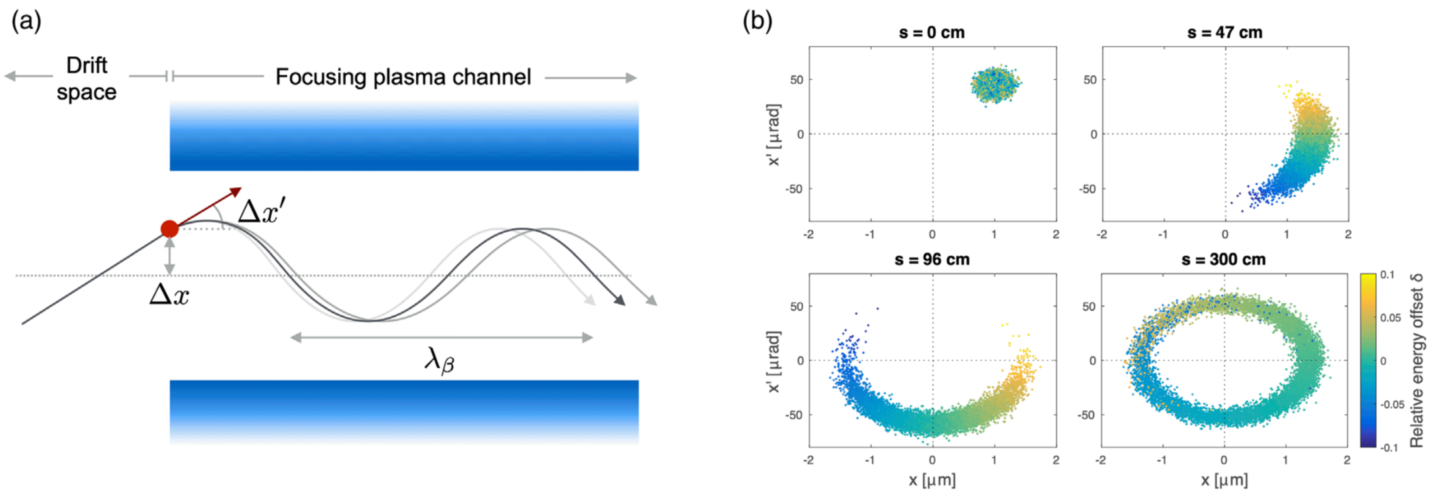
Transverse Misalignments

- The driver initiates the wake so the main beam need to overlap well in phase space
- Main beam should be aligned to the driver to a fraction of the transverse beam size and angle
- Challenging for small emittance and small betas
- Similarly to mismatch and dispersion leakage, finite energy spread beams rotate at different rates and smear out the phase space leading to projected emittance growth

$$\Delta\epsilon \approx \frac{1}{2}\left(\frac{\Delta x^2}{\beta_m} + \beta_m \Delta x'^2\right)$$

$$\Delta x \ll \sqrt{2\beta_m \epsilon} \qquad \Delta x' \ll \sqrt{2\epsilon/\beta_m}$$

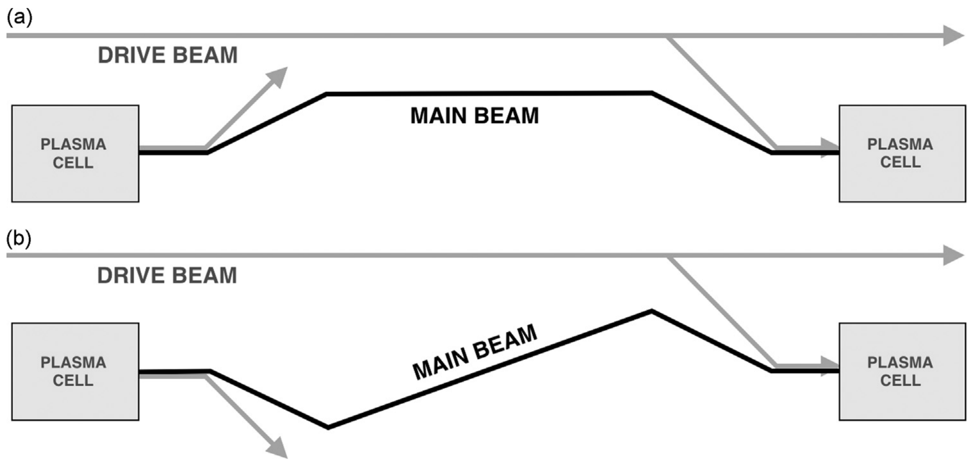
e.g. for densities with GeV/m gradients implies <100nm and <5μrad (will be true for laser drivers as well)



R. Assmann et al., Transverse beam dynamics in plasma-based linacs, Nucl. Instrum. Methods Phys. Res., Sect. A 410, 544 (1998).
S. Cheshkov et al., Particle dynamics in multistage wakefield collider, Phys. Rev. Accel. Beams 3, 071301 (2000).
D. Schulte, Application of advanced accelerator concepts for colliders, Rev. Accel. Sci. Techol. 09, 209 (2016).
C. A. Lindstrøm, E. Adli, J. Pfingstner, E. Marín, and D. Schulte, Transverse tolerances of a multi-stage plasma wakefield accelerator, in Proceedings of IPAC2016, Busan, Korea (JACoW, Geneva, 2016), p. 2561.

Drive Beam Injection and Extraction

- Extraction and injection optics should be symmetric (same optics but reverse order)
- Either mirror symmetric (C) or rotationally symmetric (S) chicane
- C: less total bending, less synchrotron radiation
- S: injection and extraction on opposite sides freeing up space for beam dumps and diverting radiation away from beam distribution and injection systems



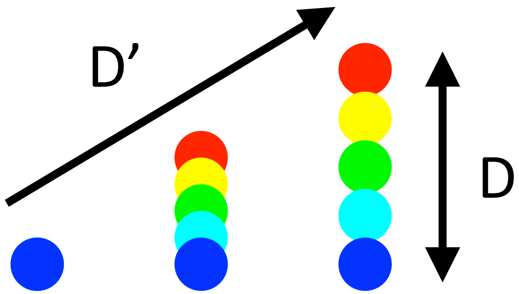
$$\Delta x = \frac{1}{2} l_d^2 Bce \left(\frac{1}{E_d} - \frac{1}{E_m} \right)$$

Some considerations:

- 10GeV drive, infinite witness, 1T, 60cm long dipole gives 1cm offset
- Dipole + septum combination?
- Kickers not fast enough (~1ns rise time)
- TCAV likely won't handle 100% energy spread
- Caution that need high power beam dumps

Dispersion Cancellation

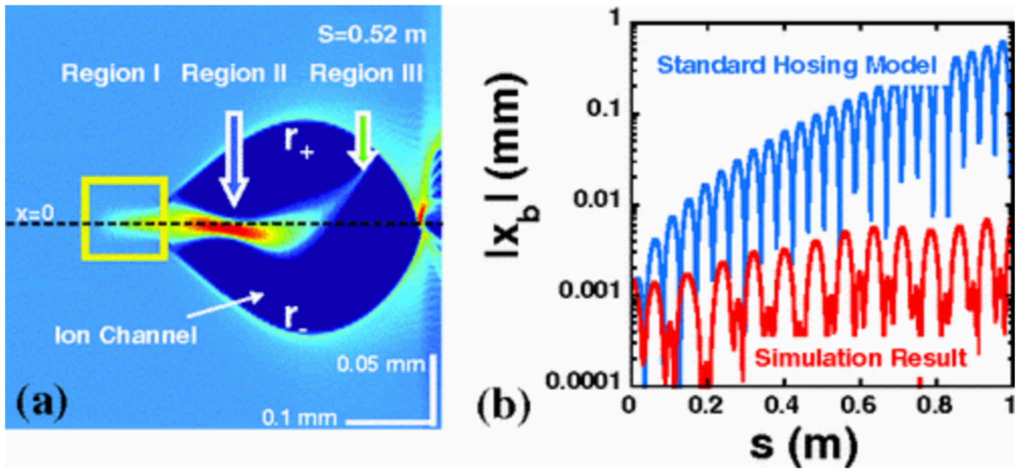
- Dipoles used for in/out coupling
- Gives a correlation between energy and position = dispersion that intentionally introduces separation between beams of different energy
- Will also disperse single beam with finite energy spread producing projected emittance growth



$$\Delta\epsilon_D \approx \frac{1}{2}\left(\frac{D_x^2}{\beta_m} + \beta_m D_{x'}^2\right)\sigma_\delta^2$$

$$D_x \ll \sqrt{2\epsilon\beta_m}/\sigma_\delta \text{ and } D_{x'} \ll \sqrt{2\epsilon/\beta_m}/\sigma_\delta,$$

Example: 1GeV, 1% dE/E, 1mm-mrad staged between two plasma cells of 10^{16}/cc requires cancellation of D and D' to better than 0.18mm and 55mrad – tough!



- Coupled with correlated energy spread can also lead to beam tilt and seed hosing

Synchrotron Radiation

- Separating drive and witness beams will likely be done with dipoles
- Synchrotron radiation becomes more important at higher energies
- Incoherent for long bunches (ISR) becoming partially coherent (CSR) as the bunches get shorter as with plasma accelerators
- Represents loss of efficiency, energy spread growth and when coupled with chicanes, intra-bunch correlations may act as seed for hosing

$$P_{\rm ISR} = \frac{e^4}{6\pi\epsilon_0 m^2 c} N\gamma^2 B^2$$

$$P_{\rm CSR} = NP_{\rm ISR} \sim N^2$$

$$\sigma_{\rm SR} = \rho/\gamma^3 = mc/Be\gamma^2$$

$$P_{\rm CSR} = \frac{\kappa e^2 c}{\epsilon_0} \frac{N^2}{\rho^{2/3}\sigma_z^{4/3}}$$

e.g. for 10GeV, 1nC charge, bunch length 10μm, 1T magnetic field radiation is partially coherent and will radiate 0.3% of its energy per meter of dipole

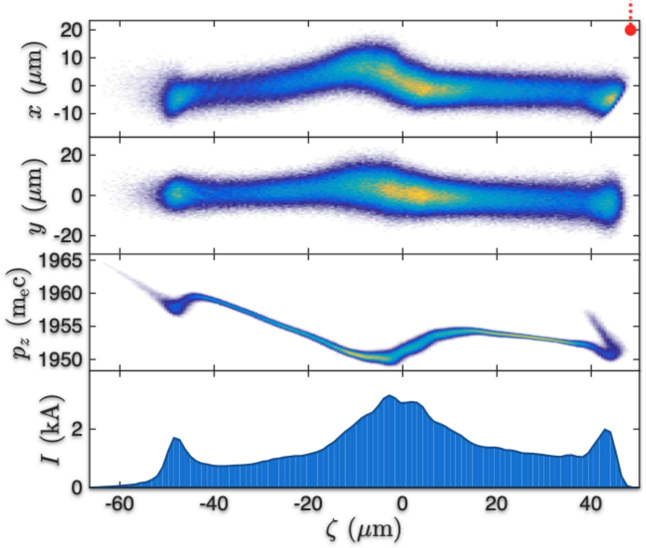


Image courtesy of J. Osterhoff

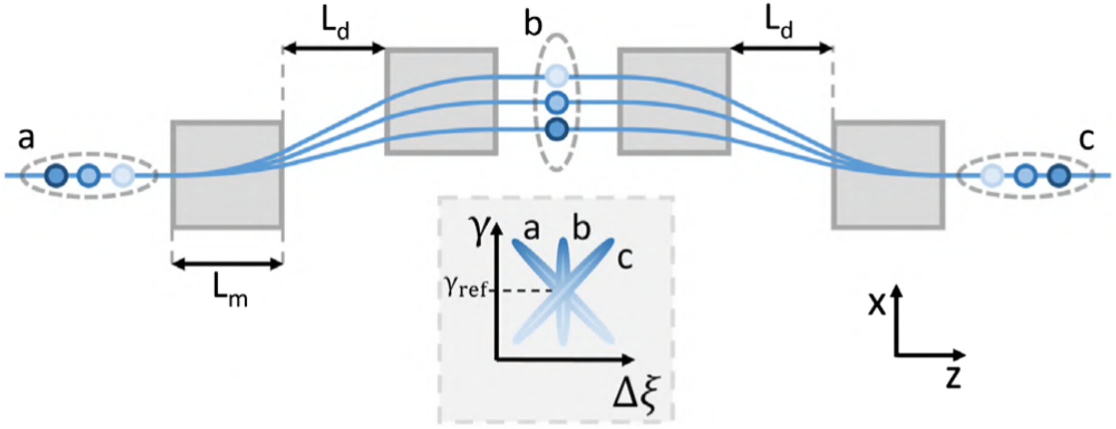
Y. Cai and Y. Ding, Three-dimensional effects of coherent synchrotron radiation by electrons in a bunch compressor, Phys. Rev. Accel. Beams 23, 014402 (2020).
Y. Jing and V. N. Litvinenko, Design of a bunch com- pressor with CSR suppression to achieve hundreds of kA peak current, in Proceedings of IPAC2019, p. 382.
S. Heifets, G. Stupakov, and S. Krinsky, Coherent synchrotron radiation instability in a bunch compressor, Phys. Rev. Accel. Beams 5, 064401 (2002).
A. D. Brynes et al., Characterisation of microbunching instability with 2D Fourier analysis, Sci. Rep. 10, 5059 (2020).

Isochronicity

- Dipole chicanes typically used to compress bunches needed for PWFA ($k_p * \sigma_z \sim 1$)
- If also use for injection/extraction, need to ensure they do not change the bunch length significantly
- Requirements might be relaxed by adjusting successive chicanes to compress, over compress, compress to compensate for imperfect beam loading and reduce energy spread
- For low energy, short beams with large divergence (LWFA), different energies can take different path length in focussing optics

$$|R_{56}| \ll \frac{\sigma_z}{\sigma_\delta}.$$

E.g. 10^{17}/cc
σ_z = 17µm
σ_E = 1%
then R_{56} < 1.7mm



$$\Delta L \approx \frac{1}{2}\sigma_{x'}^2 L \ll \sigma_z$$

E.g. 10^{17}/cc
200MeV
σ_z = 1µm
ε_n = 1 mm-mrad
$\sigma_{x'}$ = 2.3mrad
L < 37cm

Carl A. Lindstrøm, Staging of plasma-wakefield accelerators , Phys. Rev. Accel. Beams 24, 014801 (2021) .

Coulomb Scattering

- Need to confine the plasma to the plasma accelerator region with vacuum in the interstage optics in between
- Well established formulas for angular scattering in a neutral vapor
- Extend the range of Coulomb interaction to include the effects of traveling through an ion column
- Negligible for length of total acceleration in low Z materials (need differential pumping)
- Important at higher Z or when beam is mismatched

$$\Delta\varepsilon_N = \sqrt{2}\cdot r_c \cdot S \cdot \left(\sqrt{\gamma_f} - \sqrt{\gamma_i}\right)$$

$$S = Q\cdot\left(\ln\left(\frac{R_b}{R_a}\right) + \frac{1.78\cdot Z\cdot(Z+1)}{Q^2}\cdot\ln\left(\frac{287}{\sqrt{Z}}\right)\right)$$

R_b = blowout radius

R_a = range from atomic radius to blow out radius

Q = ion charge

Z = atomic number

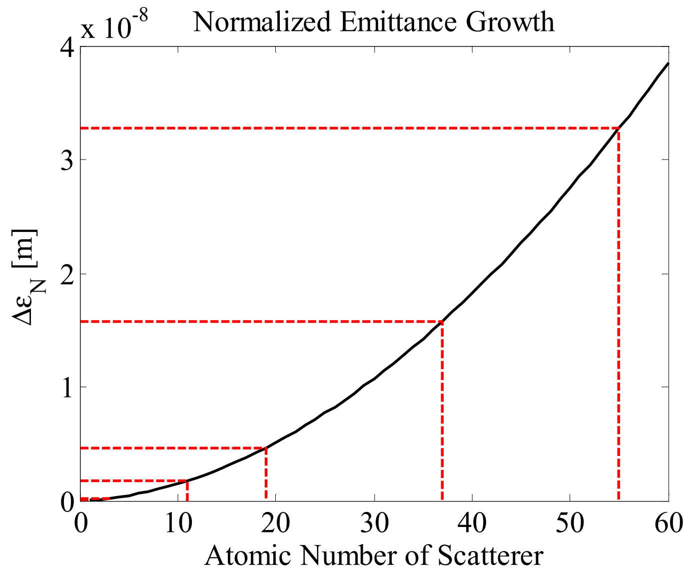
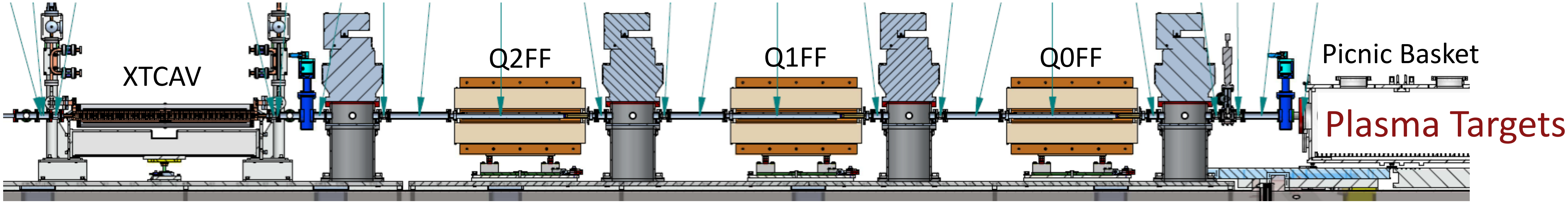


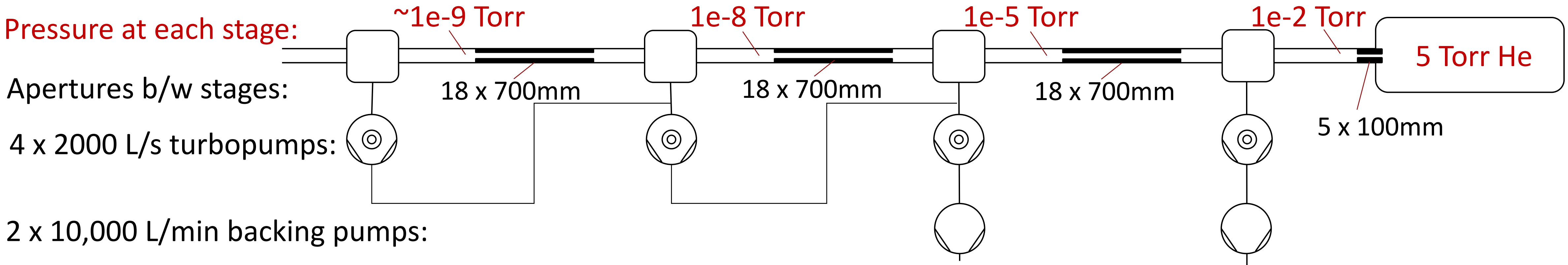
Figure 1: Normalized emittance growth from doubling the energy of an electron beam initially at 500 GeV through singly ionized materials with various atomic numbers.

N. Kirby et al., Emittance growth from multiple Coulomb scattering in a plasma wakefield accelerator, in Proceedings of the 22nd Particle Accelerator Conference, PAC-2007, Albuquerque, NM (IEEE, New York, 2007), p. 3097.

Y. Zhao et al., Modeling of emittance growth due to Coulomb collisions in plasma-based accelerators, Phys. Plasmas 27, 113105 (2020).

Design of the FACET-II Upstream Differential Pumping System





Multiple scattering not significant as long as gas/plasma is confined to the accelerating cells

- Need differential pumping system to limit scattering as illustrated at FACET-II
- Four stages of differential pumping, separated by conductance limiting apertures
- 1e-9 Torr achievable in each operating state
- Adds additional demands for beam line space between stages

Effective or Geographic Gradient

- Plasma cells have fantastic gradients but we can see there are many things to do between stages that require significant space
- 10^{17}/cc has unloaded gradient of 30GeV/m, but if need 30m to handle the beams in between, effective gradient is back down to 1GeV/m

Table 1
Energy scaling laws for the high energy regime ($E_m \gg E_d$). The same lattice is used for all energies, by scaling lengths as $\sqrt{\gamma}$, where γ is the main beam Lorentz factor.

Variable	Symbol	Energy scaling
Lattice length	L	$\sqrt{\gamma}$
Dipole, quad. length	l_d, l_q	$\sqrt{\gamma}$, $\sqrt{\gamma}$
β-functions	β	$\sqrt{\gamma}$
Spot size	σ_x	$1/\sqrt[4]{\gamma}$
Dispersion	D_x	$1/\sqrt{\gamma}$
Isochronicity	R_{56}	$1/\sqrt{\gamma}$
Chromatic amplitude	W	Const.
Emittance growth	$\frac{\Delta\epsilon}{\epsilon_0}$	Const.
Quad. field gradient	g_{max}	Const.
SR power, energy loss	P_{SR}, W_{SR}	γ, $\gamma^{1.5}$

Interstage distance will get longer at higher energies lowering effective gradient

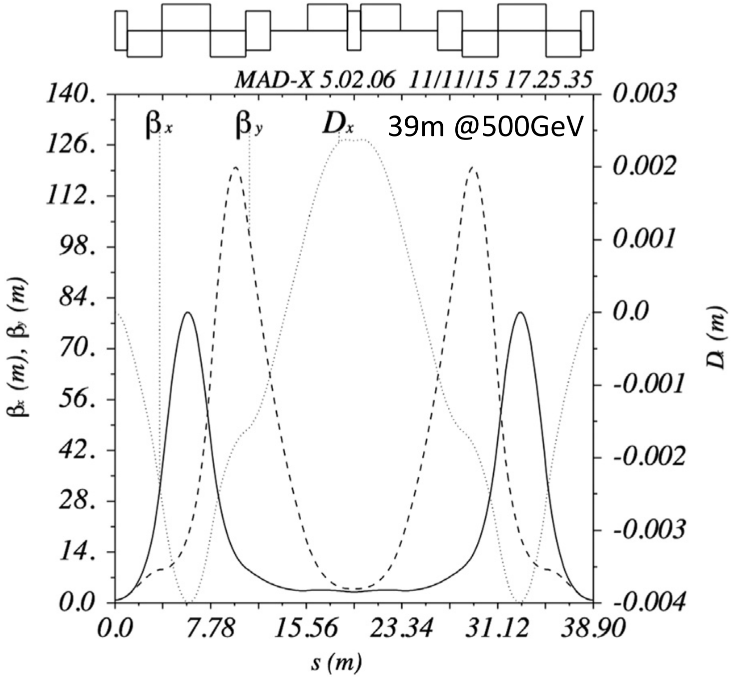
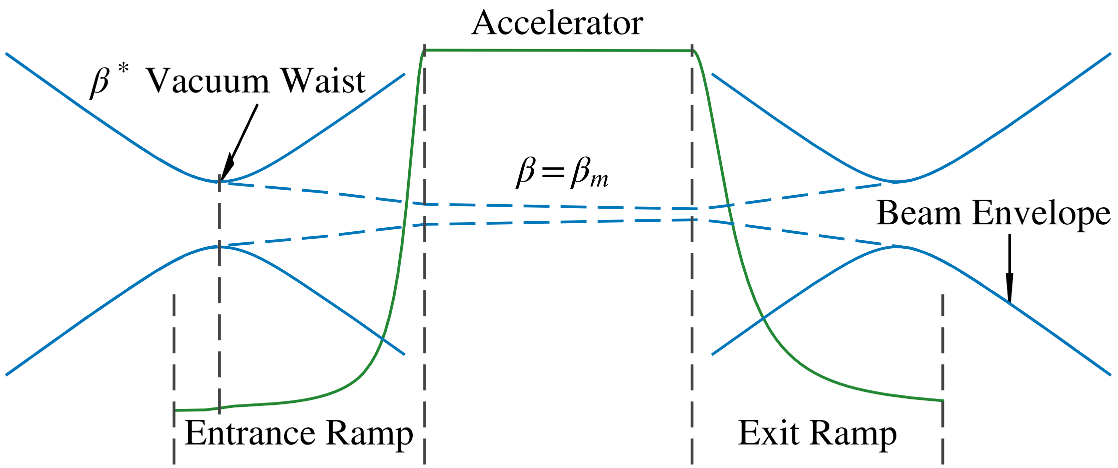


Fig. 2. Working example for 500 GeV, where 5 dipoles and 8 quadrupoles form a 39 m long C-chicane. Chromaticity is canceled by a linear lattice without sextupoles, however an uncorrected second-order dispersion leads to a 2% emittance growth.

C. A. Lindstrøm, E. Adli, J. M. Allen, J. P. Delahaye, M. J. Hogan, C. Joshi, P. Muggli, T. O. Raubenheimer, and V. Yakimenko, Staging optics considerations for a plasma wakefield acceleration linear collider, Nucl. Ins- trum. Methods Phys. Res., Sect. A 829, 224 (2016).

Plasma Density Ramps

- Gradually ramp plasma density on either side of the acceleration region
- Increases matched beta before the beam begins to diverge into vacuum
- Use adiabatic ramps to remain matched – slow change in density s.t. alpha remains ~ 0 throughout the ramp
- Ideal profiles can be calculated analytically or effects can be modeled for measured profiles



$$\left|\frac{n'(s)}{n(s)}\right| \ll \frac{1}{\beta_m(s)}$$

$$\frac{\Delta\epsilon^2}{\epsilon_0^2} \approx \frac{4L^2}{\beta_m^2}\sigma_\delta^2.$$

i.e. typical ramps are cm to 10's cm and increase beta by ~10 and dropping emittance growth by 100

K. A. Marsh et al., Beam matching to a plasma wake field accelerator using a ramped density profile at the plasma boundary, in Proceedings of the 21st Particle Accelerator Conference, Knoxville, TN, 2005 (IEEE, Piscataway, NJ, 2005), p. 2702.
I. Dornmair et al., Emittance conservation by tailored focusing profiles in a plasma accelerator, Phys. Rev. Accel. Beams 18, 041302 (2015).
R. Ariniello et al., Transverse beam dynamics in a plasma density ramp, Phys. Rev. Accel. Beams 22, 041304 (2019).
X.L. Xu et al., Physics of Phase Space Matching for Staging Plasma and Traditional Accelerator Components Using Longitudinally Tailored Plasma Profiles, Phys. Rev. Lett. 116, 124801 (2016).

Plasma Lenses

- Two types – active or passive
- Strong focussing in both planes simultaneously
- Active: drive large current through plasma to produce large magnetic field (next slide)
- **Passive**: utilize focussing from ion column (same as for PWFA, ramps etc)
- Laser or beam ionized
- Head of the drive beam will experience varying focusing as reach full blowout
- Constant focussing for the main beam
- Stronger than active plasma lens

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad 2\pi r dz E_r = \frac{\pi r^2 e n_i}{\varepsilon_0} \quad \Rightarrow \quad E_r = \frac{1}{2}\frac{e n_{e0}}{\varepsilon_0} r$$

$$E_r \equiv c B_\phi \text{ for ultrarelativistic particles} \qquad g_{\mathrm{PPL}} = \frac{en}{2c\epsilon_0}$$

$$g_{\mathrm{PPL}} = 30\ n_0[10^{18}\ \mathrm{cm}^{-3}]\ \mathrm{MT/m}$$

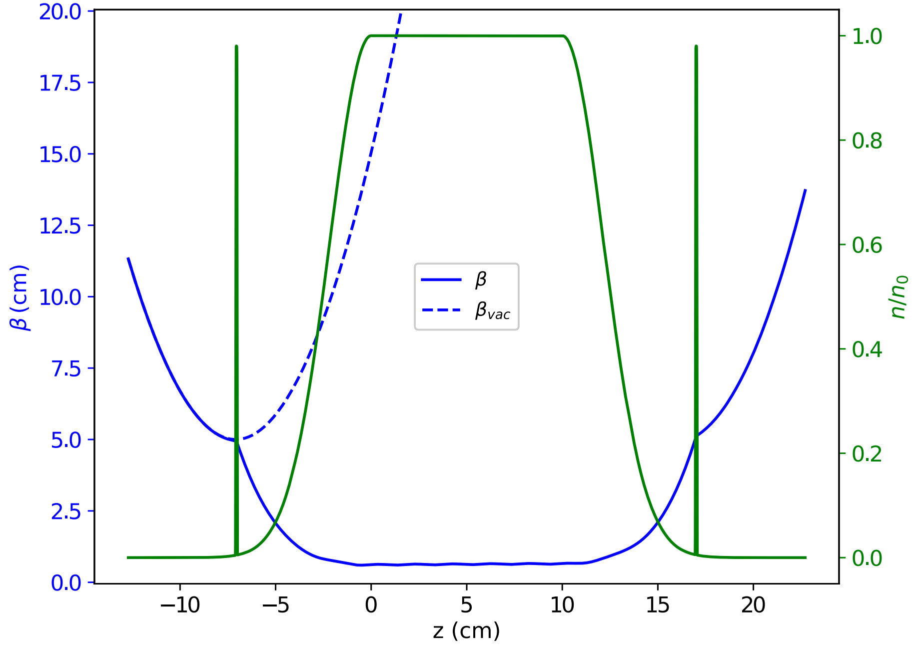


Illustration of two thin plasma lenses used to couple a 10 GeV beam in/out of a 3x10^{16}/cc plasma source used at FACET-II

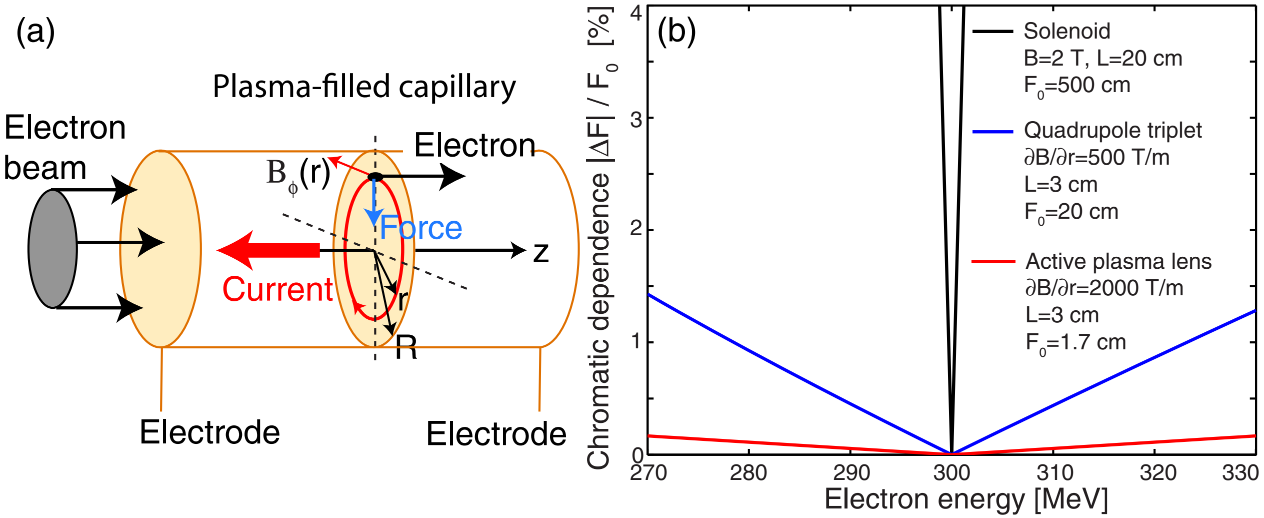
C. E. Doss et al., Laser-ionized, beam-driven, underdense, passive thin plasma lens, Phys. Rev. Accel. Beams 22, 111001 (2019).

Plasma Lenses

- Strong focussing in both planes simultaneously
- **Active**: drive current through plasma to produce large magnetic field
- kT/m symmetric focusing (less than PPL but more than PMQs)
- Ideal focusing requires uniform current density (heat flow effects)
- Current limited by z-pinch
- Beam density limited else get blow-out and active morphs to passive
- Multiple scattering at higher-z gases and large betas

J. van Tilborg et al., Active Plasma Lensing for Relativistic Laser-Plasma-Accelerated Electron Beams, Phys. Rev. Lett. 115, 184802 (2015).

Example APL comparison for 300MeV beam provides cm-scale focal length and reduced chromatic dependance vs solenoid, PMQ triplet



For cylindrical geometry, from Ampere's Law:

$$g_{\mathrm{APL}} = \frac{\mu_0 I}{2\pi R^2}$$

I.e. for R=500μm, I=500A, g=400T/m

C. A. Lindstrøm et al., Emittance Preservation in an Aberration-Free Active Plasma Lens, Phys. Rev. Lett. 121, 194801 (2018).
R. Pompili et al., Focusing of High-Brightness Electron Beams with Active-Plasma Lenses, Phys. Rev. Lett. 121, 174801 (2018)

Plasma Lenses

- Compact geometry for catching beam near exit of plasma stage but limiting aberrations in the lens places restrictions on the maximum beam density

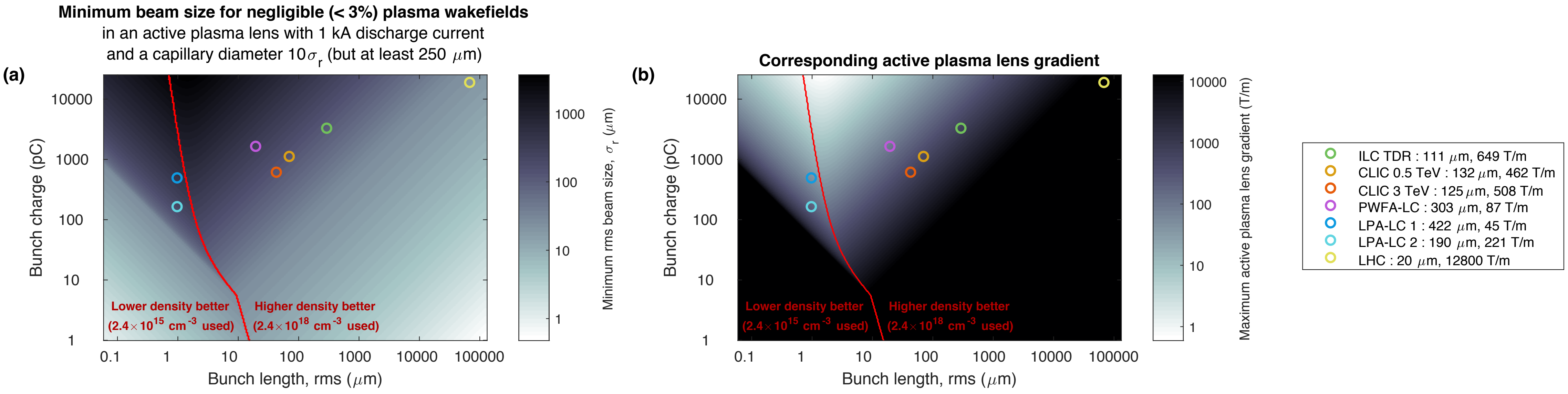


FIG. 5. (a) Minimum beam size required in an active plasma lens to have negligible distortion, i.e. when the gradient from plasma wakefields is 3% or less of the active plasma lens gradient, and (b) the corresponding active plasma lens gradient given this size. Note that the capillary diameter is constrained to 10 times the beam size, such that a smaller beam size gives a larger gradient (Eq. 8), but that this lens diameter is constrained to be at least 250 μm. The parameter space is divided in two parts, where lower (left) and higher (right) plasma densities allow smaller beam sizes, respectively. A typical discharge current of 1 kA is used, but smaller beam sizes can be tolerated if this current is increased. Collider parameters from Table II are indicated as colored circles.

C. A. Lindstrøm and E. Adli, Analytic plasma wakefield limits for active plasma lenses, arXiv:1802.02750.

Outline

- Why is staging important?
- Challenges and things to consider between stages
- Elements of a complete solution so far considered
- Benefits of having multiple stages
- Staging demonstrations
- Conclusions & Outlook

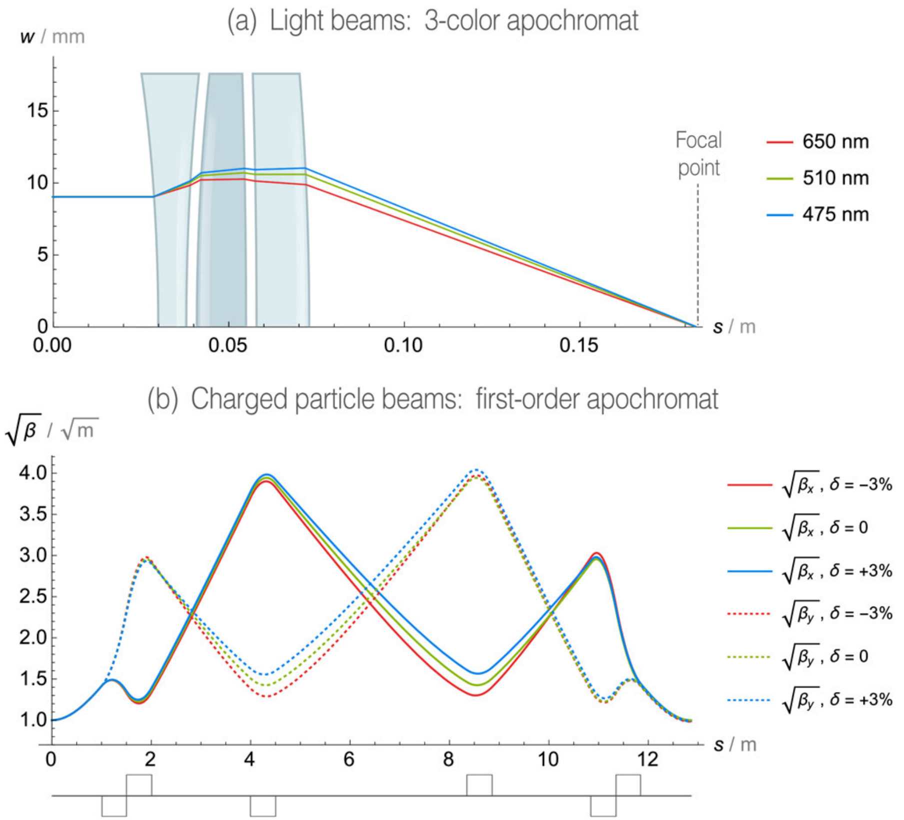
Interstage Requirements are Similar to Final Focus Systems

- Strong focussing and chromatic correction are desired to properly inject and extract the beam from the plasma
- Traditional FF designs (search literature for P. Raimondi, A. Seryi, G. White…)
  - Chromaticity is compensated in dedicated chromatic correction sections (CCX and CCY)
  - Sextupoles in high dispersion and high beta regions
  - Geometric aberrations generated by the sextuples are cancelled using -I transform between them
  - FFTB@SLAC was >200m at 50GeV
- Local chromaticity correction schemes reduce length and have been studied at ATF2@KEK and designs for 500m @500GeV
- Apochromatic focusing is a lesser-known alternative approach, whereby chromatic errors of Twiss parameters are corrected without the use of bends and sextupoles

See for example G. White et al., Experimental Validation of a Novel Compact Focusing Scheme for Future Energy-Frontier Linear Lepton Colliders, Phys. Rev. Lett. 112, 034802 (2014).

Apochromatic Correction

- Inspired by camera lenses, add additional lenses and tune so that multiple colors are in focus



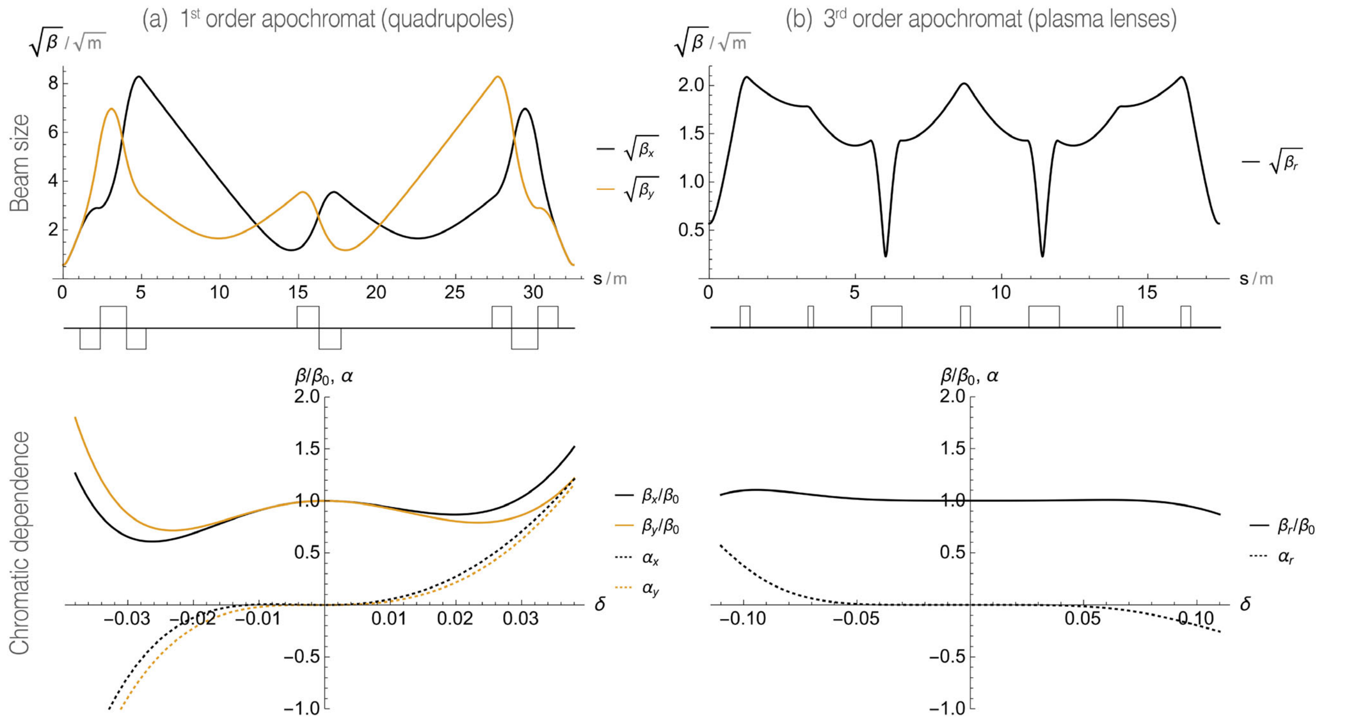
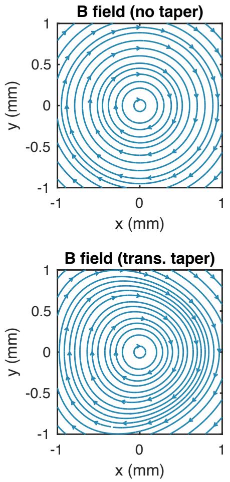


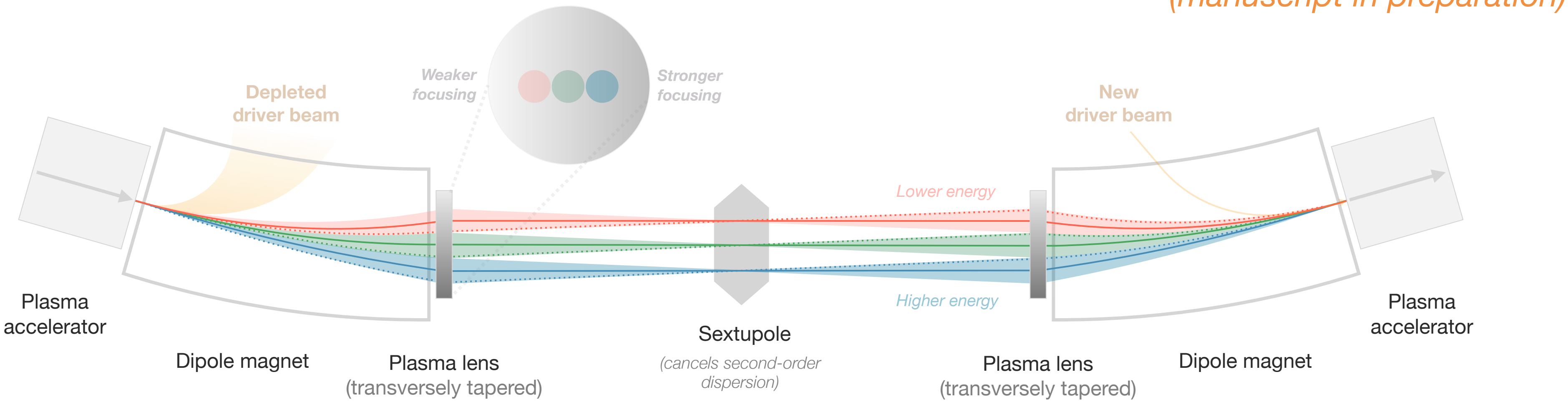
FIG. 4. Example A: PWFA staging optics, both using quadrupoles (a) and plasma lenses (b). Plots show $\sqrt{\beta}$ (proportional to rms beam size) vs beam line axis s, and chromatic dependence of $\alpha(\delta)$ and $\beta(\delta)$ vs offset δ. Both solutions capture a 100 GeV beam exiting a plasma (with density ramps) matched to $\beta_0 = 32.5$ cm and refocuses it back to 32.5 cm, with a 1 m drift space at the start and end for injection and extraction of drive beams. Solution (a) is a first-order apochromatic lattice using 8 quadrupoles with field gradient 160 T/m are placed antisymmetrically (mirrored with polarity switched), whereas solution (b) is a third-order apochromatic lattice using 7 discharge capillary plasma lenses [18] with field gradient 3000 T/m placed symmetrically. Transporting a beam with 1% rms energy spread leads to a projected emittance growth of 0.96% in lattice (a), and 0.000004% in lattice (b). Note the different δ-scales in the two chromatic dependence plots.

B. W. Montague and F. Ruggiero, Apochromatic focusing for linear colliders, CLIC Note No. 37, CERN, Geneva, 1987.
C. A. Lindstrøm et al., Design of general apochromatic drift-quadrupole beam lines, Phys. Rev. Accel. Beams 19, 071002 (2016)

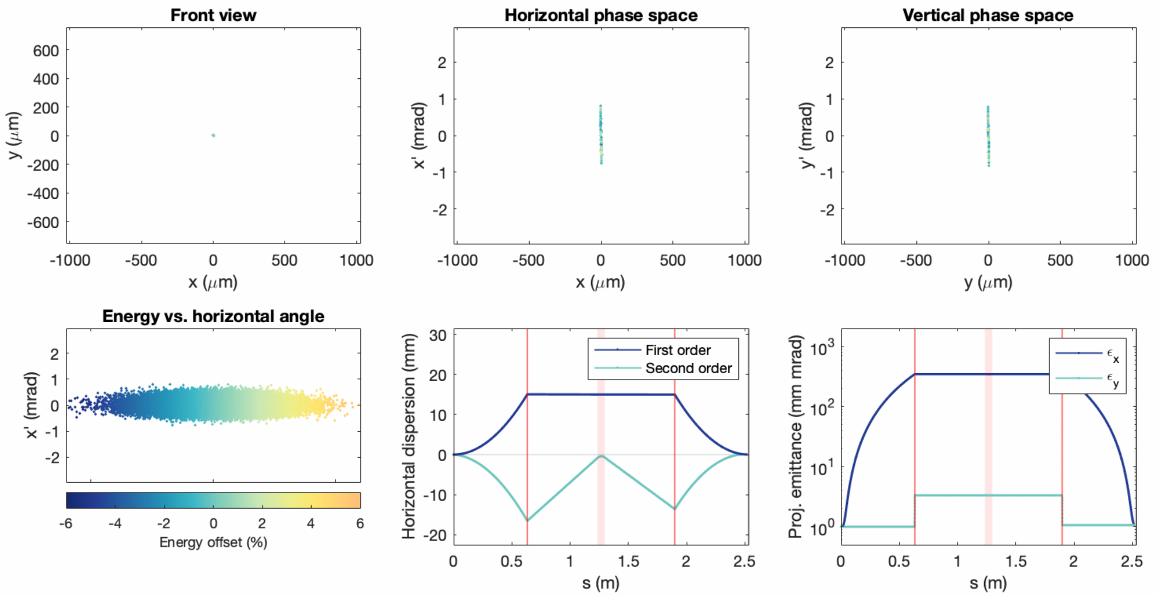
Apochromatic Transport with Transversely Tapered Plasma Lens

(manuscript in preparation)





- **<u>Transversely tapered</u> plasma lenses (APL/PPL)**
  - *Disperse the bunch into the PL with a dipole, match the focusing of each energy with a transverse taper.*
- **Local chromaticity correction*** (used in final focus systems) ** Raimondi & Seryi, Phys. Rev. Lett. 86, 3779 (2001)*
- Simple in/out-coupling of laser and beam drivers.
- Large, dispersed beams in the plasma lenses ⇒ minimal wakefield-distortion in APLs.



Outline

- Why is staging important?
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Synchronization

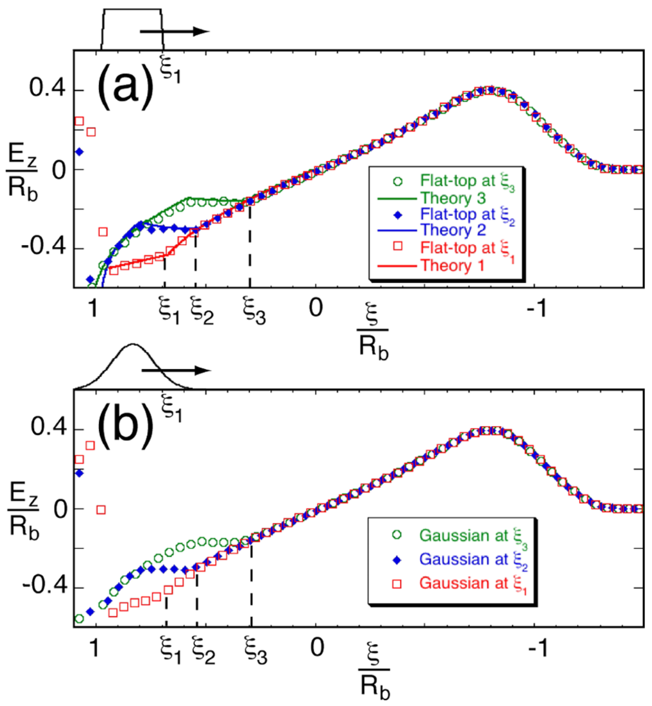
- Plasmas are high-field and high-frequency accelerators
- The driver initiates the wake so the main beam needs to be synched to the driver to a small fraction of the wakefield period
- Random timing jitter Δt will produce energy jitter

$$\frac{\Delta E_z}{E_z} \approx \omega \Delta t.$$

e.g. to remain within 1% energy bandwidth at 10^{17}/cc ($1/\omega_p$ = 177fs) would need to synchronize main beam to driver to better than 2fs. XFELs now down to ~10fs so this is challenging but does not feel much beyond state of the art

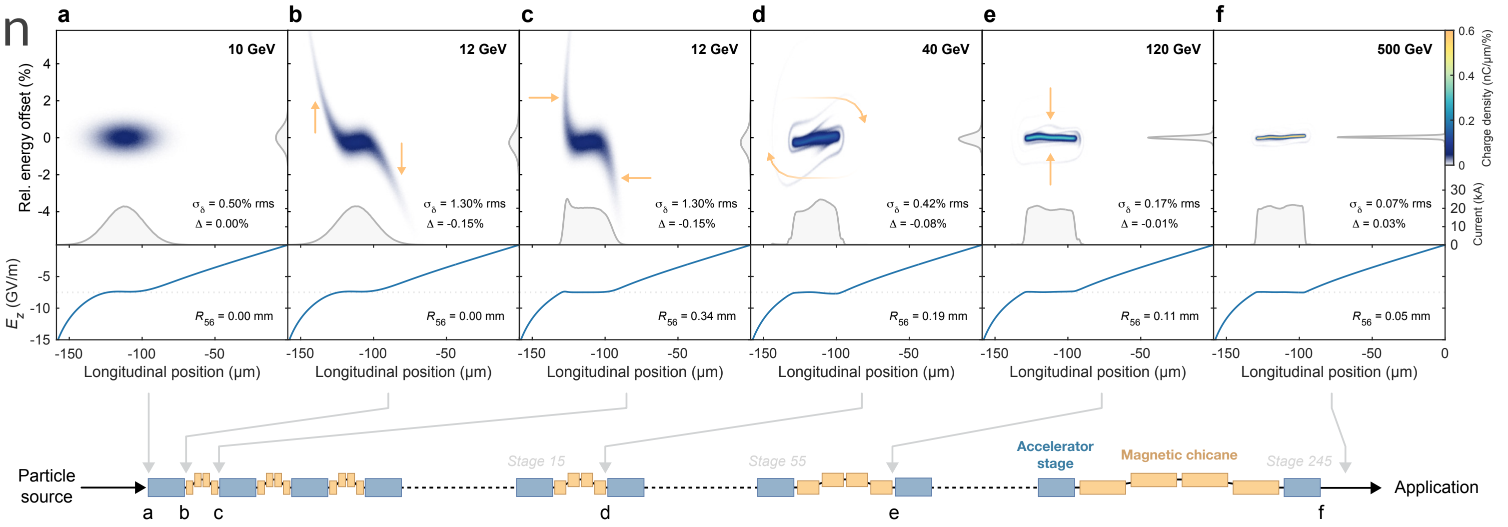
Self-correction for Stability and Energy Spread Damping

- Beam loading (Energy, energy spread) sensitive to longitudinal shape and location
- Magnetic chicanes provide energy dependent path length
- Possibility for feedback mechanism



M. Tzoufras PRL 101, 145002 (2008)

Preprint: Lindstrøm, arXiv:2104.14460 (2021)



> **Introduce a small compression** between stages (magnetic chicane; R_{56})

> (1) Synchrotron oscillations of the centroid ⇒ **phase stability.**

> (2) Feedback between beam loading and shape of current profile ⇒ **automatic wakefield flattening** (optimal beam loading).

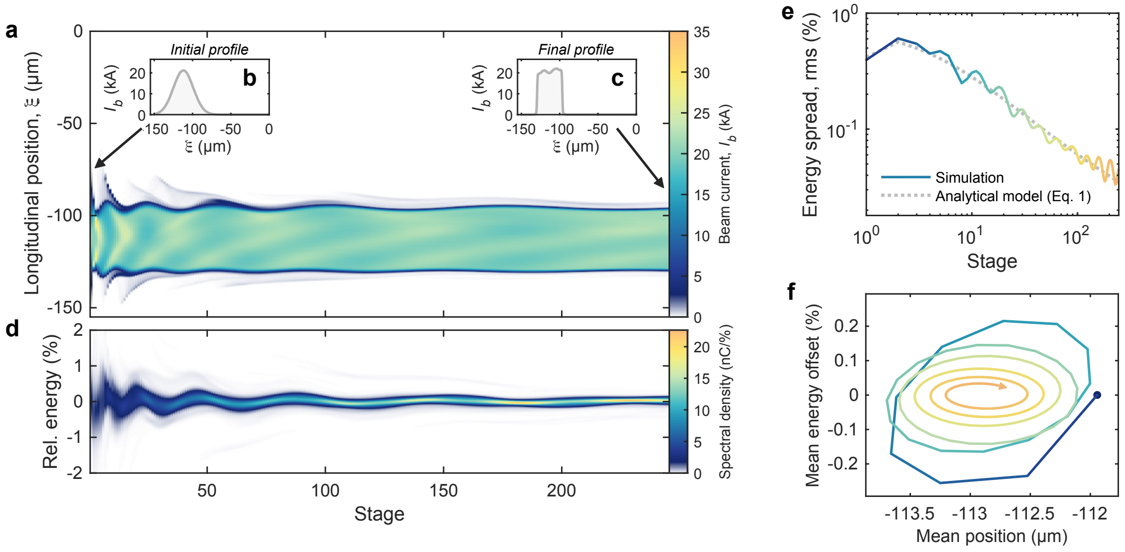
> Self-correcting long. phase space: **Damps energy spread** and **energy offset**

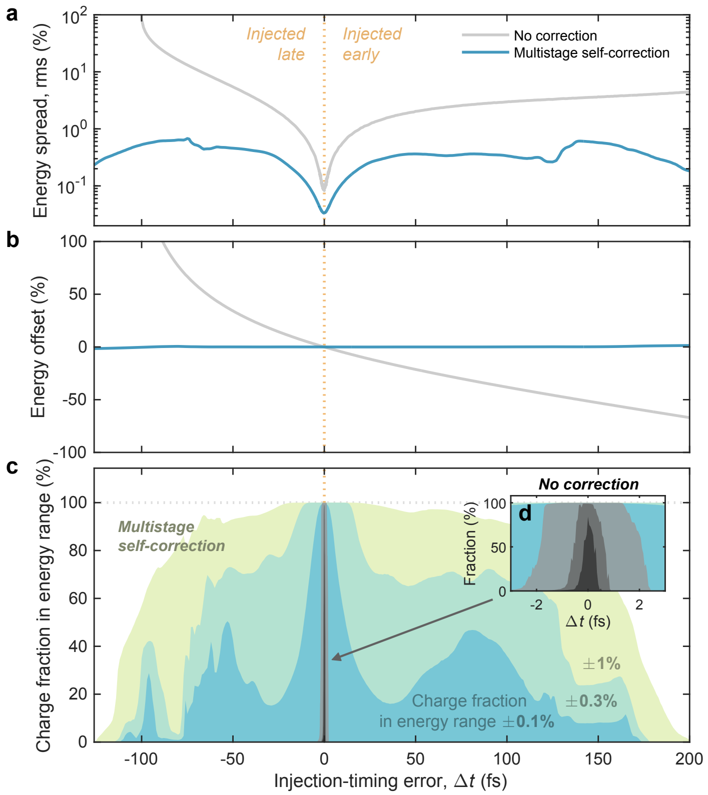
> Robust mechanism: specific wakefield regime or exact R_{56} not critical.

SLAC

Self-correction for Stability and Energy Spread Damping

Preprint: Lindstrøm, arXiv:2104.14460 (2021)

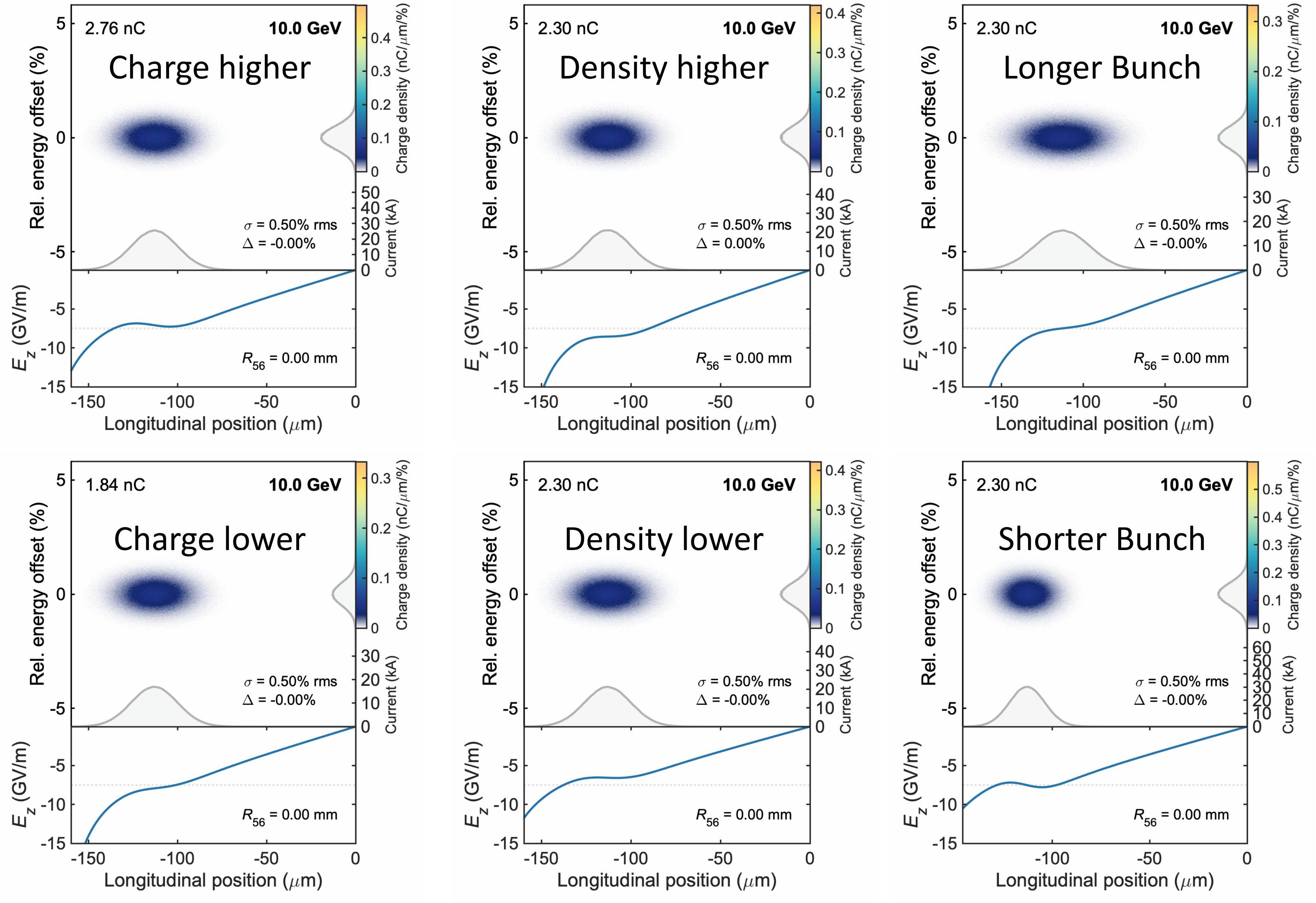




- No need for ultra-precise shaping of current profiles.
- **Improved synchronization tolerances** by several orders to magnitude.
- (Strong beam loading ⇒ natural high efficiency.)
- Implication: **Staging *not only* relevant to high energies**
  - Also beneficial for small-scale plasma accelerators.

- More R&D required to investigate…
  - …the coupling to the transverse phase space.
  - …the effect of CSR, betatron radiation, etc.

Some Nice Movies to Illustrate What is Happening



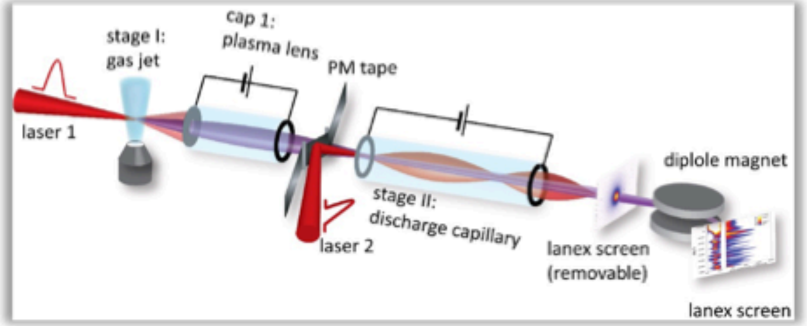
Outline

- Why is staging important?
- Challenges and things to consider between stages
- Elements of a complete solution so far considered
- Benefits of having multiple stages
- Staging demonstrations
- Conclusions & Outlook

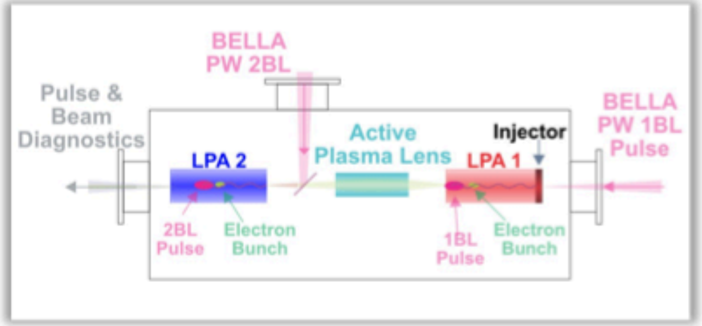
The Next Steps: Staging

A proof-of-principle demonstration of staging was performed at LBNL in 2016.

BELLA is well-positioned to demonstrate GeV-scale staging with the existing facility.



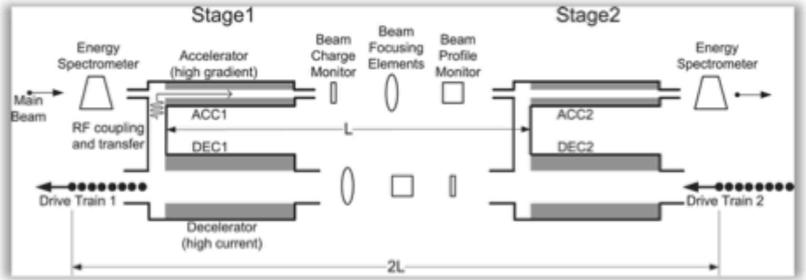
100 MeV-scale of LWFA Accelerators
S. Steinke et al. Nature (2016)



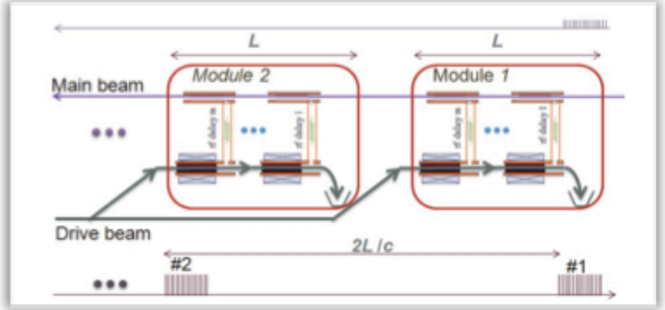
GeV-scale staging schematic

AWA plans a 0.5-GeV demo followed by a 3-GeV fully-featured module.

Ask to P5: Upgrade AWA facility for 0.5 GeV demonstrator.



SWFA Staging Experiment
C. Jing et al NIM A (2018)

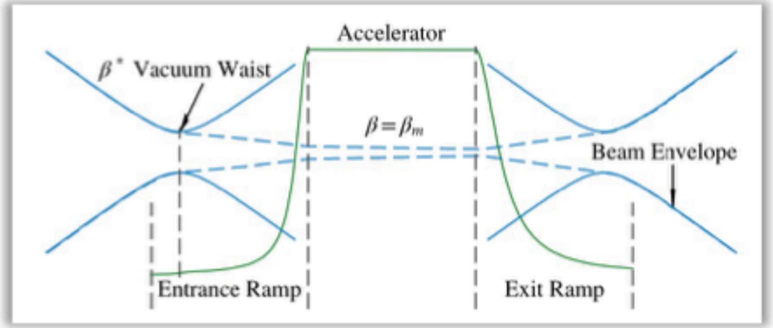


SWFA 0.5 GeV Staging Demo
C. Jing and G. Ha, JINST (2022)

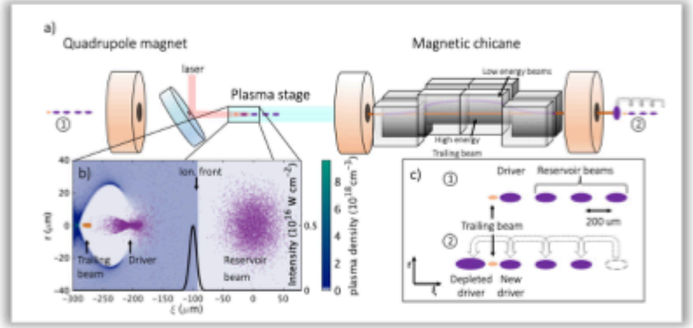
FACET-II can study beam transport in and out of a single stage.

Future Request: Facility for demonstrating two or more PWFA stages.

Note to P5: PWFA Staging experiment may be possible at C^3 Demo facility.

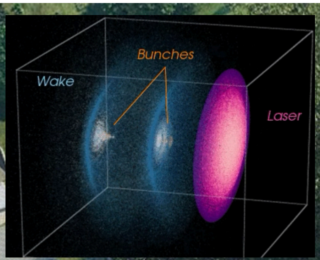


Beam matching with plasma ramps
R. Ariniello et al. PRAB (2019)



Laser-gated multistage plasma accelerator
A. Knetsch et al. arXiv:2210.02263

BELLA Center houses multiple laser facilities addressing laser, accelerator, and light source R&D and applications

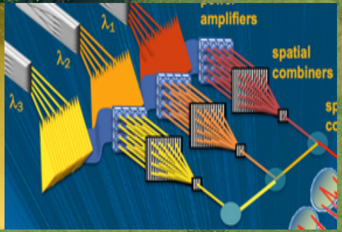


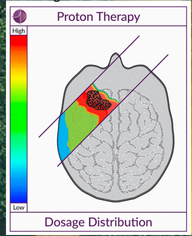
BELLA-PW 1st and 2nd beamlines
40 Joule in ~30fs (1 PW @ 1Hz)
- multi-GigaVolt e- acceleration
- Medium-intensity p+ acceleration

BELLA-HTT
3 Joule in 30fs (100 TW @ 5Hz)
- Mono-chromatic gamma rays
- Pump-probe X-rays
- Medium-intensity p+ acceleration

Future kBELLA: 100TW, operation at 1kHz

BELLA FIBER
100s mJ in <100fs (>1kHz)
- Laser R&D
- Light sources at >1kHz

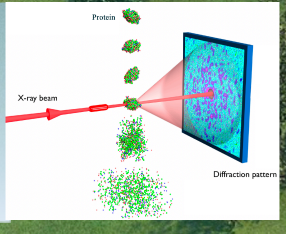




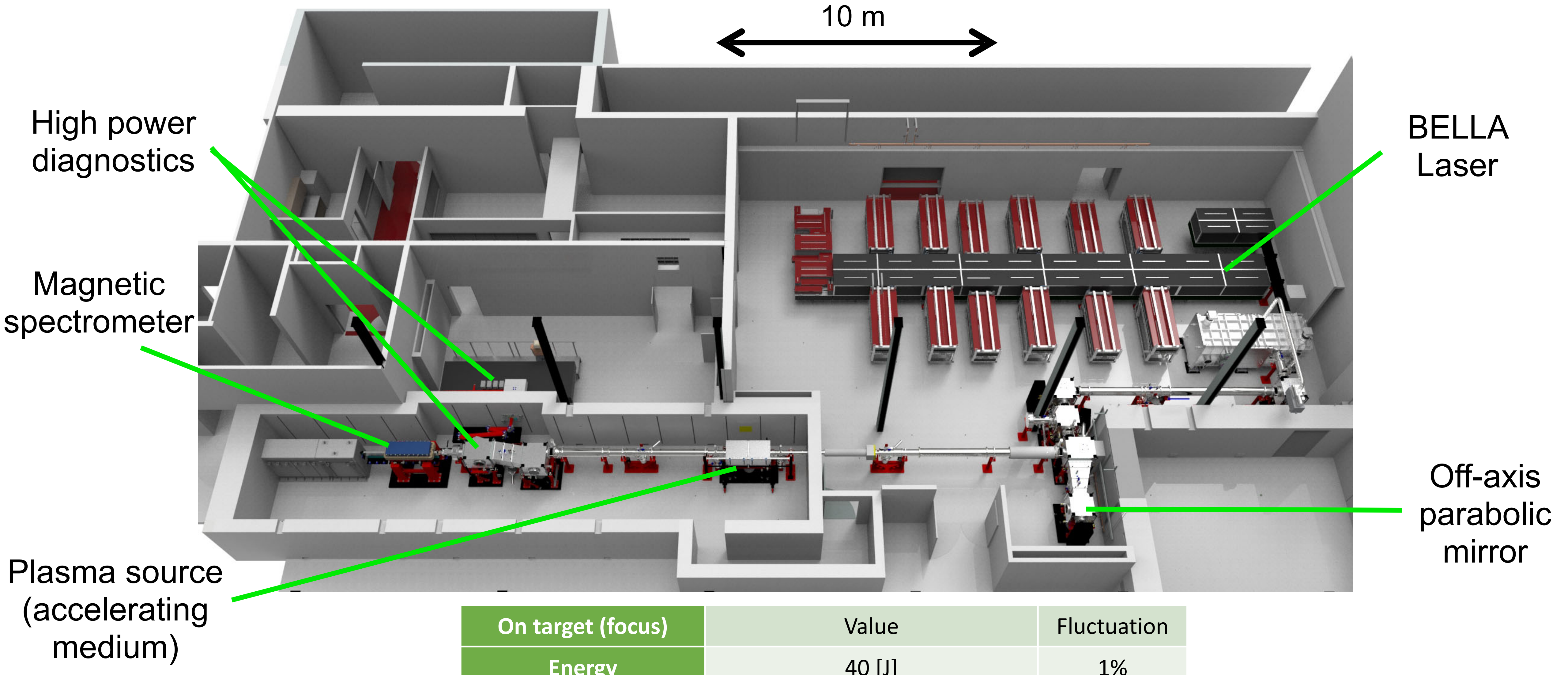
BELLA-IP2 at BELLA-PW
40 Joule in 30fs (1 PW @ 1Hz)
- High-intensity p+ acceleration
- Strong-field physics

1TW-kHz, 4mJ Joule in 4fs (1 TW @ 1kHz)
- Few-MeV electrons & X-rays
- Emergency response etc

BELLA-HTU, 3 Joule in 30fs (100 TW @ 5Hz)
- Electron transport line
- Undulator X-rays



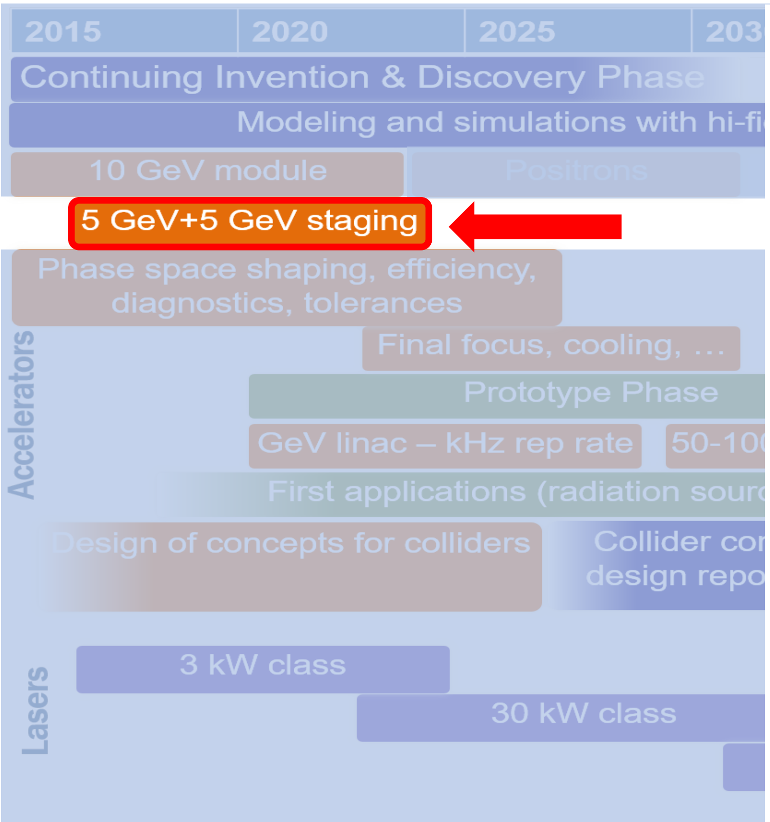
BELLA Center Houses a 1Hz Repetition Rate Petawatt Laser for LPA Science

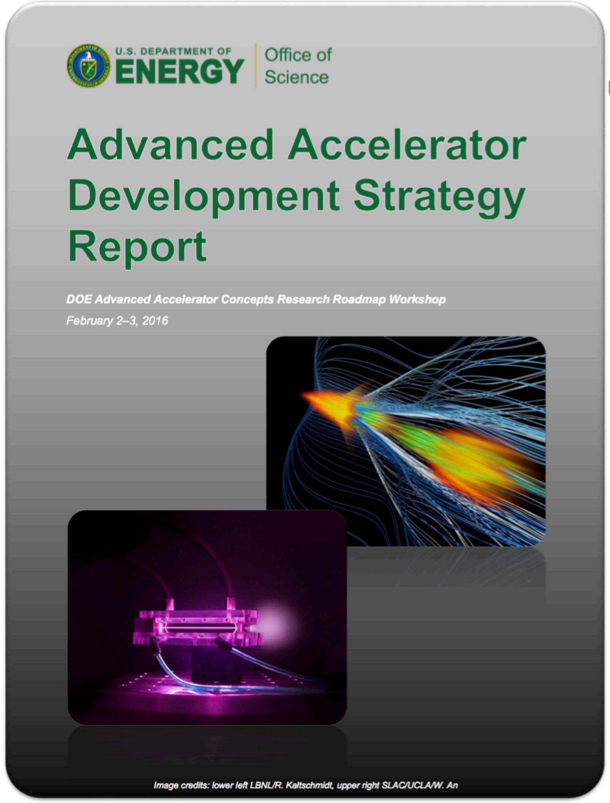


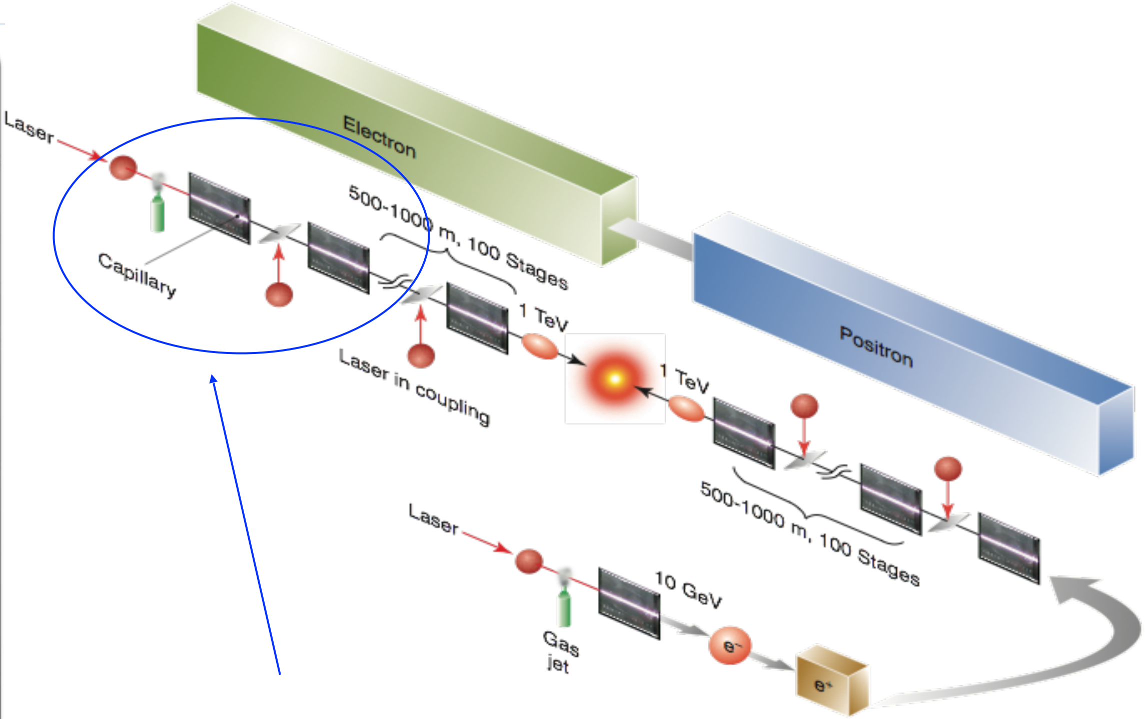
On target (focus)	Value	Fluctuation
Energy	40 [J]	1%
Peak Power	1 [PW] (for 31fs)	5%
Peak Intensity	1.5 [10^{19} W/cm^2] (for w_0 53μm)	10%

*K. Nakamura et al., IEEE JQE **53** 1200121 (2017)*

Multi-GeV staging: a key next step on the LPA collider roadmap







Upcoming BELLA PW 2nd beamline experiments

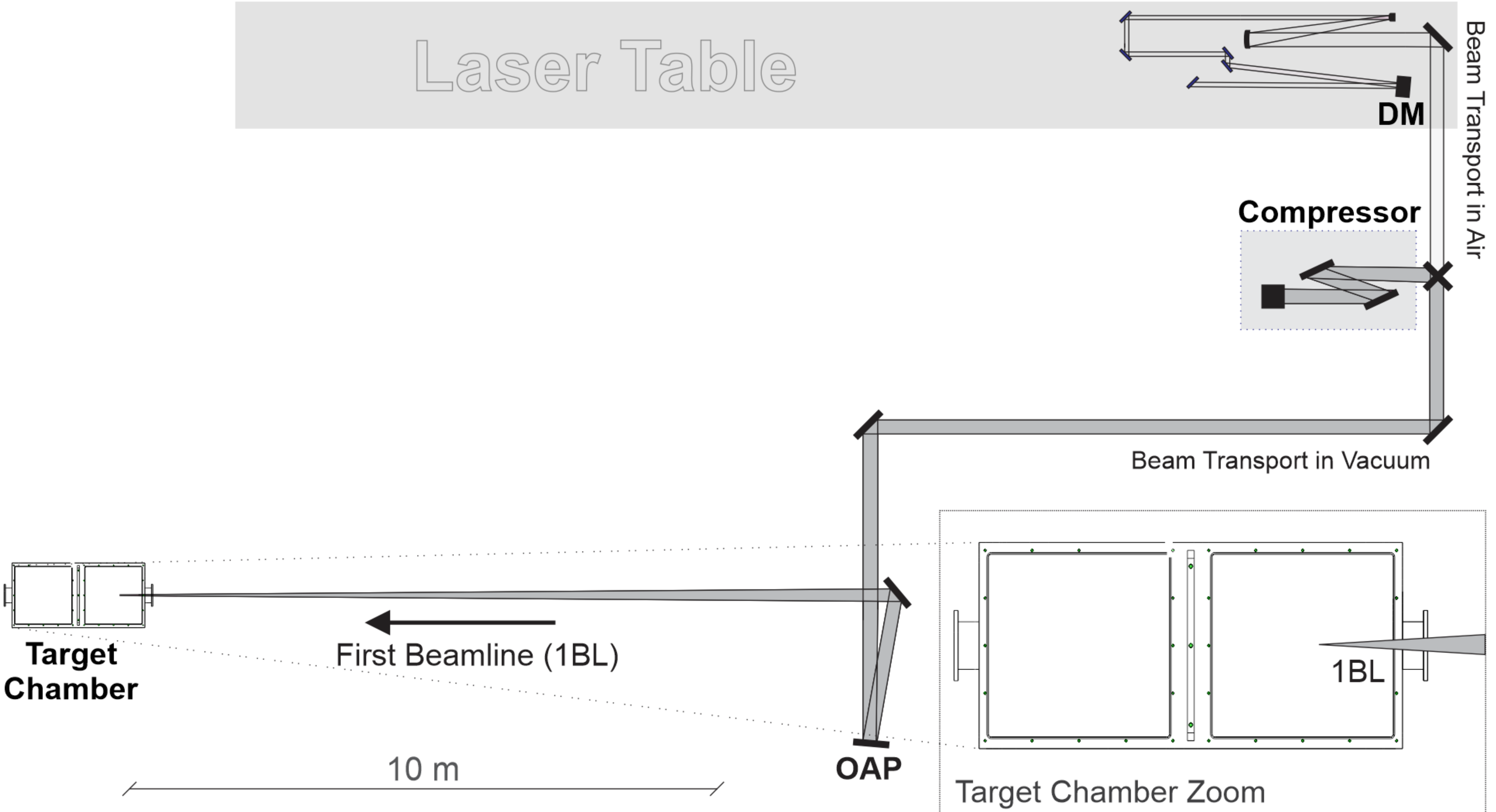
- Staging at ~100MeV using 30TW (BELLA TREX laser) in 2016, but low capture efficiency

S. Steinke PoP **23**, 056705 (2016); B. H. Shaw, PoP **23**, 063117 (2016); J. van Tilborg, PRL **115**, 184802 (2015); S. Steinke, Nature **530**, 190 (2016)

- BELLA PW laser will be used to investigate multi-GeV staging with high efficiency

BELLA PW Facility Layout before 2BL installation

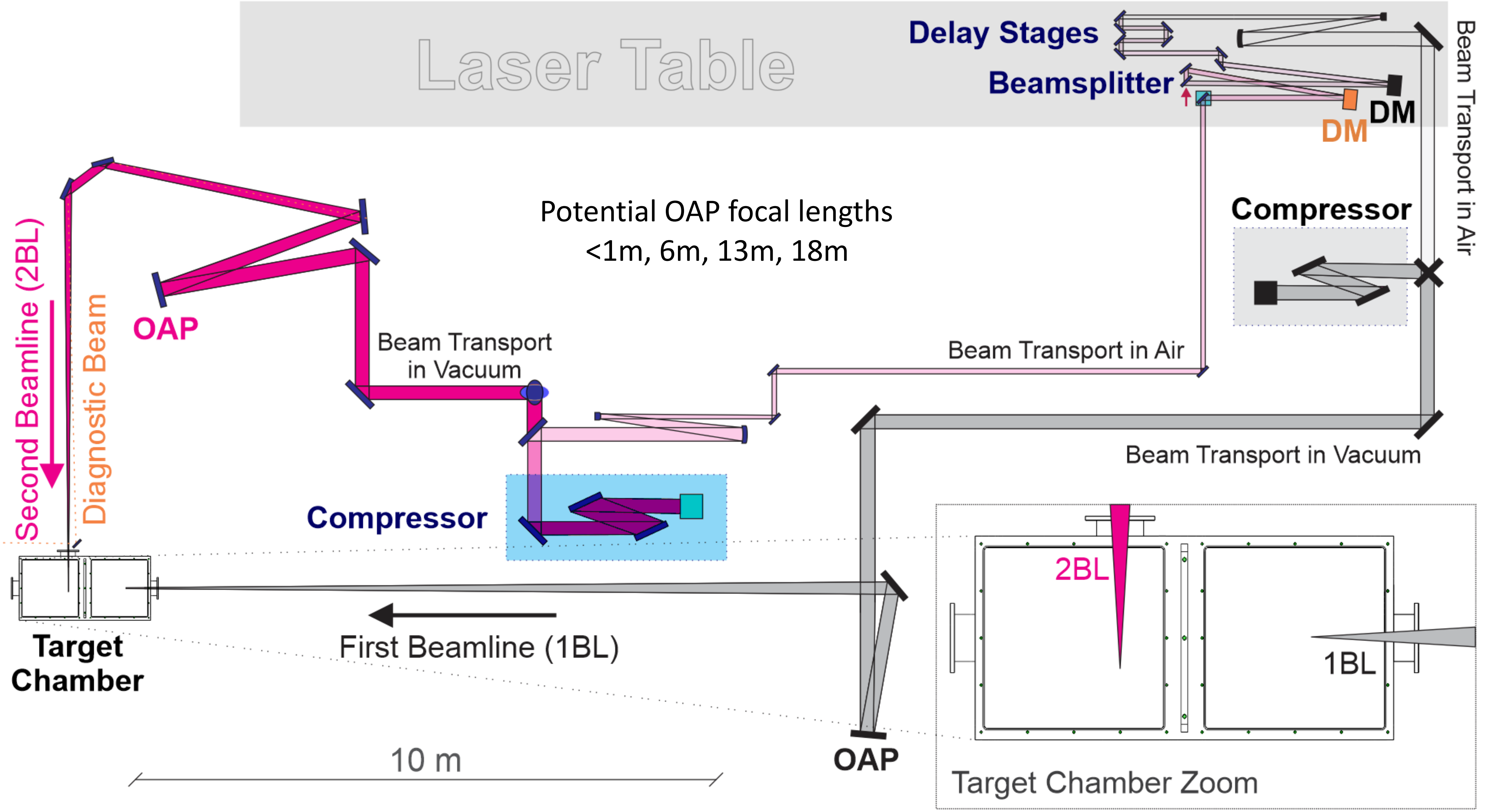
Gray / Black: elements that existed before 2BL upgrade.



BELLA 2nd bemline (2BL) adds additional laser pulse to the target chamber for the next generation of laser accelerator experiments

Gray / Black: elements that existed before 2BL upgrade.

Colored: elements installed / modified because of 2BL installation.

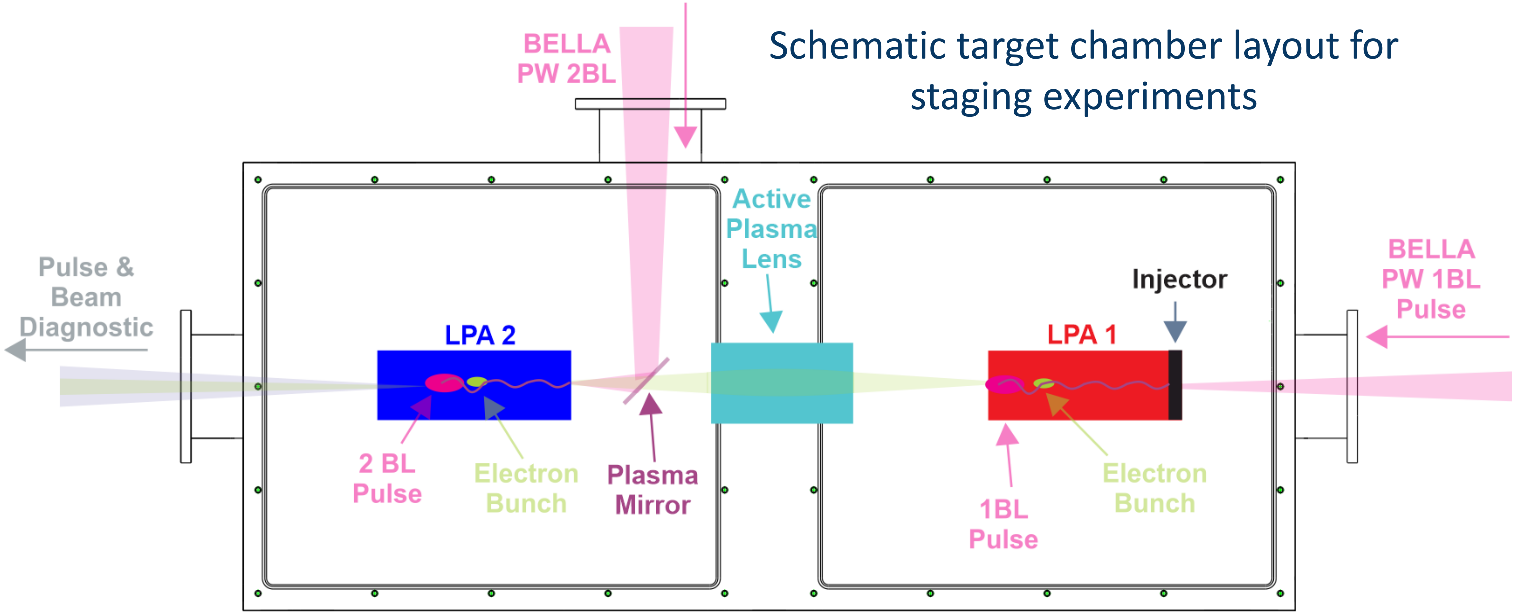


2BL enables experiments on multi-GeV staging with high efficiency

Staging elements:

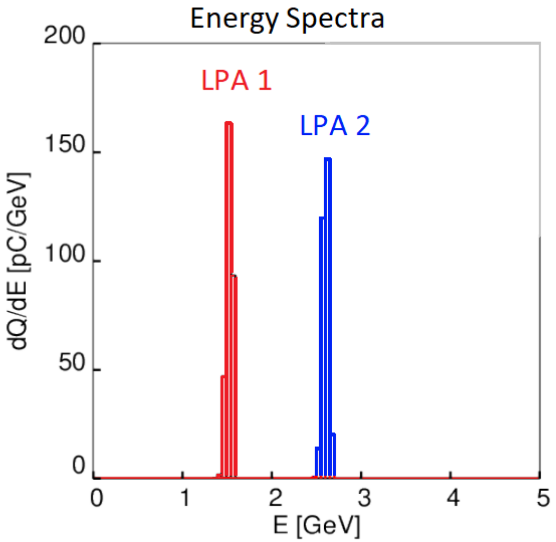
- **Injector** to produce electron bunch.
- **LPA 1** driven by 1BL pulse producing quality electron bunch (GeV-energies, ΔE/E < 10%, divergence < 2 mrad).
- **Active plasma lens** to refocus electron beam.
- **Plasma mirror** to couple in 2BL pulse.
- **LPA 2** driven by 2BL pulse.

Schematic target chamber layout for staging experiments



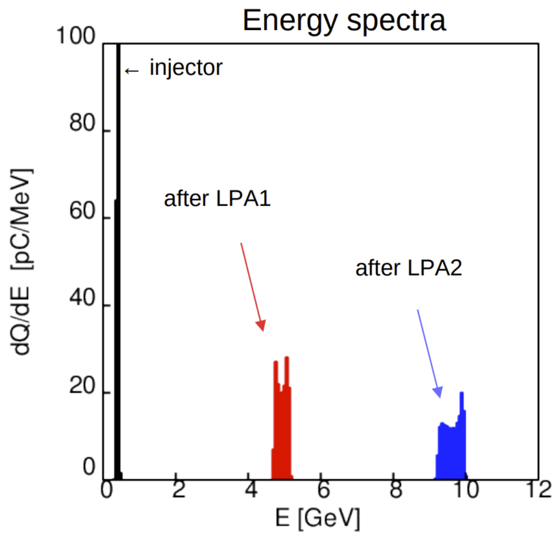
Un-guided experiments:

INF&RNO Simulations predict ~100% charge capture, multi-GeV electron energies and quality bunches.



Optimized guided experiments:

INF&RNO Simulations predict ~100% charge capture and electron energies up to ~10GeV.



Conclusion & Outlook

- Staging will be needed for collider applications of PWFA/LWFA
- Community making steady progress understanding and optimizing individual stages
- Only one staging experiment performed to date
- Next few years will see more
- A complete staging solution is challenging
- …but a great opportunity for smart young people to have an impact
- e.g. the many new and interesting ideas highlighted here from Carl Lindstrøm
- U. Oslo team is developing a workflow to model staging for HALHF concept in preparation for next European Strategy
- Ideas will inform designs for next generation facilities and demonstrations

FACET-II | Facility for Advanced Accelerator Experimental Tests

Questions?