



ETTORE MAJORANA FOUNDATION AND  
CENTRE FOR SCIENTIFIC CULTURE

TO PAY A PERMANENT TRIBUTE TO ARCHIMEDES AND GALILEO GALILEI, FOUNDERS OF MODERN SCIENCE  
AND TO ENRICO FERMI, THE 'ITALIAN NAVIGATOR', FATHER OF THE WEAK FORCES



27 July 2023 - 2 August 2023  
Ettore Majorana Center (Sicily, IT)

# Plasma Accelerators

## Overview of different schemes

DIPARTIMENTO DI SCIENZE DI BASE  
E APPLICATE PER L'INGEGNERIA

Enrica Chiadroni (Sapienza Univ. - SBAI Dept. and INFN - LNF)



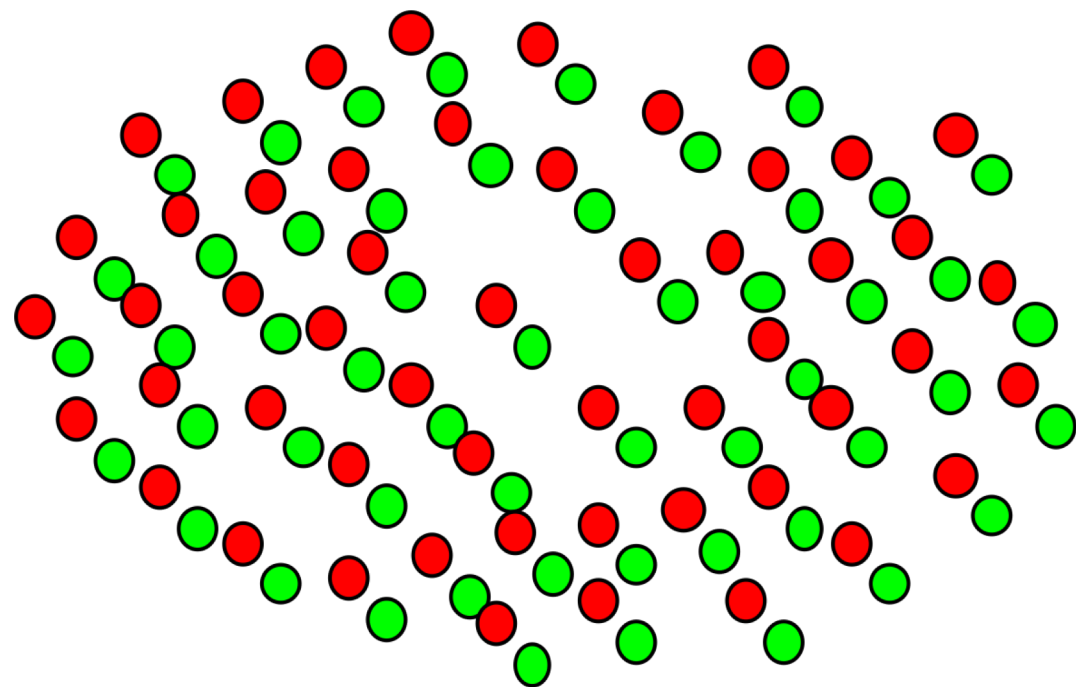
SAPIENZA  
UNIVERSITÀ DI ROMA

# Outline

- Basic principle of plasma-based acceleration
  - Issues contributing to improving acceleration techniques
- Current research highlights
- Other uses for plasma in the accelerator field
  - Plasma lenses
  - Plasma wigglers/undulators
- Conclusions



# Relativistic Plasma Waves



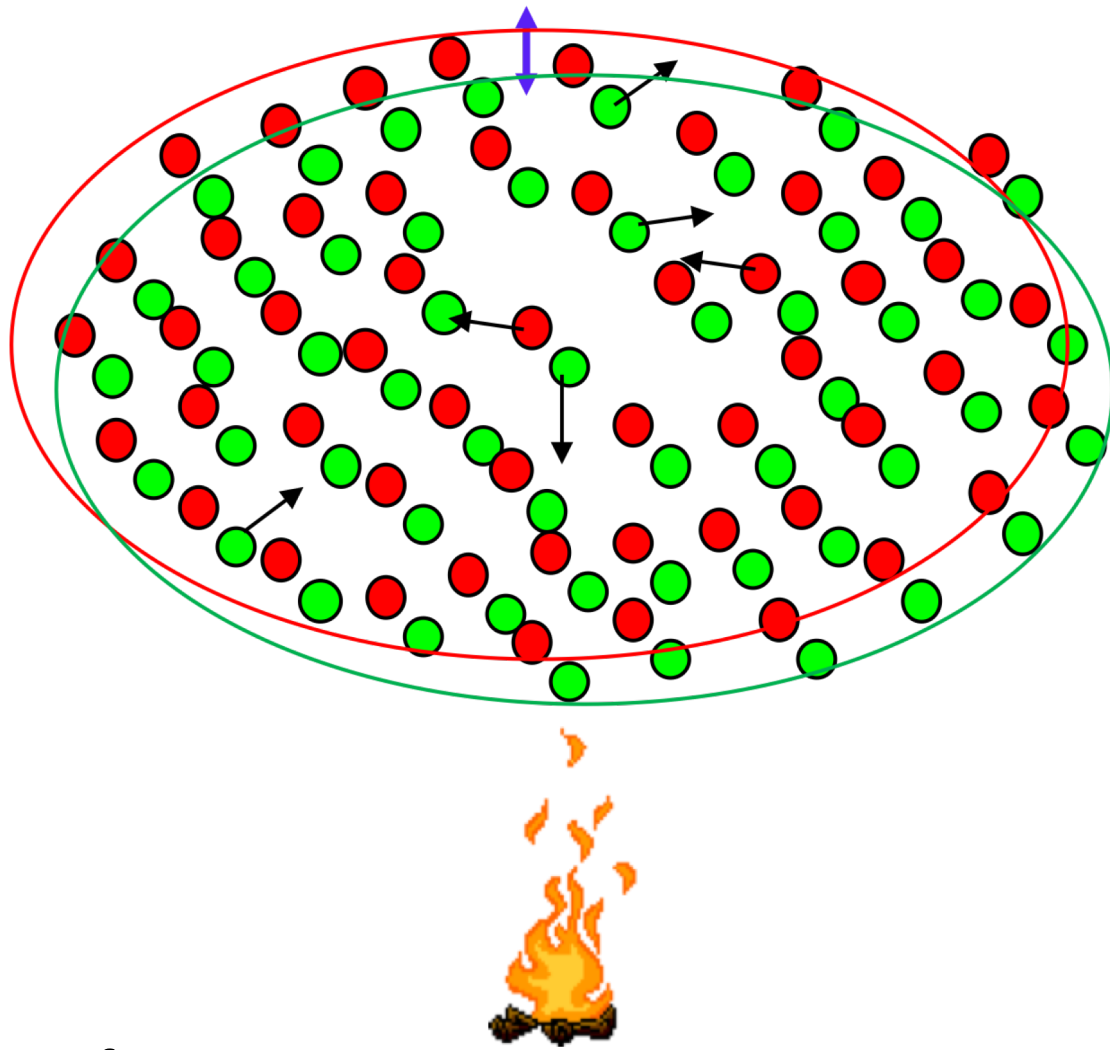
*Courtesy of M. Ferrario*

# Relativistic Plasma Waves

Surface charge density

$$\sigma = en\delta x$$

$\delta x$



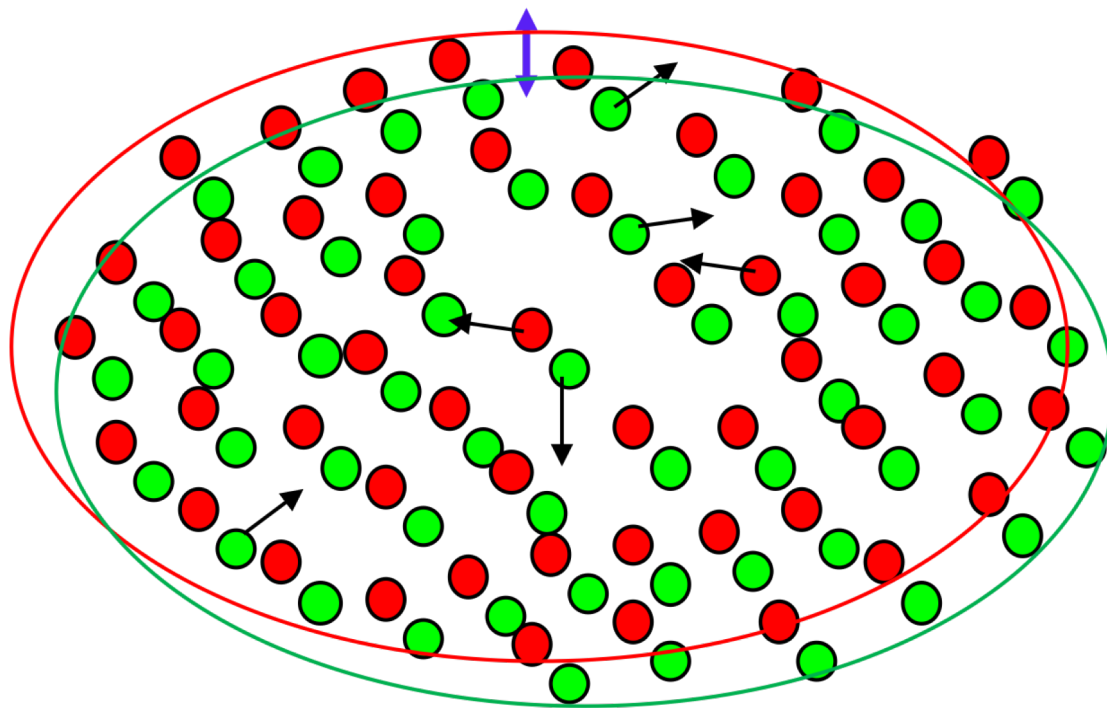
*Courtesy of M. Ferrario*

# Relativistic Plasma Waves

Surface charge density

$$\sigma = e n \delta x$$

$\delta x$



Courtesy of M. Ferrario

Surface electric field

$$E_x = -\sigma/\epsilon_0 = -e n \delta x/\epsilon_0$$

Restoring force

$$m \frac{d^2 \delta x}{dt^2} = e E_x = -m \omega_p^2 \delta x$$

Plasma oscillations

$$\delta x = (\delta x)_0 \cos(\omega_p t)$$

Plasma frequency

$$\omega_p^2 = \frac{n e^2}{\epsilon_0 m}$$

**Collective behavior !!**

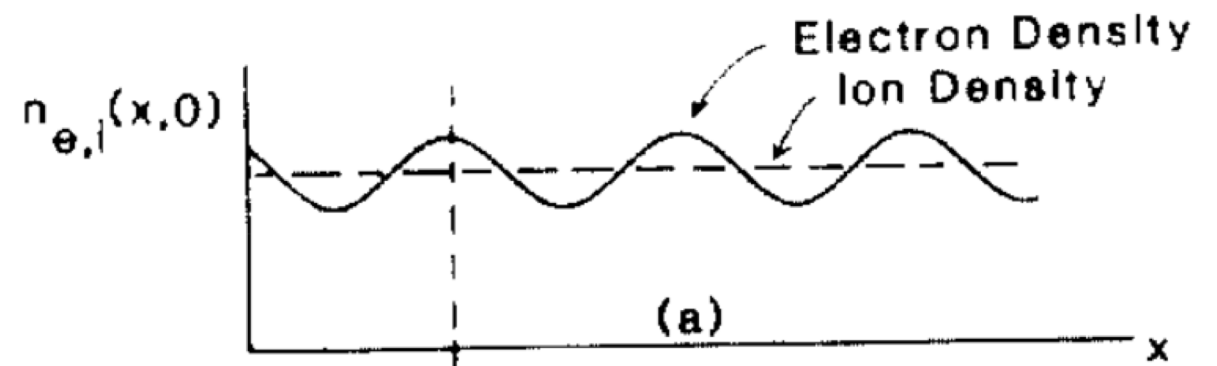
# Relativistic Electron Electrostatic Plasma Wave

$$\omega_p = \sqrt{\frac{n_0 e^2}{m_e \epsilon_0}}$$

Plasma angular frequency/regular frequency

$$f_p = \frac{\omega_p}{2\pi} \approx 9\sqrt{n_0}$$

- The oscillations lead to local compression (bunching) and rarefaction in the electron density
  - Electron oscillations are fast, ions are regarded as stationary



- The disturbance does not propagate as a wave
  - Plasma oscillation is purely an electrostatic oscillation

# Acceleration in Plasma Waves

- When electrons are disturbed by a suitable phase relation, their oscillations consist of a traveling wave whose  $v_p$  can range from a few times the electron thermal velocity to infinity
- The basic equation for waves propagation in the **cold plasmas** is the Klein-Gordon equation:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) + \frac{1}{c^2} \left[ \frac{\partial^2 \vec{E}}{\partial t^2} + \omega_p^2 \vec{E} \right] = 0$$

- Plane wave solution  $\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$   $\vec{k} \equiv$  wave vector

$$\frac{\partial}{\partial t} = -i\omega$$


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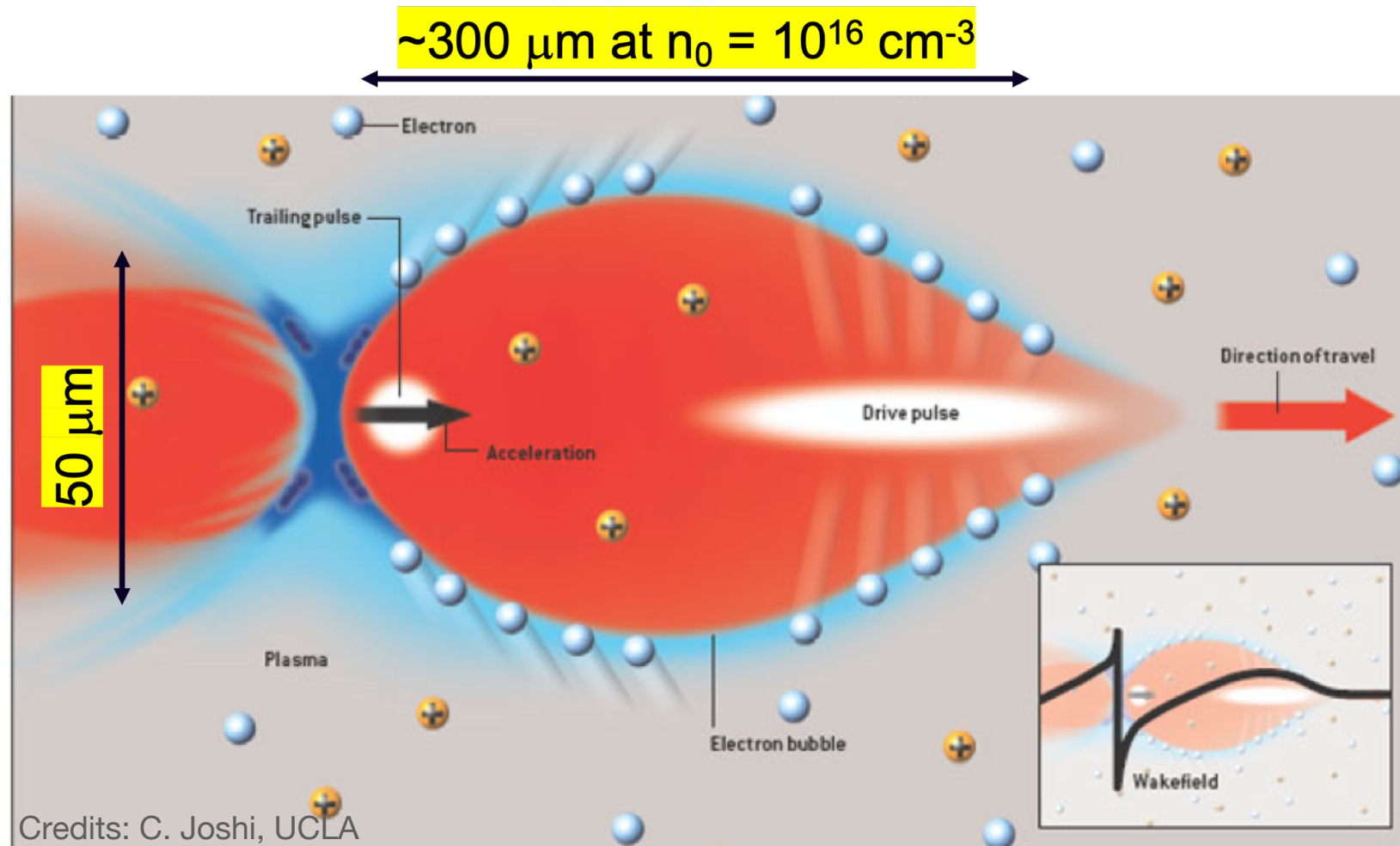

$$\vec{\nabla} \equiv i\vec{k}$$

$$i\vec{k} \times (i\vec{k} \times \vec{E}) + \frac{1}{c^2} (-\omega^2 + \omega_p^2) \vec{E} = 0$$

New Klein-Gordon equation

If  $\mathbf{E}$  and  $\mathbf{k}$  are in the same direction  $\Rightarrow$  longitudinal wave with  $\omega = \omega_p$

# Cold Wavebreaking Field



**Characteristic scale length** of the accelerating field, i.e. the plasma wake, is the plasma wavelength  $\lambda_p$

$$\lambda_p [\mu\text{m}] \approx \frac{3.3 \cdot 10^{10}}{\sqrt{n_0 [\text{cm}^{-3}]}}$$

- The **ionized plasma can sustain** accelerating gradient 2-3 orders of magnitude larger than in conventional RF-based accelerators
- Maximum accelerating field a plasma can sustain: **Cold wave breaking field**

$$E_{Max} [V/m] = \frac{m_e c \omega_p}{e} \approx 100 \sqrt{n_0 [\text{cm}^{-3}]}$$

$$n_0 = 10^{16} \div 10^{18} \text{ cm}^{-3}$$

# Plasma Definition

- A partially or completely ionized gas, globally neutral, is a plasma if it exhibits a **collective behavior**

- Coulomb shielding

- Dimensions  $\gg$  Debye length

$$\lambda_D = \sqrt{\frac{kT\epsilon_0}{ne^2}}$$

- Plasma Oscillations

- Temporal response  $\gg \omega_p^{-1}$

$$\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}}$$

- The large electric fields a plasma can sustain are supported by collective motion of plasma electrons, forming a **space charge disturbance** moving at a speed slightly smaller than  $c$



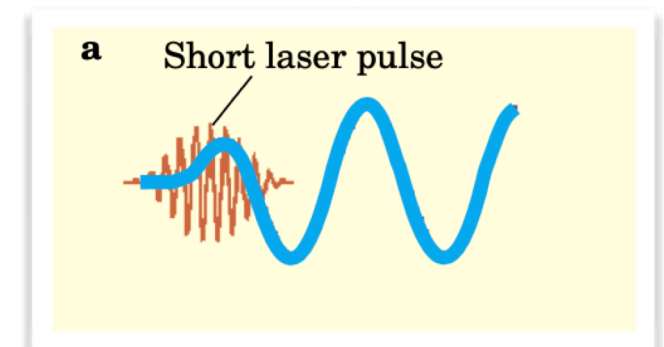
# Collective Fields to Accelerate Charged Particles

## First Ideas

- 1956, G I Budker and V I Veksler of the (then) USSR (Budker G I 1986 Proc. CERN Symp. on High Energy Accelerators vol 1 (Geneva: CERN) pp 68–80)
  - Use of the fields generated in a plasma by the passage of a medium-energy electron beam to accelerate ions to high energy
- Toshi Tajima and John Dawson of UCLA (Phys. Rev. Lett. 43 267 (1979))
  - Use of a relativistically propagating disturbance or a wake created in a plasma by the passage of a short laser pulse to accelerate electrons to ultrahigh energies in a short distance



**Laser Wakefield Accelerator**

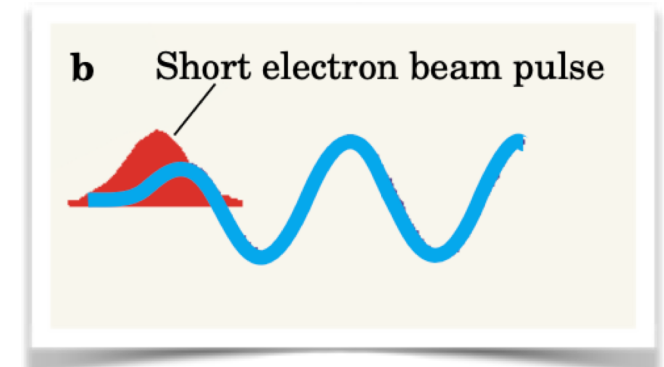


- Pisin Chen et al., (Phys. Rev. Lett. 54 693 (1985))

- instead of a laser pulse, one can use a high-current bunch of electrons to generate very high electric fields in a plasma that can be used to accelerate particles extremely rapidly



**Plasma Wakefield Accelerator**



The plasmas does not only provide high acceleration gradients,  $\sim$ GV/m, but they also serve to focus accelerated beam to micrometer scale spot size

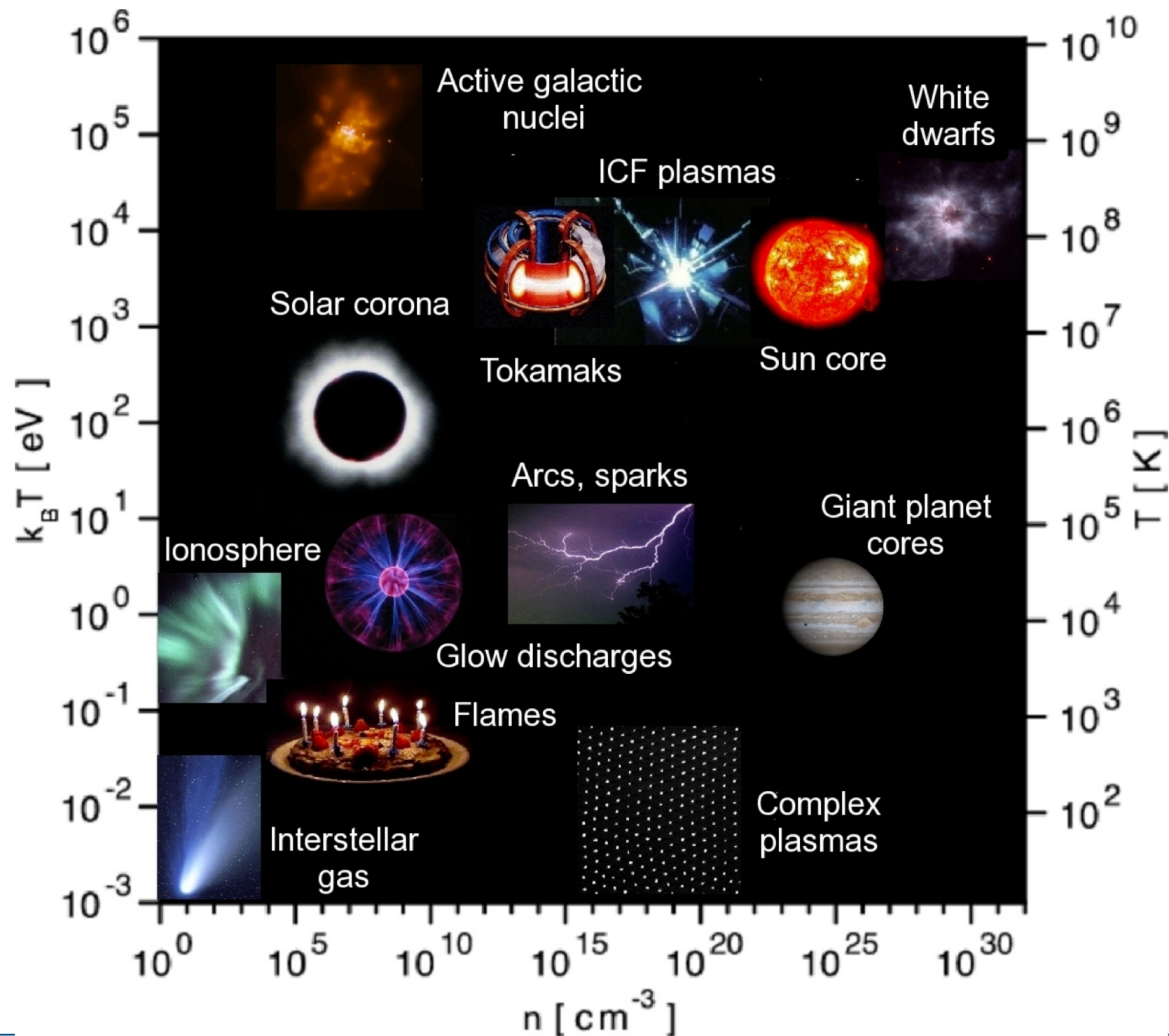


# Basic Assumptions

- Upstream from the laser or beam driver, each fluid can be treated as at rest or cold
- For **underdense plasmas**, the phase velocity of the excited wakefields is roughly the speed of light,  $c$ 
  - Compared with this speed, all the initial thermal velocities of plasma particles can be treated as zero
- Wavelike assumption
  - Fields in the wake depend on the variable  $ct-z$ , where the phase velocity of the wave is essentially  $c$
- Quasi-static approximation
  - The driver evolves on a time scale much longer than the plasma response
    - **the driver can be consider as non-evolving when calculating the plasma response.** Therefore, the wake depends weakly on the distance the driver has moved into the plasma

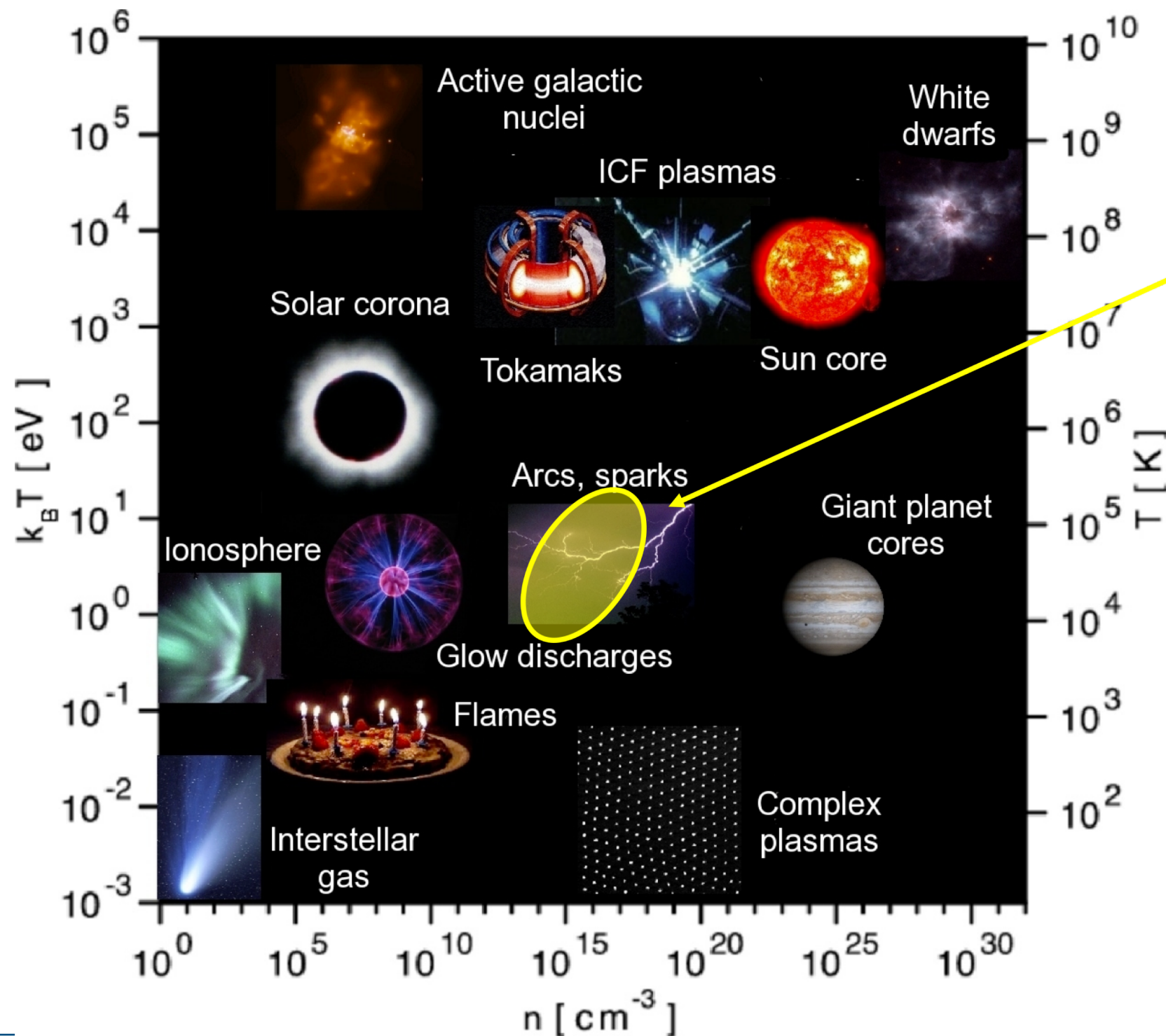
# Plasma States

*In Nature and in Laboratory*



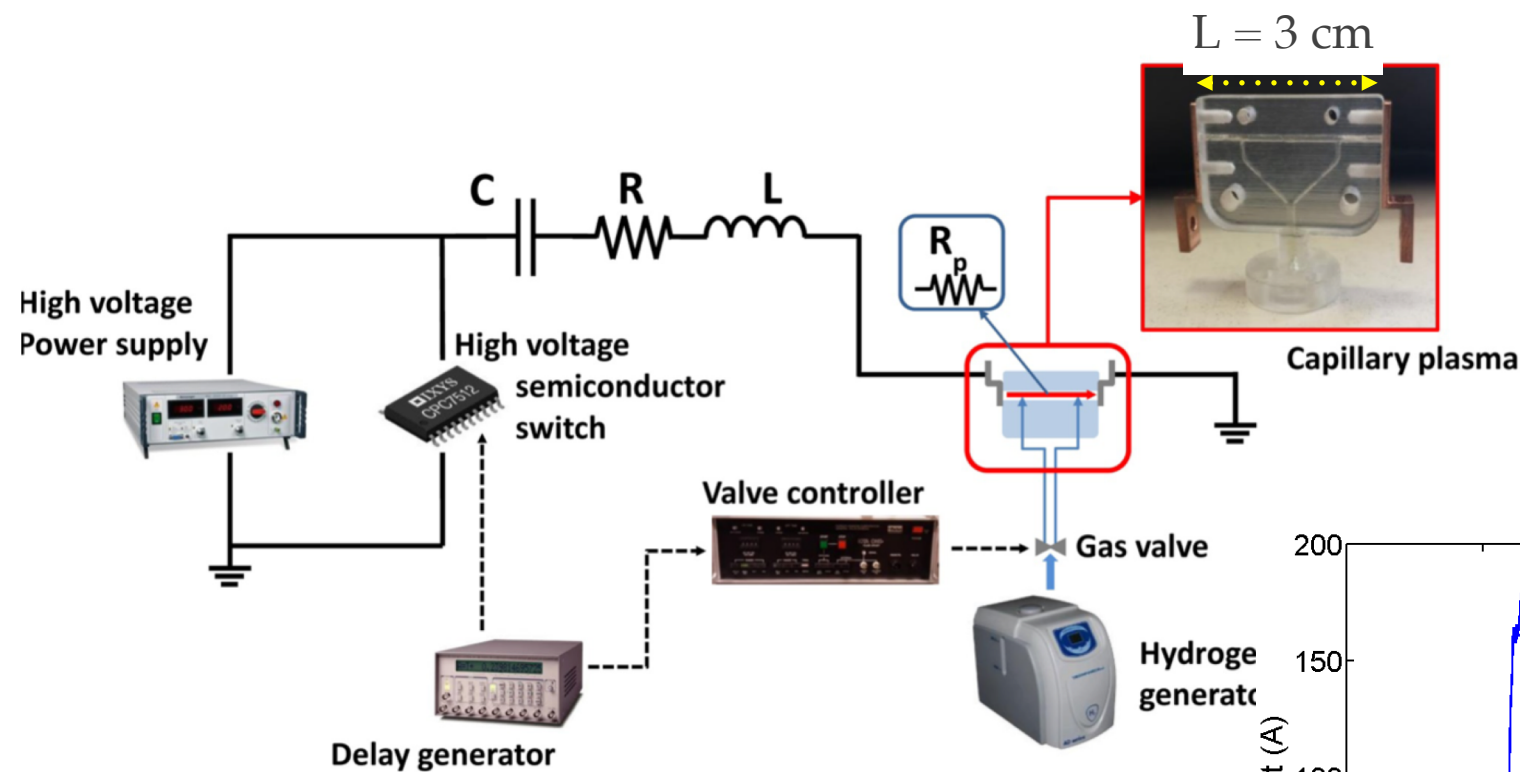
# Plasma States

*In Nature and in Laboratory*



# Plasma Source

## Gas-filled capillary



$P_{H_2} = 10$  mbar

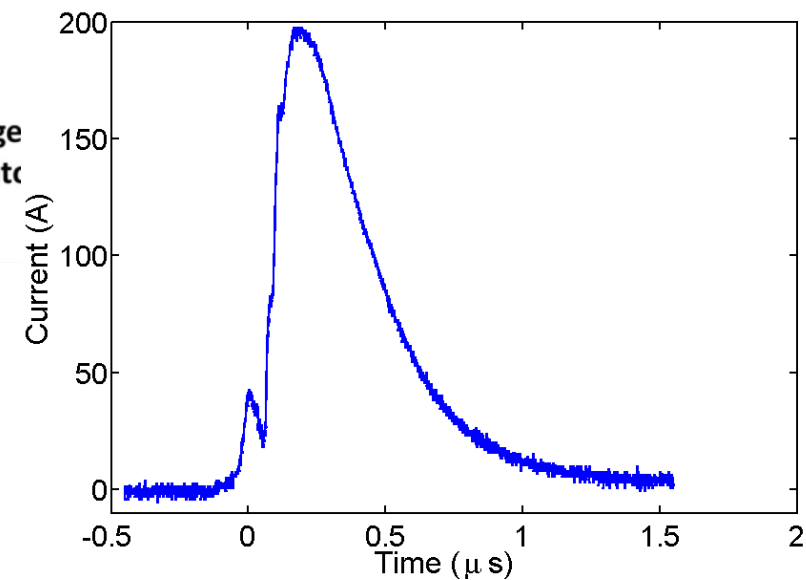
Total discharge duration : 800 ns

Voltage : 20 kV

Peak current : 200 A

Capacitor : 6 nF

### H<sub>2</sub>-filled capillary discharge



Courtesy of A. Biagioni (INFN - LNF)

# Response of a homogeneous plasma to a high frequency field

- Let's consider a charge  $q$  in an oscillating electric field with a non-uniform envelope

$$\vec{E}(\vec{r}, t) = \vec{E}_s(\vec{r}, t) \cos(\omega t) \quad \text{Laser Field (LWFA)}$$

1. Slowly varying envelope approximation (**SVEA**):  $\vec{E}_s(\vec{r}, t) \approx \vec{E}_s(\vec{r})$

2. **Non relativistic** equation of motion  $m \frac{d\vec{v}}{dt} = q \left[ \vec{E}(\vec{r}, t) + \vec{v} \times \vec{B}(\vec{r}, t) \right]$

$$\vec{v} \times \vec{B}(\vec{r}, t) \ll \vec{E}(\vec{r}, t) \quad \text{but not negligible (II order theory)}$$

3. Position of  $q$  = “**slow**” drift + “**fast**” oscillation

$$\vec{r}(t) = \vec{r}_0(t) + \delta\vec{r}_1(t)$$

$$\delta\vec{r}_1(t) \ll \vec{r}_0(t) \quad |\vec{v}_0| \ll |\vec{v}_1| \quad \left| \frac{d\vec{v}_0}{dt} \right| \ll \left| \frac{d\vec{v}_1}{dt} \right|$$

Hypotheses



# Fast Oscillations of Electrons

- Let's first neglect the **B** field and Taylor expand the **E** field around  $\vec{r}_0(t)$

$$m \frac{d\vec{v}}{dt} = m \frac{d\vec{v}_0}{dt} + m \frac{d\vec{v}_1}{dt} = q \left[ \underbrace{\vec{E}_s(\vec{r}_0) + (\delta\vec{r}_1 \cdot \nabla) \vec{E}_s(\vec{r}_0)}_{\approx \vec{E}_s(\vec{r}_0)} \right] \cos(\omega t)$$

Hyp. III

$$m \frac{d\vec{v}_1}{dt} \approx q \vec{E}_s(\vec{r}_0) \cos(\omega t)$$

$$\int \downarrow$$

$$\vec{v}_1 = \frac{q}{m\omega} \vec{E}_s(\vec{r}_0) \sin(\omega t)$$

$$\delta\vec{r}_1 = -\frac{q}{m\omega^2} \vec{E}_s(\vec{r}_0) \cos(\omega t)$$

From III Maxwell equation:

$$\nabla \times \vec{E} = \left[ \nabla \times \vec{E}_s(\vec{r}_0) \right] \cos(\omega t) = -\frac{\partial \vec{B}}{\partial t} \xrightarrow{\int} \vec{B}(\vec{r}_0, t) = -\frac{1}{\omega} \left[ \nabla \times \vec{E}_s(\vec{r}_0) \right] \sin(\omega t)$$

The particle motion equation can be written as

$$m \frac{d\vec{v}}{dt} = q \left[ \vec{E}(\vec{r}, t) + \vec{v} \times \vec{B}(\vec{r}, t) \right]$$

$$m \frac{d\vec{v}}{dt} = m \frac{d\vec{v}_0}{dt} + m \frac{d\vec{v}_1}{dt} = \underbrace{q \vec{E}_s(\vec{r}_0) \cos(\omega t)}_{q(\delta\vec{r}_1 \cdot \nabla) \vec{E}_s(\vec{r}_0) \cos(\omega t)} + \left[ \frac{-q^2}{m\omega^2} \left( \vec{E}_s(\vec{r}_0) \cdot \nabla \right) \vec{E}_s(\vec{r}_0) \cos^2(\omega t) \right. \\ \left. - \frac{-q^2}{m\omega^2} \underbrace{\vec{E}_s(\vec{r}_0) \times \left( \nabla \times \vec{E}_s(\vec{r}_0) \right) \sin^2(\omega t)}_{q\vec{v}_1 \times \vec{B}(\vec{r}, t)} \right]$$

$$m \frac{d\vec{v}_0}{dt} = \frac{-q^2}{m\omega^2} \left[ \left( \vec{E}_s(\vec{r}_0) \cdot \nabla \right) \vec{E}_s(\vec{r}_0) \cos^2(\omega t) + \vec{E}_s(\vec{r}_0) \times \left( \nabla \times \vec{E}_s(\vec{r}_0) \right) \sin^2(\omega t) \right]$$

# Ponderomotive Force

$$\left\langle m \frac{d\vec{v}_0}{dt} \right\rangle_T = \frac{-q^2}{2m\omega^2} \left[ \left( \vec{E}_s(\vec{r}_0) \cdot \nabla \right) \vec{E}_s(\vec{r}_0) + \vec{E}_s(\vec{r}_0) \times \left( \nabla \times \vec{E}_s(\vec{r}_0) \right) \right]$$

$$\frac{1}{2} \nabla [E_s(\vec{r}_0)^2]$$

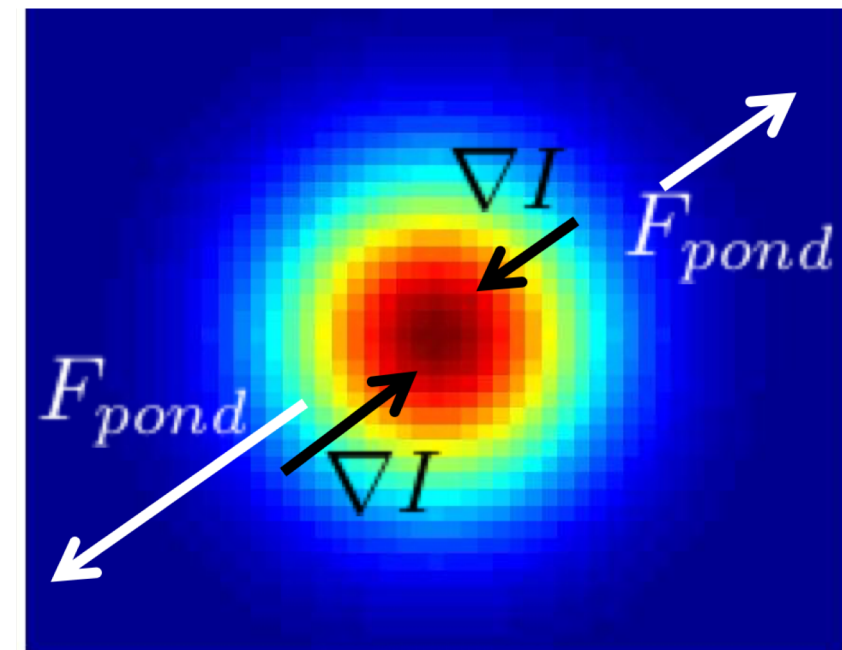
$$\left\langle m \frac{d\vec{v}}{dt} \right\rangle_T = \left\langle m \frac{d\vec{v}_0}{dt} \right\rangle_T = \frac{-q^2}{4m\omega^2} \nabla [E_s(\vec{r}_0)^2]$$

**Ponderomotive force**

**Ponderomotive force per unit volume**

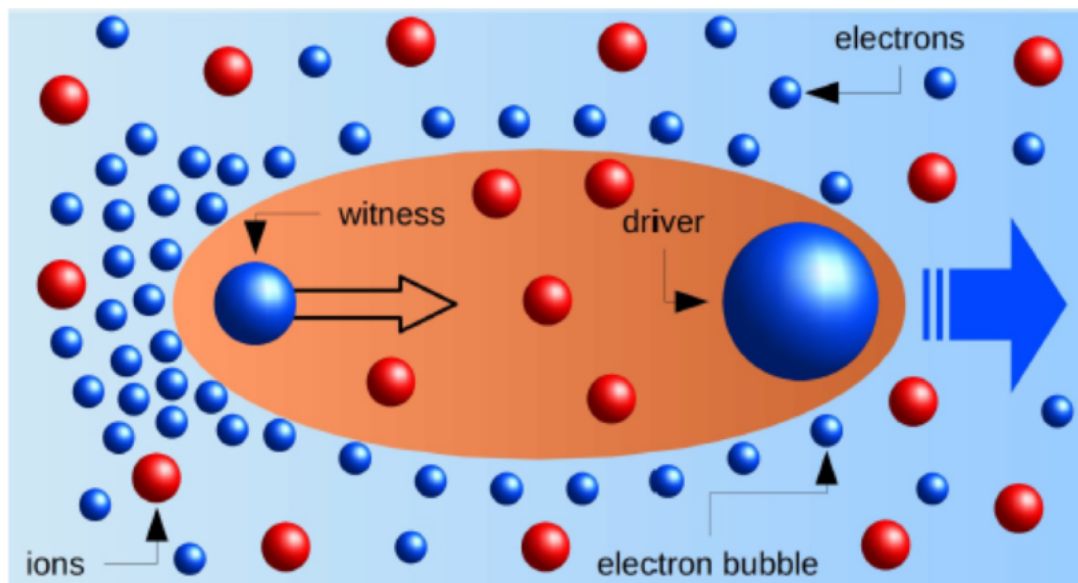
$$n \frac{-q^2}{4m\omega^2 \epsilon_0} \nabla [\epsilon_0 E_s(\vec{r}_0)^2] = -\frac{\omega_p^2}{\omega^2} \nabla \left[ \frac{\langle \epsilon_0 E^2 \rangle}{2} \right]$$

$$F_{pond} \propto -q^2 \frac{\nabla(\text{wave intensity})}{m} = -q^2 \frac{\nabla I}{m}$$





# Acceleration Mechanism



- ❖ The **driver**, creating the bubble, can be either a
  - ❖ **dense relativistic particle beam (PWFA)** of sub-ps duration and kA level peak current
  - ❖ **ultra-intense laser pulse (LWFA)**,  $\sim 10^{18} \text{ W/cm}^2$ , of few 10s fs duration
- ❖ The **witness** can be either **self-injected** or **externally injected**
- ❖ Rapid acceleration of injected electrons to ultra-relativistic energies, inside the micrometer-sized structured plasma environment
  - ❖ the trapped electron beam remains short, dense, and free of significant space-charge driven emittance degradation

# The Dawn of Compact Accelerators

September 2004

## Monoenergetic beams of relativistic electrons from intense laser-plasma interactions

S. P. D. Mangles<sup>1</sup>, C. D. Murphy<sup>1,2</sup>, Z. Najmudin<sup>1</sup>, A. G. R. Thomas<sup>1</sup>, J. L. Collier<sup>2</sup>, A. E. Dangor<sup>1</sup>, E. J. Divall<sup>2</sup>, P. S. Foster<sup>2</sup>, J. G. Gallacher<sup>3</sup>, C. J. Hooker<sup>2</sup>, D. A. Jaroszynski<sup>3</sup>, A. J. Langley<sup>2</sup>, W. B. Mori<sup>4</sup>, P. A. Norreys<sup>2</sup>, F. S. Tsung<sup>4</sup>, R. Viskup<sup>3</sup>, B. R. Walton<sup>1</sup> & K. Krushelnick<sup>1</sup>

## High-quality electron beams from a laser wakefield accelerator using plasma-channel guiding

C. G. R. Geddes<sup>1,2</sup>, Cs. Toth<sup>1</sup>, J. van Tilborg<sup>1,3</sup>, E. Esarey<sup>1</sup>, C. B. Schroeder<sup>1</sup>, D. Bruhwiler<sup>4</sup>, C. Nieter<sup>4</sup>, J. Cary<sup>4,5</sup> & W. P. Leemans<sup>1</sup>

## A laser-plasma accelerator producing monoenergetic electron beams

J. Faure<sup>1</sup>, Y. Glinec<sup>1</sup>, A. Pukhov<sup>2</sup>, S. Kiselev<sup>2</sup>, S. Gordienko<sup>2</sup>, E. Lefebvre<sup>3</sup>, J.-P. Rousseau<sup>1</sup>, F. Burgy<sup>1</sup> & V. Malka<sup>1</sup>





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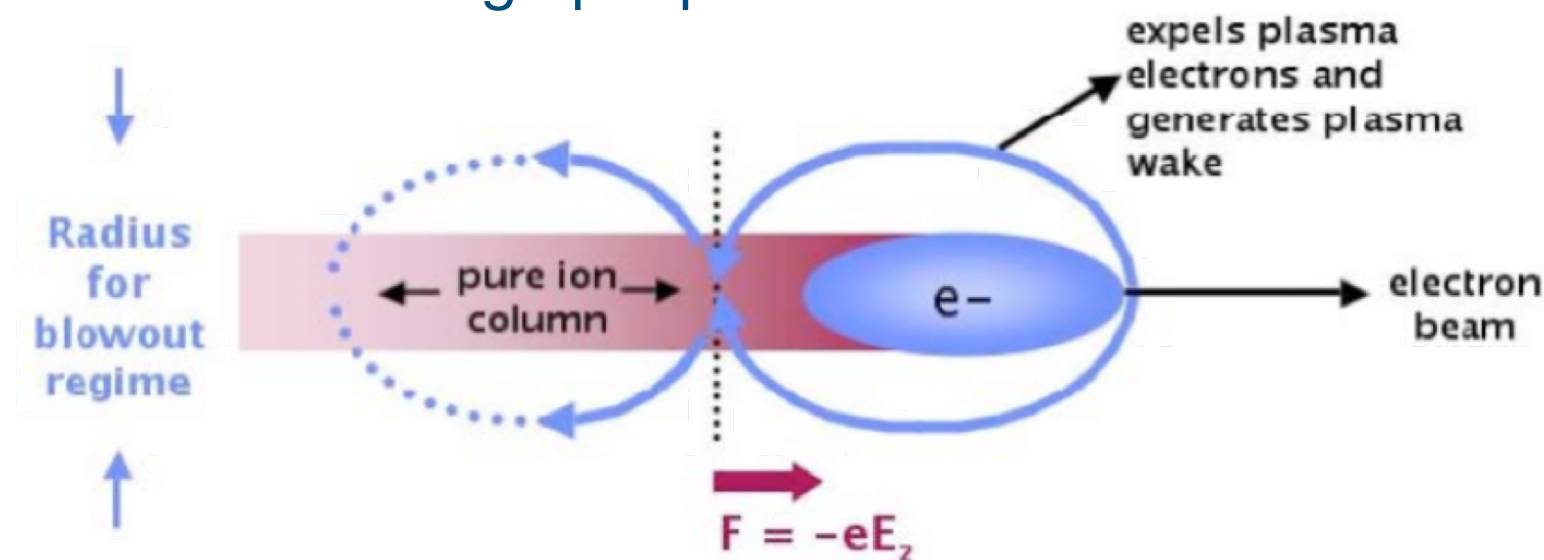
See Malka's talk for an insight on Laser Wakefield Acceleration



# Particle-driven Plasma Wakefield Acceleration

## *Single bunch*

- The high-gradient wakefield is driven by an intense, high-energy charged particle beam as it passes through the plasma.
- The space-charge of the electron bunch blows out plasma electrons which rush back in and overshoot setting up a plasma oscillation

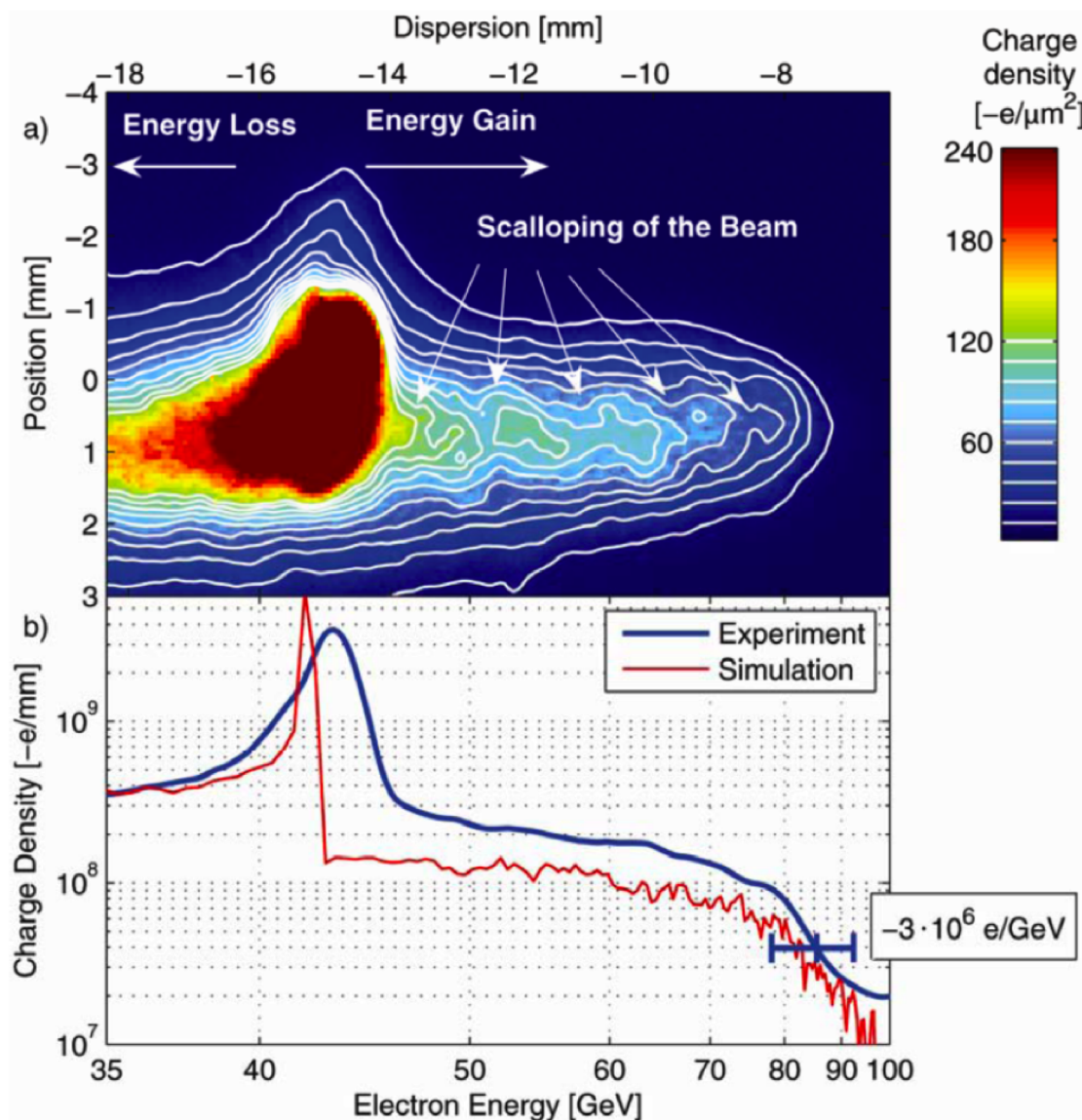
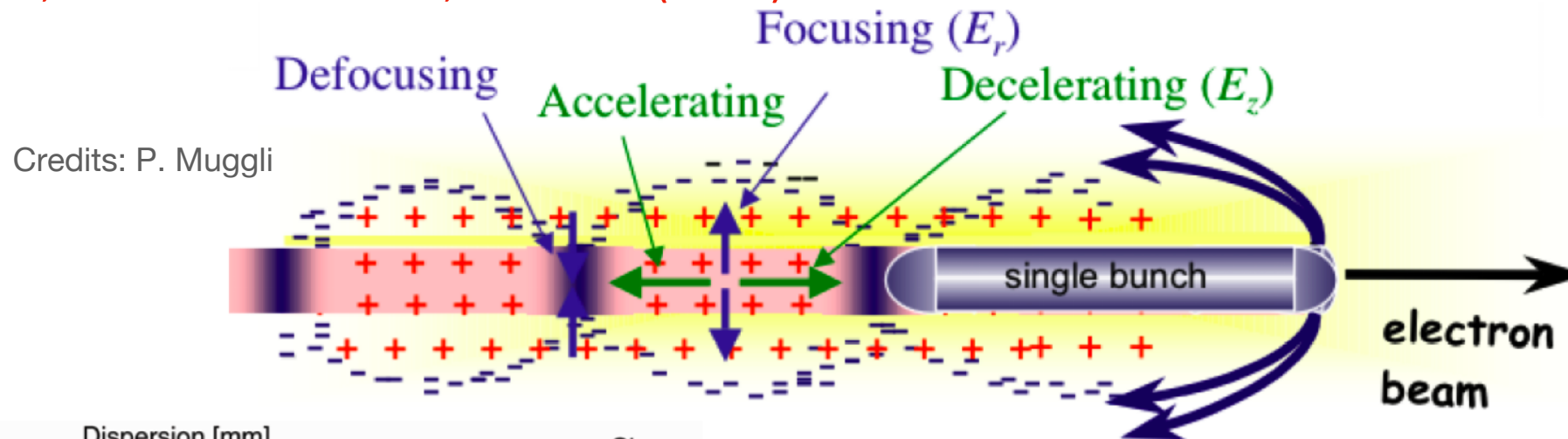


- First demonstration of the excitation of a wakefield by a relativistic beam in the linear regime, i.e. beam density typically less than the plasma density => J. Rosenzweig et al., Phys. Rev. Lett. 61, 98 (1988)
- Peak acceleration gradient  $\sim 1.6$  MeV/m, but the experiment clearly showed the wakefield persisting for several plasma wavelengths



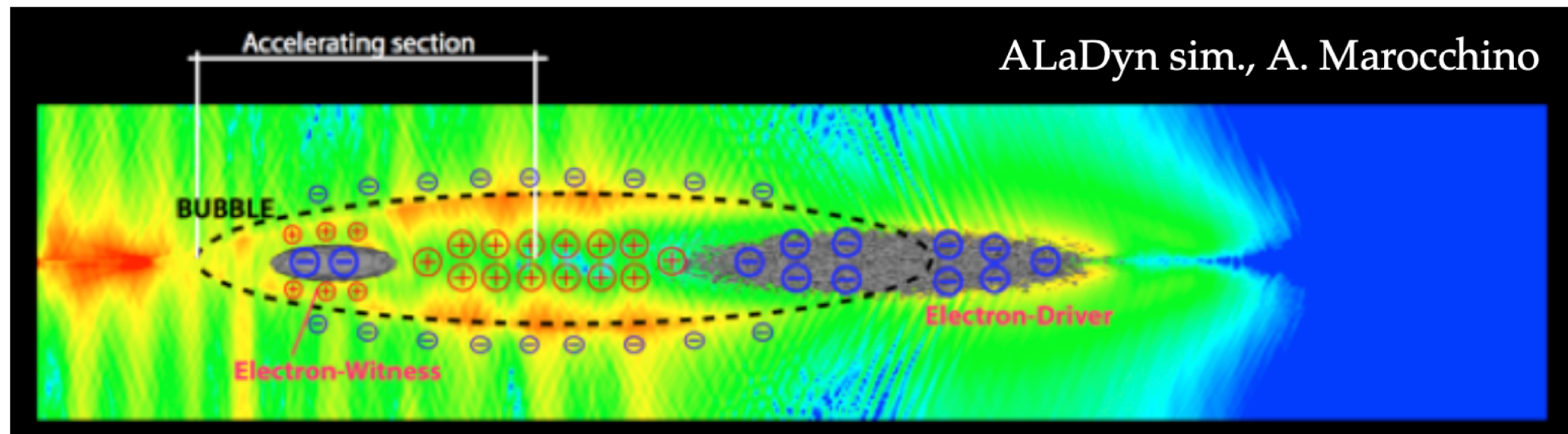
# PWFA: Energy doubling in a meter scale

Blumenfeld, I. et al. *Nature* 445, 741–744 (2007)



- Gaussian electron beam with 42 GeV, 3nC @ 10 Hz,  $s_x = 10\mu\text{m}$ , 50 fs
- 85cm Lithium vapour source,  $2.7 \times 10^{17} \text{cm}^{-3}$
- ✓ Accelerated electrons **from 42 GeV to 85 GeV in 85 cm**
- ✓ Reached **accelerating gradient of 52 GeV/m**
  - Energy gain of the 3 km-long SLAC linac
- **Single bunch**
  - $\Delta E/E \gg 1\%$

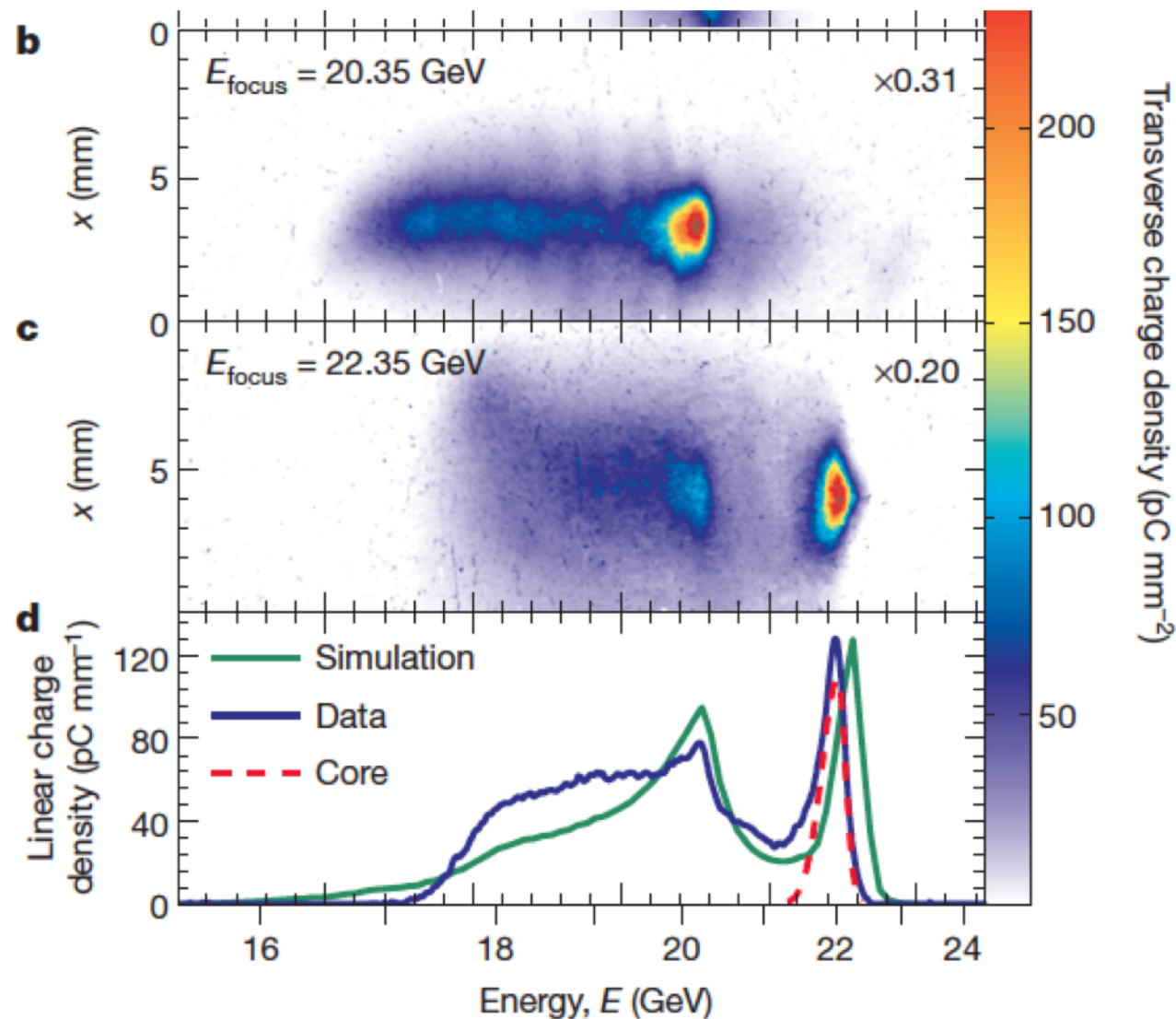
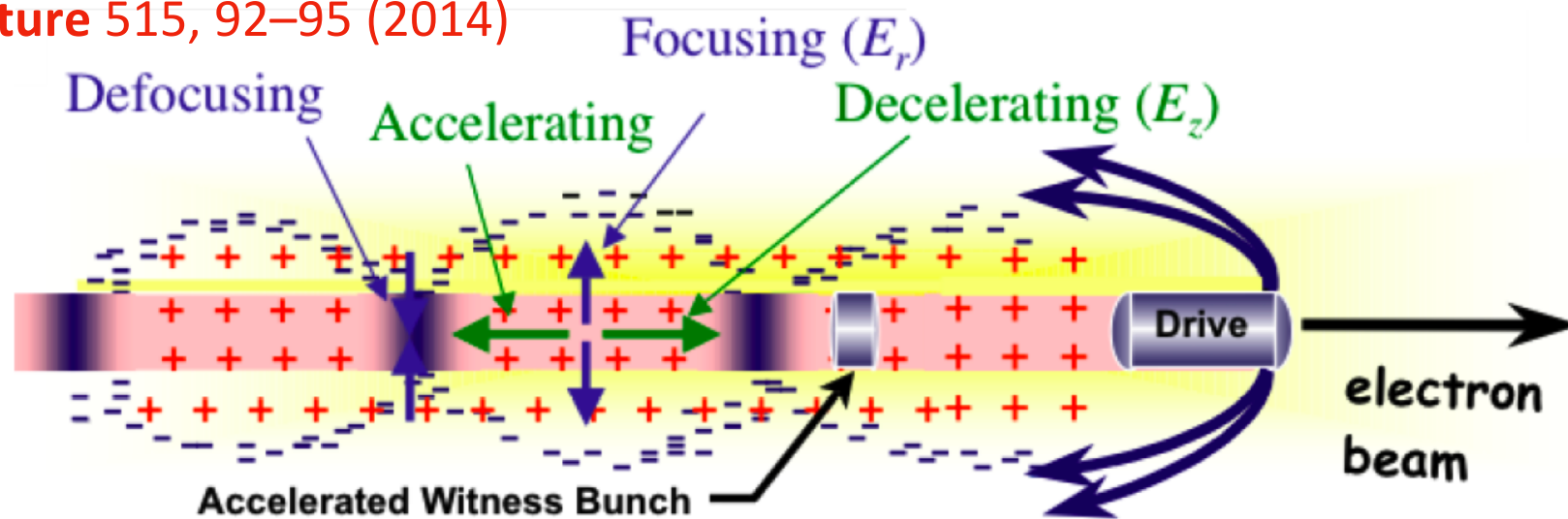
# Two-bunch Train PWFA



- Bunch length of tens of fs down to **fs scale**
- A second, appropriately phased accelerating beam (**witness beam**), containing fewer particles than the **drive beam**, is then accelerated by the wake
  - ▶ Bunch train (D+W) for bunch acceleration ( $\Delta E/E \ll 1$ )

# PWFA: High-efficiency acceleration

Litos, M. et al. *Nature* 515, 92–95 (2014)



- Injection of two beams into the plasma
- One drives the wake, *driver*, one samples the wake, *witness*
- Beam loading is key for
  - Narrower energy spread,  $\Delta E/E \ll 1\%$ , and high efficiency



# Limitation of PWFA

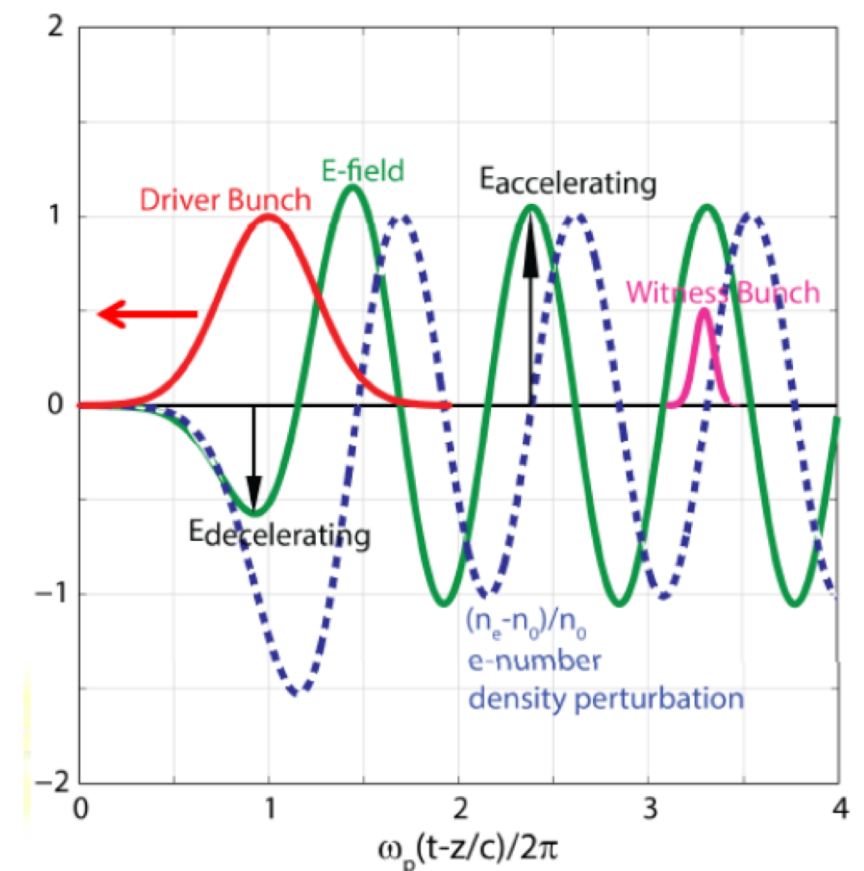
## Energy gain

- **PWFA** acting as an **energy transformer** has the great potential to double beam energy in a single stage
- The energy transfer from the drive bunch to the plasma is optimized by maximizing the transformer ratio

$$R = \frac{|E_{+,max}|}{|E_{-,max}|}$$

### Wakefield theorem\*

Symmetric drive bunch current profile in a single-mode structure: the **maximum accelerating field** behind the drive bunch **cannot exceed 2 times the maximum decelerating field** amplitude along the drive bunch



\*V. V. Tsakanov, Nucl. Instrum. Methods Phys. Res., Sect. A 432, 202 (1999)

F. Massimo et al., NIM A **740**, 242–245 (2014)

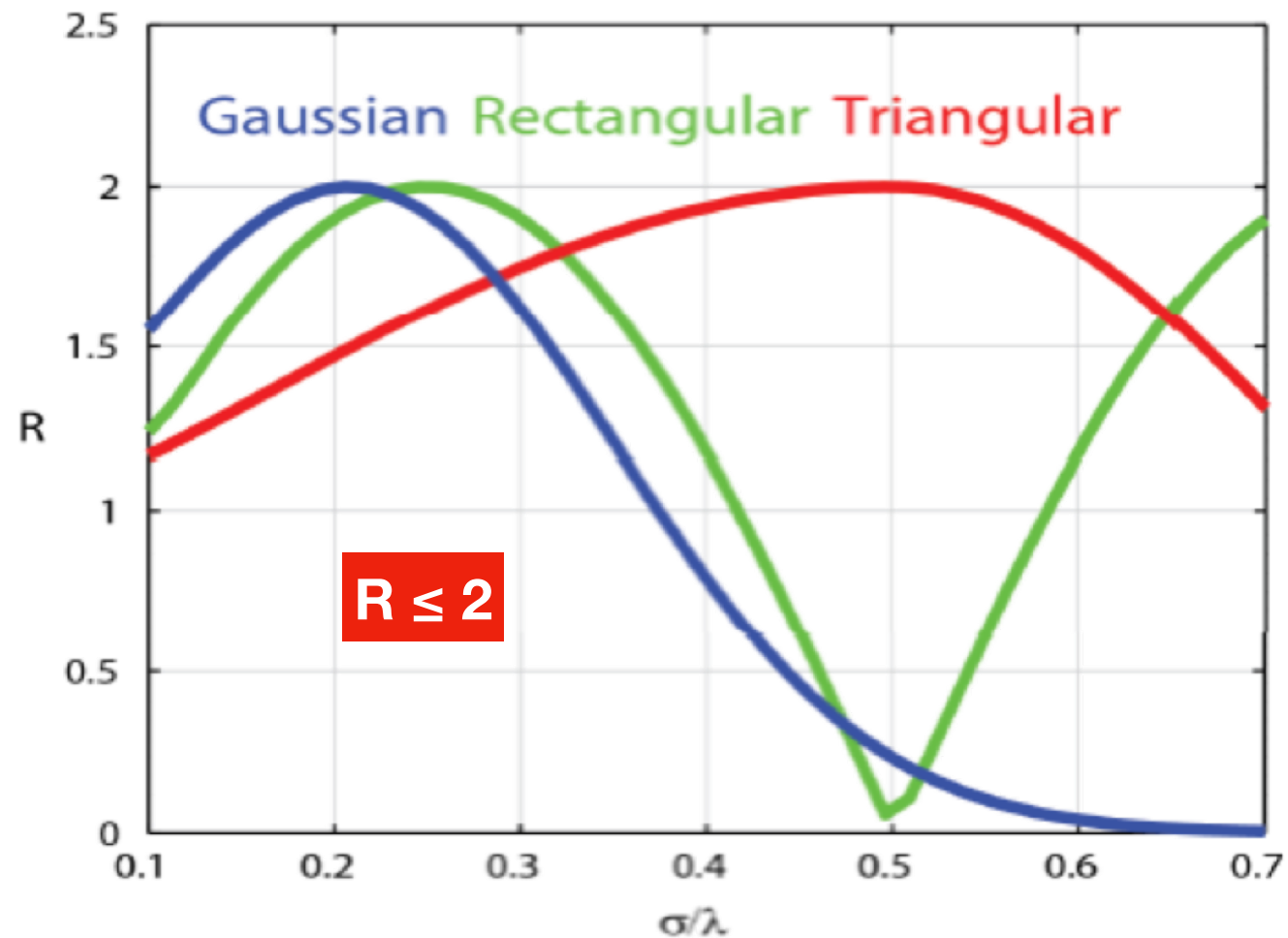
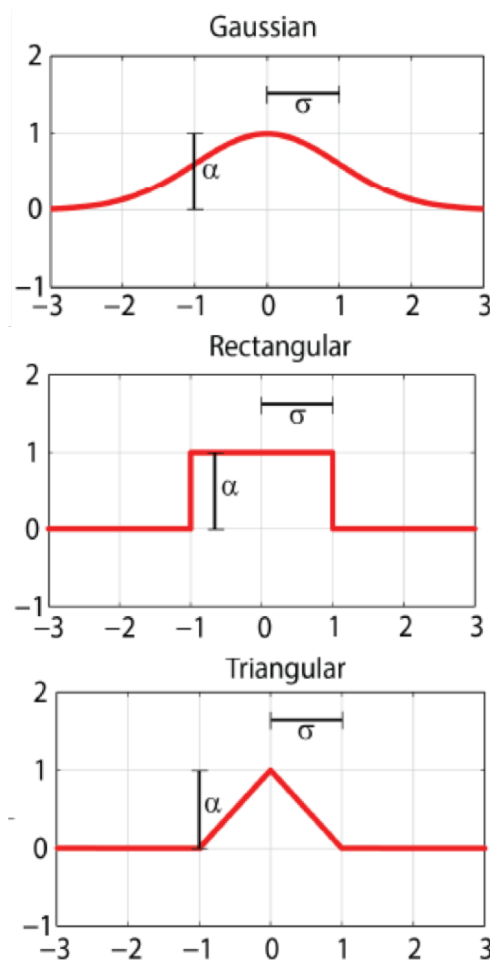


# Limitation of PWFA

## Energy gain

The transformer ratio critically depends on the bunch shape and on the density ratio

Linear regime:  $\alpha = \frac{n_{driver,peak}}{n_0} = 10^{-4} \ll 1$



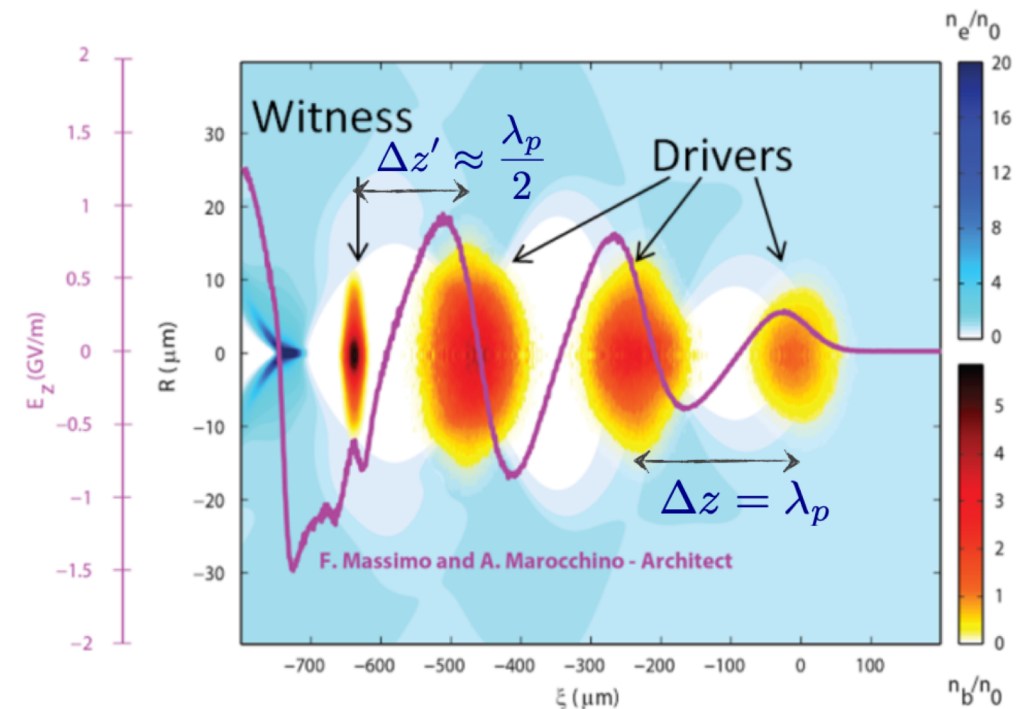
F. Massimo et al., NIM A **740**, 242–245 (2014)

# Enhancing Transformer Ratio

- By properly **tailoring the driver bunch shape**, the witness beam energy might be more than doubled when

*The maximum possible transformer ratio for a bunch with given length and total charge corresponds to that charge distribution which causes all particles in the bunch to see the same retarding field\**

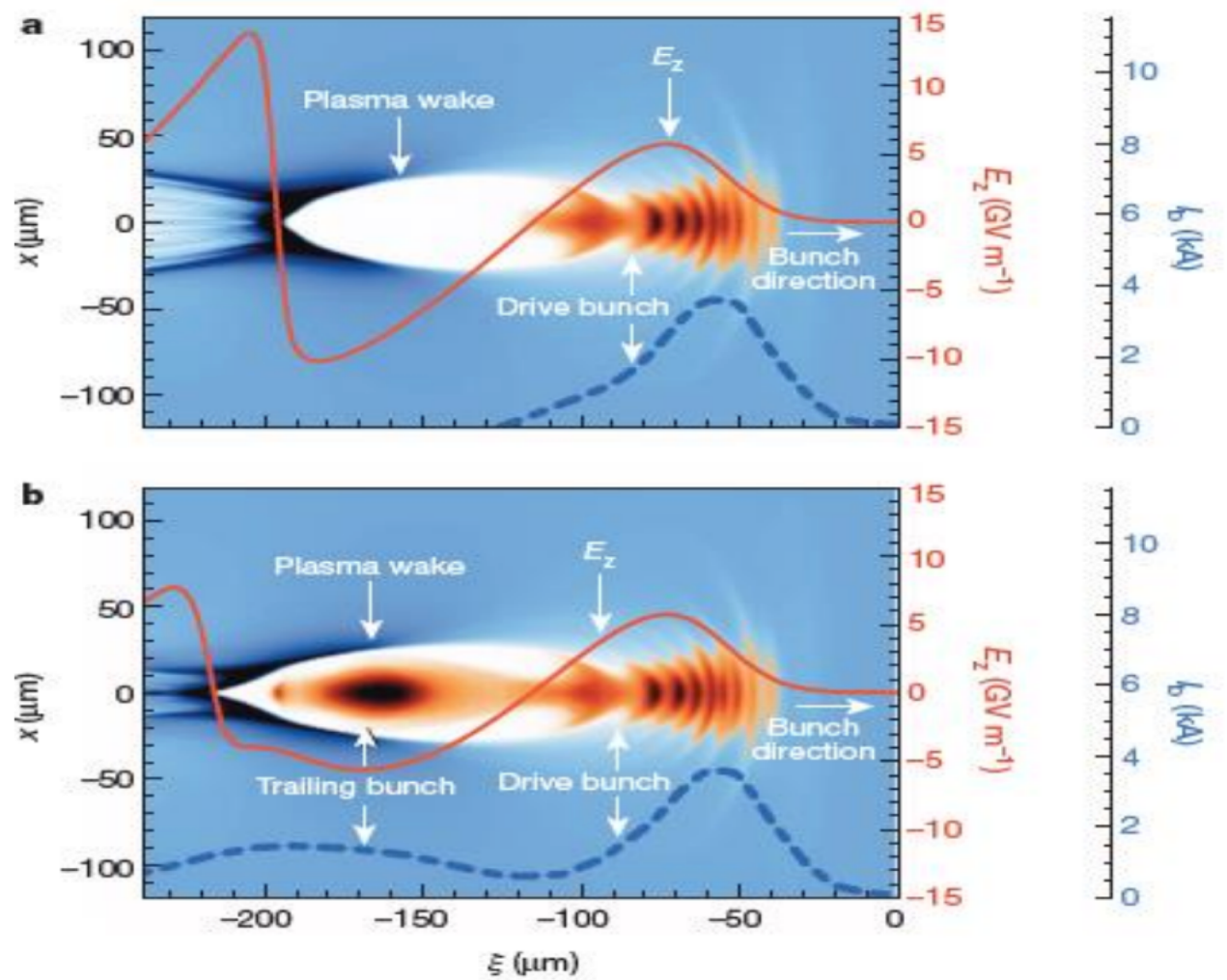
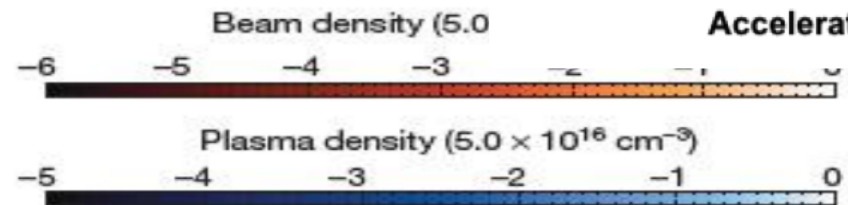
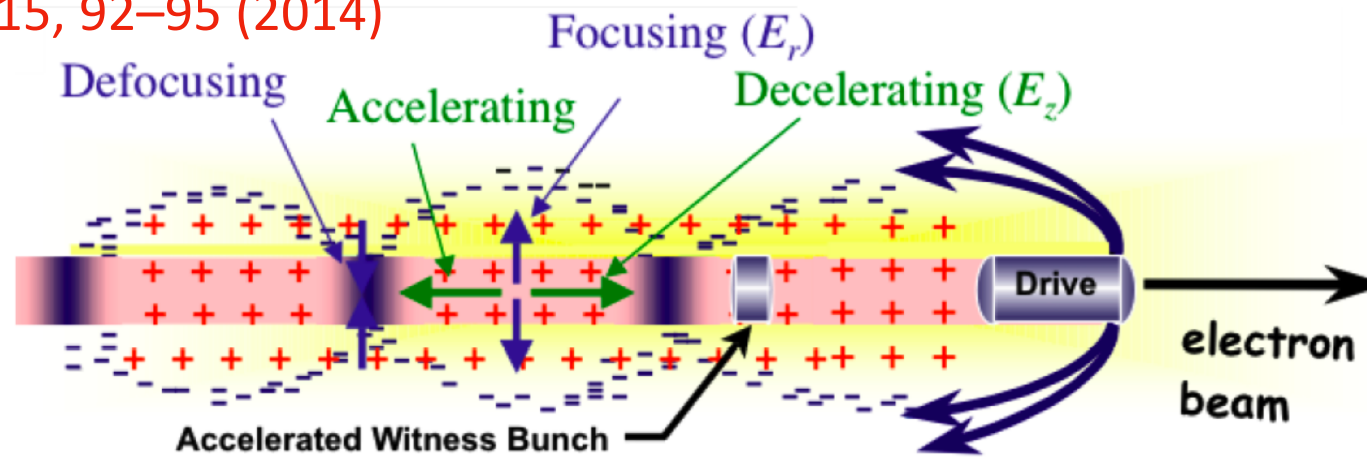
- Tailoring longitudinal current profile such that all longitudinal slices lose energy at the same rate
- Asymmetric drive bunch current profile**, i.e. triangular, double triangle, doorstep-like distributions, or **multiple ramped bunch trains**, overcome this limit (R.Ruth et al., PA 1985; W. Lu et al., PAC 2009)



\*K. Bane, P. Chen, and P. B. Wilson, *SLAC-PUB-3662*, 1985

# PWFA: High-efficiency acceleration

Litos, M. et al. *Nature* 515, 92–95 (2014)

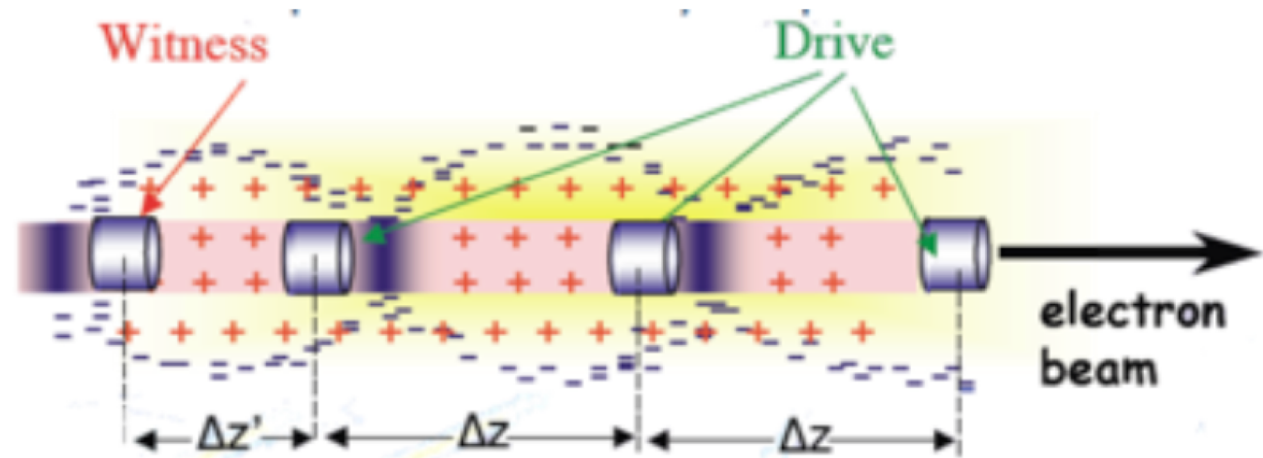


- Electric field in plasma wake is loaded by presence of trailing bunch
- Allows efficient energy extraction from the plasma wake

# Resonant PWFA

## Multi-bunch shaping

**Driver**  $\Delta z = \lambda_p$   
**Witness**  $\Delta z' \approx \frac{\lambda_p}{2}$



- Bunch spacing depends on the plasma density

**Scale length of the plasma wake**  $\lambda_p [\mu m] \approx \frac{3.3 \cdot 10^{10}}{\sqrt{n_0 [cm^{-3}]}}$

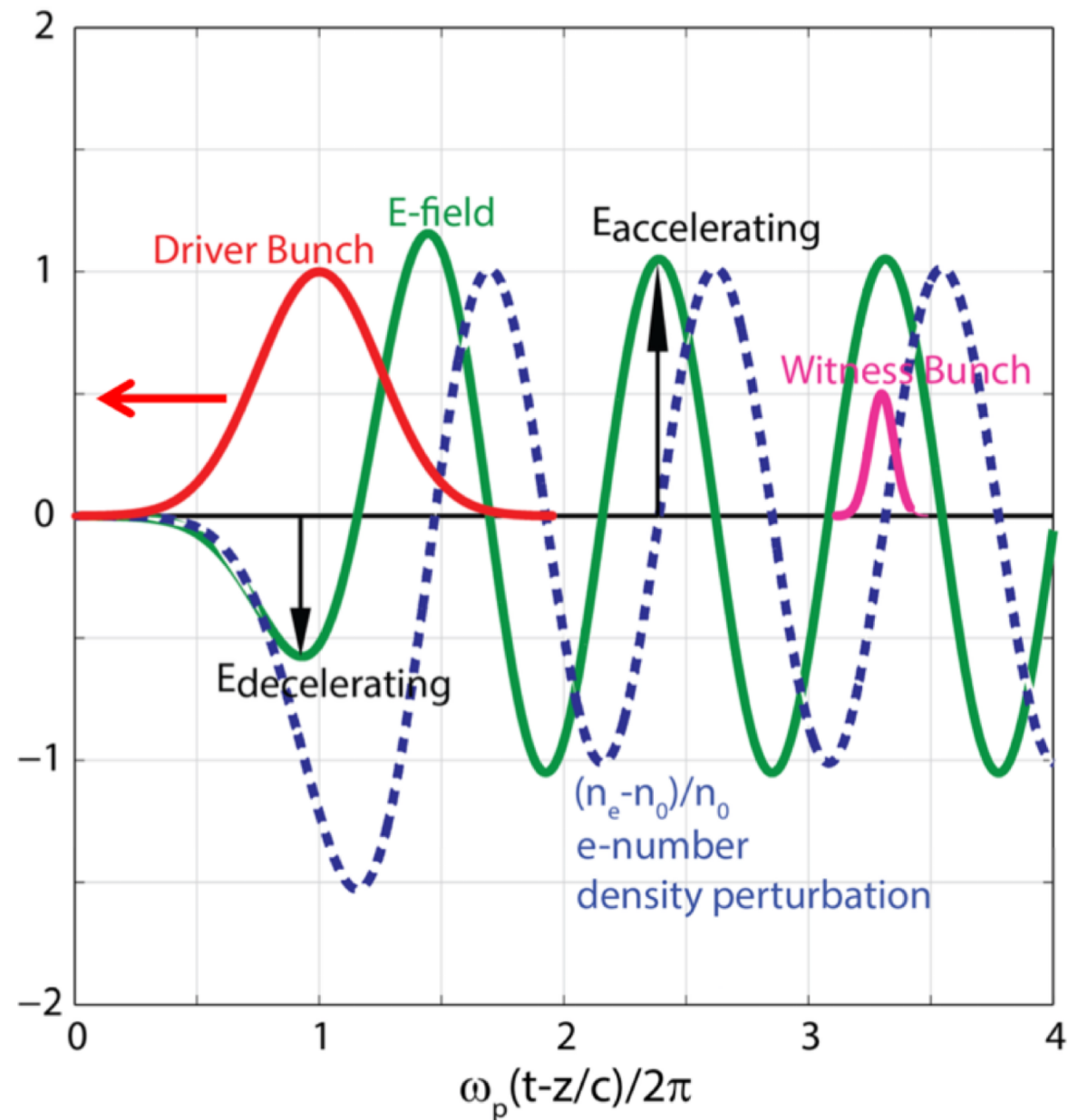
**Accelerating gradient**  $E_z \propto \left( \frac{N}{\sigma_z} \right)^2 N_T \gtrsim GV/m$

- Increase in energy of a trailing particle  $\Delta \gamma m e c^2 \sim e E_{+,max} L d = R \gamma b m e c^2$
- Preservation of witness emittance and length
- Better control of the energy spread



# From Linear Regime: $n_b \ll n_0$

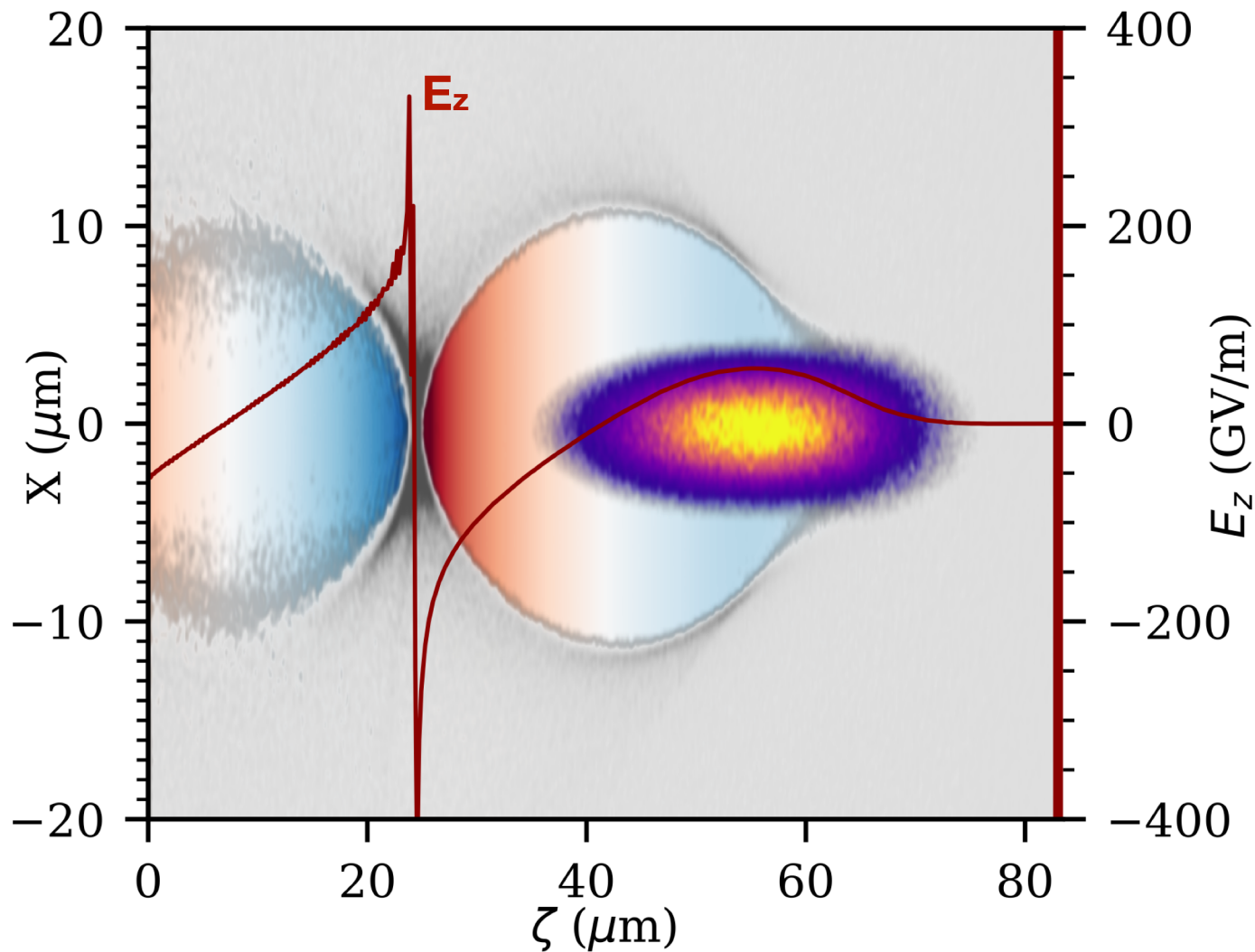
*Focusing force is sinusoidal*



- Lower wakefields
- Transverse forces not linear in  $r$
- ☑ Symmetric for positive and negative witness bunches
- ☑ Well described by theory

# to Quasi-Linear and Non-Linear Regime

*The wake structure depends on the driver pulse «intensity»*



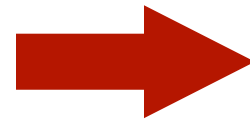
- ☑ Higher wakefields
- ☑ transverse forces linear in  $r$  (emittance preservation)
- ☑ High charge witness acceleration possible
- ☐ Requires more intense drivers
- ☐ Not ideal for positron acceleration

# Quasi-non Linear Regime

- Condition for blow-out  $\frac{n_b}{n_p} > 1$ 
  - Bubble formation w/o wavebreaking,  $\lambda_p$  is constant
  - Resonant scheme in blowout
  - Linear focusing force => emittance preservation

- A measure of non-linearity is the normalised charge

$$\tilde{Q} \equiv \frac{N_b k_p^3}{n_p} = 4\pi k_p r_e N_b$$



**<<1 linear regime**  
**>1 blowout regime**



- Using low emittance, high brightness beams

$$\tilde{Q} < 1 \text{ and } \frac{n_b}{n_p} > 1$$

- **Quasi-non Linear Regime**


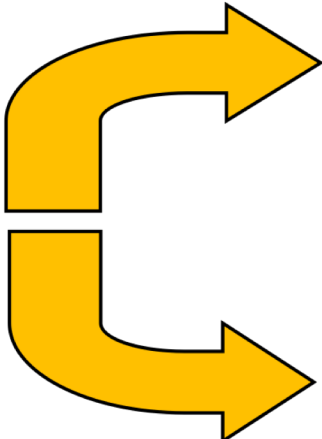
$$n_p = 10^{16} \text{ cm}^{-3}, Q_b = 200 \text{ pC}, \sigma_t = 180 \text{ fs}, \sigma_x = 5.5 \mu\text{m} \Rightarrow n_b \approx 5 n_p \text{ and } \tilde{Q} \approx 0.8$$

# High Quality Beams


$$L = \frac{N_{e+} N_{e-} f_r}{4\pi\sigma_x\sigma_y}$$


- N of particles per pulse =>  $10^9$
- High rep. rate**  $f_r$  => bunch trains

-Small spot size => **low emittance**


$$B_n \approx \frac{2I}{\epsilon_n^2}$$


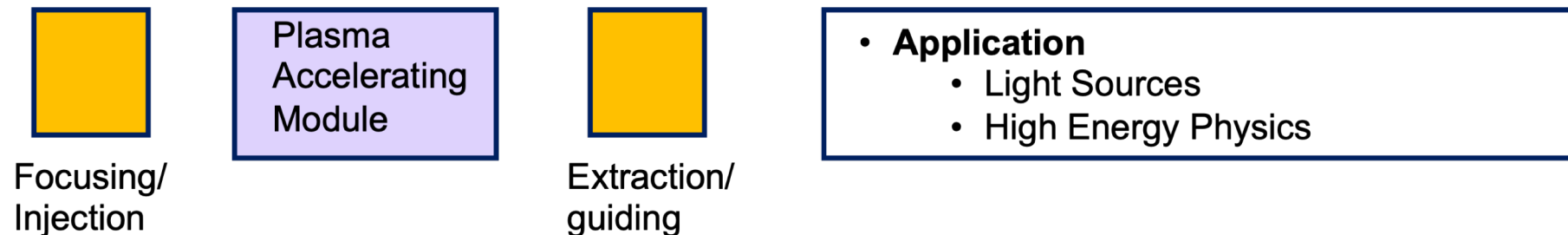
-**Short pulse** (ps to fs)

-Little spread in transverse momentum and angle => **low emittance**



# Towards the Applications

- Multi-GeV acceleration of high brightness electron beams in cm-scale plasma structures



- Injection and matching to plasma accelerating module
  - The beam has to be focused to the matching spot to prevent envelope oscillations that may cause emittance growth

- Blow-out regime**

$$\beta_{matching} = \frac{\sqrt{2\gamma}}{k_p} \quad \alpha_{matching} = 0$$

## Typical numbers

$$k_p = \frac{2\pi}{\lambda_p} \quad \gamma = 1000$$

$$\lambda_p (\mu m) \approx \frac{3.3 \cdot 10^{10}}{\sqrt{n_0 (cm^{-3})}} \quad n_0 = 10^{16} cm^{-3}$$

$$\varepsilon_n = 1 mmmrad$$

## Matching condition

$$\sigma_{matching} = \sqrt{\frac{\beta_{matching} \varepsilon_n}{\gamma}} \approx \mu m$$

# Towards the Applications

- Extraction from plasma accelerating module
  - plasma fields stronger than in conventional accelerators

$$G(MT/m) = \frac{F_r}{ecr} \approx 3n(10^{17} \text{ cm}^{-3})$$

- beams experience huge transverse size variation when propagating from the plasma outer surface to the conventional focusing optics

$$\sigma_x \sim \mu m \qquad \sigma_{x'} \sim \text{mrad}$$

- the particle transverse motion becomes extremely sensitive to energy spread
- the beam angular divergence has to be reduced and the transverse spot size increased to limit the chromatic induced emittance degradation in vacuum

$$\varepsilon_n^2 = \langle \gamma \rangle^2 (\sigma_E^2 \sigma_x^2 \sigma_{x'}^2 + \varepsilon^2) \approx \langle \gamma \rangle^2 (\sigma_E^2 \sigma_{x'}^4 s^2 + \varepsilon^2)$$

# Active Plasma Lenses

THE REVIEW OF SCIENTIFIC INSTRUMENTS

VOLUME 21, NUMBER 5

MAY, 1950

## A Focusing Device for the External 350-Mev Proton Beam of the 184-Inch Cyclotron at Berkeley

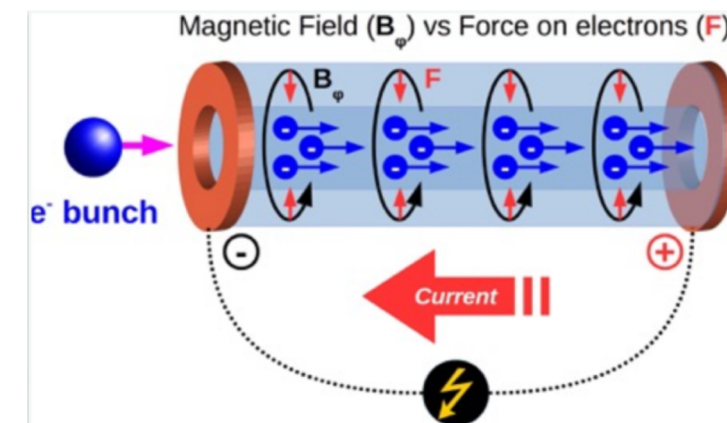
W. K. H. PANOFSKY AND W. R. BAKER  
 Department of Physics, Radiation Laboratory, University of California, Berkeley, California

(Received January 11, 1950)

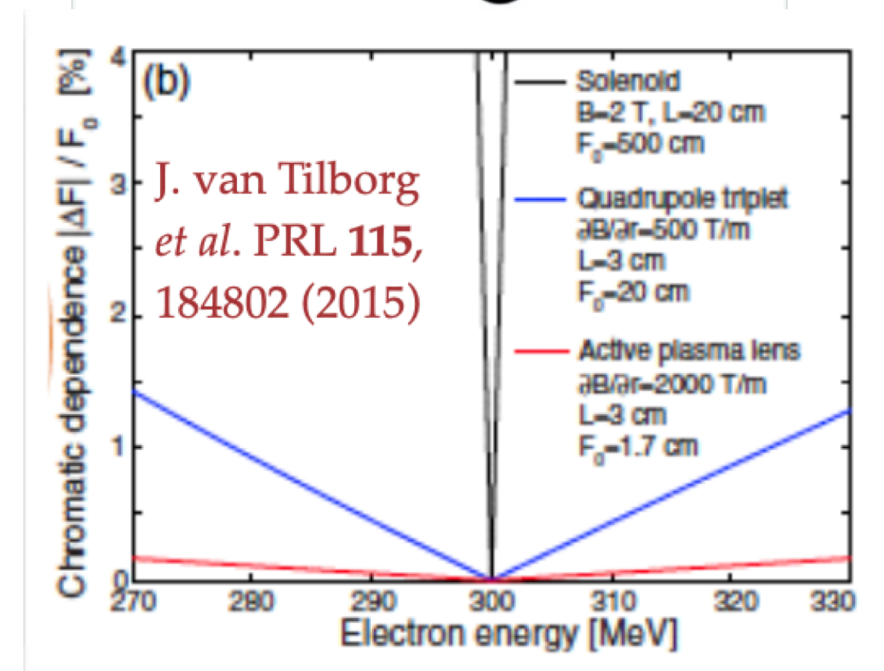
A device has been constructed to focus the external beam of the 184-in. cyclotron at Berkeley. The device consists of a cylindrical tube 4 ft. in length and 3 in. in diameter, which contains a longitudinal arc of nearly uniform current density. Such a device will focus any beam of cylindrical symmetry. Owing to the large power requirements of such a device it is applicable only to very short pulsed beams.

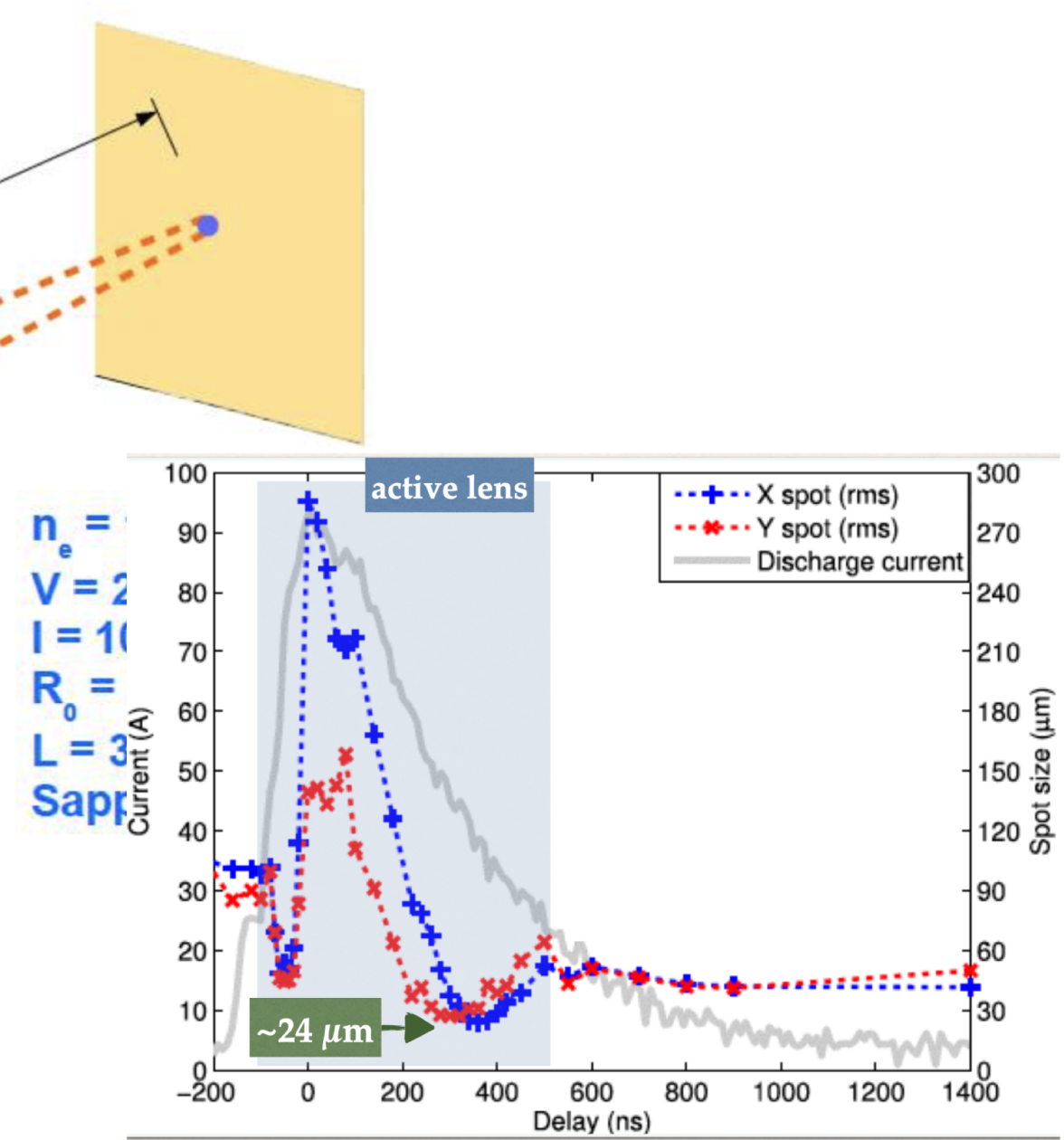
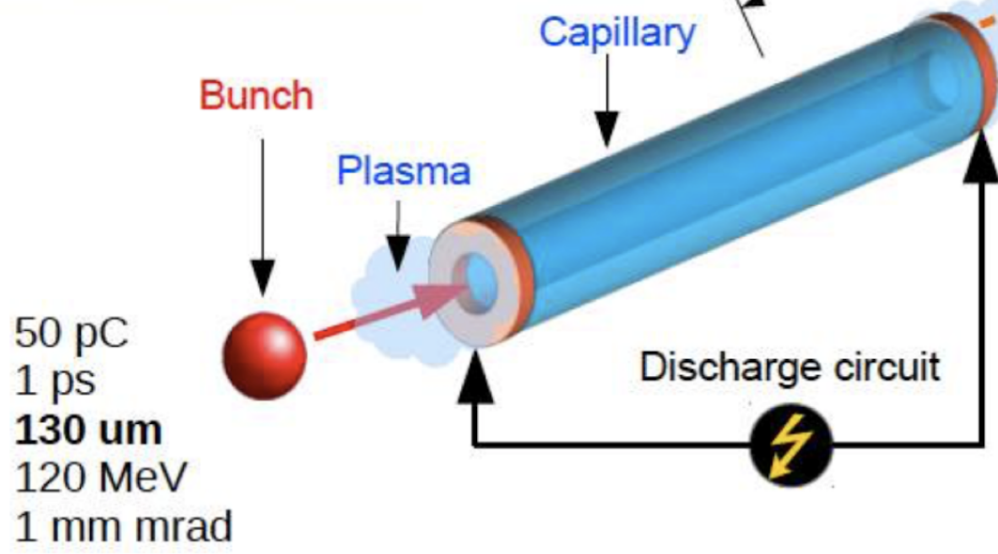
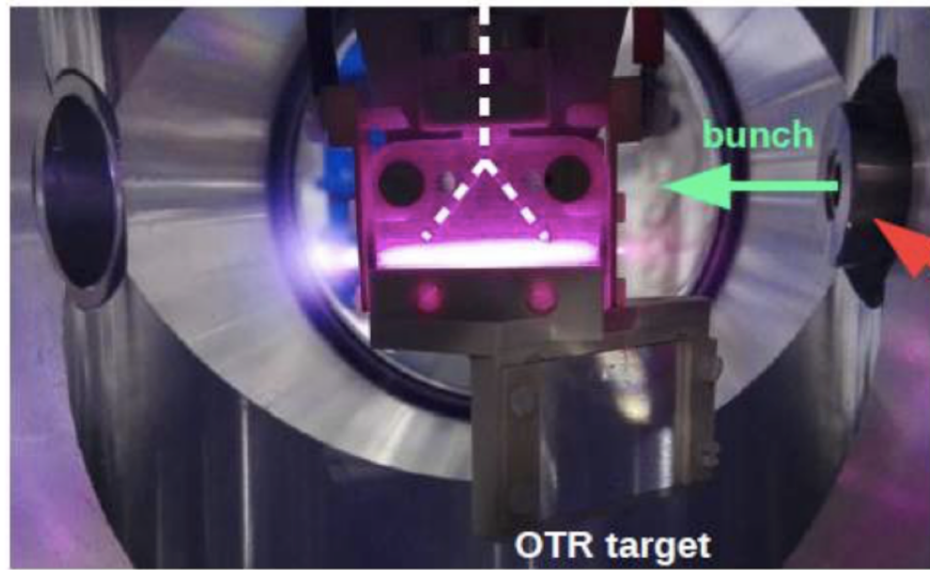
- Discharge current in gas-filled capillary

$$B_{\phi}(r) = \frac{\mu_0}{r} \int_0^r J(r') r' dr'$$



- Cylindrical symmetry
  - purely radial focusing effect
- Tunability
- Focusing strength  $k\alpha\gamma^{-1}$
- High focusing gradient  $\sim$  kT/m
  - short focal length
  - weak chromaticity



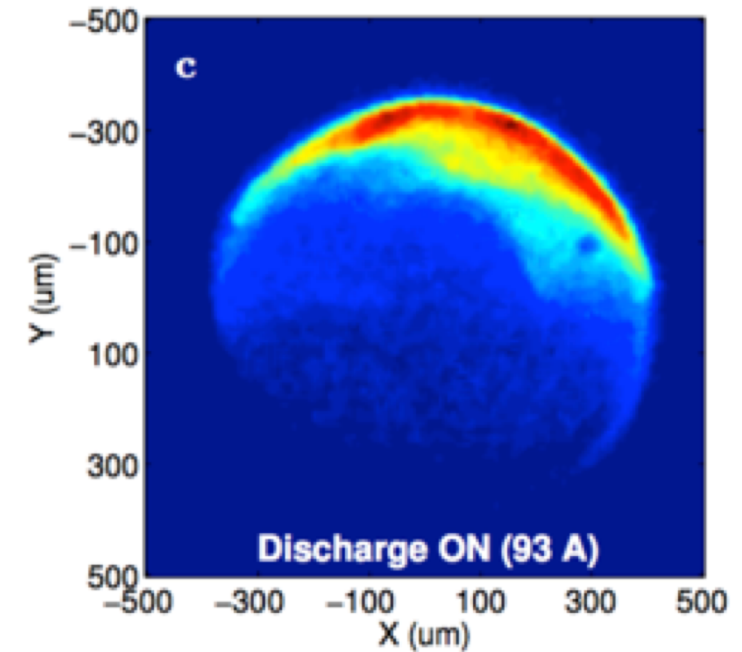
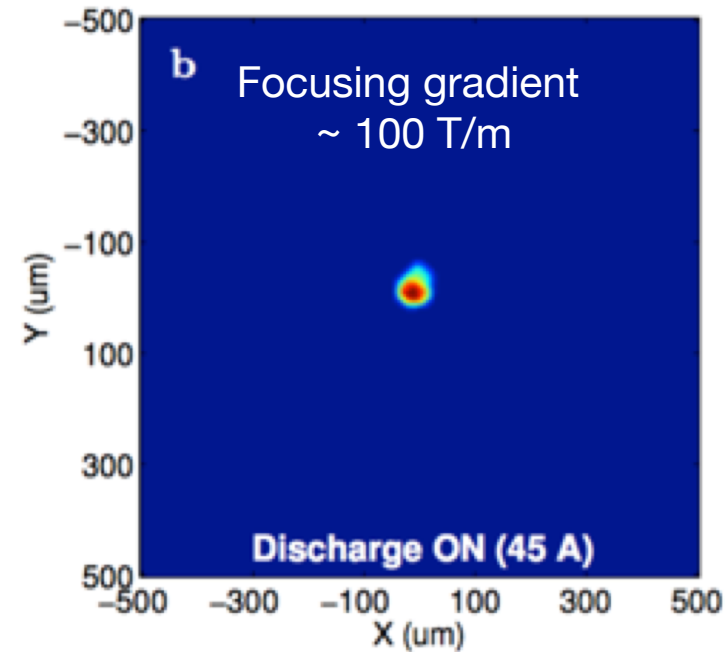
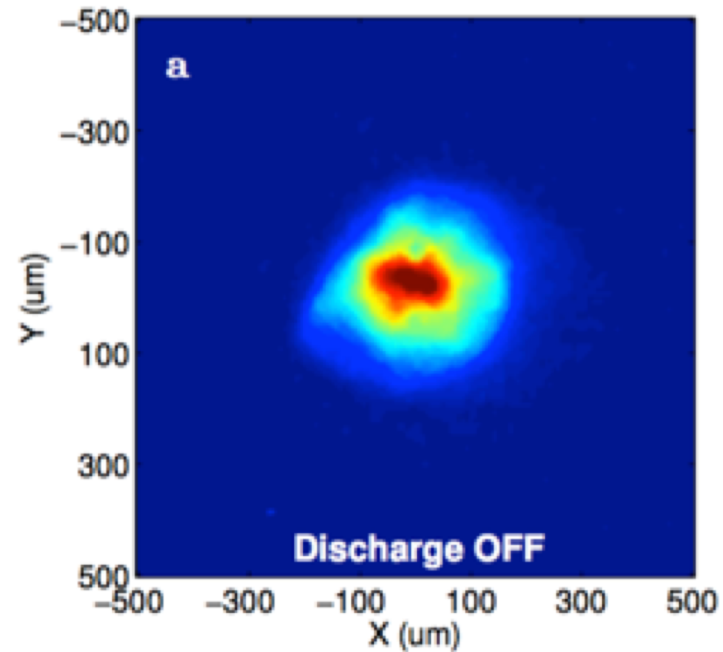




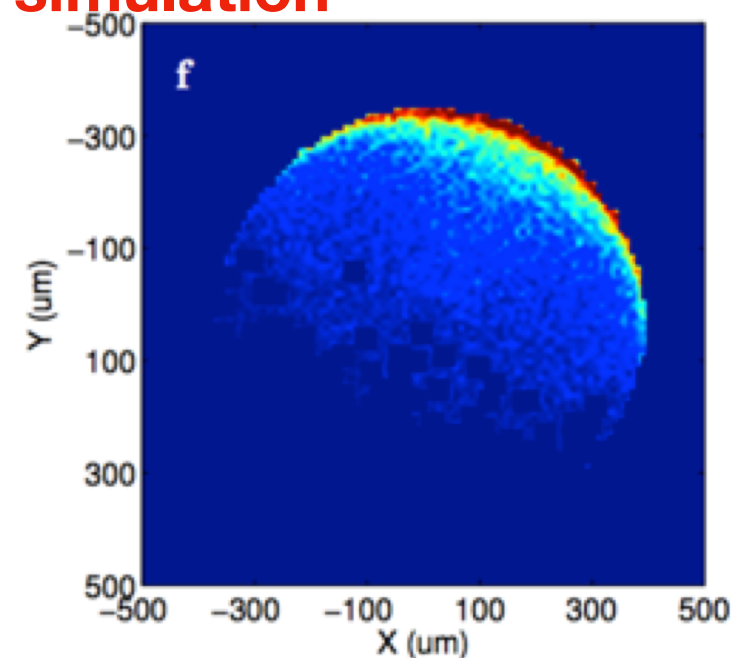
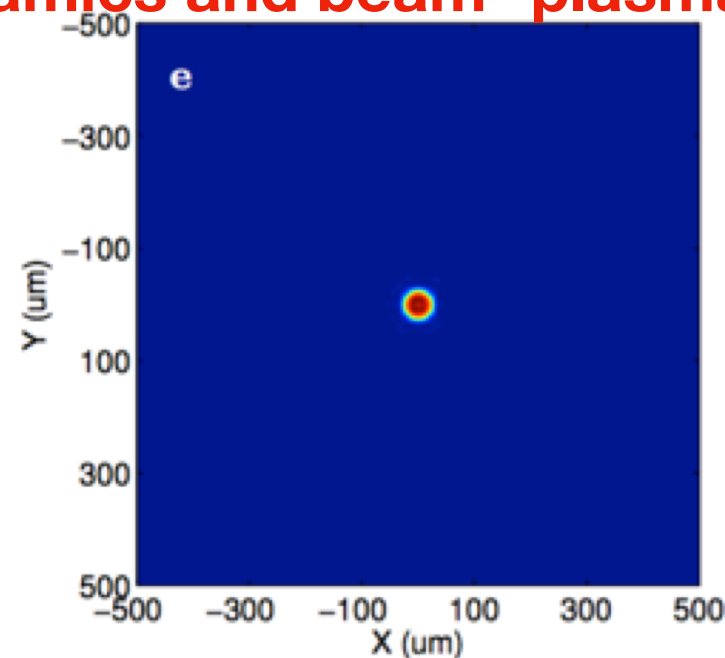
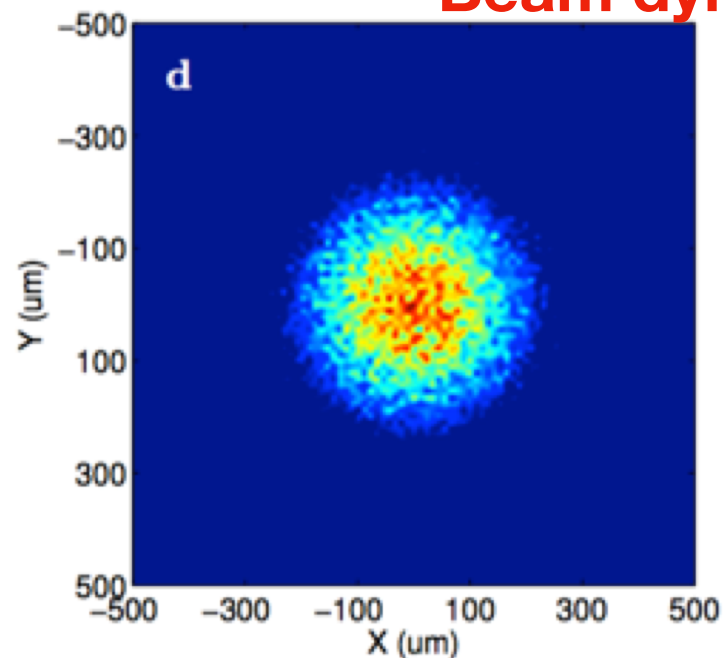
# Active Plasma Lens

## Measurements at SPARC\_LAB

### Measurements

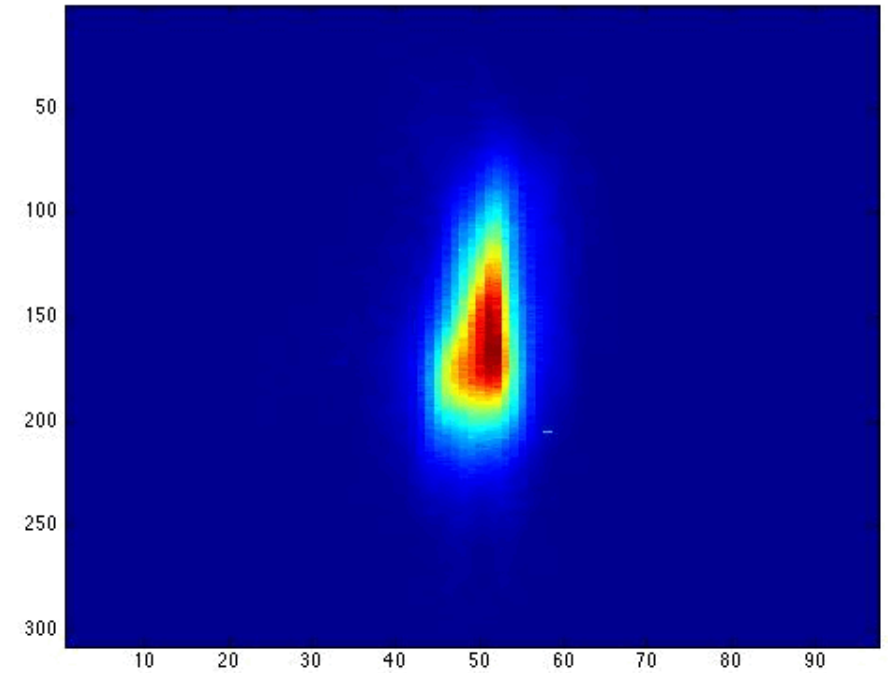
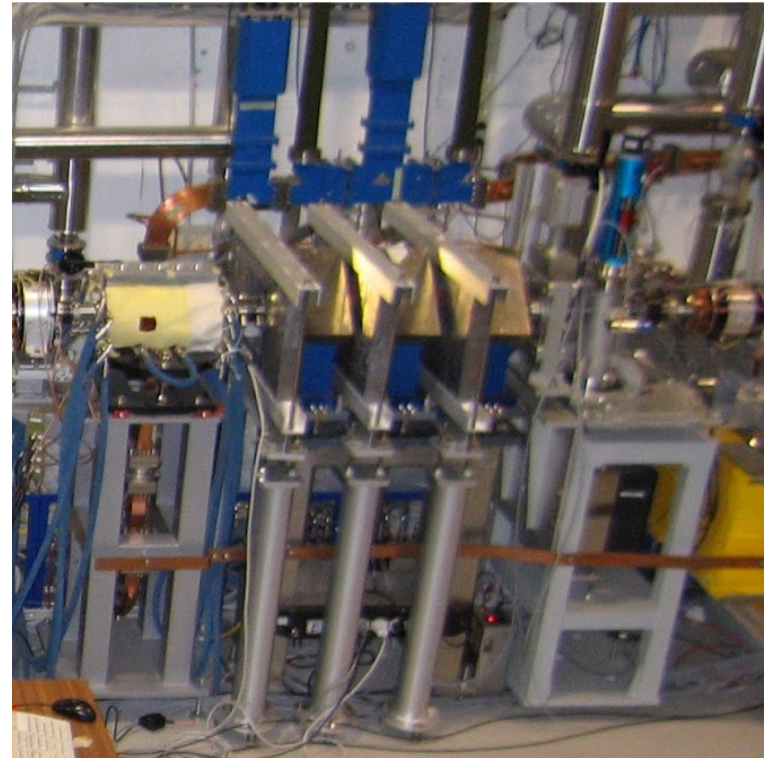


### Beam dynamics and beam-plasma simulation



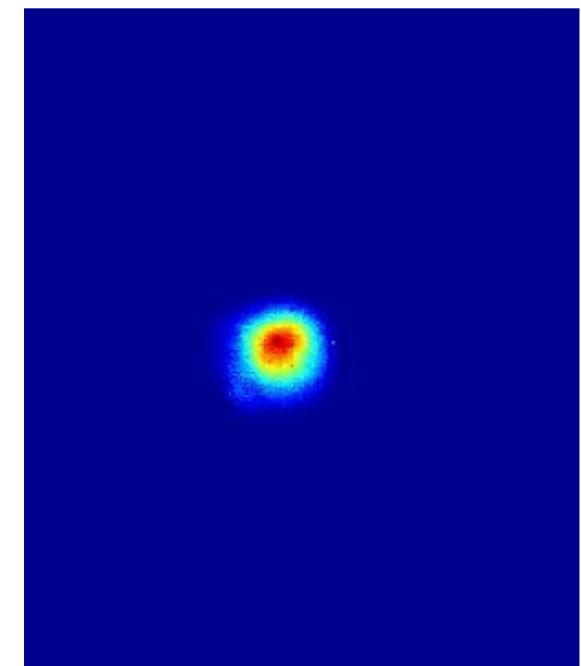
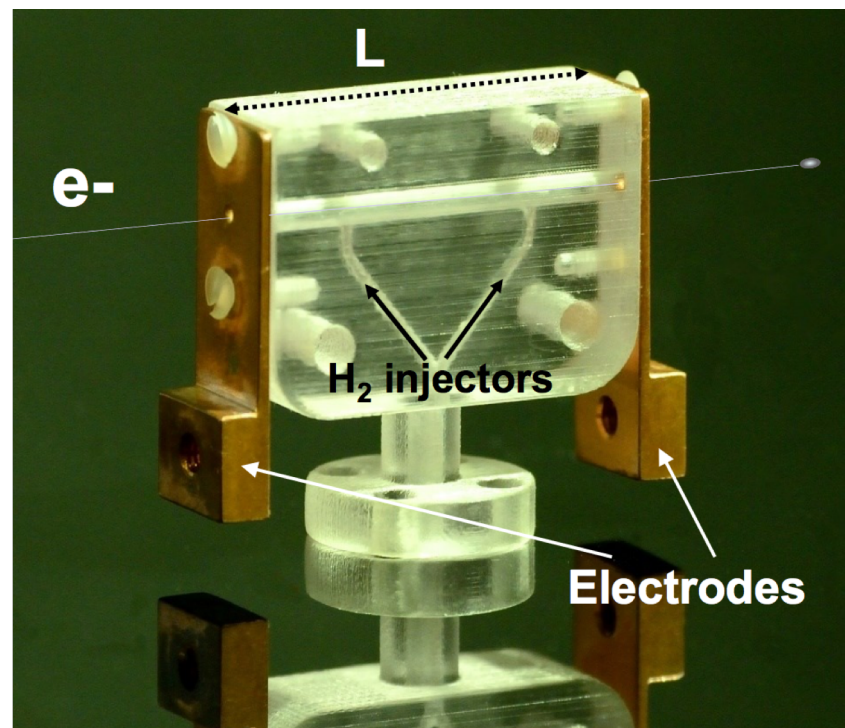
# Plasma lens vs conventional focusing

Single Quadrupole Magnet



**Experimental measurements  
at SPARC\_LAB (INFN-LNF)**

Single Plasma Lens

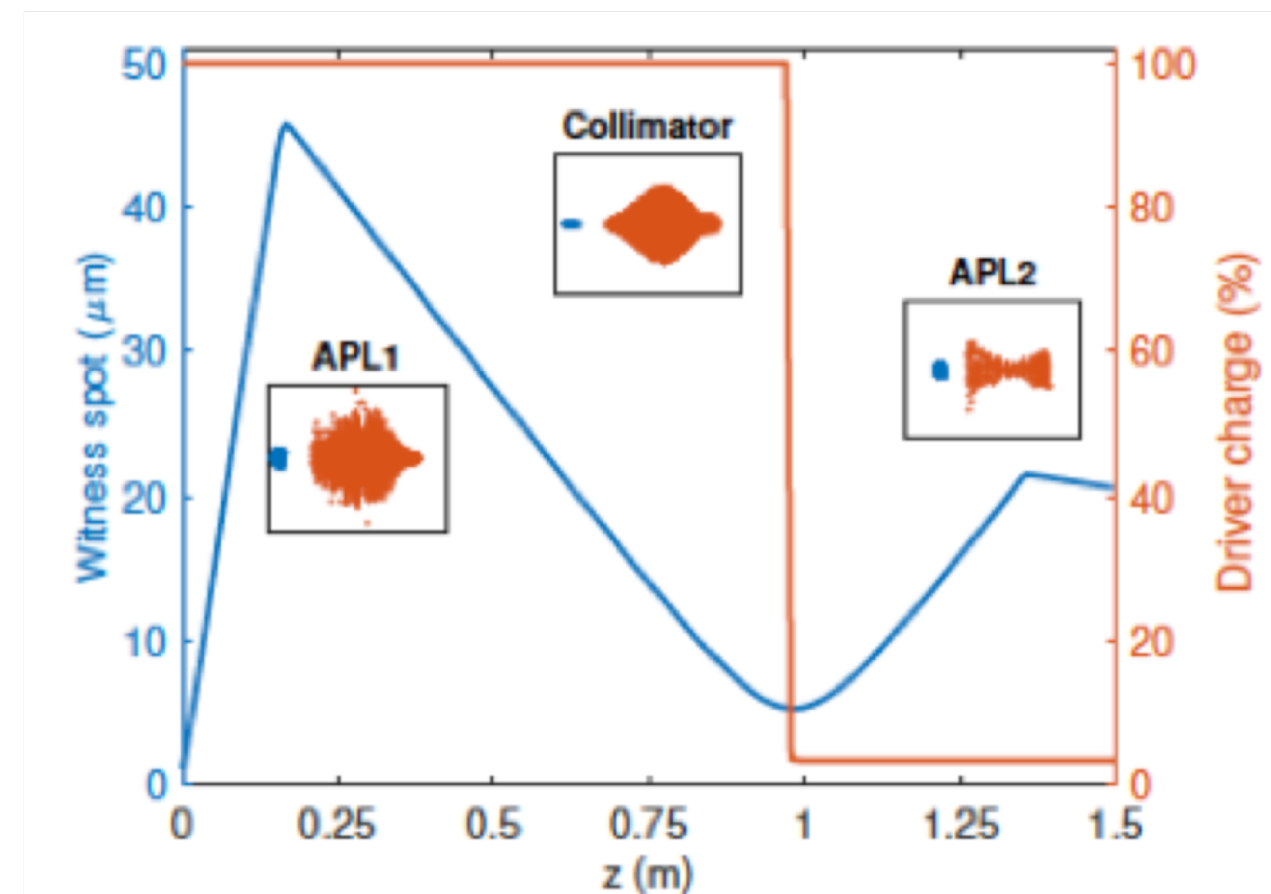
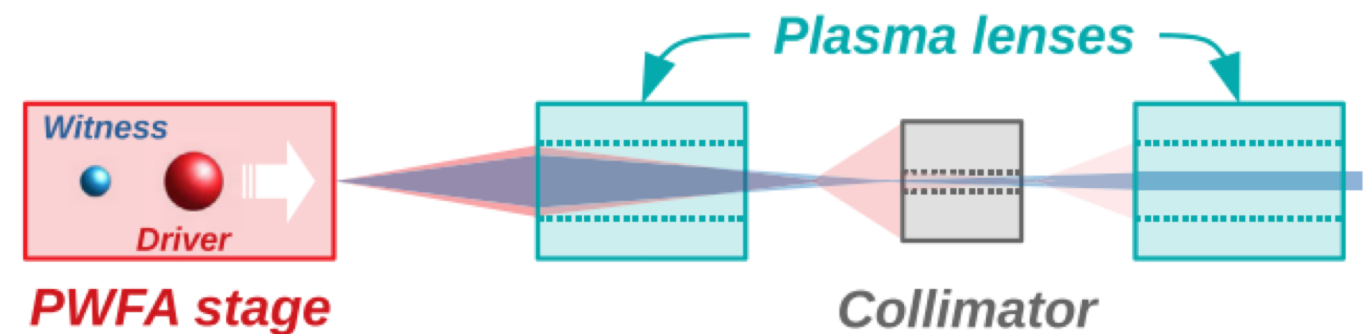


# Extraction Beamline

## *EuPRAXIA@SPARC\_LAB Case*

	Driver	Witness
Charge (pC)	200	30
Energy (GeV)	0.460	1
Energy spread (%)	16	0.73
Normalized emittance (mm mrad)	5	0.6
RMS Spot size ( $\mu\text{m}$ )	7	1.2
RMS Duration (fs)	160	11.5
Peak current (kA)	1.2	2.6

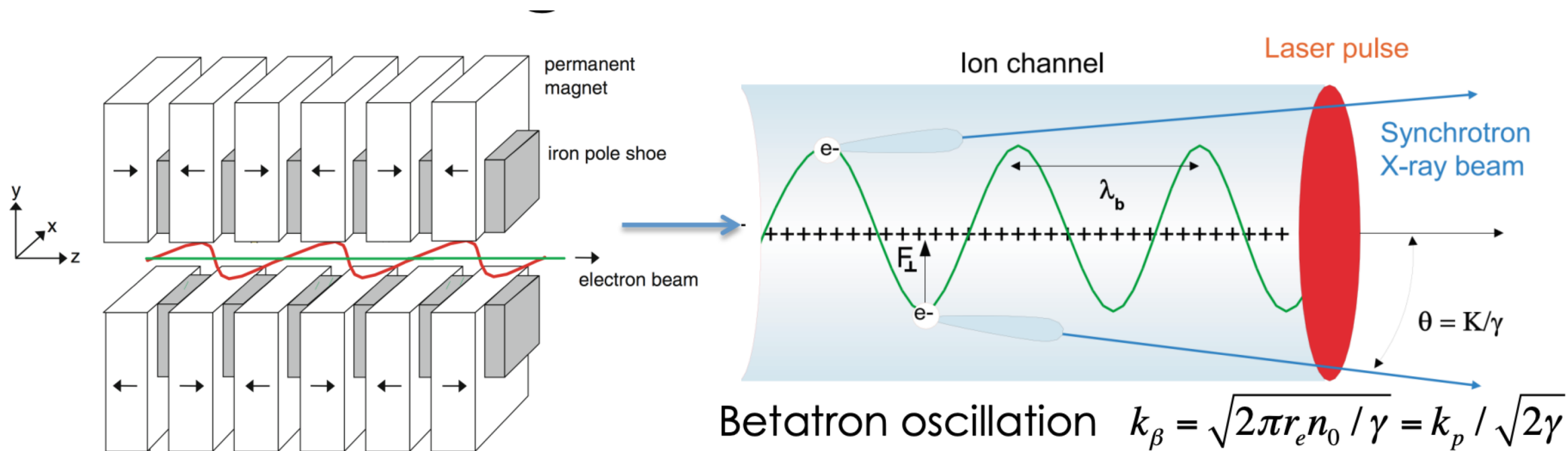
The witness is preserved in charge and quality and the driver is almost completely removed





# Plasma Wigglers

## Betatron radiation based radiator



A. Rousse *et al.*, Phys. Rev. Lett. **93**, 135005 (2004).

$$K = \frac{2\pi\gamma x_0}{\lambda_\beta} \simeq 1.33 \times 10^{-10} \gamma^{0.5} n_e^{0.5} [\text{cm}^{-3}] x_0 [\mu\text{m}] \quad \text{Amplitude dependent}$$

**$K$  can reach ~100 (Requires large offset,  $k_p x_0 \sim 1$ )**

$$E_c [\text{eV}] = 5 \times 10^{-21} \gamma^2 n_e [\text{cm}^{-3}] x_0 [\mu\text{m}] \quad \text{Photon energy up to}$$

**Can reach up to 100 MeV with dense plasma.**

**Plasma wigglers can give magnet field equivalent  $B_u > 100$  T with sub-cm wavelength**

# Conclusions

- Plasma-based acceleration techniques have demonstrated accelerating gradients up to 3 orders of magnitudes beyond presently used RF technologies
- Plasma-based acceleration techniques have provided solid feasibility proofs of FEL lasing paving the way to applications
- Successful efforts on improving beam quality
- R&D on **beam stability, staging and continuous operation**, as necessary steps towards the realization of compact plasma-based accelerator facilities
  - Challenges in high repetition rate
- Plasma-based, ultra-high gradient accelerators therefore open the realistic vision of very compact accelerators for scientific, commercial and medical applications