DEPARTMENT OF PLANETARY SCIENCES
Lunar and Planetary Laboratory

# INTERNSHIP AT LPL

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Final Report
12/2015

ISSNAF
The Italian Scientists and Scholars of North America Foundation

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# SUMMARY

- **CONCENTRIC MACLAURIN SPHEROID MODELS OF ROTATING LIQUID PLANETS**
- THE EFFECT OF DIFFERENTIAL ROTATION (DR) ON GRAVITATIONAL FIGURES
- THE TIDAL POTENTIAL: ANALOGY WITH THE THEORY OF HYDROSTATIC EQUILIBRIUM
- MISCELLANEOUS

# CONCENTRIC MACLAURIN SPHEROID MODELS OF ROTATING LIQUID PLANETS

## THEORY OF FIGURES

- The goal is to find the external gravitational potential of a liquid planet in hydrostatic equilibrium.
- Analytic methods balloon in complexity even for apparently simple cases (i.e. two constant-density layer)
- We need numerical solutions.

The theory of figure can be used for modeling the interior structure of a planet.
We are applying it to Jupiter.

# WHY JUPITER?

# CONCENTRIC MACLAURIN SPHEROID MODELS OF ROTATING LIQUID PLANETS

## ➢ JUNO MISSION

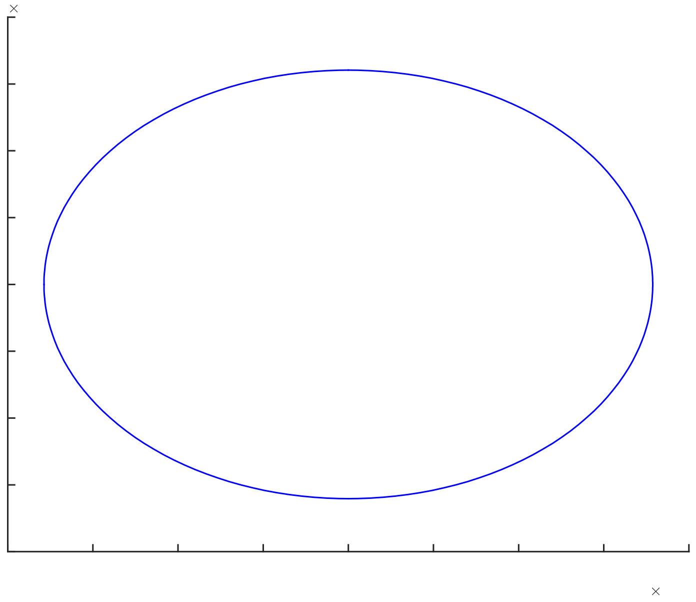
- NASA mission, it will reach Jupiter on July 2016.
- The expected precision of the measurements of Jupiter’s gravity field is one part in $10^9$ (!!).
- Such high precision data will require a gravitational modeling theory of unprecedented accuracy.

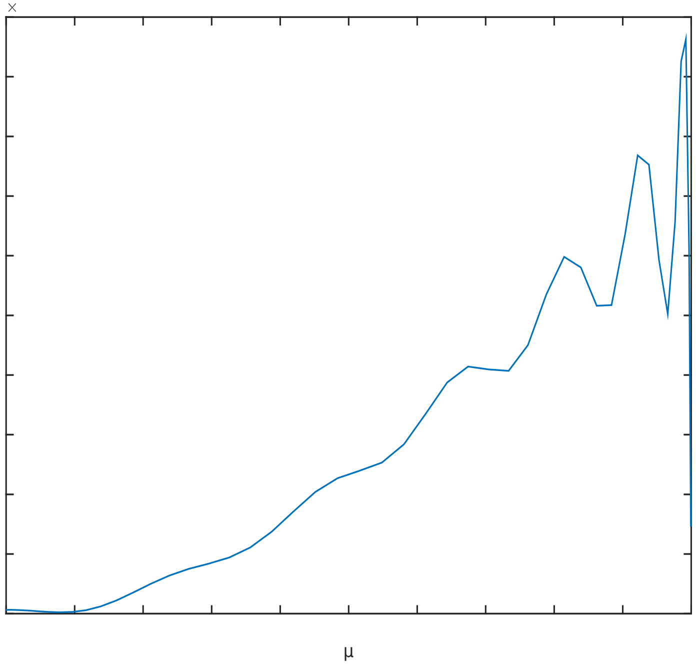
The application of Hubbard’s “Concentric MacLaurin spheroid models” to Jupiter can provide us with the gravitational moments of Jupiter with a precision of one part in $10^{12}$.

# CONCENTRIC MACLAURIN SPHEROID MODELS OF ROTATING LIQUID PLANETS

## CONSTANT DENSITY MODEL

- MATLAB code for its implementation
- Numerical resolution of the potential equation
- Resolution of $J_n$ integrals using Gaussian points
- Comparison with the analytic solution



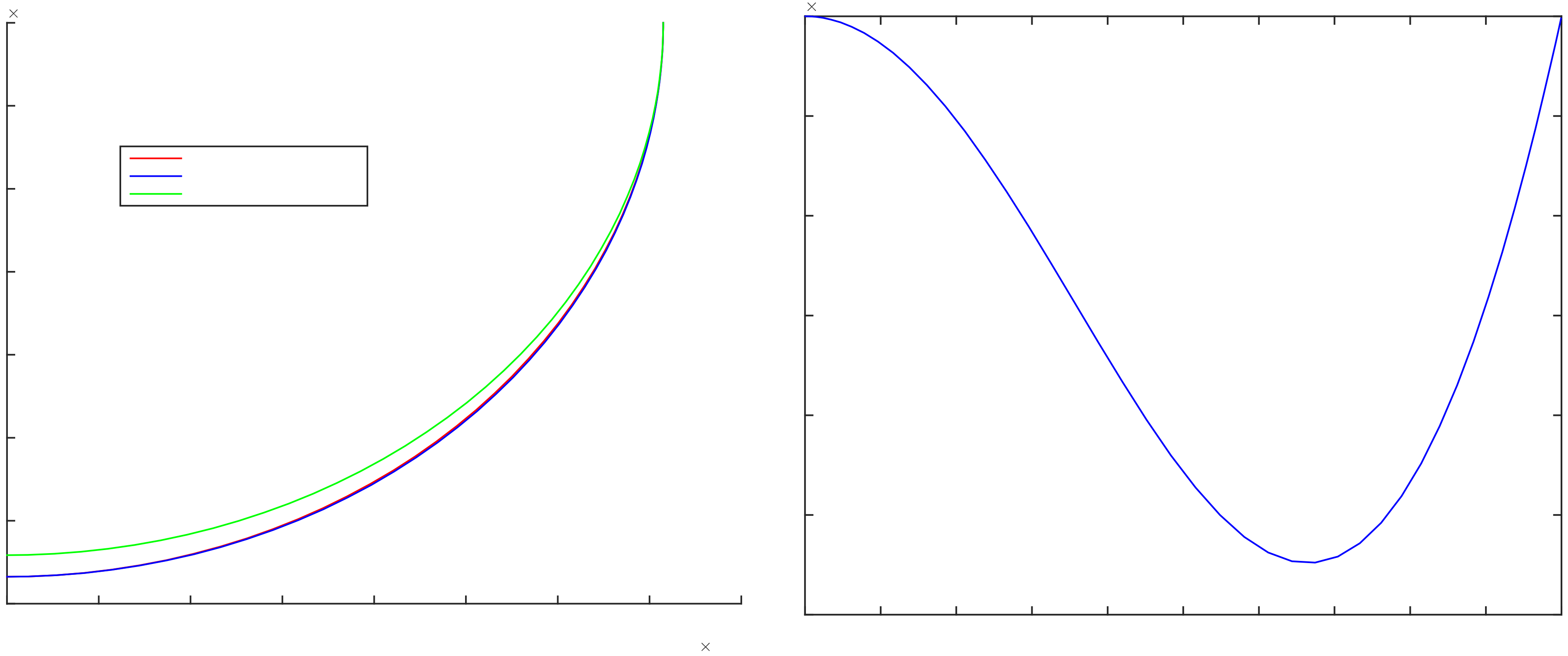


The solution in this simple case provides a first order shape of an ellipsoid. The numerical procedure uses the Gaussian points. The related integration provides exact value of the integrals for polynomials of degree 2N-1, if N is the number of quadrature points.

# CONCENTRIC MACLAURIN SPHEROID MODELS OF ROTATING LIQUID PLANETS

## VARIATION IN DENSITY PROFILE

- Any change in the density distribution provokes a departure from the ellipsoid.
- The departure is more evident at middle latitudes (max rel_dev = 2.625438e-03)
- We prove it by a massless envelope model and a delta function core model



# CONCENTRIC MACLAURIN SPHEROID MODELS OF ROTATING LIQUID PLANETS

## ➢ THE MULTILAYER MODEL

- The problem of the theory of figures can be solved in closed or partially-closed form for a small number of special barotropes.
- The internal structure is modeled as a series of concentric Maclaurin spheroids (CMSs) → a sort of "onion".
- The CMSs can be arranged in sufficient numbers to closely approximate any prescribed barotrope.

For a complete explanation of the theory, see "Hubbard, W., 2013. *Concentric Maclaurin spheroid models of rotating liquid planets.* Astrophys.J. 768"

**We developed a complete algorithm in Matlab for a general multi-layer structure.**
We test its reliability computing a two layer model for Uranus 2.

q=0.029581022

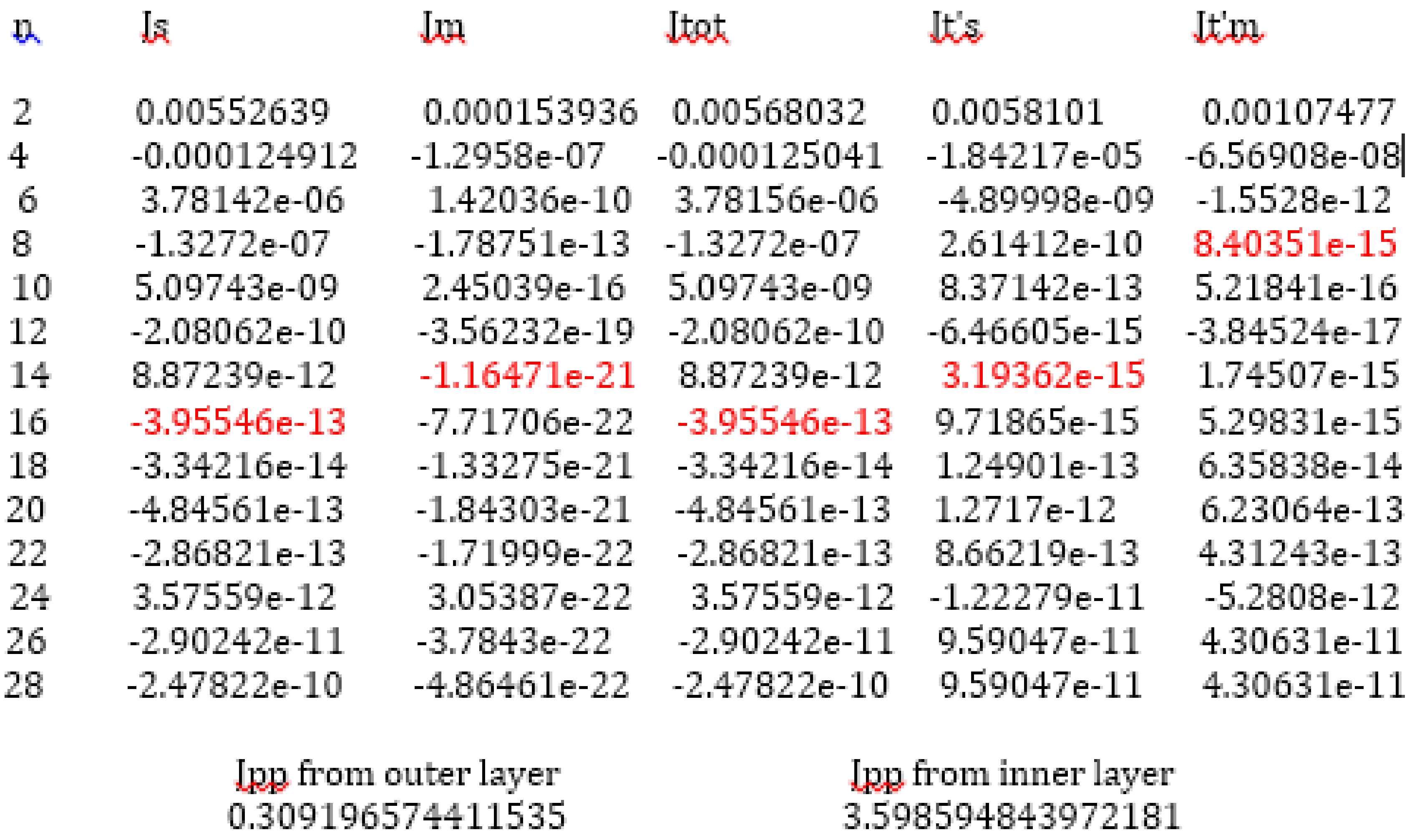
$$\text{lambda} = \frac{a_{layer}}{a_{equatorial}}$$

$$\delta\rho = \frac{\rho_i - \rho_{i-1}}{\rho_0}$$

lambda=[1.0000000, 0.38119207]

deltarho=[1.0000000, 11.638534]

# CONCENTRIC MACLAURIN SPHEROID MODELS OF ROTATING LIQUID PLANETS



| n | Js | Jm | Jtot | Jt's | Jt'm |
|---|---|---|---|---|---|
| 2 | 0.00552639 | 0.000153936 | 0.00568032 | 0.0058101 | 0.00107477 |
| 4 | -0.000124912 | -1.2958e-07 | -0.000125041 | -1.84217e-05 | -6.56908e-08 |
| 6 | 3.78142e-06 | 1.42036e-10 | 3.78156e-06 | -4.89998e-09 | -1.5528e-12 |
| 8 | -1.3272e-07 | -1.78751e-13 | -1.3272e-07 | 2.61412e-10 | 8.40351e-15 |
| 10 | 5.09743e-09 | 2.45039e-16 | 5.09743e-09 | 8.37142e-13 | 5.21841e-16 |
| 12 | -2.08062e-10 | -3.56232e-19 | -2.08062e-10 | -6.46605e-15 | -3.84524e-17 |
| 14 | 8.87239e-12 | -1.16471e-21 | 8.87239e-12 | 3.19362e-15 | 1.74507e-15 |
| 16 | -3.95546e-13 | -7.71706e-22 | -3.95546e-13 | 9.71865e-15 | 5.29831e-15 |
| 18 | -3.34216e-14 | -1.33275e-21 | -3.34216e-14 | 1.24901e-13 | 6.35838e-14 |
| 20 | -4.84561e-13 | -1.84303e-21 | -4.84561e-13 | 1.2717e-12 | 6.23064e-13 |
| 22 | -2.86821e-13 | -1.71999e-22 | -2.86821e-13 | 8.66219e-13 | 4.31243e-13 |
| 24 | 3.57559e-12 | 3.05387e-22 | 3.57559e-12 | -1.22279e-11 | -5.2808e-12 |
| 26 | -2.90242e-11 | -3.7843e-22 | -2.90242e-11 | 9.59047e-11 | 4.30631e-11 |
| 28 | -2.47822e-10 | -4.86461e-22 | -2.47822e-10 | 9.59047e-11 | 4.30631e-11 |

| Jpp from outer layer | Jpp from inner layer |
|---|---|
| 0.309196574411535 | 3.598594843972181 |

Jtot = Js (surface) + Jm (mantle), since the model is linearly proportional to density

# SUMMARY

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- **THE EFFECT OF DIFFERENTIAL ROTATION (DR) ON GRAVITATIONAL FIGURES**
- THE TIDAL POTENTIAL: ANALOGY WITH THE THEORY OF HYDROSTATIC EQUILIBRIUM
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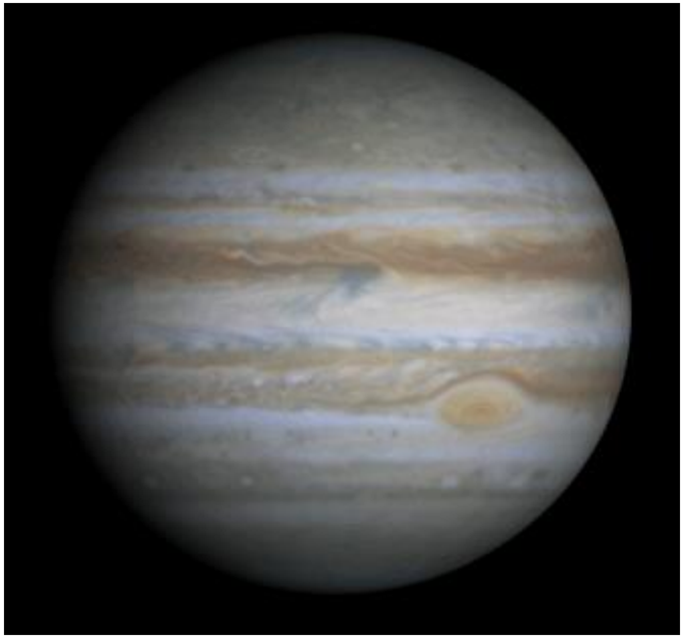
# THE EFFECT OF DIFFERENTIAL ROTATION (DR) ON GRAVITATIONAL FIGURES

## DIFFERENTIAL ROTATION

- Both Jupiter and Saturn exhibit a strong zonal flow pattern with maximum amplitude at the equator.
- The equatorial atmosphere’s rotation rate is significantly greater than that of the deep interior (as determined from the magnetospheric rotation rate).
- … while the atmosphere’s rotation rate near the pole is similar to that of the deep interior.

We consider the effect of zonal currents adding a variation term to the rotational potential

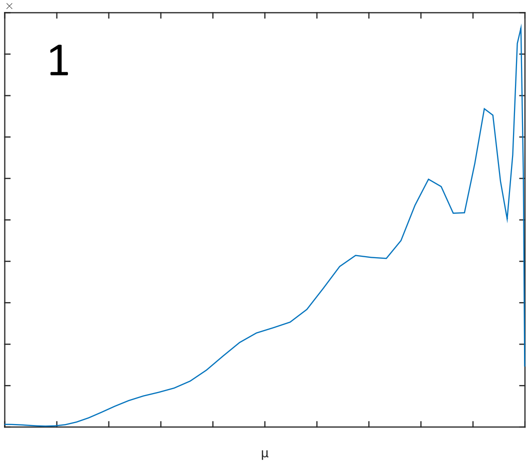
$$Q = Q_o + \Delta Q$$



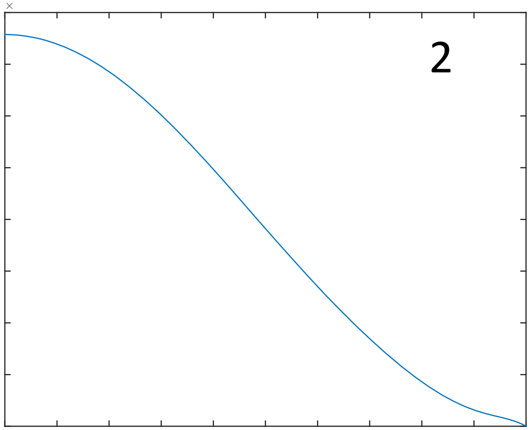
# THE EFFECT OF DIFFERENTIAL ROTATION (DR) ON GRAVITATIONAL FIGURES

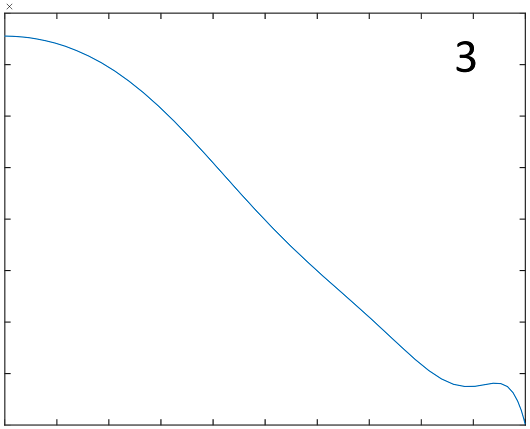
## ➢ MACLAURIN SPHEROID WITH DR

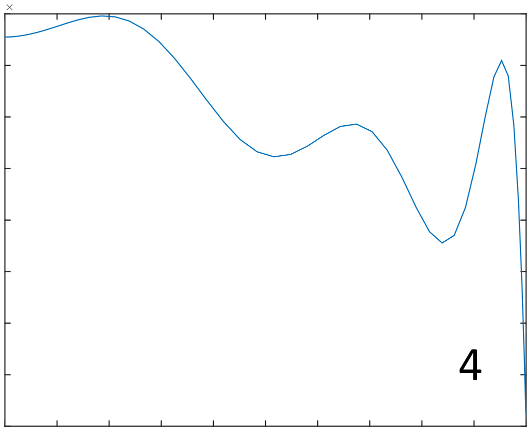
We consider several values for the perturbation $\Delta Q$ and we compared the departure of the spheroid from an ellipsoid.

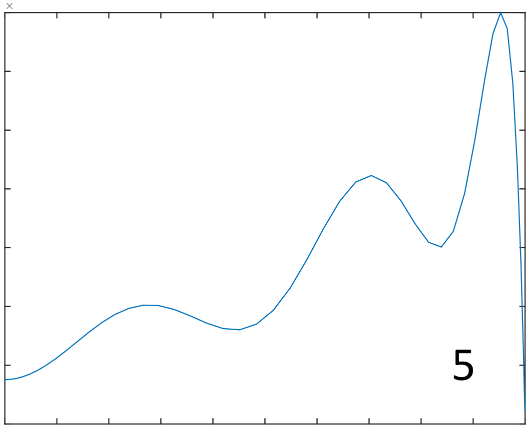


1. Maclaurin spheroid
2. DQ = 1,5 % Q0
3. DQ = 0,15 % Q0
4. DQ = 0,015 % Q0
5. DQ = 0,0015 % Q0









# THE EFFECT OF DIFFERENTIAL ROTATION (DR) ON GRAVITATIONAL FIGURES

## ➢ THE NORMALIZED SPIN ANGULAR MOMENTUM

$$w_{regress} = -\frac{3}{2}\frac{\frac{GMma^2J_2}{r^3}}{C\Omega}$$

- Juno will provide accurate measurements of pole precession rate.
- … coupled with accurate measurements of J2, we can determine the planet’s spin angular momentum without further approximation.

+

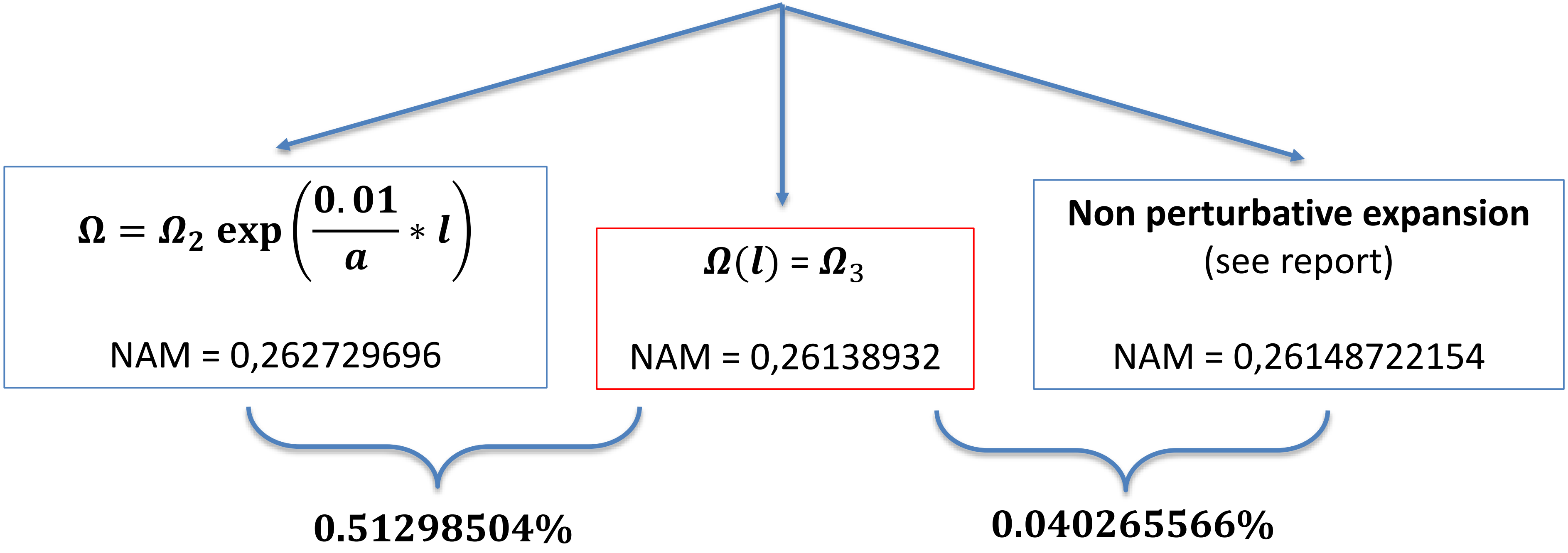
Assuming the model of rotation of cylinders, one can compute the **NAM** (Normalized Angular Momentum) as follows:

$$\boldsymbol{NAM = \frac{4\pi \int_0^a dl' l^3 \int_0^{\sqrt{a^2-l^2}} \rho(l',\xi')\Omega(l')d\xi'}{Ma^2\Omega_3}}$$

# THE EFFECT OF DIFFERENTIAL ROTATION (DR) ON GRAVITATIONAL FIGURES

## THE POLYTROPE OF UNITY INDEX

- A polytrope is a special barotrope: $P = K\rho^{\frac{n+1}{n}}$
- n = 1, K = 1.96: this is a good approximation of the equation of state for Jupiter.
- The mass distribution within the planet is: $\rho = \rho_c \frac{\sin(\pi\lambda)}{\pi\lambda}$ ($\lambda = \frac{r}{a}$; $\rho_c$ is core density).



$$\boldsymbol{\Omega = \Omega_2 \exp\left(\frac{0.01}{a} * l\right)}$$

NAM = 0,262729696

$$\boldsymbol{\Omega(l) = \Omega_3}$$

NAM = 0,26138932

**Non perturbative expansion**

(see report)

NAM = 0,26148722154

**0.51298504%**

**0.040265566%**

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- MISCELLANEOUS

# THE TIDAL POTENTIAL: ANALOGY WITH THE THEORY OF HYDROSTATIC EQUILIBRIUM

- **TIDAL POTENTIAL W → STARTING POINT FOR CMS WITH TIDES**
  - Presence of tesseral harmonics in the potential expansion (no longitudinal symmetry)
  - Juno will be able to detect $J_n^m$ (?)
  - Tidal dissipation (see Io)

$$W = \frac{GM_p}{|R-r|} + W' \cong \frac{GM_p}{R^3} r^2 P_2(cos\theta)$$

$$at\ the\ expansion\ to\ 2nd\ degree$$

**One can use the potential theory also to calculate the hydrostatic-equilibrium response to a tidal perturbation due a perturbing mass.**

$$Q_{solid-body} = \frac{1}{3} r^2 \Omega^2 [1 - P_2(cos\theta)]$$

$$-\frac{1}{3} r^2 \Omega^2 \rightarrow \frac{\mathrm{GM_p}}{R^3} r^2 \quad \longrightarrow \quad q_{tidal} = -\frac{3M_p a^3}{MR^3} \qquad J_{2,tidal} = \Lambda_2 q_{tidal}\ \text{in place of}\ \mathrm{q_{rot}}$$

# THE TIDAL POTENTIAL: ANALOGY WITH THE THEORY OF HYDROSTATIC EQUILIBRIUM

## ➢ LOVE NUMBERS

- Love numbers govern the planet’s gravitational field distortion due to tides
- Estimated dissipation function Q by analyzing the tidal evolution of the satellite orbits
- Even moments $J_{2n}$ differ slightly from their hydrostatic values
- Acquisition of the small odd moments $J_{2n+1}$ and the tesseral moments $C_{mn}$ and $S_{mn}$

$$W = \frac{GM_P}{R}\sum_{n=2}^{\infty}\left(\frac{s}{R}\right)^n P_n(\cos\theta) = \sum_{n=2}^{\infty} W_n \longrightarrow U = K_n(s)W_n$$

**Kn(s1) ≡ kn are the dimensionless Love numbers.**

To determine the tidal influence on a gravitational field, it is “sufficient” to compute the coefficient $k_n$ in the equation

$$U = \sum_{n=2}^{\infty} k_n \left(\frac{s_1}{r}\right)^{n+1} W_n = \sum_{n=2}^{\infty} U_n$$

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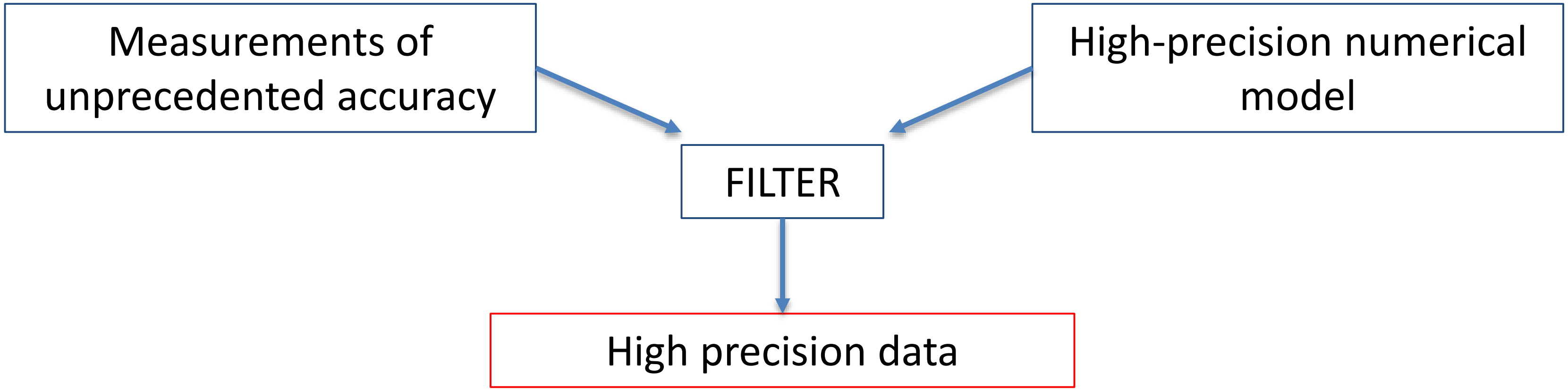
# MISCELLANEOUS

## NASA'S JUNO MISSION

Juno will improve our understanding of the solar system's beginnings by revealing the origin and evolution of Jupiter.

- Determine how much water is in Jupiter's atmosphere → which is the correct planet formation theory?
- Look deep into Jupiter's atmosphere to measure composition, temperature, cloud motions and other properties.
- Map Jupiter's magnetic and gravity fields, revealing the planet's deep structure.
- Explore and study Jupiter's magnetosphere near the planet's poles, especially the auroras → how does the planet's enormous magnetic force field affect its atmosphere?



Measurements of unprecedented accuracy

High-precision numerical model

FILTER

High precision data

# MISCELLANEOUS

## ➢ THE THEORY OF GIANT PLANETS

Theory of the nonideal thermodynamics of hydrogen and hydrogen-helium mixtures under the conditions found in giant-planet interiors.

- Jupiter is believed to be composed mostly by metallic hydrogen→ this is the cause of the Jupiter's powerful magnetic field.
- Giant planet's atmospheres and gravity field → clues for understanding the interior structure.
- From Galileo probe's data, evident abundance of noble gases in the upper atmosphere → Jupiter accreted large amounts of <u>**ICY PLANETESIMALS**</u> in addition to the surrounding solar nebula gas.

They were never heated; they formed in the solar system's nascent giant molecular cloud with little further alteration.

Amorphous ice model, enrichment of oxygen of about four times solar.

Icy planetesimals from the 5 AU region over the period of cooling of the nebula.

The clathrate model requires a factor of eight times solar.

# MISCELLANEOUS

## ➢ THE THEORY OF BROWN DWARFS AND EXTRASOLAR GIANT PLANETS

- A topic of great interest for planetary science, astrobiology, astronomy
- Many missions have been designed for observing transit objects (Kepler, Gaia..)
- The same accuracy in first recovery data for solar gas giants → many differences and analogies between solar giant planets and Jupiter-like exoplanets

**In analogy with oblate solar planets**, measuring the oblateness of exoplanets could led to a better understanding of their interior structure as well as their formation and evolutionary history.

## ➢ "CONSTRAINING THE OBLATENESS OF KEPLER PLANETS"

- On the use of Kepler short cadence light curves to constrain the oblateness of candidate planets. KOI 423.01 → estimated oblateness 0.22 ± 0.11
- The uncertainness is 50% of the value → A first attempt.
- Oblateness interpreted by authors as the evidence of the preservation of KOI's primordial spin angular momentum.
- … but the rotation rate is not consistent with what expected from tidal dissipation. Not very reliable high quality factor.

# MISCELLANEOUS

## ON THE DIFFERENTIATION OF THE ASTEROID CERES

Two different methods for determining the shape → clue for the internal structure

1. Data from **occultation observations** in order to constrain the size, shape, density and albedo of this large asteroid.
2. Image processing to compute Ceres’s shape from the measurement of **limb profiles**.

| | Equatorial radius | Polar radius |
|---|---|---|
| Method 1 | 479.6 ± 2.4 km | 453.4 ± 4.5 km |
| Method 2 | 487.3 ± 1.8 km | 454.7 ± 1.6 km |

- Considerable discrepancy regarding the equatorial radius
- Since occultation measurements are more accurate, we rely on them.
- Ceres is expected to be a globally relaxed and differentiated object → waiting for evidence from Dawn mission

# MISCELLANEOUS

## ➢ FLOWING LIQUID WATER ON MARS : INTERVIEW WITH ALFRED MCEWEN

*“Spectral evidence for hydrated salts in recurring slope lineae on Mars”*

- New findings from NASA's Mars Reconnaissance Orbiter (MRO) that provides the strongest evidence yet that liquid water flows intermittently on present-day Mars.
- The presence of liquid water on Mars today has astrobiological, geologic and hydrological implications and may affect future human exploration.
- Recurring Slope Lineae (RSL): the capability of salt in lowering the freezing point of water.

The interview has been reported on “Panorama per i giovani”, the online journal of my college in Rome (“Collegio Universitario Lamaro Pozzani”).

https://panoramaperigiovani.it/scienza-tecnologia/following-the-water.html

# MISCELLANEOUS

## ➢ LPLC

- Contributions by each member of the department during a 3 day symposium
- “Planetary Science in Rome”: presentation of the activities of the Radioscience Laboratory of Sapienza, University of Rome.

## ➢ LPL COLLOQUIUM, GRAD TALKS & MORE

- Journal club on every Friday, review of the latest papers regarding planetary science → “Constraining the oblateness of Kepler planets”
- Every Monday, Tuesday and Thursday (also evening)
- Very interesting discussions about the latest research results and missions
- Highlighting the strong synergy between space missions and planetary science
- Refreshments (a lot of good food, special occasions for sharing experiences)

## ➢ JUNO STM AT APL (3-7 NOVEMBER 2015)

- A unique experience for understanding how a real mission is run
- International contacts and collaborations
- High level scientific discussions

# FUTURE DEVELOPMENT

- My Master Thesis: the report + simulation of the orbit of Juno and its perturbation, in order to study the observability of the gravitational moments.

- Continuing the collaboration with professor Hubbard and professor Iess
  (PhD application for LPL and\or for Radioscience Laboratory @ Sapienza)

- Attending next Juno STMs (mailing list and access to mission data) → JOI on July 2016
  (? → no funding until PhD)

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# FUTURE DEVELOPMENT

**Go Juno!!!!**

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